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Spin Waves in a One-Dimensional Spinor Bose Gas

J. N. Fuchs,[1] D. M. Gangardt,[2] T. Keilmann,[3] and G. V. Shlyapnikov[2,4]

[1]*Laboratoire de Physique des Solides, Université Paris-Sud, bât. 510, F-91405 Orsay, France*
[2]*Laboratoire de Physique Théorique et Modèles Statistiques, Université Paris-Sud, bât. 100, F-91405 Orsay, France*
[3]*Max-Planck-Institut für Quantenoptik, Hans-Kopfermann Str. 1, D-85748 Garching, Germany*
[4]*Van der Waals-Zeeman Institute, University of Amsterdam, Valckenierstraat 65/67, 1018 XE Amsterdam, The Netherlands*

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We study a one-dimensional (iso)spin 1/2 Bose gas with repulsive δ-function interaction by the Bethe Ansatz method and discuss the excitations above the polarized ground state. In addition to phonons the system features spin waves with a quadratic dispersion. We compute analytically and numerically the effective mass of the spin wave and show that the spin transport is greatly suppressed in the strong coupling regime, where the isospin-density (or "spin-charge") separation is maximal. Using a hydrodynamic approach, we study spin excitations in a harmonically trapped system and discuss prospects for future studies of two-component ultracold atomic gases.

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Recent experiments have shown the possibility of studying ultracold atomic gases confined in very elongated traps [1–4]. In such geometries, the gas behaves kinematically as if it were truly one dimensional (1D). Many theoretical studies [5–10] have predicted and discussed interesting effects in 1D Bose gases, such as the occurrence of fermionization in the strong coupling Tonks-Girardeau (TG) regime, where elementary excitations are expected to be similar to those of a noninteracting 1D Fermi gas [5]. Manifestations of strong interactions have been found in the experiments [2], and recently the TG regime has been achieved for bosons in an optical lattice [3] and in the gas phase [4].

Present facilities allow one to create spinor Bose gases which has been demonstrated in experimental studies of two-component Bose-Einstein condensates [11]. These systems are produced by simultaneously trapping atoms in two internal states, which can be referred to as (iso)spin 1/2 states. Relative spatial oscillations of the two components can be viewed as spin waves [12] [see [13] for review]. A variety of interesting spin-related effects such as phase separation [14], exotic ground states [15], and counterintuitive spin dynamics [12] due to the exchange mean field, have been studied both theoretically and experimentally. However, most of these studies are restricted to the weakly interacting Gross-Pitaevskii (GP) regime. There is a fundamental question to what extent these effects survive in the strongly correlated regime characteristic of one spatial dimension. The purpose of the present Letter is to study spin excitations of an interacting 1D spinor Bose gas. This is done using an exact solution by the Bethe Ansatz.

We start with a spinor (two-component) gas of N bosons with mass m at zero temperature, interacting with each other via a repulsive short-range potential in a narrow three-dimensional waveguide. In general, the interaction depends on the internal (spin) states of the colliding particles. Here we consider the case of a spin-independent interaction characterized by a single 3D scattering length $a > 0$. This is a reasonable approximation for the commonly used internal levels of ^{87}Rb [see, e.g., [11]]. The waveguide has length L and we assume periodic boundary conditions for simplicity. The transverse confinement is due to a harmonic trapping potential of frequency ω_0. When the chemical potential of the gas is much smaller than $\hbar\omega_0$, the transverse motion is frozen to zero point oscillations with amplitude $l_0 = \sqrt{\hbar/m\omega_0}$. In such a quasi-1D geometry, the interaction between atoms is characterized by an effective one-dimensional delta potential $g\delta(x)$. For $a \ll l_0$, the coupling constant g is related to the 3D scattering length as $g = 2\hbar^2 a/m l_0^2 > 0$ [8]. The behavior of the system depends crucially on the dimensionless parameter $\gamma = mg/\hbar^2 n$, where $n = N/L$ is the 1D density. For $\gamma \ll 1$ one has the weak coupling GP regime, whereas for $\gamma \gg 1$ the gas enters the strongly interacting TG regime.

