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Reynolds number dependence of drag reduction by rodlike polymers

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We present experimental and theoretical results addressing the Reynolds number \( \text{(Re)} \) dependence of drag reduction by sufficiently large concentrations of rodlike polymers in turbulent wall-bounded flows. It is shown that when \( \text{Re} \) is small the drag is enhanced. On the other hand, when \( \text{Re} \) increases, the drag is reduced and eventually, the maximal drag reduction asymptote is attained. The theory is shown to be in agreement with experiments, explaining the universal and rationalizing some of the the nonuniversal aspects of drag reduction by rodlike polymers. © 2008 American Institute of Physics. [DOI: 10.1063/1.2931576]

I. INTRODUCTION

The aim of this paper is to present data and theoretical results concerning the phenomenon of turbulent drag reduction by rodlike polymers. Drag reduction implies an increase in the mean streamwise velocity (for the same pressure head) in turbulent flows as a result of the addition of tiny amounts of polymers. This implies a wide range of industrial applications and consequently, the effect, discovered in 1944 by Toms, has been studied intensively over the past few decades (see, e.g., Ref. 2). The main aim of this paper is to rationalize and understand the change from drag enhancement to drag reduction by rodlike polymers in turbulent wall-bounded flows as a function of the Reynolds number \( \text{Re} \). To this aim, we present new experimental data and a theoretical analysis.

A classical example of the phenomenon of interest is shown in Fig. 1 which pertains to pipe flow. We denote the velocity field as \( U(r, t) \) and its mean over time as \( V(y) \), where \( y \) is the distance from the wall. With the mean shear defined as \( S(y) = dV(y)/dy \), the friction factor is defined as

\[
f = \tau_w \left( \frac{1}{2} \rho \bar{V}^2 \right) = \left. \frac{d}{dy} \right| \left( \frac{1}{2} \rho \bar{V}^2 \right)
\]

where \( \tau_w \) is the wall shear stress at the wall

\[
\tau_w = \rho \bar{V} S(y = 0)
\]

where \( \nu, \rho, \) and \( \bar{V} \) are the kinematic viscosity, the fluid density, and the mean fluid velocity, respectively. Figure 1 presents this quantity as a function of \( \text{Re} \) in the traditional Prandtl–Kármán coordinates \( 1/\sqrt{f} \) versus \( \text{Re} \sqrt{f} \). Here, \( \text{Re}_f \) is the Reynolds number \( V \ell / \nu_0 \), where \( V \) and \( \ell \) are the channel half width and the kinematic viscosity of the solvent, respectively. The straight continuous line denoted by “N” presents the Newtonian universal law. Data points below this line are indicative of a drag enhancement, i.e., an increase in the dissipation due to the addition of the polymer. Conversely, data points above the line correspond to drag reduction, which is always bound by the maximal drag reduction (MDR) asymptote represented by the dashed line denoted by “M.” This figure shows data for a rodlike polymer (a polyelectrolyte in aqueous solution at very low salt concentration) and shows how drag enhancement for low values of \( \text{Re} \) crosses over to drag reduction at large values of \( \text{Re} \). One of the results of this paper is the theory that will be presented below, which reproduces the phenomena shown in Fig. 1 in a satisfactory manner.

It is interesting to note that with flexible polymers, the situation is very different, and there is no drag enhancement at any value of \( \text{Re} \). The reason for this distinction will be made clear below as well. At this point, we remark that the main difference between flexible and rodlike polymers is that the former need a minimal value of Reynolds number in order to stretch (i.e., undergo the coiled-to-stretched transition, whereas rodlike polymers are “stretched” at any value of the Reynolds number. This consequence of this distinction will be further clarified below.

In Sec. II, we will present new experimental results in which drag reduction is measured as a function of the Reynolds number and for different concentrations of the rodlike polymer. We determine the rheological properties of these solutions simultaneously, so that the rheology of the solutions is known. In Sec. III, we will review the basic theoretical approach to drag reduction, in general, and by rodlike polymers, in particular. In Sec. IV, we will explain how to solve the equations and present the results and their comparison to the experiments. Discussions and the conclusions will be presented in Sec. V.

