Chapter 2

A Review of the Past

Abstract

In this chapter, we discuss inventory and information models. We also turn our attention to a competitive framework and study the role of interdealer trading on the formation of prices. We end this chapter with a discussion about variables that are strongly correlated with the beliefs of market participants while (relative) easy to observe by the econometrician. Much of the theories discussed in this chapter are not specifically developed for fixed income markets but are useful for the understanding of chapter 3 and 4.

According to O’Hara (1995), market microstructure theory explores the price setting rules and the market behavior of economic agents under a specific trading mechanism. In this chapter, we take a closer look at the formation of prices and the existence of a bid-ask spread under different market mechanisms and risk factors. Notice that throughout this dissertation, we distinguish between market participants who can set new prices (market makers or dealers) and market participants that can only take prices (traders).

In the first section of this chapter, we provide a discussion of the factors that determines the price quoted by dealers. These factors are not specific for bond markets. In section 2, we turn our attention to a competitive market structure including
interdealer trading. More specific, we discuss the determinants of competitive market making and the information effects of interdealer trading. This is important as a competitive market structure prevails in the Eurozone bond market. Finally, section 3 analyzes some variables that can be observed by an econometrician and useful to test market microstructure topics. We end with a discussion about the results drawn from various empirical papers.

2.1 Determinants of Price Discovery

The interaction between investors and the resulting price formation within a financial system has been the subject of many academic studies. In traditional market microstructure theory two fundamental approaches exist. The first approach describes the role of inventory and its impact on prices formed by dealers. The second approach describes the role of information asymmetry. In the latter framework, some market participants have better information about future price developments resulting in adverse selection when trading among each other.

2.1.1 Inventory Models

Inventory models are considered to be the earliest approach to the price formation in market microstructure theory. The concept is based on the assumption that prices in equilibrium are the result of a dealer’s obligation to offer liquidity while facing uncertainty in order flow. A dealer may end up with a position in which he faces price risk and the bid-ask spread is a compensation for bearing this risk. Garman (1976) argues that the uncertainty in the order flow is the main cause of uncertainty in the market makers’ cash position as buy orders will cost money while sell orders generate cash. The bid-ask spread is an instrument used by the market maker to control his order flow, the corresponding cash position and the resulting inventory level. More specifically, the probability of buy and sell orders are endogenous and
it depends on the quoted bid and ask price. A higher bid price will increase the probability of facing a trader who is selling while a lower ask price increases the probability of facing a trader who is buying. However, a trade-off is made between controlling inventory and the cash position. A change in the spread enables the market maker to control his inventory but this in turn changes the cash position.\footnote{Stoll (1978) analyzes the implications of an investor who is willing to deviate from an optimal mean-variance efficient portfolio in order to facilitate trading. This investor can be interpreted as the dealer. The spread is therefore a compensation for disutility that arises from this deviation. The Stoll model is closely related to the Garman model as both are treating order flows as a random process, which can be controlled for by the bid-ask price. Stoll however is more concerned with the costs of offering immediacy while Garman focuses on the equilibrium price under a random arrival of trades. Because of the relative simplicity of the Stoll (1978) approach, it is worthwhile to consider his model in some detail in order to gain some insights in the pricing behavior of market makers in an inventory framework. The assumption of a risk-averse dealer assures that in equilibrium, the market maker will trade if the money compensation is enough to offset his loss in utility. Stoll assumes that buy and sell trades arrive randomly following a Poisson process.\footnote{Stoll (1978) is more concerned with the costs of offering immediacy while Garman focuses on the equilibrium price under a random arrival of trades. Because of the relative simplicity of the Stoll approach, it is worthwhile to consider his model in some detail in order to gain some insights in the pricing behavior of market makers in an inventory framework. The assumption of a risk-averse dealer assures that in equilibrium, the market maker will trade if the money compensation is enough to offset his loss in utility. Stoll assumes that buy and sell trades arrive randomly following a Poisson process.} The financing of his market making activity occurs through a trading account where the dealer receive (pay) the risk-free rate \( r_f \) in case of a surplus (deficit). In the Stoll model, the bid-ask spread \( S \) set by a dealer is asset \( i \) equals two times the transaction costs, i.e. \( S = 2C_i \) where

\[
C_i = \frac{A}{W_i} \frac{Q_i^2 \sigma_i^2}{(1 + r_f)} + \frac{2Q_p \sigma_{pi}}{(1 + r_f)} \times Q_i
\]  

(2.1)

An outline of his proof is given in the appendix. Equation (2.1) tells us that the spread \( 2C_i \) depends on the monetary value of the transaction \( Q_i \), the dealers degree of absolute risk aversion \( A \), his initial wealth \( W_i \) and the securities' characteristics
(i.e. the volatility $\sigma_i$, its covariance with the optimal portfolio $\sigma_{p}$ and the monetary value of his initial position $Q_p$). The first term in equation (2.1) is non-negative while the second term depends on the initial portfolio. Any situation that leads to a worsening of an optimal (mean-variance efficient) portfolio will lead to a positive second term and hence an increase in the bid-ask spread. For example, if $\sigma_{i,p} > 0$ and $Q_p > 0$, a long position in asset $i$ ($Q_i > 0$), will result in a larger disutility when deviating from portfolio $P$ and an increase in the bid-ask spread. Ho and Stoll (1981) extended this model to a multi-period framework where the solution is obtained using a dynamic programming approach: Ho and Stoll analyze the bid-ask price set by the market maker at time $T$ and analyze the quoted spread set at time $T-1$. Compared to the one-period model, they find that the bid-ask spreads are larger when $T$ increases.

In the inventory models suggested by Stoll (1978) and Ho and Stoll (1981), the market makers only discriminate between different types of traders based on the volume. Another drawback of inventory models is the assumption that order flows are uncorrelated with future price movements. This is very unlikely as order flows contain information about the markets’ perception of fundamentals. The earliest models that incorporate different types of traders and the role of information on future price developments are the so-called information models. This is our focus in the next section.

### 2.1.2 Information Models

The class of information models provides some important insights of the role of adverse selection in the price formation process. Copeland and Galai (1983) for example, argues that the bid-ask spread formed by dealers can be interpreted as a balancing of losses against informed traders through a gain from uninformed traders. Glosten and Milgrom (1985) showed that a spread would always exist under information asymmetry. This is even the case when the market maker is risk-neutral and
forced to make a zero-profit in a competitive environment. To see how asymmetric information affects the price process, let us start with the simplest case of a pure rational expectation model. Although this approach is simple and highly stylized, it yields some interesting conclusions and is suitable as an introduction to information models. Let us assume a model with two mean-variance traders (informed $I$ and uninformed $U$). We also assume a single-market maker and a fixed supply (equal to $X$) of the risky asset. Moreover, all traders have the same absolute risk aversion parameter $\gamma$.

1. Demand by traders: Both types of traders receive a signal $v = \bar{v} + \epsilon_v \Rightarrow v \sim N(\bar{v}, \sigma_v^2)$ while the informed group $I$ receives an additional private signal $s = v + \epsilon_s$ with $s|v \sim N(v, \sigma_s^2)$. Using this additional information, trader $I$ derives a conditional distribution

$$
E(v|s) = E(v) + \Sigma_{vs} \Sigma_{ss}^{-1} (s - E(s))
$$

$$
= (1 - \beta) \bar{v} + \beta s
$$

(2.2)

$$
Var(v|s) = \Sigma_{vv} - \Sigma_{vs} \Sigma_{ss}^{-1} \Sigma_{sv}
$$

$$
= (1 - \beta) \sigma_v^2 \leq \sigma_v^2
$$

(2.3)

where $\beta = \sigma_v^2 (\sigma_v^2 + \sigma_s^2)^{-1}$. Equation (2.2) shows that the informed trader expects a value equal to a weighted average of public and private information while equation (2.3) indicates that $I$'s information is more accurate. Both traders maximize a utility function $E(W) - 0.5 \gamma \text{var}(W)$ where wealth $W = D(v - p)$ and $p$ the price set by the market maker. Hence, the optimal demand schedules $D^*_U$ and $D^*_I$ are given by

$$
D^*_U = \frac{1}{\gamma} \frac{E(v) - p}{\text{var}(v)}
$$

(2.4)

$$
D^*_I = \frac{1}{\gamma} \frac{E(v|s) - p}{\text{var}(v|s)}
$$

(2.5)

provided that $\sigma_s \neq 0$. 

