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A comparative taxonomy of medieval and modern approaches to Liar sentences*

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Abstract:

Two periods in the history of logic and philosophy are characterized notably by vivid interest in self-referential paradoxical sentences in general, and Liar sentences in particular: the later medieval period (roughly from the 12th to the 15th century), and the last hundred years. In this paper, I undertake a comparative taxonomy of these two traditions. I outline and discuss eight main approaches to Liar sentences in the medieval tradition, and compare them to the most influential modern approaches to such sentences. I also emphasize the aspects of each tradition that find no counterpart in the other one. It is expected that such a comparison may point in new directions for future research on the paradoxes; indeed, the present analysis allows me to draw a few conclusions about the general nature of Liar sentences, and to identify aspects that would require further investigation.

Key-words: Liar sentences; medieval; modern; taxonomy; *insolubilia*.

Two periods in the history of logic and philosophy are characterized notably by vivid interest in self-referential paradoxical sentences in general, and Liar sentences in particular: the later medieval period (roughly from the 12th to the 15th century – they were then known as ‘insolubles’, *insolubilia*) and the last hundred years – roughly since the discovery of what is now known as Russell’s paradox. If only for this reason, a comparative analysis of these two frameworks, the purpose of the present study, is an endeavor worth undertaking. But beyond this historical interest, it is to be expected that

* Forthcoming in *History and Philosophy of Logic*.

we shall gain fruitful insights on the phenomenon of paradoxical sentences from such a comparison. Modern philosophers and logicians are very likely to benefit from it, insofar as the resemblances as well as disanalogies between medieval and modern treatments are likely to point at new directions on how to handle paradoxes. In fact, this has already happened: some modern approaches to paradox (to be mentioned throughout the paper) are overtly inspired by medieval solutions, attesting to the fruitfulness of the comparative approach.

Indeed, a number of modern authors have already turned to medieval authors for inspiration in their treatment of self-referential paradoxes – Arthur Prior and his keen interest in Buridan is perhaps the best example thereof. But a systematic comparison of the two traditions has not yet been undertaken, and this is precisely what I intend to do here. The comparison will proceed as follows: each of the main trends among medieval theories of *insolubilia* is presented, and immediately following each of them an analysis of modern solutions resembling the specific medieval approach in question (if any) is given.¹ It is important not only to mention such modern solutions, but also to analyze in some depth the aspects of resemblance. In particular, it may occur that a given modern approach resembles more than one medieval approach, but for different reasons. Then, in a final paragraph, I outline features of currently influential modern solutions that do not

¹ For reasons of space and for methodological coherence, I shall focus on the literature on the Liar paradox in English. However, it must be noted that there is a rich tradition on paradoxes to be found in other languages. A. Rüstorff's *Der Lügner. Theorie, Geschichte und Auflösung* (1910), for example, is a classic.

find a counterpart in the medieval tradition. Again, I am just as interested in the dissimilarities between the two traditions as I am in their similarities.

A preliminary remark: perhaps the most noticeable dissimilarity between the two traditions is the place occupied by paradoxes of the Liar family in the larger context of semantic theories and philosophical/logical systems in general. While for medieval logicians such paradoxes were among the most important logical topics (especially in the 14th-15th century), they were seen as particular cases of logical puzzles which served to test the robustness of a given semantic system or the acumen of a given logician; such logical puzzles received the general denomination of *sophismata*, and *insolubilia* were a particular genre thereof.² In other words, while *insolubilia* were considered to be a very important topic, this was not because they were thought to threaten the very possibility of logic, semantics and knowledge, as is the case in recent developments (what some have described as the modern ‘crisis mentality’ with respect to such paradoxes – see *Spade 1982, 253*).

² For the present purposes, I focus on insoluble sentences of the Liar family, that is, sentences that assert the falsity of a sentence (itself or other), leading to paradox. The typical case is of course ‘This sentence is false’, but I also consider quantification over sentences such as ‘Every sentence is false’ or even ‘Some sentence is false’ under the assumption that there are no other sentences, as well as reciprocal Liar sentences: Socrates says ‘What Plato is saying is false’ while Plato says ‘What Socrates is saying is true’. But to be sure, there is a considerable range of insoluble sentences, including epistemic and practical paradoxes, which shall not be directly dealt with here. As for the terminology, I will use the terms ‘Liar sentence’ and ‘insoluble’ interchangeably, usually (but not necessarily) the medieval term for medieval contexts and the modern term for modern contexts. When this does not occur, it should not be interpreted as having any kind of deeper meaning.

Indeed, recent treatments of paradoxes of the Liar family are largely motivated by the perception that no theory of truth, in fact no logic and perhaps even no knowledge, is possible without an adequate taming of such cases. Another term often used to refer to such phenomena is that of ‘semantic pathology’, and that gives the measure of how threatening Liar-like paradoxes are now considered to be. As Barwise and Etchemendy put it, ‘the significance of a paradox is never the paradox itself, but what it is a symptom of’ (*Barwise & Etchemendy 1987*, 4); the medieval authors, by contrast, were mostly interested in the paradoxes in themselves, as particularly difficult logical puzzles. But one should also not go as far (as Spade (1982, 253) did³) as saying that the medievals did not draw any theoretical lessons from insolubles; in fact, in many of such treatises we also find analyses of the notions of truth and sentential meaning which can be seen as the germinal stages of full-fledged theories of truth (and which were later transferred to treatises specifically on truth, such as Paul of Venice’s *Tractatus de veritate et falsitate propositionis*).

2. The Taxonomy

³ But notice that, while I may occasionally disagree with Spade’s interpretation of the material, anyone working on medieval insolubles (in fact, on later medieval logic and semantics in general) is profoundly indebted to his work. He not only catalogued manuscripts and edited/translated many of the *insolubilia* treatises which are now available in modern editions, but his analyses of the material are also exceptionally profound and challenging. In fact, it is hard to imagine what working in this field would even be like if it weren’t for Spade’s impressive contribution to it.

A taxonomy of medieval solutions to paradoxes of the Liar family is greatly facilitated by the tendency that the medieval authors themselves had towards taxonomies and towards listing other prominent views in their writings. The typical scholastic manner of arguing consisted in first laying down a nominal definition of the topic to be discussed, and then inventorying the most influential opinions on the topic held by previous authors. Subsequently, the author went on to argue against these opinions before presenting his own. Therefore, the medieval texts themselves are a valuable source for a (presumably) complete picture of the different positions held with respect to Liar sentences in the later medieval period.

Two texts that present particularly comprehensive lists are Bradwardine's treatise on *insolubilia* (written in the 1320)⁴ and (written almost a century later) Paul of Venice's treatise on *insolubilia* (part II.15 of his *Logica Magna*, which, unlike several other parts of this work, has unfortunately not yet been given a modern edition).⁵ This is important, because Bradwardine's text is widely acknowledged (see *Spade 2005*; *Read 2002*) as a watershed between two very distinct periods in the medieval *insolubilia* literature. This means that in Bradwardine's text one finds a comprehensive list of positions held thus far, which is nicely complemented by Paul's list presenting the post-Bradwardine

⁴ Edited in *Roure 1970*, but currently in the course of receiving a critical edition with translation by S. Read, to appear with Dallas Medieval Texts and Translations.

⁵ For the present analysis, I rely on Read's forthcoming edition of Bradwardine's text and on Spade's description of Paul's text in his *Spade 1975*. Simmons (1993, ch. 5) also relies significantly on Paul's text; he was able to use an unpublished edition/translation of the text (by M. McCord Adams). The text is otherwise unfortunately of difficult access.

developments (in fact, Paul includes the positions described in Bradwardine's text as well). Paul's list is rather long and contains fifteen items, of which eight are taken from Bradwardine (see *Spade 1975*, 82); not all of them will be equally important for the taxonomy to be presented here, but one could hardly hope for a better starting point.

But let us follow the natural chronology of events and start with Bradwardine's text. In total, Bradwardine considers nine different opinions, including his own, and these in fact correspond to five main opinions and their subdivisions. For the present purposes, these opinions can be classified as follows, presented roughly in their chronological order of appearance in the *insolubilia* literature – some opinions may appear more than once in this classification, as they may combine two essentially distinct approaches:

1. *Cassantes*: he who utters an insoluble sentence says nothing (the sixth and, in some senses, the fifth opinions of Bradwardine's list).
2. Solutions based on Aristotelian fallacies: *figura dictionis*, *non causam ut causam*, equivocation, *secundum quid et simpliciter* (the second, third, eighth and ninth opinions, the last one being Bradwardine's own solution).
- 2'. *Distinguentes*: two distinct acts are involved in the utterance of an insoluble (the eighth opinion).
3. *Restringentes*: restrictions on self-reference (the first, second and fourth opinions).
4. *Mediantes*: an insoluble is neither true nor false (the seventh opinion).

Bradwardine treats the opinion of the *distinguentes* as based on the fallacy of equivocation: in fact, this is precisely what he reproaches to this opinion, that it does not (as it should) solve the insolubles by means of the fallacy *secundum quid et simpliciter*.⁶ However, one of the main versions of the *distinguentes* solution known to us, that of Duns-Scotus,⁷ is explicitly (and convincingly) cast in terms of the *secundum quid* vs. *simpliciter* dichotomy. Therefore, the position of the *distinguentes* will be treated in the section on Aristotelian fallacies, since it is either based on equivocation or on *secundum quid et simpliciter*.

In the post-Bradwardine period, several other influential positions were put forward by authors such as Buridan, Heytesbury, Swyneshed, Albert of Saxony and Peter of Ailly. Some of them fit into one of the categories already proposed, but others require the creation of new categories. Let us thus proceed following Paul of Venice's list of opinions. The first seven, as already said, correspond to positions listed in Bradwardine's text (Paul omits the first one of Bradwardine's list). The eighth and ninth correspond to the second and third opinion of Heytesbury's list, the eighth being a mixture of the *cassantes* and of the *mediantes* position and the ninth a variation of the *mediantes* position: an insoluble is either-true-or-false, but it is not necessarily either true or false. These two positions (as well as Heytesbury's first position, according to which two contradictories may be both false) indicate that category 4 above, that of the *mediantes*, is

⁶ A text representing the position that Bradwardine describes, that is, the *distinguentes* position but not associated with a fallacy, is *Insolubilia Monacensia* – see Nuchelmans 1988.

⁷ In his *Quaestiones Elenchorum*, Question LIII.

more generally formulated as any rejection of bivalence, that is, of the position that any sentence is either true or false and nothing else, and of the principle of non-contradiction, according to which a sentence and its contradictory cannot both be true or both be false at the same time.

The tenth position of Paul's list is essentially Bradwardine's position, and thus belongs to the category 2 of solutions based on an Aristotelian fallacy. The eleventh position is Albert of Saxony's, and it deserves a new category of its own:

5. Solutions based on the idea that every sentence signifies/implies its own truth.

This is Albert's as well as Buridan's position with respect to insolubles.

The twelfth position, which is Heytesbury's own (and fourth of Heytesbury's list) also deserves a category of its own: it is the position according to which an insoluble must be treated with respect to a given context, what Heytesbury spells out in terms of the obligational framework and the notion of *casus* (more on obligations and *casus* below). It can be described as

6. The contextualist approach to insolubles.

The fourteenth opinion⁸ is, according to Spade, Walter Sexgrave's opinion, and it is a refined version of the *restringentes* position 3. Paul's own position is finally presented, and it is a version of Swyneshed's position, according to which the problem with insolubles is that they are self-falsifying, and hence false (following a reformulation of the notions of truth and falsity so as to include a clause concerning self-falsification). This position too deserves a category of its own:

7. Insolubles are self-falsifying.

The thirteenth position (chronologically, the last to have appeared) is, according to Spade (1975, 83), 'a schematic version of Peter of Ailly's'; the backbone of Peter's position is that insolubles do not occur in mental language because self-reference does not occur in mental language. For this position too we may want to create a new category, although in many senses it resembles the restriction to self-reference of category 3.

