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Riaz, M.

DOI

[10.1007/s00180-007-0084-6](https://doi.org/10.1007/s00180-007-0084-6)

Publication date

2008

Published in

Computational Statistics

[Link to publication](#)

Citation for published version (APA):

Riaz, M. (2008). Monitoring process variability using auxiliary information. *Computational Statistics*, 23(2), 253-276. <https://doi.org/10.1007/s00180-007-0084-6>

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Monitoring process variability using auxiliary information

Muhammad Riaz

Accepted: 13 November 2006 / Published online: 23 August 2007
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Abstract In this study a Shewhart type control chart namely V_r chart is proposed for improved monitoring of process variability (targeting large shifts) of a quality characteristic of interest Y . The proposed control chart is based on regression type estimator of variance using a single auxiliary variable X . It is assumed that (Y, X) follow a bivariate normal distribution. The design structure of V_r chart is developed and its comparison is made with the well-known Shewhart control chart namely S^2 chart used for the same purpose. Using power curves as a performance measure it is observed that V_r chart outperforms the S^2 chart for detecting moderate to large shifts, which is main target of Shewhart type control charts, in process variability under certain conditions on ρ_{yx} . These efficiency conditions on ρ_{yx} are also obtained for V_r chart in this study.

Keywords Control charts · Power curves · Simulations · S^2 chart · V_r chart · Normality · Auxiliary information

1 Introduction

The monitoring of any process output demands an early detection of shifts in process parameters. The shift may be in process variability or process mean level or both. The variability of any process is controlled first and then comes controlling of mean level. In 1920s Walter A. Shewhart introduced the idea of control charts to monitor any process for variability or process mean level. The commonly used control charts for monitoring process variability include R chart, S chart, S^2 chart etc. and for process mean level include $\bar{X}$ chart, median chart, trimmed mean chart, mid-range chart etc.

M. Riaz (✉)
Department of Statistics, Quaid-i-Azam University, Islamabad, Pakistan
e-mail: riaz76qau@yahoo.com

By identifying and observing some auxiliary variables along with variable of interest, the information on the relationship between auxiliary variables and variable of interest can be used to improve the precision with which parameters are estimated. [Olkin \(1958\)](#), [Raj \(1965\)](#), [Srivastava \(1965, 1966\)](#), [Rao and Mudholkar \(1967\)](#) and [Adhvaryu \(1975\)](#) used multi-auxiliary supplementary information for estimating population mean. [Tikkiwal \(1960\)](#) used the information on single auxiliary variable for mean estimation and also considered the case when variable of interest and auxiliary variable jointly follow bivariate normal distribution. [Isaki \(1983\)](#) compared several variance estimators using auxiliary information. [Kuk and Mak \(1989\)](#) discussed median estimation using one auxiliary characteristic. [Naik and Gupta \(1991\)](#) discussed a general class of estimators for population mean using auxiliary information. [Upadhyaya and Singh \(2003\)](#) used information on two auxiliary variables for mean estimation. [Singh et al. \(2004\)](#) used two auxiliary variables for the estimation of population mean.

The literature on control charts provides a variety of control charts to monitor dispersion and location parameters of any process. To refer a few of these: [Pearson \(1932\)](#) considered the percentage limits for the distribution of range when samples are from normal distribution. [Ferrell \(1953\)](#) used midranges and medians for constructing control charts. [Goel and Wu \(1971\)](#) determined ARL and contour nomogram for controlling means of normal populations using cusums schemes. [Chiu \(1974\)](#) discussed about controlling of means of normal populations using cusum designs. [Chiu and Wetherill \(1974\)](#) gave a simplified scheme for economic design of $\bar{X}$ chart. [Schilling and Nelson \(1976\)](#) examined the non-normality effect on the design structure of $\bar{X}$ chart. [Langenverg and Iglewicz \(1986\)](#) constructed control charts using trimmed means and ranges. [White and Schroeder \(1987\)](#) discussed the idea of simultaneous control charts. [Rocke \(1989\)](#) and [Tatum \(1997\)](#) discussed about robust control charts. [Battaglia \(1993\)](#) used the idea of regression based statistical process control. [Pappanastos and Adams \(1996\)](#) provided an alternative design for Hodges–Lehmann control chart. [Ramalhoto and Morais \(1999\)](#) developed Shewhart control charts for the scale parameter of Weibull control variable. [Gonzalez and Viles \(2000, 2001\)](#) developed designs of $\bar{X}$ and R control charts for the case of Gamma distribution. [Yeh et al. \(2003\)](#) proposed V-chart for monitoring process variability. [Woodall et al. \(2004\)](#) considered the idea of control charts for the situations where quality of product is better characterized by a relationship between quality characteristic of interest and one or more explanatory variables. [Shu et al. \(2004\)](#) considered a Shewhart control chart and an EWMA control chart for regression residuals. [Khoo and Quah \(2004\)](#) developed some new multivariate control chart designs for process dispersion by transforming standard control chart statistics and allowing runs to be incorporated into designs. [Chang and Gan \(2004\)](#) developed Shewhart control charts for variance components.

