Stable and unstable magnetic fields in stars
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Chapter 1

Introduction

1.1 Overview

The main theme of this thesis is the stability and instability of magnetic fields in stars. During the period in which the science of magnetohydrodynamics was being developed, around the middle of the twentieth century, magnetic fields were detected in Ap stars, the first fields which had been found in stars other than the Sun. This led inevitably to particular attention being given to these stars - they represented an opportunity to test the new science. The nature and origin of their magnetic fields has therefore become one of the most important topics in stellar magnetism, and it is for this reason that particular emphasis is given to them in this thesis. There are two competing explanations for their magnetism: the dynamo model and the fossil-field model; both were adapted to A-stars having been originally conceived to explain the solar field. In the dynamo model, the field observed on the surface is a manifestation of that created by dynamo action in the convective core, and in the fossil-field model, the field is left over from the star's formation. The latter obviously requires the field to be stable, and the lack of a known stable field configuration has been the main difficulty for this model. An arbitrary field configuration is generally unstable and will decay in a short time - the time taken by a magnetic (Alfvén) wave to cross the star - very short compared to the lifetime of a star. The stability of various fields has been examined, but even the most cursory of glances at the literature reveals, unfortunately, that it has been much easier to prove that a field is unstable than to show that a field configuration is stable.

While the debate over the origin of Ap-star magnetism was in full swing, magnetic fields of similar geometry and total flux were found in a number of White Dwarfs. This did a lot to strengthen the fossil-field model, as these stars do not contain any convective core. Cooler White Dwarfs do have a convective envelope, but a dynamo operating in this zone is not expected to be able to produce a field of the required strength and steadiness. Strictly speaking, of course, even if the field in a White Dwarf is a fossil, it does not mean that the field in an Ap star is also a fossil. However, the White Dwarf discovery did seem to give the fossil-field model the upper hand.

More recently, the fossil field hypothesis has been invoked to explain the properties of the so-called soft Gamma repeaters and anomalous X-ray pulsars. These objects,
which were discovered from their X-ray emission, are believed to be a class of neutron stars with a very strong (10^{15} gauss, or 10^{11} tesla) magnetic field: the magnetar model. A field of this strength would be strong enough to break the solid crust of the star—in contrast to the weaker field (10^{12} gauss, or 10^8 tesla) of a classical pulsar—and could not be held in place by the crust against its will. There is evidence that these objects retain their magnetic field for at least 10^4 years, and since an unstable field of this strength would decay on a time-scale of seconds, it seems very likely that the field resides in some kind of stable configuration.

In Sect. 1.2 I give a brief history of the observations of A-stars, as well as a summary of the theory of their magnetism. Sect. 1.3 is concerned with the instability of a toroidal magnetic field, and with an interesting consequence of this instability: a dynamo driven by differential rotation. In Sect. 1.4 an outline of the observational properties of the soft Gamma repeaters and anomalous X-ray pulsars is given, along with a description of the magnetar model. I then list the main results of this thesis in Sect. 1.5.

1.2 The magnetic A stars

A small percentage of A stars (or more precisely, stars of spectral types B8 to F0) are found to have a strong magnetic field at the surface.

1.2.1 Observations

The history of the Ap stars (meaning ‘peculiar A stars’) begins over a century ago. Maury (1897) noted that the spectrum of α^2 CVn (one of the brightest of this class, at magnitude 2.9) was peculiar, showing unusual weakness of the K line and strength of the Si II doublet at 4128Å. Variability of some of the lines was subsequently discovered and Belopolsky (1913) measured the changes in intensity and radial velocity of one of the lines (Eu at 4129Å), finding a period of 5.5 days. The photometric light curve was measured (Guthnick & Prager 1914) and similar behaviour was later found in other Ap stars (for instance Morgan 1933 and Deutsch 1947).

Only upon the discovery of variable magnetic fields (Babcock 1947) did any explanation of this interesting spectral behaviour become possible. It was found that Ap stars have an unusually strong magnetic field, with surface strengths ranging from a few hundred to a few tens of thousand gauss (0.03 to 3 tesla). The variability of the field can be most easily explained by imagining a static field not symmetrical about the rotation axis (the so-called oblique rotator model, first mentioned by Babcock (1949), although he immediately dismissed it): the spectral peculiarity is then taken to be a consequence of the effect the magnetic field has on the transport of chemical species.

