Search for heavy resonances in the dimuon channel with the D0 detector
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Chapter 6

Search for new heavy gauge bosons

Having obtained estimates for the Standard Model (SM) background and the effective luminosity, the final step in the analysis essentially consists of counting the number of events in data and comparing this with theory. This is called a ‘counting experiment’; the quantity of interest is the total cross-section and the relevant statistic is the number of observed events.

A range of models of new physics is investigated. To calculate the acceptances for the $Z'$ models, the Standard Model-like $Z'$ Monte Carlo samples described in section 4.1.2 were used; for the Randall-Sundrum model, the samples described in 4.1.2 are used.

A final selection cut on the invariant dimuon mass is made, and chosen such that it maximizes the sensitivity to the presence of a signal.

Since no significant excess over the expected Standard Model background is observed, a Bayesian 95% credibility limit on the signal cross section is calculated, taking into account systematic uncertainties on the effective luminosity and background. These limits are interpreted as a limit on the $Z'$ mass. The expected limit and its variation are calculated as well. The limits for the Standard Model-like $Z'$ cross section are also interpreted as limits for the $Z'$ in Littlest Higgs scenarios, and in a model-independent way. The limits for the Randall-Sundrum model are interpreted as limits on $M_G$ and $k/M_{\text{Planck}}$.

6.1 Counting experiment

The new physics models all involve the resonant production and decay of a new particle $Z'$ with a cross-section $\sigma_S$, which depends in some model-dependent specific way on the mass $M_{Z'}$ and other parameters. These models therefore predict an excess of events above the Standard Model background rate. We test these hypotheses by
looking at the observable \( N_R \): the number of events counted in a chosen region \( R \) of the event variable space, and compare it with the expected distributions for the number of SM events, \( B_R \). \( R \) is specified by the (pre)selection cuts described in the previous chapters, plus one more selection cut on the dimuon invariant mass, which is chosen independently for each generated signal Monte Carlo sample.

Specifically, the hypothesis \( H_{\sigma_S} \) being tested is that \( N_R \) is distributed according to a Poisson distribution as

\[
p(N_R \mid H_{\sigma_S}) = e^{-B_R-S_R} \frac{(B_R+S_R)^{N_R}}{N_R!}
\]  

(6.1)

Here, \( B_R \) is the expected (most probable) number of background (i.e., Standard Model) events in \( R \), \( S_R \) is the number of signal events in \( R \).

The number of signal events is related to the signal cross section as

\[
S_R = L_{\text{eff}}(R) \cdot \sigma_S
\]

The signal cross section \( \sigma_S \) depends on the resonance mass \( M_{Z'} \) and other parameters. For \( \sigma_S \to 0 \) we recover the Standard Model hypothesis \( H_0 \), which is the null hypothesis against which we test,

\[
p(N_R \mid H_0) = e^{-B_R} \frac{B_R^{N_R}}{N_R!}
\]  

(6.2)

The quantity \( L_{\text{eff}}(R) \) is the effective luminosity for \( R \), i.e.,

\[
L_{\text{eff}}(R) = L \cdot \epsilon(R)
\]

where \( L \) is the integrated luminosity for the data sample and \( \epsilon(R) \) is the acceptance times trigger efficiency times selection efficiency.

As described in section 5.4.1, \( L_{\text{eff}} \) is calculated with respect to the \( Z \) cross-section,

\[
L_{\text{eff}}(R) = \frac{N_{Z\text{data}}}{\epsilon_{MC}(Z) \cdot \sigma_{Z\to \mu^+\mu^-} \cdot \epsilon_{\text{sel}}(R)}
\]

where \( \frac{N_{Z\text{data}}}{\epsilon_{MC}(Z) \cdot \sigma_{Z\to \mu^+\mu^-}} \) is the normalization to the theoretical \( Z \) production cross section times branching ratio and \( \epsilon_{\text{sel}}(R) \) is the (corrected) selection efficiency for the signal.

The selection efficiencies are calculated on the Monte Carlo simulations described in 4.1.2 as

\[
\epsilon_{\text{sel}}(R) = \frac{\sum_{\{x_i \in R\}} w_i \cdot c_i}{\sum_{\{x_i\}} w_i}
\]

(6.3)

where \( w_i \) are the Monte Carlo event weights and \( c_i \) are the efficiency correction factors. The calculated efficiencies are listed in table 6.1 for the SM-like \( Z' \) model and table 6.2 for the RS model.
A discovery of a signal of new physics can be claimed if the probability of observing at least \( N_R \) events is sufficiently small given the null hypothesis

\[
\sum_{s=0}^{\infty} p(N_R + s|H_0) < \alpha
\]

where \( p(N_R|H_0) \) is given by (6.2). \( \alpha \) is the chosen significance level of the test; in the following, it is set to correspond to three standard deviations of a one-sided normal distribution, i.e., \( \alpha \approx 1.3 \cdot 10^{-3} \).

A discovery will therefore only be claimed if more events are observed than the minimum number needed for discovery,

\[
N_R > N_{\text{min}}(B_R, \alpha)
\]  

(6.4)

If, on the other hand, fewer than \( N_{\text{min}} \) events are observed, we can attempt to put an upper limit on the number of signal events that could still have contributed to \( N_R \). Because there is always a finite possibility that \( N_R \) events are observed for any value of \( \sigma_S \), one chooses a desired credibility level (CL) for the upper limit, such that \( 1 - \text{CL} \) is the plausibility that the true value of \( \sigma_S \) lies in fact above the claimed upper limit. Here we make the conventional choice of CL = 95% (or about \( 1.65\sigma \)).