Under the above conditions, the system is governed by the following spin-independent 1D total Hamiltonian:

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2} + g\sum_{i<j}\delta(x_i - x_j). \qquad (1)$$

This Hamiltonian was introduced by Lieb and Liniger [6] for describing spinless bosons, and their solution by the Bethe Ansatz (BA) has been generalized to bosons or fermions in two internal states by Gaudin and Yang [16,17]. In the case of a two-component Bose gas (spin 1/2 bosons), due to the $SU(2)$ symmetry of the Hamiltonian the eigenstates are classified according to their total (iso)spin S ranging from 0 to $N/2$. In this case, which was recently considered by Li, Gu, Yang, and Eckern [18], the ground state is fully polarized ($S = N/2$) and has $(2S + 1)$-fold degeneracy, in agreement with a general theorem [19,20]. At a fixed $S = N/2$, the system is described by the Lieb-Liniger (LL) model [6], for which elementary excitations have been studied in Ref. [7]

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for any value of the interaction constant. Spin excitations above the ground state are independent of the ground-state spin projection M_S and represent transverse spin waves. For $M_S = 0$ they correspond to relative oscillations of the two gas components.

We first give a brief summary of the BA diagonalization [16–18] of the Hamiltonian (1). An eigenstate with total spin $S = N/2 - K$ $(0 \leq K \leq N/2)$ is characterized by two sets of quantum numbers: N density quantum numbers I_j with $j = 1, \ldots, N$ and K spin quantum numbers J_μ with $\mu = 1, \ldots, K$. If $N - K$ is odd (resp. even), I_j and J_μ are integers (resp. half integers). These quantum numbers define N quasimomenta k_j and K spin rapidities λ_μ, which satisfy the following set of BA equations (we set $\hbar = 2m = 1$):

$$\frac{Lk_j}{2} = \pi I_j - \sum_{l=1}^{N} \arctan\left(\frac{k_j - k_l}{g/2}\right) + \sum_{\nu=1}^{K} \arctan\left(\frac{k_j - \lambda_\nu}{g/4}\right), \tag{2}$$

$$\pi J_\mu = \sum_{l=1}^{N} \arctan\left(\frac{\lambda_\mu - k_l}{g/4}\right) - \sum_{\nu=1}^{K} \arctan\left(\frac{\lambda_\mu - \lambda_\nu}{g/2}\right). \tag{3}$$

The energy of the corresponding state is $E = \sum_j k_j^2$, and its momentum is given by:

$$p = \sum_{j=1}^{N} k_j = \frac{2\pi}{L}\left(\sum_{j=1}^{N} I_j - \sum_{\mu=1}^{K} J_\mu\right). \tag{4}$$

As we are also interested in finite size effects, we do not take the thermodynamic limit at this point.

The ground state corresponds to the quantum numbers $\{I_j^0\} = \{-(N-1)/2, \ldots, (N-1)/2\}$ and $K = 0$, which shows that the BA equations reduce to those of LL [6]. The wave function is given by the orbital wave function of the LL ground state multiplied by a fully polarized spin-wave function. All ground-state orbital properties (energy, chemical potential, correlation functions, etc.) are therefore identical to those of the LL model. Elementary excitations in the density sector correspond to modifying the density quantum numbers I_j while leaving the total spin unchanged, i.e., $K = 0$. At low energy, the density excitations are phonons propagating with the Bogoliubov sound velocity $v_s = \sqrt{2gn}$ in the GP limit and with the Fermi velocity $v_s = 2\pi n$ in the TG regime.

We now focus on the spin sector. Elementary spin excitations correspond to reversing one spin ($K = 1$), and the total spin changes from $N/2$ to $N/2 - 1$. Thus, we have a single spin rapidity λ and the corresponding quantum number J. In general, this procedure creates a density excitation and a spin wave (isospinon) [18]. Here, we choose specific quantum numbers I_j, J in order to excite the isospinon alone [21]. Accordingly, the momentum p of the excitation is

$$p = \frac{2\pi}{L}\left(\frac{N}{2} - J\right), \tag{5}$$

which follows from the definition (4).