2.1. DETERMINANTS OF PRICE DISCOVERY

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2. **Price setting:** The total demand observed by the market maker equals $D^* = D_U^* + D_J^*$ and in equilibrium, the price $p^*$ is set such that the market clears, i.e. $D^* - X = 0$ and

$$p^* = \bar{v} (1 - \beta) + \beta s - [\gamma \sigma^2 X] (1 - \beta)$$

Equation (2.6) shows us that the equilibrium price equals a weighted average of $v$ and $s$ minus a compensation $(\gamma \sigma^2 X - \bar{v}) (1 - \beta)$. More importantly, although $U$ does not observe the private signal, he knows that $J$’s trade affects the market-clearing price. As a result, the equilibrium price $p^*$ depends on the private signal $s$. If $U$ knows the pricing rule, which in this case is linear, he can extract information about the private signal from the market price.

These results are taken from Grossman-Stiglitz (1980) and show that a rational equilibrium with asymmetric information and (known) price setting rule is fully revealing. This result is rather striking as it questions the role of superior information if it cannot be used to generate extra profit? According to Grossman-Stiglitz (1980), costly information in a fully revealing equilibrium cannot be stable. Intuitively, using the above example, this is easy to understand. Superior information can only generate some extra return if this information is not revealed immediately. Therefore, if information is costly and the price system is fully revealing, any equilibrium will break because everyone is willing to stay uninformed. However, if information is free under a fully revealing equilibrium and everyone prefers to stay uninformed, it clearly pays to become informed. This outcome is known as the Grossman-Stiglitz paradox.

A fully revealing equilibrium is the most important drawback of the rational equilibrium model. The easiest way to avoid this problem is to introduce traders who do not act on information. These are the so-called noise traders.\(^3\) Another

\(^3\)One can argue that noise traders always lose money against informed traders and therefore are not expected to survive in the long run. How noise traders survive has been the question asked
2.1. **DETERMINANTS OF PRICE DISCOVERY**

drawback of the rational equilibrium model is the non-strategic behavior of informed traders. Because the demand of informed traders has a profound impact on the equilibrium price, he may behave strategically in order to exploit this impact. The introduction of noise traders and strategic interaction of informed traders has been the starting point for the Kyle (1985) model. This model is considered to be one of the most important models in market microstructure theory and its simplest form is considered in here. The assumptions of the Kyle model are

1. The informed trader has a linear order strategy \( x(v) = \alpha + \beta v \) while uninformed and noise traders have a bidding schedule \( u \sim N(0, \sigma_u^2) \). The total bidding schedule is given by \( y = x + u \).

2. The market maker has a linear pricing schedule \( P(y) = \mu + \lambda y \).

The liquidation value of the asset is denoted by \( v \sim N(\bar{v}, \sigma_v^2) \) and is only known by the informed trader. If the informed trader expects the liquidation value to be equal to \( v = \bar{v} \), the profit optimization condition is equivalent to

\[
\max_x E(\Pi|v = \bar{v}) = \max_x [\bar{v} - \mu - \lambda (x + u)] x
\]

which under the first order condition yields \( \alpha = -\frac{\mu}{2\lambda}, \beta = \frac{1}{2\lambda} \). The market maker only observes the aggregated demand \( y \) and will quote a price \( P(y) = E(v|y) = \mu + \lambda y \). Using the projection theory under normality, this is equivalent to

\[
E(v|y) = \mu_v + \Sigma_{vy} \Sigma_{yv}^{-1} (y - Ey)
\]

\[
= \left[ \bar{v} - \frac{\beta \sigma_u^2}{\beta^2 \sigma_v^2 + \sigma_u^2} (\alpha + \beta \bar{v}) \right] + \frac{\beta \sigma_v^2}{\beta^2 \sigma_v^2 + \sigma_u^2} y
\]

Summarizing, the equilibrium in the Kyle model can be written as

\[
x(v) = \alpha + \beta v
\]

\[
P(y) = \mu + \lambda y
\]

by DeLong *et al.* (1991). They show that noise traders can trade from an overconfidence point of view and therefore are willing to take more excessive risk. More importantly, if noise traders cannot affect prices, they earn a higher expected return while dominating the market in terms of wealth in the long run.
where $\alpha = -\mu (2\lambda)^{-1}$, $\beta = \sigma_u\sigma_v^{-1}$, $\lambda = \frac{1}{2}\sigma_v\sigma_u^{-1} > 0$ and $\mu = \bar{v} - \lambda (\alpha + \beta \bar{v})$. For the market maker, the steepness of the pricing schedule ($\lambda$) gauges the implicit trading costs. The more uncertainty about the security value $\sigma_v$, the higher the quoted price and the more noise trading, the less aggressive is the quoted price unless $y = 0$. The informed trader will trade more aggressively when the number of noise traders is high but will trade less aggressively when the uncertainty about the payoff is large. More importantly, the (explicit) market maker learns about the fundamentals from the order flow as

$$E(p) = \bar{v} + \frac{\beta^2\sigma_v^2}{\beta^2\sigma_v^2 + \sigma_u^2} (v - \bar{v}) = \frac{1}{2} (\bar{v} + v) \quad (2.9)$$

$$\text{var}(p) = \frac{(\beta\sigma_v^2)^2}{(\beta^2\sigma_v^2 + \sigma_u^2)^2} (\beta^2\sigma_v^2 + \sigma_u^2) = \frac{1}{2} \sigma_v^2 \quad (2.10)$$

Regardless of the trading outcome, equation (2.10) shows that the information revelation in the Kyle model settle at precisely a half of the previous uncertainty. Although this outcome is unrealistic, it is appealing as it allows the market maker to learn from his order flow. Kyle also provides a multi-period set-up of his model and shows that a fully revealing equilibrium will occur as $\text{var}(p)$ converges to zero when the number of trading rounds is large. The Kyle model is considered to be one of the most important models within market microstructure theory for obvious reasons of learning and strategic behavior. Interestingly, the Kyle model is often being criticized as ruling out a bid-ask spread in equilibrium.\(^4\) This however is not correct as long as dealers observe the direction of a trade. The information structure in the Kyle model makes it unnecessary for the dealer to quote a bid-ask spread as the direction of the observed volume is known. The equilibrium results of a strict positive $\lambda$ implies that $P > \mu$ or $P < \mu$ when clients are buying ($y > 0$) or selling ($y < 0$). Hence, a different price arises for both buying and selling occur and this can be interpreted as a bid-ask spread. The Kyle model also faces some drawbacks. First, the batch auction approach rules out that a market maker learns from individual

\(^4\)See e.g. Lyons (2002).
orders. It also assumes one informed trader, one risk-neutral market maker and an infinite large set of uninformed traders. Some of these drawbacks were solved by the sequential trade model of Glosten and Milgrom (1985). The key assumption in their approach is that informed traders are buying under good news and selling under bad news. Learning in this model stems from the assumption that the observable order flow is correlated with the unobservable value of the asset. The price adjustment in this model arises from Bayesian learning. This implies that the updating process is done according to Bayes rule. Assume that

\[
\begin{align*}
    P(v = v_{\text{low}}) &= 1 - P(v = v_{\text{high}}) = (1 - \theta) \\
    P(sell|v = v_{\text{low}}) &= p_i, P(buy|v = v_{\text{high}}) = p_h
\end{align*}
\]

The dealer is risk-neutral and is forced to make a zero-profit policy due to market competition. In a nutshell, the following steps give the Glosten-Milgrom model:

1. \( E(v) = \theta v_{\text{low}} + (1 - \theta) v_{\text{high}} \);

2. The market maker only observes whether the direction of the trade will use Bayes rule to infer the value of an asset.

3. The probability of an event given the observed direction (e.g. \( P(v = v_{\text{low}}|sell) \)) depends on two pieces of information. It depends on \( P(sell|v = v_{\text{low}}) \) and \( P(sell|v = v_{\text{high}}) \). Using this, we can write:

\[
P(sell) = P(sell|v = v_{\text{low}}) P(v = v_{\text{low}}) + P(sell|v = v_{\text{high}}) P(v = v_{\text{high}})
\]

4. From Bayes’ rule we know that

\[
P(v_{\text{low}}|sell) = \frac{P(sell|v = v_{\text{low}}) P(v = v_{\text{low}})}{P(sell)} = \frac{p_i \theta}{p_i \theta + (1 - p_h) (1 - \theta)}
\]
5. The expected value of the asset is given by

\[ E(v|sell) = p(v = v_{low}|sell)v_{low} + p(v = v_{high}|sell)v_{high} \]

6. The market maker also considers the chance that he is facing an informed trader. Assume that a fraction \( \mu \) is uninformed and a fraction \( 1 - \mu \) is informed. Uninformed traders buy with probability \( \gamma_b \) and sell with probability \( \gamma_s \). Informed traders will buy only if \( v = v_{high} \) and sell if \( v = v_{low} \).

\[
P(sell|v = v_{low}) = (1 - \mu) + \mu\gamma_s \\
P(sell|v = v_{high}) = \mu\gamma_s
\]

and the updated probability of a high value is henceforth

\[
P(v = v_{low}|sell) = \frac{P(sell|v = v_{low})P(v = v_{low})}{P(sell)}
\]

\[
= \frac{(1 - \mu) + \mu\gamma_s \theta}{(1 - \mu) + \mu\gamma_s \theta + \mu\gamma_s (1 - \theta)}
\]

while \( P(v = v_{high}|sell) = 1 - P(v = v_{low}|sell) \). As in the Kyle model, the market maker learns from trading. After a buy, the updated probability of a high value is given by

\[
P(v = v_{high}|buy) = \frac{P(buy|v = v_{high})P(v = v_{high})}{P(buy)}
\]

\[
= \frac{(1 - \theta)(1 - \mu) + (1 - \theta) \mu\gamma_b}{\theta\mu\gamma_b + (1 - \theta)(1 - \mu)(1 - \theta)(1 - \mu)}
\]

7. Under risk-neutrality and competitive behavior, a non-negative spread \( S \) arises.

\[
S = \text{Ask} - \text{Bid} \\
= E(v|buy) - E(v|sell) \geq 0
\]
2.2. MULTIPLE DEALER TRADING

The Glosten-Milgrom model exhibits some interesting properties. First, it provides an opportunity to analyze individual orders. Second, the quoted bid-ask spread in this model is strictly positive even when the market maker is risk-neutral and forced to make a zero-profit. In contrast to the Kyle model however, it does not exploit the strategic behavior of informed traders.