II. EXPERIMENTAL

A. Flow geometry and polymer

In this section we describe the experimental results obtained using well-characterized rodlike polymer solutions. The turbulence cell has been previously described in detail by Cadot et al.6,7 A turbulent flow is generated in a closed cylindrical cell (volume of 3 l) between two counter-rotating disks spaced by one disk diameter apart. The disks are driven by dc motors. The motors are configured to keep the disks...
rotating at a constant angular velocity $\Omega$, independently of
the torque exerted by the turbulent fluid on the disks. Thus,
all the experiments presented here are performed at constant
angular velocity. All experiments were repeated with pure
water and with the water seeded with the polymers. In both
experiments presented here are performed at constant
shear thinning
viscosity decreases with increasing shear rate; i.e., the fluid is

To determine the energy dissipation in the turbulent
fluid
or drag enhancement

ture increase in time

temperature increase of the fluid: by measuring the tempera-
tion of $Re$ including the water baseline and three different

The symbols represent the
concentrations in wppm as given in the figure.

For the polymer, we use solutions of xanthane, a stiff
rodlike polymer with an average molecular weight

The rheology of the polymer solutions was studied on a
Reologica Stress-Tech (cone-plate geometry) rheometer. The
latter is equipped with a normal force transducer and has a
large cone (55 mm) with a small angle (0.4°) in order to be
able to detect small normal stress differences at high shear
rates. Changing the concentration allows us to modify the
shear-dependent viscosity of the solutions.8–10 The apparent
viscosity decreases with increasing shear rate; i.e., the fluid is
shear thinning (Fig. 2). A satisfactory description of most of
the data for high shear rates can be obtained by using the
Ostwald–de Waele power law model\(^\text{11}\) for the viscosity $\eta$:

$\eta = k_1 S^n$, which quantifies the resis-
tance to stretching of a fluid element. If sufficiently impor-
tant, $\eta_e$ can be measured\(^\text{13}\) by looking at droplet fission with
a rapid camera. The dynamics of the thinning of the filament
that connects the droplet to the orifice can be used to ob-
tain the elongational viscosity, since the stretching of the
filament corresponds to a perfectly elongational flow. By per-
foming this experiment, we find that, for the rigid polymer
solution, $\eta_e$ is so low that the dynamics of the filament is
nearly indistinguishable from that of pure water, for which
$\eta_e = 3 \text{ mPa s}$.\(^\text{14}\)

B. Results

In the upper panel of Fig. 3, the percentage drag reduc-
tion DR(%) is plotted as a function of the Reynolds number
for three different polymer concentrations. Also, in our mea-
surement geometry and with our rodlike polymer at low Re,
a drag enhancement is observed, which smoothly transforms
into a drag reduction at high Re. To verify the transition
between drag enhancement and reduction, we present in the
lower panel of Fig. 3 the dimensionless torque $\beta$ as a func-
tion of Re including the water baseline and three different
concentrations of the polymer. Finally, we present the data
also in Prandtl–Kármán coordinates in Fig. 4. In this paper,
we rationalize and explain the results presented in both Figs.
1 and 3. To this aim, we turn now to a review of the theory
of drag reduction.
III. REVIEW OF THE THEORY OF DRAG REDUCTION BY RODLIKE POLYMERS

In the presence of a small concentration of polymers, the Navier–Stokes equations for the fluid velocity \( \mathbf{U} \) gain an additional contribution to the stress tensor,\(^\text{(4)}\)

\[
\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U} = -\nabla p + \nu \nabla^2 \mathbf{U} + \nabla \cdot \mathbf{\sigma},
\]

\[\nabla \cdot \mathbf{U} = 0.\]

