Spontaneous generation of spiral waves by a hydrodynamic instability
Habibi, M.; Møller, P.C.F.; Ribe, N.M.; Bonn, D.

Published in:
Europhysics Letters

DOI:
10.1209/0295-5075/81/38004

Citation for published version (APA):
Spontaneous generation of spiral waves by a hydrodynamic instability

M. Habibi¹,², P. C. F. Møller¹, N. M. Ribe³ and D. Bonn¹,⁴

¹ Laboratoire de Physique Statistique, UMR 8550 CNRS, École Normale Supérieure - 24, rue Lhomond, 75231 Paris Cedex 05, France
² Institute for Advanced Studies in Basic Sciences (IASBS) - Zanjan 45195-1159, Iran
³ Institut de Physique du Globe de Paris and Université de Paris-7, CNRS, Tour 14 - 2, place Jussieu, 75005 Paris, France
⁴ Van der Waals-Zeeman Institute, University of Amsterdam - Valckenierstraat 65, 1018 XE Amsterdam, The Netherlands

received 25 April 2007; accepted in final form 4 December 2007
published online 3 January 2008

PACS 83.60.-w – Flow instabilities
PACS 47.54.-r – Pattern selection; pattern formation
PACS 47.35.-i – Hydrodynamic waves

Abstract – The coiling of a thin filament of viscous fluid falling onto a surface is a common and easily reproducible hydrodynamic instability. Here we report for the first time that this instability can generate regular spiral patterns, in which air bubbles are trapped in the coil and then advected horizontally by the fluid spreading on the surface. We present a simple model that explains how these beautiful patterns are formed, and how the number of spiral branches and their curvature depend on the coiling frequency, the frequency of rotation of the coiling center, the total flow rate, and the thickness of the spreading fluid film.

The study of spirals in Nature goes back at least to the seventeenth century, when Swammerdam was among the first to describe the beautiful forms of certain seashells [1]. The standard work on spontaneous pattern formation in Nature, D’Arcy Thompson’s On Growth and Form [1] describes a multitude of spiral patterns, including those of shells, sunflower seeds, and the helical structure of branches or leaves on growing plant stems. All these spirals are self-organized and obey rather strict mathematical rules. Shells, for example, are generally logarithmic spirals in which the distance between successive loops grows in a precisely determined way with increasing distance from the center [2]. In the case of sunflower seed spirals (phyllotaxis), Douady and Couder [3] used a clever laboratory experiment to show that the spirals form due to a self-organized growth process whereby new seeds are generated in the center at a fixed frequency and then repel each other by steric repulsion. The maximization of the distance between the seeds then leads to a special subtype of the logarithmic spiral pattern: the golden or Fibonacci spiral. The same authors showed how these ideas can be applied to plants, accounting for instance for the sunflower spirals [4]. Not all natural spirals are due to a steric repulsion between constitutive elements, however. Over the past few years, self-organized spiral waves have been studied extensively [5]. These dynamic spirals form spontaneously in excitable media [6, 7] and have been observed in contexts as different as catalytic surface oxidation [8], the Belousov-Zhabotinsky chemical reaction [9–13], aggregating colonies of slime mold [14, 15] and contracting heart tissue, where such waves are believed to be related to cardiac arrhythmia and fibrillation [16].

Here we demonstrate that spirals can also arise during the coiling of a thin “rope” of viscous fluid falling onto a solid surface (fig. 1) [17–19]. In previous papers we investigated how the frequency and radius of the coiling depends on the orifice diameter, the height of fall, the flow rate, and the fluid viscosity, and we showed that coiling traverses four different dynamical regimes as the fall height increases [20–24]. Here we report that in a limited portion of the parameter space, air bubbles become trapped between successive coils and are then advected radially away to form surprising and very regular spiral patterns. We also present a simple model that explains how these beautiful patterns are formed, and
how the number of spiral branches and their curvature depends on the total flow rate, the fluid film thickness, and ratio of the coiling frequency to the frequency of precession of the coiling center. We find that the spiral waves occur only when the center of the coil precesses with a frequency that is distinct from that of the coiling itself, and we show that this condition is both necessary and sufficient for the appearance of Fermat spirals in this particular case. This is in contrast to the general case where for instance the standard type (n = 1) of Archimedean spiral wave patterns forming in excitable media can exist with a single frequency, which is the primary rotation frequency of the spiral, or alternatively form non-static spirals with two frequencies resulting in a well-studied meandering instability that causes the spiral wave tip to trace out epicycloid trajectories (see for example [5]). The second frequency associated with the meandering instability is generally incommensurate with the primary rotation frequency, which can be formally eliminated by transformation to a co-rotating frame in which the stable rigidly rotating spirals appears stationary. In our experiments the two frequencies are also generally incommensurate, but the second frequency plays a quite different role than in the meandering spirals.

