Measurement of charm production in deep inelastic scattering at HERA II
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Chapter 2

Structure of the proton

2.1 Neutral current deep inelastic scattering

Deep inelastic scattering, in short DIS, is the process in which an incoming lepton collides with a constituent parton from inside the proton. Neutral current, in short NC, refers to those interactions mediated by a neutral boson, either a photon or a $Z$ gauge boson. Interactions mediated by charged gauge bosons $W^\pm$, which have a neutrino in the final state, are known as charged current interactions. The higher the virtuality of the gauge boson, the smaller the distances resolved inside the proton.

Event kinematics

Figure 2.1. Feynman diagrams for a NC DIS collision. 4-vectors of incoming and outgoing particles (a) and Lorentz invariant scalars defining the event (b)

Figure 2.1 shows an electron proton scattering interaction schematically. Taking $k$ and $P$ to be the 4-momenta of the colliding lepton and proton respectively, the
NC DIS interaction is written as:

$$e(k) + p(P) \rightarrow e'(k') + X(P')$$  \hspace{1cm} (2.1)

where \( q = k - k' \) defines the 4-momentum transfer from the lepton to the proton from which follows that \( P' = P + q \). In Eq. 2.1, \( X \) denotes any final state obeying energy momentum and quantum number conservation. In this interaction, the exchanged boson only interacts with charged constituents of the proton. These charged constituents can be identified with quarks (and anti-quarks). In this way, one can rewrite Eq. 2.1 as:

$$e(k) + q(xP) \rightarrow e'(k') + q(xP + q)$$  \hspace{1cm} (2.2)

where \( x \) is the fraction of the proton four-momentum carried by the struck quark.

The kinematics of an event can be described using the two variables \( x \) and \( Q^2 \):

$$Q^2 = -q^2 = (k - k')^2$$

$$x = \frac{-q^2}{2P \cdot q}$$

which are Lorentz scalars. The virtuality of the photon, \( Q^2 \), defines the scale of the interaction. Two other variables, which are not independent of \( x \) and \( Q^2 \), are frequently used:

- The inelasticity \( y = \frac{P \cdot q}{P \cdot k} \)
- The hadronic final state center of mass energy \( W \), defined through \( W^2 = (P + q)^2 \).

The following simple formulas allow transformation between variables:

$$W^2 = Q^2 \frac{1 - x}{x} + m_p^2$$  \hspace{1cm} (2.3)

$$Q^2 = sxy$$  \hspace{1cm} (2.4)

where \( s \) is the center-of-mass energy squared \( s = (k + P)^2 \) of the event. A combination of any these two variables define the event kinematics.
2.2 Structure functions

The most general form for the unpolarized cross section for positron proton scattering can be written as $\sigma \sim L_{\mu\nu}W^{\mu\nu}$ where $W^{\mu\nu}$ represents the hadronic tensor and $L_{\mu\nu}$ the leptonic tensor. The hadronic part can be reduced to defining three independent functions, called structure functions, which parametrize the structure of the proton as seen by the virtual boson:

$$d_{NC}^2 \frac{d\sigma}{dQ^2 dx} = 2\pi \alpha^2 \frac{xQ^4}{xQ^4} (2xyF_1(x, Q^2) + 2(1 - y)F_2(x, Q^2) - (2y + y^2)xF_3(x, Q^2))$$

(2.5)

with $\alpha$ the fine structure constant $\alpha = e^2/4\pi\epsilon_0 \simeq 1/137$. All the detailed physics of the proton is contained within the structure functions. Often, in the literature the following regrouping $F_L(x) = F_2(x) - 2xF_1(x)$ is performed on Eq. 2.5 to obtain:

$$d_{NC}^2 \frac{d\sigma}{dQ^2 dx} = 2\pi \alpha^2 \frac{xQ^4}{xQ^4} (Y_+ F_2(x, Q^2) - y^2 F_L(x, Q^2) - Y_- x F_3(x, Q^2))$$

(2.6)

with $Y_\pm = 1 \pm (1 - y)^2$. In this way, $F_L$ describes the coupling of the proton only to longitudinally polarized photons. The structure function $F_1$ is proportional to the transverse component of the cross-section and $F_2$ is the sum of both transverse and longitudinal parts of the cross-section. $F_3$ is the parity violating term due to the $Z_0$ exchange. $F_2$ is the dominant structure function. The contribution of $F_L$ to the total cross-section can only become appreciable at $y$ near 1. $F_3$ only becomes relevant when $Q^2$ is comparable to the mass squared of the heavy gauge bosons. As this thesis will deal only with $Q^2$ values that are significantly lower, the $F_3$ term will be neglected from now on.