[Chand \(1975\)](#) discussed some ratio type estimators using two or more auxiliary characteristics. [Fuller \(1980\)](#) comment on ratio estimator with respect to its smaller approximate variance. [Royall and Cumberland \(1981\)](#) provided some empirical results for ratio estimator. [Kiregyera \(1984\)](#) used two auxiliary variables for regression type estimators. [Mukerjee et al. \(1987\)](#) and [Ahmed \(1998\)](#) discussed regression type estimators using multiple auxiliary information. [Prasad \(1989\)](#) introduced some improved

ratio type estimators for population mean. Singh and Gangele (1989–1995) discussed an improved estimator with known coefficient of variation using two auxiliary variables. Reddy (1993) discussed product and ratio estimation procedures. Sahoo and Sahoo (1999) compared some regression type estimators in double sampling procedures. Singh (2001) used transformed auxiliary variables for estimating population mean in two-phase sampling. Magnus (2002) introduced an estimator for estimation of mean of normal distribution in a regression context.

There are different classifications of control charts, e.g., according to the type of data, sample size, type of control etc. Farnum (1994) classified two basic types of control: threshold control and deviation control. Threshold control is concerned with detecting large shifts while deviation control is concerned with detecting small shifts in process parameters. The Shewhart type control charts are regarded as threshold control charts while non-Shewhart control charts (e.g., CUSUM and EWMA charts) are regarded as deviation control charts.

In this study the information about an auxiliary characteristic X is introduced for improved monitoring of the process variability of a quality characteristic of interest Y . Assuming bivariate normality of (Y, X) a new Shewhart type process variability control chart namely V_r chart (a threshold control chart) is proposed which is based on regression type estimator of variance. The regression type estimator for variance of Y using a single auxiliary variable X is defined for a bivariate random sample $(y_1, x_1), (y_2, x_2), \dots, (y_n, x_n)$ of size n as:

$$V_r = s_y^2 + b(\sigma_x^2 - s_x^2). \quad (1)$$

where s_y^2 is the sample variance of Y , s_x^2 is the sample variance of X , σ_x^2 is the population variance of X (assumed to be known) and b is defined as:

$$b = \frac{s_y^2 \left(\frac{n \sum_{i=1}^n (y_i - \bar{y})^2 (x_i - \bar{x})^2}{\sum_{i=1}^n (y_i - \bar{y})^2 \sum_{i=1}^n (x_i - \bar{x})^2} - 1 \right)}{s_x^2 \left(\frac{n \sum_{i=1}^n (x_i - \bar{x})^4}{(\sum_{i=1}^n (x_i - \bar{x})^2)^2} - 1 \right)} \quad (2)$$

where $\bar{y}$ and $\bar{x}$ are sample means of Y and X , respectively.

In the following sections (i) the design structure of V_r chart is developed for improved monitoring of a process variability following the pioneering work of Shewhart (1931), Pearson (1932), Pappanastos and Adams (1996), Ramalhoto and Morais (1999) and Gonzalez and Viles (2000, 2001), (ii) the power curves are constructed as a performance measure of V_r chart following Scheffe (1949), Duncan (1951) and Nelson (1985), (iii) the performance of V_r chart is compared with that of well-known Shewhart control chart used for the same purpose namely S^2 chart following Tuprah and Ncube (1987), Acosta-Mejia et al. (1999) and Ding et al. (2005) and (iv) the efficiency conditions are obtained where V_r chart outperforms the S^2 chart for detecting shifts (especially of large amounts) in process variability.

2 The proposed chart

Assuming the bivariate normality of (Y, X) a relationship between σ_y^2 (the unknown process variability of quality characteristic of interest Y which is to be monitored) and V_r (the regression type estimator of σ_y^2 defined in (1)) is required to develop the structure of the proposed V_r chart. Let $(y_1, x_1), (y_2, x_2), \dots, (y_n, x_n)$ be a bivariate random sample of size n from bivariate normal distribution, and let D be a random variable that defines a relationship between σ_y^2 and V_r as:

$$D = \frac{V_r}{\sigma_y^2}, \quad (3)$$

which helps in determining the parameters (i.e., centerline, lower control limit and upper control limit) of the proposed V_r chart.

Now if the distributional behavior of D is available then the sample statistic V_r can easily be used for testing hypothesis about shifts in σ_y^2 . When (Y, X) follow bivariate normal distribution, the distributional behavior of D depends only on ρ_{yx} (the correlation between Y and X) and n . The distributional behavior of D , in terms of its mean, standard error and quantile points, is required for the development of V_r chart and is explored in the following paragraphs when (Y, X) follow bivariate normal distribution.