The extra stress tensor \( \mathbf{\sigma} \) is due to the interaction between the polymers and the fluid. The actual form of the stress tensor \( \mathbf{\sigma} \) depends on whether the polymers are flexible or rodlike. The former case was dealt with frequently in literature,\(^\text{15–17}\) however, these attempts to understand the phenomenon tended to be rather qualitative. A quantitative theory for flexible polymers with a predictive power which can explain the measured profiles of the mean velocity as a function of the distance from the wall was achieved only recently.\(^\text{18,19}\) For rodlike polymers, the stress tensor \( \mathbf{\sigma} \) takes the form:

\[
\sigma_{ab} = 6\nu_p n_a n_b (n \cdot \nabla) S_{ij},
\]

where \( \nu_p \) is the viscosity contributed by the polymer in the limit of zero shear, \( n = n(r,t) \) is a unit vector describing the orientation of the polymer, and \( S_{ij} = \partial U_i / \partial r_j \) is velocity gradient. The equation of motion for the orientations of the polymers is approximated by the rigid-dumbbell model which reads

\[
\frac{D R_{ab}}{Dt} = S_{ai} R_{ib} + S_{bi} R_{ia} - 2 S_{ij} R_{ij} R_{ab} - 6 \gamma \left( R_{ab} - \delta_{ab} / 3 \right),
\]

where \( R = nn \) and \( \gamma \) is the Brownian rotational frequency, proportional to temperature.

A. High Re flows

The phenomenon of drag reduction in highly turbulent channel flows can be understood on the basis of the balance equations of the mechanical momentum and turbulent energy.\(^\text{18}\) For a channel flow, we choose the coordinates \( x, y, \) and \( z \) as the lengthwise, wall-normal, and spanwise directions, respectively. The length and width of the channel are usually taken much larger than the midchannel height \( L \), making the latter a natural rescaling length for the introduction of dimensionless (similarity) variables, also known as “wall units.”\(^\text{21}\) By denoting the pressure gradient...
\[ p' = -\frac{\partial p}{\partial x}, \] we define the friction Reynolds number \( \text{Re}_f \), the normalized distance from the wall \( y^* \), and the normalized mean velocity \( V'(y^*) \) (which is in the \( x \) direction with a dependence on \( y \) only) by
\[ \text{Re}_f = \frac{L}{\nu' L_{\nu}}, \quad y^* = \text{Re}_f / L, \quad V^* = V / \nu' L. \]

The balance equations are derived on the basis of the Reynolds decomposition
\[ U_i(r,t) = V(y) \delta_i + u_i(r,t), \]
\[ \sigma_{ij}(r,t) = S(y) \delta_{ij} \delta_y + s_{ij}(r,t). \]

In addition to the mean shear \( S(y) \), we need to introduce the mean turbulent kinetic energy \( K = \langle u'^2 \rangle / 2 \) and the Reynolds stress \( \nu' S = -\langle u_i u_j \rangle \). The momentum balance equation is obtained by averaging Eq. (4) and integrating with respect to \( y \), ending up with the exact equation,
\[ v_0 S + W + \langle \sigma_{xy} \rangle = p' (L - y). \]

The physical meaning of this equation is transparent: the momentum generated by the fixed pressure gradient \( p'(L-y) \) is transported to the wall by the momentum flux (Reynolds stress) \( W \) and dissipated by the Newtonian viscosity \( v_0 S \). We will see below that the mean stress induced by the polymer \( \langle \sigma_{xy} \rangle \) is positive definite and, therefore, is an additional mechanism of dissipation due to the polymers.

In wall units, Eq. (10) can be written in the form
\[ S^* + W^* + \langle \sigma_{xy}^* \rangle = (1 - y^*) / \text{Re}_f, \]
where \( S^* = v_0 S / (\nu' L) \), \( W^* = W / (\nu' L) \), and \( \sigma_{xy}^* = \sigma_{xy} / (\nu' L) \). When \( y^* \) is not too large, e.g., in the log layer, we neglect the second term on the right-hand side (RHS), approximating the RHS as unity.