The theories presented so far might suggest that determinants of price formation in financial markets are addressed either to inventory or asymmetric information and hence represents a dichotomy. This however is not correct. For example, Madhavan and Smidt (1993) argue that both inventory and information effects play an important role in the spread quoted by dealers. Moreover, dealers act not only as liquidity providers but also as speculators. Using transaction data from the NYSE on 16 stocks with an entire sample period from January 1987 to 31 December 1987, they show that there exist substantial and persistent deviations from inventory means over the entire sample period. This suggests the possibility of periodical shifts in the desired inventory holding for speculative motives. In addition, Manaster and Mann (1996) test for mean-reversion in inventory holding using a cross-sectional data set on futures trading on the Chicago Mercantile Exchange (CME) in 1992. As predicted by inventory models, they find that the most active sellers (buyers) have a long (short) position. Also, market maker tends to widen the spread when volatility in the market increases. Interestingly, they also find that the most active buyer does not always show price concessions as they often sell at higher prices (instead of lower prices). These findings contradict the predictions of inventory models and suggest that speculation is an important motive in the dealer's behavior.

2.2 Multiple Dealer Trading

The pricing models presented in the previous section do not explicitly explore the implications of a multiple-dealer setup. The Stoll and Kyle model for instance, focuses
only on a single market maker. Their approach is reasonable for stock exchanges with quasi-monopoly dealer structures like the NYSE but it is less applicable for markets with an over the counter structure like the options, bond and foreign exchange market. Some important differences in pricing should be expected when a market maker is facing competition. The probability of winning a trade from an outside client against another dealer should be taken into account. In addition, there is also the aspect of interdealer trading as dealers may trade with each other and this creates room for strategic behavior among dealers themselves.

As we saw in the previous section, the price formed by a market maker depends on the price risk associated with an unwanted position (inventory models) or from adverse selection (information models). Hence, price risk can be reduced dramatically when there is an option to trade with another market maker instead of waiting for an incoming customer order. Although modeling multiple dealer trading is complicated involving game theory, it provides useful insights into the dynamics of the market. Lyons (2002) provides arguments why the analysis of interdealer trading is important:

1. Dealer inventories and customer order flows are sources of private information:

2. Private information and strategic interdealer behavior reduce the information captured in prices;

3. Private information and the dealers' risk-aversion reduce the information revealed by prices.

It is therefore important to analyze the price and order strategies in a multiple dealer setup. For this we analyze two types of models. The first model which is originated from Ho and Stoll (1980, 1983) focuses on the determinants of the bid-ask spread under competitive trading and the conditions in which multiple dealer trading will occur. The second model is originated from Lyons (1997) and Cao-Lyons (1999) and focuses on the information effects of interdealer trading.
2.2.1 Spread Dynamics under Competitive Market Making

One of the earliest attempts to analyze the formation of a bid-ask spread in a multiple dealer market was given by Ho and Stoll (1980, 1983). Their model analyzes the conditions of interdealer trading and spread dynamics when dealers have homogeneous preferences and expectations. The model generates useful insights and is a suitable starting point for the understanding of competitive market making. The authors assume a framework with \( N \) dealers who are making the market in one asset. All dealers have the same information about the value \( p \) of the asset and no uncertainty lies in the variability of \( p \). Uncertainty enters their model through the inventory size, the cash flow and the time until the next transaction. The construction of the optimal bid and ask price in a one-period model is comparable with the Stoll (1978) model.

The main assumption of the model is that a risk neutral dealer will set a bid and ask price such that his utility will not be lowered when he buys at the bid \( p(1 - b) \) or sells at the ask \( p(1 + a) \). Ho and Stoll (1980, 1983) show that the reservation fee (i.e. the price difference between the quoted price and the true price of the security) quoted by dealer \( i \in N \) is given by

\[
\begin{align*}
b_i &= \gamma_i \sigma^2 \left[ \frac{1}{2} Q_i + X_i \right] \\
a_i &= \gamma_i \sigma^2 \left[ \frac{1}{2} Q_i - X_i \right]
\end{align*}
\]

(2.11)  (2.12)

The dealers' reservation fee is determined by the asset volatility \( \sigma^2 \), the dealers constant absolute risk aversion \( \gamma_i \), his inventory level \( X_i \) and the value of the transaction \( Q_i \). First of all, the higher the value of the transaction \( Q_i \), the larger the fee demanded by the dealer. To see how the inventory position affects the bid and ask component, assume that dealer \( i \) is short, i.e. \( X_i < 0 \). As a result, we must have \( b_i \) (\( a_i \)) becoming smaller (larger) resulting in an increase in the bid (ask) price. The crucial factor in determining the dealer offering the best bid-ask price depends on the interaction between inventory and the constant absolute risk-aversion. If dealers
have the same inventory, the dealer with the lowest absolute risk aversion will trade as he can offer the lowest buying and selling fee to the trader. On the other hand, if the inventory positions differ, the best dealer is not necessarily the dealer with the lowest absolute risk aversion. In other words, when all dealers have the same risk preference we must have the dealer with the smallest (largest) inventory being the first buyer (seller). The following propositions are important to gain understanding in competitive market making.

**Proposition 1** The dealer with the lowest reservation fee does not have the incentive to quote his own fee but rather the fee of his closest competitor (minus a small fraction). The bid price in a competitive market \( N > 2 \) is always determined by the second buyer while the ask price is determined by the second seller.

**Proof.** Consider \( (b_{i2} < b_{i3} < \ldots) \) and \( (a_{i1} < a_{i5} < \ldots) \) which means that dealer \( i_2(i_1) \) has the lowest reservation bid (ask) fee followed by dealer \( i_3(i_5) \). We call dealer \( i_2(i_1) \) being the first buyer (seller). By ranking the dealers in ascending order according to their reservation fee we have

\[
\begin{array}{c|c|c}
1^{st} \text{ buyer} & b_{i2} & 1^{st} \text{ seller} \ a_{i1} \\
2^{nd} \text{ buyer} & b_{i3} & 2^{nd} \text{ seller} \ a_{i5} \\
\vdots & \vdots & \vdots & \vdots \\
\end{array}
\]

Clearly, the best quote is \[ p(1 - b_{i2}), p(1 + a_{i1}) \]. However, the 1\textsuperscript{st} buyer and seller will not quote these prices but the price closest to its nearest competitor minus the minimum tick size. In other words, the market price is given by

\[ p(1 - b_{i3} + \varepsilon), p(1 + a_{i5} - \varepsilon) \]

**Proposition 2** Under a continuous tick size and homogeneous preferences, inter-dealer trading is only possible when \( N > 2 \).
Proof. This follows directly from the previous proposition. If prices are continuous we have $\varepsilon \to 0$ and the market spread is determined by the $2^{nd}$ dealer. Assume that under $N = 2$ interdealer trading exists and that dealer 1 is the first seller (i.e. $a_1 < a_2$). Dealer 1 can now choose to trade with dealer 2 (by paying a fee $b_2$) or wait for an incoming trade to sell at a price $p(1 + a_1)$. From proposition 1, it follows directly that the fee paid to dealer 2 is set by 1 himself and hence equals $b_1 - \varepsilon = b_1$. Therefore, dealer 1 will not sell at a price for which he is also willing to buy. When $N > 3$, the buying fee can be set by another dealer and the first seller is not the necessarily the second buyer. ■

Proposition 3 Under homogeneous preferences and expectations, the bid-ask spread $S = (a + b)$ is given by

\[
S \geq \gamma a^2Q \quad \text{when} \quad N = 2 \\
S = \gamma a^2Q \quad \text{when} \quad N = 3 \\
S \leq \gamma a^2Q \quad \text{when} \quad N > 3
\]

Proof. The appendix gives an outline of the proof ■

For $N > 2$, Ho and Stoll also analyze the decision of a dealer to conduct an interdealer trade with certainty or to wait for an incoming market order. They calculate the reservation fee $\pi^*$ that the first buyer has to pay in case he is buying from the first seller in an interdealer trade. This reservation fee is compared with his own bid price in case he is buying at an incoming market order. As long as the bid fee from the first buyer is larger than $\pi^*$ the first seller will prefer an interdealer trade rather than bearing the uncertainty of waiting for an incoming market order. This setup is identical to the model of Cohen et al. (1980) who analyze the decision made by a dealer who is considering placing a limit order rather than a sure execution using a market order at the prevailing market price. Cohen et al. (1980) call this the "gravitational pull" effect and arises as market orders are placed rather than the orders are executed at the concurrent market price rather than being listed in the limit order book against better limit prices. Note that the Ho-Stoll model
does not provide any formal game structure for equilibrium as they assume myopic dealers. In a multiple period framework, strategic interaction between dealers is of importance as it creates room for strategic behavior. Lyons (1997) and Cao-Lyons (1999) analyze a model including information asymmetry for interdealer trading. We examine their results in the next section.