First for mean, applying expectations to (3) gives:

$$E(D) = E(V_r/\sigma_y^2) = E(V_r)/\sigma_y^2. \quad (4)$$

Here $E(V_r)$ can safely be replaced by its estimate $\bar{V}_r$ (the mean of sample V_r s) using an appropriate number of random samples, as discussed in Hillier (1969) and Yang and Hiller (1970), from the process under study when the process is in the state of statistical control as written in Shewhart (1939, p. 26) just like $\bar{R}$ replaces $E(R)$ for R -Chart. Thus from (4) an estimate of σ_y^2 , after rearranging the terms, is given as:

$$\hat{\sigma}_y^2 = \bar{V}_r/E(D). \quad (5)$$

Let

$$E(D) = v_0. \quad (6)$$

It is not easy to get the analytical results for v_0 because $E(V_r)$ is difficult to obtain analytically. So the simulation results are obtained for v_0 in this paper. (In practice, simulation methods are often used to evaluate the expectation of a statistic, see Ross 1990). The coefficient v_0 entirely depends on ρ_{yx} and n for the case of bivariate normal distribution. Using 10,000 random samples generated from standard bivariate normal distribution without loss of generality, the results of v_0 have been obtained, for different combinations of ρ_{yx} and n , 1,000 times each. Based on these results the mean values

of v_0 , along with their respective standard errors, are provided in Appendix Table 1 for $n = 5, 6, \dots, 15, 20, 25, 30, 50, 100$ at some representative values of ρ_{yx} . The similar results can easily be obtained for any combination of ρ_{yx} and n .

Now using (6) in (5), the estimate of σ_y^2 is given as:

$$\hat{\sigma}_y^2 = \bar{V}_r/v_0. \quad (7)$$

The expression for $\hat{\sigma}$ given in (7) is similar to $\hat{\sigma} = \bar{R}/d_2$ derived for R chart assuming normal distribution in Alwan (2000, p. 393), $\hat{\sigma} = \bar{S}/c_4$ derived for S chart assuming normal distribution in Alwan (2000, p. 396) and $\hat{\sigma} = \bar{R}/m_1$ derived for the design of R chart assuming gamma distribution by Gonzalez and Viles (2001).

Now combining (4) and (6) yields the following:

$$E(V_r) = v_0\sigma_y^2. \quad (8)$$

Replacing the estimate of σ_y^2 (given in (7)) in (8) and simplification gives:

$$E(V_r) = \bar{V}_r. \quad (9)$$

A first order approximation for $E(V_r)$, when (Y, X) follow a bivariate normal distribution, is given as (see Garcia and Cebrian 1996):

$$E(V_r) \simeq \sigma_y^2. \quad (10)$$

Consequently

$$v_0 \simeq 1.00, \quad (11)$$

and (7) can be written as:

$$\hat{\sigma}_y^2 \simeq \bar{V}_r. \quad (12)$$

Asymptotically this first order approximation result (12) works very well, and even for smaller values of n it works fairly good as can be seen from Appendix Table 1. Thus V_r can be safely used as an unbiased estimator of σ_y^2 , and hence (12) can be used for unbiased estimation of unknown process variability. Thus V_r chart, not only asymptotically but even for smaller values of n , can work without constants, like d_2 for R chart and c_4 for S chart, for unbiased estimation of process variability.

Secondly for standard error, let the standard deviation of D (i.e., σ_D) be

$$\sigma_D = v_1. \quad (13)$$

For the same reason given above for v_0 , the analytical results for v_1 are difficult to obtain. So the simulation results are obtained for v_1 in this paper. The coefficient v_1 entirely depends on ρ_{yx} and n for the case of bivariate normal distribution. Using

the same 10,000 simulated random samples, the results of v_1 have been obtained, for different combinations of ρ_{yx} and n , 1,000 times each. Based on these results the mean values of v_1 , along with their respective standard errors, are provided in Appendix Table 2 for $n = 5, 6, \dots, 15, 20, 25, 30, 50, 100$ at some representative values of ρ_{yx} . The similar results can easily be obtained for any combination of ρ_{yx} and n .

Also taking variance of D and then simplification finally gives the expression for σ_D as:

$$\sigma_D = \sigma_{V_r} / \sigma_y^2, \quad (14)$$

where σ_{V_r} represents the standard deviation of distribution of sample statistic V_r .

Using (13) in (14) and rearranging yields the following result for σ_{V_r} :

$$\sigma_{V_r} = v_1 \sigma_y^2. \quad (15)$$

Substituting the estimate for σ_y^2 , given in (7), into (15), the estimate for σ_{V_r} is given as:

$$\hat{\sigma}_{V_r} = v_1 \bar{V}_r / v_0. \quad (16)$$

The expression in (16) is similar to the expression for $\hat{\sigma}_R$ of R chart as provided in Alwan (2000, p. 394).