The balance equation for the turbulent kinetic energy is calculated by taking the dot product of the fluctuation part of Eq. (4) with \( \mathbf{u} \),
\[ WS = \frac{\partial}{\partial y} \langle u_i u_j + u_i p - \sigma_{ij} u_j \rangle + v_0 \langle s_{ij} s_{ij} \rangle + \langle \sigma_{ij} s_{ij} \rangle. \]

Also this equation is exact. We simplify it by noting that the first term on the RHS involves the spatial flux of turbulent energy. It is well-known that in the Newtonian case, this term is negligible in the log layer.\(^{13}\) It was also shown that this term is negligible near the MDR simply because \( K \to 0 \) near the MDR. As we expect \( K \) to decrease with any amount of drag reduction, it is safe to assume that the spatial flux is still negligible also with the polymer added compared to the terms discussed next. This assumption had been verified by direct numerical simulations in Ref. 22. The second term on the RHS represents the Newtonian dissipation which is modeled (in wall units) as\(^8\)
\[ \langle s_{ij}^* s_{ij}^* \rangle = K^* \left( \frac{a}{y^*} \right)^2 + b \left( \frac{K^*}{y^*} \right)^{3/2}, \]
where \( a \) and \( b \) are dimensionless coefficients of the order of unity and \( s_{ij}^* = v_0 p_{ij} / (\nu' L) \) and \( K^* = K / (\nu' L) \). The physical reasons of this model are as follows: When \( y^* \) is small, the dissipation of the turbulent energy is dominated by the viscosity (the first term). When \( y^* \) is large, an eddy with size \( y^* \) and energy \( K^* \) loses its energy in the time scale of \( y^* / \sqrt{K^*} \), which is the eddy turnover time scale. We note that the summation sign in Eq. (13) is merely a pseudosum. It only means that when \( y^* \) is small (large), the first (second) term is more important. Writing in wall unit, Eq. (12) becomes
\[ W^* S^* \sim a^2 \left( \frac{K^*}{y^*} \right)^{3/2} + b \left( \frac{K^*}{y^*} \right)^{3/2} + \langle \sigma_{ij}^* s_{ij}^* \rangle. \]

Finally, we quote the experimental fact that in the log layer, \( W^* \) and \( K^* \) are proportional to each other,
\[ K^* c^2 = W^*. \]

Experimentally, it was found that \( c \approx 0.5 \) in the Newtonian case and \( c \approx 0.25 \) in the MDR.

To estimate the polymer term in the RHS of Eq. (12) (the energy dissipation due to the polymers), we use the results of Ref. 23 where it was shown that
\[ \langle \sigma_{ij} s_{ij} \rangle \sim \nu^* R_{yy} S_{yy} \frac{K^*}{y^*}. \]

Putting into Eq. (12) and writing in wall unit, we have
\[ W^* S^* \sim a^2 \left( 1 + \nu^* R_{yy} \right) S^* + b \left( \frac{K^*}{y^*} \right)^{3/2}. \]

Equations (17) and (19) imply that the polymers change the properties of the flows by replacing the viscosity by
\[ \nu_{eff} = 1 + \nu^* R_{yy}. \]

This means that polymeric flows can be reasonably described by changing the "effective viscosity" of the solution. In the fully developed turbulent flow, it was shown in Ref. 23 that \( R_{yy} \) depends on \( K^* \) and \( S^* \),
\[ R_{yy} = \frac{K^*}{y^* S^*}. \]

It was argued in Ref. 23 that for large \( \text{Re} \), \( K^* \) grows linearly with \( y^* \) and, thus, the viscosity profile is linear. Furthermore, it was shown theoretically in Ref. 24 that if the effective viscosity varies linearly with \( y^* \), i.e.,
\[ \nu_{\text{eff}} = 1 + \alpha(y^+ - y^+_p), \]  

then \( c \) must satisfy the relation

\[ \frac{a}{c} = \frac{\delta}{1 - \alpha \delta}. \]  

(23)

Here, \( \delta \approx 6 \) is the width of the viscous sublayer in the Newtonian flows.

For a given \( \nu_{\text{eff}} \), Eqs. (15), (17), (19), and (23) form a complete set of equations for variables \( S^* \), \( W^* \), \( K^* \), and \( c \). This model has been studied extensively for Newtonian flows\(^{23} \) and for the flows with linear viscosity profile.\(^{24} \) The resulting velocity profiles agree reasonably with the experimental results. It was found that in the viscous sublayer, the resulting velocity profiles agree reasonably with the experimental data. Moreover, an edge solution for \( \alpha = 1/12 \) was observed.\(^{24} \) If \( \alpha > 1/12 \), the model no longer has a solution corresponding to a turbulent flow and, therefore, the flow must be laminar. The special case of \( \alpha = 1/12 \) was identified as the MDR of the polymeric drag reduction.

