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Unconditional Aid and Green Growth

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Abstract

Environmentally motivated aid can help developing countries to achieve economic growth while mitigating the impact on emission levels. We argue that the usual practice of giving aid conditionally is not effective, and we therefore study aid that is given unconditionally. Our framework is a differential open-loop Stackelberg game between a fully developed leader country and a developing follower country. The leader chooses the amount of mitigation aid given to the follower, which the follower either consumes or invests in either costly non-polluting capital or cheap high-emission capital. We find that the leader gives unconditional mitigation aid only when it is sufficiently rich or when it cares sufficiently about environmental quality, and aid is found more effective if the follower cares about environmental quality to some extent. If aid is given in steady state, it decreases the steady state level of high-emission capital and capital investments in the recipient country as well as the global pollution stock, but it has no effect on the levels of non-polluting capital and non-polluting investments. Transitional aid accelerates the economic growth of the follower. Moreover, we find that the increase in growth takes place in the non-polluting sector.

Keywords: Strategic transfers, Development aid, Green growth, Conditionality, Open loop Stackelberg equilibrium

JEL codes: H41, Q56, O13

1 Introduction

Through the 2015 Paris Climate Agreement all countries acknowledge the negative impact of climate change to each country regardless of its development level. Growth is historically accompanied with high levels of pollution; as climate change is a global rather than national problem, it is in the global interest to direct the growth path of least developed countries towards building low-emission rather than high-emission industrial capital.

It is generally accepted that the first best solution to the climate change problem is to implement a unique carbon tax among all countries, but the political difficulties are as yet overwhelming. Another solution is that developed countries donate environmentally motivated aid voluntarily: the Paris agreement envisages this mechanism, as each signatory country specifies a 'Nationally Determined Contribution'. The agreement has been criticised precisely because these contributions are voluntary and there is no enforcing mechanism in place.

To investigate this criticism, in this study we build a model of environmentally motivated development aid. We find that a fully developed country that has exhausted its domestic abatement possibilities can have an environmental incentive to provide a developing country with mitigation aid, making both countries better off. At present, developed countries have still ample scope for domestic abatement, as these opportunities become scarcer, the question investigated in this paper will become more relevant.

Generally, a donor country can give aid either unconditionally, or conditional on the recipient country investing in certain kinds of non-polluting capital. Adam and O'Connell (1999) highlight institutional failures that may prevent aid conditionality to work optimally. In this article we therefore investigate the conditions under which a donor will give unconditional aid. We use an infinite horizon Ramsey growth model for the developing country; we model the relation between donor and recipient as a Stackelberg differential game with the donor as leader, and we determine the equilibrium amount of aid. We also analyse the effects of this aid on the growth of the recipient country and the direction of the resulting growth.

We find that the donor country gives aid if it is rich or if it values environmental quality highly, and if at the same time the valuation of environmental quality by the recipient country is neither too high nor too low. In these situations, equilibrium aid programmes extend over an infinite time period, and the giving of aid is weakly time-consistent: the donor country has no incentive to renege on its commitment to give aid. Giving aid Pareto-improves the situation of both countries. The recipient country uses most of the aid to increase consumption, which relieves it from the need to invest strongly in a high-emissions 'brown' industry, and allows it to build up a non-polluting 'green' industry instead.

The conditions we find are natural: the donor country has to care much about environmental quality or little about additional consumption in order to have an incentive to give aid. Conversely, if the recipient country does care much about environmental quality, it will abate of its own accord and the donor will not transfer aid; if the recipient cares too little, unconditional aid is an insufficient inducement to develop green industries. This is the main result of our paper: transfers to the follower are effective only if they are aiding a willingness to abate that is already present.

Analysis of the long run steady state shows that unconditional aid decreases the optimal steady state levels of the brown capital in the recipient country, while it has no effect on the optimal steady state level of the green capital: effectively, it substitutes output from the fully developed country, whose production uses by assumption only green capital, for the output of brown capital of the developing country. Moreover, giving aid decreases the steady state level of the global pollution stock.

2 Literature

Our model does postulate a purely environmental motivation of development aid, it explicitly recognises the incentive and the practical possibilities of the recipient country to re-allocated the aid received as it sees fit, and it takes a long-term dynamical perspective. While there are several papers that look at one or two of these aspects, the present paper is the first to combine all three.

2.1 Aid and development

Rajan and Subramanian (2008) and Alesina and Dollar (2000) discuss possible incentives for donor countries to give aid: ethical international equity concerns, historical relations, political and strategic reasons, or poverty alleviation and growth promotion in the recipient country. Sometimes the need to secure a global agreement might include transfers between countries. Other motives include strategic environmental concerns, donors caring about global environmental quality. The 2015 Paris Climate Agreement represents an example of these motives, where environmentally motivated transfers were an essential aspect to secure the agreement. For other kinds of donor's incentives we refer to Lahiri and Raimondos-Moller (2000).

The literature on development aid focuses mainly on identifying the effectiveness of foreign aid on the economic growth of recipient countries. Hansen and Tarp (2000) note that most studies base themselves on three basic theoretical models: the Harrod-Domar growth model with a stable linear relationship between growth and investment in physical capital (Harrod 1939, Domar 1946), the two gaps model of Chenery and Strout (1966), and the Solow (1956) model.

Empirical work has provided conflicting evidence about the effect of development aid on growth. Boone (1996) found that there is no effect, or even if there is, it is lower than what the Harrod-Domar model predicts, and that the recipient consumes most of the aid. Mosley (1986) found a positive effect of aid on the micro level, but could not determine systematic effects of aid on growth on the macro level, which resulted in a micro-macro paradox. Rajan and Subramanian (2008) found no systematic effect of aid on growth regardless of the estimation approach, the time horizon, or the types or sources of aid. Hansen and Tarp (2000) classified the empirical cross-country work on aid effectiveness, concluding that the existing literature supports the proposition that aid improves economic performance, and that there is no macro-micro paradox to resolve. Doucouliagos and Paldam (2008) conducted a meta-analysis of the effectiveness of development aid on growth; they found no significant positive effects. Mekasha and Tarp (2013) re-examined key hypotheses of Doucouliagos and Paldam and concluded that the effect of aid on growth is positive and statistically significant, and that there is no evidence to suggest presence of publication bias.

When one country grants aid to another country for a specific purpose, credibility is an issue, as the actions of — typically — the recipient country may deviate from what is initially agreed on after the aid payment has taken place. Conditionality is the typical mechanism to deal with moral hazard situations between recipient and donor countries (Svensson 2000). Using conditionality, donors try to influence policy and to induce reforms in recipient countries; they also try to make sure that the recipient country uses the promised aid flow effectively, at least in terms of the donor's criteria (Azam and Laffont 2003).

There are however numerous problems associated with conditionality: donors are reluctant to enforce sanctions (Svensson 2000); both success and failure of the recipient to satisfy conditions are cited to justify giving more aid (Easterly 2003); new governments are often given a 'clean slate' from aid agencies (Easterly 2003); enforcing conditionality may be in conflict with other goals of the aid agency, cf. Mosley et al. (1995) who quotes an example from the World Bank. Both Svensson (2003) and Mosley et al. (1995) moreover argue that the current working system is biased towards disbursing aid regardless of the reform effort.

In our dynamic Stackelberg approach, conditionality ceases to be an issue: the donor country envisages that the optimal response of recipient will use most of the aid for other purposes than the development of green capital, but that the remaining green investments are sufficient to justify the transfer. There is still a credibility problem, though of a different kind: the institutions of the recipient country have to be sufficiently strong to guarantee that aid will not be misallocated. A certain amount of misallocation could be modelled by assuming that a fraction of the aid is 'lost' during the transfer. We have not taken this into account explicitly, but our results are robust to small losses.

As our approach is dynamic, it highlights a second credibility aspect, related to the credibility of the donor, that has not been mentioned in the conditionality literature. When an aid programme is announced, the recipient changes its investment policy to use the aid optimally. In our context, this means that capital investments shift towards green industries early on, and even before the start of the aid programme. As these investments are taken to be irreversible, the donor country might have then an incentive to renege on its aid promise. We therefore investigate only those programmes that are weakly time consistent, that is, where the donor country has in no point of time an incentive to renege.

2.2 Aid and climate

Under the terms of the 2009 Copenhagen Accord, which were later re-emphasised by the 2015 Paris Climate Agreement, developed countries engaged in providing climate finance up to $100 billion per year, starting from 2020 onwards, to help developing countries reduce their emissions and adapt to the consequences of climate change (Eyckmans et al. 2016).

As a consequence of these pledges, a theoretical literature on the effectiveness of climate-motivated transfers has emerged. Eyckmans et al. (2016) found, in a two period Stackelberg game, that a large part of the intended effect of transfers dissipates as the follower reallocates its own resources to achieve the balance it prefers. Pittel and Rübbelke (2013) underline the sensitivity of the results on the productivity of mitigation and adaptation technologies. Heuson et al. (2015) show that there are many instruments of climate funding which possibly yield Pareto improvements for donor and recipient countries, and that therefore transfers might induce an implicit cooperation between regions.[1]

A major mitigation instrument is the building of 'green' industries, which "reduce greenhouse gas and air pollutant emissions, without significantly reducing the production and consumption of non-energy goods" (Eyraud et al. 2011). Rozenberg et al. (2014) find that a climate tax is optimal to induce the switch to green capital, but if the environmental conditions are not at a critical level, subsidies are a good long term policy. Claude et al. (2012) use a dynamic model with two jurisdictions to discover the properties of price-based policies to control environmental externalities, introducing temporary heterogeneity between jurisdictions in the initial stocks of infrastructure which diminish over time. They conclude that the optimal policy scheme may require to simultaneously tax one jurisdiction and subsidise the other for a period of time. The policy chosen in each jurisdiction depends on the degree to which stocks are complements or substitutes.

Strategic transfers to induce the private provision of public goods have been investigated using one-shot game theoretic models, starting with Warr (1983), Buchholz and Konrad (1995) and Ihori (1996). The typical result is that if the players have different public good productivity, the less productive agent has an incentive to make unconditional transfers to the more productive agent. Global environmental quality has been a prime motivation for this literature almost from the start. Several situations have been described featuring environmentally motivated positive levels of unconditional transfers, see Stranlund (1996), Ono (1998), Ono and Maeda (2002), Vicary and Sandler (2002) and Altemeyer-Bartscher et al. (2010). Of direct interest to our analysis is Vicary and Sandler (2002), who compared in-cash — unconditional transfers — to technology — conditional — transfers, and find that the former can Pareto-dominate the latter. The conflicting demand of policies for developing countries between policies that improve living conditions and those that address climate change has been noted in many places, see Klein et al. (2005) and Rübbelke (2011) and their references.

Van Soest and Lensink (2000), Fredj et al. (2004), Martín-Herrán et al. (2006) analyse a Stackelberg differential game with financial transfers to help developing countries to

[1] Bowen et al. (2012) discuss development, climate vulnerability, and adaptation.

preserve their rainforests. Cabo et al. (2002) used a differential infinite horizon game to analyse the feasibility and optimality of sustainable economic growth in a North–South trade model.

2.3 Our contribution

In our model, a possible donor is a developed sovereign country for which greenhouse gas emissions by another sovereign country constitute an externality, and which gives aid in order to induce the developing country to reduce these emissions. We therefore investigate endogenously given aid in a noncooperative differential game framework where the donor, North, is a Stackelberg leader and the recipient, South, a Stackelberg follower. We have argued that the practical effectiveness of conditionality is questionable. The leader, motivated by environmental considerations, therefore announces an aid programme, where aid is given unconditionally and independently of the actual actions of the recipient follower. This gives the recipient country the choice to use the aid in a way that achieves its best interest. If under these conditions, there is a positive aid flow towards the follower, it will be a Pareto-improvement. At the same time, it provides a lower bound on the effect aid can have on the growth of the recipient (Azam and Laffont 2003).