Measurement of the magnetic field on these stars is made difficult by rotational and Doppler broadening of the spectral lines, and by the fact that the spectrum we see is the sum of the spectra from each point on the visible part of the surface, with
no way of separating them. However, it is possible, with the aid of polarimetry, to retrieve a small number of quantities from the spectrum, each of them some average over the visible disc of the star of some part of the magnetic field. The easiest to measure is called the longitudinal field, the line-of-sight component. If the star is rotating slowly (so that Doppler broadening is small) then it may be possible to see the lines separated into their Zeeman components, in which case one can measure the field modulus, an average of $|\mathbf{B}|$. By measuring such quantities as the star rotates, it is possible to reconstruct the field on the surface (at least, that part of the surface which is visible from the Earth), given some assumptions about the shape of the field. Having made, for instance, the assumption that the field consists of a dipole and quadrupole only, one can reconstruct observations as a function of the free parameters in the model, (i.e. dipole and quadrupole strength and orientation), looking through parameter space to find the point of least disagreement with observations.

Various models can be used and the typical result is that the field consists mainly of a dipole, with smaller but significant contributions from higher orders. [See for instance Landstreet & Mathys 2000, Bagnulo et al. 2002 and Gerth et al. 1997.]

A few properties of the Ap stars have yet to be explained:

- The Ap stars rotate in general much more slowly than the normal A stars: they have periods ranging from 1 day to over 10 years, compared to the typical A star with a period of between a few hours and 1 day (see Abt 1979, Wolff 1981 and Abt & Morrell 1995).

- A correlation has recently been discovered between the rotation period and the angle between the rotation and magnetic axes. It is found that in stars with periods of greater than 25 days, the two axes are close to each other; in stars with $P < 25$ days the distribution of angles is apparently random (Landstreet & Mathys 2000 and Bagnulo et al. 2002).

- It has been suggested recently that there is a correlation amongst A stars between magnetism and age (Hubrig et al. 2000a). It was found that no A-stars are magnetic during the first 30% of their main-sequence lifetime, so it seems that some small proportion of A stars must become observably magnetic only after a certain time, the others not becoming observably magnetic at all.

The last of these three observations connects nicely with the results obtained in Chapter 3. Unfortunately less can be said about the other two, as the effects of rotation have not yet been properly studied.

**1.2.2 The fossil-field and core-dynamo models**

There are two possible reasons why these stars have a strong magnetic field on the surface. Either the field is being continually regenerated by some process, possibly feeding off convection in the core (A-stars have a small convective core and a radiative
envelope) – the core-dynamo model, or the field was present when the star was born and has managed to survive ever since, the fossil-field model. The two models have been competing against each other ever since their conception over five decades ago. The main difficulties faced by the dynamo model are the failure to explain the absence of any strong correlation between rotation period and field strength, the lack of a model linking the field generated in the core to the field seen on the surface, and the failure to explain why only a small proportion of A stars are magnetic. The latter is also a problem in the fossil-field model, although this model seems to be gradually gaining the upper hand. The major difficulty has always been the lack of any equilibrium field which can be proven to be stable.

More strong evidence in favour of the fossil-field model comes in the form of magnetic White Dwarfs (see Putney 1999 and Wickramasinghe & Ferrario 2000 for recent reviews). Around 5% of isolated White Dwarfs are observed to be magnetic, with field strengths ranging from $3 \times 10^5$ up to $10^9$ gauss (30 to $10^5$ tesla). It is thought that these stars have evolved from Ap and Bp stars, the evidence including, for instance, their abundance and range of field strengths. If a main-sequence Ap star with a field of $3 \times 10^3$ G collapses to a White Dwarf, while conserving flux, the field will grow to around $3 \times 10^7$ G, which is a typical value observed. The same question of dynamo or fossil-field exists here, except that a White Dwarf contains no convective core; no known method exists to regenerate magnetic fields of the kind observed. This makes the fossil field model look enormously more likely.