Whether or not a signal can be excluded at a certain credibility level depends of course on the choice of the selection criteria \( R \). The selection cuts described in the previous chapter were not optimized other than that they were chosen to be efficient (except the anti-cosmic cuts); heuristically, this makes sense because the number of signal events is expected to be small in comparison with the number of Drell-Yan background events for any value of these cuts, while other remaining backgrounds are negligible.

However, this is not true for the cut on the invariant mass of the selected muon pair \( M_{\mu\mu} \). This cut,

\[
M_{\text{lower}} < M_{\mu\mu} < M_{\text{upper}}
\]

is therefore optimized to maximize the sensitivity to the presence of a signal over the Drell-Yan (and other remaining) background.

It is a priori not clear how sensitivity should be defined and what is then the optimal choice for \( M_{\text{lower}}, M_{\text{upper}} \). Common approaches include: the choice that leads to the highest significance in the case of a discovery, the largest expected range of discovery for a given model, or the largest expected range for exclusion in case there is no signal. These are not in general identical choices, and can lead to counter-intuitive results in pathological cases.

Here, the definition of [86] is used instead, which avoids these complications. The argument from that paper is summarized and applied in the following.
### 6.1.1 Sensitivity Optimization

Let \(1 - \beta_\alpha(R; \sigma_S, M_{Z'}, \ldots)\) be the power of the test (6.4). This is the probability of discovering the signal at a significance level \(\alpha\) in the case that \(H_{\sigma_S}\) is actually true, i.e., that a new gauge boson actually exists. The “sensitivity region” of an experiment is defined as the region in the space of signal parameters \((\sigma_S, M_{Z'}, \ldots)\) for which holds that

\[
1 - \beta_\alpha(R; \sigma_S, M_{Z'}, \ldots) > CL
\]

where CL is the chosen confidence level (95%). In words, the sensitivity region of an experiment is the region in parameter space for which the probability to observe a signal at a significance level \(\alpha\) when that signal indeed exists is greater than or equal to the confidence level chosen for the exclusion limits.

That is, if the true values of the parameters are such that equation (6.5) holds, there is a probability of at least CL that a discovery will be made at a significance \(\alpha\). If, on the other hand, no discovery is made, at least this region can be excluded at a credibility level equal to or greater than CL.

Thus equation 6.5 describes the values of the signal parameters for which the experiment is “sufficiently sensitive” to definitely give an answer, either as a discovery, or as an exclusion at a predefined CL.

The “optimal” sensitivity is then reached for the choice of \(R\) (in this case, the choice of \(M_{\text{lower}}\) and \(M_{\text{upper}}\)) which leads to the largest sensitivity region.

In all models under investigation the cross section \(\sigma_S\) decreases monotonically with \(M_{Z'}\); also, the effective luminosity is only dependent on \(M_{Z'}\), and is calculated for a number of discrete points in \(M_{Z'}\). Therefore, the problem is reduced to finding for each mass point the choice of \(M_{\text{lower}}\) and \(M_{\text{upper}}\) which leads to largest sensitivity range in \(\sigma\).

Since the power of a counting experiment increases monotonically with the number of signal events \(S\), equation (6.5) is satisfied if

\[
S_R(\sigma_S, M_{Z'}, \ldots) > S_{\text{min}}(B_R, \alpha, CL)
\]

for some number of events \(S_{\text{min}}\) which only depends on \(B_R\) and the choice of \(\alpha\) and CL. Both \(S_R\) and \(B_R\) depend on the choice of \((M_{\text{lower}}, M_{\text{upper}})\), but since the signal effective luminosity is independent of \(\sigma_S\), this can be simplified to

\[
\sigma_S(M_{Z'}, \ldots) > S_{\text{min}}(B_R, \alpha, CL)/L_{\text{eff}}(R) \equiv \sigma_{\text{min}}
\]

The maximal sensitivity is then obtained for the choice of \(M_{\text{lower}}\) and \(M_{\text{upper}}\) which leads to the lowest value of \(\sigma_{\text{min}}\).

\(S_{\text{min}}\) can in principle be calculated from (6.1) or computed numerically, but this leads to unworkable complicated expressions. Instead, in [86] an expansion orders of \(\alpha\) and CL was fit to the exact answer. This leads to the following approximate expression for \(S_{\text{min}}\),
Figure 6.1: (a) Standard Model background (open circles) and Standard Model-like $Z'$ with $M_{Z'} = 400 \text{ GeV}/c^2$ (squares), fitted with the functions (6.8) and (6.7) respectively; (b) $\epsilon/S_{\text{min}}$ (equation (6.6)) as a function of $M_{\text{lower}}$ and $M_{\text{upper}}$ around the maximum with the other variable held fixed.

$$S_{\text{min}} = \frac{1}{8} a^2 + \frac{9}{13} b^2 + a \sqrt{B_R} + \frac{1}{2} b \sqrt{b^2 + 4 \sqrt{B_R} + 4 B_R}$$

Here, $a$ and $b$ are the number of standard deviations corresponding to one-sided Gaussian tests for $\alpha$ ($3\sigma$) and $\text{CL (95\% \sim 1.65\sigma)}$ respectively.

The quantity that has to be maximized is then

$$\frac{\epsilon_{\text{mass}}}{S_{\text{min}}(M_{\text{lower}}, M_{\text{upper}})} \quad (6.6)$$

where $\epsilon_{\text{mass}}$ is the efficiency of the cut on the invariant mass. The difference with the exact calculation is negligible.