The failure to achieve a stable global agreement on climate change is the starting point of the paper. Also, it is assumed that each country knows the extent to which its decisions impose costs on itself and on the other country (for informational considerations cf. Morath 2010). These two assumptions justify the use of a Stackelberg setting. In addition, as aid is a gift, moving sequentially is natural, that is, aid is a unilateral action that can be followed by other unilateral actions.

We consider open-loop Stackelberg equilibria. It is known that these often fail to be time consistent. We extend the game by giving South the option to use a trigger strategy in case North deviates from its originally announced aid programme. Section (6.3) shows that a subset of open-loop Stackelberg equilibria is time consistent under this extension.

Eyckmans et al. (2016), which employs a two-period framework, is closest to our approach in spirit. Unlike them, we analyse unconditional aid in a noncooperative infinite horizon differential game. We employ a Ramsey framework to model South's growth, with endogenous capital investment processes for green and brown capital respectively. Damage flows from global pollution, affecting the welfare of both countries, address the pollution externality. We treat unconditional aid as a component of national income of the recipient country, rather than assume a direct relationship between aid and investments as in Rosenstein-Rodan (1961) and in the Harrod and Domar model. From the donor's perspective, in our model unconditional aid is a mitigation transfer. To the recipient, aid acts as development aid, influencing consumption and total investment, as well as mitigation aid, by influencing the relative investments in green and brown capital. We follow Eyckmans et al. (2016) by not considering global welfare: each country takes only its own welfare into account when taking its decisions. Our model also links the green investments literature and climate finance literature; to our knowledge only Claude et al. (2012) study a similar link.

The next section presents the model and the dynamic optimisation problem of each player. Then we give theoretical results about the steady state of the resulting Stackelberg equilibrium dynamics. To analyse the transient dynamics, numerical techniques are necessary. We discuss their methodology before turning to the results and the conclusions.

3 The model

3.1 The aid game

In our framework, all countries care about the consumption and the quality of the environment of their citizens, which translates to an intertemporal tradeoff between short term consumption benefits and long term environmental costs. A country can grow by either investing in costly, non-polluting, 'green' capital or cheap, high-emission, 'brown' capital. We assume that both kinds of capital are equally productive. Brown capital contributes through emissions to degradation of the global environmental quality; the latter is a public good affecting both developed and developing countries alike. Developing countries are assumed to have credit constraints, as their high probability of defaulting on debts restrict their access to financial markets. We consider the situation, which in practice has not been realised yet, that the developed country has exhausted all domestic abatement possibilities. To avoid future environmental degradation it may be motivated to give aid to developing countries, helping these countries to achieve economic growth with minimal effect on the environment. Since the adverse effects of climate change are felt over a long time, it is natural to study this problem in an infinite horizon framework.

We study a Stackelberg differential game between two countries: The leader, which will be called 'North', is a developed country; the follower, 'South', a developing country. North's decision variable is the amount of aid that it gives to South; this lessens North's consumption budget. North is assumed to be unable to observe how South uses the aid it receives; aid therefore automatically becomes unconditional. South's decision is how to allocate its output and the aid it receives from North between consuming, investing in brown capital, or investing in green capital.

We solve for open-loop Stackelberg equilibria: this means that North's aid schedule is fixed at the initial time, and that the amount of aid given depends merely on the date, but not on any other variable. A closed-loop approach, where the amount of aid would depend on the current state variables, would involve similar problems as discussed above in the context of conditionality: South would need strong institutions to measure and report the capital stocks correctly, and in practice the lowering of aid as a consequence of an adverse stock evolution might easily give rise to political tensions. The open-loop approach avoids this, as the aid schedule is fixed and known beforehand. Of course, for an announced aid schedule to be credible, it needs to be time consistent. This issue will be addressed in Section 6.3.

3.2 South's decision problem

We begin by describing South's decision problem, given North's aid schedule a_t.

3.2.1 Consumption

South's citizens are assumed to be identical and to be represented by an infinitely lived representative agent, who gains utility $u(c_t)$ from consuming c_t of a generic good and dis-utility $D(E_t)$ from environmental degradation represented by a damage function of a global pollution stock E_t. The discounted intertemporal welfare of South's representative consumer can be written as:

$$W = \int_0^\infty e^{-\rho t}(u(c_t) - D(E_t))dt. \tag{1}$$

Here ρ denotes the time preference rate. The utility function u and the damage function D are assumed to be, respectively, increasing and concave and increasing and convex, i.e. $u' \geq 0$ and $u'' \leq 0$; $D' \geq 0$ and $D'' \geq 0$. Furthermore, we assume that the Inada conditions hold, that is, $u'(c) \to \infty$ as $c \downarrow 0$ and $u'(c) \to 0$ as $c \to \infty$.

3.2.2 Production

Output comes from production processes using brown capital $K_{b,t}$, green capital $K_{g,t}$ and labour L as factors of production. In the present article we fix labour supply and focus on physical capital as variable inputs for production, leaving the inclusion of labour as a variable input for future research. South's total output is therefore a function of the stocks of brown and green capital

$$Y_t = F(K_{b,t}, K_{g,t}) \tag{2}$$

The function F is assumed to satisfy $F'_{K_b} \geq 0$, $F'_{K_g} \geq 0$, and F'' negative semi-definite. Then output is increasing and jointly concave in both arguments.

In much of the analysis below, we shall think of capital stocks as energy plants. As energy is in general equally productive regardless of its source, we assume that production is separable in the two inputs. In that situation, the production function is the sum of the two production functions F_b and F_g, for brown and green capital respectively

$$F(K_{b,t}, K_{g,t}) = F_b(K_{b,t}) + F_g(K_{g,t}) \tag{3}$$

The functions F_b and F_g then satisfy $F'_i \geq 0$ and $F''_i \leq 0$ for $i \in \{b, g\}$.

South's invests, per unit time, $I_{b,t}$ in brown capital and $I_{g,t}$ in green capital. The investment costs $C_i(I_{i,t})$ are assumed to be increasing from 0 and convex: $C_i(0) = 0$, $C'_i \geq 0$ and $C''_i \geq 0$ for $i \in \{b, g\}$. Both types of investment are assumed to be irreversible: once an investment has been made, the resulting capital cannot be transformed to a different type of capital. Moreover, we shall assume that brown investments are cheaper than green investments, that is $C_b'(I) \leq C_g'(I)$ for all $I > 0$. In most of the analysis, we shall assume investment costs to be quadratic:

$$C_i(I_{i,t}) = \frac{\beta_i}{2} I_{i,t}^2, \qquad i \in \{b, g\},$$

where $\beta_i > 0$ is the rate of increase of the marginal investment costs. As we assume that brown investments are cheaper than green investments, we have $\beta_b \leq \beta_g$. The price of the generic good is normalised to 1.

Along with its output from the production process, South may receive aid from North. At each point of time South allocates its output and the aid it receives between consuming, investing in green capital and investing in brown capital, taking into account its budget constraint

$$F(K_{b,t}, K_{g,t}) + a_t = c_t + C_b(I_{b,t}) + C_g(I_{g,t}). \tag{4}$$

We model the situation that part of North's aid is given conditionally on it being spent on green investments. If $0 \leq \varepsilon \leq 1$ is the fraction of aid to be spent in this way, South faces the restriction that

$$C_g(I_{g,t}) \geq \varepsilon a_t. \tag{5}$$

Capital dynamics are assumed to take the same form for both kinds of capital

$$\dot{K}_{i,t} = I_{i,t} - \delta K_{i,t}, \quad K_{i,0} \quad \text{given}, \qquad i \in \{b, g\}. \tag{6}$$

Each type of capital increases with new investments and depreciates with a uniform capital depreciation rate δ.

Production processes involving brown capital emit greenhouse gases, which accumulate in the atmosphere. Pollution is therefore transboundary, affecting consumers in both countries. The dynamics of the pollution stock is given as:

$$\dot{E}_t = \alpha K_{b,t} - \vartheta E_t, \qquad E_0 \quad \text{given}. \tag{7}$$

That is, pollution emissions are proportional to the amount of installed brown capital, with an emission intensity α; without emissions, the pollution stock decreases at the natural decay (absorption) rate ϑ.

3.2.3 South's policy

South maximizes its intertemporal welfare, taking into account capital and pollution dynamics. That is, South maximizes the objective functional (1) subject to the budget constraint (4), the investment constraint (5) and the dynamic constraints (6) and (7).

The current value Lagrangian of the intertemporal maximisation problem is

$$\begin{aligned}L &= u(c_t) - D(E_t) + \mu_t(\alpha K_{b,t} - \vartheta E_t) + \nu_{b,t}(I_{b,t} - \delta K_{b,t}) + \nu_{g,t}(I_{g,t} - \delta K_{g,t}) \\ &\quad + \Lambda_t \left(F(K_{b,t}, K_{g,t}) + a_t - C_b(I_{b,t}) - C_g(I_{g,t}) - c_t\right) + \zeta_t(\varepsilon a_t - C_g(I_{g,t})).\end{aligned}$$

Here μ_t, $\nu_{b,t}$, ν_{g_t} and Λ_t are the shadow valuations of, respectively, the pollution stock, South's brown capital, South's green capital and South's consumption; ζ_t is the multiplier of the conditionality restriction. The first order necessary conditions for an optimum yield

$$0 = \nu_{b,t} - \Lambda_t C_b'(I_{b,t}), \quad 0 = \nu_{g,t} - (\Lambda_t + \zeta_t)C_g'(I_{g,t}), \quad 0 = u'(c_t) - \Lambda_t, \tag{8}$$

which determine the optimal actions $I_{b,t}$, $I_{g,t}$ and c_t, and

$$\dot{\mu}_t = D'(E_t) + (\rho + \vartheta)\mu_t, \tag{9}$$

$$\dot{\nu}_{b,t} = -\Lambda_t F_{K_b}'(K_{b,t}, K_{g,t}) + (\rho + \delta)\nu_{b,t} - \alpha\mu_t, \tag{10}$$

$$\dot{\nu}_{g,t} = -\Lambda_t F_{K_g}'(K_{b,t}, K_{g,t}) + (\rho + \delta)\nu_{g,t}, \tag{11}$$

which determine the shadow valuations. These conditions are complemented by the budget equation

$$0 = F(K_{b,t}, K_{g,t}) + a_t - C_b(I_{b,t}) - C_g(I_{g,t}) - c_t, \tag{12}$$

the complementary slackness condition

$$\zeta_t(\varepsilon a_t - C_g(I_{g,t})) = 0, \tag{13}$$

the non-negativity condition $\zeta_t \geq 0$, initial conditions for the states E_t, $K_{b,t}$ and $K_{g,t}$ and, since there are no terminal conditions on the states, by the transversality conditions

$$\lim_{t\to\infty} e^{-\rho t} E_t = 0, \quad \lim_{t\to\infty} e^{-\rho t} K_{i,t} = 0, \qquad i \in \{b, g\}, \tag{14}$$

which hold whenever the state variables are uniformly bounded away from 0.

3.3 North

3.3.1 Consumption

North's citizens are, analogously to South's, represented by an infinitely lived agent who gains utility from consumption and dis-utility from environmental degradation. We use a superscript n to denote North's variables. The discounted intertemporal welfare of North's representative agent is

$$W^n = \int_0^\infty e^{-\rho t}(u^n(c_t^n) - D^n(E_t))dt, \tag{15}$$

where u^n and D^n are respectively assumed to be increasing and concave and increasing and convex: $(u^n)' \geq 0$, $(u^n)'' \leq 0$; $(D^n)' \geq 0$, $(D^n)'' \geq 0$. Moreover, we assume that the Inada conditions hold: $(u^n)'(c) \to \infty$ as $c \downarrow 0$ and $(u^n)'(c) \to 0$ as $c \to \infty$.

North can only affect the pollution stock through influencing the investment decision of South by giving it aid. As motivated in the introduction, we assume that North is unable to observe South's state variables, and it can therefore condition aid only on time.