When comparing Eq. 2.5 with the cross section for the interaction between a positron and a free spin 1/2 quark the structure function can be identified with:

$$F_2(x) = \sum_q e_q^2 x f_i(x)$$

where $e_q$ is the charge of the quark and $f_i(x)$ are quark densities inside the proton, i.e. the probability of finding a parton $i$ with the momentum fraction $x$ in the proton. The summation is carried out over the different quark flavors. In the case of massless spin 1/2 quarks, the conservation of helicity in the interaction precludes an interaction with a longitudinally polarized photon. In this static model of the
proton, \( F_L = 0 \). This is commonly referred to as the “Callan Gross” relationship. This static model of the proton predicts that the structure functions are independent of \( Q^2 \). This behavior was confirmed at SLAC at \( Q^2 \simeq 4\text{GeV} \) [1] and lead to the general acceptance of the quark model of hadrons [2, 3].

Subsequent measurements revealed that the total four-momentum carried by the quarks was far below the value of 1, predicted by the static model described above. The solution to this problem arrived with the introduction of Quantum Chromodynamics as the theory of strong interaction. In this theory, the strong interaction, which holds the quarks together inside the proton, is mediated by the exchange of gluons. These gluons are neutral particles and so do not directly participate in the DIS interaction. They do however carry a substantial fraction of the proton momentum. Experimental evidence for the existence of the gluons was given by the TASSO, JADE and MarkJ experiments at the PETRA collider at DESY in 1979 [4].

### 2.3 QCD and \( ep \) interactions

Quantum Chromodynamics (QCD) is a non-abelian gauge theory, based on the SU(3) color symmetry group. It describes the interactions of quarks and gluons. Quarks manifest one of the six possible color charges (three fundamental colors and three anti-colors) and interact by exchanging gluons. Gluons carry one color and one anti-color and can therefore interact with each other as well. This is a direct consequence of the non-abelian nature of the underlying symmetry group. The strong force decreases at small distances, a phenomenon called asymptotic freedom. The Standard Model describes the strength of the quark-gluon interaction in terms of the strong coupling constant \( \alpha_s \). The value of \( \alpha_s \) varies with \( Q^2 \). As \( Q^2 \) rises, smaller distances can be resolved inside the proton and smaller values of \( \alpha_s \) are measured. A world summary of measurements of \( \alpha_s \)[5] is shown in Fig. 2.2. Theoretically, the running of \( \alpha_s \) is expressed as:

\[
\alpha_s(\mu_r^2) = \ln \left( \frac{12\pi}{(33 - 2N_f) \ln \left( \frac{\mu_r^2}{\Lambda_{QCD}^2} \right)} \right)\]

(2.7)

where \( \mu_r \) is the renormalization scale, \( N_f \) is the number of active quark flavors in the interaction and \( \Lambda_{QCD} \) is a parameter which is determined experimentally. As \( \mu_r^2 \) increases, for \( \mu_r^2 >> \Lambda_{QCD}^2 \) it holds that \( \alpha_s << 1 \). In this regime, QCD can be
described completely perturbatively such that each higher order Feynman diagram involving a higher power of \( \alpha_s \) will contribute less to the total cross-section than the previous orders in the calculation. In the case when \( \mu_r^2 \sim \Lambda_{QCD}^2 \), \( \alpha_s \sim O(1) \), higher order terms in the perturbative expansion may not converge and therefore non-perturbative approaches are taken, such as phenomenological models based on measurement or numerical methods. In DIS, \( \mu_r^2 \) can be equated to \( Q^2 \).

![Figure 2.2](image)

**Figure 2.2.** World summary of measurements of the running coupling constant \( \alpha_s \), as measured in DIS, \( e^+e^- \) annihilation, hadronic collisions and heavy quarkonia, as a function of \( Q^2 \). The curves are the QCD predictions for the combined world average value of \( \alpha_s(M_{Z^0}) \) (2006).