In the long wavelength limit, where $|p| \ll n$, due to the $SU(2)$ symmetry one expects [22] a quadratic dispersion for the spin-wave excitations above the ferromagnetic ground state:

$$\varepsilon_p \equiv E(p) - E_0 \simeq p^2/2m^*, \tag{6}$$

where $E(p)$ is the energy of the system in the presence of a spin wave with momentum p, E_0 is the ground-state energy, and m^* is an effective mass (or inverse spin stiffness). This quadratic behavior is due to a vanishing inverse spin susceptibility, which is a consequence of the $SU(2)$ symmetry [22]. A variational calculation in the spirit of Feynman's single mode approximation [20] shows that $\varepsilon_p \leq p^2/2m$ implying that $m^* \geq m$. Below we show that strong interactions greatly enhance the effective mass.

In the strong coupling limit it is possible to solve the BA Eqs. (2) and (3) perturbatively in $1/\gamma$ [6]. We solve these equations both for the ground state $\{I_j^0\}$ and the excited state $\{I_j; J\}$. We anticipate that in the limit of strong interactions, for small momenta ($|p|/n \ll 1$) and a large number of particles ($N \gg 1$), the dimensionless spin rapidity is $\tilde{\lambda} \equiv 2\lambda/g \gg 1$ and the dimensionless quasimomenta are $|k_j|/g \ll 1$. This allows us to expand Eqs. (2) and (3) to first order in $1/\gamma$ and $1/N$. The ground-state quasimomenta are then given by:

$$k_j^0 L = 2\pi I_j^0 (1 - 2/\gamma). \tag{7}$$

Here we used the relation $\sum_l \arctan[2(k_j^0 - k_l^0)/g] \simeq 2Nk_j^0/g - 2(\sum_l k_l^0)/g = 2Nk_j^0/g$, which is a consequence of the vanishing ground-state momentum. Similarly, the excited state quasimomenta obey the equations:

$$k_j L = \left(1 - \frac{2}{\gamma}\right) 2\pi I_j + \frac{2pL}{N\gamma} - \pi + \frac{1}{\tilde{\lambda}}\left(1 + \frac{k_j L}{\gamma N \tilde{\lambda}}\right), \tag{8}$$

where p is given by Eq. (4). Neglecting quasimomenta k_l in the argument of arctangent in the BA Eq. (3), we obtain the excited state spin rapidity:

$$2\pi J = 2N \arctan(2\tilde{\lambda}) \simeq \pi N - N/\tilde{\lambda}. \tag{9}$$

Equations (5) and (9) then give:

$$\tilde{\lambda} = N/pL, \tag{10}$$

which justifies that $\tilde{\lambda} \gg 1$ for $|p|/n \ll 1$. Combining this result with Eq. (8) shows that $|k_j|/g \ll 1$, as anticipated. Let us now define the shift of the quasimomenta $\Delta k_j \equiv k_j - k_j^0$. Taking the difference between Eqs. (8) and (7), we find:

$$\Delta k_j = \frac{1}{L\tilde{\lambda}} + \frac{k_j^0}{\gamma N \tilde{\lambda}^2} + \frac{2p}{\gamma N} - \frac{2\pi}{L\gamma}, \tag{11}$$

where we used that $I_j - I_j^0 = 1/2$. We can now compute the energy of the spin wave, as defined in Eq. (6):

$$\varepsilon_p = \sum_{j=1}^{N} [2k_j^0 \Delta k_j + (\Delta k_j)^2]. \quad (12)$$

Using Eq. (11) for Δk_j and Eq. (10) for $\tilde{\lambda}$ gives $\varepsilon_p = p^2(1/N + 2\pi^2/3\gamma)$. Note that the last two terms in the right-hand side of Eq. (11) give no contribution, as the ground-state momentum is zero. According to the definition (6), the inverse effective mass is therefore:

$$\frac{m}{m^*} = \frac{1}{N} + \frac{2\pi^2}{3\gamma}, \quad (13)$$

where we restored the units. Remarkably, the effective mass reaches the total mass Nm for $\gamma \to \infty$: the bosons are impenetrable and therefore a down spin boson can move on a ring only if all other bosons move as well.