**B. Low Re flows**

According to Eq. (20), the value of \( \nu_{\text{eff}} \) depends on \( \nu_p^* \) and \( R_{yy} \). The value of \( \nu_p^* \) is determined by the polymer properties such as the number of monomers, their concentration, etc., and thus, \( \nu_p^* \) should be considered as an external parameter in the equation. The value of \( R_{yy} \), on the other hand, depends on the properties of the flow. In the case of laminar flow with a constant shear rate, i.e., \( \nu_{\text{eff}} = W^* = 0 \) and \( S^* = \text{constant} \), it was shown theoretically in Ref. 20 that

\[ R_{yy} = \frac{2^{1/3}}{\text{De}^{2/3}}, \]

(24)

where the Deborah number \( \text{De} \) is defined by \( \text{De} = S/\gamma \). Thus, the effective viscosity is reduced if \( \gamma \) is increased and, therefore, the rodlike polymers solution is a shear-thinning liquid. Naturally, the value of \( \text{De} \) changes with \( \nu_{\text{eff}} \). To clarify this dependence, we consider the momentum equation [Eq. (10)] at \( y = 0 \) in the Newtonian case,

\[ \nu_0 S = p^* L. \]  

(25)

Usually in experiments, system size and the working fluid remain to be the same. Therefore, \( \nu_0 \) and \( L \) are constants and so, \( \text{Re}_\tau \) depends on \( p^*/L \) only. According to Eq. (7), \( \text{Re}_\tau \) grows as \( \sqrt{p*/L} \), and therefore,

\[ \text{De} = \frac{\nu_0}{\gamma L^2 \text{Re}_\tau^2}, \]

(26)

by Eq. (25). Putting into Eq. (24), we have

\[ \nu_{\text{eff}} = 1 + \nu_p^* \frac{\lambda}{\text{Re}_\tau^{4/3}}, \]

(27)

where \( \lambda = \nu_0/\gamma L^2 \) is a constant.

**C. Intermediate Re flows**

In the case of intermediate \( \text{Re} \), we need an interpolation between Eqs. (21) and (24). To do this, we note that when \( y^+ \) is small, the solution of Eqs. (15), (17), and (19) result in \( W^* = K^* = 0 \) in the viscous sublayer. This implies that the flow cannot be highly turbulent in the viscous sublayer. Thus, it is reasonable to employ Eq. (24) as long as \( y^+ \) is small. On the other hand, as the upper bound of \( y^+ \) is \( \text{Re}_\tau \) when \( y^+ \) is large, it automatically implies that \( \text{Re}_\tau \) is large. The laminar contribution is, therefore, negligible as it varies inversely with \( \text{Re}_\tau \). The effective viscosity due to the polymer is dominated by the turbulent estimate [Eq. (21)]. To connect these two regions, we simply use the pseudosum,

\[ \nu_{\text{eff}} = 1 + \nu_p^* \left[ \frac{\lambda}{\text{Re}_\tau^{4/3}} + \frac{K^*}{(y^+ S^*)^2} \right] = g + \nu_p^* \frac{K^*}{(y^+ S^*)^2}, \]

(28)

where \( g = 1 + \nu_p^* \lambda/\text{Re}_\tau^{4/3} \). One can see that the limits for both high and low \( \text{Re}_\tau \) are satisfied.

**IV. SOLUTION AND RESULTS**

**A. Setting up the equations**

Having the expression for \( \nu_{\text{eff}} \), we have to specify the value of \( c \) in Eq. (15) to complete the equations. This variable naturally depends on \( \alpha \) and \( g \). The latter dependence, however, can be eliminated by rescaling the dimensional variables. Define

\[ \text{Re}_\tau = \frac{L \sqrt{p^* L}}{\nu_0}, \quad \frac{y^+}{\text{Re}_\tau} = \frac{y}{\nu_0 \sqrt{p^* L}}, \quad v^+ = \frac{V}{\sqrt{p^* L}}. \]

