Search for heavy resonances in the dimuon channel with the D0 detector
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Chapter 2

Theory

In this chapter I will describe the central ideas behind the Standard Model and various possible extensions. The Drell-Yan interaction, which is studied in the experimental part, is discussed in more detail.

Some familiarity with the subject matter is assumed. A detailed and thorough standard reference work is [1].

2.1 The Standard Model

The Standard Model is the relativistic quantum field theory (QFT) that predicts the strong, weak and electromagnetic interactions of elementary particles.

Relativistic quantum field theory arises from the combination of quantum mechanics, Lorentz invariance and the locality of the fundamental interactions.

The idea that nature is described by a relativistic QFT has many consequences beyond relativity and quantum mechanics, the most important of which may be that different elementary particles have to be exactly indistinguishable.

Another consequence is that the world is symmetric under combined charge conjugation, change of parity (left- or right handedness) and time reversal, called CPT invariance. For one thing, this implies the existence of an anti-particle field for each particle field which transforms under CPT.

The specific relativistic QFT that turns out to describe nature extremely well, called the Standard Model [2, 3, 4], is subject to other symmetry principles which will be discussed in the following sections. The other idea that is central to the Standard Model is that of renormalization and will be discussed afterwards.

2.1.1 Gauge symmetry

Since the gauge symmetries of the Standard Model determine its structure and predictions up to a finite number of coupling constants, it can be argued that they represent
the most fundamental aspect of the Standard Model. Indeed, the idea of local gauge symmetry has proven to be so powerful in the development of the Standard Model, that in addition most hypotheses beyond the Standard Model rely on symmetry arguments; some of these models will be discussed at the end of this chapter.

Local gauge symmetries are symmetries of the theory under continuous transformations of the fields which form a group and which are different at each point in space or moment in time. Gauge symmetries arise necessarily in quantum field theories that describe the dynamics of particles with integer spin, i.e., bosons. By a general argument [5], the joint requirements of Lorentz invariance and the ground state energy being bounded from below lead to the conclusion that a Lagrangian for \( N_V \) spin one fields \( A^a_{\mu} \) at high energies and momenta approaches the free vector field Lagrangian

\[
\mathcal{L}_V = \sum_{a=1}^{N_V} -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}, \quad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu
\]

Since the four-component vector fields \( A^a_\mu \) describe the dynamics of a (massless) spin one particle, which is specified by two possible polarizations\(^1\), there are many field degrees of freedom in this Lagrangian. However, the physical predictions of the theory should not depend on the superfluous degrees of freedom. The unique way to force this to be the case is to require that the theory is invariant under local redefinitions of the fields that can be written as

\[
A^a_\mu(x) \rightarrow A^a_\mu(x) + \partial_\mu \phi^a(x) + \ldots \quad (2.1)
\]

for arbitrary scalar fields \( \phi^1 \ldots N_V \) (where one allows for terms not contributing to the kinetic terms in \( \mathcal{L}_V \)), since then \( \mathcal{L}_V \) transforms as

\[
\mathcal{L}_V \rightarrow \mathcal{L}_V + \text{total derivative}
\]

Each gauge transformation must be an element of a \( N_V \)-dimensional continuous (Lie) group \( G \). For quantum electrodynamics (QED), the quantum version of classical electrodynamics, \( G \) is the Abelian Lie group \( U(1) \), and the additional terms in equation (2.1) vanish exactly. More generally however, and for the case of the Standard Model, \( G \) can be a non-Abelian Lie group and the gauge transformation is

\[
A^a_\mu \rightarrow A^a_\mu + \partial_\mu \phi^a - g f_{abc} \phi^b A^c_\mu
\]

where \( f_{abc} \) are the structure constants of the group in question and \( g \) is a freely adjustable coupling constant. To make sure the Lagrangian is still gauge invariant, the field strength \( F^a_{\mu\nu} \) gets an additional contribution \( g f_{abc} A^a_\mu A^b_\nu \). This is a self-interaction term, which is absent in the Abelian (electromagnetic) case. Gauge theories with non-Abelian gauge symmetry groups are called Yang-Mills theories.

\(^1\)Or it can be seen as the high energy and momentum limit of a massive vector field, which is (in its rest-frame) specified by three possible polarizations.
2.1. THE STANDARD MODEL

Formally, the other elementary particle fields $\psi_i(x)$ corresponding to the fermions and scalars in the theory can be written as (combinations of) irreducible representations of $G$, so that under an infinitesimal gauge transformation

$$\psi_i(x) \rightarrow \psi_i(x) + \sum_{a=1}^{N_V} i\phi^a(x)T^a\psi_i(x) + O(\phi)^2$$

(2.2)

where the $T^a$ are the $N_V$ infinitesimal generators of the gauge group, with $[T_a, T_b] = \sum_c ig f_{abc} T_c$. It is clear that not every possible term in the Lagrangian will be invariant under these transformations by itself.

If one starts with a Lagrangian $L_{\text{matter}}$ for these fields and requires that it becomes invariant under gauge transformations, in order to make the kinetic terms gauge invariant one is forced to introduce couplings to the $N_V$ vector fields in such a way that each space time derivative in $L_{\text{matter}}$ gets replaced by a gauge covariant gradient,

$$\partial_\mu \psi_i(x) \rightarrow D_\mu \psi_i(x) \equiv \left( \partial_\mu + \sum_{a=1}^{N_V} i g A_\mu^a(x) T^a \right) \psi_i(x)$$

In the general case (Abelian and non-Abelian), the vector fields $A_\mu^a$ must then transform under the gauge group as

$$A_\mu^a(x) \rightarrow A_\mu^a(x) - \frac{1}{g} \partial_\mu \phi^a(x) + \sum_{b,c} f_{abc} \phi^b(x) A_\mu^c(x)$$

The vector fields correspond to a special representation (called adjoint) related to the way the group acts on itself,

$$(A_\mu(x))_{bc} = A_\mu^a(x) (f_a)_{bc}$$

The vector bosons are representations of the gauge group on real vector spaces, whereas the fermions are representations on complex vector spaces. For any complex vector space there is a conjugate vector space related by the usual complex conjugation, and the representations on that space correspond with the anti-particle fields. Lorentz invariance then implies that for every interaction term its conjugate must be included as well, so that the sum is gauge invariant.

In this way, specifying the symmetry group and the representations corresponding to all the fields in the theory determines all the possible interactions in the massless theory.

We can now sketch how this works in the Standard Model. The non-Abelian Lie group that determines the gauge interactions of the Standard Model, up to coupling constants, is

$$\text{ISpin}(3,1) \times \text{U}(1) \times \text{SU}(2) \times \text{SU}(3)$$
The ISpin(3, 1) factor refers to the symmetries that are geometrical in nature, i.e., that come from the symmetries of space-time, the Lorentz group plus translations. It is therefore called the ‘external’ symmetry group. The fields transform under this group as vector (spin-1) fields or scalar (spin-0) fields in the case of bosons\(^2\), or bi-spinors for fermions. In the latter case, there are always two orthogonal representations, which differ only in the sign they have under the parity transformations (chirality) and are called ‘right-handed’ and ‘left-handed’.

The ‘internal’ gauge group \(U(1) \times SU(2) \times SU(3)\) is a compact Lie group, that is, a direct product of three simple compact Lie groups, which corresponds to rotations in complex vector spaces. The fields in the Lagrangian are written as products of the irreducible representations of these three groups (and of the external group). The SU(3) part corresponds with the strong interactions, and the \(U(1) \times SU(2)\) corresponds to the electromagnetic and weak nuclear interactions.

There is a slight twist in that the \(U(1)\) and SU(2) parts do not refer directly to the electric and weak parts respectively (in the following, \(U(1)_Y\) denotes the obvious subgroup). Instead, another \(U(1)\) subgroup of \(U(1)_Y \times SU(2)\) corresponds to the electromagnetic force (we will call this one \(U(1)_Q\)).

The electric charge \(Q\) is related to the charge under the product groups\(^3\) as \(Q = I_3 + Y/2\), where \(I_3\) is the charge under the ‘third’ component of the SU(2) group, acting by matrix multiplication on the defining representation as

\[
I_3 \equiv \frac{1}{2} \sigma_3 = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}
\]

(called \textit{weak isospin} for historical reasons) and \(Y\) is the charge under the \(U(1)_Y\) factor (called \textit{hypercharge}). In the same way, the adjoint representation of the ‘slanted’ subgroup \(U(1)_Q\) corresponds to the physical photon, with the adjoint representation of \(U(1)_Y\) and the third component of the adjoint representation of SU(2) mixing to give the neutral weak vector boson, and the other two components mixing to give the \(W^+\) and \(W^-\).

The electromagnetic and weak coupling strengths are therefore specified by a common coupling strength and a mixing angle. In this way the electromagnetic and weak forces are said to be \textit{unified}. (The idea of unification also arises as a motivation for physics beyond the Standard Model. This will be discussed in section 2.3.2).

The left-handed fermions (and right-handed anti-fermions) are combined in one representation, whereas the right-handed fermion fields are in separate representations.