In the opposite limit of weak interactions it is possible to compute the effective mass from the Bogoliubov approach [23]. The validity of this procedure when considering a 1D Bose gas, i.e., in the absence of a true Bose-Einstein condensate, is justified in [24]. The Hamiltonian of the system can be written as $H_0 + H_{\text{int}}$, where H_0 is the Hamiltonian of free Bogoliubov quasiparticles and free spin waves:

$$H_0 = \sum_p \epsilon_p \alpha_p^\dagger \alpha_p + \sum_p e_p \beta_p^\dagger \beta_p, \quad (14)$$

with α_p, β_p being the Bogoliubov quasiparticle and the spin-wave field operators, $\epsilon_p = \sqrt{e_p(e_p + 2gn)}$ the Bogoliubov spectrum, and $e_p = p^2/2m$ the spectrum of free spin waves [25]. The Hamiltonian H_{int} describes the interaction between Bogoliubov quasiparticles and spin waves and provides corrections to the dispersion relations ϵ_p and e_p. The most important part of H_{int} reads:

$$H_{\text{int}} = g\sqrt{\frac{n}{L}} \sum_{k,q\neq 0} (u_q \alpha_q^\dagger - v_q \alpha_{-q}) \beta_{k-q}^\dagger \beta_k + \text{H.c.}, \quad (15)$$

where u_q and v_q are the u, v Bogoliubov coefficients satisfying the relations $u_q + v_q = \sqrt{\epsilon_q/e_q}$ and $u_q - v_q = \sqrt{e_q/\epsilon_q}$ [23]. Neglected terms contribute only to higher orders in the coupling constant. To second order in perturbation theory, in the thermodynamic limit the presence of a spin wave changes the energy of the system by:

$$\Delta E(p) = e_p + \frac{g^2 n}{2\pi\hbar} \int dq \frac{e_q}{\epsilon_q} \frac{1}{e_p - [\epsilon_q + e_{p+q}]}. \quad (16)$$

In order to calculate a correction to the effective mass of the spin wave, we expand Eq. (16) in the limit of $p \to 0$. Terms which do not depend on p modify the ground-state energy, linear terms vanish, and quadratic terms modify the spin-wave spectrum as follows:

$$\varepsilon_p = e_p \left(1 - \frac{4g^2 n}{\pi\hbar} \int_0^\infty dq \frac{e_q}{\epsilon_q} \frac{e_q}{[\epsilon_q + e_q]^3}\right), \quad (17)$$

where the main contribution to the integral comes from momenta $q \sim \sqrt{mgn}$. Using the definition (6), we then obtain the inverse effective mass:

$$\frac{m}{m^*} = 1 - \frac{2\sqrt{\gamma}}{\pi} \int_0^\infty dx \frac{(\sqrt{1+x^2} - x)^3}{\sqrt{1+x^2}} = 1 - \frac{2\sqrt{\gamma}}{3\pi}, \quad (18)$$

which clearly shows nonanalytical corrections to the bare mass due to correlations between particles. This result can also be obtained directly from the BA equations.

For intermediate couplings, we obtained the effective mass by numerically solving the BA Eqs. (2) and (3). Our results are displayed in Fig. 1. Note that when solving the BA equations, one should take care of choosing $N^{-2} \ll \gamma \ll N^2$. Indeed, if $\gamma < N^{-2}$, the potential energy per particle in the weak coupling limit is lower than the zero point kinetic energy $\hbar^2/mL^2$ and the gas is therefore noninteracting (effectively $\gamma = 0$). In the strong coupling limit and for the same reason, if $\gamma > N^2$, the system behaves as a TG gas (effectively $\gamma = \infty$).

We now turn to harmonically trapped bosons in the TG regime and rely on spin hydrodynamics introduced for uniform systems [22]. As the ground state is fully polarized we assume the equilibrium (longitudinal) spin density $\vec{S}(x) = n(x)\hat{e}_3$ and study small transverse spin density fluctuations $\delta\vec{S}(x,t) = \delta S_1 \hat{e}_1 + \delta S_2 \hat{e}_2$, where $\hat{e}_1, \hat{e}_2, \hat{e}_3$ form an orthonormal basis in the spin space. For a large N, the equilibrium density profile $n(x)$ in a harmonic trapping potential $V(x) = m\omega^2 x^2/2$ is given by the Thomas-Fermi expression