Comparing the quantity (6.6) with a more usual quantity (a measure of the expected significance in the presence of a signal), $S/\sqrt{B}$, one can see that maximizing (6.6) will lead to a higher efficiency. The difference is larger for the case that the number of background events is small. This is in itself desirable because $S/\sqrt{B}$ diverges when $B \rightarrow 0$, so that maximizing it biases to low efficiency for small number of background events. For $B \rightarrow \infty$ the two expressions converge.

**Mass cut optimization**

To find the maximum of the expression (6.6), suitable functions are fitted to the signal and background distributions. The signal distributions were fitted with the following...
Figure 6.2: (a) Standard Model background (open circles) and the lowest mass graviton excitation in the Randall-Sundrum model with $M_{G[1]} = 200 \text{ GeV/c}^2$ (squares), fitted with the functions (6.8) and (6.7) respectively; (b) $\epsilon/S_{\text{min}}$ (equation (6.6)) as a function of $M_{\text{lower}}$ and $M_{\text{upper}}$ around the maximum with the other variable held fixed.

ad-hoc expression,

$$\frac{dS}{d\sqrt{s}} = (C_1 \cdot L(\sqrt{s}; \mu_l, \sigma_l) + C_2 \cdot G(\sqrt{s}; \mu_g, \sigma_g))$$  \hspace{1cm} (6.7)$$

where $L$ is the Landau distribution with median $\mu_l$ and width $\sigma_l$, and $G$ is a Gaussian distribution with mean $\mu_g$ and width $\sigma_g$. The background with the sum of two exponentials,

$$\frac{dB}{d\sqrt{s}} = \left( C_1 e^{-p_1 \sqrt{s}} + C_2 e^{-p_2 \sqrt{s}} \right)$$  \hspace{1cm} (6.8)$$

As an example, figures (6.1) show the result for a Standard Model-like $Z'$ model with $M_{Z'} = 400 \text{ GeV/c}^2$ and figures (6.2) for the Randall-Sundrum model with $M_{G[1]} = 200 \text{ GeV/c}^2$. Also shown are graphs of the quantity $S$ around the maximum. The results are qualitatively similar for all mass points and the differences of the fitted optimal cut with the binned result are small.

For all models, the actual solution has $M_{\text{upper}} \rightarrow \infty$, the only exception being the 200 GeV/c$^2$ Randall-Sundrum model, where the optimum lies at $M_{\text{upper}} = 268 \text{ GeV/c}^2$. Tables (6.1) and (6.2) show $M_{\text{lower}}$ for each $Z'$ and Randall-Sundrum sample respectively.
6.2 Bayesian credibility limits

The 95% Bayesian upper credibility limit on the signal cross section is calculated for each mass point under consideration. In the absence of a signal, the credibility of the true value of the signal cross-section \( \sigma \) (in the following, the subscript \( S \) is implicit) to lie in the range \((\sigma, \sigma + d\sigma)\), given the number of observed events \( N \), is the posterior likelihood density \( L(\sigma|N) \), with

\[
L(\sigma|N) = \frac{P(N|\sigma)\pi(\sigma)}{\pi(N)}
\]

where

\[
\pi(N) = \int P(N|\sigma)\pi(\sigma)d\sigma
\]

is a normalization factor, independent of \( \sigma \). \( \pi(\sigma) \) is called the ‘prior density’. In the following, it is taken to be uniform in \( \sigma \),

\[
\pi(\sigma) = \begin{cases} 
\frac{1}{\sigma_{\text{max}}} \quad &0 \leq \sigma < \sigma_{\text{max}} \\
0 &\text{otherwise}
\end{cases}
\]

with \( \sigma_{\text{max}} \) large enough so that \( P(N|\sigma_{\text{max}}) \) is arbitrarily small for any value of \( N \). This is not the least informative (or Jeffrey’s) prior for p.d.f. (6.1), but it has the advantage that the coverage is equal to the classical (“frequentist”) definition of coverage. \( P(N|\sigma) \) is given by the Poisson distribution \( p(N|\sigma, L_{\text{eff}}, B) \) of equation (6.1), convoluted with the p.d.f.’s for \( L_{\text{eff}} \) (called \( \pi(l) \)) and \( B \) (called \( \pi(b) \));

\[
P(N|\sigma) = \int\int p(N|\sigma, b, l)\pi(l)\pi(b)dbdl
\]

where \( \pi(l) \) and \( \pi(b) \) are are given by two-sided truncated Gaussians,

\[
\pi(l) = \frac{2}{\sqrt{2\pi} \left( \sigma_{+} + \sigma_{-} \right)} \begin{cases} 
\exp \left( -\frac{1}{2} \left( \frac{(l-L_{\text{eff}})}{\sigma_{+}} \right)^2 \right) &l \geq L_{\text{eff}} \\
\exp \left( -\frac{1}{2} \left( \frac{(l-L_{\text{eff}})}{\sigma_{-}} \right)^2 \right) &0 \leq l < L_{\text{eff}} \\
0 &l < 0
\end{cases}
\]

where \( \sigma_{+} \) and \( \sigma_{-} \) are the positive and negative systematic uncertainties on \( L_{\text{eff}} \), quoted below, and \( N \) is a normalization constant. Similarly,

\[
\pi(b) = \frac{2}{\sqrt{2\pi} \left( \sigma_{+} + \sigma_{-} \right)} \begin{cases} 
\exp \left( -\frac{1}{2} \left( \frac{(b-B)}{\sigma_{+}} \right)^2 \right) &b \geq B \\
\exp \left( -\frac{1}{2} \left( \frac{(b-B)}{\sigma_{-}} \right)^2 \right) &0 \leq b < B \\
0 &b < 0
\end{cases}
\]
with $\sigma_+$ and $\sigma_-$ the systematic uncertainties on the expected background $B$

The upper 95% credibility limit on the signal cross-section, $\sigma_{\text{upper}}$, is then found by solving numerically

$$\int_0^{\sigma_{\text{upper}}} P(N|\sigma) d\sigma = 0.95$$

I.e., finding $\sigma_{\text{upper}}$ so that there is a plausibility of 95% that the true value of $\sigma$ lies inside the one-sided interval $[0, \sigma_{\text{upper}}]$.