8. Insolubles do not occur in mental language.

These eight positions seem to encompass the main trends within the medieval literature on insolubles. Naturally, it may very well be that solutions which have not been

⁸ I change the order of presentation with respect to Paul's list because his thirteenth position is chronologically the last one to have appeared. He presents Swyneshed's position as the last one in his list because it is the one he favors himself, but chronologically Swyneshed's position should even come before Heytesbury (indeed, Spade believes Swyneshed's treatise to have been written before Heytesbury's – cf. Spade 2005).

considered for this analysis will contain yet different approaches to insolubles which are not accounted for by these eight categories. Moreover, as with all taxonomies, there is a considerable degree of arbitrariness in the manner in which these categories have been divided; indeed, in many cases specific solutions are in fact a combination of more than one of such categories. Nevertheless, it seems fair to say that this taxonomy represents the most influential positions with respect to insolubles in the period from the rediscovery of such self-referential paradoxes, in the 12th century, to the late 14th/early 15th century.⁹

3. The categories

I now analyze in more detail each of these positions, including a brief inventory of the authors actually having held them as well as a comparison of each of them with some modern approaches to Liar-like paradoxes.

1. *Cassantes*

The earliest known treatise on insolubles, known as *Insolubilia Monacensis*, (*De Rijk* 1966; de Rijk dates it at the end of the 12th century) maintains the position on such paradoxes known as that of the *cassantes*, that is, the position according to which he who utters an insoluble ‘says nothing’, fails to make a statement. The term *cassantes* is related

⁹ Other influential texts tend to list roughly the same positions. Ralph Strode, for example, analyzes the positions of Bradwardine, Heytesbury, Swyneshed and Robert Fland, basically accepting Fland’s position (a variation of Bradwardine’s and Heytesbury’s solutions).

to cancellation, annulment; it is originally a legal term, but which was used in the context of the regimented form of oral disputation known as *obligationes* (more on which in section 6 below – see also *Martin 2001*, 78).¹⁰ Interestingly, we know of only one other treatise also maintaining this position (see *Spade 1975*, 43-44), but it was nevertheless extensively mentioned (and criticized) in different medieval treatises, including Bradwardine's and Paul of Venice's. In fact, one may say that the *cassantes* position was never really given much of a chance in the medieval period after around 1225 (until then it was said to be widely held – see *Spade 1982*, 246), as it was usually fairly quickly dismissed. Bradwardine, for example, says that “because these nullifiers [*cassantes*] [...] appear so very asinine, we do not need to argue with them any further” (5.6 in Read's forthcoming edition). Bradwardine is content to notice that, if Socrates speaks and is heard, then obviously he has said something. A slightly more charitable version of the *cassantes* position is discussed (but also dismissed) in the text formerly ascribed to William of Sherwood (edited in *Roure 1970*). However, the position as presented in the anonymous *Insolubilia Monacensis* is in fact much more sophisticated than the strawman attacked by Bradwardine (see *Goldstein 2007*, 76-77).

Obviously, it all depends on one's interpretation of the phrase ‘says nothing’. Bradwardine and others took it quite literally and thus thought it absurd and contrary to senses that he who utters an insoluble ‘says nothing’, since sounds, syllables and words

¹⁰ The fact that this term, imported from the obligational framework, is used to name the earliest medieval position with respect to Liar sentences is one of the strongest pieces of evidence presented by Martin (2001) to corroborate his (very convincing) thesis that the origin of the medieval *insolubilia* literature is to be found in connection with obligations.

come out of his mouth. Of course, to ‘say nothing’ can mean many things beyond that; to say nothing may mean that one has failed to say anything meaningful (such as if I utter ‘babubabu’). But even then it seems that the utterer of an insoluble cannot be said to have said nothing; apparently, his statement is meaningful, as it is clearly understood by the speakers of the language in question. Finally, on a third level, to say nothing may be understood as the speaker failing to make a statement, even though he has uttered a meaningful and well-formed sentence. The challenge for the advocate of this latter approach is to explain why the speaker failed to make a statement even though he has uttered something meaningful.

A few modern authors have been significantly more charitable towards the ‘says nothing’ approach than the post-1225 medieval authors. Some decades ago, Ryle (1950) defended views on paradoxes quite akin to the *cassantes* position. More recently, Smiley (1993), Beall (2001) and mainly Goldstein (2001; 2006; 2007) have defended versions of this position. Beall (2001) enlists the second sense of ‘saying nothing’ outlined above to conclude that the usual paradoxical sentences are meaningless, which he does in connection with deflationist theories of truth (see *Armour-Garb 2001* for a critique of Beall’s position). In other words, the different modern approaches to Liar sentences which maintain that such sentences are meaningless – fail to express a proposition, fail to make a statement etc. – are thus all variations of the *cassantes* position.

Most notably, in a series of papers, Goldstein presents a sustained defense of the *cassantes* approach; indeed, he never fails to mention that his approach to paradoxes is

overtly inspired by the medieval *cassantes*. As I read him, he is stressing that there are ways of making sense of ‘saying nothing’ even with respect to meaningful sentences. He is thus defending the third sense of ‘saying nothing’ as described above. For Goldstein, a Liar sentence fails to make a statement because it fails to say something having a truth-value (albeit meaningful); another way of putting it is that the utterer of a Liar sentence has uttered a sentence but has failed to express a proposition. This kind of failure can be compared to instances of presupposition-failure (see *Goldstein 2006*, fn. 5) or even to noun-phrases that, even though they are well-formed and meaningful, fail to pick out a referent (because it happens not to exist, such as the current king of France, or because it violates the very nature of the object mentioned, such as ‘The natural number that is the successor of itself’ – *Goldstein 2006*, 874). Goldstein’s task is then to spell out the principles of ‘saying something’ that are being violated in the case of Liar sentences such that they fail to express a proposition and hence to be truth-apt (*Goldstein 2007*, 76). To my mind, he does it in quite a convincing way, showing for example that the sentences of the Yablo paradoxical sequence fail to make statements due to underspecification (*Goldstein 2006*, 874).

In sum, while it was a widely known position in the medieval period, the *cassantes* position was not really given the attention it deserved after circa 1225. It is based on the sound idea that grammaticality and even meaningfulness are not sufficient conditions for making statements; one of its strengths is that it gives rise to discussions on the conditions for making statements (a topic very dear to Austin, for example) in order to demonstrate how the paradoxical sentences of the Liar family fail these conditions.

2. Solutions based on Aristotelian fallacies

Contrasting with the not so influential (at least in the 14th and most of the 13th century) *cassantes* position, the approach to *insolubilia* based on different Aristotelian fallacies was certainly the most influential well until the 14th century – Bradwardine for example dismisses a few of the solutions he examines simply because they are not, as they ought to be, based on the fallacy *secundum quid et simpliciter*.

To be sure, with respect to *insolubilia*, the fallacy *secundum quid et simpliciter* was by far the most important one. Other fallacies that are mentioned in connection with *insolubilia* are *figurae dictionis*, *non causam ut causam* and equivocation (all three mentioned by Bradwardine and Paul of Venice) and the fallacy of accident (the fourteenth opinion of Paul's list). Here, however, I will only comment on those solutions based on *secundum quid et simpliciter* and on equivocation, as these offer the most interesting opportunities for comparisons with modern solutions to Liar sentences. (Moreover, according to Bradwardine, solutions based on the fallacies *figurae dictionis* and *non causam ut causam*, at least as he describes them, are in fact variations of the *restringentes* approach).

But before that, let me make a general remark concerning approaches based on fallacies and a few modern solutions to the Liar. First, let us recall what a fallacy is: it is an **argument** which appears to be cogent but is not, and the different kinds of fallacies are

meant to identify the source and cause of this merely apparent correctness. So, clearly, what is at stake with fallacies are (erroneous) inferential moves. In particular with respect to *insolubilia*, the starting point for using the fallacies framework is the recognition that there is a problem with the **inference** that a Liar sentence is false from the assumption that it is true, or that it is true from the assumption that it is false (the two inferential legs of the paradox). Different authors will claim to show, based on the different fallacies, what is wrong with these apparently sound inferences.

Interestingly, a few modern authors have also identified an inferential problem as the cause of paradox in the case of Liar sentences. In terms of the very important distinction between a diagnosis and a solution to the Liar paradox, introduced by Chihara (1979), the diagnosis in these cases would be that there is an inferential problem, and the solution would consist of somehow blocking the inferential step(s) that is (are) the source of trouble. For most of these modern authors, it is not so much that fallacious, i.e. mistaken, principles of inference are being used in the reasoning that leads to paradox, but rather that a naïve theory of truth plus a naïve theory of inference, i.e. one containing the usual principles of classical logic, is bound to lead us straight into paradox. Hence, according to this view, we must either give up our naïve theory of truth (and this is the gist of the Tarskian approach, which presents a hierarchical notion of truth), or we must reformulate our logic, that is, the principles of inference that we recognize as valid. The principles of inference which are then to be excluded are not ‘fallacious’ properly speaking; rather, they are ‘sacrificed’ in order to maintain consistency and avoid paradox. A proponent of this approach is H. Field, who in several papers (such as 2002 and 2003) has investigated

what the minimal reformation of classical logic ought to be so that the naïve theory of truth can be kept, while paradox is also avoided. Other non-classical solutions to the Liar, such as those based on paraconsistent systems, also belong to this general approach. (See also *Restall 2007* for the limitations of the general enterprise of reforming the logic in order to avoid paradox, most notably the thread posed by Curry's paradox.)

But notice that, while there is a perceptible similarity between medieval approaches to *insolubilia* based on fallacies and the non-classical revisions of classical logic which seek to maintain the naïve theory of truth, some medieval solutions based on fallacies (in particular *secundum quid et simpliciter*) are nevertheless revisions of the concept of truth themselves. This is for instance the case of Bradwardine's solution, which we will discuss in more detail below.

2.a Equivocation – *Distinguentes*

The eighth opinion discussed by Bradwardine is that of the *distinguentes*, those that claim that, when Socrates utters 'Socrates utters a falsehood' and nothing else, this occurrence of 'utters' is equivocal and thus must be distinguished. It is equivocal in that there are two different senses of 'utters' involved, '*dicere exercitum*' and '*dicere conceptum*'. As it stands, Bradwardine's description of this position seems rather odd in that it is not clear why it can even be presented as a solution to insolubles in the first place. But as we have already seen, Bradwardine is not always very charitable in his description of the different opinions to which he objects. The position seems to consist in attributing truth to

Socrates' utterance insofar as 'utters' refers to the *actus exercitum*, but falsity to it if read according to the other sense of uttering, '*dicere conceptum*'.

As for other authors: although Duns Scotus, for example, does not explicitly associate his treatment of the insolubles with the fallacy of equivocation (in his *Quaestiones Elenchorum*, Question LIII) – but rather with the fallacy *secundum quid et simpliciter* – his solution is also a version of the *distinguentes* position. He distinguishes the '*actus significatus*' from the '*actus exercitus*' involved in the assertion of 'I say a falsehood'. In the case of '*ego dico falsum*', the *actus significatus* is what is signified by the sentence, in this case the act of saying a falsehood, while the *actus exercitus* is what is accomplished by the speaker (its performative dimension). When the speaker utters 'I am saying a falsehood', he performs a true *actus exercitus*, since he is indeed saying a falsehood; but that means that his utterance is true only *secundum quid*, that is, with respect to the *actus exercitus*; but the truth *simpliciter* of a sentence depends on the *actus significatus* involved in it, and since what it signifies is false, it is a false sentence. Notice that, although it resembles the eighth opinion of Bradwardine's list, the distinction introduced by Scotus is between *actus significatus* and *actus exercitus*, while the distinction as described by Bradwardine is between '*dicere exercitum*' and '*dicere conceptum*'.

In terms of modern solutions, those that identify an ambiguity in the meaning of the words involved in the paradox – 'to say', 'true', 'false' – follow thus a similar strategy (such as *Parsons 1974*). Below we will refer to solutions that identify a change in the

extension of the truth predicate; but some go as far as saying that it is not only its extension that changes, but in fact its very meaning (for a variety of reasons).

For example, the approach to the Liar proposed by Barwise and Etchemendy, based on the idea of situational semantics, consists in recognizing that the very utterance of a Liar sentence has different meanings (must be distinguished, in medieval parlance) in different situations. Here is V. McGee's account of their position:

The Liar sentence "This proposition is false" does not express a uniquely determined proposition (even taking for granted that "this" is used reflexively).

The proposition expressed by uttering the sentence varies, depending upon the situation. In situation s , the Liar sentence expresses a proposition f_s that says that f_s is false. The falsity of f_s is not a part of situation s , and so f_s is false.

