Exclusion and cooperation in networks

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Chapter 3

The network dilemma game

In this chapter we introduce and discuss our model of a social dilemma with endogenous choice of partners: the network dilemma game. All subsequent chapters deal with this game or its variations. In section 3.1 we introduce notation and define the basic game. In section 3.2 we discuss some related examples and discuss what real-life features we have omitted in the zeal for clarity and parsimony. We conclude with a brief review of the related literature.

3.1 Definition

We consider an \( n \)-person prisoner’s dilemma game with endogenous partner selection. In this game each of the \( n \) players simultaneously makes two decisions:

**LINKING DECISION:** Each player proposes links between herself and other players. Linking is costless. Each player can propose a link to any other player in the group. Proposing a link may or may not be sufficient to establish it. When a link between two players is established we say the players are neighbors.

**CHOICE OF ACTION:** Each player chooses an action in a prisoner’s dilemma game. One game is played by each pair of neighbors. Players, however, cannot discriminate in their action choices. They can either cooperate with all their neighbors or defect on all of them. A player receives an outside option for each player she is not linked with.

The payoff to a player is the sum of the earnings from the prisoner’s dilemma games played with her neighbors and of the outside options received for not playing with the remaining players.

At most one link can be established between each pair of players. We say that linking is unconstrained when each player is free to propose and establish
any number of links, up to \( n - 1 \). Otherwise we say that linking is constrained. We introduce here, for the game with unconstrained linking, the notation used throughout all chapters. This game is a special case of the game with linking constraints, which we define and analyze in chapter 4.

To distinguish between different kinds of choices we use the following notation:

- a *linking choice* describes the links proposed by a player,
- an *action* refers to a player’s choice between cooperation and defection,
- a *move* consists of a linking choice and an action and thus describes all choices made by the player in the game described above.

Let \( N = \{1, \ldots, n\} \) be the set of players. The linking choice of player \( i \) can be captured by a binary vector \( p_i = (p_{ij})_{j \in N} \in \{0, 1\}^n \), such that \( p_{ii} = 0 \).

If player \( i \) proposed a link to player \( j \) then \( p_{ij} = 1 \), otherwise \( p_{ij} = 0 \). The constraint \( p_{ii} = 0 \) is assumed for convenience, and indicates that a player cannot establish a link with herself. If \( p_{ij} = 0 \) we say that player \( i \) refused to link to player \( j \). Linking choice \( p_i \) is *trivial* if \( p_{ij} = 0 \) for all \( j \), that is, when no links are proposed by player \( i \).

For each profile of linking choices \( p = (p_1, \ldots, p_n) \) let \( g(p) \) denote the corresponding network of established links. We consider two models of link formation. Each describes how the network is established from a profile of proposed links. The two models of link formation are formalized as follows:

**Mutual Link Formation:** A link between players \( i \) and \( j \) is established when \( p_{ij} = 1 \) and \( p_{ji} = 1 \). That is, \( g_{ij} = \min\{p_{ij}, p_{ji}\} \).

**Unilateral Link Formation:** A link between players \( i \) and \( j \) is established when \( p_{ij} = 1 \) or \( p_{ji} = 1 \). That is, \( g_{ij} = \max\{p_{ij}, p_{ji}\} \).

In the mutual linking model consent of both players is needed to establish a mutual link. That is, a link between two players is established if and only if it is proposed by both players. In the unilateral linking model no second party consent is needed to establish a link. A link between two players is then established whenever it is proposed by at least one of them.

Whenever a link between two players is established each of them interacts with the other, regardless of how the link was established and by whom it was proposed. For each network \( g \) let \( L_i(g) = \{j \mid g_{ij} = 1\} \) be the set neighbors of player \( i \) and let \( l_i(g) = |L_i(g)| \) be the size of her neighborhood. We say that two players without an established link are *separated*, that is, they are not neighbors. For convenience we use the shorthand notation \( L_i(p) \equiv L_i(g(p)) \) and \( l_i(p) \equiv l_i(g(p)) \) for a profile of linking choices \( p \).

The action of player \( i \) in the prisoner’s dilemma game is denoted by \( a_i \in \{C, D\} \). Let \( v(a_i, a_j) \) denote the payoff to player \( i \) choosing action \( a_i \) for playing
the game with player \( j \) choosing action \( a_j \), where the payoff function \( v \) is given by the following payoff matrix,

<table>
<thead>
<tr>
<th></th>
<th>Player ( j )</th>
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<tbody>
<tr>
<td>Player ( i )</td>
<td>C</td>
</tr>
<tr>
<td>C</td>
<td>( c, c )</td>
</tr>
<tr>
<td>D</td>
<td>( f, e )</td>
</tr>
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where \( f > c > d > e \) and \( 2c > e + f \). As mentioned above, player \( i \) plays a prisoner’s dilemma game with all her neighbors and receives an outside option for each other player that is not her neighbor. Let \( o \in \mathbb{R} \) be the value of the outside option. We distinguish between

- high outside option: \( o > d \), the outside option is more valuable than mutual defection; and
- low outside option: \( o < d \), the outside option is less valuable than mutual defection.