**Expected Limits**

In addition, the expected limit is calculated; specifically, the mean upper limit on the signal cross-section one would expect to find in the absence of a signal,

$$\langle \sigma_{\text{upper, exp}} \rangle = \sum_n e^{-B} \frac{B^n}{n!} \sigma_{\text{upper}}(n, B, \mathcal{L}_{\text{eff}})$$  \hspace{1cm} (6.9)

where $\sigma_{\text{upper}}(n, B, \mathcal{L}_{\text{eff}})$ is the upper 95% CL on the signal cross-section for $n$ observed events, and $n$ is distributed as in (6.1) with $\sigma \rightarrow 0$.

The (positive and negative) systematic variations on the average expected limit due to the systematic uncertainty on the expected number of observed events, $\Delta_{(\text{syst})} \langle \sigma_{\text{upper, exp}} \rangle$, are then calculated as

$$\langle \sigma_{\text{upper, exp}} \rangle \pm \Delta_{(\text{syst})} \langle \sigma_{\text{upper, exp}} \rangle = \sum_n e^{-(B \pm \Delta B)} \frac{(B \pm \Delta B)^n}{n!} \sigma_{\text{upper}}(n, B, \mathcal{L}_{\text{eff}})$$  \hspace{1cm} (6.10)

where $\Delta B$ is the total positive/negative systematic uncertainty on the background, described below.

The variation around the average expected limit due to random fluctuations of the stochastic variable $n$, the number of observed events, is given by

$$\Delta^2_{(\text{stat})} (\sigma_{\text{upper, exp}}) = \sum_n e^{-B} \frac{B^n}{n!} (\sigma_{\text{upper}}(n, B, \mathcal{L}_{\text{eff}}) - \langle \sigma_{\text{upper, exp}} \rangle)^2$$  \hspace{1cm} (6.11)

**6.2.1 Systematic uncertainties on signal efficiency**

The systematic uncertainties on the muon identification, track reconstruction and isolation efficiencies were described in the previous chapter. Since the Monte-Carlo is normalized to the $Z$ peak, only the dimuon mass-dependent systematic effects are taken into account, i.e.,

$$\Delta \mathcal{L}_{\text{eff}} = |\mathcal{L}_{\text{eff}} - N \pm \epsilon \pm|$$
### 6.2. Bayesian Credibility Limits

#### Table 6.1: For each mass point in the Standard Model-like $Z'$, is shown the lower mass cut, the mass cut efficiency $\epsilon_{\text{mass}}$, the total signal selection efficiency $\epsilon_{\text{tot}}$, and the relative uncertainties on the total signal selection efficiency.

<table>
<thead>
<tr>
<th>$M_{Z'}$</th>
<th>$M_{\text{lower}}$</th>
<th>$\epsilon_{\text{mass}}$ (%)</th>
<th>$\epsilon_{\text{tot}}$ (%)</th>
<th>muon ID isolation</th>
<th>track</th>
<th>PDF</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>317</td>
<td>71</td>
<td>31</td>
<td>$\pm 0.5%$ $\pm 0.1%$ $\pm 0.1%$</td>
<td>+0.3%</td>
<td>+0.7%</td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>399</td>
<td>63</td>
<td>27</td>
<td>$\pm 0.6%$ $\pm 0.1%$ $\pm 0.2%$</td>
<td>+0.3%</td>
<td>+0.7%</td>
<td></td>
</tr>
<tr>
<td>600</td>
<td>481</td>
<td>57</td>
<td>25</td>
<td>$\pm 0.6%$ $\pm 0.1%$ $\pm 0.2%$</td>
<td>+0.4%</td>
<td>+0.8%</td>
<td></td>
</tr>
<tr>
<td>650</td>
<td>514</td>
<td>55</td>
<td>23</td>
<td>$\pm 0.7%$ $\pm 0.1%$ $\pm 0.2%$</td>
<td>+0.8%</td>
<td>+1.1%</td>
<td></td>
</tr>
<tr>
<td>700</td>
<td>566</td>
<td>51</td>
<td>22</td>
<td>$\pm 0.7%$ $\pm 0.1%$ $\pm 0.2%$</td>
<td>-0.6%</td>
<td>-0.9%</td>
<td></td>
</tr>
<tr>
<td>750</td>
<td>588</td>
<td>51</td>
<td>22</td>
<td>$\pm 0.7%$ $\pm 0.1%$ $\pm 0.3%$</td>
<td>+1.0%</td>
<td>+1.2%</td>
<td></td>
</tr>
<tr>
<td>800</td>
<td>618</td>
<td>48</td>
<td>20</td>
<td>$\pm 0.5%$ $\pm 0.1%$ $\pm 0.2%$</td>
<td>+1.3%</td>
<td>+1.4%</td>
<td></td>
</tr>
<tr>
<td>850</td>
<td>643</td>
<td>48</td>
<td>20</td>
<td>$\pm 0.6%$ $\pm 0.2%$ $\pm 0.2%$</td>
<td>+1.7%</td>
<td>+1.8%</td>
<td></td>
</tr>
<tr>
<td>900</td>
<td>669</td>
<td>45</td>
<td>18</td>
<td>$\pm 0.6%$ $\pm 0.1%$ $\pm 0.2%$</td>
<td>-2.4%</td>
<td>-2.5%</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>718</td>
<td>37</td>
<td>13</td>
<td>$\pm 0.7%$ $\pm 0.1%$ $\pm 0.3%$</td>
<td>+4.9%</td>
<td>+4.9%</td>
<td></td>
</tr>
</tbody>
</table>