We performed our experiments by ejecting a thin jet of silicone oil from a syringe, driven by a syringe pump with a computer-controlled stepper motor. In a typical experiment, the fluid was ejected continuously at a constant rate Q while the fall height H was varied over a range of discrete values. Silicone oils of viscosities \( \nu = 100, 300, 1000, \) and 5000 cm\(^2\)s\(^{-1}\) were used, but we observed spiral patterns only for \( \nu = 300 \) cm\(^2\)s\(^{-1}\). We also used different orifice diameters \( d = 0.68, 1.5, 1.6, \) and 2.5 mm. While we saw some irregular patterns for \( d = 0.68 \) mm with \( Q = 0.02 \) cm\(^3\)s\(^{-1}\) and \( H = 30 \) mm, clear spiral patterns were observed only for \( d = 1.5 \) and 1.6 mm, \( Q = 0.047−0.137 \) cm\(^3\)s\(^{-1}\), and \( H = 32−50 \) mm.

Figure 1 shows two pictures of steady “liquid rope coiling”. Depending on the fall height and the fluid viscosity, the pile of coils can have different shapes. For low fall heights and high viscosities (“viscous” regime), the pile remains intact for several coiling periods, becomes quite high, and has a shape like a corkscrew (fig. 1a). For somewhat greater heights and/or lower viscosities (called the “gravitational regime”), the pile disappears within one or two coiling periods, and remains low (fig. 1b). No bubbles are generated in either of these cases.

At still larger fall heights (called the “inertial regime”), fluid inertia becomes important. Because the coiling period is much shorter than the time required for an individual coil to coalesce completely with its predecessor, the coiling filament forms a tall liquid tube that builds up, buckles under its own weight at a critical height, and starts rebuilding again with a characteristic period [23]. In this regime we observed bubbles of two different sizes: bubbles smaller than the filament radius that form with a period comparable to that of the coiling; and larger bubbles with sizes comparable to that of the liquid tube that form during the secondary buckling. However, the patterns formed by both types of bubbles are very irregular.

Within a quite narrow portion of the gravitational coiling regime, however, encapsulated air bubbles are observed to form very regular and beautiful spiral patterns (fig. 2). The origin of this behavior is as follows. In all other coiling regimes, each newly formed coil falls exactly on top of the one laid down previously. In this small part of the gravitational regime, by contrast, the center of coiling precesses along a separate circle of its own, with a frequency much smaller \((\approx 25\%)\) than that of the coiling itself. As a result, successive coils do not land exactly on top of one another; and it is at the intersections of two such coils that small air bubbles are formed and trapped.
in the liquid due to its high viscosity. The spiral patterns are then generated as the bubbles are advected radially away from the pile of coils by the flow associated with the pile’s gravitational collapse (fig. 3).

In our experiments, the behavior of the bubbles showed a clear progression as the fall height was increased. At relatively low heights corresponding to the lower-frequency part of the gravitational regime [20–23], the center of coiling precessed and some irregular bubbles were formed. At somewhat greater heights, the bubble pattern became more regular and some rather unclear spiral patterns were observed. At still greater heights, the patterns become clear spirals. Finally, at heights corresponding to the upper end of the gravitational regime the patterns once more became unclear and finally disappeared. The correspondence between spiral patterns and the gravitational regime is illustrated in fig. 4, which shows a numerical prediction of the steady coiling frequency vs. height for the parameters of one of our laboratory experiments [20]. The portions of the curve corresponding to the gravitational regime and the multivalued “inertio-gravitational” regime are labelled (G) and (IG), respectively [22].

In our experiments, the behavior of the bubbles showed a clear progression as the fall height was increased. At relatively low heights corresponding to the lower-frequency part of the gravitational regime [20–23], the center of coiling precessed and some irregular bubbles were formed. At somewhat greater heights, the bubble pattern became more regular and some rather unclear spiral patterns were observed. At still greater heights, the patterns became clear spirals. Finally, at heights corresponding to the upper end of the gravitational regime the patterns once more became unclear and finally disappeared. The correspondence between spiral patterns and the gravitational regime is illustrated in fig. 4, which shows a numerical prediction of the steady coiling frequency vs. height for the parameters of one of our laboratory experiments [20].