Let us now consider some more inclusive situation t that includes s and also includes the semantic facts about s . In t to say, referring to f_s , "That proposition is false" will be true, for the falsity of f_s is part of t . Thus in t , unlike s , to assert that f_s is false will be true. (*McGee 1991*, 472)

Barwise and Etchemendy then avoid the paradoxical circle of the falsity of a Liar sentence being inferred from its truth and vice-versa from the fact that the situation in which f_s comes out as false is different from the situation in which it comes out as true. This approach is also fundamentally a contextual approach to Liar sentences, more of

which will be discussed below, but the idea that a Liar sentence does not express a unique proposition very much resembles the medieval notion of sentences that must be distinguished, and the fact that different uses of ‘true’ and ‘false’ in the usual paradoxical reasoning refer to different situations seems to indicate in the direction of a fallacy of equivocation.

Indeed, Gauker (2006, 396) notices that this strategy of revealing the apparent paradoxical nature of Liar sentences by claiming that the context in which the sentence is judged to be false is not the same as the context in which it is judged to be true is essentially the attribution of the fallacy of equivocation to the presumed-to-be paradoxical reasoning that infers the truth of a Liar sentence from its falsity and vice-versa. “In one way, this argument is just a fallacy of equivocation.”

2.b *Secundum quid et simpliciter*

As already mentioned, most of the first medieval treatments of *insolubilia* were explicitly related to the fallacy *secundum quid et simpliciter* – well until Bradwardine, who also claims that his solution is fundamentally based on this fallacy. Indeed, in the period until Bradwardine (thus roughly from the end of the 12th century, when the first treatments of *insolubilia* were proposed, until the first decades of the 14th century), the wide majority of treatments of *insolubilia* were to be found either in commentaries or in sets of questions on Aristotle’s *Sophistical Refutations* (see a representative list of texts in Spade 1987).

However, Spade has claimed in several places that such references to the fallacy *secundum quid et simpliciter* were in most cases purely ‘honorary’, and that this in any case held of the later solutions such as Bradwardine’s. But it has been argued (cf. Dutilh Novaes & Read forthcoming) that many of these solutions, Bradwardine’s in particular, are indeed very much in the spirit of this fallacy.¹¹ In other words, the framework of the fallacy is actively being put to use in these solutions to Liar sentences; therefore, it seems unjustified to view such mentions to *secundum quid et simpliciter* as purely honorary.

It is important to notice that the various solutions based on this fallacy differ from each other in significant ways, and thus should not be treated as a homogeneous block.¹² However, for reasons of space, it is impossible here to treat many such solutions, and for this the reader is referred elsewhere (cf. Dutilh Novaes & Read forthcoming). We shall focus on Bradwardine’s solution, not only because it was very influential in the 14th century (see Read’s ‘Introduction’, section 6, in his forthcoming edition and translation of Bradwardine’s *Insolubilia*), but also because it is an exceptionally sophisticated solution to Liar sentences – one that, when transposed to modern discussions, yields an appealing solution to the paradox also from a modern point of view.

¹¹ Simmons (1987 and 1993, 5.2.2) has made a similar point with respect to Pseudo-Sherwood’s and Ockham’s respective solutions, both overtly cast in terms of this fallacy.

¹² Some of the other authors who explicitly use this fallacy in their solutions to the Liar are Simon of Faversham, Lambert of Auxerre/Lagny, and Duns-Scotus, as already mentioned. For the first two, see the relevant texts in Pozzi 1987.

The cornerstone of Bradwardine's solution is the claim that sentences may, in fact typically do, say more than one thing. In particular, Bradwardine proves, on the basis of a few solid theoretical assumptions, that all sentences that say of themselves that they are false also say of themselves that they are true. Moreover, he reformulates the traditional correspondentist definition of truth – a sentence is true iff it signifies things to be as they are – so as to include universal quantification in it: a sentence is true iff it signifies things to be **only/howsoever** as they are. In other words, if sentence P says two things, namely A and B, and A is the case whereas B is not, then the sentence is false absolutely (*simpliciter*), even though it does say something that is the case, namely A, and thus is true in a certain respect (*secundum quid*). Now, since the Liar sentence says of itself that it is false and also that it is true, it is true only *secundum quid*, in a certain respect, namely when it says that it is false, but it is false *simpliciter*, since two of the things that it says¹³, namely that it is false and that it is true, cannot both obtain. Therefore, there is at least one thing that it says that does not obtain, and thus it is not a true sentence (as it violates the universal clause that defines truth). In sum, for a sentence to be true *simpliciter*, all the things it says must obtain; if only one or some of the things it says do obtain but not the others, then it is true merely *secundum quid*.

S. Read has recently published several articles (2002, 2006, forthcoming a and forthcoming b) where he investigates and argues in favor of Bradwardine's solution. He also goes on to show how the schema of Bradwardine's solution can be transposed to

¹³ One should not exclude the possibility of it saying yet other things.

modern discussions and even handle several other paradoxes related to the Liar, such as Curry's paradox, the postcard paradox and Yablo's paradox.

Besides Read's recent 'campaign' in favor of a Bradwardinian solution to paradoxes of the Liar family, which is thus overtly inspired by a medieval solution, there is another group of modern solutions to the Liar that resembles the medieval *secundum quid et simpliciter* approach. These are described by Robert L. Martin in the following terms: "the truth predicate is indexical, changing extensions at various levels without changing meaning." (*Martin 1984*, 3) Burge (1979; 107 in *Martin 1984*) for example sees the truth predicate as a "schematic predicate expression [true_i] [...]. The numerals substituted for ' i ' mark not new predicate constants, but contextual applications of the indexical 'true'."

In a sense, the idea that the truth predicate can be ascribed to something *secundum quid* or *simpliciter* amounts precisely to holding that there are different extensions for the predicate 'true'. There is the extension of 'true' *simpliciter*, absolutely, and there are different extensions of 'true' *secundum quid*, in a certain respect – for each specific '*quid*', there would be a different extension, echoing the idea that the truth predicate is indexical. Note that in this case the term 'truth' is not thought to change meaning, in which case we would have an occurrence of equivocation, as already discussed above; the fallacy *secundum quid et simpliciter* allows for the formulation of the idea that a predicate such as 'true' may have different extensions while retaining the same meaning.

3. Restrictions on self-reference

The *restringentes* approach, the gist of which is restriction on self-reference (hence the name), was indeed one of the most popular approaches to *insolubilia*, certainly until Bradwardine; but even after Bradwardine's sharp critique of this position, it remained influential.

The cornerstone of the *restringentes* position was to restrict (or altogether ban) the possibility of a phrase in a sentence standing for the very sentence in which it occurred. In modern terms, this 'standing for' is usually understood as the relation of reference, and thus this position can be roughly described as a restriction on self-reference. For the medieval authors, this position was cast in terms of the concept of supposition. It was often justified by means of the fallacy *secundum quid et simpliciter* (for example in Lambert of Auxerre/Lagny and Ockham), but it was also often formulated on its own, such as in Burley.¹⁴ Pozzi 1987, which is an extremely useful collection of medieval *insolubilia* texts, lists 6 *restringentes* texts: an anonymous *Sorbonnensis*, (Pseudo-) Sherwood, Lambert of Auxerre/Lagny, Simon of Faversham, Walter Burley and William of Ockham.

The general principle supporting this position goes as follows: 'the part cannot signify or supposit for the whole' (to be found thus formulated and later criticized in Bradwardine's treatise, in the first paragraph of ch. 3). The key idea of applying this principle to a Liar sentence is that, in 'This sentence is false', 'false' cannot supposit for this very sentence,

¹⁴ The relevant texts can all be found in Pozzi 1987 (in Latin and Italian).

which is the *suppositum* of the subject-term; and since this is an affirmative sentence, if subject and predicate do not supposit for the same, the sentence is false.¹⁵ Similarly, in ‘This sentence is not true’, ‘true’ cannot supposit for this very sentence; since this is a negative sentence, if subject and predicate do not supposit for the same, the sentence is true (see *Simmons 1993*, 86). The paradox is thus avoided, because the (affirmative) Liar sentence clearly infringes a necessary condition for truth (co-supposition), and thus it is plainly false.

It is clear that several problems can be raised concerning the restrictionist principle. First of all (and this is a point that I have not yet seen raised elsewhere), if ‘true’ and ‘false’ (the predicates) cannot supposit for the very sentence in which they occur, how come the subject, ‘This sentence’, does supposit for the very sentence in which it occurs? If the restrictionist principle is to be applied in an unrestricted way, then since the phrase ‘This sentence’ is also a part of the whole sentence, it could not supposit for it either. Not that the medieval authors explicitly say that the subject ‘This sentence’ can supposit for itself while the predicate cannot; but their discussions are invariably on the supposition of the predicate and simply ignore the subject of a Liar sentence. That the same problem could

¹⁵ In the 14th century, it was very common to spell out the truth-conditions of sentences in terms of the supposition of its terms. In the first chapters of the second part of his *Summa Logicae*, Ockham presents the following truth-conditions: in the case of affirmative sentences, they are true if the predicate supposits (at least) for the same thing(s) as the subject; in the case of negative sentences, they are true if this does not occur – which can happen in two ways, namely if the subject does not actually supposit for anything (in the case of empty names) or if it does but there is no intersection between its supposita and the supposita of the predicate.

be raised concerning the subject is, to my knowledge, a fact never explicitly noticed by a medieval author.¹⁶

Secondly (and this is a point often raised in the literature), there are several entirely unproblematic, i.e. non-paradoxical, cases of terms occurring in sentences suppositing for the very sentence in which they occur. ‘This sentence has five words’ is unproblematic; the sentence has indeed five words, so it is a rather plain true sentence, even though subject and predicate supposit for the whole in which they occur. ‘Every sentence is negative’ is a bit more problematic, but this is because it falsifies itself every time it is uttered; it is, however, quite straightforward to establish its truth value: it is false in and of a context where the very sentence exists, but it will be true of the counterfactual situation in which there are only negative sentences (and thus the sentence itself does not exist).

In other words, the problem is obviously that the restrictionist principle generally applied bans perfectly fine examples of self-reference alongside with the problematic ones. This was indeed one of Bradwardine’s objections to this approach (which he corroborates by means of a passage from Aristotle’s *Metaphysics* where the Philosopher is clearly allowing for the part to supposit for the whole – see 3.1.1 in Read’s forthcoming edition of the text). Many medieval authors were aware of this problem, and thus put forward

¹⁶ What comes closest to such a realization is the position known as *transcasus* (essentially in the same spirit as the restrictionist approach) (see *Spade 2005*, 2.2; Bradwardine comments on it in 5.4 of his treatise), which consisted in maintaining that, when uttering (L) ‘This sentence is false’, what is referred to is a sentence previously uttered by the speaker, and not the very sentence (L).

weaker versions of the restrictionist position, where the part could not supposit for the whole only in some cases; the issue was then to specify such cases, and to provide a rationale for excluding only such cases that would not be entirely ad-hoc – ‘such-and-such cases are excluded because they are problematic’ is clearly not a very satisfying explanation since we are still not told what the origin of the problem is (see *Spade 2005*, 2.4 for a short but incisive description of the problem, and *Spade 1974* on Ockham’s unsatisfying reformulation of the restrictionist approach).

It has been argued (in *Simmons 1993*, 5.2.1) that, in the cases of Ockham and Burley, what we have is not the use of the restrictionist principle but in fact a contextual approach to *insolubilia* (in the sense that the context broadly understood would determine the supposition of the terms of a sentence). This interpretation is appealing, but there is no doubt that, broadly understood, any restriction to the supposition of a term in a sentence for the whole sentence is a variation of the restrictionist approach. So it still seems reasonable to classify these positions among the restrictionists; we shall return to their contextualist element in the section on contextualism below.