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\(^2\)Graviton fields, which do not enter in the Standard Model but will be discussed later, are described by (spin-2) tensor fields

\(^3\)The irreducible representations are specified by \(N\) numbers, where \(N\) is the number of generators, which correspond to the charges of the field under the group. For instance, for every integer \(q\) there is a unitary irreducible representation \(\alpha\) of \(U(1)\) (which has a single generator) on \(\mathbb{C}\) given by

\[
\alpha \phi = e^{iq\theta} \phi, \quad \phi \in \mathbb{C}, \, \theta \in \mathbb{R}
\]

One can then define the self-adjoint operator \(Q \phi = -i(\alpha)\mid_{\theta=0} \phi = q \phi\). The observable corresponding to \(Q\) is the electric charge. Similar (but not necessarily commuting) quantities exist to specify the irreducible representations of the other groups.
2.1. THE STANDARD MODEL

<table>
<thead>
<tr>
<th>Electroweak</th>
<th>Strong</th>
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<tbody>
<tr>
<td>$\text{ISpin}(3,1)$</td>
<td>$U(1)_Y$</td>
</tr>
<tr>
<td>left-handed spin-$\frac{1}{2}$</td>
<td>$-1$</td>
</tr>
<tr>
<td>right-handed spin-$\frac{1}{2}$</td>
<td>$-2$</td>
</tr>
</tbody>
</table>

Leptons

$\begin{pmatrix} \nu_L \\ e_L \\ e_R \end{pmatrix}$

Quarks

$\begin{pmatrix} u_L \\ d_L \\ u_R \\ d_R \end{pmatrix}$

Gauge bosons

$\gamma^*/Z \left\{ \begin{array}{c} W_0 \\ (W_3, W_1, W_2) \end{array} \right\}$

Higgs

$\begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix}$

Table 2.1: Fermion fields in one generation, vector boson fields, their representations under the gauge group, as well as their charge under the unbroken $U(1)_Q$. Omitted are the anti-particles of the fermions and Higgs-boson. $su(2)\text{ etc.}$ indicate the adjoint representations.

that transform trivially under $SU(2)$. That is to say, an element of $SU(2)$ transforms a left-handed electron in a left-handed electron-neutrino and a left-handed up-type quark in a left-handed down-type quark, but does nothing on right-handed spinors. This means that the $W^+$ and $W^-$ only couple to left-handed fermions, so that the weak force breaks parity maximally.

Table 2.1 lists the force carriers and the fermions in one generation. There exist two more generations of fermions in the same representations, differing only by their masses (cf. table 2.2).

Also omitted is the right-handed neutrino, which was traditionally not included in the Standard Model because a right-handed neutrino with a mass similar to that of the left-handed neutrino has not been observed. Because of the observation of the mixing (see 2.1.4) of neutrinos, it is now thought that right-handed neutrinos may exist after all; however, this will not be discussed here.

There is also a spin-0 field which was not yet discussed. It plays a central role in the Standard Model, generating the masses of all elementary particles in the theory.
2.1.2 The Brout-Englert-Higgs mechanism

The conserved quantity corresponding to the $U(1)_Q$ subgroup is the electric charge, and we know from experiments that, indeed, it is conserved. However, the symmetry under the entire $U(1) \times SU(2)$ group does not obviously exist in nature. Since left-handed electrons and neutrinos are orthogonal components of the same representation of $SU(2)$, conservation of weak isospin would imply symmetry under the continuous mixing of left-handed electrons with neutrinos (and similarly for their anti-particles). However, in reality neutrinos and electrons are not exchangeable: electrons have electric charge while neutrinos are electrically neutral and their masses differ by, as far as is known today, more than 6 orders of magnitude.

There is another thing about the massless gauge theory which would seem to be in disagreement with observation. We know that some fields are associated with particle states that have a mass: the leptons, the quarks and the weak vector bosons. However, a mass term for the weak vector bosons is not invariant under (2.1).

In addition, the naive way of adding fermion masses to a theory with a non-Abelian symmetry, by simply putting a bilinear scalar term in the Lagrangian for the fermion fields, is made impossible because the two chiral fermion fields have different representations under $SU(2)$, viz., a right-handed fermion $SU(2)$ singlet must combine with a left-handed anti-fermion $SU(2)$ doublet and vice versa to form a scalar.

It can however be shown that if the masses are dynamically generated, they can be accommodated in the theory. The way this happens in the Standard Model is through the same mechanism that breaks the $U(1) \times SU(2)$ group down to its $U(1)_Q$ subgroup, and is called the Brout-Englert-Higgs mechanism after a number of its inventors [3, 4, 6, 7, 8, 9, 10, 11, 12].

The mechanism consists of introducing two new complex spin-0 (i.e., Lorentz scalar) fields which together form a defining (doublet) representation of $SU(2)$. It self-interacts according to a gauge invariant potential

$$V(\Phi) = \frac{1}{2} \mu^2 \Phi^\dagger \Phi + \frac{1}{4} \lambda (\Phi^\dagger \Phi)^2, \quad \mu^2 < 0, \quad \lambda > 0$$

(2.4)

with $\Phi$ as in table 2.1. This results in a non-zero vacuum expectation value

$$\langle \Phi \rangle = \left( \begin{array}{c} 0 \\ \sqrt{-\mu^2/\lambda} \end{array} \right) \text{ modulo } U(1)_Q$$

i.e., it is not locally symmetric under the entire $U(1)_Y \times SU(2)$ group, but it has a remaining redundancy in a phase which corresponds to the unbroken $U(1)_Q$ subgroup.

The inclusion of this Higgs field allows for a number of additional invariant terms in the Lagrangian. The leading fermion terms are the trilinear combinations

$$\psi_R \Phi \bar{\psi}_L + \text{conjugate}$$

(2.5)

which are gauge invariant, unlike the bilinear mass terms, because of the doublet representation of $\Phi$; such terms are called Yukawa interactions. Expanding $\Phi = \begin{pmatrix} v \\ H \end{pmatrix}$,
the constant vacuum expectation value \( v = 246 \text{ GeV} \) acts as a mass term, whereas the remaining degrees of freedom \( H \) correspond to a new predicted particle, the Higgs particle. The mass of the Higgs particle is an unknown (but constrained) parameter of the theory.

An important difference of the interaction of the Higgs field with Dirac fermions compared to the gauge interactions is thus that the latter preserve chirality, whereas the Higgs force changes a left-handed particle into a right-handed one, and the other way around. Therefore, if a particle exists only in one chirality, it does not couple to the Higgs field and does not acquire a mass. In the traditional Standard Model, this is the case for the neutrinos, which were thought to be massless and hence exist only in a left-handed variety.

The gauge fields which transform non-trivially under the broken part of \( U(1)_Y \times SU(2) \) interact with \( \Phi \) through the gauge covariant derivatives in its kinetic potential; these correspond to the weak vector bosons. Again, the masses are found by considering the terms proportional to powers of \( v \) only.

One also finds in this way how the remaining field \( H \) couples to the fermions and massive gauge bosons. The Higgs field is a singlet under the \( SU(3) \) gauge symmetry, i.e., it does not interact through the strong force.

This mechanism is usually called ‘symmetry breaking’, although it is more precise to say that the symmetry is \textit{hidden}, since the laws of physics still have the same underlying symmetry.

### 2.1.3 Renormalization

In the modern view, the Standard Model is not a ‘fundamental’ theory in the sense that it could describe all interactions at arbitrarily small distance scales or at arbitrarily large energies. Apart from the fact that this is not testable experimentally, there are several theoretical arguments which preclude this idea.

First, only the part of the Standard Model which describes the strong interaction, QCD, is known to be asymptotically free, i.e., the coupling constant \( \alpha_S \) approaches zero at small distances. Only asymptotically free theories can be useful to arbitrarily small distances scales, since at a sufficiently large energy scale, for theories that are not asymptotically free all interactions become strong. Secondly, at energies of the order of the Planck scale

\[
M_{\text{Planck}} \approx 1.2 \cdot 10^{19} \text{ GeV}/c^2
\]

gravity becomes as strong as the other forces and needs to be incorporated in the theory.

For these reasons the Standard Model is thought of as an \textit{effective} field theory, which describes the physics of a system only up until some large energy scale (or, equivalently, short distance scale) \( \Lambda \). At scale \( \Lambda \) the theory is defined by all interactions that are consistent with the symmetry group. Since, in general, an infinite number of such terms can be found, this only defines the theory up to an infinite number of coupling constants, which means such a theory is not in general predictive.
One can then relate the observables at the scale $\Lambda$ to the observables at some other scale $\mu$ by ‘renormalizing’ the interactions: starting with the Lagrangian specified at some energy scale $\Lambda$ by the symmetry group $G$ and coupling constants $g_i$ with mass dimension $\Delta_i$, one can express the coupling constants at the scale $\mu$ as a function of the coupling constants at the scale $\Lambda$, and $\Lambda$ and $\mu$ as

$$\mu^{-\Delta_i} g_i(\mu) = F_i(\Lambda^{-\Delta_i} g_i(\Lambda), \mu/\Lambda)$$

(2.6)

using the fact that the left-hand side is dimensionless. In general, the coupling strengths at the scale $\mu$ will depend on all coupling strengths at the scale $\Lambda$. However, there exist special theories for which the number of physically distinguishable terms stays finite and fixed. These are called renormalizable theories. It has been shown that the only predictive theories with vector bosons that have this property are gauge theories [13, 14].