A move of player \( i \) is a pair \((a_i, p_i)\), where \( a_i \) is her action and \( p_i \) her linking choice. Let \( A_i = \{C, D\} \) be the set of actions and let \( P_i = \{p_i \in \{0, 1\}^n \mid p_{ii} = 0\} \) be the set of linking choices of player \( i \). The set of moves of player \( i \) is denoted by \( J_i = A_i \times P_i \). Let \( J = \times_{i \in N} J_i \). A profile of moves is the \( n \) tuple of pairs denoted by \((a, p) \in J\). The components of a profile of moves, \( a \) and \( p \), are the action profile and the linking profile, respectively.

Let \((a_{-i}, p_{-i})\) denote a profile of moves of player \( i \)'s opponents \( N \setminus \{i\} \), and let \( J_{-i} = \times_{j \in N \setminus \{i\}} J_j \) be the set of all such profiles. For each profile \((a, p) \in J\) the payoff to player \( i \) is given by her payoff function \( \pi_i : J \to \mathbb{R} \), defined by

\[
\pi_i(a, p) = \sum_{j \in L_i(p)} v(a_i, a_j) + (n - 1 - l_i(p))o. \quad (3.1)
\]

We also define \( \pi : J \to \mathbb{R}^n \) as the function whose \( i \)-th component is \( \pi_i \).

The set \( L_i(p) \) and its size \( l_i(p) \) by definition depend only on the links that are established. By implication, for a given action profile \( a \) the payoff \( \pi_i(a, p) \) of player \( i \) also depends only on the network of established links \( g(p) \). With this in mind we may slightly abuse notation and define \( \pi_i(a, g) \) for any network \( g \) by

\[
\pi_i(a, g) = \sum_{j \in L_i(g)} v(a_i, a_j) + (n - 1 - l_i(g))o. \quad (3.2)
\]

We refer to the stage game \( \Gamma = (N, J, \pi) \) as the network dilemma game with unconstrained linking, or shortly, the basic game.

Two particular profiles of actions are convenient to define. Let \( D = (D, ..., D) \) and \( C = (C, ..., C) \) denote the profiles of uniform defection and of uniform cooperation, respectively.
An established link between two cooperative players represents a *cooperative relation*. An established link between a cooperative and a defective player represents a *semi-cooperative relation*. An established link between two defective players represents a *defective relation*.

### 3.2 Discussion

The network dilemma game is a simple, abstract, but fundamental model of a situation where agents are free to select their interaction partners and face an inherent free-rider problem in their interactions. The game-theoretic models of social dilemmas traditionally assume that participation in the game is non-voluntary. However, in many real situations, for example in human social networks, agents are free to choose whether or not and with whom to interact. It is, therefore, important to develop and study models of interaction in endogenous networks. Of course, our game is an abstraction from reality and might not be an accurate representation of all related real-life situations. It does, however, capture the essential features of many such situations better than models of social dilemmas that assume an exogenously imposed interaction structure.

We are aware of only few other game-theoretic models of interactive situations with endogenous interaction structures. Droste et al. (2000), Jackson and Watts (2002) and Goyal and Vega-Redondo (2005) describe games of coordination in endogenous networks which are similar to our network dilemma game except that interaction is modeled as a coordination game instead of the prisoner’s dilemma game. Corbae and Duffy (2003) consider a slight variation of these models by assuming that links change relatively less frequently than behavior. The multiple prisoner’s dilemma games with outside option in Orbell and Dawes (1993) and Hauk and Nagel (2001) differ from our network dilemma game in that discrimination in actions is possible across neighbors. This essentially reduces the game to a collection of two-player prisoner’s dilemma games with outside option, with little or no significance of the network structure. Smucker et al. (1994) and Hauk (2001) consider a further variation of this model by assuming a two-phase process of link formation: in the first phase each player proposes one link and then decides which of the proposed links to accept in the second phase. A related game is studied in an evolutionary setting by Hanaki et al. (2004). We allude to these models in more detail in subsequent chapters. In chapter 4, for example, we indicate that our theoretical results hold regardless of whether or not discrimination in actions across the neighbors is possible.