(29)

In these units, Eq. (28) is written as

\[ \tilde{\nu}_{\text{eff}} = 1 + \tilde{\nu}_p \frac{\tilde{K}^*}{(\tilde{y}^+ \tilde{S}^*)^2}, \]

(30)

where \( \tilde{\nu}_p = \nu_p/g \), \( \tilde{S}^* = S^* g \), and \( \tilde{K}^* = K^* \). We can find the \( \alpha \) dependence by comparing Eq. (30) to Eq. (22) and identifying the width of the viscous sublayer \( y^+_p \) with \( a/c(\alpha) \). This stems from the continuity of \( \tilde{S}^* \) at the boundary of the viscous sublayer. This means that

\[ a/c(\alpha) = \tilde{y}^+_p. \]

(31)

Combining Eqs. (22), (23), and (31), the relationship between \( \tilde{\nu}_{\text{eff}} \) and \( c \) is

\[ \tilde{\nu}_{\text{eff}} = 1 + \frac{a - c \delta}{a \delta} \left( \tilde{y}^+ - \frac{a}{c} \right). \]

(32)

We note also that Eqs. (22) and (30) can be written as

\[ \tilde{K}^* = A^2 (\tilde{S}^* \tilde{y}^+)^2, \]

(33)

with

\[ A^2 = \frac{\tilde{\nu}_{\text{eff}} - 1}{\tilde{\nu}_p}. \]

(34)

By using Eqs. (15) and (33), we can rewrite Eqs. (17) and (19) as two equations for the two variables \( \tilde{\nu}_{\text{eff}} \) and \( \tilde{S}^* \),
the variables in wall unit by using Eq. 065108-6 Amarouchene et al. using Eqs. H20849/H9261

is reproduced within the model, as shown in Fig. 5. H9261

y and then joins the MDR until it begins to cross back to the H9263

drag reduction. Once MDR but with a smaller intercept, signaling less efficient H9270

in size, but then the velocity profile becomes parallel to the H20849

viscous layer becomes rather large, of the order of 20. The H20849

results for H9261

we first solve the model with Eq. H9263

B. Comparison of analysis with experiments

To compare our analytical results with the experiments, H20849

we need to relate Re, to Re as used in that figure. The relation is H20850

ReL = Re, VL,

where VL is the velocity at the center of the channel. Figure H20850

7 shows f−1/2 as a function of log10(ReLf1/2) with various values of νp. One can see that for large νp, we obtain a drag enhancement when ReL is small. This enhancement decreases with increasing ReL, and eventually for large enough ReL, there is drag reduction. The maximum amount of drag reduction, however, cannot exceed the MDR. For small νp, the drag enhancement is nearly unnoticeable. However, drag reduction in this case is smaller for large ReL because the Newtonian plug occurred earlier. It should be noted that the curves in Fig. 7 exhibit the “ladder” characteristics for different values of νp. This was identified as “type B” drag reduction by Virk et al.4 and is the fundamental characteristic of drag reduction by rodlike polymers. We see that our re-

Finally, we can solve Eqs. (32) and (38) to get \( \bar{v}_{\text{eff}}(y^*) \) for different values of \( \nu_p \). Then, we can obtain \( \bar{S}^* \) and \( K^* \) by using Eqs. (33) and (37), respectively. Finally, we reexpress the variables in wall unit by using Eq. (29).

B. Comparison of analysis with experiments

To compare our analytical results with the experiments, we first solve the model with Eq. (28) for Re,=590 and for \( \lambda=0 \), i.e., the high Re limit. This demonstrates that the MDR is reproduced within the model, as shown in Fig. 5.

We observe that \( V^* \) is increasing with a constant rate for small \( y^* \) and then follows the log law of the MDR. The results for \( \lambda=1 \) are shown in Fig. 6. We note that for \( \nu_p^* =100 \), the velocity profile first follows the viscous profile and then joins the MDR until it begins to cross back to the Newtonian plug. For \( \nu_p^* =1000 \), the viscous layer is increased in size, but then the velocity profile becomes parallel to the MDR but with a smaller intercept, signaling less efficient drag reduction. Once \( \nu_p^* \) reaches the value of 10,000, the viscous layer becomes rather large, of the order of 20. The velocity profile still succeeds to run parallel the MDR but with a much reduced intercept.