The non-renormalizable terms in the original Lagrangian are suppressed with powers of $\mu/\Lambda$, so that at a scale $\mu \ll \Lambda$, these terms become vanishingly small and only the renormalizable interactions remain relevant [1]. In fact, this means that at any scale sufficiently smaller than $\Lambda$, only the renormalizable interactions are present in the theory, so that after we calculated the renormalization flow (or ‘running’) of the various constants in the theory, a finite number of measurements at a particular energy which constrain these constants are enough to make predictions at any other scale sufficiently below $\Lambda$, thereby retaining predictive power in this range.

An additional requirement is that the action of fields with energies $\sim \mu$ satisfies the same gauge symmetries as the original action. In general, a quantum field theory with a bare Lagrangian that obeys certain symmetries, can have those symmetries broken due to quantum effects, so-called anomalies.

In gauge theories with chiral fermions, such as the Standard Model, gauge anomalies arise through certain self-interactions of the vector fields mediated by chiral fermions. In order for the gauge theory to be renormalizable, the gauge anomalies must exactly vanish. Since left-handed and right-handed fermions contribute to the anomalies with opposite sign, this poses a constraint on the number of left- and right-handed fermions in the theory.

In the Standard Model, this leads to the requirement that each generation of elementary particles has the same number of quarks and leptons. Thereby anomaly cancellation ties together the different forces. In grand unified theories the anomaly cancellation follows from the fact that the leptons and quarks are combined in one representation of a larger gauge group; this will be discussed later on.

An intuitive physical picture of renormalization arises in perturbative calculations. Suppose one wants to calculate the measured charge $q$ of an electron at scale $\mu$. The increasing difference between $q(\mu)$ and $q(\Lambda)$ as $\mu$ decreases are due to the increasing number of virtual particle-anti-particle pairs in the vacuum being ‘seen’ over the relevant distance scales which screen or anti-screen the bare charge, i.e. $q(\mu) = q(\Lambda)/\epsilon(\mu)$, where $\epsilon(\mu)$ is the running ‘dielectric constant’ of the medium that is the sea of virtual particle-anti-particle pairs. The fact that the electromagnetic and weak forces are governed by one coupling constant at sufficiently small distance scales but
differ at longer distances arises from the fact that the ‘vacuum’ is polarized differently for them.

The fact that no processes are observed that, in the Standard Model, would be due to non-renormalizable interactions, can therefore be explained by the fundamental energy scale being very large compared to the energies at which we do measurements, so that any physics due to non-renormalizable interactions is highly suppressed. Therefore, in the calculations of observable quantities it suffices to take only the renormalizable interactions into account. The requirement of renormalizability is stringent enough that only a finite number of such interactions are allowed, and in addition gives rise to additional, ‘accidental’ symmetries, such as lepton and baryon number conservation (which may be broken explicitly in models with larger symmetry groups, discussed below).

Alternatively, one can make the additional assumption that the Lagrangian at the fundamental scale $\Lambda$ for some reason or another only contains the renormalizable terms to begin with. This means that the physics becomes insensitive to $\Lambda$ and that the absence of non-renormalizable interactions is exact. The accidental symmetries implied by renormalizability then also become exact [1].

In calculations, one basically attempts to calculate a quantity at some scale $\mu$ using the ‘bare’ Lagrangian at scale $\Lambda$, and subsequently calculates counter-terms to account for the difference with the renormalized couplings at scale $\mu$. Since the calculation is done only to some finite accuracy, usually to some finite order in the coupling constants, it can still depend on $\mu$. A conceptual problem arises in that both the calculated quantities and the counter-terms are sometimes infinite, so that regularization is needed to quantify these infinities during the calculation, finding that in the end the result is independent of the regularization procedure that was used. However, it is possible, if not practical or trivial, to calculate the renormalized terms in the perturbative expansion directly, without encountering any infinities [15].

### 2.1.4 Quark mixing

Another feature of the Standard Model is the fact that the fermions are reproduced among three generations, exactly equal except for the masses of the particles. It is not known whether this structure follows from any underlying symmetry principle; it is, for instance, possible to build a consistent Standard Model-like theory with two or four families$^4$.

Since the related fermions in the three generations are of the same representation of the gauge group, they are not necessarily independent and can be ‘mixed’: the mass eigenstate of a (free) particle is then a linear combination of the representations.

Specifically, in the Standard Model the weak representations of the quarks are mixed$^5$ through the Higgs mechanism. In effect, the trilinear terms (2.5) are replaced by a complex matrix

$^4$Although for more than 16 quark flavors the renormalization flow reverses sign and the strong sector is not anymore asymptotically free.

$^5$Lepton mixing will not be discussed here.
Table 2.2: Lepton masses (rounded off for typographical purposes; the relative error on electron and muon mass is $< 10^{-7}$) and weak vector boson mass, full width and muonic decay mode branching fraction [16].

\[
\begin{array}{ccc}
  m_e & m_\mu & m_\tau \\
  0.511 \text{ MeV}/c^2 & 105.6 \text{ MeV}/c^2 & 1.776 \pm 0.0002 \text{ GeV}/c^2 \\
  M_{W^\pm} & \Gamma_{W^\pm} & \Gamma_{W^\pm \to \mu^\pm \nu}/\Gamma_{\text{tot}} \\
  80.403 \pm 0.029 \text{ GeV}/c^2 & 2.141 \pm 0.041 \text{ GeV}/c^2 & (10.57 \pm 0.15) \times 10^{-2} \\
  M_Z & \Gamma_Z & \Gamma_{Z \to \mu^+ \mu^-}/\Gamma_{\text{tot}} \\
  91.1876 \pm 0.0021 \text{ GeV}/c^2 & 2.4952 \pm 0.0023 \text{ GeV}/c^2 & (3.366 \pm 0.007) \times 10^{-2}
\end{array}
\]

\[Y_{ij}^d \bar{q}_L \Phi d_{Rj} + Y_{ij}^d \bar{q}_L \Phi^* u_{Rj} + \text{conjugate}, \quad i, j = 1 \ldots 3\]

where the $i, j$ indices run over the generations, and $u$ and $d$ indicate the up- and down-type weak singlets. The physical eigenstates are found by diagonalizing the terms proportional to $v$,

\[M_{\text{diag}}^{u,d} = V_L^{u,d} V^{u,d\dagger} (v/\sqrt{2})\]

Since the weak interactions formally occur in the quark kinetic terms, which are diagonal with respect to the weak eigenstates, the weak interactions are modified to couple quarks of different generations. The unitary matrix $V_{CKM} = V^u_L V^d_R$ called the \textit{CKM} (Cabibbo-Kobayashi-Maskawa) \textit{matrix}, specifies (together with the gauge coupling strength $g$) the coupling strengths of the $W^\pm$. These are extra constants that enter the theory and are not determined through symmetry principles.

\section*{2.1.5 Phenomenology}

We have seen that the symmetries and renormalizability of the Standard Model dictate its structure up to a number of constants. Specifically, these are 19 parameters, which can be counted as the 3 coupling constants, the Higgs mass and self-coupling, the 3 lepton masses and 6 quark masses, and the 4 quark mixing angles, as well as a quantity $\theta_S$, related to the the topology of the QCD vacuum. The relative masses and widths of the gauge bosons are predicted by the Standard Model. While it is believed that the masses of baryons are predicted by QCD, they can not be calculated (not counting lattice calculations) and are determined experimentally.

Table 2.2 lists the lepton masses and vector boson properties.

The electromagnetic and weak nuclear forces are weakly interacting, in the sense that cross sections and other observables can be calculated to good accuracy by taking into account only the leading terms in a perturbative expansion in the coupling constants. A specific example of this will be presented in the next section.
The strong force is described by the unbroken SU(3) theory. At low energies, the strong force is strongly coupled. The result is that free quarks are not seen at lower energies, but only inside hadrons, because the force between two quarks increases with separation rather than decreases; this is called confinement. One important effect of confinement in collider physics is that whenever a light quark or gluon is produced in a hard collision, it will shower (by emitting gluons which decay into quark-anti-quark pairs, etc.) and form a narrow jet of colorless combinations (hadrons and mesons).

Conversely, at high energies, QCD approaches a free field theory. Highly energetic quarks and gluons inside colliding hadrons can scatter off each other like free particles. One way in which this happens in particular at proton-anti-proton colliders is through the Drell-Yan interaction, which is discussed in the next section.

2.2 The Drell-Yan interaction

I will now turn to a specific process that occurs in hadron colliders such as the Tevatron (see next chapter), namely, the Drell-Yan production of lepton pairs.[17] By this process, a quark of one (anti-)hadron and an anti-quark of the other hadron annihilate to produce a lepton and its anti-particle in a neutral current with a high invariant dilepton mass.