In terms of modern approaches, the restrictionist approach is often compared to the Tarskian-hierarchy of languages approach (*Tarski 1983*). Indeed, there is an undeniable similarity between the medieval diagnosis that the predicates ‘true’ and ‘false’ cannot supposit for the sentence where they occur, and the aspect of Tarski’s ‘sanitation’ of language for scientific purposes that concerns the semantic predicates such as ‘true’ and ‘false’, namely that they cannot be satisfied by the very sentences in which they occur. In

both cases the basic idea is the ban or restriction on sentential self-reference. But while the medieval solution simply bans or restricts supposition of the part for the whole and otherwise leaves the language as is, Tarski's proposal consists, as is well known, in a whole reformation of the language in hierarchies, such that the semantic predicates of a given level (in particular 'true') would only be present in the higher levels of the language. There is nothing among medieval solutions that really mirrors the Tarskian hierarchical structure. As Simmons (1993, 93) puts it:

Ockham, Burley, and Pseudo-Sherwood have no need of a distinctions between levels of discourse. They proceed by placing restrictions on certain occurrences of the semantic predicates and by this alone.

Nevertheless, the kinship between the *restringentes* solution and the Tarskian approach (whose starting point is to restrict self-reference; the hierarchy of languages is **how** this is done) should be duly acknowledged. They start with the same diagnosis, namely that the source of problem in a Liar sentence is the fact that a semantic predicate occurring in such a sentence has the very sentence in its range of application, but propose different treatments: a ban or restriction on the supposition of the part for the whole vs. a hierarchy of languages. The result in the latter case is that the sentence 'This sentence is not true_{*i*}' pertaining to level *i* of the language is simply ill-formed, as the predicate 'true_{*i*}' is not a part of level *i* of the language.

4. Rejection of bivalence and non-contradiction: non-classical solutions

In the medieval *insolubilia* literature, the position according to which an insoluble has no truth-value, or else a third truth-value, is very much underrepresented. Bradwardine refers to it under the name of ‘*mediantes*’, and dismisses it swiftly by making appeal to the principle of bivalence (§ 5.7):

[...] the middle way, whose proponents are so called because they say that an insoluble is neither true nor false, but in the middle indifferent to both. But they are mistaken, for every proposition¹⁷ is true or false, so since an insoluble is a proposition, an insoluble is true or false.

In our current knowledge of the medieval *insolubilia* literature, the only author who seems to have held such a position (and even that is open to discussion) was Swyneshed; however, Swyneshed’s text was composed after that of Bradwardine, so there must surely have been other authors (albeit apparently not many of them, considering that we have so far been unable to uncover texts representing this position) who held such a position so as to justify its inclusion in Bradwardine’s list.

Swyneshed seems to claim that bivalence fails only in very few cases (not even in all Liar-like sentences); one of such cases is ‘This sentence signifies other than things are’

¹⁷ Notice that, in the quotations of translations of medieval texts, the term ‘proposition’ appears in its Latin acceptance of *propositio*, i.e. equivalent to what I elsewhere in this text refer to as declarative sentences.

(see §§2-3 of Swyneshed's treatise (*Spade 1979*)).¹⁸ Spade (1983, 107-8; 2005, 3.2) notes that it is not entirely clear how Swyneshed's purported solution to this paradoxical sentence actually avoids paradox, so one can say that a more thorough investigation of Swyneshed's position would be required at this point.¹⁹ In any case, with Swyneshed we have the explicit statement that there are sentences which are 'neither true nor false' (§2)²⁰, and the statement that some Liar sentences such as the one above are of this kind (other Liar sentences are plainly false on account of infringing his condition that a sentence should not be self-falsifying in order to be true – see *Spade 1983* and section 8 below).

Furthermore, Swyneshed drew three conclusions from his postulates that were widely seen as troubling by his contemporaries (see *Spade 2005*, 3.2). The one that matters most for our present purposes is: 'in the case of insolubles, two mutually contradictory propositions [sentences] are false at the same time' (*Spade 2005*, 3.2). This position is interesting in that it bears some resemblance with modern dialethism, i.e. the idea that two contradictory sentences can both be true at the same time (see *Priest 2004*). With Swyneshed we have the reverse of dialethism, what we could call 'di-pseudism'²¹: some

¹⁸ On the use of the notion of what a sentence signifies to define truth, in particular in connection with insolubles, see *Read 2002*.

¹⁹ Given the overview purpose of this piece, this remains a project for future research.

²⁰ In §§28-30, Swyneshed also examines and rebuts a possible objection to this truth-value gap approach based on an authoritative text by Aristotle; he presents an alternative reading for the Aristotelian text according to which the necessity of bivalence does not necessarily follow from it.

²¹ Unfortunately, the word for 'falsity' in Greek is significantly less nice than the word for 'truth'.

pairs of mutually contradictory sentences can be both false at the same time, in particular insolubles.

Moreover, as already mentioned, some of the positions in Paul of Venice's list resemble the *mediantes* position in that they reject what we would now call some basic principles of classical logic. Obviously, this is an anachronistic characterization of these positions, but for the purposes of the taxonomy being presented here, this appears to be a useful classification, in particular with respect to the comparison with modern solutions to be presented shortly. The eighth position of Paul's list is a mixture of the *cassantes* and of the *mediantes* position and the ninth is a variation of the *mediantes* position: an insoluble is either-true-or-false, but it is not necessarily either true or false. These two positions; Swyneshed's claim (also the first position in Heytesbury's list) that two contradictories may be both false; and Bradwardine's description of the '*mediantes*', seem to be the closest medieval counterparts to modern non-classical solutions to Liar-like paradoxes.²²

This is indeed an important contrast with modern treatments of Liar sentences. As we know, the truth-value gap approach has been one of the most popular approaches in the last 30 years, in particular since the publication of Kripke's 'Outline of a theory of truth'

²² Here I refer to the non-classical position with respect to truth-values, namely the rejection of bivalence and of non-contradiction. Earlier in this paper I have referred to the non-classical approach which consists of reformulating or rejecting classical rules of inference; obviously, these two non-classical approaches are related, since a rejection of the traditional view on truth-values often (but not necessarily) leads to a revision of the classical rules of inference; but these are nevertheless essentially different non-classical approaches.

in 1975.²³ Before that, van Fraassen (1968) had already advanced similar views, and Martin and Woodruff (1975) reached results similar to Kripke's independently, at roughly the same time. Since then, many have taken further the idea of truth-value gaps for Liar sentences, in particular Gaifman (1992, 2000). The goal of the gappy approach is to respect some of our intuitions about the languages we speak, in particular that they are semantically closed (i.e. they contain their own semantic predicates such as 'true' and 'false', something that is lost in the traditional Tarskian approach) while also avoiding inconsistency, by means of partial truth-evaluations.

The basic idea of this approach is very similar to the gist of the *mediantes* position: some sentences, for instance Liar sentences, are neither true nor false; they either have no truth-value at all, or they have an alternative, third truth-value, GAP for example (see Gaifman 1992). Of course, this approach is known to be prey to the so-called strengthened Liar: paradox strikes again if the Liar sentence is formulated in slightly different ways, such as 'This sentence is not true' or 'This sentence is gappy'. Its proponents must then find ways to block the paradoxical circle that arises again with the strengthened Liar, what is usually done using different technical devices, and with varying results.²⁴

²³ As is well known, the concept of truth-value gaps is only one of the elements of Kripke's mathematically sophisticated theory of truth. The other key concept in the construction is that of a fixed-point, but this latter concept has no particular relevance for the comparison being presented here.

²⁴ It has been argued that the problem that gap-solutions have with the strengthened Liar in fact arises just as much with the weaker versions of the paradox (Rieger 2001). The real problem for any version of the paradox would be indeed that of revenge.

But besides the challenge posed by the strengthened Liar, the main problem faced by the proponents of the gap approach is to provide a convincing explanation for why certain sentences do not have one of the traditional truth-values; that this is the case is but a partial solution to the problem. One must also give an account of what is special about the sentences that are gappy, i.e. why they fail to have a truth-value.²⁵ The most widely accepted account of this phenomenon is the idea that they are **ungrounded**, that is, that there is no basic sentence (with an unproblematic truth-value) on which their truth-value depends: the process of evaluation of their truth-value never hits the bottom line (see *Kripke 1975*). This occurs with classical Liar sentences which are circular (and thus ungrounded), but also with sequences of sentences such as those of Yablo paradox (see *Yablo 1993*), which form an unending chain of sentences each of which has a truth-value that depends on other sentences which are equally ungrounded. Another way to express the intuition that gappy sentences, Liar ones in particular, are ungrounded is to compare sentences to computer programs, and to associate gappy sentences to non-terminating programs: they just keep running without ever being able to terminate, i.e. to establish their truth-values (see *Gaifman 1992 and 2000*).

Interestingly, dialethism (which we have already encountered in connection with Swyneshed's di-pseudism) purports to offer an answer to the issue of paradoxical Liar sentences that is the dual of the truth-value gap approach: a truth-value glut approach. While according to the gap approach Liar sentences would have no (classical) truth-

²⁵ By contrast, I am not aware of any medieval discussion on what differentiates sentences lacking a truth-value from the 'well-behaved' ones.

value, the glut approach contends that such sentences do have a classical truth-value, in fact **both** classical truth-values; they are true **and** false. Within dialethism, this is of course not a problem, since it is perfectly alright for a Liar sentence and its contradictory to be both true, which in practice is tantamount to the Liar sentence having both truth-values. Proponents of the glut approach claim that, unlike the gap approach, they avoid being dragged back into paradox by the strengthened Liar (see *Priest 1979*); but it has been argued recently (*Restall 2004*) that the gap and the glut approaches are in fact symmetric and thus equivalent in the results obtained with respect to Liar sentences.

In sum, we have a neat symmetry between ‘non-classical’ medieval and modern treatments of the Liar: both traditions feature truth-value gap approaches (the ‘Kripkean’ approach and the *mediantes*) as well as truth-value glut approaches (dialethism and Swyneshed’s di-pseudism). The main difference is that, while non-classical approaches are quite popular now, their medieval counterparts never matched the level of popularity of other approaches such as the *restringentes*’ approach.

5. Every sentence signifies/implies its own truth

In the last few decades, the medieval solution to Liar sentences that attracted the most attention was John Buridan’s solution, even though recent years have witnessed growing interest for other authors as well (in particular Bradwardine). Buridan’s main text where *insolubilia* are discussed, his treatise on *sophismata*, has received quite a few different

translations and editions (Scott, Hughes, Klima, Pironet), and has been extensively discussed by influential authors such as Prior.

At first sight, it seems that Buridan's solution is in fact in the same spirit as Bradwardine's (even though Buridan does not mention the fallacy *secundum quid et simpliciter* as Bradwardine so emphatically does). Bradwardine's solution consists in proving that a sentence that says of itself that it is false also says of itself that it is true, and thus is false (in fact, impossible), since it says (at least) two things which cannot obtain simultaneously. At an earlier stage, Buridan had also claimed that a Liar sentence signifies of itself that it is true (in his *Quaestiones Elenchorum*), but he later rejected the notion of what a sentence signifies in his mature theory, and offered an account of the truth of a sentence in terms of the co-supposition of its terms instead. However, the co-supposition criterion is not sufficient to block paradox in the case of a Liar sentence; for such sentences, Buridan adds, the co-suppositional criterion must hold also of the sentences implied by the sentence in question. Now, a Liar sentence virtually implies²⁶ its own truth, and therefore the strengthened co-suppositional criterion cannot obtain: 'This sentence' cannot both supposit for 'false', in the original Liar sentence, and for 'true', in its implication 'This sentence is true' (see *Read 2002*). Therefore the Liar sentence fails to satisfy the (strengthened) criterion for truth, and is thus plainly false.

²⁶ Buridan's notion of virtual implication is related to his staunch commitment to actually formed tokens: "The point of the qualification 'virtual' in this phrase seems to be that for this implication to hold the consequent need not actually be formed, although, of course, the antecedent has to exist (for otherwise it could not be said to be true)." (*Klima 2004*, 105)

But there is a fundamental difference between Buridan's and Bradwardine's solutions: Buridan attributes the property of signifying/virtually implying its own truth to **each and every** sentence, a claim nowhere made by Bradwardine. Unlike Bradwardine, who proves of **some** sentences that they signify their own truth (namely those that say of themselves that they are false), Buridan seems to take it to be a general and intuitive fact about sentences and the act of uttering them that they (virtually) imply their own truth. In particular with respect to the Liar, it turns out to be false because (as already diagnosed by Bradwardine) it signifies/implies two things that cannot obtain simultaneously, its truth and its falsity.