#### Table 6.2: For each mass point in the Randall-Sundrum model is shown the lower mass cut, the mass cut efficiency $\epsilon_{\text{mass}}$, the total signal selection efficiency $\epsilon_{\text{tot}}$, and the relative uncertainties on the total signal selection efficiency. For $M_{G^{[1]}} = 200 \text{ GeV}/c^2$, the dimuon invariant mass is also required to be below $M_{\text{upper}} = 268 \text{ GeV}/c^2$.

<table>
<thead>
<tr>
<th>$M_{G^{[1]}}$</th>
<th>$M_{\text{lower}}$</th>
<th>$\epsilon_{\text{mass}}$ (%)</th>
<th>$\epsilon_{\text{tot}}$ (%)</th>
<th>muon ID isolation</th>
<th>track</th>
<th>PDF</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>178</td>
<td>67</td>
<td>29</td>
<td>$\pm 0.6%$ $\pm 0.1%$ $\pm 0.1%$</td>
<td>+0.8%</td>
<td>+1.0%</td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>238</td>
<td>80</td>
<td>34</td>
<td>$\pm 0.7%$ $\pm 0.2%$ $\pm 0.2%$</td>
<td>+0.3%</td>
<td>+0.8%</td>
<td></td>
</tr>
<tr>
<td>400</td>
<td>321</td>
<td>70</td>
<td>29</td>
<td>$\pm 0.8%$ $\pm 0.1%$ $\pm 0.1%$</td>
<td>+0.6%</td>
<td>+2.0%</td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>405</td>
<td>64</td>
<td>27</td>
<td>$\pm 0.8%$ $\pm 0.1%$ $\pm 0.1%$</td>
<td>+1.5%</td>
<td>+1.7%</td>
<td></td>
</tr>
<tr>
<td>600</td>
<td>498</td>
<td>57</td>
<td>23</td>
<td>$\pm 0.7%$ $\pm 0.1%$ $\pm 0.2%$</td>
<td>+0.1%</td>
<td>+0.8%</td>
<td></td>
</tr>
<tr>
<td>700</td>
<td>569</td>
<td>56</td>
<td>24</td>
<td>$\pm 0.7%$ $\pm 0.1%$ $\pm 0.2%$</td>
<td>+1.4%</td>
<td>+1.6%</td>
<td></td>
</tr>
<tr>
<td>800</td>
<td>642</td>
<td>55</td>
<td>23</td>
<td>$\pm 0.7%$ $\pm 0.1%$ $\pm 0.1%$</td>
<td>+2.1%</td>
<td>+2.2%</td>
<td></td>
</tr>
<tr>
<td>900</td>
<td>688</td>
<td>56</td>
<td>23</td>
<td>$\pm 0.7%$ $\pm 0.1%$ $\pm 0.3%$</td>
<td>+0.6%</td>
<td>+1.0%</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>749</td>
<td>54</td>
<td>22</td>
<td>$\pm 0.7%$ $\pm 0.1%$ $\pm 0.2%$</td>
<td>+0.8%</td>
<td>+1.1%</td>
<td></td>
</tr>
</tbody>
</table>
PDF uncertainty

The uncertainty on the signal efficiency due to the PDF uncertainties was calculated following the prescription of [72]. The CTEQ collaboration provides a collection of $2 \times 20$ error sets with the CTEQ6.1M PDF. These represent a basis of “barely tolerable” fits, which have a $\chi^2 = \chi^2_0 + T^2$, where $\chi^2_0$ is of the optimal fit (i.e., the CTEQ6.1M PDF) and $T$ is the “tolerance” which is chosen to be $T \sim 10$. For more details see [72].

Each of the 40 CTEQ6.1 error sets CTEQ6.1.xx ($xx = 1\ldots40$) is summed over, counting the squared, respectively positive and negative, differences with the central value estimate of the efficiency.

$$\Delta_{\text{PDF}} \epsilon_{\text{sel}} = \sqrt{\sum_{xx=1}^{40} (\epsilon_{\text{sel}}(\text{CTEQ6.1.xx}) - \epsilon_{\text{sel}}(\text{CTEQ6.1M}))^2}$$

summing over the PDF error sets for which $\epsilon_{\text{sel}}(\text{CTEQ6.1.xx}) > \epsilon_{\text{sel}}(\text{CTEQ6.1M})$, and summing over the other sets for $\Delta_{\text{PDF}} \epsilon_{\text{sel}}$. Here, the central estimate $\epsilon_{\text{sel}}(\text{CTEQ6.1M})$ is given by equation 6.3, and the displaced values $\epsilon_{\text{sel}}(\text{CTEQ6.1.xx})$
6.2. BAYESIAN CREDIBILITY LIMITS