Such hard scatters are not at all the most prevalent events at a hadron collider. Most elastic collisions produce only particles with a low energy. However, for such processes long range effects are dominant, and the cross section becomes essentially incalculable in perturbation theory. Fortunately, the hard scatters are not only easier to detect and to calculate, but they also are the most likely place to find new physics (that is to say, physics not accommodated in the Standard Model). The hard scattering processes that occur most frequently are $2 \rightarrow 2$ QCD events. The Drell-Yan processes studied here have the additional experimental advantage of being relatively clean events with a clear signature of two high energetic muons, which leads to high detection probability which partly offset the relatively low production cross section. Thus, these type of processes are ideal for discovering new neutral currents.

To lowest order, the Drell-Yan process is described by the resonant production and decay of a neutral vector boson, a Z or virtual photon. It may receive additional contributions from new neutral vector bosons that are predicted by a slew of theories extending the Standard Model (see next section), the search for which are the subject of this thesis. The Drell-Yan type processes therefore describe both the production of the particles searched for and the most important, irreducible background to this search.

Most of the material in the following section was compiled from [18].

2.2.1 The Drell-Yan production of muon pairs

The leading order contribution to the Drell-Yan production of muon pairs is depicted in figure 2.1.
To a first approximation, at high $M^2$ the cross section is given by the lowest order electroweak $q\bar{q}\rightarrow l\bar{l}$ amplitude convoluted with the quark distributions in the proton and anti-protons. This is an immediate consequence of the asymptotic freedom of the strong force: at high energies, the quarks behave as free particles inside the proton and parton-parton interactions can be ignored.

Far below the $Z^0$-pole, the hard scattering sub-process cross section is given by

$$\hat{\sigma}(q(p_1)\bar{q}(p_2)\rightarrow \gamma^*\rightarrow \ell^+\ell^-) = \frac{4\pi\alpha^2}{3s} \frac{1}{N_c} Q_q^2$$

to lowest order in the fine structure constant $\alpha$, where $s = (p_1 + p_2)^2$ is the four-momentum of the virtual photon, which by momentum conservation equals the dilepton invariant mass squared, $N_c$ is the number of colors of the initial state quarks (i.e., three). The factor $1/N_c$ comes from the fact that the initial state quark and anti-quark must be of the same color, since the weak vector bosons are colorless.

The sub-process differential cross section $\frac{d\hat{\sigma}}{dM^2}$ for the production of a muon pair with mass $M$ is given by

$$\frac{d\hat{\sigma}}{dM^2} = \frac{\sigma_0}{N_c} Q_q^2 \delta(s - M^2), \quad \sigma_0 = \frac{4\pi\alpha^2}{3M^2}$$

The rapid falling off of the cross section with the invariant dilepton mass is the most striking (if not particularly exciting) phenomenological feature far away from the $Z$-pole. On the $Z$-pole,

$$\hat{\sigma} \propto \frac{s^2}{(s - M_Z)^2 + \Gamma_Z^2 M_Z^2} + O(s - M_Z)$$

i.e., a Breit-Wigner resonance shape with the narrow width of $\Gamma_Z$ (see table 2.2). The peak cross section is
\[ \hat{\sigma}(q\bar{q} \to Z \to \mu^+\mu^-) = \frac{\pi}{3} \sqrt{2} G_F M_Z^2 (V_q^2 + A_q^2) \]

with

\[ V_q = I_q^3 - 2Q_q \sin^2 \theta_W, \quad A_q = I_q^3 \]

(where \( I_q^3 \) is the charge of the quark under weak isospin (cf. equation 2.3) and \( \theta_W \) is the weak mixing angle), neglecting the width of the Z.

To obtain the leading Drell-Yan production cross section, the electroweak amplitude \( \hat{\sigma}(p_1, p_2) \) must be multiplied by the probability \( f_q(x_1) dx_1 \) of finding a quark with momentum \( p_1 = x_1 P_1 \) in one hadron and \( f_{\bar{q}}(x_2) dx_2 \) of an anti-quark with momentum \( p_2 = x_2 P_2 \) in the other hadron, where \( P_{1,2} \) are the total momenta of the incoming hadrons. Then we integrate over the momentum fractions and sum over the quark - anti-quark pairs.

\( Q \) is the characteristic scale of the hard scattering; this is usually taken to be the mass of the vector boson which mediates the scattering. The dependence on the ultraviolet (i.e. high-energy, short-range) renormalization scale \( \mu_R \) comes from the fact that \( \hat{\sigma} \) is calculated at some fixed order in the coupling parameters.

The factorization scale \( \mu_F \) is in some sense the infrared (i.e. low energy, long range) analogue of the renormalization scale \( \mu_R \), and in a perturbative calculation it is related to infrared divergences rather than ultraviolet divergences. Infrared divergences arise if one tries to calculate the contributions involving gluons emitted by the quark or anti-quark that are soft (low energy, hence long range) and collinear (emitted almost along side the quark). Because the partons are interacting strongly at low energies, perturbation theory can not be used to calculate such processes. The divergence of the perturbation series in the soft/collinear limit can be understood to come from the fact that in this limit, a vanishing amount of energy is required to emit an on-shell (since massless) gluon. On-shell propagators imply propagation over long distances, but at distances of the order of the hadron size, \( \sim 1 \text{ fm} \), non-perturbative confinement and hadronization takes place, which makes the apparent divergences disappear.

It would seem that perturbation theory is therefore useless to calculate processes that are sensitive to infrared physics. However, the situation is slightly better for infrared safe quantities, which are insensitive to long distance effects, and for factorizable quantities, of which the Drell-Yan cross section is an example. The latter have the property that the long distance effects can be absorbed in an overall non-perturbative factor, which can be determined experimentally.

Specifically, one can show that all infrared divergences that arise in the calculation of the subprocess cross section \( \hat{\sigma}_{q\bar{q} \to l^+l^-} \) can be absorbed in the parton distribution functions (PDF) \( f_q \), which can be determined from inelastic scattering experiments.
The idea is that the hadron structure as seen by an incoming particle is obtained from an unknown, ‘bare’ quark distribution, and the probabilities of this quark to interact with the other partons in the hadron, calculated perturbatively. The long-range effects, which cannot be calculated perturbatively, are absorbed in the ‘bare’ quark distribution above some scale \( \mu_F \). The factorization scale thus determines the scales at which one chooses to separate long-distance effects from short-distance effects. Measuring the hadron structure at \( \mu_F \) and comparing with the ‘renormalized’ quark distribution calculated to a particular accuracy gives enough information to obtain the hadron structure at any scale with that accuracy, analogously to the renormalization procedure in the Standard Model at large.

In practice, one often sets the renormalization and factorization scales equal.

In the approximation that the partons have vanishingly small transverse momentum relative to the direction of the parent hadron in the infinite momentum frame, the lepton and anti-lepton are produced exactly back-to-back in the plane transverse to the direction of the incoming proton. However, in reality the partons do have some finite distribution within the proton. Since the distribution of partons inside the proton cannot be calculated from first principles, it has to be determined experimentally. A measurement by the CFS collaboration from fixed-target \( pN \) collisions at \( \sqrt{s} = 27.4 \) GeV measured it to be Gaussian distributed with \( \langle k_T \rangle = 760 \) MeV [18]. Therefore the lepton - anti-lepton pair is on average not produced exactly back-to-back and the boson will have a, on average small, transverse boost.\(^6\)

Moreover, the inclusive \( q\bar{q} \rightarrow \gamma^*/Z + X \) process receives additional contributions of higher order in the coupling parameters, eg. from \( 2 \rightarrow 2 \) processes where the incoming partons scatter strongly, i.e. \( q + \bar{q} \rightarrow g + \gamma^*/Z \) and \( q + g \rightarrow q + \gamma^*/Z \). Such contributions enhance the cross section at high transverse vector boson momentum.

**K-factors**  A process such as the Drell-Yan process may be calculated to some finite order in a coupling constant. The orders are denoted by LO (leading order), NLO (next to leading order) and so on. Often, a process may be implemented to some order in a simulation, say LO, but be already known to a higher order, for instance NNLO. The predicted LO cross section from the simulation, with a parton distribution function measured with respect to a fixed order as well, can then be scaled to the cross section at the higher order, neglecting the kinematical details of the difference between the LO and NNLO predictions; the scale factor is called the “K-factor”. See section 4.2 for more details.

### 2.2.2 Experimental measurements

The Drell-Yan and Z boson cross sections have been measured by many experiments on different setups. For a 1993 overview, see [21].