3.3.2 Production

We simplify North's problem as much as possible in order to focus on analysing the aid effects on South and to make the model more tractable. Baldwin et al. (2016) show that the expectation of a carbon tax that increases over time reduces irreversible investments in polluting capital, thus no or little new investments in this kind of capital are expected to take place in the developed world. This means, for our framework, that North's brown capital only depreciates over time, which is equivalent to setting the initial pollution stock at the beginning of the time horizon to a higher level. Furthermore, we are interested in the case where North has already exhausted all domestic mitigation opportunities; that is, its mitigation productivity is zero (cf. Buchholz and Konrad 1995, Ihori 1996) and South is the only one that can reduce global emissions. Therefore, we simplify North's problem by assuming that North is a fully developed country having only green capital at the steady state level. This level is assumed to be at least equal to the sum of the steady state levels of South's brown and green capitals. North's production processes use only green capital and produce the output Y^n, which is net of depreciation costs. North has to decide at each point of time how to allocate its output between consumption and unconditional aid to South. Its budget constraint takes the form:

$$Y^n = c_t^n + a_t. \tag{16}$$

Moreover, North can choose whether or not to give aid to South, but it cannot force South to pay aid back; hence there is a positivity constraint on aid:

$$a_t \geq 0. \tag{17}$$

Finally, we do restrict the analysis of North's problem to the unconditional case $\varepsilon = 0$.

3.3.3 North's dynamic optimisation problem

Since we have a Stackelberg open-loop game, North will choose the amount of aid that maximizes the intertemporal welfare of its representative consumer, subject to its budget constraint (16), the aid positivity constraint (17), as well as South's first order conditions (8)–(14).

For North's problem, we make a number of simplifying assumptions. As we assume that $\varepsilon = 0$, we set $\zeta_t = 0$ as the conditionality constraint is always satisfied. Also we use the explicit functional forms $C_i(I_i) = (\beta_i/2)I_i^2$ for $i \in \{b, g\}$, we assume separable production

$F(K_b, K_g) = F_b(K_b) + F_g(K_g)$, and we use (8) to express the $I_{i,t}$ in terms of $\nu_{i,t}$ and Λ_t in terms of $u'(c_t)$.

The current value Lagrangian associated to the maximisation of North's social welfare is then

$$\begin{aligned}
L^n &= u^n(Y^n - a_t) - D^n(E_t) \\
&\quad + \kappa_{b,t}\left(\frac{\nu_{b,t}}{\beta_b u'(c_t)} - \delta K_{b,t}\right) + \kappa_{g,t}\left(\frac{\nu_{g,t}}{\beta_g u'(c_t)} - \delta K_{g,t}\right) + \psi_t(\alpha K_{b,t} - \vartheta E_t) \\
&\quad + \lambda_{b,t}\Big((\rho+\delta)\nu_{b,t} - u'(c_t)F_b'(K_{b,t}) - \alpha\mu_t\Big) \\
&\quad + \lambda_{g,t}\Big((\rho+\delta)\nu_{g,t} - u'(c_t)F_g'(K_{g,t})\Big) + \tau_t\Big((\rho+\vartheta)\mu_t + D'(E_t)\Big) \\
&\quad + \Lambda_t^n\left(F_b(K_{b,t}) + F_g(K_{g,t}) + a_t - \frac{\nu_{b,t}^2}{2\beta_b u'(c_t)^2} - \frac{\nu_{g,t}^2}{2\beta_g u'(c_t)^2} - c_t\right) + \xi_t a_t.
\end{aligned}$$

The variables $\kappa_{b,t}$, $\kappa_{g,t}$ and ψ_t are North's shadow valuations of South's brown capital, green capital, and global pollution respectively, whereas $\lambda_{b,t}$, $\lambda_{g,t}$ and τ_t are North's shadow valuations of South's shadow valuations of brown capital, green capital and the pollution stock. The Lagrange multipliers associated to South's budget constraint and to the aid positivity constraint are denoted Λ_t^n and ξ_t respectively.

The first order necessary conditions yield

$$0 = -u''(c_t)\left(\lambda_{b,t}F_b'(K_{b,t}) + \lambda_{g,t}F_g'(K_{g,t}) + \frac{1}{u'(c_t)^2}\left(\frac{\kappa_{b,t}\nu_{b,t}}{\beta_b} + \frac{\kappa_{g,t}\nu_{g,t}}{\beta_g}\right)\right) \tag{18}$$

$$\quad + \Lambda_t^n\left(\frac{u''(c_t)}{u'(c_t)^3}\left(\frac{\nu_{b,t}^2}{\beta_b} + \frac{\nu_{g,t}^2}{\beta_g}\right) - 1\right), \tag{19}$$

$$0 = \xi_t - (u^n)'(Y^n - a_t) + \Lambda_t^n, \tag{20}$$

which determine the optimal actions a_t and c_t: note that in the Stackelberg problem, South's consumption is an action of North. North's shadow values are determined by

$$\dot\kappa_{b,t} = (\rho+\delta)\kappa_{b,t} + \lambda_{b,t}u'(c_t)F_b''(K_{b,t}) - \Lambda_t^n F_b'(K_{b,t}) - \alpha\psi_t \tag{21}$$

$$\dot\kappa_{g,t} = (\rho+\delta)\kappa_{g,t} + \lambda_{g,t}u'(c_t)F_g''(K_{g,t}) - \Lambda_t^n F_g'(K_{g,t}) \tag{22}$$

$$\dot\psi_t = (\rho+\vartheta)\psi_t + (D^n)'(E_t) - \tau_t D''(E_t) \tag{23}$$

$$\dot\lambda_{b,t} = -\delta\lambda_{b,t} - \frac{\kappa_{b,t}}{\beta_b u'(c_t)} + \Lambda_t^n\frac{\nu_{b,t}}{\beta_b u'(c_t)^2} \tag{24}$$

$$\dot\lambda_{g,t} = -\delta\lambda_{g,t} - \frac{\kappa_{g,t}}{\beta_g u'(c_t)} + \Lambda_t^n\frac{\nu_{g,t}}{\beta_g u'(c_t)^2} \tag{25}$$

$$\dot\tau_t = -\vartheta\tau_t + \alpha\lambda_{b,t}. \tag{26}$$

These equations are complemented by the complementary slackness condition

$$0 = \xi_t a_t, \tag{27}$$

and the non-negativity constraint $\xi_t \geq 0$.

Finally, there are initial and terminal conditions. We already have the initial conditions for the states E_t, $K_{b,t}$ and $K_{g,t}$ and the terminal conditions (14) on the co-states of South's problem. Moreover, both South's states and South's co-states are states of North's problem. Since there is no terminal condition on South's states and no initial condition on South's co-states, there will be a terminal transversality condition on North's co-states of South's states, that is, on $\kappa_{i,t}$ and ψ_t, and an initial transversality condition on North's co-states of South's co-states, that is, on $\lambda_{i,t}$ and τ_t. These conditions read as

$$\lim_{t\to\infty} e^{-\rho t}\psi_t = 0, \quad \lim_{t\to\infty} e^{-\rho t}\kappa_{b,t} = 0, \quad \lim_{t\to\infty} e^{-\rho t}\kappa_{g,t} = 0, \tag{28}$$

again assuming that the corresponding states are uniformly bounded away from 0, and

$$\lambda_{b,0} = 0, \quad \lambda_{g,0} = 0, \quad \tau_0 = 0. \tag{29}$$

4 Steady state analysis

In this section we present a comparative statics analysis for the steady state levels of South's capital and consumption, and we give sufficient conditions for a positive aid flow to occur at steady state.

The first result we show is the imperviousness of the steady state level of green capital to aid, while the steady state level of brown capital decreases if aid increases. The underlying mechanism is best understood by considering both kinds of capital separately; that is, from (4) we write consumption as $c = F(K) - C(K) + Y_{\text{other}}$, where $F(K) - C(K)$ is net income from capital K, which is subsequently taken equal to green and brown capital, and Y_{other} the income from the other kind of capital and from aid.

If K is green capital, steady state welfare depends only on utility from consumption, and the optimal steady state level is that value of K which maximises production. In particular, it is independent of the additional income Y_{other}.

However, if K is brown capital, welfare depends on both utility from consumption and the adverse direct welfare effects brought about by pollution, that is, $W = u(c) - D(K)$. The optimal level of brown capital satisfies $W'(K) = 0$, which can be written as

$$0 = F'(K) - C'(K) - \frac{D'(K)}{u'(c)}. \tag{30}$$

As marginal disutility $D'(K)$ from pollution is positive, K is optimal at a level for which the net marginal productivity $F'(K) - C'(K)$ is positive as well. But then an increase in Y_{other}, for instance by foreign aid, will lower the marginal utility of consumption, increase the net marginal productivity and hence lower the optimal value of polluting capital.

This simple argument also shows that foreign aid is ineffective if pollution only affects the production function, for then aid would not change the optimal capital level.

To analyse the steady state levels of South's capital and consumption in the full model, it is sufficient to study a rest point of the evolution equations (6) – (11) of South's states and shadow prices. The solution procedure for the steady state can be found in Appendix A.1. If the investment requirement (5) is not binding, the steady state level of green capital is determined by the relation

$$F'_{K_g}(K_b, K_g) = (\rho + \delta) C'_g(\delta K_g) \tag{31}$$

which states that discounted marginal productivity of green capital equals marginal cost of green investments. In particular if production is separable the steady state level of green capital, and consequently that of green investments, is not affected by the aid received from North is aid is given unconditionally, or if the condition is not binding.

If the constraint is binding, we have that

$$I_g = C_g^{-1}(\varepsilon a) \quad \text{and} \quad K_g = \frac{C_g^{-1}(\varepsilon a)}{\delta}. \tag{32}$$

In either situation, equation (7) implies that the steady state level of emissions E is a function of the steady state level of brown capital

$$E = \frac{\alpha}{\vartheta} K_b. \tag{33}$$

4.1 South's decision problem

4.1.1 Binding investment requirements

We first investigate the situation that the investment requirement is binding. The question of interest here is whether North, by imposing the requirement that part of the aid has to be invested into green capital, achieves its aim of reducing South's emissions.

Clearly, conditionality will have no effect if South's optimal investment level exceeds the part of the aid budget earmarked for green investment. For the other situation, we have the following result.

Theorem 1. *Assume that the investment constraint* (5) *is binding.*

a. *If green and brown capital are complements and the conditionality restriction is only lightly binding, that is, if $F''_{K_b K_g} \leq 0$ and $\eta \geq 0$ is sufficiently small, then steady state brown capital and steady state emissions decrease as the level of conditionality increases.*

b. *If green and brown capital are substitutes and either South's steady state consumption level is sufficiently low, or the conditionality restriction is sufficiently binding, or the restriction is binding and South's time preference rate is sufficiently small, then steady state brown capital level K_b, and hence steady state emissions E, increase with the level of conditionality ε.*

To understand this result, consider the relation equating marginal steady state investment benefits to marginal steady state investment costs for brown capital

$$F'_{K_b}(K_b, K_g) - (\rho+\delta)C'_b(\delta K_b) - \frac{\alpha}{\rho+\vartheta}\frac{D'(\frac{\alpha}{\vartheta}K_b)}{u'(c)} = 0. \tag{34}$$

If we assume that the investment restriction is binding, we have $K_g = C_g^{-1}(\varepsilon a)/\delta$. Consider now what happens if the level of conditionality ε increases. First we need to discuss what happens to the consumption level. From the budget equation we derive, under the assumption that the restriction is binding, that

$$\frac{\partial c}{\partial \varepsilon} = a\left(\frac{F'_{K_g}}{\delta C'_g} - 1\right). \tag{35}$$

This indicates that consumption increases if marginal long term investment benefits — in the absence of discounting — outweigh marginal investment costs. The ratio F'_{K_g}/C'_g we derive from the relation equating marginal steady state investment benefits to marginal steady state investment costs for green capital

$$F'_{K_g}(K_b, K_g) - (\rho+\delta)C'_g(\delta K_g) + (\rho+\delta)\frac{\eta}{u'(c)} = 0, \tag{36}$$

whence

$$\frac{\partial c}{\partial \varepsilon} = \frac{a}{\delta}\left(\rho - \frac{(\rho+\delta)\eta}{u'(c)C'_g(\delta K_g)}\right). \tag{37}$$

We see that there are two opposite tendencies at work. The term ρ corresponds to a forced saving effect: consumption increases as the capital level through forced investment is higher than it would have been South would have invested voluntarily, as it would have discounted future benefits. The second term, involving η, shows that consumption will decrease if the constraint binds hard, as South cannot achieve its optimal investment plan. The effects of conditionality are exacerbated if the consumption c level is smaller.