<table>
<thead>
<tr>
<th>( M_Z' )</th>
<th>( p_T ) resolution</th>
<th>PDF</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>-3.0% +5.1%</td>
<td>-3.4% +2.4%</td>
</tr>
<tr>
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</tr>
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<td>-4.0% +2.8%</td>
</tr>
<tr>
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<td>-4.1% +2.8%</td>
</tr>
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<td>-4.2% +2.9%</td>
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<tr>
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<td>-8.2% +13.1%</td>
<td>-4.4% +3.0%</td>
</tr>
<tr>
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<td>-10.1% +11.3%</td>
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</tr>
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<td>-4.4% +3.0%</td>
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<td>-4.5% +3.0%</td>
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<tr>
<td>1000</td>
<td>-9.5% +10.8%</td>
<td>-4.5% +3.0%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
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<th>( p_T ) resolution</th>
<th>PDF</th>
</tr>
</thead>
<tbody>
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</tr>
<tr>
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<td>-2.7% +1.9%</td>
</tr>
<tr>
<td>400</td>
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<td>-4.1% +2.8%</td>
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<tr>
<td>700</td>
<td>-9.9% +8.8%</td>
<td>-4.2% +2.9%</td>
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<tr>
<td>800</td>
<td>-11.4% +12.2%</td>
<td>-4.4% +3.0%</td>
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<tr>
<td>900</td>
<td>-10.0% +8.3%</td>
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</tr>
<tr>
<td>1000</td>
<td>-10.0% +9.9%</td>
<td>-4.6% +3.0%</td>
</tr>
</tbody>
</table>

Table 6.3: Uncertainties on number of background events for each mass point, (a) Standard Model-like \( Z' \) and (b) Randall-Sundrum model.

are calculated with scaled Monte Carlo weights \( w'_i = w_i \cdot \frac{W(\text{CTEQ6.1.xx})}{W(\text{CTEQ6.1.M})} \) where the \( W \)'s are the appropriate PDF weights.

Tables 6.1 and 6.2 list these systematic uncertainties for the \( Z' \) and RS models respectively. As an example, figures 6.3 show for each error set the ratio of the computed efficiency to the central value, for an 800 GeV/c² SM-like \( Z' \) and an 800 GeV/c² \( G[\text{I}] \). The first is not dominated by one error set in particular, but for the RS model the PDF error is dominated by CTEQ6.1.30 (the error set CTEQ6.1.30 is driven by the gluon density, which is not well constrained at high \( x \)). This picture is qualitatively similar for \( Z' \)'s and \( G[\text{I}] \)'s of other masses.

6.2.2 Systematic uncertainty on background

The following systematic uncertainties on the number of background events are taken into account:

- The uncertainty on the normalization,

  \[ \Delta^- n = 4.0\%, \; \Delta^+ n = 4.3\% \]

  taken from the \( Z \) peak fit error, the error on the theoretical \( Z \) peak cross section and the difference between the normalization with and without using the kinematic fit (see 5.4.1).

- The uncertainty on the normalized invariant mass distribution due to the statistical plus systematic uncertainties on the muon resolution (figure 5.4(b)); see 5.3.1.
• The uncertainty on the normalized Drell-Yan cross section due to the PDF uncertainty (see figure 4.3).

All systematic uncertainties are taken to be uncorrelated. If instead the uncertainty on the normalized Drell-Yan cross-section due to the PDF uncertainty is taken to be fully (positively) correlated with the error on the signal efficiency due to the PDF uncertainties, the resulting cross-section limits are \( \lesssim 1.5\% \) better than the found result (depending on the mass point); the difference on the found mass limit on the Standard Model-like \( Z' \) is negligible.

The results are summarized in tables 6.3.

### 6.2.3 Limits on Standard Model-like \( Z' \) cross section

<table>
<thead>
<tr>
<th>( M_{Z'} )</th>
<th>( M_{\text{lower}} )</th>
<th>( N_{\text{obs}} )</th>
<th>( N_{\text{bkg}} )</th>
<th>( \mathcal{L}_{\text{eff}}(\text{fb}^{-1}) )</th>
<th>( \sigma_{\text{upper}}(\text{fb}) )</th>
<th>( \sigma_{\text{upper}}(\text{exp.})(\text{fb}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>317</td>
<td>36</td>
<td>31.5(^{+7.5%}_{-6.5%} )</td>
<td>30.8(^{+0.7%}_{-0.6%} )</td>
<td>56</td>
<td>45(^{+16}_{-15} )</td>
</tr>
<tr>
<td>500</td>
<td>399</td>
<td>12</td>
<td>12.7(^{+7.9%}_{-7.6%} )</td>
<td>26.7(^{+0.7%}_{-0.7%} )</td>
<td>32</td>
<td>35 ( \pm ) 12</td>
</tr>
<tr>
<td>600</td>
<td>481</td>
<td>5</td>
<td>5.9(^{+9.9%}_{-9.4%} )</td>
<td>24.9(^{+0.8%}_{-0.8%} )</td>
<td>24</td>
<td>28 ( \pm ) 9</td>
</tr>
<tr>
<td>650</td>
<td>514</td>
<td>3</td>
<td>4.5(^{+10.7%}_{-8.6%} )</td>
<td>23.4(^{+1.1%}_{-0.9%} )</td>
<td>21</td>
<td>27 ( \pm ) 9</td>
</tr>
<tr>
<td>700</td>
<td>566</td>
<td>1</td>
<td>3.1(^{+10.8%}_{-11.2%} )</td>
<td>21.7(^{+1.0%}_{-1.2%} )</td>
<td>17</td>
<td>26 ( \pm ) 9</td>
</tr>
<tr>
<td>750</td>
<td>588</td>
<td>1</td>
<td>2.5(^{+13.5%}_{-10.4%} )</td>
<td>22.0(^{+1.2%}_{-1.7%} )</td>
<td>17</td>
<td>24 ( \pm ) 8</td>
</tr>
<tr>
<td>800</td>
<td>618</td>
<td>1</td>
<td>2.1(^{+12.7%}_{-12.0%} )</td>
<td>20.2(^{+1.4%}_{-2.0%} )</td>
<td>19</td>
<td>25 ( \pm ) 8</td>
</tr>
<tr>
<td>850</td>
<td>643</td>
<td>1</td>
<td>1.8(^{+13.5%}_{-13.1%} )</td>
<td>19.8(^{+2.8%}_{-2.5%} )</td>
<td>20</td>
<td>24 ( \pm ) 8</td>
</tr>
<tr>
<td>900</td>
<td>669</td>
<td>1</td>
<td>1.5(^{+10.7%}_{-12.3%} )</td>
<td>17.6(^{+2.5%}_{-3.4%} )</td>
<td>23</td>
<td>26 ( \pm ) 8</td>
</tr>
<tr>
<td>1000</td>
<td>718</td>
<td>1</td>
<td>1.1(^{+12.3%}_{-11.5%} )</td>
<td>13.5(^{+4.9%}_{-6.4%} )</td>
<td>31</td>
<td>32 ( \pm ) 10</td>
</tr>
</tbody>
</table>