\(^6\)Conversely, a measurement of the \( Z p_T \) distribution constrains the \( k_T \) distribution; in the analysis of this thesis, the corresponding parameters in the simulation of the Standard Model and new physics prediction are ‘tuned’ to match the measured boson \( p_T \) distribution in \( Z \rightarrow e^+e^- \) decays. See section 4.2 for details.
2.2. THE DY INTERACTION

Figure 2.2: DØ Run I result for the differential inclusive dielectron production cross section [19]. The 68% uncertainty intervals are shown for the data points. The last three bins, which have no events, show the 84% C.L. upper limit on the cross section corresponding to the upper end of the error bars in the preceding bins. Also shown is the prediction of the SM at NNLO, and SM + contact term process (this plot was published in a paper on the limit on leptoquark interactions), at LO corrected with a NNLO K factor.

Figure 2.3: CDF Run I Drell-Yan dimuon production cross section extracted from the combined 1992-1993 and 1994-1995 data [20]. The solid line is the NLL (next-to leading logarithmic) QCD prediction using the MRS (A) PDFs. The dashed line is the LO QCD prediction with a K factor to account for higher order effects, calculated with the CTEQ 3L PDFs. The dotted line is the NLL QCD MRS (A) prediction without the contribution from Z exchange.
At the Tevatron Run I (see next chapter) the first results were obtained on the Drell-Yan differential cross section at and above the Z-resonance in $p\bar{p}$ collisions at $\sqrt{s} = 1.8$ TeV. Figure 2.2 shows the DØ results in the electron channel, figure 2.3 shows the CDF results.

### 2.3 Extensions to the Standard Model

Even although, as to date, the Standard Model and its calculable derivatives predict correctly the result of many high energy experiments (a significant deviation being the discovery of neutrino mixing), and some to an unprecedented precision, there are physical reasons why we are certain that the Standard Model can not describe physics to arbitrarily small distance scales.

One is that it does not include gravity. Therefore, when the gravitational force between two particles becomes strong, the Standard Model breaks down. This happens at a comfortably small distance scale known as the Planck scale, corresponding to an energy $M_{\text{Planck}} \sim 10^{19}$ GeV.

Another physical reason to suspect the Standard Model can not be the complete is the gauge hierarchy problem, explained below.

Other reasons to look for extensions to the Standard Model are mentioned as well. Simply put, the fact that it describes such a multitude of observations, yet can be constructed from only a few principles, while at the same time being quite rigid in the kind of modifications that are allowed, suggests that the few aspects that do allow for arbitrary or not severely constrained modifications could in an extended theory also follow from underlying symmetries.

#### 2.3.1 Naturalness

An effective theory with respect to a scale $\Lambda$ is said to be ‘natural’ if the values of its free parameters are not sensitive to the physics on distance scales smaller than $\Lambda$. Unfortunately, the Standard Model ceases to be natural for the description of processes with energies larger than around 1 TeV [22, 23]. The reason is the large difference between the electroweak breaking scale and the Planck scale.

The electroweak breaking scale is of the order of $v \propto m_H(\mu)$, with $\mu \ll \Lambda$. $m_H(\mu)$ multiplies a renormalizable term in the effective Lagrangian, and depends in some complicated way on all fields of energies $\mu'$ with $\mu < \mu' < \Lambda$, but depends on the physics beyond $\Lambda$ only through the values of the couplings and masses $m_H(\Lambda)$, $\lambda(\Lambda)$, $g(\Lambda)$, $m_q(\Lambda)$, etc., up to effects of order $\Lambda^{-1}$. Similarly, the value of $m_H(\Lambda)$ only depends on the physics beyond $\Lambda$.

The leading corrections to $m_H$ (or in fact to the mass parameter of any scalar field) are of the form

$$m_H^2(\mu) = m_H^2(\Lambda) + \Lambda^2(c_1 \lambda(\Lambda) + \ldots)$$

Dividing (2.7) by $\Lambda^2$, we get
2.3. EXTENSIONS TO THE SM

\[ m_H^2(\mu)/\Lambda^2 = O(10^{-26}) = m_H^2(\Lambda)/\Lambda^2 + (c_1 \lambda(\Lambda) + \ldots). \]

The \( c_i \) depend on the physics between \( \mu \) and \( \Lambda \) in some complicated way, whereas the dimensionless term \( m_H^2(\Lambda)/\Lambda^2 \) depends only on the physics beyond the scale \( \Lambda \). The requirement that the first and the second term cancel to 26 orders of magnitude therefore amounts to a “conspiracy” between the low and high energy physics. This is deemed “unnatural”, because we expect the physics at large distances to follow from the physics at small distances, and not the other way around.

This does not mean that any small parameter is unnatural. If there exists some approximate symmetry, forming a group with all other symmetries in the theory, that becomes exact (at the quantum level) if some parameter \( \alpha \) approaches zero, all corrections to \( \alpha \) must necessarily be proportional to \( \alpha \) itself. In that case no cancellations are necessary (because there really is a “conspiracy”).

One could hope that there is such an approximate symmetry protecting \( m_H \). However, the only symmetry consistent with gauge symmetry is a symmetry under transformations

\[ \Phi(x) \rightarrow \Phi(x) + \Omega(x) \]

which would imply that the Higgs field is an unphysical degree of freedom, since a gauge can be chosen in which it vanished at each point in space-time. Therefore, the Standard Model with \( m_H \ll \Lambda \) is ‘unnatural’ and one speaks here of the gauge hierarchy problem. The unnaturalness of the Standard Model is therefore a hint that it will be modified at a scale that is much lower than \( M_{\text{Planck}} \), not more than an order of magnitude larger than \( v \) [23].

Experimental constraints favor a light Higgs mass, which leads to a relatively light scale of new physics. However, electroweak precision measurements match the Standard Model very well and therefore seem to favor a large scale for any new physics that modifies the relevant low-energy observables. Therefore, the new physics has to be weakly coupled and affect the low-energy observables only through radiative corrections.

An attractive way of modifying the Standard Model is by embedding the gauge group in one simple gauge group. However, in general these models introduce other gauge hierarchies. Such extensions are detailed in the next section.

Another theory, in which the gauge hierarchy arises naturally, is the Randall-Sundrum model, on which limits are presented in this thesis, and which is discussed at the end of this chapter. This model postulates an extra space dimension, where the apparent relative weakness of gravity is caused by the fact that it propagates in a larger space than the gauge fields.

In theories with super-symmetry an extra (hidden) symmetry between fermions and bosons forces the higher order contributions to \( m_H \) to vanish exactly (up to super-symmetry breaking terms), so that \( m_H(\mu) = m_H(\Lambda) \). While super-symmetry is the most popular extension of the Standard Model, it will not be discussed further because it is not directly relevant for the searches presented here.
Unification of the Couplings of the Electromagnetic, Weak and Strong Forces

Standard Model

Minimal Supersymmetric Model

Figure 2.4: Evolution of the inverse of the three coupling constants in the Standard Model, where $\alpha_1, \alpha_2, \alpha_3$ are the electromagnetic, weak and strong couplings respectively. The width of the lines represent the error in the coupling constants. The evolution of the couplings was calculated to second order [24].

In ‘little Higgs’ theories, also described in more detail later, the Higgs particle arises as a pseudo Nambu-Goldstone boson associated with a spontaneously broken global symmetry, so that its mass is protected and the hierarchy is no longer unnatural.

2.3.2 Extended Gauge groups

Motivation

The motivation for an extended gauge group is twofold.

First, if one follows the renormalization flow towards higher energies there is an energy $\Lambda_{\text{GUT}} \sim 10^{15}$ GeV at which all three effective coupling constants are nearly (but not quite) equal; see figure 2.4. That they do so rather than cross at entirely different scales, or not at all, can be seen as a hint that at $\Lambda_{\text{GUT}}$ the three forces combine into one force with one coupling strength, corresponding to one gauge group $G$ which is broken down to $U(1) \times SU(2) \times SU(3)$ below $\Lambda_{\text{GUT}}$, analogously to the way QED fits into the electroweak part of the Standard Model. Gravity becomes strongly coupled at the Planck energy, which is ‘only’ $2 - 3$ orders of magnitude larger than $\Lambda_{\text{GUT}}$.

Secondly, the way in which $U(1) \times SU(2) \times SU(3)$ can be embedded in larger simple groups provides additional relations between the different representations under the subgroups and can therefore be said to ‘explain’ these relations, which appear to be ‘coincidental’ in the Standard Model. I will sketch how this works.

$U(1) \times SU(2) \times SU(3)$ has a so-called ‘normal’ subgroup that acts trivially on all fields in the Standard Model. It is isomorphic to $\mathbb{Z}/6$ (the integers modulo 6), so that the smallest symmetry group of the Standard Model is in fact $U(1) \times SU(2) \times SU(3)/(\mathbb{Z}/6)$. This group has the nice property that it is in fact isomorphic to a subgroup of $SU(5)$ which has a representation as matrices that can be written as
where \( g \) is \( 3 \times 3 \) and \( h \) is \( 2 \times 2 \) (see, for instance, [25]). One might call it \( S(U(2) \times U(3)) \). This formulation suggests that the ‘true’ gauge group of the Standard Model is \( SU(5) \). In fact, it was shown by Georgi and Glashow [2] that \( SU(5) \) is the smallest suitable Lie group in which the Standard Model group fits. The only other group that allows complex representations, which are needed for chiral fermions, is \( SU(3) \times SU(3) \), but this group cannot accommodate representations with relatively ‘fractional’ charges, needed for the quarks to have multiples of \( 1/3 \) of the electron charge.