In our experiments, the behavior of the bubbles showed a clear progression as the fall height was increased. At relatively low heights corresponding to the lower-frequency part of the gravitational regime [20–23], the center of coiling precessed and some irregular bubbles were formed. At somewhat greater heights, the bubble pattern became more regular and some rather unclear spiral patterns were observed. At still greater heights, the patterns became clear spirals. Finally, at heights corresponding to the upper end of the gravitational regime the patterns once more became unclear and finally disappeared. The correspondence between spiral patterns and the gravitational regime is illustrated in fig. 4, which shows a numerical prediction of the steady coiling frequency vs. height for the parameters of one of our laboratory experiments [20].
Fig. 6: Effect of fall height on the shape of the spiral branches, for an experiment with $\nu = 300 \text{ cm}^2\text{s}^{-1}$, $d = 1.6 \text{ mm}$, and $Q = 0.137 \text{ cm}^2\text{s}^{-1}$. (a) $H = 3 \text{ cm}$; (b) $H = 3.5 \text{ cm}$; (c) $H = 3.7 \text{ cm}$; (d) $H = 4 \text{ cm}$. In all photographs, the coiling is in the same direction and the number of spiral branches is 5. Photos were taken from below; reflection of light from the glass substrate is the cause of the extra “ghost” branches.

On the basis of our experimental observations we now propose a simple model for the formation of the spirals. We have seen that the slow precession of the coiling center causes successive coils to be slightly displaced from each other, leading to the trapping of air bubbles which are subsequently transported radially with the stagnation flow. Assumptions of volume conservation and constant height of the fluid film implies that the radial position of a bubble obeys $\frac{dr}{dt} \sim 1/r$ or $r \sim t^{0.5}$. Since the bubble generator moves with constant angular speed, this gives $r = \pm (0.5) t$, where $r$ is the radius, $a$ some constant, and $\theta$ the angle. Spirals obeying this type of equation are called Fermat’s spirals. To model this we assume that the coiling center moves with frequency $f_p$ on a circle with radius $r_p$. If the radius and frequency of the coiling about this center are $r_c$ and $f_c$, respectively, then the path laid down by the coiling filament is

$$x(t) = r_p \cos(2\pi f_p t) + r_c \cos(2\pi f_c t), \quad (1a)$$
$$y(t) = r_p \sin(2\pi f_p t) - r_c \sin(2\pi f_c t). \quad (1b)$$

We observe experimentally that the coiling and precession are always in opposite directions and since we want to keep $f_c$ and $f_p$ positive we include the minus sign in (1b). An example of the trajectory given by eq. (1) is shown in fig. 7.

Our experiments show that $f_c/f_p \approx r_c/r_p \approx 4$. Now if $f_c/f_p = 4$ exactly, the path defined by eq. (1) will repeat itself after one precession period and the “spirals” will be straight lines pointing outwards from the origin. If however $f_c/f_p$ differs slightly from 4, the path will shift slightly with each precession period and a curved spiral pattern will emerge (fig. 8). As already mentioned, our numerical code does not include the interaction between the coiling rope and the pile so we do not yet understand what causes this precession. We are currently attempting to correctly include this interaction and understand how the precession frequency varies with the experimental parameters and why $f_c/f_p \approx r_c/r_p \approx 4$ for all experiments we performed. This is however far beyond the scope of the present paper. We did observe, however, that the spiral patterns change smoothly with system parameters, indicating that frequency locking does not occur. Our experimental observations indicate that during coiling bubbles are trapped at points 1 through 5 in fig. 8, so that five bubbles will be generated for each four coils. Geometrically speaking, a bubble is formed each time the vector $r_p = r_p(\hat{x} \cos 2\pi f_p t - \hat{y} \sin 2\pi f_p t)$ from the rotation center to the coiling center is antiparallel to the vector $r_c = r_c(\hat{x} \cos 2\pi f_c t + \hat{y} \sin 2\pi f_c t)$ from the coiling center to the filament laid down (see fig. 7). The frequency of bubble formation is therefore just that of the dot product

$$r_c \cdot r_p = r_c r_p \cos 2\pi (f_c + f_p) t, \quad (2)$$

or $f_c + f_p$. The number of bubbles generated per coil is therefore $(f_c + f_p)/f_c$. For the measured value...
In the context of liquid rope coiling, ordered spiral bubble patterns can be formed. The spiral patterns are formed and why two frequencies are needed in this particular case. The specific spiral we observed is a particular type of an Archimedean spiral \( r = a \theta^{1/n} \), namely Fermat’s spiral \( r = a \theta^{1/2} \), which arises because the radial and angular positions of the bubbles obey \( r \sim t^{1/2} \) and \( \theta \sim t \).

**REFERENCES**