Many other types of star are observed to have magnetic field which are ordered on a large scale. For instance, dipole-like fields have been measured on main-sequence B and O stars, (e.g. Henrichs et al. 2003) and it is also known that neutron stars have strong magnetic fields – any stable field could presumably also exist in these stars. In neutron stars, though, we have the added complication of a solid crust. More on neutron stars below in Sect. 1.4.

At first glance, it may seem remarkable that a magnetic field can exist for a long time in a star. To do this, it has to be stable, but it is sitting in a fluid which can move about in any direction, in contrast to a magnetic field in the laboratory where the boundaries are solid. Stability in the radial direction is provided by the stratification, but the Lorentz forces need also to be balanced in the horizontal directions.

There is another reason why one expects a stable field to be possible. Magnetic helicity, a quantity calculated by integrating the dot product of the field and its vector potential, is conserved, as long as the field resides in an environment free of diffusion. The principle of magnetic helicity conservation has been shown to be useful in many contexts, for instance in the solar corona (Zhang & Low 2003). We can expect reconnection to be relatively unimportant in a stellar interior, so the principle of magnetic helicity conservation should be applicable. An arbitrary initial field present in the interior of a star will in general be unstable, and will begin to decay on an Alfvén time-scale, its total energy falling. However, its magnetic helicity cannot change. This means that the field will eventually find its way into a configuration from which it cannot to decay further without its magnetic helicity falling, at which point it will have to stop decaying. The field will then be in stable
1.3 Magnetohydrodynamic instabilities

As discussed above, the stability of a magnetic field in a star is an interesting and important topic. Whether a field is stable or not depends on its geometry, so a good starting point is to look at the stability of fields of various geometries. The stability of many possible field configurations has been looked at, and particularly interesting is the stability of those fields which one might consider to be likely to exist. A simple field configuration which could be produced by the 'winding-up' of field lines in a differentially rotating star is the toroidal (azimuthal) field.

Such a field, described in cylindrical coordinates \((\varpi, \phi, z)\), has only an azimuthal component, so that \(\mathbf{B} = B\mathbf{e}_\phi\). It has been shown (for instance Tayler 1973 and Acheson 1978) that this toroidal field is unstable and will decay on a time-scale comparable to the time taken by an Alfvén wave to travel around the star on a field line. This is very short compared to the lifetime of a star, at only a few years, assuming the field strength is 1000 G. The instability is local in the meridional plane, meaning that the conditions for instability have only to be satisfied at one point in the \((\varpi, z)\) plane and an unstable eigenfunction can be fitted into this point in space; the instability can then grow. It is global in the azimuthal direction, typically only small wavenumbers \(m\) being unstable and, most importantly, the mode \(m = 1\). Fig. 2.1 shows the form of the instability – the gas is displaced mainly in the horizontal direction, since this avoids doing work against buoyancy forces, and the divergence of the displacement is zero, as this avoids doing work against pressure forces. Chapter 2 looks at the nature of this instability (called the ‘Tayler instability’) in some detail, the purpose being to verify the results obtained through analytic methods, and to check that no other stronger instabilities were missed, which might render this instability irrelevant.

1.3.1 A dynamo driven by differential rotation

The notion that a magnetic field may be generated by movement in a conducting body is not new; it was first applied, in the early part of the twentieth century, in

the astrophysical context to explain the magnetic fields observed in sunspots. It was suggested that the Sun’s magnetic field is generated in the convective envelope, and powered by the energy in the convection. The field is generated in the following way. A weak initial field is wound up by differential rotation into a toroidal configuration (since the field lines are ‘frozen’ into the plasma). The movement of the convective cells upwards and downwards bends the toroidal field lines, producing a new radial component. This radial component is then wound up by the differential rotation, closing the ‘dynamo loop’.