Besides Buridan, another influential author to have held this position was Albert of Saxony. Albert returns to the Bradwardine/young Buridan notion of what a sentence signifies in order to define its truth, and he argues for the claim that every sentence signifies its own truth by combining the notion of signification of a sentence with that of co-supposition of its terms with respect to truth (see *Read 2002*).²⁷

Read (2002) has recently argued that Buridan's solution is problematic, leading to a number of counterintuitive results, and that Bradwardine's solution is vastly superior. This is not the time or place to discuss the flaws of Buridan's solution; what is important for the present purposes is to stress the fact that, while it has received the most attention from modern logicians and philosophers, Buridan's solution is certainly not the only one deserving our attention.

²⁷ Buridan had argued in a similar fashion for this position in his *Quaestiones Elenchorum*, p. 92.

There are a few modern solutions to the Liar paradox which resemble Buridan's, some of which are overtly inspired by the medieval author. As is well known, Arthur Prior on many occasions turned to Buridan for inspiration for new developments, such as his distinction between 'possible' and 'possibly-true' (in *Prior 1969*) which in many senses anticipates two-dimensional semantics (see *Dutilh Novaes 2005b*). With respect to the Liar, inspired by Buridan and Peirce, Prior (*1976*) discusses the view that every statement includes an implicit assertion of its own truth; from this he concludes that a Liar sentence is equivalent to the statement 'This sentence is false and this sentence is true', which is a plain contradiction, as much as any of the form 'A & ~A', and thus is plainly false (see also *Prior 1962*, 298). In (*Prior 1976*, 141) he even professes a preference for Buridan's earlier theory, in terms of the signification of the sentence.

The issue is thus about the status of this 'implicit assertion' of truth, which Buridan formulated in terms of a 'virtual implication'. Does it belong to the level of the speech act being performed, to be understood as a Gricean maxim to the effect that asserting includes the implicature that what is being asserted is true (thus following from the Maxim of Quality)? Or is the relation between an assertion and the assertion that it is true of a logical nature, indeed one of the directions of Tarski's T-schema? If the former, then the assertion of the Liar sentence is something like a performative contradiction, similar to someone uttering 'I cannot speak' – the very act of asserting it contradicts the content of the assertion. But many logicians would rather have a purely 'logical' solution to the Liar paradox, i.e. one that does not identify a failure of the speech-act performed but

rather attributes a definitive truth-value to the Liar sentence and thus escapes paradox (for example, by identifying it as an outright contradiction). A fairly recent solution to the Liar based on the idea that every sentence (logically) implies its own truth (and, for as far as I know, with no special connection to Buridan's solution) can be found in (*Mills 1998*). Epstein (*1992*) presents a theory of truth/solution to paradoxes that is overtly inspired by Buridan's solution, and where two traits of the latter figure prominently: the fact that tokens are truth-bearers (to be discussed in more detail in the next section) and the validity of the principle of truth-entailment, as Epstein names it – i.e. that every sentence implies its own truth.

6. Contextualist approaches to the Liar

If there is one approach whose medieval popularity is on a par with its current popularity, it is most certainly the contextual approach. But notice that there are significant differences between the two contextualist traditions, as much as there are fundamentally two distinct levels of contextualism with respect to paradoxes.

The first important difference is that, in the modern tradition, the contextualist turn in general, and with respect to the Liar in particular, is a reasonably recent phenomenon. Kripke's remark to the effect that, given the right setting, just about any sentence can turn out to be paradoxical (*Kripke 1975*, 54/5) was something of an eye-opener for those working on paradoxes. Until then, virtually all the work on Liar-like paradoxes had completely disregarded contextual elements, perhaps due to the focus on the paradigmatic

case ‘This sentence is false’, which *prima facie* seems to express the same proposition in every one of its utterances. By contrast, in the medieval tradition, it was arguably the very realization that contextual elements may make a sentence paradoxical or not that is at the origin of the medieval literature on *insolubilia*, in the 12th century (see *Martin 2001*). In sum, while in the modern tradition attention to contextual elements came only at a later stage, in the medieval tradition it may have been the very source for its development.

The two distinct levels of being a contextualist with respect to paradoxes alluded to a few lines above are related to the already mentioned distinction proposed by Chihara (1979) between diagnosing a paradox and solving it. One may hold that an important cause of paradox is the situation-setting of the kind described by Kripke, and this would amount to being a diagnostic contextualist. But one may also hold that a **solution** to paradoxes of the Liar family will lie precisely in considering the different contexts in which Liar sentences are uttered and/or evaluated. These two forms of contextualism are related to but independent of one another: it is possible to be a diagnostic contextualist but to propose a solution to the Liar that is not essentially based on the notion of context-shifting, while it is also possible to focus on context-independent forms of paradoxical sentences (those that are not contingently paradoxical, i.e. not depending on the situation in which they are uttered) and yet to offer a context-based solution to the Liar.

The medieval contextualist tradition is best described as essentially diagnostic, in the sense that the very awareness of the paradoxical nature of some sentences that arose in the 12th century was intimately related to the kind of contextualist stage-setting offered by

obligationes (to be discussed below). By contrast, the modern contextualist tradition is less interested in how different contexts are the cause of the paradoxical nature of sentences, and more interested on how context-shifting may offer a solution to the paradoxes.

The medieval contextualist tradition is represented by the very early treatises where Liar sentences are discussed in connection with the regimented form of oral disputation known as obligations, and later by the resurgence of the use of the obligational framework to treat *insolubilia* in the 14th century with Heytesbury and those that were influenced by him (see *Spade 2005*, fn. 29 for a list of such authors). Besides the analyses related to the obligational framework, those medieval authors that explicitly favored a token-based semantics, such as Buridan (see *Klima 2004*), in practice also adopted a contextualist approach in their treatments of *insolubilia*. As for the modern contextualist tradition, it was in many senses sparked by Kripke's comments, but Kripke himself did not emphasize the contextualist approach as much as others such as Burge (1979), Parsons (1974), Barwise and Etchemendy (1987), and more recently Gaifman (1992, 2000), Glanzberg (2001, 2004) and Simmons (1993).

Let us first examine the medieval contextualist approach to *insolubilia* within the framework of obligations. Obligations were, as already said, a regimented form of oral disputation opposing two participants, Opponent and Respondent. Opponent would put forward sentences, and Respondent was to accept, deny or doubt them on the basis of several rules, in particular whether the sentence being put forward (or its contradictory)

followed from the previously granted or denied sentences (see *Dutilh Novaes 2005a* and *2006a*; *Martin 2001*). What is of interest for our purposes is that, besides the sentences being put forward, most versions of obligation also had an artificial stage-setting, called a *casus*, which was the description of the hypothetical situation that was to be taken as true for the purposes of the disputation. An example would be: suppose that Socrates is white, and then propose the *positum* (the *positum* was the first sentence put forward) ‘Socrates is black’. In this case, the *casus* served to specify that the *positum* was a false sentence (see *Dutilh Novaes 2006b* for more on the notion of *casus*).

Now, the point is that Respondent was not to accept a *positum* which was paradoxical or impossible, otherwise he would not stand a chance of winning the disputation (his task was mainly that of consistency maintenance – see *Dutilh Novaes 2005a*). And this is (as argued by *Martin (2001)*) where the medieval interest in *insolubilia* came from: with the stage-setting that the *casus* provided, it was very easy to produce a contingently paradoxical *positum*, but at the same time it was essential for Respondent to be able to identify such cases in order to reject them as *posita*. The obviously paradoxical cases such as ‘I am lying’ were in fact easily identified; more difficult were the cases of contingently paradoxical sentences with a complex *casus*. The self-reference that is nowadays usually obtained with indexicals or proper names was, in the case of obligations, obtained by using the term ‘the *positum*’ in the very *positum*. A good indication that a *positum* was likely to be problematic was indeed if it contained the term ‘*positum*’ as its subject; in an obligational context, there was only one *positum* per disputation, so in practice what was obtained by the use of the term ‘*positum*’ in the very

positum was the same unique denotation obtained with indexicals or other devices such as Gödel numbers or proper names for sentences.

However, simply banning the occurrence of the term '*positum*' in the *positum* does not altogether avoid the kind of problems that Respondent is to avoid (cf. *Martin 2001*, 75). Some perfectly fine examples are thereby excluded ('The *positum* has three words', a contingently false sentence), while other problematic examples are not. In practice, the problematic cases are the ones which, for necessary or contingent reasons (in the latter case related to the *casus* or to how the disputation progresses), are convertible with (equivalent to) sentences expressing the content (*enunciabile*) that the *positum* is false. The good performance in an obligational disputation consisted thus, among other things, in correctly identifying the sentences that, against the background of a given *casus*, did become equivalent to those expressing such content. *Martin (2001)* discusses several examples, found in these early treatises on obligations and *insolubilia*, of sentences that are or become (given the stage-setting) paradoxical *posita*, which should thus be cancelled or annulled as *positum* – hence the use of the term '*cassatio*' to refer to the earliest medieval approach to *insolubilia*. For example, the *positum* 'The *positum* has the same truth-value as that you are a bishop' will be problematic only if you (the Respondent) are not the bishop (see *Martin 2001*, 75); 'The *positum* has the same truth-value as the next proposed sentence' becomes paradoxical (and thus should be 'cassed' as a *positum*) only if the next proposed sentence is false.

In terms of the diagnosis vs. solution distinction proposed by Chihara, it is clear that, on such early approaches to *insolubilia* based on the obligational framework, the diagnosis of the problem is contextual – virtually any sentence can be made paradoxical with the appropriate stage-setting – but no attempt is made to actually solve such paradoxes. Because the goal in such cases is the good performance in an obligational disputation, diagnostic skills were essential but no special solution was required; simply rejecting the paradoxical *positum* was sufficient. Eventually, this cancellation of the *positum* was developed into the (already discussed) *cassantes* approach to *insolubilia*, but even this latter approach is more diagnostic than solution-oriented (it diagnoses the problem as a failure of certain sentences to express content).

In the 13th century literature on *insolubilia* the ties between *insolubilia* and obligations are not so clearly perceived; in this century, the dominating approach was the emphasis on the fallacy *secundum quid et simpliciter* (Lambert of Auxerre/Lagny, Simon of Faversham, John Duns Scotus) and on the *restringentes*' principles. It was in the 14th century, and most notably with Heytesbury, that *insolubilia* were again analyzed predominantly against the background of obligations²⁸; the relevant text is the first chapter of his *Rules for solving sophismata* (Heytesbury 1979).

²⁸ But notice that, as noted by Pironet (forthcoming), the range of application of Heytesbury's rules for solving *insolubilia* was not reduced to performing in an obligational disputation. The notion of *casus*, for example, while arguably originally stemming from the obligational framework, was widely used for *sophismata* as well (cf. Pironet forthcoming, 245). But Pironet (forthcoming, 243) confirms that Heytesbury was indeed the first in the 14th century to analyze *insolubilia* explicitly against the obligational background. (Pironet forthcoming) is an in-depth analysis of the background of Heytesbury's treatise as

Indeed, for Heytesbury, it did not even make sense to view a sentence as an insoluble (a paradoxical sentence) if not within a given context. This is because he, as did most 14th century authors, viewed language as essentially conventional in its meaning. How this affects the issue of insolubles is aptly described by Spade:

Heytesbury treated insolubles as paradoxical only *with respect to* certain assumed circumstances (what he calls the *casus* or "case"). For example, the proposition [sentence] 'Socrates is saying a falsehood' is not paradoxical in the abstract, all by itself, but only in contexts where, say, it is Socrates who utters that proposition, the proposition is the *only* proposition Socrates utters (it is not an embedded quotation, for instance, part of some larger statement he is making), and where his proposition signifies just as it normally does. Spoken and written languages are thoroughly conventional, for medieval authors, so that the vocal sequence or inscription 'Socrates is saying a falsehood' could theoretically signify any way you want. It might, for example, signify that $2 + 2 = 4$, in which case it would not be insoluble at all but straightforwardly true. (*Spade 2005*, 3.3).

Indeed, one of the obligational forms recognized in the 14th century was *impositio*, and its point of departure was the change of meaning (imposition) of a certain word or sentence for the sake of the disputation. So a sentence can only be an insoluble if it is clearly specified the meaning that it has in a given situation (§48). In other words, Heytesbury's

well as of its influence in later authors; it also offers several examples of specific insoluble sentences and the *casus* required for each of them to be a real insoluble.

diagnosis of the problem is also essentially contextual: he considers the issue of the *casus* already discussed, but he goes even further and demands that the contextual meaning of a given sentence also be explicitly dealt with.