The result given in (16) can safely be approximated, for any values of n , by using (12) in (15) as:

$$\hat{\sigma}_{V_r} \simeq v_1 \bar{V}_r. \quad (17)$$

An approximation for σ_{V_r} , when (Y, X) follow a bivariate normal distribution, is given as (see Isaki 1983):

$$\sigma_{V_r} \simeq \sqrt{2\sigma_y^4(1 - \rho_{yx}^4)/(n - 1)}. \quad (18)$$

Consequently

$$v_1 \simeq \sqrt{2(1 - \rho_{yx}^4)/(n - 1)}. \quad (19)$$

This approximation result (19) works well only asymptotically. For the case of smaller n it is a poor approximation as can be seen from Appendix Table 2.

Lastly for the quantile points of the distribution of D , let D_a represents the a th quantile point of the distribution of D (i.e., the point where D completes $a\%$ area). The analytical results for D_a are difficult to obtain so the simulation results are obtained for D_a . For bivariate normal distribution of (Y, X) the quantile points of the distribution of D entirely depend on both ρ_{yx} and n . Using the same 10,000 simulated random samples, the results of D_a have been obtained (like quantile points of $W = R/\sigma$ that determine the values of control limits of R chart and power of the chart) following

Pearson (1932), for different combinations of ρ_{yx} and n , 1,000 times each. Based on these results the mean values of some commonly used quantile points, along with their respective standard errors, are provided for $n = 5, 6, \dots, 15, 20, 25, 30, 50, 100$ in Appendix Tables 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12 at some representative values of ρ_{yx} . The similar results can easily be obtained for any combination of ρ_{yx} and n . These quantile points help in determining the control limits and power of the proposed V_r chart to detect shifts in process variability. The distributional behavior of D is not symmetrical at least for small values of n as obvious from Appendix Tables 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12. Asymptotically D is normally distributed, $N(1, 2(1 - \rho_{yx}^4)/(n - 1))$.

Now based on results obtained in Sect. 2 the parameters of the proposed V_r chart are discussed in the following section.

3 Parameters of proposed chart

The central line (CL), lower control limit (LCL) and upper control limit (UCL) are the three parameters of any Shewhart type control chart. There are two approaches to express these parameters namely probability limits approach and 3-sigma limits approach. In case of asymmetric distributional behavior of a relevant estimator the probability limits approach is preferred. If the distributional behavior of a relevant estimator is nearly symmetric then 3-sigma limits approach is a good alternative. The parameters of the proposed V_r chart using both the approaches are expressed in the following two subsections.

3.1 Probability limits approach

The value $\bar{V}_r$ corresponds to CL of the proposed V_r chart just like $\bar{R}$ for R chart provided in Alwan (2000, p. 347) and $\bar{S}$ for S chart provided in Alwan (2000, p. 362). Assuming the probability of making a Type-I error to be less than a specified value say α , the control limits (which are actually the true probability limits) for the proposed V_r chart are defined as:

$$\left. \begin{aligned} LCL &= V_{r_l} \text{ with } P_n(V_r = V_{r_l}) \leq \alpha_l \\ UCL &= V_{r_u} \text{ with } P_n(V_r = V_{r_u}) \geq 1 - \alpha_u \end{aligned} \right\}, \quad (20)$$

where $\alpha = \alpha_l + \alpha_u$ and P_n represents the cumulative distribution function for a given value of n .

Now using (3) and (7) in (20) and simplification finally gives the following:

$$\left. \begin{aligned} LCL &= V_{r_l} = D_l \bar{V}_r / v_0 \text{ with } P_n(D = D_l) \leq \alpha_l \\ UCL &= V_{r_u} = D_u \bar{V}_r / v_0 \text{ with } P_n(D = D_u) \geq 1 - \alpha_u \end{aligned} \right\}, \quad (21)$$

or using (12) instead of (7) in (20), result (21) can safely be converted into following:

$$\left. \begin{aligned} LCL &= V_{r_l} = D_l \bar{V}_r \text{ with } P_n(D = D_l) \leq \alpha_l \\ UCL &= V_{r_u} = D_u \bar{V}_r \text{ with } P_n(D = D_u) \geq 1 - \alpha_u \end{aligned} \right\}. \quad (22)$$

Thus quantile points of the distribution of D and average of sample V_{r_s} (i.e., $\bar{V}_r$) allow setting the true probability limits of the proposed V_r chart.