2.2.2 The Informational Effects of Interdealer Trading

The Ho and Stoll findings presented in the previous section show the spread dynamics when dealers have the same expectations and preferences. In their setup, the optimal quoting strategy is conditioned on the observable action taken by competitors (i.e. other dealers). The outcome would change drastically when all dealers move simultaneously. In this section, we try to understand the consequences of a simultaneous move. Lyons (1997) 'hot-potato' model provides us some insights when a simultaneous game with asymmetric information is played. In contrast to the Ho-Stoll model however, he assumes that dealers do not quote a spread but just a single price for both buying and selling. Before we start with a formal introduction of the interdealer model, let us discuss the main conclusions briefly. The term 'hot-potato' comes from repeated passing of inventory among dealers. Risk-averse dealers may pass their inventory imbalance to another dealer without offsetting the receiving dealers' position. The intuition behind this stems from the obligation of a market maker to quote and he therefore may end up with an unwanted position. More importantly, 'hot-potato' trading creates additional noise in the trading process, making the market less efficient. Lyons (1997) and Cao-Lyons (1999) argue that the reason for this noise effect arises as dealers act as information intermediaries while at the same time being risk-averse speculators. The combination of risk aversion and speculation is enough to motivate dealers to distort the information that they receive. The following example shows how dealers may distort the information they receive:
Example: Assume a market in which $N > 2$ risk-averse dealers are quoting one price for the same asset. At time 0, assume that the market price is given by $P_0$ and dealer $A$ has a long position of this asset bought at a price $P_0$. Right after the purchase of these securities, dealer $A$ receives $K$ private signals $S_1, \ldots, S_K$ about the value $V_1$ of this asset at time 1. Hence, dealer $A$'s information set is $\Omega_A = \{S_i\}_{i=1}^K$. Assume that $E(F_{\Omega}) = \frac{1}{K} \sum_{i=1}^K S_i < P_0$ and dealer $A$ prefers to sell his position in order to limit his loss. Assume that dealer $A$ conducts an interdealer trade with dealer $B$. Dealer $A$ may signal a price e.g. $P_{A-B} = 3s_1 + \frac{1}{K} \sum_{i=2}^K s_i$ where $E(F_{\Omega}) \leq P_{A-B} \leq P_0$ to dealer $B$. If dealer $A$ decides to signal $\beta = \frac{1}{K}$, the signal is sufficient for $E(F_{\Omega})$ but anything else would give a noisy signal of $E(F_{\Omega})$.

The above example shows us that, even in a market dominated by public information (as in the sovereign fixed income market), there is room for private information in the form of order flows. This private information creates adverse selection because dealers are able to distort the price signal towards other dealers easily due to their direct impact on the price formation process. Let us now construct a formal setup of the price process under interdealer trading. The proposed model is a simplified version based on the results of the Lyons (1997) model. Let us consider an asset having a stochastic payoff $F \sim N(\mu, \Sigma_F)$ and a market with $N > 2$ risk-averse dealers and customers with identical negative exponential utility function defined over terminal wealth $W_T$. The trading day consists of 3 periods and in the beginning of each period, the dealer makes a decision. In the first period, when the market opens, we have customer trades. In the second and third period only interdealer trading occurs. The timing is as follows:

In round one, (outside) customer based trading, dealer $i$ decides to post a single price $P_{1i}$:

1.1 The price $P_{1i}$ is for both buying and selling (any amount). The information

\footnote{\(P_{1i-B}\) denotes the price signal that dealer $A$ is sending to dealer $B$.}
set available at the beginning of period 1 equals $\Omega_{1i} = \{\mu, \sigma_F\}$ and reflects the public information of the expected value and associated uncertainty at the end of time 3. This price is valid for both (other) dealers and outside customers:

1.2 Given price $P_{1i}$, every dealer receives a number of customer orders $C_i$. These orders enter the market only in period 1 and have a distribution $C_i \sim N(0, \Sigma_c)$ which is private information (can only be observed by dealer $i$). If $C_i < 0$, customers are selling while $C_i > 0$ means that customers are buying.

**Round two, only interdealer trading.** Dealer $i$ decides his number of interdealer trades for period 2:

2.1 Let $T_{i-\to}^{(2)}$ be the outgoing interdealer order flow placed by dealer $i$ for period 2. Let $T_{-i-\to}^{(2)}$ reflects the orders placed by other dealers and received by dealer $i$. We have

$$
T_{i-\to}^{(2)} =
\begin{cases}
> 0 & \text{if dealer } i \text{ purchases} \\
< 0 & \text{if dealer } i \text{ sells}
\end{cases}
$$

$$
T_{-i-\to}^{(2)} =
\begin{cases}
> 0 & \text{if dealers } -i \text{ are purchasing} \\
< 0 & \text{if dealers } -i \text{ are selling}
\end{cases}
$$

Denote dealer $i$’s target inventory by $D_{2i}(\mu)$ and is a function of the expected value at the end of period 3. The information set is given by $\Omega_{2i} = \{|\Omega_i, C_i\}$. We assume that the initial position is 0. Because dealer $i$ has to decide his outgoing interdealer flow $T_{i-\to}$ based on the expected incoming order flow $E(T_{-i-\to}|\Omega_{2i})$ we have

$$
T_{i-\to}^{(2)}D_{2i}(\mu) = D_{2i} + c_i + E\left(T_{-i-\to}^{(2)}|\Omega_{2i}\right) \tag{2.13}
$$

2.2 At the close of period two, all dealers observe the aggregated interdealer order flow $V = \sum_{i=1}^{n} T_{i-\to}^{(2)}$. They cannot address the fraction bought and sold by individual dealers.
2.2. MULTIPLE DEALER TRADING

Round three, only interdealer trading. Dealer $i$ decides to quote a single price $P_{3i}$:

3.1 Again, the price $P_{3i}$ is for both buying and selling (any amount). The information set available at the beginning of period 3 equals $\Omega_3 = \{\Omega_1, \Omega_2, V\}$.

3.2 Denote $T_{-i-i}^{(3)}$ as the interdealer order placed by dealer $i$ and $T_{-i-i}^{(3)}$ the incoming orders received by dealer $i$.

$$T_{-i-i}^{(3)} = D_{3i}(\mu) - D_{2i}(\mu) + E\left(T_{-i-i}^{(3)} | \Omega_3\right) + T_{-i-i}^{(2)} - E\left(T_{-i-i}^{(2)} | \Omega_2\right)$$ \hspace{1cm} (2.14)

Let us take a look at $T_{-i-i}^{(2)} - E\left(T_{-i-i}^{(2)} | \Omega_2\right)$. Because trading will occur simultaneously, he can only base his strategy based on the expected incoming order interdealer order flow $E\left(T_{-i-i}^{(2)} | \Omega_2\right)$.