Return now to equation (34). If ε increases, it affects the first term, through the increase of green capital K_g, and the third term, through the effect of the level of conditionality on total consumption.

There are two situations where there is a univocal response. If brown and green capital are complements and consumption increases as conditionality increases, through the forced savings effect, marginal productivity of brown capital decreases and marginal environmental costs increase. As a consequence, the optimal brown capital level and its emissions decrease.

The other situation with a univocal response is if brown and green capital are substitutes and consumption decreases as the level of conditionality increases. Then marginal productivity of brown capital increases and environmental costs decrease, hence brown capital and the emissions level increases.

This is an important result of our investigation. If brown and green capital are substitutes and aid conditionality has a real impact on South's investment decisions, then the same aid conditionality is counterproductive if North's aim is to lower global emissions by giving aid.

4.1.2 Non-binding investment requirements

In the remainder of the article, we therefore consider the situation that there are no investment requirements, that is, $\varepsilon = 0$. We also restrict to the situation that the production function is separable 3, as in the case of production of electricity with either green or brown technology. The steady state levels of consumption and brown capital are then determined jointly by the equations

$$0 = F_b'(K_b) - (\rho + \delta)C_b'(\delta K_b) + \frac{\alpha}{\rho + \vartheta} \frac{D'(\frac{\alpha}{\vartheta} K_b)}{u'(c)}, \tag{38}$$

$$0 = F_g'(K_g) - (\rho + \delta)C_g'(\delta K_g), \tag{39}$$

and South's unconditional budget equation

$$c = F(K_b, K_g) + a - C_b(\delta K_b) - C_g(\delta K_g). \tag{40}$$

From the assumptions that $\beta_g \geq \beta_b$ and that brown and green capital have the same productivity, it follows that the steady state level of brown capital with no pollution is at least equal to the steady state level of green capital if either $\alpha = 0$ or $D'(E) \equiv 0$. This is natural, as green investments are more expensive than brown ones.

Equations (38) and (40) readily furnish information about the effects of parameter changes on the steady state levels c and K_b of consumption and brown capital. We begin with the effect of an increase in the aid flow a.

Theorem 2. *If aid is unconditional and the aid flow a rises, then the steady state level K_b of brown capital falls, the steady state consumption level c rises, while the steady state level K_g of green capital is unaffected.*

Consequently, the steady state levels I_b of brown investment and E of the pollution stock fall as well, whereas green investments I_g are also unaffected.

Finally, South's total welfare rises.

This theorem is proved in Appendix A.4. The underlying mechanism is the same as in the simple static case described above.

The next result investigates the effects of changing the investment cost parameters β_g and β_b and the capital depreciation rate δ.

Theorem 3. *Assume that $C_i(I_i) = (\beta_i/2)I_i^2$, for $i \in \{b, g\}$. If the cost parameter β_g of green investments falls, the consumption level c and the level of green capital K_g rise, while the level K_b of brown capital falls.*

If the cost parameter β_b of brown investments falls, the consumption level rises, while the green capital level is unaffected.

If the capital depreciation rate δ falls, the green capital level and the consumption level rise.

Finally, for small positive values of the emission intensity α, the brown capital level rises if either the cost of brown investments or the capital depreciation rate fall.

Again, the proof of this theorem can be found in Appendix A.4. The result conforms fully to economic intuition.

Finally, we have a result on parameters affecting the pollution stock. Again the results for the brown capital stock are intuitive, and the behaviour of the green capital stock is explained by the mechanism above.

Theorem 4. *Assume that $D(E) = (\eta/2)E^2$. If either the natural decay rate ϑ falls, the emission intensity α rises, or the weight η of environmental quality rises, then both the consumption level c and the brown capital level K_b fall. The green capital level K_g is unaffected.*

Moreover, for small positive values of the emission intensity α, the pollution level rises with increasing values of α, while it falls with increasing values of the natural decay rate ϑ.

These theorems are proved in Appendix A.4, except the last statement of Theorem 4, which we shall discuss now.

The effect on the global steady state pollution depends on the elasticity of brown capital at steady state with respect to emission intensity, for

$$\frac{\partial E}{\partial \alpha} = \frac{K_b}{\vartheta}\left(\frac{\alpha}{K_b}\frac{\partial K_b}{\partial \alpha} + 1\right).$$

The elasticity $\epsilon_\alpha = \frac{\alpha}{K_b}\frac{\partial K_b}{\partial \alpha}$ is negative, therefore the effect of α on E is positive if and only if $\epsilon_\alpha > -1$. Clearly this elasticity is 0 if $\alpha = 0$, yielding that E rises with α for small values of α.

The dependence of the steady state level of pollution on the natural decay rate can be written as

$$\frac{\partial E}{\partial \vartheta} = \frac{\alpha}{\vartheta^2}K_b\left(\frac{\vartheta}{K_b}\frac{\partial K_b}{\partial \vartheta} - 1\right);$$

the effect of ϑ on E is positive if and only if the elasticity $\epsilon_\vartheta = \frac{\vartheta}{K_b}\frac{\partial K_b}{\partial \vartheta} > 1$. If the emission intensity α is zero, industrial production does not affect the pollution level. Conversely the natural decay rate cannot affect the steady state level of brown capital: this results in the fact that $\partial K_b/\partial \vartheta = 0$, and hence that $\epsilon_\vartheta = 0$ if $\alpha = 0$. By continuity, for small but positive values of α, we have that ϵ_ϑ is close to zero, which results in the steady state pollution level decreasing as ϑ increases.

4.2 North's decision problem

We turn to the question under which conditions North will give aid in steady state. Note that ξ_t is the shortfall of net marginal welfare benefits from aid; as long as this quantity is positive, it is not in North's interest to give aid. In Appendix B.1, the following relation is derived for its steady state value:

$$\xi = (u^n)'(Y^n - a) - \frac{(-u'')}{u'} \frac{\alpha (D^n)'/(\rho + \vartheta)}{(\rho+\delta)\delta\beta_b - F_b'' + \frac{\alpha^2}{(\rho+\vartheta)\vartheta} \frac{D''}{u'}} \left(\rho\delta\beta_b K_b + \frac{\alpha}{\rho+\vartheta} \frac{D'}{u'} \right). \tag{41}$$

The first term on the right hand side is the direct marginal welfare cost to North, the second the direct welfare benefits. For the latter term to be nonzero, it is necessary that $\alpha > 0$ — brown capital generates pollution — $(D^n)'(E) > 0$ — North suffers from pollution — and either $D'(E) > 0$ or $\rho > 0$ — South suffers from pollution or there is time discounting and South shifts investments towards the future if it receives aid.

The next result summarises this discussion.

Theorem 5. *Take $D^n(E) = \eta^n E^2/2$. Assume that $\alpha > 0$ — brown capital pollutes — $\eta^n > 0$ — North suffers from pollution — and either $\rho > 0$ — North and South are impatient — or $D'(E) > 0$ for all $E > 0$ — South suffers from pollution. Then the following are true in steady state.*

a. $\xi - u'(Y^n - a) < 0$: the indirect welfare benefits of North's giving aid are positive.

b. If η^n is fixed and $Y^n > 0$ is sufficiently large, or if Y^n is fixed and η^n is sufficiently large, then $a > 0$: if North is sufficiently rich or sufficiently concerned about the environment, it will give aid.

To conclude, aid decreases the steady state level brown capital, brown investments, and the stock of global pollution, it increases South's consumption and total welfare, and it has no effect on the steady state level of green capital or green investments. Moreover, in certain circumstances it is in North's interest to provide South with mitigation aid, which effectively amounts to North buying off the need to build brown capital, or at least to build much of it in the short term, and by that, buying off the resulting pollution.

5 Methodology

Next to the steady state, we are also interested in the growth path towards it, and its dependence on the parameter change, its 'comparative dynamics'. If there are to be any aid transfers, we expect the bulk to be effected during the growth phase of South, which is, by definition, not in steady state. Solving the model analytically is however beyond our capabilities; we have therefore resorted to numerical simulations.

This section discusses the numerical methods with which the Stackelberg open loop equilibria of the dynamic game are determined and motivates the calibration of the model parameters.

5.1 Numerical Solution

Section 3.2 formulated the necessary conditions of South's decision problem in the form of a boundary value problem over an infinite time interval, involving six nonlinear differential equations, together with initial and terminal conditions; North's boundary value problem features twelve nonlinear differential equations. We adapt a numerical approach taken from Grass (2012).

In general, boundary value problems deriving from infinite horizon optimisation problems with m state variables are characterised by the following elements: a $2m$-dimensional system of differential equations, determining solution paths $z_t = (x_t, y_t) \in \mathbb{R}^m \times \mathbb{R}^m$, where x_t is the state evolution and y_t the co-state evolution; a specification of the initial states x_0, which yields m initial conditions; a specification of m asymptotic transversality conditions, which are typically satisfied by a solution of the system that tends to steady state values $\hat{z} = (\hat{x}, \hat{y})$.

In order to solve for such solution paths numerically, we approximate the asymptotic conditions by conditions that hold for a large, but finite, time T. Following Grass (2012), we impose the following 'asymptotic transversality condition'

$$M^T \begin{pmatrix} x_T - \hat{x} \\ y_T - \hat{y} \end{pmatrix} = 0; \tag{42}$$

here the columns of the matrix M form a basis spanning the orthogonal complement to the stable eigenspace at steady state, and M^T denotes the transpose of M. The geometrical content of (42) is that the vector $z_T = (x_T, y_T)$ is contained in the stable eigenspace of the steady state $\hat{z}$, and therefore approximately in the stable manifold of the steady state. Note that (42) consists of m scalar conditions on the $2m$-dimensional vector z_T. The $2m$ differential equations, together with m initial state conditions and m asymptotic transversality condition then form a boundary value problem over the finite time interval $[0, T]$. As $T \to \infty$, the solution curves of the approximate problem tend uniformly to solution curves of the original problem.

Specifically, South's boundary value problem consists of equations (6)–(11), together with initial conditions at $t = 0$ for the three states $K_{b,t}$, $K_{g,t}$, and E_t, and the transversality conditions (14). The initial conditions are South's initial capital stocks $K_{b,0}$ and $K_{g,0}$, and the initial pollution stock E_0.

North's problem involves twelve differential equations: the state equations (6)–(11) and the co-state equations (21)–(26), as well as twelve boundary conditions. The first six of these are equal to South's boundary conditions, the initial conditions for the states and the transversality conditions (14) for the co-states. In addition, boundary conditions on North's co-states are furnished by the transversality conditions (28) and the initial conditions (29).

5.2 Functional forms

We assume that both South's and North's representative agent have a constant intertemporal elasticity of substitution utility

$$u(c) = u^n(c) = \frac{c^{1-\sigma}}{1-\sigma}.$$

In computations, we take $\sigma = 0.5$, which yields an intertemporal elasticity of substituion of 2. We take Cobb–Douglas production functions with the factor labour taken constant; we assume moreover that green and brown technology are equally productive, yielding

$$F_b(K) = F_g(K) = \frac{\Omega}{1-\gamma} K^{1-\gamma} \qquad \text{for all } K.$$

In computations we set $\Omega = 0.6$ and $\gamma = 0.75$.

The damage functions are assumed to be quadratic:

$$D(E) = \frac{\eta}{2} E^2, \qquad D^n(E) = \frac{\eta^n}{2} E^2, \qquad \text{for all } E.$$

The parameters η and η^n govern the weight of the environmental quality in the welfare of each country.

5.3 Calibration

To calibrate the parameters in our model, we take a wind energy plant as a model for green industrial capital, and a traditional coal or gas energy plant as a model for brown capital.