3.2 3-Sigma limits approach

If normal approximation to the distribution of D is used then the parameters of V_r chart with the usual 3-sigma control limits are given as:

$$\left. \begin{aligned} UCL &= \bar{V}_r + 3\sigma_{V_r} \\ CL &= \bar{V}_r \\ LCL &= \bar{V}_r - 3\sigma_{V_r} \end{aligned} \right\}, \quad (23)$$

Using (15) in (23) and then substituting result (16) finally gives the following result:

$$\left. \begin{aligned} UCL &= \bar{V}_r + 3v_1 \bar{V}_r / v_0 \\ CL &= \bar{V}_r \\ LCL &= \bar{V}_r - 3v_1 \bar{V}_r / v_0 \end{aligned} \right\}, \quad (24)$$

or using (17) instead of (16) gives the following result:

$$\left. \begin{aligned} UCL &= \bar{V}_r + 3v_1 \bar{V}_r \\ CL &= \bar{V}_r \\ LCL &= \bar{V}_r - 3v_1 \bar{V}_r \end{aligned} \right\}, \quad (25)$$

where the values of v_0 and v_1 are provided in Appendix Tables 1 and 2, respectively.

The validity of these 3-sigma limits based parameters of the proposed V_r chart depends on how close the normal approximation is to the true distribution of D .

A problem of LCL : For small values of n sometimes the LCL (using either 3-sigma limit approach or probability limit approach) results into a negative value as can be seen in starting rows of Appendix Tables 3, 4, 5, 6, 7, 8, 9, 10, and 11. A negative value for variability measure has no realistic meanings. Therefore in such situations it is assigned the value of 0 (as is done for range statistic in R chart, see Alwan 2000, p. 355).

After deciding the control structure, for given significance level, by either probability limit approach or 3-sigma limit approach, the sample statistic V_r is plotted against time order of samples. If all sample V_{r_s} lie within control limits, there is reasonable

evidence to conclude that there is no shift in process variability and process is stable at $\bar{V}_r/v_0 (\simeq \bar{V}_r)$. Otherwise some assignable cause or causes are at work causing shift in process variability.

To address small and moderate shifts using the developed structure of V_r chart, the runs rules (as discussed by Nelson 1984; Wheeler 1995; Quesenberry 1997) may be supplemented to its basic structure. As a result the risk of false alarms is increased.

Now for comparison purposes a similar structure for the conventional S^2 chart is given in the following section.

4 S^2 Chart

The S^2 chart does not use the information on auxiliary variable so the distributional assumption for S^2 chart is normality of Y (the marginals of bivariate normal distributions are always normal). The similar relationship, as defined in (3) for V_r chart, for S^2 chart is given as:

$$J = \frac{s_y^2}{\sigma_y^2}. \quad (26)$$

The relationship defined in (26) helps in determining the parameters of S^2 chart on the similar pattern as the relationship defined in (3) helps in determining the parameters of V_r chart. On similar pattern as for V_r chart, let $E(J) = u_0$ and $\sigma_J = u_1$. Then u_0 and u_1 (using the well-known properties of sample statistic S^2) are given as:

$$u_0 = 1. \quad (27)$$

and

$$u_1 = \sqrt{2/(n-1)}. \quad (28)$$

For comparison purposes the results of u_1 are tabulated in Appendix Table 13 for $n = 5, 6, \dots, 15, 20, 25, 30, 50, 100$. The parameters for S^2 chart are discussed by Montgomery (1996, p. 221).

5 Comparisons

In this section a comparison of means (v_0 and u_0), standard errors (v_1 and u_1) of the random variables used in V_r and S^2 charts and the power curves of these charts is provided.

The values of v_0 and u_0 do not differ much as can be seen from the results (11), (27) and Appendix Table 1.

The values of v_1 and u_1 differ depending on ρ_{yx} and n . It is observed that for small values of ρ_{yx} , v_1 is larger than u_1 for a given value of n , and when ρ_{yx} increases v_1 becomes smaller than u_1 as can be seen from Appendix Tables 2 and 13.

Fig. 1 **a** Power curves of V_r and S^2 charts for $n = 15$, $|\rho_{yx}| = 0.30$ and $\alpha = 0.002$. **b** Power curves of V_r and S^2 charts for $n = 15$, $|\rho_{yx}| = 0.70$ and $\alpha = 0.002$. **c** Power curves of V_r and S^2 charts for $n = 15$, $|\rho_{yx}| = 0.90$ and $\alpha = 0.002$

The efficiency of V_r chart as compared to S^2 chart has been examined using power curves as a performance measure. As the distributional behavior of D and J are not symmetrical, at least for smaller values of n so we have preferred to use the probability limits approach for the two charts to set control limits for a given significance level (α). Using their respective control structures, the probability limits of V_r and S^2 charts have been obtained for different combinations of ρ_{yx} and n with different significance levels, and power curves for the two charts have been constructed. The power curves, for $n = 15$ and 25 , are produced here for one low, one moderate and one high value of ρ_{yx} in the following Figs. 1a–c and 2a–c, respectively (using $\alpha = 0.002$).