His solution, however, is quite minimalistic and in fact not altogether satisfying: as in the earlier texts, he holds that, if in a given *casus* a sentence turns out to be paradoxical and it is said to signify exactly as its terms pretend, then it must be rejected as a *casus* (§49). Such a *positum* can only be admitted if it is not specified that it must signify precisely and only as its terms pretend, but then it **must** have a signification other than its usual one. Take the sentence ‘Socrates says a falsehood’ considered in the *casus* that it is uttered by Socrates and nothing else is uttered by Socrates; Heytesbury is saying that in this case the sentence simply cannot signify only as it usually does, it must have a secondary signification. What is somewhat disappointing in Heytesbury’s ‘solution’ is that he seems to think that it is not up to him (or Respondent – in any case whoever is dealing with the insoluble) to specify what secondary signification that would be (§51); unlike Bradwardine, who argued that sentences that say of themselves that they are false also say of themselves that they are true, and Albert of Saxony, who claimed that all sentences signify their own truth, Heytesbury is silent concerning the secondary signification of paradoxical sentences. In other words, he does not offer a uniform solution to paradoxes; instead, he seems to think that they should be examined one by one (if at all!) with respect to their possible secondary signification.²⁹

²⁹ In his quietism with respect to Liar paradoxes, Heytesbury goes as far as acknowledging that even the solution he favors (the fourth solution examined by him) is not entirely satisfactory (§9, §43), and that

Finally, another group of medieval solutions should be mentioned with respect to contextualism, namely the solutions proposed by Ockham, Burley and Pseudo-Sherwood. At first sight, they are not as obviously contextual as those that are explicitly formulated against the background of obligations; they are indeed essentially solutions that restrict self-reference – *restringentes* solutions. However, one of the main modern proponents of contextualist solutions to paradoxes, Keith Simmons, claims to have been overtly inspired by the aforementioned medieval solutions, and he makes a compelling case for his contextualist interpretation of them (*Simmons 1993*, ch. 5). These solutions are in any case not diagnostically contextual: they do not emphasize the fact that a sentence is paradoxical only with respect to the appropriate situation (even though in practice they often have to specify the situation such that the paradoxical nature of a sentence emerges).³⁰ If Simmons' interpretation is to be accepted, then these are a good example of approaches that are not contextual with respect to the diagnosis but which are contextual with respect to the solution proposed – and the gist of this solution will be discussed in more detail shortly when I discuss Simmons' proposal.

insolubles are indeed **insoluble** (§64). Among modern authors, such quietism is rarely found, precisely because we now think that there is an awful lot at stake with such paradoxes (the very possibility of a theory of truth and semantics). But perhaps the revision theory of truth (proposed independently by Herzberger and Gupta – see *Kremer 2006*) is what comes closest to Heytesbury's attitude towards insolubles; rather than solving these paradoxes, the revision theory of truth simply seeks to model the reasoning leading to paradox.

³⁰ An example from Ockham: Socrates begins to talk and says 'Socrates says a falsehood', and then says nothing else (in *Pozzi 1987*, 138). Without the specification that he says nothing else, the sentence may not be paradoxical after all.

Another aspect in which the medieval approaches to Liar sentences were in general essentially contextual is the fact that the medieval authors often considered tokens as truth-value bearers; they presented numerous examples of tokens of the same type being uttered by different people, some of which turned out to be paradoxical while others did not, yielding different truth-values for different utterances of the same type. Take for example Buridan's 'reciprocal Liar' (in his *Summulae de Dialectica*, Buridan 2001, 971-974) (analyzed in detail in Klima 2004): Plato says 'Socrates says something true'; Socrates says 'Plato says something false'; Robert says 'Plato says something false', and nobody says anything else. It is easy to see that the utterances by Plato and Socrates form a loop, while Robert's is outside the loop; it is in fact **about** the loop.³¹ Buridan's conclusion is that the utterances by Plato and Socrates are false (using the principle that every sentence implies its own truth), while Robert's is true (since it says that Plato's utterance is false, and this is indeed the case). The important conclusion to be drawn is that the utterances by Socrates and Robert are perfectly equiform, but they have different truth-values, and this because the circumstances of their utterance are different.

On the modern side, several philosophers and logicians also stressed the fact that sentence-tokens and not sentence-types should be truth-value bearers. For instance, Gaifman (1992, 2000) is interested in precisely this kind of phenomenon. His starting point is an example analogous to Buridan's, of two equiform sentences one of which is paradoxical while the other is not. This motivates the need to consider tokens and their

³¹ This is a terminology borrowed from Gaifman's pointer semantics (1992, 2000), which is particularly suitable to help us see what is going on in this example.

context in the assignment of truth-values. But Gaifman's system is contextual in a way that goes beyond what is usually observed with indexicals:

The proposed system introduces an essential non-compositional element into the semantics: the meaning of a sentence-token is determined not only by the structure of the sentence (as a type) and by the meanings its components, but also by the token's place within a global network of mutually referring tokens.
(*Gaifman 2000*, 84)

The idea is thus that many tokens 'point' at tokens (different ones or even to themselves), forming a complex web of mutual relations, and the relative position of a token with respect to the other tokens is essential for the determination of its truth-value. Many tokens do not point at tokens – i.e. sentences that are not about other sentences, such as 'The grass is green' – but some do, in particular (but not exclusively) those with semantic predicates such as 'true' and 'false'. Not all tokens that point at tokens are problematic: in many cases the program corresponding to a token terminates and yields a truth-value, such as with 'This sentence has four words' (it yields the truth-value false in an unproblematic way). But in other cases a loop is formed, and the process cannot terminate; these are the cases of ungroundedness identified by Kripke, and in Gaifman's system they also yield a truth-value gap (or the non-classical truth-value GAP).

Indeed, one of the main common traits of modern contextualism with respect to the Liar paradoxes is a discussion of what is to count as truth-value bearers. Prior to the contextualist turn, sentence-types were usually taken to be truth-value bearers without further ado; but more careful reflection shows that one of the points that most definitely

deserve to be examined in connection with Liar sentences is precisely what is to count as truth-value bearers.

Generally speaking, the most popular contextualist candidates for the position of truth-value bearers are: sentence types in a context (Simmons' choice); sentence tokens understood contextually (the choice of most medieval authors, and in some senses Gaifman's choice); and propositions, that is, contents expressed by sentences in a context. It has been argued for example that naïve inscriptionalism – physical tokens taken as truth-value bearers, which some have termed 'token relativism' (Weir 2000) – is not entirely unproblematic (Clark 1999), or even that it is outright unsatisfactory (Weir 2000). It is clear in any case that a larger part of the context should be considered: for example, if the stone adorning a tomb says 'Here lies Socrates', and Socrates is indeed buried there, it is a true statement; but if the same stone is moved to a different tomb, the same inscription no longer makes a true statement (see Simmons 1993, 101). But the inscriptionalist approach has the metaphysical advantage of dealing with objects in time and space: inscriptions and utterances. By contrast, the proposition/content option is metaphysically problematic – given the dubious ontological status of propositions – but semantically extremely convenient, and this is most certainly why it remains a popular approach (the choice of Barwise and Etchemendy (1987, 11), and Glanzberg (2001, 226)). (Notice that although most 14th century authors are indeed inscriptionalists, the 12th century texts discussed by Martin (2001) attribute truth-value to *enunciabilia*, a notion that comes very close to the modern notion of proposition, corresponding to the asserted content of a sentence.)

Besides differing with respect to truth-value bearers, modern authors also differ in how they use the basic contextualist concepts to approach Liar sentences. In fact, explaining why a contextual approach is required at all to understand what is at stake with Liar sentences is also an important part of some modern treatments, in particular Glanzberg (2001, 2004). Since Kripke's remark to the effect that just about any sentence can be made paradoxical given the right stage-setting, it has become almost a common-place to hold that the phenomenon of such paradoxes is context-related; but in truth Liar sentences do not seem to be contextual in any of the usual ways. If it is formulated as '(1) Sentence (1) is false', for example, it seems to contain no indexicals, no demonstratives, no gradable adjectives, no quantifiers – in sum, none of the grammatical categories that are usually associated with context-dependence. So the proponent of the contextual approach must provide a precise diagnosis of how exactly the contextual element comes into play with such Liar sentences; the solution proposed will then obviously depend on the diagnosis made.

Some (Parsons, Burge) emphasize the fact that the terms forming Liar sentences (and, in the case of Parsons, not only the semantic predicates) shift meaning (and thus extension) depending on the context, and in particular (Burge) that the predicate 'true' is indexical. Others, such as Simmons, reject the traces of a hierarchical language structure in these earlier proposals (cf. *Simmons 1993*, 96). Simmons casts his solution in terms of the notion of singularity: in some very specific cases, 'there are minimal restrictions on occurrences of 'true' and 'false'' (106) – a view that he claims is inspired by the Ockham/Burley/Pseudo-Sherwood position. This is why Simmons argues that their solutions are also contextual: what determines when these minimal restrictions occur is

fundamentally the context of utterance of a sentence. The challenge is then to characterize these minimal and special circumstances when the predicate ‘true’, for example, cannot refer to the very sentence in which it occurs, in a non-ad-hoc way.

By contrast, Glanzberg is happy to combine contextual and hierarchical elements in his approach. He focuses on the relation of expressing that exists between a sentence and a content/proposition, and identifies a hidden existential quantifier as the source of context-sensitivity in the case of Liar sentences: **there is** a proposition *p* expressed by sentence *s* in context *c*.³² According to him (following Parsons) ‘there is a quantifier domain restriction phenomenon at work, though an extraordinary one’ (Glanzberg 2004, 30) with respect to the hidden quantifier ranging over propositions. Given this diagnosis, Glanzberg then moves on to present a contextual-hierarchical solution to the Liar.

In sum, it is evident that the contextual approach is of great importance in both medieval and modern solutions to the Liar paradox, but in quite different ways; in other words, there are many ways of being a contextualist. As much as some modern solutions, medieval approaches (especially in the 14th century) emphasize the context-sensitivity of tokens (which most 14th century authors take to be the truth-value bearers). Just as importantly, at the very heart of the medieval literature on *insolubilia* is its relation to the obligational framework. In spite of the undeniable strangeness of this framework to modern eyes, these analyses bear resemblance to modern discussions of contingent

³² Notice that, in this particular aspect, Glanzberg’s approach has a remarkable similarity with Bradwardine’s analysis. The difference is that, for Bradwardine, a Liar sentence typically expresses not one but several contents.

paradoxes (those first noticed by Kripke) and to discussions of how different tokens are related to one another ('point' at one another, as Gaifman would put it). Modern solutions that identify a context-dependence in uses of the predicate 'true' in turn are more akin to medieval solutions based on the *secundum quid et simpliciter* distinction (which in some senses is also a contextual distinction), such as Ockham's.

But again, in the modern context, it is not sufficient to simply state that Liar sentences are context-sensitive and that this is the source of their paradoxical nature; it is required also to show **how** this context-sensitivity operates.³³ The medieval authors, by contrast, had the contextual approach as their starting point, given their commitment to tokens as truth-value bearers and the fact that their interest in *insolubilia* was to a large extent prompted by obligations, a situation where contextual elements were inherently important.

7. Insolubles are self-falsifying

³³ Gauker (2006) offers a sustained critique of the contextualist approach to paradox. According to him, the essence of such (modern) solutions is what he calls 'stepping back': "Every one of these authors, as far as I am aware, has supposed that what we have to explain, above all, is how it is that once we judge that a liar sentence such as 'This sentence is not true' is not true, we can in some sense 'step back' and judge that the liar sentence is after all true. This is supposed to be possible because the context in which we judge that the liar sentence is true is not the context in which the liar sentence says of itself that it is not true." (Gauker 2006, 393). Gauker claims that this description fits in particular the proposals by Simmons, Glanzberg and Barwise& Etchemendy. To me, though, it seems that, while it is indeed a powerful critique of the 'stepping back' approach, the different contextualist approaches, even these three mentioned by him, go well beyond this basic principle, and thus his critique is not as far-reaching as it claims to be.