The relative cost β_g/β_b of green investments with respect to brown is calibrated as the ratio between investment costs of a wind plant to that of a coal/gas plant. Salvadore and Keppler (2010) estimate that the specific overnight construction costs of most coal-fired plants range between 1000 and 1500 USD/kWe, while those of a gas-fired plants range between 400 and 800 USD/kWe. In contrast, for nuclear and wind generating technologies overnight construction costs range between 1000 and 2000 USD/kWe. Accordingly, we calibrate β_g/β_b to range between 1 and 2.5.

For the emission intensity α of brown capital we use the average emission intensity of a coal energy plant, which is estimated to be 0.888 tonnes CO_2/MWh, while for a gas plant those estimates average at 0.499 tonnes CO_2/MWh, as reported by WNA (2011). Salvadore and Keppler (2010) reported an investment cost between 9–18 USD/MWh at a 5% discount rate, while at a 10% discount rate the investment costs range between 17.5 and 30 USD/MWh. Therefore, at a 5% discount rate we get an emission intensity of 5% – 10% per unit of capital invested in a coal plant, while at a 10% discount rate, the emission intensity ranges from 3% to 6%. For a gas plant, investment costs range between 5.5 – 9 USD/MWh at 5% discount rate, and therefore, the emission intensity ranges between 5% – 9% of each unit of capital invested in a gas plant.

Damage from global pollution stock is likely to be a persistent problem for a long time, and small values, between 1% (Stern (2007)) and 4% (Nordhaus (2014)), are usually used for the time discount rate ρ. However, in order to be consistent with the calibration of other parameters we use ρ between 5% - 10%. This does not greatly affect the results obtained.

The investment cost parameter β_b represents the rate of increase of the marginal investment cost in brown capital per unit of investment. We use values of β_b ranging between 2% and 8%.

Depending on the estimated life time for a wind energy plant (around 40 years), we use the same depreciation rate for both types of capital, resulting in a range for δ between 2.5%–5%.

Higher values of the parameters η and η^n imply that governments care more about the environmental quality of their consumers when taking decisions. We choose different values of these parameters to test different assumptions about the weight of environmental quality between North and South.

Annual carbon emissions from burning fossil fuels in the United States are about 1.6 gigatons (billion metric tons), whereas annual uptake is only about 0.5 gigatons, resulting in a net release of about 1.1 gigatons per year. This implies that only 31% of the U.S carbon emissions are absorbed naturally (Sundquist et al. 2008). Using this, and an estimated emission rate between 5% and 9% of installed capital at a 5% discount rate, we arrive at a natural absorption rate of installed capital between 1.55% and 2.8% at a 5% discount rate. The resulting benchmark values for parameters can be found in Appendix C.

6 Results

For the analysis of the growth dynamics, we set low initial values for brown and green capital as well for the pollution stock, as we are interested in the situation that South initially falls in the 'least developed' class of countries.

6.1 North's allocation of aid

We start the analysis with investigating the aid allocation of North in equilibrium, and how it is affected by parameter changes.

We know from Section 4 that North will give aid in steady state either if its output is sufficiently high, or if it values environmental quality highly enough. If South does not care about the environment and South and North are patient, that is if η and ρ are close to 0, there will be no incentive for North to give any aid to South, as South will never make green investments. On the other hand, we find that if South cares a lot, that is, if η is sufficiently large, then again there will be no incentive for North to give aid, as South will make sufficient green investments on its own accord. The benchmark parametrisation describes therefore an intermediate situation.

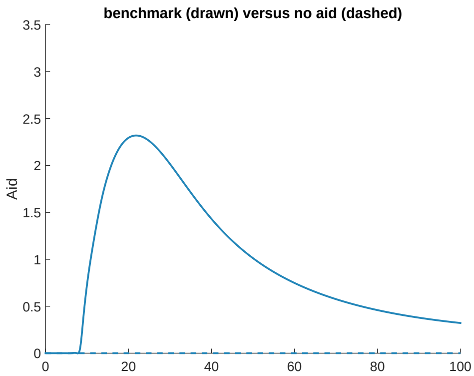


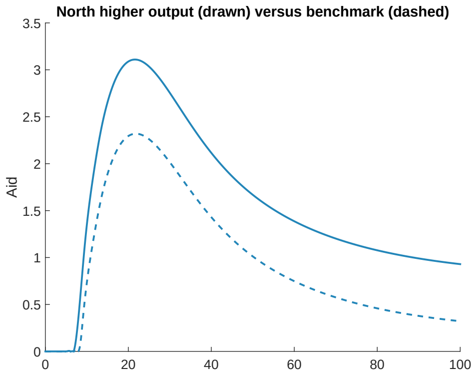
Figure 1: Aid profile over time (benchmark)

Figure 1 shows the benchmark aid profile over time. There is an initial time interval where no aid is given: this is when South's stocks of brown capital and global pollution are still at low levels. It is only when South's brown capital stock is sufficiently large that North starts giving aid. Although most of the aid is consumed, a part of it enables South to invest in green capital and thereby to lessen its emissions. North's decision to give aid is motivated only by environmental reasons — there is no 'warm glow' term in its utility function — and therefore it should be considered as mitigation aid. Whenever $\eta > 0$, South cares about pollution from brown capital and has an incentive to invest relatively less in brown capital if income is higher (i.e. environmental quality is a normal good, so that demand for environmental quality rises with income). The hump shaped aid profile follows from the profile of South's brown investments, and thus emissions, in South. These correspond to an Environmental Kuznets curve: countries at a low development level tend to increase their emission until average income reaches a certain point over the course of their development.

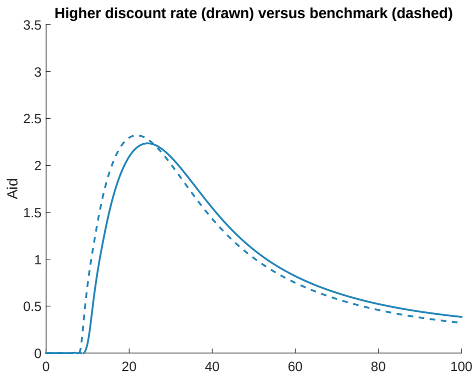
Figures 2 and 3 show changes in the aid profile with respect to changes to different parameters, compared to the benchmark profile. In these figures, a dashed curve represents the benchmark aid profile, while the solid curve indicates the aid profile after the change. In all cases, the parameter has been increased or decreased by 20% with respect to its benchmark value.

6.1.1 Effects of changing capital parameters

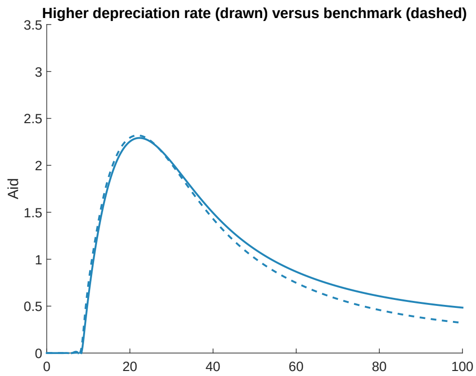
Figure 2 illustrates the effects of changing parameters that affect the industrial output of North or South. Figure 2a shows the effect of increasing North's output: the level of aid is higher. This finding is in line with the result of Theorem 5 on steady state aid.



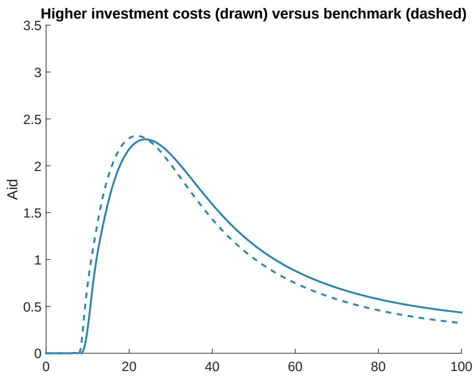
(a) Increasing North's output Y^n



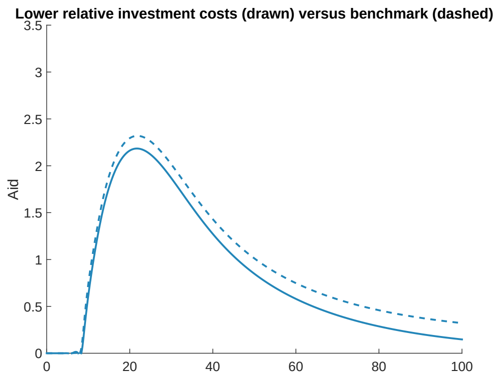
(b) Increasing time discount rate ρ



(c) Increasing depreciation rate δ

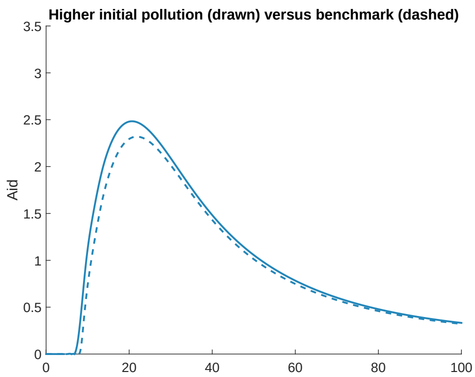


(d) Increasing both β_b and β_g while keeping β_g/β_b constant

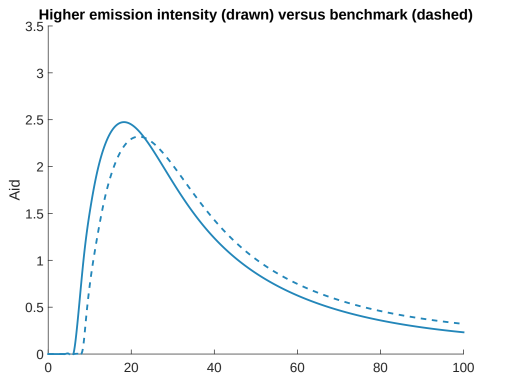


(e) Decreasing β_g while keeping β_b constant

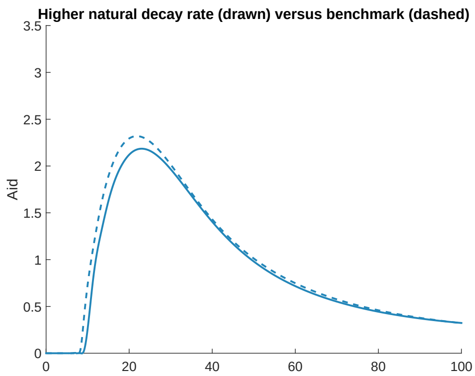
Figure 2: Influence of capital-related parameter changes on the aid profile



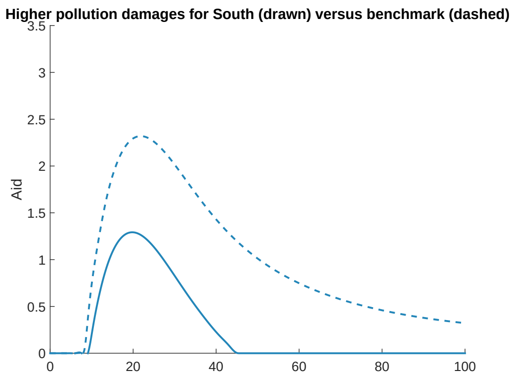
(a) Increasing initial pollution E_0



(b) Increasing emission intensity α



(c) Increasing natural decay rate ϑ



(d) Increasing South's weight of environmental quality η

Figure 3: Influence of environmental parameter changes on the aid profile

Figure 2b increases the discount rate, which both decreases aid and shifts the aid profile to the future, because the long term effects of environmental pollution impact North's welfare less. Increasing the depreciation rate δ, as in Figure 2c, has a similar but smaller effect, although the explanation is different: if capital depreciates quickly, brown capital is less quickly at a critical level. Moreover, it is inefficient to start enabling South too early to invest in green capital.

Higher values of the rate of increase in the marginal cost of investments β_g and β_b, while keeping their ratio constant, imply again that South needs more time to build up capital towards critical levels, implying a shift of the aid profile into the future, as seen in Figure 2d.