In the above figures solid curve represents the power curve of V_r chart while the curve with dashes represents the power curve of S^2 chart. Figure 1a–c show that for $n = 15$ the suggested V_r chart is less powerful than S^2 chart for $|\rho_{yx}| = 0.30$, almost equally powerful as S^2 chart for $|\rho_{yx}| = 0.70$, and more powerful than S^2 chart for $|\rho_{yx}| = 0.90$.

The similar behavior is observed in Fig. 2a–c for $n = 25$ as in Fig. 1a–c for $n = 15$.

In general for each value of n there exists a value of $|\rho_{yx}|$ below which the suggested V_r chart remains less powerful than S^2 chart and above which it becomes more powerful as obvious from above Figs. 1a–c and 2a–c. Let l_n be the smallest value of $|\rho_{yx}|$, for a sample of size n , above which V_r chart outperforms the S^2 chart for detecting the shifts (especially moderate to large shifts) in process variability. Analytically l_n is difficult to obtain therefore the simulation results are obtained. Using the same 10,000 simulated random samples of Sect. 2, the results of l_n have been obtained for different values of n , 1,000 times each. Based on these results the mean values of l_n ,

Fig. 2 **a** Power curves of V_r and S^2 charts for $n = 25$, $|\rho_{yx}| = 0.30$ and $\alpha = 0.002$. **b** Power curves of V_r and S^2 charts for $n = 25$, $|\rho_{yx}| = 0.70$ and $\alpha = 0.002$. **c** Power curves of V_r and S^2 charts for $n = 25$, $|\rho_{yx}| = 0.90$ and $\alpha = 0.002$

along with their respective standard errors, are provided in Appendix Table 14 for $n = 5, 6, \dots, 15, 20, 25, 30, 50, 100$. The similar results for l_n can easily be obtained for any value of n .

In Appendix Table 14, $l_n = 0.80$ for $n = 10$ (e.g.), which means that the smallest value of $|\rho_{yx}|$, required for better performance of V_r chart as compared to S^2 chart, is 0.80. By examining other rows of this table it is obvious that as n increases, l_n becomes small.

6 Conclusion

The proposed V_r chart which is a Shewhart type control chart for monitoring process variability uses the information on a single auxiliary variable for monitoring process variability of a quality characteristic of interest. The V_r chart outshines the S^2 chart, under certain conditions on ρ_{yx} (provided in Appendix Table 14), for detecting shifts (not of smaller amount because Shewhart control charts do not target smaller shifts) in process variability.

Appendix

Note: In the following tables the results are reported up to four decimal places. The values reported in brackets are standard errors (reported up to five decimal places) for the results of each cell, reported to show how precise the results of each cell are.