### 2.4 QCD dynamics and evolution

Inside the proton, quarks continuously exchange gluons and gluons fluctuate in \( q_i\bar{q}_i \) pairs or even interact among themselves (a direct consequence of the fact that gluons also carry color charge). The number of quarks and gluons changes depending on the scale of interaction. Also, the strong force decreases rapidly at small distances:
this is called asymptotic freedom. This allows DIS to be described as a photon interaction with a free quark. Processes like a gluon radiating a quark or a quark splitting into a quark and a gluon can be computed in QCD. The calculation of the cross-section $\gamma^* q \to q g$ yields:

$$\frac{d\sigma}{dp_T^2} = \frac{4\pi\alpha^2}{s} e_q^2 \frac{1}{p_T^2} \alpha_s P_{qq}(z)$$

where $z$ is the momentum fraction of the outgoing quark (w.r.t. its incoming momentum) and $p_T$ is its transverse momentum. The function $P(z)$ is called the splitting function and is proportional to the probability for the quark to split into a quark with a momentum fraction $z$ by radiating a gluon [6]:

$$P_{q\to q}(z) = P_{qq}(z) = \frac{4}{3} \left( 1 + \frac{z^2}{1-z} \right)$$

The divergence at $z = 1$ is due to radiation of very soft gluons (very low energy) and is canceled by virtual loop contributions. Integrating Eq. 2.8 over $p_T$, one obtains:

$$\sigma_{\gamma^* q \to qq} = \frac{4\pi\alpha^2}{s} e_q^2 \frac{\alpha_s}{2\pi} P_{qq}(z) \log \frac{Q^2}{\mu^2}$$

with $\mu$ some minimum momentum of the outgoing quark. The contribution of this process to the structure functions is $Q^2$ dependent. $F_2(x, Q^2)$ becomes:

$$F_2(x, Q^2) = \sum_{q,\bar{q}} e_q^2 \int_x^1 dy \frac{y}{y} q(y) \left( \delta \left( 1 - \frac{x}{y} \right) + \frac{\alpha_s}{2\pi} P_{qq}(\frac{x}{y}) \log \frac{Q^2}{\mu^2} \right)$$

where $y$ is the fraction of the incoming quark w.r.t. the proton momentum and $x$ is the momentum fraction of the secondary quark w.r.t. the proton $z = x/y$. The integral runs over all possible momentum fractions $y$ larger than $x$.

One can also include quark contributions coming from gluon splitting into quarks or other gluon contributions from gluon splitting and so on. The parton (quark and gluon) densities will evolve as a function of the probe scale due to the increase in detail with increasing $Q^2$. This evolution of the PDF’s is given by the following equation:

$$\frac{\partial}{\partial \ln Q^2} \left( q_i(x, Q^2) \right) = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} P_1 \left( \frac{x}{y} \right) \left( q_i(y, Q^2) \right)$$

(2.12)
where $q_i$ denotes all the active (light) quark flavors. This equation is known as the DGLAP equation, after its authors Dokshitzer, Gribov and Lipanov and Altarelli and Parisi[6]. It describes the coupled evolution of quark and gluon densities in the proton. The kernel $P_1$ is given by:

$$
P_1(z) = \begin{pmatrix}
P_{qq}(z) & P_{qg}(z) \\
P_{gq}(z) & P_{gg}(z)
\end{pmatrix}
$$

(2.13)

where one of the entries was made explicit in Eq. 2.9. Each of the splitting functions that enter the kernel are represented schematically in Fig. 2.3.

Through the QCD induced evolution, the parton distributions depend on $Q^2$. By inspecting the proton with finer and finer probes (higher $Q^2$) more and more quarks and gluons can be resolved. We have taken into account so far only leading order (LO) splitting functions. These can be extended to next to leading order (NLO):

$$
\frac{\partial}{\partial \ln Q^2} \left( q_i(x, Q^2) \right) = \frac{\alpha_s}{2\pi} \int_0^1 dy \frac{y}{x} P_1 \left( \frac{x}{y} \right) \left( q_i, g_i \right) + \frac{\alpha_s^2}{4\pi^2} \int_0^1 dy \frac{y}{x} P_2 \left( \frac{x}{y} \right) \left( q_i, g_i \right) + ...$

and so on. The splitting functions for higher orders gain in complexity as they involve more diagrams. The NLO splitting functions are known for some time [6] and recently NNLO splitting functions have been calculated numerically [7]. Possible contributions from NLO diagrams are shown in Fig. 2.7.