If the cost β_g/β_b of green investments relative to brown investments falls, aid goes down, for South is less constrained when building up its green capital.

6.1.2 Effects of changing environmental parameters

Figure 3 documents the consequences of changes to environmental parameters. The first panel, Figure 3a, shows the effect of an increase in the initial pollution stock: this aggravates the environmental conditions and leads North to start giving more aid more quickly, as already a smaller stock of green capital build by South improves the situation.

Higher emission intensity of brown capital makes the aid programme start sooner, Figure 3b: as brown capital emits more pollution, more damages from pollution are realised sooner by North.

If the natural decay rate of pollution increases, Figure 3c, the pollution stock decreases faster and South's emissions take longer to reach critical levels. Together this makes the problem less urgent for North, whose aid programme is reduced.

Finally, Figure 3d shows that if South's consumers put more weight on environmental quality, its incentive to build green capital increases, which in turn lowers North's incentive to give aid dramatically.

6.2 South's use of the aid

We turn to South consumption and investment decisions. First we analyse these as function of the model parameters. Then we study the how South allocates the aid it receives from North between consumption and total investments, and how it allocates investment aid between brown and green investments.

6.2.1 Aid increases consumption and green growth

Transboundary pollution is considered as a natural asset necessary for development and for economic growth. Keeping global pollution under tolerable levels, such as keeping the average global temperature below 2 degrees, is necessary to achieve sustainable growth, and thus green growth. Therefore, in this study we measure green growth in the long run by a decrease in global pollution levels. In the short run, investing in non-polluting capital means that these investments are not taking place in the polluting sector, and therefore, lower transitional pollution and higher green growth.

The decisions of South how to allocate aid show how efficiently aid promotes economic growth of the recipient country as well as the effect of aid on the direction of growth.

In order to identify the choice of South for both decision processes, we compare the time paths of South's controls when it receives aid to those when it does not, holding all parameters constant.

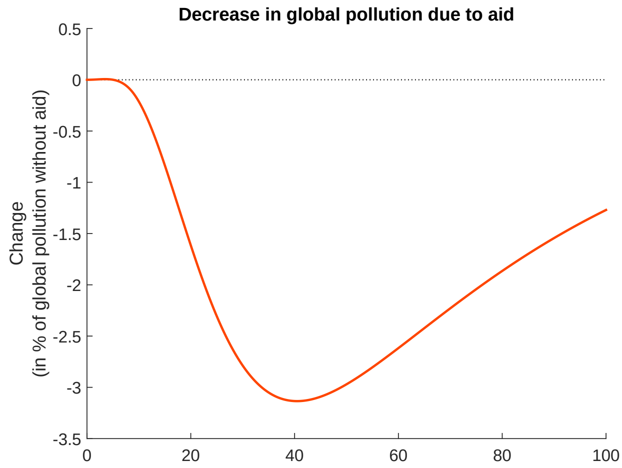
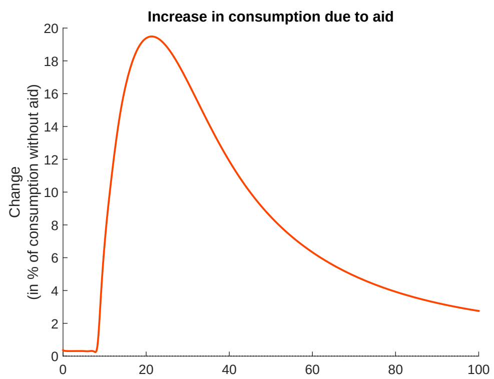
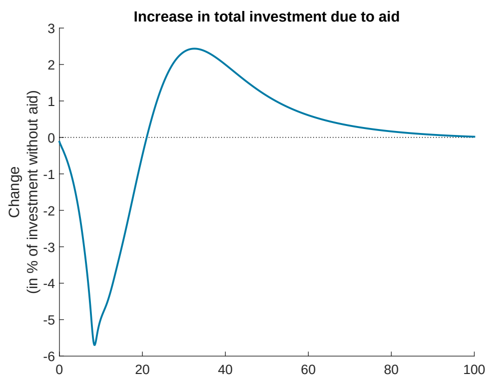


Figure 4: Effect of aid on global pollution

As mentioned in Section 6.1, in the benchmark situation North starts giving aid when the environmental conditions become critical from its perspective. Figure 4 shows the relative change of global pollution level from the benchmark without aid. Giving aid decreases the pollution stock, mainly by shifting brown capital levels downwards. The latter effect become clearer when we study the effect of aid on brown investments.



(a) Effect of aid on consumption

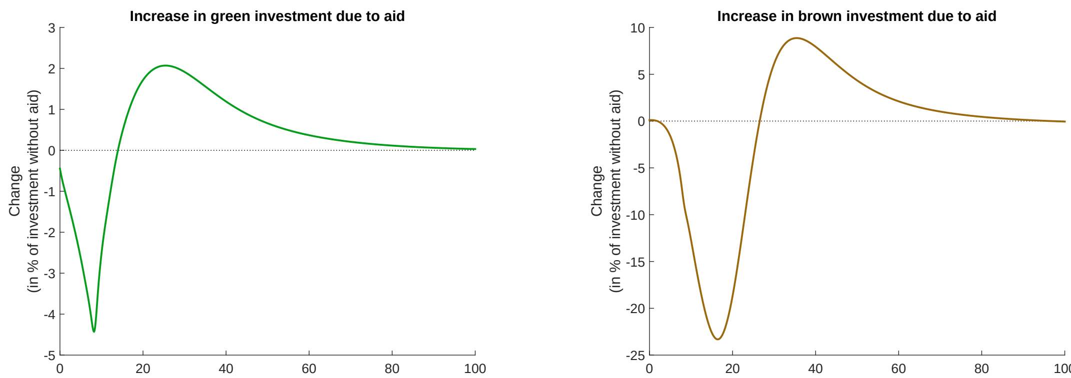


(b) Effect of aid on investments

Figure 5: South's allocation of aid between consumption and investments

Figure 5a depicts the relative increase of South's consumption when receiving aid compared to the situation where no aid is received; Figure 5b gives the corresponding increase in total investments.

The figures show that it is optimal for South to use most of the aid to smooth out its consumption schedule. This is clear from panel 5a as investments are postponed, consumption increases a little before aid is received. However, as South starts receiving aid, the rise in consumption takes a hump shaped similar to that of the aid profile. This seem at first sight to agree to the findings of Boone (1996), who concludes that aid primarily goes to consumption and that there is no relationship between aid and growth. Figure 5b depicts how South's total investments change over time with aid: it shows that investments fall steadily relative to the situation where no aid is expected, until the moment aid starts to arrive. Investments increase again and are then for a substantial period of time over the no-aid levels. Therefore we argue that the conclusion of Boone (1996) about the relationship between aid and growth is imprecise: in our situation, aid has a positive effect on growth, but this is modest and lower than what the Harrod and Domar model would predict. These findings are in line with Chatterjee et al. (2003) who find that a temporary pure transfer has only modest short-run growth effects compared to a transfer tied to investment in public infrastructure. We note that a second effect of aid is to push investments into the future.



(a) Effect of aid on green investments

(b) South's allocation of aid between brown and green investments

Figure 6: South's allocation of aid between brown and green investments

Figure 6 depicts the change of South's investment schedule due to aid for, respectively, brown and green capital. There is a decrease of investments before the aid period begins. The maximal decrease of green investments respective to the case that no aid is received tops out at about 4.5%, before it starts to increase again and ends up at its highest about 2.3% higher than in the no-aid situation. Investments in brown capital fall much more strongly, to a minimum of 26% of investments in the no-aid regime. Also here we see that later on, investments in brown capital pick up again, topping out at an increase of 9% over the no-aid levels. Note however that these effects are small in absolute terms, as the brown capital level is much lower than the green capital levels. Moreover, since pollution stock is proportional to South's brown capital, figure 4 illustrates that aid is effective in lessening brown capital.

We summarise these findings by noting that aid has two effects on investments: it modestly increases total eventual growth, in the benchmark situation mainly for green capital, and it pushes growth farther into the future, by enabling South to increase consumption earlier.

6.3 Time consistency

The Stackelberg equilibria we have investigated so far are open-loop equilibria: that is, at time $t = 0$ North announces an aid schedule a_t, and South subsequently makes its plans taking this schedule for granted. At any given point in time, North may reconsider its decision, which then can result in a change in the announced aid policy.

To model South's reaction to such a policy change, we extend the original differential game by introducing a binary state variable, *trust*, which can take the values 0 and 1. At the beginning of the game, *trust* is assumed to take the value 1, which is interpreted as South trusting North to stick to its announced aid schedule. When, at some time $t > 0$, North deviates from the announced schedule — this can be observed by South — *trust* switches from 1 to 0, and South falls back to that growth policy which is optimal if it will receive no aid from North. North will then switch to giving no aid at all, as in the 'no trust' regime giving aid will not alter South's behaviour. This is analogous to the trigger strategy mechanism in repeated games.

In order to find out whether North will stick to its original aid schedule, we have to compare, for each time $t > 0$, North's payoff over the time interval $[t, \infty)$ when sticking to the announced aid schedule versus its payoff when cutting aid at time t. More precisely, let $(E_t, K_{b,t}, K_{g,t})$ be the evolution of pollution level, brown and green capital stock, under the aid schedule a_t announced by North at time $t = 0$, and let

$$W^n(t_0) = \int_{t_0}^{\infty} \mathrm{e}^{-\rho(s-t_0)} \Big(u^n(Y^n - a_s) - D^n(E_s) \Big) \, \mathrm{d}s$$

the corresponding present value of North's welfare at time t_0. If North changes its aid payment at time t_0, South falls back to its optimal growth policy starting at time t_0, with initial values $(E_{t_0}, K_{b,t_0}, K_{g,t_0})$, under the assumption that it will receive no aid. This results, amongst other things, in a different evolution E_t^0 of the pollution stock and a different present value

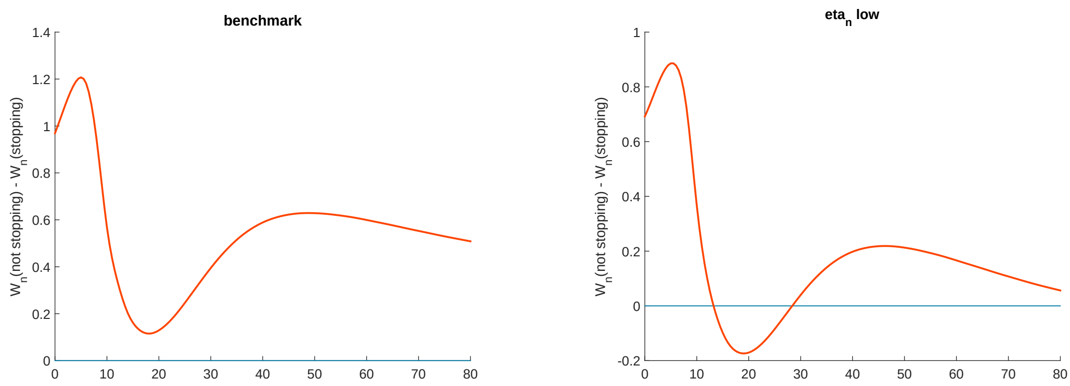
$$W^{n,0}(t_0) = \int_{t_0}^{\infty} \mathrm{e}^{-\rho(s-t_0)} \Big(u^n(Y^n) - D^n(E_s^0) \Big) \, \mathrm{d}s$$

of North's welfare. If the difference

$$\Delta_t = W^n(t) - W^{n,0}(t)$$

is negative for some $t > 0$, North has an incentive to reconsider its aid policy at that date, and the announced policy is not time-consistent.

Panel 7a shows the evolution of Δ_t for the benchmark parametrisation. In Section 6.2 we saw that in anticipation of the aid transfers, South reduces production, resulting in lower



(a) Benchmark: time consistent

(b) Lower η^n: not time consistent

Figure 7: Time consistency of the Stackelberg equilibrium

emissions which benefits North. We conclude that in the benchmark parametrisation, giving aid is a time-consistent policy.