The Georgi-Glashow \( SU(5) \) model

While the simplest \( SU(5) \) model, the Georgi-Glashow model, is ruled out by experiment, it serves as a good example of a Grand Unified Theory (GUT). One can write down an \( SU(5) \) gauge theory in the manner indicated in the previous section. One 5 and one 10 dimensional chiral representation together with their duals suffice for all fermions and their anti-particles in one generation:

\[
(d^r, d^g, d^b, e^+, \tilde{\nu})_R, \quad \begin{pmatrix}
0 & \tilde{u}^r & u^r & d^r \\
-\tilde{u}^r & 0 & \tilde{u}^g & u^g & d^g \\
-\tilde{u}^g & -\tilde{u}^b & 0 & u^b & d^b \\
-u^r & -u^g & -u^b & 0 & e^+ \\
-d^r & -d^g & -d^b & -e^+ & 0
\end{pmatrix}_L
\]

Nice features of this theory are that it automatically gives an explanation for the fact that the quarks have an electric charge of \( 1/3 \) relative to the leptons, since this follows directly from the fact that the trace over the 5-dimensional representation must be zero. The theory is also anomaly-free, and does not need new fermions.

In addition to the Higgs field, one has to introduce extra scalar fields analogous to the Higgs field which acquire a vacuum expectation value that breaks \( SU(5) \rightarrow U(1) \times SU(2) \times SU(3)/(\mathbb{Z}/6) \) at some energy scale \( \Lambda_{GUT} \). The new gauge bosons, which correspond to the broken part of \( SU(5) \), thereby acquire a mass of the order of \( \Lambda_{GUT} \).

There will be \( 5^2 - 1 \) gauge bosons, \((3^2 - 1) + (2^2 - 1) + (1)\) of which are identified with the 8 gluons, 3 weak bosons and the photon respectively. The other half are new predicted gauge bosons which mediate diquark and lepton-quark interactions because they transform both under the adjoints of \( SU(2) \times U(1) \) and \( SU(3) \).

These kind of processes lead to baryon \((B)\) and lepton \((L)\) number violation, which were ‘accidental’ symmetries in the Standard Model, in the sense that they are broken only by non-renormalizable interactions. A consequence of this is that the \( SU(5) \) model predicts a finite proton lifetime of [26]
\[ \tau_p \sim \frac{1}{\alpha_{(5)}^2} \frac{m_X^4}{m_p^2} \]

where \( \alpha_{(5)} \) is the SU(5) coupling constant and \( m_X \) is the mass of the gauge bosons of the broken part, proportional to the SU(5) breaking scale. The experimental limit \( \tau_p \gtrsim 10^{33} \) years [16] gives \( m_X \gtrsim 10^{20} \) GeV. However, for reasons outlined above, the GUT scale should be smaller than the Planck scale \( M_{\text{Planck}} \approx 1.2 \cdot 10^{19} \) GeV.

The theory also predicts a value of the ratio of the electromagnetic and weak nuclear force. This is found by requiring that at or above the unification scale \( \Lambda_{\text{GUT}} \), the theory is described by one coupling, which fixes the relative strength. The ratio at a lower energy is then found by renormalizing the GUT values. It turns out that the predicted value is ruled out by (but quite close to) the measured one (\( \sin^2 \theta_W \approx 0.21 \) vs. the measured value of 0.23120(5)).

**Larger groups**

It is clear that the Georgi-Glashow model is therefore experimentally excluded. In addition, the theory still suffers from an explicit large hierarchy \( m_X \gtrsim 10^{12} m_W \), which means that this theory is unnatural, like the Standard Model itself. Moreover, it does not include a right-handed neutrino, the existence of which is indicated by the observation of neutrino mixing. Arguably, another deficit is that two representations are needed to fit all the particles, rather than one.

For these reasons, when conceiving of GUTs one is forced to consider larger gauge groups. These larger groups necessarily introduce representations with new fermions. Moreover, any such further enlarged gauge symmetry introduces additional *neutral* massive gauge bosons in addition to the Z.

For instance, the next larger interesting group is SO(10). There, all fermions of a generation plus a right-handed neutrino fit in one irreducible 16 dimensional chiral spinor representation of SO(10). Because of the right-handed neutrino, neutrino masses can be accommodated in this theory.

The new neutral gauge boson comes from the extra Abelian subgroup which fits into SO(10) besides the Standard Model group. This can be seen through the maximal subgroup decomposition \( \text{SO}(10) \supset \text{SU}(5) \times \text{U}(1). \) The new U(1) symmetry corresponds to the conservation of baryon number minus lepton number, \( B - L. \)

Even larger groups can be considered, which introduce more new gauge bosons and new ‘exotic’ fermions. For instance, super-string theories favor the exceptional group \( E_6 \) which has a maximal decomposition \( E_6 \supset \text{SO}(10) \times \text{U}(1). \) The breaking pattern

\[
E_6 \rightarrow \text{SO}(10) \times \text{U}(1)_{\psi} \rightarrow \text{SU}(5) \times \text{U}(1)_{\chi} \times \text{U}(1)_{\psi} \rightarrow \text{SU}(3) \times \text{SU}(2) \times \text{U}(1)_{Y} \times \text{U}(1)_{\chi} \times \text{U}(1)_{\psi}
\]

gives rise to a physical \( Z' \) which is a mixture of the two new U(1) gauge bosons,

\[
Z'(\theta) = Z_{\psi} \cos \theta' + Z_{\chi} \sin \theta'
\]
Commonly considered choices for the mixing angle $\theta'$ are $0$, $\arctan \sqrt{3/5}$ and $\pi/2$, called $Z_{\psi}$, $Z_{\eta}$ and $Z_{\chi}$ respectively. These models do require some extra fermions and need fine tuning to avoid the electroweak precision bounds.

In the research presented in this thesis, only the phenomenology of the extra neutral gauge bosons is studied, not the effects of possible extra fermions.

### 2.3.3 Generic U(1) extension

Motivated by the above, one can also look at generic extensions of the Standard Model group with a new neutral spin-1 (gauge) boson. The new particle is generally called $Z'$. Free parameters are the $Z'$ mass $M_{Z'}$, the angles which determine the mixing between the $Z'$ and the Standard Model $Z$, and the gauge couplings of the extended gauge group. The generic $Z'$ Lagrangian can then be written as [16]

$$
\mathcal{L}_{Z'} = -\frac{1}{4} F_{\mu\nu}' F'^{\mu\nu} + M_{Z'}^2 Z'^2 - \frac{e}{2c_W s_W} \sum_i \bar{\psi}_i \gamma^\mu (f_V^i - f_A^i \gamma^5) \psi_i Z'_\mu + \delta M_{ZZ'}^2 Z'_\mu Z_\mu - \frac{1}{2} \sin \chi F_{\mu\nu}' F'^{\mu\nu}
$$

where $F^{(i)}$, $M_{Z^{(i)}}$ are the $Z^{(i)}$ field strength and mass, $c_W$ and $s_W$ are the cosine and sine of the weak mixing angle. The mass terms are assumed to be generated by a symmetry breaking mechanism, as in the Standard Model, with the term proportional to $\delta M_{ZZ'}^2$ due to scalar field condensates that are charged under both groups.

The last (‘kinetic mixing”) term is only gauge invariant for the case that the extended gauge group is Abelian, since for non-Abelian extensions $F_{\mu\nu}'$ is not gauge invariant by itself. The effect of this term is to change the $Z - Z'$ mixing and the U(1)' charges of all fields [27]. However, in most analyses, as in the searches presented here, $\chi$ is taken to be zero, even for Abelian extensions.

The mixing angle between the $Z$ and $Z'$, $\theta_{Z-Z'}$, is related to the entries of the mass matrix as

$$
\tan 2\theta_{Z-Z'} = \frac{2\delta M_{ZZ'}^2}{M_{Z'}^2 - M_Z^2} + O(\chi)
$$

The $Z - Z'$ mixing has the effect of modifying $Z$-pole observables. Low-energy measurements and electroweak precision measurements (discussed below) severely constrain mixing to be very small.

The couplings to fermions are described by 15 new parameters in the theory corresponding with the charges $z_f$ under U(1)', however, the charges must obey certain relations so that the U(1)' anomalies cancel (or in the case there are extra exotic

---

*In theories with additional symmetries these parameters can be constrained, for instance in super-symmetric GUTs, super-gravity or theories motivated by string theory, but these will not be discussed here.*
fermions that the $U(1)'$ anomalies and the anomalies of the exotic fermions under the SM group cancel. Under mild assumptions it can then be shown that far above the $Z$ mass the cross section for $Z'$ production in $p\bar{p}$ collisions with a decay to two muons is given by [28]

$$\sigma(p\bar{p} \to Z'X \to \mu^+\mu^-X) = \frac{\pi}{488} \left[ c_u w_u(s, M_{Z'}^2) + c_d w_d(s, M_{Z'}^2) \right]$$

where $w_{u,d}$ are the parts of the hadronic structure function which do not depend on any coupling, and

$$c_{u,d} = g_z^2(z_q^2 + z_{u,d}^2) \times Br(Z' \to \mu^+\mu^-)$$

contain all dependence on the $Z'$ couplings to fermions, with $g_z$ being the $U(1)'$ gauge coupling and $z_f$ the fermion charges.