Table 6.4: Summary of results for Standard Model-like \( Z' \).

The result of the calculations described before are shown in table 6.4 and figure 6.4. Shown is the computed upper 95\% credibility limit on the cross-section as a function of the \( Z' \) mass. In the same figure the cross-section times branching ratio of a Standard Model-like \( Z' \) was plotted. The lower 95\% credibility limit on the SM-like \( Z' \) mass is \( 838^{+5}_{-8} \) GeV/\( c^2 \), where the error is due to the error on the signal cross-section coming from the PDF uncertainty relative to the \( Z \) peak cross section [78] (see figure 4.3).\(^1\)

Also shown is the cross section of a \( Z' \) in the littlest Higgs scenario (with the same acceptances as a SM-like \( Z' \)) for \( \cot \theta = 1 \). The mass limit for this model is \( M_{Z_H} > 863 \) GeV/\( c^2 \).

\(^1\)The effect of the variation of the cross section limit with the \( Z' \) mass was neglected in the calculation of this error, because for \( M_{Z'} \) between 830 – 843 GeV/\( c^2 \) the cross section limit happens to be nearly constant.
Figure 6.4: 95% credibility limit on Standard Model-like $Z'$ cross-section as a function of the $Z'$ mass, and the average expected limit, the systematic uncertainty on the average expected limit and the systematic + statistical uncertainty. Also shown is the Standard Model-like NNLO $Z'$ cross section times branching ratio, the error due to the PDF uncertainty and the (LO) Littlest Higgs $Z'$ cross section times branching ratio for $\cot \theta = 1$. 
6.2.4 Limits for generic $Z'$ models from extended gauge groups

For a generic $Z'$ model with an extra $U'(1)$ extension to the Standard Model gauge group (see 2.3.3 for details), the production cross-section times branching ratio can be expressed as [28]

$$
\sigma(p\bar{p} \rightarrow Z'X \rightarrow \mu^+\mu^-X) = \frac{\pi}{48s} \left[ c_u w_u(s, M_{Z'}^2) + c_d w_d(s, M_{Z'}^2) \right]
$$

where $w_{u,d}$ are the parts which do not depend on any $Z'$ coupling, and were calculated for each $M_{Z'}$ point under consideration using the CTEQ6.1M structure functions\(^2\).

The parameters $c_u$ and $c_d$ contain all dependence on the $Z'$ couplings to fermions. Therefore, assuming the signal acceptance is independent of $c_{u,d}$, an upper limit on the SM-like $Z'$ cross-section as a function of $M_{Z'}$ corresponds to a limit in the $(c_u, c_d)$ plane, which can then be interpreted as a limit for any generic $Z'$ model. The dependence of the signal acceptance on $c_{u,d}$ was investigated in [28] and found to be small.

Figure 6.5 shows the result. Also indicated are the regions in the $(c_u, c_d)$ plane corresponding to specific classes of models described in 2.3.3; the limit on the masses of the $Z_\psi$, $Z_\eta$ and $Z_\chi$ (see section 2.3.3) were 665 GeV/$c^2$, 654 GeV/$c^2$ and 790 GeV/$c^2$ respectively.