3.3 At the end of period 3, the terminal value $F$ is paid out.

In order to find the equilibrium prices $(P_1, P_{3i})$, one must optimize the utility of these $N$ dealers. Because the set-up of this trading mechanism follows a dynamic game with imperfect information, we must work backwards to find $(P_1, P_{3i})$. The equilibrium prices are depicted in the following proposition:

**Proposition 4** Let the trading strategy be linear functions of order flow, i.e. $T_{-i-i}^{(2)} = \beta_i C_i \Rightarrow V = \sum_{i=1}^{N} \beta_i C_i$. The third and first round quoting strategy is a perfect Bayesian equilibrium if and only if

$$P_3 = \mu + \lambda_{\text{Lyons}} V \hspace{1cm} (2.15)$$

$$P_1 = \mu \hspace{1cm} (2.16)$$

where $\lambda_{\text{Lyons}} = \frac{\gamma}{\beta N} \times \text{var}(F | \Omega_3)$. A proof of this proposition is outlined in the appendix. Let us take a closer look at the second component of equation (2.15). First, in round 1 (where no interdealer trading has occurred) we have $P_1 = \mu$ while in round 3, we have $P_3 \neq \mu$ as long as $V \neq 0$. Hence, interdealer trading creates a price
that is different from the unconditional expectation and depends on the steepness of the pricing schedule \( \lambda_{Lyons} \). Equation (2.15) reflects the hot-potato effect of Lyons. The higher the interdealer flow \( (V) \), the higher the implicit trading costs \( (\lambda V) \) associated with interdealer trading. Second, because all dealers are assumed to have identical utility functions and risk aversion, we expect their risk aversion and interdealer strategy being equal, i.e. \( \gamma_i = \gamma \) and \( \beta_i = \beta \). Although interdealer trading creates noise, all dealers know that one part of the noise is certain, namely their own part which is determined by \( \beta \). Hence, we can interpret \( \gamma \beta^{-1} \) in equation (2.16) as a correction factor for its constant absolute risk aversion. Third, the more dealers are active in the market, the less uncertainty exist about the asset’s value conditioned on the information at the beginning of period 3 because uncertainty is averaged out.

It is interesting to see what the differences are in the pricing strategy in the Lyons model compared to the equilibrium results of Kyle (1985). Both dealers have a linear pricing strategy in the observable batch order flow (or interdealer flow). From the previous section, we know that Kyle’s equilibrium (using the same notation as the Lyons model) is given by

\[
P_{Kyle} (V) = \mu + \lambda_{Kyle} V
\]  

Comparing equation (2.15) with equation (2.17) under \( N = 1 \), we see that a dealer in both models learns about the fundamentals from its order flow. In the Kyle model, the order flow stems from both informed and uninformed traders while in the Lyons model, the order flow is a pure interdealer flow. The main difference lies in the liquidity indicator \( \lambda \), which gauges the implicit trading costs. In both models, the more uncertainty about the security value (through \( \sigma \), and \( var (F|\Omega_3) \)), the higher the quoted price. Note that the liquidity indicator in the Lyons model depends on his risk-aversion parameter \( \gamma \) because dealers are risk-averse. In contrast, the Kyle model assumes that the dealer is risk-neutral.
2.2.3 Empirical Evidence on Interdealer Trading

Empirical research on interdealer trading focuses mainly on stock markets. Reiss and Werner (1998) provide a detailed study of inventory control among market makers on the London Stock Exchange. Using trading data, they test several hypotheses with respect to interdealer trading and find that 65% of all interdealer trades are used to reverse positions. Hansch, Naik and Viswanathan (1998) also use trading data from the London Stock Exchange and find that the mean reversion component in interdealer trades varies over time. There are periods in which inventory moves stronger back to its long run average. Overall, they find that this mean reversion component is stronger compared to the traditional specialist markets as found by e.g. Madhavan and Schmidt (1993). These findings suggest (i) market makers use interdealer trades mainly to reduce inventory risk and (ii) it is easier to manage inventory using interdealer trading. Reiss and Werner (2001) also studied the role of adverse selection in an interdealer system. Interdealer trading can be conducted through an interdealer market order or through an electronic interdealer system. In contrast to the electronic system, posting dealers are not anonymous through a market order. Because of the anonymity of the posted trader, informed dealers prefer to trade in an anonymous system. However, in order to avoid a market breakdown in the electronic system, uninformed dealers must participate as well. In order to induce noise trading in the electronic system, a smaller bid-ask spread is required. Using this idea, Reiss and Werner compare the spread in a non-anonymous system (the Interdealer Market Maker system) with an anonymous system (the Interdealer Brokers system) on the London Stock exchange using data on 25 FTSE-100 stocks. They indeed find better price improvements and lower price impact in an anonymous system.⁶ This finding suggests that dealers will only use a non-anonymous system when adverse selection in an anonymous system is high. Locke and Sarajoti (2001)

⁶In here, price improvement is defined as the difference between the transaction price and the best bid- or ask price surrounding the trade.
examine interdealer trading in the future markets and relate this to the inventory control problem. They use 1995 trading data from the Commodity Futures Trading Commission (CFTC).\footnote{They analyze Swiss Francs, German Marks, live cattle and pork bellies.} Using the pricing skill measure introduced by Manaster and Mann (1996), they find that pricing skills are worsened when initiated in an interdealer environment. Also, building a futures position results in paying a premium while unwinding a position results in receiving a premium. Some of their results are somewhat surprising. First, they find that pricing skills are negative when buying and positive when selling. Second, there exists some pricing skill advantage when a dealer has a short position while we expect a priori no difference. Moreover, interdealer trading is more used when unwinding a position as it gives a better price execution. This effect is even stronger when the counterpart is a dealer with an opposite inventory position. Manaster and Mann (1996) use CME Futures transactions and find evidence that futures floor traders manage their inventory on a daily basis. They find that active sellers have most likely the largest long position supporting the competitive dealer model of Ho-Stoll (1983). In contrast to what inventory models predict, they find that an increase in the market makers position is done at less favorable prices. This suggests that market makers not only provide a service to their clients for providing liquidity, but also are active investors willing to increase their position to speculate. Massa and Simonov (2001) finds that interdealer trading also generates reputation. They estimate the degree of information when dealer \( j \) is trading with dealer \( i \) in bond \( k \). This degree of information is defines as

\[
\Delta P_{k,t} = \gamma_{ij} T_{ijk,t} + \varepsilon_{ijk,t} \tag{2.18}
\]

In here, the parameter \( \gamma_{ij} \) reflects the degree of information of the \( j^{th} \) dealer when placing an order \( T_{ijk,t} \) at dealer \( i \). A non-negative \( \gamma_{ij} \) means that the \( j^{th} \) trader is well informed as he is consistently buying asset \( k \) at time \( t \) before the price goes up while selling before the price goes down. Hence, dealer \( j \) has a good reputation. Using
2.3.2.3. The Information in Variables

The regression results, they separate the traders into five exogenously determined groups. The first group is described as confident traders with good reputation while low confident traders with bad reputation reflect the last group. Based on these groups, they analyze the strategic behavior of dealers in every group by estimating the number of trades of a dealer in every posted to other dealers. They find that the volatility created by a trade posted by a smart dealer is higher than a trade posted by a bad dealer indicating a different price impact from individual dealers.

2.3 On the Information of Variables

In this section, we give a review of some empirical aspects of market microstructure. Empirical studies are growing due to the growing availability of data. We discuss some observable variables that are helpful in testing the importance of inventory effects and adverse selection in data. Empirical analysis is by means of understanding the factors that can contribute in understanding the price movements quoted by dealers. There exist a number of variables that are strongly correlated with the beliefs of market participants while being (relatively) easy to observe by the econometrician. The conclusions drawn from the theoretical part outlined previously showed that an important role can be given to order flow, the volatility of prices and the bid-ask spread. In this subsection, we discuss these variables in greater detail.

2.3.1 The Information in the Bid-Ask Spread

The role and properties of bid-ask prices have been analyzed extensively in academic literature. The probability of buy and sell orders are not random but endogenously determined by the bid-ask price quoted by market makers. In order to control for inventory, a dealer can use the bid and ask price as an instrument to induce a buy or a sell. From the perspective of information asymmetry, a dealer can use the bid-
ask spread as a natural compensation for the losses that arise when trading against informed traders.

There are a number of measures for the bid-ask spread and we discuss the average quoted spread, the average effective spread and the realized spread. The average quoted spread ($\overline{QS}$) is given by

$$\overline{QS} = T^{-1} \sum_{t=0}^{T} (A_t - B_t)$$  \hspace{1cm} (2.19)

This measure is not always correct for calculating the average transaction costs because the quoted bid-ask prices are often indicative. Negotiation and changing market circumstances may result in a transaction within or outside the quoted spread. The average effective spread ($\overline{ES}$) reflects the spread between the transaction price $P_t$ and the midpoint $m_t$ of the current quoted spread

$$\overline{ES} = 2T^{-1} \sum_{t=0}^{T} I_t (P_t - m_{t-1}) \geq 0$$  \hspace{1cm} (2.20)

Where $m_t = \frac{1}{2} (A_t + B_t)$ is the mid-price and $I_t$ the trade indicator, i.e. $I_t$ is -1 or +1 for buying and selling respectively. The effective spread indicates the position of the transaction price relative to the midpoint and therefore is capable in analyzing the actual transaction price. The average realized spread $\overline{RS}$ reflects the spread between the transaction price $P_t$ and the midpoint $m_{t+1}$ of the subsequent quoted spread

$$\overline{RS} = 2T^{-1} \sum_{t=0}^{T} I_t (m_{t-1} - P_t)$$  \hspace{1cm} (2.21)

The realized spread shows the impact of a trade on following trades. The basic idea is that trades will have an effect on future prices and dealers will amend accordingly depending on previous order flow (like the Glosten-Milgrom model). Chordia et al (2001) use these spread measures to calculate the costs of trading NYSE stocks for the period 1988 to 1998. They show that trading has become cheaper throughout
the years as the effective and quoted spread fell. They also find that $\overline{ES} < \overline{QS}$ which is evidence of negotiations and the existence of within trading.