This type of dynamo has been the subject of extensive research over the last few decades, and perhaps because of this historical connection between magnetic field and convective zones, relatively little research has been carried out into dynamos operating in a non-convective region. Some kind of small-scale instability is required for a dynamo, which in the case of the dynamo described above comes from the convection itself, which imposes a small-scale velocity field on the plasma, in addition to the large-scale velocity field of the differential rotation. It is also possible to produce a dynamo by imposing just the large-scale velocity field, if the small-scale perturbations come from the magnetic field itself. This can happen if the magnetic field produced by winding-up from the large-scale field is unstable itself - there is no need for another instability to be imposed on the magnetic field. This principle was first demonstrated in the context of accretion discs, where a dynamo was produced when differential rotation wound up a field which was then subject to magnetohydrodynamic instability (Hawley et al. 1996). The same principle was applied by Spruit (2002) in the context of a differentially rotating star. In this scenario, a toroidal field is wound up by differential rotation from a weak seed field. The field remains predominantly toroidal, subject to decay by Tayler instability (described above), but is continuously regenerated by the winding-up of irregularities produced by the instability.

A dynamo powered by differential rotation could be important not just for the magnetic field of a star, but also for the rotation. Whilst a convection-powered dynamo is running off the luminosity of the star, the energy source in this dynamo is limited and can be used up entirely, killing the differential rotation. Indeed, fluid viscosity itself is far too small to have much effect on rotation within any time-scale of interest: a magnetic field looks to be the only plausible explanation for, for instance, the near-uniform rotation of the solar core (Schou et al. 1998 and Charbonneau et al. 1999).

The field strength resulting from such a dynamo process depends on the balance between the decay of the toroidal field and the winding up of irregularities, and there is some uncertainty as to its precise dependence on the parameters of the system. This, and a desire to know more about the potentially important role that angular momentum transfer resulting from this dynamo could have, are the motivation for Chapter 5 of this thesis.
1.4 Magnetars

The magnetar model was invented in an attempt to explain the behaviour of a small number of objects in our galaxy which emit X rays at high luminosity.

1.4.1 Observations

Four objects have been found which emit X rays at a luminosity of $10^{35} - 10^{36}$ erg s$^{-1}$ as well as temporally non-regular X-ray outbursts lasting from a fraction of a second to several minutes, but in total accounting for a similar luminosity to the continuous emission. This group has been given the name soft gamma repeaters (SGRs). A further six objects have been found which appear virtually identical except for the absence of these outbursts. They were first classified as a group by Mereghetti & Stella (1995). These have been called anomalous X-ray pulsars (AXPs), and are assumed to be dormant or extinct SGRs (Thompson & Duncan 1996, Mereghetti 2000 and Frail 1998).

The outbursts are extremely bright ($10^{12}$ erg s$^{-1}$) (Rothschild et al. 1994 and Hurley et al. 1999a) and therefore super-Eddington ($10^4 L_{\text{Edd}}$), and in two of these objects much brighter outbursts have been observed – in total $1 \times 10^{44}$ erg and $4 \times 10^{44}$ erg were released respectively (Cline 1982 and Mazets et al. 1999).

Regular variability in the X-ray flux from these objects has been detected, which can most easily be interpreted as the rotational period, $P$. The periods range from 5 to 12 seconds (e.g. Hurley et al. 1999b). It is possible to measure the rate of change of this period $\dot{P}$ and hence calculate a characteristic age $P/\dot{P}$. Typical values measured are $10^3$ to $10^5$ years (Kouveliotou et al. 1998 and Woods et al. 2000), the SGRs tending to have lower characteristic ages. These ages are much smaller than the typical values for classical pulsars, the oldest specimens of which have characteristic ages of around $10^{10}$ years.

Around half of these objects are observed in or near young supernova remnants (e.g. Kulkarni & Frail 1993 and Vasisht et al. 1994). This gives us a useful way to estimate their distance and age, since there are various ways to infer the age of an SNR. This has been done by, for instance, Parmar et al. (1998) and Helfand et al. (1994). This confirms that the characteristic ages do correspond at least vaguely to the real ages.

1.4.2 The magnetar model

Their association with supernova remnants immediately suggests that these objects are young neutron stars. Their high spin-down rates imply the presence of strong magnetic fields, with a dipole component of the order of $10^{15}$ gauss ($10^{11}$ tesla). [See, for example, Shapiro & Teukolsky (1983) p.277 for an explanation of the relevant physics.] A field of this strength would contain enough energy, around $10^{47}$ erg, to account for the observed X-ray luminosity for a period comparable to the characteristic age. The spin-down luminosity (the rate of change of rotational kinetic
energy), on the other hand, is at least one or two orders of magnitude smaller than the observed luminosity. This marks a fundamental difference from classical pulsars, in which the field causing spin-down is inferred to be much weaker (typically around $10^{12}$ gauss, or $10^8$ tesla) and whose luminosity can be accounted for with spin-down alone.