$$n(x) = n_0 \sqrt{1 - (x/R)^2}. \quad (19)$$

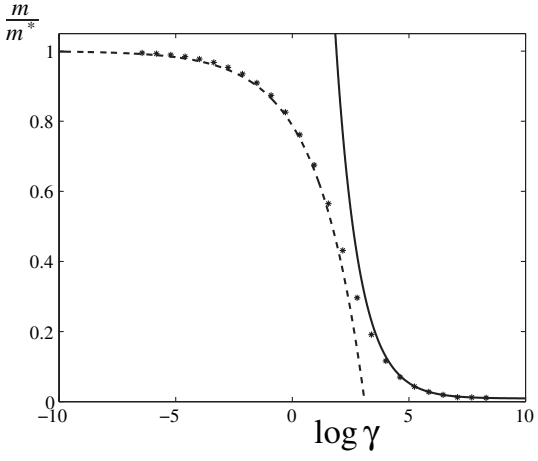

FIG. 1. Inverse effective mass m/m^* as a function of the dimensionless coupling constant γ (logarithmic scale). The stars (*) show numerical results for $N = 111$ particles, the solid curve represents the behavior in the strong coupling limit [Eq. (13)], and the dashed curve the behavior for a weak coupling [Eq. (18)].

Here $n_0 = n(0)$ is the density in the center of the trap and $R = \sqrt{2\hbar N/m\omega}$ is the Thomas-Fermi radius. For a strong but finite coupling Eq. (19) represents the leading term, with corrections proportional to inverse powers of $\gamma_0 = mg/\hbar^2 n_0$. The spin density fluctuations $\delta\vec{S}$ obey the following linearized Landau-Lifshitz equations [22]:

$$\delta\dot{S}_{1,2} = \mp\frac{\hbar}{2}\,\partial_x \frac{n(x)}{m^*(x)}\,\partial_x \frac{\delta S_{2,1}}{n(x)}. \tag{20}$$

In the TG regime the effective mass entering the equation of motion (20) depends on the density profile $n(x)$ as

$$m^*(x)/m \approx 3\gamma(x)/2\pi^2 = 3mg/2\pi^2\hbar^2 n(x). \tag{21}$$

Using the density profile (19) and introducing a complex function

$$n(x)\Phi(x,t) = \delta S_1(x,t) + i\delta S_2(x,t), \tag{22}$$

one obtains from Eq. (20):

$$i\dot{\Phi} = \Omega\Phi = -\frac{\pi^2}{6}\,\frac{\omega}{\gamma_0 N}\,\frac{1}{\sqrt{1-X^2}}\,\partial_X(1-X^2)\partial_X\Phi, \tag{23}$$

where $X = x/R$ is the dimensionless coordinate, and we assumed the stationary time dependence $\Phi(X,t) = e^{-i\Omega t}\Phi(X)$. Equation (23) shows that the typical frequency scale of the isospin excitations is given by $\omega/\gamma_0 N$, which is smaller than the scale ω of acoustic frequencies by a large factor $\gamma_0 N$. The exact solution to this equation was obtained numerically using the shooting method, and the spectrum shows only a small difference from the semiclassical result

$$\Omega_j = \frac{A\omega}{\gamma_0 N}\left(j+\frac{1}{2}\right)^2, \qquad j = 0, 1, 2, \ldots, \tag{24}$$

where the numerical factor is $A = \pi^5/48\Gamma^4(3/4) \approx 2.83$. For $\omega \sim 100$ Hz, $\gamma_0 \sim 10$ and $N \sim 100$ as in the experiment [4]; the lowest eigenfrequencies Ω_j are two or three orders of magnitude smaller than acoustic frequencies and are $\sim$0.1 Hz.

In conclusion, we have found extremely slow (iso)spin dynamics in the strong coupling TG regime, originating from a very large effective mass of spin waves. In an experiment with ultracold bosons, this should show up as a spectacular isospin-density separation: an initial wave packet splits into a fast acoustic wave traveling at the Fermi velocity and an extremely slow spin wave [26]. One can even think of ''freezing'' the spin transport, which in experiments with two-component 1D Bose gases will correspond to freezing relative oscillations of the two components.

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