Estimating the spread using bid-ask prices used to be difficult involving large datasets and these where not available in earlier years. To see how transaction prices can be used to estimate transaction costs, consider the Huang and Stoll (1994) model with market friction $I_tC$

$$ p_t = p_t^* + I_tC $$  \hspace{1cm} (2.22)

$$ p_{t-1}^* = p_t^* + \beta I_t + v_{t+1} $$  \hspace{1cm} (2.23)

Here $p_t$ is the observable transaction price and $p_t^*$ the unobservable efficient price. The parameter $C$ reflects the cost of trading and is comparable with a time-invariant Stoll (1978) component. Again, $I_t$ is the trade indicator with equal probability of observing $\{-1, +1\}$ and $v_{t+1} \sim N(0, \sigma_v^2)$ is the information shock in period $t+1$.

In the Huang and Stoll model a buy trade has two types of impact. Not only does it influences the transaction price as the cost impact is $C > 0$ but it also influences the fundamental price as future trades occur at a higher price $\Delta p_t^{t+1} = \beta > 0$. If $\beta = 0$, we can use the Roll estimator to estimate the bid-ask spread using only the sample auto-covariance. To see this, note that under these conditions $p^*$ equals a fixed fundamental price

$$ p_t = p^* + I_tC $$  \hspace{1cm} (2.24)

where $\text{var}(\Delta p_t) = 2C^2 + \sigma_v^2$, $\text{Cov}(\Delta p_t, \Delta p_{t-k}) = -C^2$ for $k = 1$ and zero otherwise.

If we define $S = 2C$, we can use the first-order autocovariance to estimate the spread

$$ S = 2\sqrt{-\text{Cov}(\Delta p_t, \Delta p_{t-1})} $$  \hspace{1cm} (2.25)

Equation (2.25) is the Roll estimator and provides a method to estimate the spread based only on transaction data.\footnote{The Roll estimator can only be estimated when $\Delta p_t$ exhibits negative serial correlation and volatility even when the efficient price is fixed. This is called the bid-ask bounce.} Note that if $\beta \neq 0$, we cannot use equation (2.25) to
estimate the bid-ask spread as \( \text{var}(\Delta p_t) = (3 - C)^2 + C^2 + \sigma^2 \) and \( \text{cov}(\Delta p_t, \Delta p_{t-k}) = 3C - C^2 \) for \( k = 1 \) and zero otherwise. Hence, by using only the sample autocovariance we cannot identify the parameters. However, by taking first differences we can express the price change \( \Delta p_t \) as

\[
\Delta p_t = 3I_{t-1} + C (I_t - I_{t-1}) + \epsilon_t
\]  

(2.26)

and the parameters \( \beta, C \) and \( \sigma^2 \) can be estimated in a regression of \( \Delta p \) on \( I_{t-1} \) and \( \Delta I_t \). Glosten and Harris (1988) assume that the spread can be decomposed into a transitory costs component \( C_t \) and an adverse selection costs component \( \beta_t \). The price dynamics are equal to the general model suggested by Huang and Stoll (1997). The dynamics of the spread components are assumed to be functions of the quantity \( Q_t \) traded.

\[
\beta_t = b_0 + b_1 Q_t
\]

(2.27)

\[
C_t = c_0 + c_1 Q_t
\]

(2.28)

Although the basic model cannot be estimated as most components are unobservable, we can estimate the parameters using a regression of \( \Delta p \) on \( I_t, Q_t I_t, \Delta I_t \) and \( \Delta Q_t I_t \).

\[
\Delta p_t = \Delta p^*_t + C_t I_t - C_{t-1} I_{t-1}
\]

\[
= b_0 I_t + b_1 Q_t I_t + c_0 \Delta I_t + c_1 \Delta Q_t I_t + \epsilon_t
\]

Glosten-Harris (1988) estimate the cost components for stocks traded on the NYSE. For this purpose, they use 800 transactions from 20 stocks excluding opening trades using data from December 1981. They assume \( C_1 = b_0 = 0 \). The find that the fixed costs around 4.44 cent per trade while adverse selection costs around 1.13 cents per 1000 shares. This means that the estimated spread is around \( 2 \times (4.44 + 1.13) = 2 \times 5.57 = 11.04 \) cents per 1000 shares and around \( 2 \times (4.44 + 11.3) = 2 \times 15.74 = 31.48 \) cents per 10000 shares.
2.3.2 The Information in Volatility

Inventory and information models also put an important role aside for price volatility. Inventory models argue that volatility increases the price risk in a dealer's position. On the other hand, volatility may reflect the information asymmetry in markets. Wang (1993) for example shows that changing expectations about future cash flow and noise trading contribute to price volatility. Biais (1993) analyzed the equilibrium number of traders in a competitive market setup and shows that the number of interdealer trades depends on the volatility of the security and the trading activity in the market. He also finds that the quoted spread around the reservation price is a decreasing function of the inventory, supporting the findings of Ho and Stoll (1980, 1983). French and Roll (1986) analyze the volatility smile and argue that this pattern may arise due to the arrival of information (either public or private). The authors assume an efficient market (i.e. returns should not be uncorrelated) and a time proportional variance. In order to test the differences between public and private information, they separate the dataset into days in which the markets are open or closed. If information is public, the return variance should not depend on trading days. Hence, the return variance does not reduce during holidays. On the other hand, if volatility is mainly due to the arrival of private information, the fall in volatility should be large whenever the markets are closed.

Using daily data from the NYSE between January 1963 through December 1982, they find that the volatility during trading hours is some seventy times compared to days in which the markets are closed and conclude that the arrival of private information is an important contributor to the daily price volatility. Ito, Lyons and Melvin (1997) adapted the French and Roll approach and analyze the existence of private information in the foreign exchange market using data from the Tokyo trading floor. Instead of using daily data, they turn their attention to the use of intraday spot market data running from 29 September 1994 to 28 March 1995 with a time

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4 Recall for example the Stoll model.
interval of one minute. The authors conclude that the arrival of private information is not only an important factor for daily price variation but is also an important contributor to intraday price volatility.

2.3.3 The Information in Order Flows

The transaction size and its trade direction are also important variables within inventory and information models. Indeed, large volumes induce lower prices as it will shift the level of inventory towards less favorable positions. See for example Easley and O'Hara (1987) who analyzes the impact of block trading on the transaction price. Also, large transactions incorporate a form of adverse selection because private information is positive correlated with the traded quantity. This stems from the fact that informed market participants prefer to trade larger amounts at any price. Another variable that is closely related to volume is order flow and is given by the signed volume and therefore more able to capture market sentiment. For example, a positive accumulative of order flow over a certain period reflects a buy pressure while a sell pressure is being depicted by a negative accumulation of order flow. Lyons (2002) argues that order flow will contain no information if one of the two following criteria is fulfilled:

- No private information exist. For the market maker, order flows contain information about the market perception of fundamentals as it contains trades from participants who analyze these fundamentals. Also, order flows are in essence private information as it is only known by the market maker. Learning from market flows is therefore important.

- The interpretation of a signal by market participants is the same and known publicly. Order flows also conceal information to traders even if all information is public as one can extract information about other traders' interpretation of the information by observing its action, i.e. order flow.
The explanatory power of intraday order flow for daily price changes has been analyzed extensively within the foreign exchange markets. Evans and Lyons are prominent examples in this field. Lyons (1993) also test for the information and inventory component in quoting behavior and assumes that a market maker is a speculative liquidity provider. He finds that trading volume and order flow affects quoted prices through inventory and information effects. Evans and Lyons (2001, 2002) also find that order flows have an important impact on FX markets. Some 60% of the daily price variation can be explained using order flow data and almost 70% of public news is transmitted to the prices through order flow. Hence, a part of the public news is channeled through the prices and expectations based on interpretations embedded in order flow. This conclusion is very important as it shows that there is room for private information coming from order flow even when market movements stems from publicly available news. Existence of information asymmetry in the foreign exchange market has also been the main objective in De Jong, Mahieu and Schotman (1998). In this paper, the authors analyze the price leadership hypothesis of German banks in the DM/USD Foreign exchange market. If there exist order flow information in the foreign exchange market, then some banks must outperform in the long run because they face the largest order flows. The authors indeed find some banks being market leaders.