Let us now turn to Roger Swyneshed's treatment of *insolubilia*. We have already encountered a few aspects of it when discussing non-classical approaches to such paradoxes, but we have not yet dealt with the core of his solution, namely the claim that insolubles are false because they are self-falsifying. This position was (for as far as we know) first put forward by Swyneshed, but it was later defended by quite a few authors (including Paul of Venice).³⁴

The cornerstone of Swyneshed's solution, as with Bradwardine, is a reformulation of the usual correspondentist definition of truth. We have seen that Bradwardine defines a true sentence as a sentence that signifies things to be **only** as they are, which can be understood as a universal quantification over the different things that a sentence says (they must all obtain for the sentence to be true). Like Bradwardine, Swyneshed strengthens the necessary condition for a sentence to be true, but in a different way: a sentence must not only signify as things are, but it must also not be self-falsifying.³⁵

The second (definition) is this: A true sentence is a sentence that does not falsify itself and that principally signifies as is the case [...].

³⁴ See *Spade 2005*, note 26.

³⁵ Thus, with this definition of truth, just as much as with Bradwardine's, the standard correspondentist semantic ascent from 'p' to 'p is true' is not valid, as noticed by Spade (1983, 105).

The third definition: A false sentence is an expression that falsifies itself, or else an expression that does not falsify itself and that principally signifies otherwise than is the case [...].³⁶

With this move it becomes evidently more difficult for a sentence to be true (just as with Bradwardine), and thus the class of true sentences becomes smaller. The goal is clearly that of showing that most insolubles fail the second clause of the necessary condition to be a true sentence. In particular, the standard Liar ‘Socrates says a falsehood’ (when uttered by Socrates and when nothing else is uttered by Socrates) comes out as false according to this definition because, even though it does signify things as they are (namely that the insoluble is false), it is self-falsifying. So it is plainly false, and from the fact that it says of itself that it is false (which is the case) does not follow that it is true, given that the other necessary condition for truth (not being self-falsifying) is not obtained; thereby the second leg of the paradox (the inference that the sentence is true from the fact that it says that it is false and the assumption that it is indeed false) is blocked.

Of course, the main task for the proponent of such a solution is to explain the notion of ‘self-falsifying’; it all depends on this if the solution is to avoid being ad-hoc. Swyneshed does offer a discussion of this notion, including a series of subdivisions of different ways

³⁶ Latin text in *Spade 1979*, §§ 14-15; translation from *Spade 1983*, 105. For an explanation of the notions involved in this definition which are not essential for the present discussion, such as ‘to signify principally’, ‘to signify naturally’ etc., see *Spade 1983*.

for a sentence to be self-falsifying. However, the crucial concept to understand the notion of a self-falsifying sentence is that of a **pertinent/relevant** sentence. This is again a loan from the obligational framework; so again, as with Heytesbury, one can only fully understand Swyneshed's solution to the Liar against the background of obligations. Here is Swyneshed's definition of a self-falsifying sentence:

A self-falsifying sentence [...] is a sentence signifying principally as is the case or other than as is the case, which is pertinent for the inference that it will itself be false.³⁷ (*Spade 1979*, §5)

Let us thus turn again to *obligationes*. The rules determining whether a sentence should be accepted, denied or doubted varied per author, but in all versions of *obligationes* the logical relations between the first-proposed sentence, the *positum* (in the case of position, the main form of *obligationes*) and the sentences proposed afterwards were decisive. In the case of Swyneshed, a sentence that either follows from the *positum* or whose contradictory follows from the *positum* (known as repugnant to the *positum*) is known as pertinent (*Spade 1977*, §4); a sentence not having this property is known as impertinent. In the case of pertinent sentences, Respondent is obliged to accept those that follow from the *positum* and to deny those that are repugnant to the *positum*; if he does not do so, he responds badly.

³⁷ Latin text in *Spade 1979*, §5; my translation.

The notion of a self-falsifying sentence is thus easily formulated in terms of the notion of pertinent sentence and the obligational framework; it is a sentence P_1 that, if it were to be the *positum* in an obligation, and if its contradictory P_2 were to be proposed afterwards, P_2 would follow from the *positum* and thus would have to be accepted. This is why such a *positum* would have to be rejected (as in the very early texts on *insolubilia* and *obligationes* already discussed), because it would force Respondent to accept its very contradictory as a *propositum*. In other words, if there is a valid inference such that

$$\Gamma, \varphi \Rightarrow \sim\varphi$$

then φ is a self-falsifying sentence. Swyneshed does not require that from φ alone $\sim\varphi$ should be derived; auxiliary premises Γ may also be required (*Spade 1979*, §§5-7; *Spade 1983*, 108). He does require though that φ not be idle in the inference, i.e. he requires that $\Gamma, \varphi \Rightarrow \sim\varphi$ be valid but not $\Gamma \Rightarrow \sim\varphi$ (cf. *Spade 1983*, 109).

Naturally, several issues arise from this account (many of which are also discussed in *Spade 1983*). First of all, what is the exact nature of this relation of ‘following from’ which defines the notion of self-falsification? This is of course not a problem only for Swyneshed’s account of *insolubilia*, but for the whole obligational framework, where inferential relations play a crucial role but whose canons of soundness are usually not explicitly discussed. To be sure, this was indeed a topic of much interest to medieval logicians, discussed in treatises on *consequentia*; but it is still not an entirely

straightforward matter to justify the validity of the inference from ‘This sentence is false’ to ‘That sentence is not false’ within a tight logical system.³⁸

The second aspect worth noticing is that the phenomenon of self-falsification with respect to a sentence can be understood in different ways. Medieval authors were very interested in cases of what we could call performative contradictions, that is, situations in which the very act of uttering a sentence would falsify its content; such cases were often discussed in obligational treatises, and also by Buridan in his discussions on the notion of consequence (in his treatise on consequence (*Buridan 1976*) as well as in his *Sophismata* (*Buridan 2004*; transl. in *Buridan 2001*)). One of Buridan’s examples is: ‘No sentence is negative’. This sentence, when uttered, modifies the context in such a way that the context automatically falsifies what it says to be the case. So there is a sense in which it is self-falsifying. But are we prepared to admit as valid the inference: ‘No sentence is negative, therefore some sentence is negative’? Most likely not: Buridan calls it a sophism (*Buridan 2001*, 956), and indeed according to his reformulation of the notion of consequence (so that it is not cast in terms of truth but rather in terms of things being (or

³⁸ One of Swyneshed’s conclusions in his treatise that was viewed with much skepticism in his time was precisely the claim that, in some valid inferences, falsehood follows from truth, in the case of inferences which are valid according to the modal criterion (things cannot be as the antecedent signifies them to be without being as the consequent signifies them to be), but which have a false consequent that does signify things as they are and is thus false on account of being self-falsifying (*Spade 1979*, §26). In sum, the very notion of ‘follows from’ must be reconsidered in view of his redefinition of the notion of truth so as to include non-self-falsification, but the very notion of self-falsification presupposes some previous notion of ‘follows from’ to be defined. There is thus an imminent threat of circularity in these definitions.

not) as the sentences signify them to be), this is not a valid consequence. ‘No sentence is negative’ is not self-falsifying in the sense that ‘This sentence is false’ is; the former case is that of a performative contradiction, i.e. the sentence is false whenever it is uttered simply because, when uttered, it automatically makes things be other than it signifies them to be (that there are no negative sentences, whereas there obviously is a negative sentence, namely this very sentence).

Similar examples of pseudo self-falsifying sentences can be found in obligational treatises, and were discussed with the purpose of determining which sentences should indeed be rejected as *posita* – actual paradoxes – as opposed to those with only the appearance of being paradoxical, and which therefore should not be rejected as *posita*. One example is: ‘Nothing is posited to you’; if this is the *positum*, obviously the very fact that it is the *positum* falsifies its content. But this does not mean that it is truly paradoxical; it is only a performative paradox, similar to ‘I am not speaking now’ and the like. Most authors argued that such *posita* should indeed be accepted.

The reason why the distinction between pseudo self-falsifying sentences and actual self-falsifying sentences is important is the following; Swyneshed seems to hold that to be self-falsifying is the defining characteristic of truly paradoxical sentences. As we know from modern discussions, one of the main risks of a demarcating criterion of what is to count as a paradoxical sentence is that of overgeneration, that is, that of deeming paradoxical cases of perfectly fine sentences. We have encountered this problem already when discussing the *restringentes* approach; a mere ban on self-reference is not an

adequate solution to Liar-like paradoxes because there are also many virtuous cases of self-referential sentences. Here we have a similar problem; the notion of self-falsification should not, it seems, be understood as encompassing also cases of what we could call performative contradictions, which are, or so it seems, not truly paradoxical. Alternatively, if truly self-falsifying sentences are indeed to be seen on a par with performative contradictions, then this view should be supported by a considerable amount of semantic machinery in order to demonstrate that the phenomena at stake are really of the same kind.

In contemporary circles, the notion of performative contradictions has been discussed by a wide range of theorists, ranging from speech-act theorists such as Searle (see *Johansson 2003*) to the proponents of the theory of communicative action, most notably Habermas, as well as by those interested in the phenomenon of self-reference in general. Interestingly, it seems that little work has been done relating Liar sentences and the notion of performative contradictions and speech-act theory in general, even though there appears to be scope for a pragmatic approach to Liar paradoxes – one that identifies the source of the problem as essentially pragmatic and not semantic (one rare example of such an approach is *Martinich 1983*). That is, although I have argued that true paradoxes are *prima facie* distinct from mere performative contradictions, a theoretical framework treating truly paradoxical sentences as failures of the performative kind seems conceivable and in fact possibly quite appealing.

Other than the possibility of developing Swyneshed's idea of a self-falsifying sentence into a pragmatic approach to paradoxes (as done for example by van Benthem (2004) with respect to Fitch's paradox³⁹), perhaps within the broader context of speech-act theory, there do not appear to be many points of contact between modern solutions to the Liar and Swyneshed's. Indeed, a purely logical approach to self-falsification would have to deal with the serious issues raised in (*Spade 1983*).

8. Insolubles do not occur in mental language

An approach to Liar sentences with a distinctive 14th century flavor is the view that insolubles occur only in 'imperfect' languages such as spoken and written language, but not in mental language. The idea of a mental language dates back to Aristotle and Boethius, but it was much more thoroughly developed in the 14th century, especially with the nominalists, starting with Ockham (see *Panaccio 1999*). The gist of the concept of mental language is that thought is organized in a structure not fundamentally different from spoken and written language; this hypothesis was used to account for all kinds of different semantic phenomena, such as the signification of terms, ambiguity, equivocation etc. The main difference between mental and spoken/written languages would be that the former is natural, the result of purely causal processes, while the latter is strictly conventional, i.e. the meaning of its terms is conventionally established and can be changed at will.

³⁹ I owe the reference to *Van Benthem 2004* to Sara Uckelman.

What exactly mental language is and ought to be like is a topic of debate among medieval authors as well as among the modern scholars who try to reconstruct their views (see *Trentman 1970; Spade 1980a; Normore 1997; Chalmers 1999*). Two basic lines of interpretation among scholars are: 1- mental language really is very much like our non-mental languages, including some of their ‘imperfections’ such as synonymy and connotative terms (cf. *Panaccio 1999*); 2- if mental language is to perform the semantic tasks that authors such as Ockham want it to perform, then it must be some kind of ideal language and thus not contain these imperfections (cf. *Normore 1997*).

At any rate, what matters for our present purposes is the idea that mental language is not conventional, and that it is less (or not at all) prone to imperfections and problematic semantic phenomena. We have seen that Heytesbury maintained that an insoluble sentence must signify not only as its terms pretend, but it must also have some kind of secondary signification; now, in the case of sentences of mental language, given the natural and causal mechanisms that establish the signification (meaning) of mental terms, this cannot occur, since mental sentences can only signify exactly as their terms pretend. So a consequence of Heytesbury’s position (which he did not draw himself) is that insoluble sentences cannot be formulated in mental language.

