On a unified description of non-abelian charges, monopoles and dyons
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Summary

Symmetry

One of the most powerful principles in physics is symmetry. This does not refer to the symmetries of objects in the world around us but instead to the symmetries of the laws of physics themselves. Nonetheless, these two categories share some examples. A sphere is symmetric under rotations just as the laws of nature are invariant under rotations. Another example is that the laws of physics are invariant under translations or shifts in space. The laws that hold in Amsterdam are the same as those that hold in Athens or anywhere else in the universe. Similarly, the laws that apply now are the same as those that applied centuries ago or at any other moment in the history of our universe. This symmetry under time shifts is not only logical, but also essential for the predictive value of the laws of nature.

Besides rendering the laws of physics into true laws that apply always and everywhere, symmetries are also related to conservation laws. For example, translation symmetry is related to conservation of momentum while time translation symmetry is related to the conservation of energy. Vice versa, conservation laws, or their violation, show us what the laws of nature must look like.

A fascinating observation is that translations and rotations are actually symmetries of space, to be precise of space-time. Time translations are no exception and also leave space-time invariant.

In addition to the space-time symmetries mentioned above there are also symmetries of physical laws that do not have a counterpart in our daily experience. These may therefore seem much more abstract. A specific subclass of such internal symmetries are so-called gauge symmetries which are closely related to a certain arbitrariness in the mathematical description of physical laws. A good example is that electrical forces can be described in terms of, among other things, an electric potential. Roughly speaking, it is only relevant how the potential changes from place to place. From that perspective the absolute value of the potential can be chosen freely. Similar gauge freedom appears in theories that describe the weak and strong nuclear forces between fundamental particles.
Summary

Gauge symmetries are also related to conserved quantities. For the electric gauge symmetry the conserved quantity is the electric charge, whereas the weak and strong nuclear forces each have their own type of charge. A final aspect is that particles that carry the latter kind of charges can interact via nuclear forces, just like electrically charged particles interact via electric forces.

Unification

The idea behind unification is to cast different theories into one big theory. The advantage of the unified theory over the fragmented theory is that seemingly unrelated phenomena can be explained from a single point of view. It could also be that certain phenomena can be explained only in terms of the unified theory. An example is Maxwell theory. This set of four laws shows that electricity and magnetism are two aspects of one interaction. Only with this unified theory it could be predicted that for example radio waves and visible light are both electro-magnetic radiation.

The electric-magnetic force and the weak force can be merged together into a bigger theory describing the electro-weak force. An interesting property of this theory is that its symmetry is spontaneously broken, e.g. violated in an elegant way, so that one can indeed distinguish the two individual forces. Someday, physicists hope to show that the fundamental forces are correctly described by a spontaneously broken ‘Grand Unified Theory’ which also incorporates the strong nuclear force.

Magnetic monopoles

Even though electricity and magnetism are part of one theory, they are not completely equal within this theory. While electric field lines can be closed as well as open, magnetic field lines can only be closed. In other words, only electric charges may exist and no magnetic charges can exist. However, for a theoretical physicist it is a piece of cake to change Maxwell’s laws and add magnetic sources. For an experimental physicist on the other hand, it is virtually impossible to falsify this adjustment. The fact that a magnetic monopole has never been observed does not imply that it may not exist. One should worry, though, whether it makes sense to write down a theory for a phenomenon that never occurs. The specific power of the assumption that monopoles may exist lies somewhere else. Namely, monopoles play a crucial role in understanding the phase structure of gauge theories. Just like a collection of atoms can be in a particular phase such as a gas phase, a fluid phase or a solid phase, or even in a more exotic one such as a superconducting phase, a large number elementary particles can also be in a certain phase. As far as it concerns for example quarks, a common phase is the ‘confinement’ phase. In this phase
quarks cannot exist separately but are locked together in small groups. This is why one
never observes quarks on their own in everyday life but instead only encounters nuclear
particles such as protons and neutrons each of which consist of three such quarks.

**Magnetic symmetry**

Magnetic charge may be conserved. However, the mechanism for this is not the same
as that for the conservation of electric charge. The conserved quantity for monopoles is
a topological charge as discussed in chapter 2. An example of a topological charge is
the hole in a rubber donut. However we squeeze and stretch the donut, as long as we
do not tear it apart, the hole remains. Similarly, any magnetic field configuration can be
deformed smoothly without changing the topological charge of a monopole. Despite the
difference with the conservation of electric charge it is appropriate to ask if a conserved
magnetic charge also gives rise to a magnetic symmetry. It is easy to guess what this
symmetry should be by analysing which magnetic charges may occur in a given gauge
theory. Chapter 2 contains a comprehensive description of these magnetic charge lattices.
By analysing the lattices one can show that for every given gauge theory with a certain
gauge symmetry there is a dual symmetry whose conserved charges correspond to the
magnetic charge lattice. In chapter 3 we study to what extent this magnetic symmetry is
seen to occur in a spontaneously broken gauge theory. The approach used here is inspired
on recent results by Kapustin and Witten involving more exotic theories. First, by using
the magnetic charge lattice, we show that every magnetic charge can be decomposed in
terms of the charges of a finite set of monopoles with unit topological charges. This leads
to so-called classical fusion rules for monopoles. Further support for these fusion rules
is found by analysing the dimensions of the moduli spaces, the spaces of monopole solu-
tions that correspond with a given magnetic charge. Additional evidence is also found by
drawing on existing results concerning the patching of monopoles. Finally, we compare
our results with those of Kapustin and Witten. The classical fusion rules and the underly-
ing magnetic charges we obtain are essentially the same as those of Kapustin and Witten.
Hence, it is certainly possible that also the semi-classical fusion rules agree. If this can be
shown, it would mean that the dual symmetry is indeed present in a spontaneously broken
gauge theory.

**Electric-magnetic unification**

Just like there may exist objects with magnetic charge in a gauge theory, there may also
be objects with both electric and magnetic charge. These are called dyons. A fascinat-
ing aspect of dyons is that the electric symmetry is partially broken by the presence of
magnetic charge which amounts to limitation in the possible electric charges for each
given magnetic charge. This, in turn, will lead to a complicated set of fusion rules. What
these fusion rules exactly are is not known but they should be related to an underlying
electric-magnetic symmetry. In this way a new unification challenge emerges. Is there a
symmetry that contains both the known electric and magnetic symmetries and which is
consistent with what is known about the dyonic fusion rules? In chapter 4 of this thesis
we give a proposal for such a symmetry. Starting from the set of dyonic charges we give
a first motivation for this unified electric-magnetic symmetry which we call the skeleton
group and continue with a detailed discussion of its construction. Next, we consider the
fusion rules of the skeleton group, show that they are consistent with fusion rules of the
electric as well as the magnetic degrees of freedom and extract new predictions for the
fusion rules of dyons in some particular cases. Finally, we show that the skeleton group
plays the role of an effective symmetry in the so-called skeleton gauge. This gauge choice
turns out to be quite relevant in the description of certain phases of a gauge theory.