Panel 7b illustrates a contrasting situation: if North is less sensitive to pollution damages than in the benchmark parametrisation, giving aid is not time-consistent. Unlike the benchmark parametrisation, here aid is given only temporarily and there is no aid in steady state. Moreover, the panel shows that this quantity starts taking negative values at the moment where North should be starting making aid payments. In the benchmark situation, the long term gains in pollution reduction in steady state are always more important to North than the short time savings by not sticking to the announced aid transfers.

Accordingly, we conclude that giving unconditional aid in the open loop Stackelberg equilibrium is weakly time consistent if η^n is sufficiently high.

6.4 Main effects of giving aid

The discussion in the previous section highlights some of the effects of aid on South's decision over time. We distinguish three effects: the first one is that South chooses to postpone a small amount of its investments until it starts receiving aid, increasing consumption instead. This is coherent with the life-cycle theory of consumption. However, the intertemporal substitution of consumption is effectively small as South poor at the initially.

The second effect is that South consumes most of the aid received. This squares with much anecdotal evidence of development aid 'leaking away'. The present analysis shows however that apart from corruption and mismanagement, which undoubtedly play a role in practice and which are not addressed by our model, there is also the purely economic motivation that the aid is simply better employed elsewhere from South's point of view.

Thirdly, South stops developing its brown capital when receiving aid. Effectively, giving aid results in a reduction in global emissions.

7 Conclusion

This study theoretically identifies the dynamic effects of unconditional aid on the growth and the direction of the growth of a recipient country. We studied a differential Stackelberg game between a leading donor country and a following recipient country. The decision of the donor to give aid in our model is motivated by environmental concerns, and should be classified as mitigation aid. Our model identifies circumstances under which the donor is motivated to give unconditional mitigation aid.

We conclude that if the recipient is sufficiently concerned about environmental quality, there is no incentive for the donor to give aid, as the recipient takes its decisions in a way that preserves the environment whether it receives assistance or not. If the recipient is not concerned about environmental quality at all, again there is no incentive for the donor to give aid, as the behaviour of the recipient will not be influenced by it. In between these two extreme situations, when the recipient is weakly concerned about environmental quality, the donor has an incentive to give aid.

In particular, since we argue that most 'conditional' aid is in practice given virtually unconditionally, our study provides an explanation for the empirical evidence that indicates the relative ineffectiveness of aid on growth of the recipient country: our model indicates that it is optimal for the recipient to consume most of the aid and only to allocate a minor part to investments. Still, even giving unconditional aid can be a Pareto-improvement over giving no aid at all.

Our model also shows that unconditional development aid has a modest positive short term effect on growth. This effect seems however much lower than what the Harrod and Domar model predicts. At least for our benchmark case we investigated, we found that most of the increase of growth caused by aid takes place in the green sector.

We propose two possible extensions to our model. The first is to include demographical changes in the recipient country by adding labour as a second input for production. This would help to complete the analysis, to study whether high population growth rate in these countries necessitates a higher growth rate to meet the demographical changes: the possible effect would be that aid is more effectively used to increase growth. The second extension would be to introduce a parameter that captures aid being given under the condition that it is used only for green investments. We expect then to find an intertemporal trade-off between consumption and investments, resulting in a higher consumption ex-ante and consequently a *de facto* failure of conditionality.

As a policy recommendation, results analysed in this study suggest that even when aid conditionality cannot be fully achieved, a donor country can gives mitigation aid if it is sufficiently rich or if it cares sufficiently about the environmental quality of its citizens. The effectiveness of aid if environmental awareness of the recipient country is sufficiently high. Note that the effects of aid on green growth might be more than that analysed here, as the analyzed impacts represent the minimum expected effects since aid is given unconditionally.

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A South

A.1 South's steady state

Here, we compute and analyse the steady state of South's decision problem under the assumption that the aid schedule $a_t = a$ is constant in time. To denote a steady state value of a dynamic quantity, we drop the subscript t.

A.2 Green capital

First, we derive the steady state level of green capital. At steady state, we obtain from the capital evolution equation (6) the steady state condition

$$I_i = \delta K_i, \qquad \text{for} \quad i \in \{b, g\}. \tag{43}$$

The costate evolution equation (11) yields the steady state condition

$$(\rho + \delta)\nu_g = u'(c) F'_{K_g}(K_b, K_g). \tag{44}$$

Equation (8) implies at steady state

$$\nu_g = C'_g(I_g)(u'(c) - \zeta). \tag{45}$$

Finally, we have the restriction $C_g(I_g) \geq \varepsilon a$ and the associated complementary slackness condition

$$\zeta(C_g(I_g) - \varepsilon a) = 0. \tag{46}$$

We use (45) to eliminate ν_g from (44), to obtain

$$u'(c) F'_{K_g}(K_b, K_g) = (\rho + \delta) C'_g(I_g)(u'(c) - \zeta). \tag{47}$$

First consider the situation that the green investment restriction is not binding. Then $\zeta = 0$ and we can divide out $u'(c)$ from both sides of (47). Using (43) to eliminate I_g yields then

$$F'_{K_g}(K_b, K_g) = (\rho + \delta) C'_g(\delta K_g) \tag{48}$$

equating marginal productivity to marginal discounted investment costs.

If production is separable, $F(K_b, K_g) = F_b(K_b) + F_g(K_g)$, then the equation simplifies to

$$F'_g(K_g) = (\rho + \delta) C'_g(\delta K_g). \tag{49}$$

This equation determines the steady state level K_g of green capital as a function of the system's parameters; K_g in turn determines the steady state level I_g of green investments. Note in particular that in this situation green capital and green investments at steady state do not depend on aid.

If on the other hand the green investment restriction is binding, then

$$I_g = C_g^{-1}(\varepsilon a) \quad \text{and} \quad K_g = \frac{C_g^{-1}(\varepsilon a)}{\delta}, \tag{50}$$

while

$$\zeta = u'(c) \left(1 - \frac{F'_{K_g}(K_b, K_g)}{(\rho + \delta) C'_g(I_g)} \right) \tag{51}$$

In this case, the steady state levels of investment and green capital depend directly on the amount of aid given.

A.3 Brown capital

We turn to brown capital. From the budget constraint (4), we write steady state consumption c as a function of aid a and brown capital K_b

$$c = F(K_b, K_g) + a - C_b(\delta K_b) - C_g(\delta K_g). \tag{52}$$

From (7) and (9) it follows that

$$E = \frac{\alpha}{\vartheta} K_b \tag{53}$$

and

$$\mu = -\frac{D'(E)}{(\rho + \vartheta)} = -\frac{D'(\frac{\alpha}{\vartheta} K_b)}{(\rho + \vartheta)}. \tag{54}$$

This yields E and μ as functions of K_b.

Eliminating μ from (10) using (54), both at steady state, yields

$$(\rho+\delta)\nu_b = u'(c)F'_{K_b}(K_b,K_g) - \frac{\alpha}{\rho+\vartheta}D'\left(\frac{\alpha}{\vartheta}K_b\right). \tag{55}$$

Using (8) and (43), we obtain a second expression

$$\nu_b = u'(c)C'_b(\delta K_b)$$

for ν_b. After eliminating ν_b from (55), we finally obtain

$$F'_{K_b}(K_b,K_g) = (\rho+\delta)C'_b(\delta K_b) + \frac{\alpha}{(\rho+\vartheta)}\frac{D'\left(\frac{\alpha}{\vartheta}K_b\right)}{u'(c)}, \tag{56}$$

equating marginal productivity gains to the sum of marginal discounted investment costs and marginal environmental damage costs. Note that the environmental damage term depends inversely on the marginal utility of consumption, and, all other things constant, will increase with the consumption level, and hence with aid.

If production is separable, this relation simplifies to

$$F'_b(K_b) = (\rho+\delta)C'_b(\delta K_b) + \frac{\alpha}{(\rho+\vartheta)}\frac{D'\left(\frac{\alpha}{\vartheta}K_b\right)}{u'(c)}, \tag{57}$$

We see from this relation that even in the separable case, the steady state brown capital level K_b is sensitive to aid, that is, changes in c.

A.4 Proofs of theorem 2–4

A.4.1 Proof of theorem 2

Proof.
It follows from (49) that aid does not affect the steady state level of green capital, and hence that $\frac{\partial K_g}{\partial a} = 0$. Consumption c and brown capital K_b are jointly determined by (52) (56), which can be written as

$$G_1 = c + C_b(\delta K_b) + C_g(\delta K_g) - F(K_b,K_g) - a = 0,$$

$$G_2 = (\rho+\delta)C'_b(\delta K_b) - F'_{K_b}(K_b,K_g) + \frac{\alpha}{(\rho+\vartheta)}\frac{D'\left(\frac{\alpha}{\vartheta}K_b\right)}{u'(c)} = 0.$$

Introduce the vector-valued functions $G = (G_1, G_2)$, whose components are net expenses and net marginal productivity costs, the variable $X = (c, K_b)$, and introduce the derivative

$$D_X G = \begin{pmatrix} \frac{\partial G_1}{\partial c} & \frac{\partial G_1}{\partial K_b} \\ \frac{\partial G_2}{\partial c} & \frac{\partial G_2}{\partial K_b} \end{pmatrix}.$$

We shall need the elements of the matrix $D_X G$ and its inverse. These are

$$\frac{\partial G_1}{\partial c} = 1, \qquad \frac{\partial G_1}{\partial K_b} = \delta C_b' - F_{K_b}',$$
$$\frac{\partial G_2}{\partial c} = \frac{\alpha}{\rho + \vartheta} D' \frac{(-u'')}{(u')^2}, \qquad \frac{\partial G_2}{\partial K_b} = (\rho + \delta)\delta C_b'' - F_{K_b K_b}'' + \frac{\alpha^2}{(\rho + \vartheta)\vartheta} \frac{D''}{u'}.$$

It follows from the assumptions of F_b, u and D that $\frac{\partial G_1}{\partial c} > 0$, $\frac{\partial G_2}{\partial c} > 0$ and $\frac{\partial G_2}{\partial K_b} > 0$. Using (56) to eliminate F_{K_b}', we find that

$$\frac{\partial G_1}{\partial K_b} = \delta C_b' - F_{K_b}' = -\rho C_b' - \frac{\alpha}{(\rho + \vartheta)} \frac{D'}{u'} < 0.$$

This implies that the determinant $\Delta = \det D_X G$ is positive. Setting

$$-(D_X G)^{-1} = B = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}$$

these results imply that $B_{11} < 0$, $B_{12} < 0$ and $B_{22} < 0$, while $B_{21} > 0$.