Table 1 Control chart coefficient v_0 of V_r chart

n	ρ_{yx}									
	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.99
5	0.97105 (0.00013)	0.9847 (0.00069)	0.9866 (0.00014)	0.9889 (0.00037)	0.9811 (0.00012)	0.9927 (0.00017)	1.0111 (0.00056)	0.9954 (0.00081)	0.9995 (0.00061)	1.0011 (0.00055)
6	0.9780 (0.00021)	0.9838 (0.00033)	0.9811 (0.00052)	0.9806 (0.00016)	1.0294 (0.00002)	0.9998 (0.00051)	1.0033 (0.00097)	1.0013 (0.00015)	0.9907 (0.00034)	0.9974 (0.00047)
7	0.9858 (0.00066)	0.9882 (0.00024)	0.9879 (0.00035)	1.0096 (0.00087)	0.9890 (0.00049)	1.0001 (0.00038)	1.0006 (0.00011)	0.9948 (0.00072)	1.0028 (0.00046)	0.9995 (0.00099)
8	1.0003 (0.00043)	1.0010 (0.00051)	0.9954 (0.00068)	1.0060 (0.00037)	0.9938 (0.00029)	0.9977 (0.00045)	0.9981 (0.00052)	0.9939 (0.00037)	0.9953 (0.00089)	1.0000 (0.00008)
9	1.0041 (0.00017)	0.9969 (0.00074)	1.0026 (0.00047)	1.0013 (0.00019)	1.0002 (0.00053)	0.9997 (0.00048)	1.0026 (0.00063)	1.0028 (0.00094)	1.0029 (0.00023)	1.0002 (0.00055)
10	0.9916 (0.00038)	0.9996 (0.00062)	0.9959 (0.00053)	1.0020 (0.00014)	1.0044 (0.00082)	1.0014 (0.00039)	1.0040 (0.00092)	0.9987 (0.00011)	1.0018 (0.00077)	0.9994 (0.00037)
11	1.0007 (0.00073)	1.0028 (0.00021)	1.0027 (0.00054)	0.9888 (0.00064)	0.9953 (0.00028)	0.9999 (0.00082)	0.9959 (0.00096)	1.0029 (0.00081)	1.0019 (0.00095)	1.0001 (0.00045)
12	1.0000 (0.00083)	1.0029 (0.00058)	0.9992 (0.00022)	1.0044 (0.00095)	1.0008 (0.00077)	1.0020 (0.00086)	1.0018 (0.00055)	0.9967 (0.00043)	0.9977 (0.00082)	0.9994 (0.00061)
13	0.9953 (0.00063)	0.9975 (0.00054)	1.0075 (0.00009)	1.0006 (0.00002)	0.9967 (0.00033)	1.0060 (0.00067)	0.9989 (0.00082)	0.9984 (0.00074)	0.9996 (0.00087)	1.0009 (0.00057)
14	1.0046 (0.00072)	1.0024 (0.00031)	0.9965 (0.00054)	0.9990 (0.00064)	1.0018 (0.00081)	0.9996 (0.00063)	1.0017 (0.00027)	0.9980 (0.00049)	1.0018 (0.00057)	0.9995 (0.00082)
15	1.0053 (0.00093)	1.0080 (0.00054)	0.9986 (0.00049)	1.0007 (0.00080)	1.0041 (0.00087)	1.0005 (0.00067)	1.0028 (0.00014)	1.0005 (0.00024)	1.0014 (0.00057)	1.0007 (0.00044)
20	1.0081 (0.00086)	1.0064 (0.00067)	1.0042 (0.00024)	0.9933 (0.00043)	0.9987 (0.00082)	0.9976 (0.00052)	1.0055 (0.00064)	0.9997 (0.00025)	0.9981 (0.00074)	0.9982 (0.00061)
25	1.0018 (0.00095)	1.0034 (0.00068)	1.0030 (0.00027)	0.9981 (0.00023)	0.9987 (0.00057)	1.0079 (0.00082)	0.9998 (0.00055)	1.0016 (0.00049)	1.0046 (0.00037)	1.0001 (0.00052)
30	1.0016 (0.00063)	1.0047 (0.00014)	1.0047 (0.00054)	1.0002 (0.00067)	0.9985 (0.00051)	1.0019 (0.00028)	0.9993 (0.00081)	1.0013 (0.00075)	0.9999 (0.00059)	0.9999 (0.00072)
50	1.0005 (0.00088)	0.9994 (0.00073)	1.0029 (0.00027)	1.0005 (0.00049)	1.0000 (0.00082)	0.9997 (0.00054)	0.9988 (0.00034)	1.0001 (0.00061)	0.9996 (0.00064)	0.9995 (0.00085)
100	0.9996 (0.00011)	0.9987 (0.00067)	0.9974 (0.00075)	0.9984 (0.00035)	0.9983 (0.00057)	1.0002 (0.00082)	0.9993 (0.00063)	0.9995 (0.00081)	1.0001 (0.00051)	0.9997 (0.00066)

Table 2 Control chart coefficient v_1 of V_r chart

n	ρ_{yx}									
	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.99
5	2.6484 (0.00061)	2.4178 (0.00057)	2.2898 (0.00034)	2.2147 (0.00044)	1.8967 (0.00058)	1.6297 (0.00024)	1.4014 (0.00048)	1.29147 (0.00071)	0.8164 (0.00041)	0.2237 (0.00037)
6	1.5159 (0.00071)	1.4122 (0.00096)	1.3541 (0.00060)	1.2952 (0.00081)	1.2010 (0.00059)	1.0793 (0.00067)	1.0249 (0.00057)	0.8172 (0.00035)	0.5818 (0.00087)	0.1866 (0.00061)