There are some real-life situations that are captured well by our network dilemma game, such as for example the exchange networks in Papua New Guinea and Ghana, described by Lyon (2000) and Healey (1990). These are instances of voluntary exchange networks with incomplete contracts. For an illustration of such a network consider a village exchange economy that consists of a number of villagers each uniquely specialized in producing one good. A villager can produce either low or high quality goods and can obtain other goods through bilateral exchange. Producing goods of different quality requires dif-
ferent tools and investments which makes simultaneous production of goods of
different quality unfeasible for an individual villager. No villager can be forced
to produce goods of high quality but it is possible to refuse bilateral exchange
with another villager. The network dilemma game captures this situation if we
assume that villagers do not know the quality of other villager's goods before
the exchange. Contemporary instances of voluntary exchange networks with
incomplete contracts can be found in production networks among specialized
firms whose production processes require advance investments that permit only
periodic adjustments of quality, and in markets for "experience goods" (Huck
and Tyran, 2004). On a larger scale, the international trade network may be
viewed as another such instance: we may interpret the players as countries,
defection as the decision of a country to subsidize its economy, and exclusion as
the imposition of tariffs or quotas for goods imported from another country.

Peer-to-peer computer file-sharing networks can also be seen as situations that
carry features of a network dilemma game. A file sharing network consists of
a (usually large) number of members, each able to share some of her computer
files with other members by making them freely available for download. A
typical peer-to-peer network faces an inherent free-rider problem: each member
benefits from the increased number of files shared by the other members but
prefers not to share her own files, for example because of the resulting copyright
violation. To counter this problem the file-sharing services typically provide
public classification of its members according to their past or current volume
of shared files and permit each member to use this classification to bar selected
other members from downloading her shared files. Recent research suggests
that selective choice of file-sharing partners based on their reputation increases
the volume of sharing (Ranganathan et al., 2003) and that free-riding may be
reduced even if information about reputation is decentralized and flows through
bilateral links between the file-sharing partners (Morselli et al., 2004). The
volume of sharing may be further increased if exclusion of free-riders is enforced
by the file-sharing service itself (Kung and Wu, 2003).

There are, of course, limits to the applicability of our network dilemma game
for exploration of real-life situations. For example, our game cannot capture the
sequential nature of decisions in human social networks, where the possibilities
to serve or help occur at different times for different persons. Moreover, the
real process of formation of social links is more complex than presumed by our
simple network formation models. Finally, in contrast to many real-life situa-
tions, but in common with virtually all similar theoretical models, we assume
that all players are homogeneous (Haller and Sarangi, 2002 and Galeotti et al.,
2005 study formation of networks among heterogeneous players). We intend
to address these points in future research by tailoring the models to capture
specific social networks, such as networks of informal insurance.

We conclude this chapter with a brief overview of models of network forma-
tion in the game-theoretic literature (see Jackson, 2004 for a more thorough
discussion). The mutual and unilateral link formation models that we consider
in the previous section were introduced by Myerson (1991) and Bala and Goyal
(2000), respectively. The mutual link formation model seems more realistic and
is more frequently applied. Unfortunately, it is less suitable for analysis with off-the-shelf game theoretic tools because it induces a problem of multiplicity of equilibria. We will discuss this in detail in Chapter 4.

Jackson and Wolinsky (1996) avoid the problem of multiple equilibria by investigating the stability of established networks rather than the equilibria of the network formation game. They define a network to be *pairwise-stable* if no player can benefit by removing one of her links and there is no pair of separated players such that each of them benefits if they establish the mutual link. Watts (2001) shows that the following adaptive dynamic process always converges to a pairwise-stable network: at each time two players are selected at random and myopically decide whether to add or remove their link, or to make no changes. Jackson and Watts (1999) introduce perturbations to this process and use the criterion of stochastic stability to identify the most robust among the pairwise-stable networks. Several related results are reviewed and synthesized in Jackson (2004). Interesting extensions of the concept of pairwise-stability are discussed by Gilles and Sarangi (2004).

Aumann and Myerson (1988) model network formation as a finite game in extensive form. The game is described by an exogenously given sequence over all possible pairs of players. In order of this sequence two players are selected in each period who then decide whether or not to establish the mutual link, having perfect foresight about the links that will be established in subsequent turns. Variants of this game are analyzed by Curarini and Morelli (2000), Deroian (2003), Watts (2002) and Dutta et al. (2005). The drawbacks of this approach are that this game is difficult to analyze even in the one-shot setting and that the set of equilibria is sensitive to the choice of the initial sequence.

A related line of research deals with cooperative games that have communication structures. It originated with coalition formation models of Aumann and Drèze (1974) and Myerson (1977) and assumes that payoffs depend only on which players are directly or indirectly connected, but not on how they are connected. This literature is surveyed in van den Nouweland (2004). Such an approach is not really suitable for modeling network formation because it neglects the effects of the structure of networks (Slikker and van den Nouweland, 2000 introduce such effects through linking costs).

Finally, a distinct approach is taken by Skyrms and Pemantle (2000) who model network links as continuous rather than binary variables. In their model a link is characterized with the probability that the two involved players will interact in any given period. They investigate the network dynamics if the probability of future interactions depends on the rewards from past interactions.