Since

$$D_a X = -\left(D_X G\right)^{-1} D_a G, \tag{58}$$

we also have to compute the elements of $D_a G$, which evaluate to $\frac{\partial G_1}{\partial a} = -1$ and $\frac{\partial G_2}{\partial a} = 0$. Equation (58) then implies that $\frac{\partial c}{\partial a} = -B_{11} > 0$ and $\frac{\partial K_b}{\partial a} = -B_{21} < 0$. This shows the results about consumption and dirty and green capital. The results about investments and the pollution stock follow from equations (43) and (53). If consumption increases and the pollution stock decreases, South's welfare increases. □

A.4.2 Proof of theorem 3

Retaining the notations from the previous proof, we note that

$$\begin{pmatrix} \frac{\partial c}{\partial \beta_i} \\ \frac{\partial K_b}{\partial \beta_i} \end{pmatrix} = B\, D_{\beta_i} G = B \begin{pmatrix} \frac{\partial G_1}{\partial \beta_i} \\ \frac{\partial G_2}{\partial \beta_i} \end{pmatrix} \qquad \text{for} \quad i \in \{b, g\}.$$

For green investment costs, we have

$$\frac{\partial G_1}{\partial \beta_g} = \frac{\delta^2}{2} K_g^2 > 0, \quad \frac{\partial G_2}{\partial \beta_g} = 0,$$

hence

$$\frac{\partial c}{\partial \beta_g} = B_{11} \frac{\delta^2}{2} K_g^2 < 0, \qquad \frac{\partial K_b}{\partial \beta_g} = B_{21} \frac{\delta^2}{2} K_g^2 > 0.$$

Then, for brown investment costs

$$\frac{\partial G_1}{\partial \beta_b} = \frac{\delta^2}{2} K_b^2 > 0, \quad \frac{\partial G_2}{\partial \beta_b} = (\rho + \delta)\delta K_b > 0,$$

which implies

$$\frac{\partial c}{\partial \beta_b} = B_{11} \frac{\delta^2}{2} K_b^2 + B_{21}(\rho + \delta)\delta K_b < 0,$$

and

$$\frac{\partial K_b}{\partial \beta_b} = B_{21} \frac{\delta^2}{2} K_g^2 + B_{22}(\rho + \delta)\delta K_b.$$

In the last expression, the two terms on the right hand side have opposite signs. However, if the emission intensity $\alpha = 0$, then $B_{21} = 0$ and

$$\left.\frac{\partial K_b}{\partial \beta_b}\right|_{\alpha=0} < 0,$$

which implies, by continuity, that $\frac{\partial K_b}{\partial \beta_b} < 0$ for values of α close to 0.
Finally, for the capital depreciation rate

$$\frac{\partial G_1}{\partial \delta} = \beta_g \delta K_g^2 + \beta_b \delta K_b^2 > 0, \quad \frac{\partial G_2}{\partial \delta} = (\rho + 2\delta)\beta_b K_b > 0.$$

Analogously to the situation of brown investment costs, this implies

$$\frac{\partial c}{\partial \delta} < 0$$

whereas the sign of $\frac{\partial K_b}{\partial \delta}$ is undetermined in general, but for α taking values close to 0, we have that $\frac{\partial K_b}{\partial \delta} < 0$.

A.4.3 Proof of theorem 4

Again retaining the notations of the proof of theorem 2, and using $E = \frac{\alpha}{\vartheta} K_b$, we find

$$\begin{aligned} D_\vartheta G &= -\left(\frac{\alpha}{(\rho + \vartheta)^2} \frac{D'(E)}{u'(c)} + \frac{\alpha}{\rho + \vartheta} \frac{D''(E)}{u'(c)} E \right) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ &= -\frac{\alpha}{(\rho + \vartheta)^2} \frac{D'(E)}{u'(c)} \left(1 + (\rho + \vartheta) \frac{ED''(E)}{D'(E)} \right) \begin{pmatrix} 0 \\ 1 \end{pmatrix} = -C \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \end{aligned}$$

where $C > 0$: that is, net expenses are not affected by changes in the natural decay rate, but net marginal productivity costs decrease if ϑ increases. It follows that

$$\frac{\partial c}{\partial \vartheta} = -B_{12} C > 0, \qquad \frac{\partial K_b}{\partial \vartheta} = -B_{22} C > 0.$$

From

$$D_\alpha G = \frac{1}{\rho+\vartheta}\frac{D'(E)}{u'(c)}\left(1+\frac{ED''(E)}{D'(E)}\right)\begin{pmatrix}0\\1\end{pmatrix},$$

the factor in brackets being positive, it follows in the same manner that $\frac{\partial c}{\partial \alpha} < 0$, $\frac{\partial K_b}{\partial \alpha} < 0$. Using the functional form $D(E) = \eta E^2/2$, we find

$$D_\eta G = \left(\frac{\alpha}{\rho+\vartheta}\frac{\frac{\alpha}{\vartheta}K_b}{u'(c)}\right)\begin{pmatrix}0\\1\end{pmatrix}.$$

In the same manner as before, we obtain $\frac{\partial c}{\partial \eta} < 0$, $\frac{\partial K_b}{\partial \eta} < 0$.

B North

B.1 North's aid decision

This section proves theorem 5 by demonstrating the validity of equation (41).

In the analysis of South's steady state, aid a was treated as an external parameter. From the steady state conditions of North's co-state equations, we derive an equation that links North's steady state aid level to South's consumption level c and South's brown capital level K_b.

The equation

$$\xi = (u^n)'(Y^n - a) - \Lambda^n \tag{59}$$

shows that ξ are North's net marginal shadow costs of giving aid, while Λ^n are North's marginal shadow benefits of increasing South's budget; as long as $\xi > 0$, North cannot increase its welfare by giving aid.

Note that if $a = 0$, then Λ^n is independent of ξ and the restriction $\xi \geq 0$ together with equation (59) implies an upper bound $\bar{Y}^n$ for Y^n that is such that $(u^n)'(\bar{Y}^n) = \Lambda^n$. Conversely, if $Y^n > \bar{Y}^n$, then necessarily $\xi = 0$, $a > 0$ and it is optimal for North to give aid. This proves the first statement in the theorem if $\Lambda^n > 0$.

We turn to the sign of North's shadow values of South's capital in steady state. We first solve the λ_i from the steady state versions of (24) and (25), to obtain

$$\lambda_i = \frac{\nu_i\Lambda^n - \kappa_i u'}{\delta\beta_i(u')^2}. \tag{60}$$

Equations (8), together with (43) provide

$$\nu_i = \delta\beta_i K_i u'. \tag{61}$$

We solve for τ, North's shadow value of South's shadow value of the pollution stock, and ψ, North's shadow value of the pollution stock, as

$$\tau = \frac{\alpha}{\vartheta}\frac{\nu_b\Lambda^n - \kappa_b u'}{\delta\beta_b(u')^2}$$

and

$$\psi = \frac{1}{\rho+\vartheta}\left(\frac{\alpha}{\vartheta}\frac{\nu_b\Lambda^n - \kappa_b u'}{\delta\beta_b(u')^2}D'' - (D^n)'\right).$$

This yields equations for North's shadow values of South's capital stocks

$$\begin{aligned}
0 &= (\rho+\delta)\kappa_g + \frac{\nu_g\Lambda^n - \kappa_g u'}{\delta\beta_g u'}F_g'' - \Lambda^n F_g' \\
0 &= (\rho+\delta)\kappa_b + \frac{\nu_b\Lambda^n - \kappa_b u'}{\delta\beta_g u'}F_b'' - \Lambda^n F_b' - \frac{\alpha}{\rho+\vartheta}\left(\frac{\alpha}{\vartheta}\frac{\nu_b\Lambda^n - \kappa_b u'}{\delta\beta_b(u')^2}D'' - (D^n)'\right).
\end{aligned}$$

Rearranging yields

$$\begin{aligned}
\left(\rho+\delta-\frac{F_g''}{\delta\beta_g}\right)\kappa_g &= \Lambda^n\left(F_g' - K_g F_g''\right) \\
\left(\rho+\delta-\frac{F_b''}{\delta\beta_b}+\frac{\alpha^2}{(\rho+\vartheta)\vartheta\delta\beta_b}\frac{D''}{u'}\right)\kappa_b &= \frac{\alpha}{\rho+\vartheta}(D^n)' \\
&\quad + \Lambda^n\left(F_b' - K_b F_b'' + \frac{\alpha^2}{(\rho+\vartheta)\vartheta}\frac{D''}{u'}K_b\right).
\end{aligned}$$

The first equation is simplified by using (49) to

$$\left(\rho+\delta-\frac{F_g''}{\delta\beta_g}\right)\kappa_g = \Lambda^n\left((\rho+\delta)\beta_g\delta K_g - K_g F_g''\right) = \Lambda^n\beta_g\delta K_g\left(\rho+\delta-\frac{F_g''}{\delta\beta_g}\right),$$

whence we conclude that

$$\kappa_g = \Lambda^n\beta_g\delta K_g.$$

Taking our cue from this solution, we attempt to find κ_b in the form $\delta\beta_b(\Lambda^n K_b + \alpha X)$. The quantity X is then determined by the equation

$$\left((\rho+\delta)\delta\beta_b - F_b'' + \frac{\alpha^2}{(\rho+\vartheta)\vartheta}\frac{D''}{u'}\right)X = \frac{1}{\rho+\vartheta}(D^n)'.$$

This yields finally

$$\kappa_b = \delta\beta_b\left(\Lambda^n K_b + \frac{\alpha}{\rho+\vartheta}\frac{(D^n)'}{(\rho+\delta)\delta\beta_b - F_b'' + \frac{\alpha^2}{(\rho+\vartheta)\vartheta}\frac{D''}{u'}}\right).$$

Finally, we compute Λ^n, which satisfies

$$\Lambda^n \left(\frac{1}{(u')^2} \left(\frac{\nu_b^2}{\beta_b} + \frac{\nu_g^2}{\beta_g} \right) + \frac{u'}{(-u'')} \right) = u'(\lambda_b F_b' + \lambda_g F_g') + \frac{\kappa_b \nu_b}{\beta_b u'} + \frac{\kappa_g \nu_g}{\beta_g u'}. \tag{62}$$

The right hand side of this equation is

$$\begin{aligned}
&u'(\lambda_b F_b' + \lambda_g F_g') + \frac{\kappa_b \nu_b}{\beta_b u'} + \frac{\kappa_g \nu_g}{\beta_g u'} \\
&= u' \left(\frac{\nu_b \Lambda^n - \kappa_b u'}{\delta \beta_b (u')^2} F_b' + \frac{\nu_g \Lambda^n - \kappa_g u'}{\delta \beta_g (u')^2} F_g' \right) + \frac{\kappa_b \nu_b}{\beta_b u'} + \frac{\kappa_g \nu_g}{\beta_g u'} \\
&= \frac{\Lambda^n}{u'} \left(\frac{\nu_b F_b'}{\delta \beta_b} + \frac{\nu_g F_g'}{\delta \beta_g} \right) + \frac{\kappa_b}{\delta \beta_b} (\delta \nu_b / u' - F_b') + \frac{\kappa_g}{\delta \beta_g} (\delta \nu_g / u' - F_g') \\
&= \Lambda^n \left(K_b F_b' + K_g F_g' \right) + (\delta^2 \beta_b K_b - F_b')(\Lambda^n K_b + \alpha X) + (\delta^2 \beta_g K_g - F_g') \Lambda^n K_g \\
&= \Lambda^n \left(\beta_b \delta^2 K_b^2 + \beta_g \delta^2 K_g^2 \right) + \alpha X (\delta^2 \beta_b K_b - F_b').
\end{aligned}$$

We bring the terms involving Λ^n to the left hand side of (62) and use 57 to rewrite the remaining term on the right hand side. The equation then reads as

$$\begin{aligned}
&\Lambda^n \left(\frac{1}{(u')^2} \left(\frac{\nu_b^2}{\beta_b} + \frac{\nu_g^2}{\beta_g} \right) + \frac{u'}{(-u'')} - \beta_b \delta^2 K_b^2 - \beta_g \delta^2 K_g^2 \right) \\
&\quad = -\alpha X \left(\rho \delta \beta_b K_b + \frac{\alpha}{\rho + \vartheta} \frac{D'}{u'} \right).
\end{aligned}$$

We note finally that (61) implies that the left hand side simplifies further to $\Lambda^n u'/(-u'')$, whence we obtain the result that

$$\Lambda^n = \frac{(-u'')}{u'} \frac{\alpha (D^n)'/(\rho + \vartheta)}{(\rho + \delta)\delta \beta_b - F_b'' + \frac{\alpha^2}{(\rho+\vartheta)\vartheta} \frac{D''}{u'}} \left(\rho \delta \beta_b K_b + \frac{\alpha}{\rho + \vartheta} \frac{D'}{u'} \right),$$

which is clearly positive if $\alpha > 0$.

This conclude the proof of theorem 5.

C Benchmark parametrisation

Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
$K_{b,0}$	1	δ	0.025	γ	0.75	α	0.05
$K_{g,0}$	1	T	300	ρ	0.05	ϑ	0.016
E_0	15	Ω	0.6	β_b	0.05	η	0.0006
ε	0.5	σ	0.5	β_g	0.125	η^n	0.023
Y^n	12.123						