For US bond markets, the importance of order flow has been shown e.g. by Fleming (2001) and Brandt and Kavajecz (2004) who conducted an analysis comparable to Evans and Lyons (2001, 2002). They find that order imbalances and liquidity are strongly correlated with contemporaneous return. Fleming and Remolona (1999) and Balduzzi, Elton and Green (2001) showed that macroeconomic news has an important impact on bond prices as the largest price movements arises in days with economic announcements. These papers find that before the announcement, trading intensity and price volatility is low while bid-ask spreads are high. Green (2004) documented a higher adverse selection component after the announcement of news and argues that this arises from an increase in trading activity. He also finds
that prices are more sensitive to order flow in a period of increased liquidity after a scheduled announcement and he argues that dealers absorbing a large portions of order flow may have superior information about short-term price directions. This informational advantage will result in a dispersion of information among dealers and an increase in information asymmetry in the market. The same pattern is also documented by Cohen and Shin (2003). They analyze the impact of an order shock on the price and find that (unexpected) order flows have an impact on prices. This effect is even larger when news arrives.

2.4 Conclusions

This chapter shows that inventory and information asymmetries are important determinants in the pricing behavior of dealers in financial markets. The equilibrium price formed by dealers compensates for non-optimal inventory positions and protects against adverse selection. In addition, this chapter shows that it is worthwhile to consider the price effects in a multiple-dealer framework. In the latter case, inventory and information asymmetry still play an important role because dealers' inventory and customer order flows are sources of private information. This private information combined with risk-averse speculation among dealers, will result in strategic interdealer behavior. We showed that interdealer trading influences the implicit trading costs.

In order to test market microstructure issues, we can use many variables that are relatively easy to access by the econometrician. In this chapter we have turned our attention to the bid-ask spread, price volatility and order flow. The bid-ask spread is an important instrument for dealers to control the incoming and outgoing order flow. Not only does it compensate the dealer for his market making activity, it also serves as a protection against adverse selection. The bid-ask spread can also be interpreted as the cost associated with trading and it is therefore important to find an appropriate measure for these costs. Based on quote data, one can
calculate the average, effective and realized spread. Based on transaction data, one can calculate implicit trading costs. An important role is also put aside for price volatility. According to theory, the more informed traders we have, the more price volatility will occur as market makers will change their quotes more often to protect themselves against adverse selection. The empirical papers indeed support this idea and show that the arrival of information is an important contributor to intraday price volatility. Monitoring order flow is also crucial in financial markets. Even if markets are driven by publicly available news, there exist information asymmetry in the form of private order flow as it can observed only by the parties involved in the transaction. In addition, the impact of order flow is time varying. Not only does it depend on the size and its direction, but also on the arrival of information in the market.
CHAPTER 2. A REVIEW OF THE PAST

2.A Appendix to Chapter 2

2.A.1 Appendix: Stoll’s Inventory Model

Stoll assumes in his paper that buy and sell trades arrive randomly following a Poisson process with arrival rates $\lambda_b$ and $\lambda_a$. The financing of his market making activity is through a trading account in which any surplus yields a risk free rate $r_f$. If the dealer is indifferent between trading and no trading, we must have

$$E_t \left[ U(W_{t+1}^*) \right] = E_t \left[ U(W_{t-1}) \right]$$

(2.29)

where $W_{t-1}$ is his $t+1$ wealth of his initial portfolio and $W_{t-1}^*$ the $t+1$ wealth of his portfolio after a transaction. Price dynamics of this security is given by $\Delta S_i = \mu_i \Delta t + \sigma_i \sqrt{\Delta t} \zeta$ where $\zeta$ is standard normal distributed. If no transaction occurs in the period $[t, t+1)$, then his expected initial wealth at time $t+1$ is given by

$$E(W_{t+1}) = W_t(1 + R)$$

(2.30)

where $(1 + R) = \left[ 1 + kR_e + \frac{Q_p}{W_t} R_p + \left( 1 - k - \frac{Q_p}{W_t} \right) r_f \right]$. Here $k$ is the fraction of wealth in the efficient portfolio with expected return $R_e$ and $Q_p$ the true dollar value of trading account with expected return $R_p$. However, if a transaction in security $i$ occurs, the expected wealth of his initial portfolio changes into

$$E(W_{t+1}^*) = W_t (1 + R) + Q_i (1 + r_i) - (Q_i - C_i) (1 + r_f)$$

(2.31)

where $Q_i = \{-1, +1\}$ is the monetary value of security $i$ being traded in period $(t, t+1)$ in case of a sell or purchase. Moreover, $r_i$ is its expected return, $C_i$ the cost of trading the amount of $Q_i$. Note that $Q_i - C_i$ can be interpreted as the amount the market maker needs to borrow to finance a purchase and $Q_i + C_i$ the amount in which he earns interest on a (short) sale. Clearly, the objective is to find $C_i$, which is the spread component.

A Taylor expansion of equation (2.29) around its expected wealth and dropping terms
higher than 2 yields\(^{10}\)

\[
U \left( E(W_{t+1}^*) \right) + \frac{1}{2} U'' \left( W_{t+1}^* \right) E_t \left( W_{t-1}^* - E \left( W_{t-1}^* \right) \right)^2
\]

\[
= U \left( E(W_{t+1}) \right) + \frac{1}{2} U'' \left( W_{t-1} \right) E_t \left( W_{t-1} - E \left( W_{t-1} \right) \right)^2
\]

where the first order term equals zero as the market maker behaves optimally. Writing \(W^*\) and \(W\) in terms of initial wealth and return yields

\[
U \left( E(W_{t+1}^*) \right) + \frac{1}{2} U'' \left( W_{t+1}^* \right) \left( W_t^2 \sigma^2_t + Q_t^2 Q_t^2 + 2W_t Q_t \text{cov}(R, r_t) \right)
\]

\[
= U \left( E(W_{t+1}) \right) + \frac{1}{2} U'' \left( W_{t-1} \right) W_t^2 \sigma^2_t
\]

The dynamics of the security allows us to make the following approximations:

\[
\frac{U'' \left( E \left( W_{t+1}^* \right) \right)}{U' \left( E \left( W_{t+1} \right) \right)} = \frac{U'' \left( E \left( W_{t-1} \right) \right)}{U' \left( E \left( W_{t-1} + 1 \right) \right)}
\]

\[
E \left( W_{t+1}^* \right) - E \left( W_{t+1} \right) = E \left( W_{t+1} \right) - E \left( W_{t+1} \right)
\]

which means that

\[
0 = \frac{U \left( E(W_{t+1}^*) \right) - U \left( E(W_{t+1}) \right)}{U' \left( E \left( W_{t+1} \right) \right)} + \frac{1}{2} \frac{U'' \left( W_{t+1} \right)}{U' \left( E \left( W_{t+1} \right) \right)} \left( Q_t^2 \sigma^2_t + 2W_t Q_t \text{cov}(R, r_t) \right)
\]

\[
= \frac{1}{2} A \left( Q_t^2 \sigma^2_t + 2W_t Q_t \text{cov}(R, r_t) \right) - \left[ E \left( W_{t+1}^* \right) - E \left( W_{t+1} \right) \right]
\]

where \(A = \frac{-U'' \left( W_{t+1} \right)}{U' \left( E \left( W_{t+1} \right) \right)} W_t\) is the index of relative risk aversion.

\[
- \left[ E \left( W_{t+1} \right) - E \left( W_{t-1} \right) \right] = -C_t \left( 1 + r_f \right) - Q_t \left( r_t - r_f \right)
\]

\[
\text{cov}(R, r_t) = k \sigma_e + \frac{Q_p}{W_t} \sigma_p = \frac{1}{A} \sigma_e^2 \left( R_e - r_f \right) + \frac{Q_p}{W_t} \sigma_p
\]

Here, the last equality reflects the optimal fraction held in a risky portfolio by a mean-variance investor with risk-aversion \(A\). Substituting these results into equation (2.32), use the CAPM condition \(r_t - r_f = \sigma_e \sigma_e^{-2} \left( R_e - r_f \right)\) and rearrange gives us the percentage trading costs

\[
C_t = \frac{A \frac{1}{2} Q_t \sigma_t^2 + Q_p \sigma_p}{W_t \left( 1 + r_f \right)}
\]

\(^{10}\)The dynamics in the stock tells us that that terms involving \((\Delta t)^{i/2}\) is neglectable for \(i > 2\).
2.A.2 Appendix: Spread Dynamics under Competitive Market Making

The spread quoted by dealers is the sum of the reservation fees, i.e. the sum of equations (2.11) and (2.12). This in turn depends on the dealers’ inventory. Under $N = 2$, assume that trader $i_1$ is the first seller and hence the second buyer. The same result (with an appropriate change in notation) holds when dealer 2 is the first seller. The ranking is given by

\[
\begin{array}{ccc}
1^{st} \text{ buyer} & b_{i_2} & 1^{st} \text{ seller} & a_{i_1} \\
2^{nd} \text{ buyer} & b_{i_1} & 2^{nd} \text{ seller} & a_{i_2}
\end{array}
\]

and the spread is determined by the reservation fees of the second dealer, i.e.