FIG. 5. (Color online) The velocity profile as predicted by the model with \( \nu_p^* =10^4 \) and Re,=590.

FIG. 6. (Color online) The velocity profile predicted by the model with Re,=590 and \( \lambda=1 \) for various values of \( \nu_p^* \).

FIG. 7. (Color online) \( f^{-1/2} \) as the function of log10(ReLf1/2) with \( \lambda=1 \) with various values of \( \nu_p^* \).
results agree reasonably well with the experiments presented in Prandtl–Kármán coordinates.

To compare the theory to the experimental results in DR(%) versus Re coordinates, we use the experimentally measured values of $g$ and $r_p$ to predict the amount of drag reduction. As we mentioned before, the effective viscosity in laminar flow of xanthan gum (XG) solutions is well approximated by Ostwald–de Waele model. This implies that in the low-Re flow, $g$ is given by (in MKS units),

$$g = k_1 S^{-1} = k_1 \left( \frac{\nu_0 \text{Re}^n}{L^2} \right)^{-1},$$

(40)

where the last equality is obtained by multiplying Eq. (26) by $\gamma$. The experimental value of $r_p$ for XG solution is given by (in MKS units)

$$r_p = 0.011 \ 147 C^{1.422} \nu_0,$$

(41)

where $C$ is the concentration of XG in wppm. The value of $f$ can then be calculated by (i) obtaining the experimental values of $C$, $k_1$, and $n$ for Eqs. (40) and (41), (ii) rescaling the variables $S$, $K$, and $W$ by Eq. (29), and finally, (iii) solving Eqs. (32) and (38) to get $f$ as a function of Re. It should be noted that in the present experiments, the definition of Reynolds number is, in general, different from our definition in Eq. (39). For the sake of comparison of the theory with the experiments, we assert that two Reynolds numbers must be proportional to each other. The easiest way to find the proportionality factor is to compare the point of transition from drag enhancement to drag reduction for a given polymer concentration (making sure that this ratio of proportionality does not depend on the polymer concentration). The result of this is the relation

$$12 \text{Re}_L \approx \text{Re}_\text{exp}.$$

(42)

Figure 8 shows the comparison of the percentage of drag reduction (enhancement) between the theoretical predictions and the experimental results. The two data sets shown pertain to $c = 250$ wppm ($k_1 = 11.04$, $n = 0.727$) and $c = 500$ wppm ($k_1 = 40.62$, $n = 0.607$). The agreement between theory and experiment is very satisfactory. We reiterate that in the comparison of theory and experiment, there is no free parameter employed, all the constants were measured directly in the experimental apparatus as explained above.

V. DISCUSSION AND CONCLUSIONS

In summary, we studied the Re dependence of the drag reduction by rodlike polymers. We showed that when the value of Re is small, the drag is enhanced due to the homogeneous increase in the effective viscosity. When Re is sufficiently large, due to the turbulent activity, the effective viscosity varies as a function of the distance to the wall. As the result, the amount of drag is reduced. We use a simple interpolation between low Re and the high Re flows to account for this. The resulting theoretical results agree semiquantitatively with the experiments both in Prandtl–Kármán coordinates and in DR(%) versus Re coordinates.

It should be noted that only rodlike polymers exhibit drag enhancement for low Re. For flexible polymers, the drag is the same as that of the Newtonian flow for low Re, and only after a critical value of Re, the drag reduction sets in. This difference in behavior from rodlike polymers is because the flexible polymers are coiled when Re is small. They do not affect the flow unless Re increases enough to allow the shear to develop to affect the coil-to-stretch transition in the flexible polymers. In contrast, rodlike polymers are always extended and, therefore, they can affect the flow for all Re.

We note that in this paper, the theory had been employed to understand the mean velocity and percent drag reduction only, but the same theory can also provide without further experimental input the profiles of Reynolds stress and kinetic energy. For such comparisons of theory with numerical simulation, the reader is referred to Ref. 22.

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