**Figure 2.3.** Leading order contribution of different splitting functions

Processes involving higher order QCD interactions give rise to a non-zero coupling between the longitudinally polarized incoming photon and quarks off the mass-shell and consequently $F_L \neq 0$. 


2.5 The improved quark parton model

It was shown that QCD can describe the evolution of the parton distributions inside the proton with $Q^2$. It is however not possible to predict the values of the parton distributions and these must therefore be determined experimentally.

In practice, the DIS structure function data is utilized for this. The method used is to parametrize all parton densities (quark and gluon) at a single value of $Q^2$, $Q_0^2$, as a function of $x$. Given the parton densities, QCD is used to predict the structure functions over a large kinematic range. By comparing the predictions with the measured structure functions, the parameters describing the parton density distributions at $Q_0^2$ can be fitted.

This method has been used by the ZEUS collaboration to determine the parton distributions. The inclusive structure functions measured by ZEUS and several fixed target experiments are given in Fig. 2.5, together with the QCD fit. Excellent agreement is obtained. The parton distributions extracted from these fits are shown in Fig. 2.4. The results are compared to PDFs from several PDF fitting groups, MRST [8] and CTEQ [9].

Figure 2.4. Standard ZEUS NLO QCD fit for $Q^2 = 10 \text{ GeV}^2$. The gluon sea and the up and down valence quark distributions are shown. The shaded band represents the uncertainty. For comparison, MRST and CTEQ fits are also shown.
Figure 2.5. $F_2$ as a function of $Q^2$ for several values of $x$. ZEUS data are compared to fixed-target experiments NMC, BCDMS and E665 as well as to the ZEUS NLO QCD fits.
2.6 Heavy quark production

Production of heavy quarks can be determined at least at moderate $Q^2$ perturbatively, as their masses are larger than $\Lambda_{\text{QCD}}$. The focus will be on $c$ and $b$ quark production as $t$ production is beyond the reach of the HERA accelerator. Above the charm threshold, effectively $Q^2 \sim (2m_c)^2$, $c\bar{c}$ pair production increases steeply, contributing more and more to the $F_2$ structure function with increasing $Q^2$ and decreasing $x$. For instance, at $x \sim 0.01$ and $Q^2 \sim 100 \text{ GeV}^2$, charm contributes approximately 25% to the total NC DIS cross-section.

The dominating process for creating such $c\bar{c}$ pair at HERA is known as boson gluon fusion and schematically represented in Fig. 2.6. In the pQCD inspired picture of the proton, the gluon splits (at leading order) in a off mass-shell $c\bar{c}$ pair which subsequently interacts with the photon. The interaction transfers enough energy to the quark system such that the charm quarks can become on mass-shell. Leading order and next to leading order diagrams are shown in Fig. 2.7.

![Figure 2.6](image)

**Figure 2.6.** (a) The boson gluon fusion process. (b) Different flavor contributions to the NC ep cross section for $0.005 < x < 0.02$. Charm and beauty production decreases steeply at low $Q^2$.

The charm leading-order contribution to the structure function is directly proportional to the gluon density in the proton. The next-to-leading-order contribution also contains a term that follows the gluon density. Therefore, charm production is directly sensitive to gluon density inside the proton. The cross-section for charm in DIS follows from eq. (2.5) (restricted to charm only):}

\[
\frac{d^2\sigma^{c\bar{c}}}{dQ^2dx} = \frac{2\pi\alpha^2}{xQ^2} \left( (1 + (1 - y)^2)F_2^{c\bar{c}}(x, Q^2) - y^2F_L^{c\bar{c}}(x, Q^2) \right)
\] (2.14)
The charm structure functions dependence on the parton densities are known to the next-to-leading order [10]. The $b$ quark is heavier: its production threshold is effectively $Q^2 = (2M_b)^2 \approx 100 \text{GeV}^2$. As $F_2$ is proportional to the square of the quark charge, the beauty production is a factor of $(e_c/e_b)^2 = 4$ suppressed w.r.t. charm production, at infinite $Q^2$. For a $\sqrt{s} = 318 \text{GeV}$, the cross-sections for $ep \rightarrow ec\bar{c} + X$ and $ep \rightarrow eb\bar{b} + X$ are about $0.5 \mu b$ and $1 \text{nb}$ respectively.