This view was indeed explicitly held by later authors, such as Gregory of Rimini (or so it seems, as we no longer possess a copy of his text but only second-hand accounts of it) and Peter of Ailly. Spade (*1980b*, 6-7; *2005*, 3.4) conjectures that Gregory must have reached this conclusion based on Heytesbury’s position; Read (in his introduction to

Bradwardine forthcoming), however, believes that Bradwardine's influence can also be perceived. Indeed, the gist of Gregory's position is that a spoken insoluble such as 'This sentence is false' corresponds to the conjunction of two mental sentences, namely the sentences 'That sentence [the spoken one] is false' and 'The first conjunct ['That sentence is false'] is false'; given bivalence, the latter amounts to the claim that the original sentence is true. In other words, the Bradwardinean idea that a sentence that says of itself that it is false also says of itself that it is true is here reformulated in terms of mental language: a spoken sentence that says of itself that it is false corresponds to the conjunction of two mental sentences, one of which saying that the original sentence is false and the other saying that the original sentence is true, exactly as Bradwardine had described.

In his treatise on insolubles (*Spade 1980b*), Peter of Ailly describes Gregory's position, but he does more than that; Peter's is a sophisticated and argumentative treatise. In a nutshell, his main disagreement with Gregory's position is that, while Gregory claimed that an insoluble sentence corresponded to the conjunction of two mental sentences, Peter maintains that the insoluble sentence corresponds to both these mental sentences, and not to their conjunction. In other words, for Peter an insoluble sentence is a sentence that is ambiguous and thus must be distinguished (*distinguenda* – see *Ashworth 1991* and *Dutilh Novaes 2007*, 1.3.2.2 on this notion). As *Spade (2005, 4.2)* puts it,

Peter's theory has the phenomenological advantage that it accounts for the psychological "flip-flop" sense we have when thinking about insolubles. When

we look at them one way they seem true; when we look at them another way, they seem false.⁴⁰

One of the implications of this difference is that, while for Gregory an insoluble is then plainly false (as it corresponds to a conjunction of two mental sentences which cannot both be true together), for Peter it can be true or false according to the reading given to it – just as much as a sentence such as ‘Man is a noun’ can be either true or false, depending on whether the subject has material or personal supposition.

Of course, the greatest challenge for this position is to justify why an insoluble sentence simply cannot be formed in mental language – if it could, then this approach would lead nowhere, as there is no supra-mental level to account for an insoluble sentence in mental language in the way that mental language accounts for an insoluble in spoken and written language. We know from modern discussions that one usually has to go to great lengths to put restrictions on a language so that such paradoxical sentences cannot be construed in it (such as excluding semantic predicates); but mental language is not the kind of thing one can put restrictions on, precisely because it is natural. It is what it is, whether we like it or not. So Peter must argue for the non-existence of such paradoxical sentences in mental language on the basis of a series of principles that (presumably) hold unproblematically of mental language – in other words, a descriptive rather than a normative approach.

⁴⁰ Notice that this psychological “flip-flop” is also found in the revision theory of truth (cf. *Kremer 2006*), where a Liar sentence is taken successively to be false, true, false, true etc.

This is indeed what he does in Chapter 3 of his *Concepts and Insolubles* (Spade 1980b); he deploys a web of sophisticated distinctions, corollaries and principles in order to conclude (§§ 307-310) that self-reference cannot occur in mental language. His claim that insolubles cannot occur in mental language is proved by means of the auxiliary claim that self-reference cannot occur in mental language, based thus on the tacit (and contentious) premise that self-reference is the real cause of paradox in the case of insolubles. In Peter's own terms: 'No mental sentence properly so called can have reflection on itself.' (Spade 1980b, 79).

There are, of course, several possible objections to this view. Peter examines some of them, such as the objection to the effect that an insoluble can very well arise in mental language; if I form the mental sentence 'Some mental sentence is false', and then for some strange reason (God's will, for example) all my other mental sentences are destroyed, then it seems that this is a paradoxical mental sentence after all (§311). His reply to this objection is (somewhat disappointingly) to resort to the *restringentes* principle applied to mental language: 'a part of a mental sentence properly so called cannot supposit for that very sentence of which it is a part' (§313).

In sum, Peter's solution to Liar sentences based on the idea that self-reference cannot occur in mental language, and thus that such paradoxical sentences cannot be formed in mental language, is appealing, but it also raises several issues. In Peter's text, in any case, it is not satisfactorily worked out; as Spade says in his introduction, 'one should not

expect a fully coherent doctrine to emerge from Peter's discussion. His flair for the clever argument does not seem to be accompanied by an equal sensitivity to overall consistency.' (*Spade 1980b*, 8). Peter was, though, to be quite influential on authors writing after him; his approach to insolubles in particular was often discussed and sometimes overtly defended in subsequent writings (see *Spade 1980b*, 1).

How can we relate this distinctively medieval approach to Liar sentences (since it is based on the idea of mental language as it was widespread in the 14th century) to modern solutions? At first sight, there isn't much of a connection, except for the fact that some modern authors, Tarski in particular, set out to develop an ideal language where Liar sentences could not be formed, just as mental language would presumably be an ideal language for Peter and other medieval authors. But Tarski's is a project of sanitation of ordinary language for scientific purposes; by contrast, according to the medieval concept of mental language, the latter is not the result of deliberation – it is not something artificially created to fulfill certain tasks. Much to the contrary, mental language is entirely natural, the result of processes that occur independent of our deliberations, and which are a consequence of the kind of beings we are (humans), with a certain cognitive apparatus. To be sure, there is something of a hierarchy of languages in the distinction between written, spoken and mental language (in that order), insofar as, according to the influential Ockhamist view, written terms are subordinated to spoken terms, which in turn are subordinated to mental terms. But this is very different from the Tarskian hierarchy of languages, where higher-order languages serve the main purpose of allowing one to talk about lower-order languages.

There is, however, an aspect of this medieval approach to Liar sentences which deserves some attention from modern authors. We are all familiar with the currently very influential hypothesis of a language of thought (LOT), developed by Fodor and others, which is thought to have an explanatory role especially in the philosophy of mind. But, to my knowledge, the issue of whether Liar-like paradoxes can arise in LOT has thus far not been thoroughly explored; it is however (or so it seems to me) a crucial issue for the characterization of LOT. Do paradoxical thoughts occur? If yes, in which form (assuming that they are expressed in LOT)? If yes, how does the mind operate when confronted with paradoxical thoughts? If no (as Peter of Ailly would have it), why is it so, given that paradoxical sentences can easily be formed in spoken and written language?

In other words, it appears that the issue raised by this medieval approach to Liar sentences, namely whether such paradoxes do or do not occur in thought, should be addressed by the modern advocates of the LOT hypothesis. To be sure, some intriguing work on the ramifications of paradoxes for the philosophy of mind has been done (e.g. *Patterson forthcoming*), but there seems to be scope and need for a much more thorough discussion of the matter.

4. Differences

Because the starting point for the taxonomy presented here were the medieval solutions to Liar-like paradoxes, it is important now to ponder on aspects of the modern solutions

which may have been left out of the discussion so far, on account of having no clear counterpart in the medieval solutions. Some of these differences between the two traditions have already been mentioned at different places of the present study, but let us now list them in a more orderly manner.

- The medieval authors never proposed to reformulate the language radically so as to exclude semantic predicates (or any other kind of predicate, for that matter), as proposed by the influential Tarskian approach to paradoxes. And this was not because their conception of language did not allow for such ‘artificial’ procedures; as already mentioned, for many of the medieval authors (especially in the 14th century), spoken and written languages are essentially conventional, and new conventions can be introduced (especially for scientific and philosophical contexts) whenever the need for them is felt. But just as much as rejection of semantic closure is nowadays often seen with reservation (on account of going against some of our strong intuitions concerning our languages), one may conjecture that excluding semantic closure would have been seen by the medieval authors as too artificial a step.
- One of the main breakthroughs in the analysis of paradoxical sentences of the last decades (dating back to Kripke’s seminal work) is the realization that, if there is anything really distinctive about paradoxical sentences (as opposed to merely circular sentences), what makes them paradoxical is essentially their ungroundedness. The notion of ungroundedness is powerful in that it allows for

the common diagnosis of paradoxes that seem *prima facie* very different from each other, such as the ones explicitly involving circularity as opposed to those where the cause of ungroundedness is of a different nature (such as Yablo's paradox). However, the idea of ungroundedness as such is nowhere to be found in the medieval writings on insolubles; in fact, it seems that they would not have had the conceptual and technical tools necessary for the formulation of this notion. To be sure, many medieval texts seem to be headed in the right direction with respect to ungroundedness when they point out the circularity involved in many insolubles (see *Martin 2001*). But a full-fledged formulation of this notion seems quite inconceivable in the medieval context. In the recent literature, by contrast, one of the main lines of investigation is the structural and model-theoretical (meta-)analysis of paradoxes (in particular *Priest 1994*). Examples of this approach are: Barwise and Etchemendy's use of non-classical set theory in their 1987; Gaifman's (1992, 2000) many-leveled models of mutually referring pointers; and the debate on whether there exist non-circular paradoxes (such as Yablo's 1993) or whether (as claimed by Priest in 1997) paradoxes that are apparently not circular are circular after all (see also *Yablo 2004*, and *Ketland 2005* on Yablo's paradox being in fact an ω -paradox). Nothing of this structural approach to paradoxes is to be found in the medieval literature, even if definitions of what is to count as an insoluble are indeed an important subject in the medieval treatises.

- As already pointed out, the main difference between the two traditions seems to be the role occupied by such paradoxes in the greater context of logic, semantics and theories of knowledge. For the medievals, Liar-like sentences remained essentially difficult puzzles; clearly, puzzles from which important lessons could be learned, but nothing more than puzzles. Modern authors, by contrast, often refer to them as ‘semantic pathologies’ and other threatening terms, giving the measure of how much they are thought to infect and decimate the very possibility of knowledge if they are to remain untreated. Consequently, for a medieval author to say that an insoluble is in fact insoluble (as Heytesbury) is a much lighter theoretical blow than it is for a modern author to make such an acknowledgement. This disanalogy also follows from the different perceptions of the structure of a theory: modern authors basically take for granted that theories are structured in a foundational way, and that, if from its very fundamental notions a contradiction can be derived, then the whole castle of cards crumbles.⁴¹ Medieval authors, by contrast, do not subscribe to the same extent to this foundational model (even though their model of science is admittedly essentially Aristotelian, and thus to some extent foundational/axiomatic) and as a consequence do not assign the same theoretical role to paradoxes.

⁴¹ This is obviously not true of dialethic theorists; however, the reason why they reject the idea that a theory explodes if a contradiction is derived from it is related to their views on the very notion of contradiction, and not to a non-foundational view of logic and semantics as that held by the medieval authors.

- The different roles occupied by such paradoxes in the greater context of logic, semantics and theories of knowledge for medieval and modern theorists may also explain the readiness to resort to non-classical principles of logic in order to deal with them among the latter. Medieval authors, by contrast, may have felt less compelled to deviate significantly from well-established logical principles because less was at stake, i.e. because, to their minds, the threat posed by such paradoxes was not as great as modern theorists think it is. This being said, one does encounter traits of non-classical logical principles among medieval authors, albeit to a modest extent, in particular with Swyneshed and the solutions discussed in section 3.4 above.

5. Conclusion

What does such a comparative taxonomy teach us about Liar-like paradoxes in general? It seems that one lesson to be learned from the medieval authors is not to be so afraid of these paradoxes and of the damage they can cause – something like learning to stop worrying and love the paradoxes. Perhaps a consistent and coherent notion of truth is not as essential to our philosophical and logical investigations as we may think. As some have put it, the very concept of truth may after all be mere pretense (cf. *Woodbridge 2005*). This does not mean that we shouldn't deal with these paradoxes at all – much to the contrary, they are very instructive puzzles – but perhaps not with the same sense of urgency.

Moreover, it seems that the modern contextualist approach to paradoxes could only benefit from a more thorough investigation of the medieval contextualist tradition, in particular with respect to *obligationes*. Unlike other modern approaches to Liar sentences which have been overtly inspired by medieval solutions, the modern contextualist approach is an entirely independent development. In the section above on contextualism I have sketched some of these possible points of contact, but there is most definitely scope for a more robust comparative investigation between the two contextualist traditions. In any case, from the present analysis it seems manifest that paradoxes must always be discussed against the background of a context; and indeed, this paper is also meant to provide historical corroboration for the necessity of a contextual approach to paradoxes.

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