Many anomaly-free solutions for the $z_f$ exist. In [28], four classes of one-parameter solutions are found where the charge under the $U(1)'$ is generation-independent.

One solution is the $U(1)_{B-L}$ model, where the fermion charges are proportional to $B - L$, baryon number minus lepton number. This symmetry occurs in the left-right symmetric models, which requires a right-handed neutrino for each generation, whose existence is indicated by neutrino mixing. If extra fermions are allowed, a larger class of one-parameter solutions exists for which the fermion charges are proportional to $B - xL$ (where $x$ is an unconstrained rational number). For these models, the $Z - Z'$ mixing vanishes at tree-level, which is desirable because it avoids the constraints from electroweak precision data.

For another one-parameter set of solutions, $U(1)_{d^+xu}$, the fermion charges commute with the representations of the SU(5) group; it can arise from the breaking pattern $E_6 \to SU(5) \times U(1)_{\psi} \times U(1)_\chi$ mentioned above. The specific cases $U(1)_\chi$, $U(1)_{\psi}$ and $U(1)_\eta$ correspond to $x = -3, 1, -1/2$ respectively.

Another set of solutions is $U(1)_{d^+xu}$, where the left-handed weak quark doublets are neutral under the new group, but the right-handed quark singlets carry a charge, with the ratio of the $u_R$ charge to the $d_R$ charge given by $-x$.

Finally, the most general generation-independent solution with no extra fermions that are charged under the SM group is $U(1)_{q^+xu}$, where the charges are a linear combination of the hypercharge $Y$ and $B - L$.

A model that is often used as a ‘benchmark’ in searches is the so-called Standard Model-like $Z'$ model or sequential Standard Model (SSM) $Z'$, for which the couplings of the $Z'$ to the Standard Model fermions are identical to those of the $Z$ boson. This is an ad-hoc model; the extension is not, by it self, gauge invariant, and is used mainly for reasons of convenience. However, it can be used as a starting point for model-independent limit setting, which is what is done here.

---

8 There are models with extra dimensions in which a sequential $Z'$ arises as a resonance of the Standard Model $Z$ [29].
Table 2.3: Present 95% confidence limits on the mass of a hypothetical heavy neutral vector boson [16].

<table>
<thead>
<tr>
<th>Experiment/author</th>
<th>Type</th>
<th>limit on $M_{Z'SSM}$ (GeV/$c^2$)</th>
<th>limit on $M_{Z'}$ (GeV/$c^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDF [32]</td>
<td>$pp$ with $Z' \rightarrow e^+e^-$</td>
<td>&gt;850</td>
<td>&gt;740</td>
</tr>
<tr>
<td>D∅ [33]</td>
<td>$p\bar{p}$ with $Z' \rightarrow e^+e^-$</td>
<td>&gt;780</td>
<td>&gt;640</td>
</tr>
<tr>
<td>D∅ [34]</td>
<td>$p\bar{p}$ with $Z' \rightarrow \mu^+\mu^-$</td>
<td>&gt;680 (not given)</td>
<td>&gt;780</td>
</tr>
<tr>
<td>DELPHI [30]</td>
<td>$e^+e^-$ with $Z' \rightarrow f \bar{f}$</td>
<td>&gt;1305</td>
<td>&gt;545</td>
</tr>
<tr>
<td>OPAL [31]</td>
<td>$e^+e^-$ with $Z' \rightarrow f \bar{f}$</td>
<td>&gt;1018</td>
<td>(not given)</td>
</tr>
<tr>
<td>Cheung [35]</td>
<td>Combined electroweak precision</td>
<td>&gt;1500</td>
<td>&gt;781</td>
</tr>
</tbody>
</table>

2.3.4 Little Higgs

A special class of theories with larger symmetry groups are the little Higgs models [36]. Here, the symmetry groups and breaking pattern are chosen in a special way to protect the gauge hierarchy. In these models the Higgs particle is a pseudo-Nambu-Goldstone boson associated with a continuous global symmetry that is broken at a scale $\Lambda_S \sim 10-30$ TeV. Its mass is protected because it is proportional to the breaking scale of this global symmetry. The radiative corrections to $m_H$ by Standard Model
particles are cancelled by new particles with masses $\sim 1 - 3$ TeV that contribute with an opposite sign at the 1-loop level.

The “littlest” Higgs model [37] is the simplest example of such a theory. It has a continuous global symmetry group SU(5), and a $[SU(2) \times U(1)]_1 \times [SU(2) \times U(1)]_2$ gauge symmetry group which is a subgroup of the global symmetry group SU(5).

At the scale $\Lambda_S$, the SU(5) multiplet $\Sigma$ acquires a non-zero vacuum expectation value

$$\Sigma_0 \propto f \cdot \begin{pmatrix} I \\ 1 \\ I \end{pmatrix}$$

where $I$ is the $2 \times 2$ identity matrix. The vacuum expectation value $f$ is related to the breaking scale as $\Lambda_S \sim 4\pi f$. This breaks the global symmetry group SU(5) down to SO(5), which results in 14 new particle degrees of freedom. Under the SM electroweak group these transform as a real singlet, a real triplet, a complex doublet and a complex triplet.

The 4 degrees of freedom in the real singlet and the real triplet become the longitudinal components of the gauge bosons associated with the broken part of the full gauge group, which thereby acquire a mass $M \sim gf \sim 1 - 3$ TeV (where $g$ is the gauge coupling strength). This breaks the gauge group $[SU(2) \times U(1)]_1 \times [SU(2) \times U(1)]_2$ to the diagonal subgroup $SU(2)_L \times U(1)_Y$, which is the Standard Model electroweak gauge group.

The gauge and Yukawa couplings that break the SO(5) symmetry induce a potential of the form of equation (2.4) for the remaining pseudo-Nambu-Goldstone bosons. The neutral part of the complex doublet gets a non-zero vacuum expectation value $v$ which results in the electroweak symmetry breaking.

In addition, a new set of heavy fermions is introduced which have their couplings chosen such that their quadratic corrections to the Higgs mass cancel those of the top/anti-top quarks.

One (or in some little Higgs models, two) of the new heavy gauge bosons is a new neutral massive gauge boson, i.e., a $Z'$, associated with the extra SU(2) or U(1) gauge groups; there is also a $W'$, associated with the broken part of the subgroup. The $Z'$ couples left-handedly and universally to all Standard Model fermions with a coupling strength $g_V = -g_A \sim \frac{g}{2} \cot \theta T_3$, where $\theta$ is a mixing angle and $T_3$ is the charge under the new group. The production cross-section through a Drell-Yan process is then proportional to the partial width

$$\Gamma(qq' \rightarrow Z') = \frac{C}{12\pi} \left( g_V^2 + g_A^2 \right) M_{Z'} \propto \cot^2 \theta$$

Figure 2.5 shows the dependence of the branching ratios on the mixing angle [38]. For $\cot \theta \gtrsim 0.5$ the branching ratio to leptons is nearly independent of $\cot \theta$. For $\cot \theta \lesssim 0.5$ the BR to fermion-anti-fermion pairs drops sharply and other decay channels dominate.
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Figure 2.5: Branching ratios of the $Z'$ into SM particles in the littlest Higgs scenario, as a function of $\cot \theta$, neglecting final-state mass effects [38].

2.3.5 Extra dimensions

A different approach is taken by a number of models with extra (space) dimensions. The primary motivation for this approach is again to resolve or hide the gauge hierarchy.

The central idea is that gravity appears to be weak (i.e., the Planck mass appears to be large) because there exist one or more extra space dimensions in which only gravity propagates, i.e. gravity waves propagate in a $(3+n)$ dimensional space but the Standard Model fields are localized on a 3 dimensional surface in the full $3+n$ dimensional space$^9$.

In the models discussed here, the extra space dimensions are very small compared to the 4 normal dimensions, so that the effective strength of gravity at macroscopic scales can remain unmodified.

Large extra dimensions

The first proposal [39, 40] that exploits this effect, was the theory of large extra dimensions or the ADD-model$^{10}$. Here, the $n \geq 1$ extra dimensions are chosen to be compactified to a finite volume, and the 4 dimensional metric does not depend on the extra coordinates. This means the Einstein-Hilbert action in $4+n$ dimensions,

$$S(g_{4+n}) = \frac{1}{2} \tilde{M}_{4+n}^{2} \int d^{4}x \int d^{n}y \sqrt{-g_{4+n}} R(g_{4+n})$$

(where $\tilde{M}_{4+n}$ is the (reduced) $4+n$-dimensional Planck mass ($\tilde{M}_{4+n} = M_{4+n}/(8\pi)^{(4+n)/2}$), $g_{4+n}$ is the determinant of the $4+n$ -dimensional Lorentz metric and $R$ is the scalar

$^9$In the minimal models discussed here; more generally, they may propagate in all dimensions but be suppressed outside the 3 dimensional surface to satisfy experimental contraints.