### 6.2.5 Limits on Randall-Sundrum model

<table>
<thead>
<tr>
<th>$M_G[1]$</th>
<th>$M_{lower}$</th>
<th>$N_{obs}$</th>
<th>$N_{bkg}$</th>
<th>$L_{eff}(fb^{-1})$</th>
<th>$\sigma_{upper}(fb)$</th>
<th>$\sigma_{upper}(exp.)(fb)$</th>
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<td>222.4</td>
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<td>167</td>
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<tr>
<td>300</td>
<td>238</td>
<td>112</td>
<td>94.8+6.1</td>
<td>33.6+0.8</td>
<td>115</td>
<td>72+25</td>
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<tr>
<td>400</td>
<td>321</td>
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<td>30.0+7.3</td>
<td>29.1+0.8</td>
<td>54</td>
<td>47+16</td>
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<tr>
<td>500</td>
<td>405</td>
<td>12</td>
<td>11.7+8.5</td>
<td>26.6+1.7</td>
<td>34</td>
<td>34+12</td>
</tr>
<tr>
<td>600</td>
<td>498</td>
<td>4</td>
<td>5.2+10.5</td>
<td>23.3+0.8</td>
<td>23</td>
<td>28+10</td>
</tr>
<tr>
<td>700</td>
<td>569</td>
<td>1</td>
<td>3.0+10.5</td>
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<td>1.8+13.5</td>
<td>22.6+2.2</td>
<td>17</td>
<td>21+7</td>
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<td>1</td>
<td>1.3+10.2</td>
<td>23.3+1.0</td>
<td>17</td>
<td>19+6</td>
</tr>
<tr>
<td>1000</td>
<td>749</td>
<td>1</td>
<td>0.9+11.5</td>
<td>22.2+2.2</td>
<td>19</td>
<td>19+6</td>
</tr>
</tbody>
</table>

Table 6.5: Summary of results for the Randall-Sundrum model.

The same calculations were done for the Randall-Sundrum model and are summarized in table 6.5. Figure 6.6 shows the 95% credibility limit on the presence of the lightest massive Randall-Sundrum graviton as a function of $M_G[1]$, the expected limit and the

\(^2\)Code kindly provided by M. Carena [28].
Figure 6.5: Excluded regions in $(c_u, c_d)$ plane; the area above the curved lines for fixed $M_{Z'}$ is excluded. Also indicated are the regions of the $(c_u, c_d)$ plane allowed for certain model types; the thick solid line at $c_u = c_d$ covers the $B-xL$ models, the area between the thin solid lines covers the $q+xu$ models, and the area above the dashed line covers the $10+x^5$ models and the entire plane covers the $d-xu$ models (see section 2.3.3 for an explanation). Also indicated are the specific $E_6$ inspired models $Z_\psi$, $Z_\eta$ and $Z_\chi$. 
Figure 6.6: 95% credibility limit on Randall-Sundrum production cross section times branching ratio as a function of the $G^{[1]}$ mass, and average expected limit. Also shown is the cross section in the RS model for different values of the parameter $k/M_{Pl}$. 
RS cross section for several values of $k/\bar{M}_{Pl}$. Here, the variation of the width of the RS resonance was neglected; this is a good approximation because the reconstructed width is (by far) dominated by the mass resolution for $k/M_{Pl}$ between 0.01 and 0.1. The difference of the limit for $M_{G[1]} = 300$ GeV/c$^2$ with the expected limit is $1.7\sigma$; this can be compared to the ‘excess’ of data events over the expected Standard Model background for a reconstructed dimuon mass of around 280 GeV/c$^2$, as can be seen in figure 5.24.

For $k/\bar{M}_{Pl} = 0.1$ the limit is at $M_{G[1]} > 693$ GeV/c$^2$.

Figure 6.7 shows the observed and expected limits in the $(M_{G[1]}, \frac{k}{\bar{M}_{Pl}})$ plane.
Figure 6.7: Observed and expected 95% credibility limit on $(M_{G[1]}, \frac{k}{M_{Pl}})$. The area above the curve is excluded.
6.3 Highest mass event

Here we look in some detail at the event (run 213675, event 67615820) that had the highest reconstructed dimuon mass after the kinematic fit. Figure 6.8 schematically shows the configuration of the reconstructed objects and their transverse energies. Figure 6.9 shows a frontal view of the DØ detector, showing reconstructed tracks extrapolated to the muon system and calorimeter objects.

As events in hadron colliders go, this event is relatively clean. Although a large number of low energy tracks were present, only three tracks had a $p_T > 10$ GeV/c, the (by far) most energetic of which were matched to muon objects. One muon object ($\mu_1$) was reconstructed from a large number of hits in all three layers, while the central track of the second muon object ($\mu_2$) was matched only with forward scintillator hits, also in all three layers. $\mu_2$ thus satisfied the “loose” selection criteria that were used in this analysis, but not tighter requirements due to a number of missing hits in the muon system. Looking at figure 6.9 we see that the extrapolated central track and the matched calorimeter object point in the direction of the crack between two octants,
Figure 6.9: X-Y view of the DØ detector. For a detailed description, see the text.
which may explain why the MDTs are missing some hits for this muon. The muon objects were matched with extremely straight central tracks which were very nearly back-to-back ($\Delta\phi_{1,2} = 3.12$) in the transverse plane, exactly what one would expect from a high mass dimuon event. However, the track of $\mu_1$ had a reconstructed transverse momentum of $p_{T,1}^{\text{reco}} = 628$ GeV/$c$ while the track of $\mu_2$ was reconstructed with only $p_{T,2}^{\text{reco}} = 209$ GeV/$c$, even though no other high-energy objects were reconstructed. The resulting reconstructed dimuon invariant mass was 1140 GeV/$c^2$. Note that at $p_{T}^{\text{reco}} = 628$ GeV/$c$ the predicted (inverse) track resolution for a track with SMT hits is $\gtrsim 100\%$ (see figure 5.25), so that an error like this is not unexpected at these energies. The missing calorimeter $E_T$ (omitting the muons) was only 8 GeV. Consequently, after the kinematic fit the fitted track $p_T$’s were $p_{T,1}^{\text{fit}} = 312$ GeV/$c$ and $p_{T,2}^{\text{fit}} = 306$ GeV/$c$ respectively, with a fitted $p_T(Z) = 10$ GeV/$c$ and a resulting dimuon invariant mass of 975 GeV/$c^2$. 