The model according to which SGRs and AXPs are neutron stars powered by the decay of a strong magnetic field is called the magnetar model, and was first proposed by Duncan & Thompson (1992). The energy in the magnetic field can be released slowly, heating up the interior of the star as well as the atmosphere: this accounts for the quiescent emission (Thompson & Duncan 1996). It can also be released quickly, if Lorentz forces are great enough to produce cracks in the solid crust of the star. If parts of the crust rotate, the field lines in the atmosphere will become twisted: this means that large currents will flow through the tenuous atmosphere. The resulting Ohmic heat will produce a fireball which we observe as an outburst of X rays (Thompson & Duncan 1995).

A dynamically unstable field of this strength will be strong enough to break the crust and will consequently decay on an Alfvén time-scale, which is of the order of a second or less. However, these fields are observed to persist in neutron stars for at least $10^4$ years. A key ingredient of the magnetar model, therefore, is a stable field configuration which can reside inside the star. The issue of what, if any, stable field can exist inside a neutron star is the same as that studied in the context of A stars and White Dwarfs, except for the presence of the solid crust, the cracking of which causes the outbursts observed. In Chapter 4 the evolution is studied of a stable magnetic field in a star with a solid crust, under the influence of diffusion.

1.5 Summary of main results

In this section I summarise the highlights of this thesis.

- Using numerical MHD simulations to follow the evolution of the Tayler instability (described above in Sect. 1.3), the predictions made from the analytic work concerning the conditions for instability in a toroidal field are largely confirmed. It is confirmed, for instance, that the instability conditions, which depend amongst other things on the form of the field, correspond to those predicted, and that the growth rate of the instability agrees with that predicted.

- An arbitrary initial field in the core of a star evolves, on an Alfvén time-scale, into a stable field of linked poloidal-toroidal torus form, either left-handed or right-handed. This appears to be the only stable field configuration which can exist in a stellar interior.

- This stable linked poloidal-toroidal field diffuses slowly outwards. During this time, the field strength on the surface of the star increases. In an A star, this is found to take place over a time-scale of the order of $2 \times 10^9$ years. This helps
to account for the observations of Hubrig et al. (2000a), which suggest that Ap stars become observably magnetic only after 30% of their main-sequence lifetime has passed.

- At some point, after the stable field has been diffusing outwards for a time, it loses its dipolar shape and begins to decay quickly, dying away completely.

- It is confirmed that a purely poloidal (meridional) field (a uniform field inside the star and a potential field outside it) is unstable and decays to zero on an Alfvén time-scale.

- An important principle in the evolution of magnetic fields in the stellar interior is that of magnetic helicity conservation – a field evolves, in a situation where reconnection can be ignored, into the lowest energy state with the same magnetic helicity. The resulting field is then stable. A field with zero magnetic helicity or a field with strong connection to the atmosphere (where reconnection cannot be ignored) does not find this stable state. This helps to explain the results listed above: the formation of the stable field in the stellar interior, the decay of that field when it becomes too strongly connected to the atmosphere, and the decay of the purely poloidal field.

- If a neutron star contains a stable linked poloidal-toroidal torus field at the time when the solid crust is formed, stress will build up in the crust, into which the field lines are frozen, while the field in the interior continues to evolve under the influence of diffusion. The stress patterns will produce rotational displacements of parts of the crust; this is consistent with the model of SGR outbursts developed by Thompson & Duncan (1995).

- A dynamo is constructed in the differentially rotating non-convective part of a star. Differential rotation winds up an existing magnetic field into a predominantly toroidal field, which is unstable and decays, creating as it does so a new poloidal component, which can itself be wound up by the differential rotation. Unlike a dynamo in a convective region, where hydrodynamic instability creates a small-scale velocity field, the instability comes from the magnetic field itself. In this sense, it is similar to the dynamo found in accretion discs.
Chapter 1 Introduction