\[S_{N=2} = a_{i_2} + b_{i_1} = A\sigma^2 [Q + (X_{i_1} - X_{i_2})]\]

and the conditions $a_{i_1} \leq a_{i_2} (b_{i_2} \leq b_{i_1})$ imply $X_{i_2} \leq X_{i_1}$ and this proofs the condition for $N = 2$. For $N = 3$, consider the following ranking

\[
\begin{array}{ccc}
1^{st} \text{ buyer} & b_{i_2} & 1^{st} \text{ seller} & a_{i_1} \\
2^{nd} \text{ buyer} & b_{i_3} & 2^{nd} \text{ seller} & a_{i_2} \\
3^{rd} \text{ buyer} & b_{i_1} & 3^{rd} \text{ seller} & a_{i_3}
\end{array}
\]

Again, the bid-ask spread is determined by the second dealer, i.e. $S_{N=3} = A\sigma^2 Q + A\sigma^2 (X_{i_3} - X_{i_2})$ while the ranking conditions yields

\[
0 \leq b_{i_2} \leq b_{i_3} \leq b_{i_1} \Rightarrow X_{i_2} \leq X_{i_3} \leq X_{i_1}
\]

\[
0 \leq a_{i_1} \leq a_{i_2} \leq a_{i_3} \Rightarrow X_{i_3} \leq X_{i_2} \leq X_{i_1}
\]

which implies $X_{i_2} = X_{i_3}$ and hence $S_{N=3} = A\sigma^2 Q$. The same approach can be used for $N = 4$. Consider the following ranking

\[
\begin{array}{ccc}
1^{st} \text{ buyer} & b_{i_2} & 1^{st} \text{ seller} & a_{i_1} \\
2^{nd} \text{ buyer} & b_{i_3} & 2^{nd} \text{ seller} & a_{i_4} \\
3^{rd} \text{ buyer} & b_{i_1} & 3^{rd} \text{ seller} & a_{i_3} \\
4^{th} \text{ buyer} & b_{i_4} & 4^{th} \text{ seller} & a_{i_2}
\end{array}
\]
and the bid-ask spread equals \( S = A\sigma^2 Q + A\sigma^2 (X_{i3} - X_{i4}) \) while the ranking conditions yields

\[
0 \leq b_{i2} \leq b_{i3} \leq b_{i1} \leq b_{i4} \Rightarrow X_{i2} \leq X_{i3} \leq X_{i1} \leq X_{i4} \\
0 \leq a_{i1} \leq a_{i4} < a_{i3} < a_{i2} \Rightarrow X_{i2} \leq X_{i3} \leq X_{i4} \leq X_{i1}
\]

and this implies \( X_{i1} = X_{i4} \) and \( X_{i3} \leq X_{i4} \) with an equality if and only if the inventory of the second dealers are equal.

### 2.A.3 Appendix: Informational Effects of Interdealer Trading

For notational convenience, let us drop the asset’s expected value at time 3 and rewrite \( D_{3i}(\mu) = D_{3i} \) and \( D_{2i}(\mu) = D_{2i} \). The decision of quoting \( P_{3i} \) takes place at the beginning of period 3. Because all traders are quoting one price, we must have \( P_{3i} = P_3 \) (\( \forall i \)) in equilibrium in order to avoid arbitrage. Moreover, all agents have the same utility function and hence \( \beta_i = \beta \). The information set available at the beginning of period 1 equals \( \Omega_{g1i} = \{\mu, \sigma^2\} \) and is public available information. The information set in the subsequent periods are a combination of public and private information. We have \( \Omega_{gi} = \{\Omega_{0i}, C_t, T_{i-1-i}^{(2)}, T_{i-1-i}^{(3)}\} \) and \( \Omega_{3i} = \{\Omega_{0i}, \Omega_{2i}, V\} \). Under market-clearing, we must have \( \sum_{i=1}^{N} E[T_{i-1-i}^{(3)} - T_{i-1-i}^{(2)}|\Omega_{3i}] = 0 \). Using equation (2.14) and the law of iterated expectations, one can write this market clearing property as

\[
0 = \sum_{i=1}^{N} [E(D_{3i}|\Omega_3) - E(D_{2i}|\Omega_3)] + \sum_{i=1}^{N} [E(T_{i-1-i}^{(2)}|\Omega_3) - E(T_{i-1-i}^{(3)}|\Omega_3)] \\
= \sum_{i=1}^{N} [E(D_{3i}|\Omega_3) - E(D_{2i}|\Omega_3)] + \sum_{i=1}^{N} \varepsilon_i^{(2)}
\]

where \( E(T_{i-1-i}^{(2)}|\Omega_3) - E(T_{i-1-i}^{(3)}|\Omega_3) = T_{i-1-i}^{(2)} - T_{i-1-i}^{(3)} = \varepsilon_i^{(2)} \) is the inventory shock and non-stochastic conditioned on \( \Omega_3 \). In equilibrium, we must have \( \sum_{i=1}^{N} \varepsilon_i^{(2)} = 0 \) as the number of dealers having a positive inventory shocks must equal the number of dealers.
with a negative shock. Hence, $\sum_{t=1}^{N} E(D_{3t} | \Omega_3) = \sum_{t=1}^{N} E(D_{2t} | \Omega_3)$ and using equation (2.13) we have

$$\sum_{t=1}^{N} E(D_{3t} | \Omega_3) = \sum_{t=1}^{N} \left[ T_{t,t-1}^{(2)} - T_{t-1,t}^{(2)} \right] - \sum_{i=1}^{N} C_i$$

which conditioned on $\Omega_3$ must be non-stochastic. Note that $\sum_{t=1}^{N} \left[ T_{t,t-1}^{(2)} - T_{t-1,t}^{(2)} \right] = 0$ as the number of outgoing trades must equal the number of incoming trades. Hence, the inventory demanded by all the dealers in period 3 must equal the customer order flow observed in period 1:

$$\sum_{t=1}^{N} E(D_{3t} | \Omega_3) = - \sum_{i=1}^{N} C_i$$

(2.34)

Under a negative exponential utility function and normally distributed wealth, the optimal demand is given by\(^{11}\)

$$\sum_{t=1}^{N} E(D_{3t} | \Omega_3) = \sum_{i=1}^{N} \frac{E(F | \Omega_3) - P_3}{\gamma \text{var}(F | \Omega_3)} = N \frac{\mu - P_3}{\gamma \text{var}(F | \Omega_3)}$$

where $\gamma$ is the risk aversion of the dealers. Substituting this into equation (2.34) yields

$$P_3 = \mu + \frac{V}{\beta N} \times \gamma \text{var}(F | \Omega_3)$$

(2.35)

Equation (2.35) tells us that the price quoted at the beginning of period 3 depends on the expected value of the security at the end of period 3, the risk aversion of the dealers $\gamma$ and the uncertainty in the payoff $\text{var}(F | \Omega_3)$. Moreover, a higher order flow is induced by either (i) a higher demand $D_{2t}$ from the dealers or (ii) more customer buying $C_i$ or (iii) more dealers buying $T_{t-1,t}^{(2)} | \Omega_2$. All these have an upward price effect.

Let us now turn our attention to the quoting decision $P_{1t}$. This decision takes place at the beginning of period 1. First of all, we must have $P_{1t} = P_1 (\forall t)$ in order to avoid

\(^{11}\)As in the rational equilibrium models.
arbitrage. The information-set at the beginning of period 1 equals $\Omega_0$. Under market clearing, we have $\sum_{i=1}^{N} E \left[ T_{1-i}^{(2)} - T_{1-i}^{(2)} | \Omega_0 \right] = 0$. Using equation (2.13) and the law of iterated expectations, one can write this market-clearing property as

$$0 = \sum_{i=1}^{N} E \left( D_{2i} | \Omega_0 \right) + E \left( c_i | \Omega_0 \right) + E \left( T_{1-i}^{(2)} | \Omega_0 \right) - E \left( T_{1-i}^{(2)} | \Omega_0 \right)$$

$$= \sum_{i=1}^{N} E \left( D_{2i} | \Omega_0 \right)$$

as $E \left( c_i | \Omega_0 \right) = 0$. Again, under a negative exponential utility function and normally distributed wealth, the optimal demand is given by

$$\sum_{i=1}^{N} E \left( D_{2i} | \Omega_0 \right) = N \frac{\mu - P_1}{\gamma \text{var} (F | \Omega_0)} = 0 \Rightarrow P_1 = \mu$$

(2.36)

This proofs the proposition.