$^{10}$No limits are derived for this model in this thesis, but it is summarized here by way of introduction.
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curvature) gives rise to an effective action for the 4-dimensional gravity field $g$ of the form

$$S(g) = \frac{1}{2} V_n \tilde{M}_{4+n}^2 \int d^4 x \sqrt{-g} R(g)$$

where the integral over the extra dimensional coordinates $y^{1\ldots n}$ has simply become the volume of the extra dimensions, $V_n$. From this, one can conclude\textsuperscript{11} that the Planck mass in 4 dimensions, $\tilde{M}_{Pl}$ is related to the one in $4 + n$ as

$$\tilde{M}_{Pl}^2 = V_n \tilde{M}_{4+n}^2$$

Therefore, the true Planck scale $\tilde{M}_{4+n}$ can be of the order of the electroweak scale $m_H \sim 1$ TeV if the volume of the extra space is large enough. For compactification on a torus so that $V_n = (2\pi R)^n$ where $R$ is the radius of the extra dimension, one gets

$$R^{-1} \sim 2\pi \tilde{M}_{4+n} \left( \tilde{M}_{4+n}/\tilde{M}_{Pl} \right)^{2/n}$$

For $n = 2$ and $\tilde{M}_{4+n} \sim 1$ TeV this works out to $R \sim 1$ mm which is just excluded by (among others) direct measurements of gravity; for $n > 2$ the size of the extra dimensions is too small to be detected by precision gravity experiments. For a review of direct measurements, see [42].

Although the gauge hierarchy has apparently disappeared from the theory, requiring that $\tilde{M}_{4+n} \sim 1$ TeV does give rise to a new hierarchy, namely that $V_n^{-1} \ll M_{4+n}^n$. This hierarchy has to be stabilized in some way, but this would supposedly rely on some dynamical theory of quantum gravity.

Warped extra dimensions

Another approach is taken by the Randall-Sundrum (RS) model [43], where the geometry of the extra dimensions is chosen such that no new hierarchy arises. The extra dimensions are compactified to an $S_1/\mathbb{Z}_2$ orbifold, i.e., a circle with opposite points identified. The fixed points of $\mathbb{Z}_2$ on $S_1$ are the location of branes with equal but opposite tension; the Standard Model fields are localized on the negative tension brane, whereas gravity is localized on the ‘hidden’ positive tension brane. The solution of the Einstein equations in vacuum gives the following solution for the metric describing the space,

$$ds^2 = e^{-2k|y|} dx_\mu dx_\nu \eta^{\mu\nu} - dy^2$$  \hspace{1cm} (2.8)

with $y \in [0, \pi R]$, the Standard Model brane located at the $y = \pi R$ and the other at $y = 0$. The curvature of this 5-dimensional space is $R_5 = -20k^2$, i.e., it has constant negative curvature; such spaces are called “Anti-deSitter” spaces. Unlike in the ADD model, the 4-dimensional metric depends on the extra coordinate $y$.

\textsuperscript{11}[41] presents a physical and more intuitive derivation using Gauss’ law for gravitation.
The action for the Higgs potential (see (2.4)) in 5 dimensions, after integrating over the $y$-coordinate, looks like

$$S = \int dx^4 \left( e^{-2\pi kR} \eta^{\mu\nu} \partial_\mu H^\dagger \partial_\nu H - \lambda e^{-4\pi kR} (H^2 - v^2)^2 \right)$$

where $v$ is the vacuum expectation value of the Higgs field. Rewriting this in the canonical form by substituting $H \rightarrow e^{\pi kR} h$ we get

$$S = \int dx^4 \left( \eta^{\mu\nu} \partial_\mu h^\dagger \partial_\nu h - \lambda (h^2 - e^{-2\pi kR} v^2)^2 \right)$$

The effective value of the Higgs v.e.v. for the 4-dimensional theory is then $v' = e^{-\pi kR} v$. The effective 4-dimensional Planck scale $\bar{M}_{Pl}$ is related to the true 5-dimensional Planck scale $\bar{M}_5$ as

$$\bar{M}_{Pl}^2 = (1 - e^{-2\pi kR}) \bar{M}_5^3 / k$$

This means that large hierarchies can be avoided, i.e., $1 \text{ TeV} \sim \bar{M}_{Pl} \sim \bar{M}_5 \sim v \sim k$ if $kR \sim 11$

This has been shown to be a ‘natural’ hierarchy, which can be realized without fine-tuning. Therefore, in the RS model the hierarchy problem truly disappears; all masses are of the order $\bar{M}_{Pl}$, but on the Standard Model brane they appear to be $\sim 1 \text{ TeV}$ because they are scaled by a “warp” factor $e^{-\pi kR} \sim \frac{1 \text{ TeV}}{\bar{M}_{Pl}}$.

The next question is what the effective 4D action for the 5-dimensional graviton looks like. Although the graviton is described by a 5D massless spin-2 field, qualitatively the effective 4D action is similar to that of a 5D massless scalar [44]. The action for such a field is

$$S[\Phi] = \int dx^4 \int_0^{\pi R} dy \left( e^{-2k|y|} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - \partial_y \Phi \partial_y \Phi \right)$$

Because in the 5th coordinate $y$ the field is constrained by the geometry of the extra dimension to be periodic, using Fourier decomposition the $y$-dependence of $\Phi$ can be written in terms of an orthonormal basis of periodic functions $\chi_n(y)$ as $\Phi = \sum_n \chi_n(y) \phi_n(x)$ with $\int_0^{\pi R} dy \chi_n \chi_m = \delta_{nm}$.

The equations of motion for the fields $\chi_n(y)$ derived from this action can then be written as

$$-e^{k\pi y} \partial_y (e^{-2k\pi y} \partial_y \chi_n) = m_n^2 \chi_n$$

which have a solution in terms of the Bessel functions $J_2, Y_2$, with

$$m_n = x_n k e^{-k\pi R}$$ (2.9)
where \( x_n \) is the \( n \)-th zero of the Bessel function \( J_1 \), \( J_1(x_n) = 0, x_n = 0, 3.38, 7.02 \ldots \). The effective 4D action for the fields \( \phi_n \) is then

\[
\int dx^4 \frac{1}{2} \sum_n \left( \partial^\mu \phi \partial_\mu \phi - m_n^2 \phi^2 \right)
\]

Thus, this theory describes one massless scalar field \( \phi_0 \) and an infinite number of massive scalar fields \( \phi_n \). From equation (2.9) we see that the masses \( m_n \) are also of the order of \( \sim 1 \) TeV.

These scalar fields are the analogues of the RS gravitons \( G[n] \); the massless graviton \( G[0] \) is the normal graviton that mediates gravity and the rest are massive spin-2 resonances. The theory has 2 free parameters; these are usually taken to be the mass of the lightest massive graviton, \( M_{G[1]} \) and \( \frac{k}{M_{Pl}} \).

A realization of the RS model in string theory restricts the value of \( \frac{k}{M_{Pl}} \) to be greater than 0.01 (see [45]). Requiring the curvature of the extra dimension to be less than the 5-dimensional Planck scale restricts \( \frac{k}{M_{Pl}} \) to be less than \( \approx 0.1 \) (see [46, 47]).

The width of the \( n \)-th resonance is given by

\[
\Gamma_n = \rho m_n x_n \left( \frac{k}{M_{Pl}} \right)^2
\]

where \( \rho \) is a constant that only depends on the number of decay channels that are open.

For \( \frac{k}{M_{Pl}} \) less than 0.1 the lightest massive resonance has a width of not more than a couple of GeV.

The RS graviton fields \( h^{(n)}_{\alpha\beta} \) interact with the Standard Model fields by coupling to the energy-momentum tensor,

\[
\mathcal{L} = -\frac{1}{e^{-k\pi R M_{Pl}}} T^{\alpha\beta} \sum_{n=1}^{\infty} h^{(n)}_{\alpha\beta}
\]

At the Tevatron, the resonances are produced through \( q\bar{q} \rightarrow G[n] \) and \( gg \rightarrow G[n] \) and can decay to \( G[n] \rightarrow \ell^+\ell^-, gg, \gamma\gamma, q\bar{q} \). Discovering and measuring the properties of the first resonance \( G[1] \) would allow the determination of all parameters in the model [45].

Existing experimental constraints

The most stringent constraint on the presence of a RS graviton comes from a direct search at DØ with 1 fb\(^{-1}\) of data in the combined \( e^+e^- \) and \( \gamma\gamma \) channels [48], which excludes a massive graviton with \( M_{G[1]} < 300 \text{ GeV}/c^2 \) for \( \frac{k}{M_{Pl}} = 0.01 \) and \( M_{G[1]} < 900 \text{ GeV}/c^2 \) for \( \frac{k}{M_{Pl}} = 0.1 \).