**2.7 Charm hadrons**

The calculations of cross-sections for $b$ and $c$ quark production are performed in perturbative QCD. Nevertheless, due to color confinement in QCD\(^1\), the experimentalist measures colorless hadrons (mesons and baryons) which are produced in the fragmentation process. Hadronization cannot be described by pQCD. Therefore, theoretical predictions at hadron level depend on data already collected at other experiments and on empirical models.

Charm hadrons are hadronization products of the charm quark. Some of their properties are listed in the Table 2.1. The lighter mesons, with masses below 2 GeV, are pseudo-scalars. The heavier mesons, containing a "*" in their names, represent excited states of their pseudo-scalar counterparts and are vector mesons with the same quark content but different quark spin alignment. Pseudo-scalars decay weakly: for instance, the two main contributing diagrams in $D^0$ decay are given in Fig. 2.9. There, a charm meson decays into non-charm mesons due to the flavor changing property of the weak decay. The weak decay also dictates the timescale for this decay to happen. In Fig. 2.8, the two main contributions to $D^{*+}$ vector me-

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\(^1\)Free quarks or gluons cannot exist individually. The process of formation of hadrons out of free quarks or gluons is called hadronization.
The probability of charm to hadronize to a particular charm hadron is described by the charm fragmentation fractions $f(c \rightarrow D, \Lambda)$. The fragmentation fractions are assumed to be universal[11]. The fragmentation of the $c$ quark is shown schematically in Fig. 2.10. The branching ratio’s represent world averages, as reported by

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**Table 2.1.** Charm mesons and their properties as listed by Particle Data Book 2006: the valence quarks content, the mass (in GeV) and the mean lifetime (in seconds) are given.

<table>
<thead>
<tr>
<th>meson</th>
<th>$D^0$</th>
<th>$D^+$</th>
<th>$D_s^+$</th>
<th>$D^{*+}$</th>
<th>$D^{*0}$</th>
<th>$D_s^{*+}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>quarks</td>
<td>$c\bar{u}$</td>
<td>$c\bar{d}$</td>
<td>$c\bar{s}$</td>
<td>$c\bar{d}$</td>
<td>$c\bar{u}$</td>
<td>$c\bar{s}$</td>
</tr>
<tr>
<td>$m$ (GeV)</td>
<td>1.864</td>
<td>1.869</td>
<td>1.968</td>
<td>2.010</td>
<td>2.007</td>
<td>2.112</td>
</tr>
<tr>
<td>$\tau$ (s)</td>
<td>$4.1 \cdot 10^{-13}$</td>
<td>$10.4 \cdot 10^{-13}$</td>
<td>$5.0 \cdot 10^{-13}$</td>
<td>$O(10^{-18})$</td>
<td>$O(10^{-18})$</td>
<td>$O(10^{-18})$</td>
</tr>
</tbody>
</table>
Figure 2.9. Main contributions to the $D^0 \rightarrow K^- + \pi^+$ weak decay. (a) The $c$ quark emits a $W^+$ which solely decays into a $\pi^+$ (b) an internal $W^+$ exchange and a creation of $u\bar{u}$ pair out of the vacuum. As these two diagrams interfere constructively, the cross-section for $D^0 \rightarrow K^- + \pi^+$ increases and the $D^0$ lifetime decreases.

the Particle Data Group [12]. A recent ZEUS measurement [11] is shown in Fig. 2.11. After the subsequent strong/electromagnetic decays of $D^{*0}$ and $D^{*+}$, the $D^0$ charm meson is the most abundant.
The numbers indicate the world average values for the fragmentation fractions\cite{12}. The table on the right sums the different branching contributions to the fragmentation fractions of charm to $D^+$, $D^0$ and $D^+_s$.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{charm_fragmentation_tree.png}
\caption{The charm fragmentation tree into vector and pseudo-scalar charm mesons.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{charm_fragmentation_histogram.png}
\caption{Charm fragmentation fractions as measured by ZEUS in photo-production and DIS, compared to H1 and $e^+e^-$ results. More than half of charm quarks fragment into $D^0$ mesons. \cite{11}}
\end{figure}