Table 2 continued

n	ρ_{yx}									
	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.99
7	1.0537 (0.00064)	1.0234 (0.00072)	0.9912 (0.00034)	0.8861 (0.00059)	0.8647 (0.00063)	0.8545 (0.00075)	0.7425 (0.00034)	0.6483 (0.00087)	0.4727 (0.00022)	0.1531 (0.00097)
8	0.7788 (0.00067)	0.7669 (0.00082)	0.7596 (0.00072)	0.7409 (0.00019)	0.7253 (0.00024)	0.6934 (0.00042)	0.6314 (0.00087)	0.5391 (0.00052)	0.4224 (0.00061)	0.1408 (0.00063)
9	0.6809 (0.00076)	0.6764 (0.00037)	0.6670 (0.00014)	0.6575 (0.00067)	0.6447 (0.00019)	0.6234 (0.00051)	0.5531 (0.00055)	0.4935 (0.00034)	0.3755 (0.00018)	0.1237 (0.00043)
10	0.6387 (0.00061)	0.6234 (0.00054)	0.61245 (0.00082)	0.6036 (0.00071)	0.5738 (0.00059)	0.5496 (0.00037)	0.5146 (0.00019)	0.4440 (0.00043)	0.3450 (0.00086)	0.1138 (0.00073)
11	0.5413 (0.00017)	0.5345 (0.00019)	0.5271 (0.00073)	0.5125 (0.00082)	0.5035 (0.00046)	0.4890 (0.00053)	0.4692 (0.00015)	0.4141 (0.00075)	0.3176 (0.00095)	0.1059 (0.00068)
12	0.5078 (0.00049)	0.5013 (0.00067)	0.4960 (0.00081)	0.4873 (0.00035)	0.4792 (0.00016)	0.4659 (0.00042)	0.4442 (0.00059)	0.3846 (0.00017)	0.2938 (0.00084)	0.0990 (0.00037)
13	0.4820 (0.00011)	0.4791 (0.00054)	0.4661 (0.00095)	0.4513 (0.00058)	0.4454 (0.00073)	0.4203 (0.00051)	0.4082 (0.00048)	0.3592 (0.00010)	0.2797 (0.00066)	0.0956 (0.00083)
14	0.4541 (0.00025)	0.4456 (0.00081)	0.4417 (0.00055)	0.4331 (0.00072)	0.4276 (0.00091)	0.4077 (0.00084)	0.3872 (0.00057)	0.3461 (0.00049)	0.2645 (0.00073)	0.0918 (0.00012)
15	0.4286 (0.00091)	0.4219 (0.00073)	0.4129 (0.00048)	0.4020 (0.00021)	0.3953 (0.00077)	0.3847 (0.00084)	0.3575 (0.00037)	0.3286 (0.00042)	0.2546 (0.00066)	0.0856 (0.00080)
20	0.3534 (0.00052)	0.3486 (0.00017)	0.3427 (0.00050)	0.3375 (0.00061)	0.3303 (0.00089)	0.3237 (0.00013)	0.3061 (0.00094)	0.2759 (0.00061)	0.2116 (0.00048)	0.0716 (0.00077)
25	0.3175 (0.00067)	0.3116 (0.00064)	0.3083 (0.00081)	0.3011 (0.00093)	0.2921 (0.00082)	0.2789 (0.00018)	0.2697 (0.00029)	0.2380 (0.00083)	0.1837 (0.00067)	0.0623 (0.00043)
30	0.2802 (0.00037)	0.2790 (0.00052)	0.2713 (0.00082)	0.2698 (0.00032)	0.2643 (0.00067)	0.2571 (0.00097)	0.2428 (0.00052)	0.2190 (0.00061)	0.1653 (0.00095)	0.0557 (0.00033)
50	0.2112 (0.00088)	0.2090 (0.00034)	0.2067 (0.00028)	0.2034 (0.00016)	0.2017 (0.00092)	0.1949 (0.00058)	0.1831 (0.00035)	0.1624 (0.00019)	0.1234 (0.00037)	0.0422 (0.00068)
100	0.1437 (0.00062)	0.1428 (0.00038)	0.1417 (0.00057)	0.1407 (0.00044)	0.1379 (0.00069)	0.1325 (0.00054)	0.1260 (0.00062)	0.1114 (0.00067)	0.0860 (0.00057)	0.0289 (0.00092)

Table 3 Quantile points of the distribution of D (when $|\rho_{yx}| = 0.10$)

n	$D_{0.01}$	$D_{0.05}$	$D_{0.10}$	$D_{0.20}$	$D_{0.25}$	$D_{0.50}$	$D_{0.75}$	$D_{0.80}$	$D_{0.90}$	$D_{0.95}$	$D_{0.99}$
5	-3.2964 (0.00037)	-0.1873 (0.00033)	0.0785 (0.00052)	0.2434 (0.00067)	0.3327 (0.00023)	0.8016 (0.00025)	1.43219 (0.00051)	1.6248 (0.00046)	2.2611 (0.00031)	3.0814 (0.00061)	6.2105 (0.00015)
6	-1.7250 (0.00059)	0.0211 (0.00037)	0.1592 (0.00084)	0.3312 (0.00055)	0.4138 (0.00034)	0.8193 (0.00067)	1.4128 (0.00036)	1.5616 (0.00085)	2.0720 (0.00073)	2.6408 (0.00049)	4.9011 (0.00036)
7	-0.6279 (0.00069)	0.1156 (0.00097)	0.2267 (0.00021)	0.3944 (0.00049)	0.4639 (0.00056)	0.8522 (0.00062)	1.3836 (0.00075)	1.5111 (0.00038)	1.9602 (0.00064)	2.4000 (0.00055)	3.9601 (0.00040)
8	-0.2610 (0.00049)	0.1607 (0.00081)	0.2697 (0.00019)	0.4486 (0.00068)	0.5179 (0.00078)	0.8799 (0.00028)	1.3507 (0.00063)	1.4792 (0.00095)	1.8502 (0.00046)	2.2567 (0.00051)	3.3840 (0.00076)
9	-0.0406 (0.00035)	0.2338 (0.00060)	0.3469 (0.00063)	0.4834 (0.00082)	0.5477 (0.00091)	0.9059 (0.00091)	1.3299 (0.00042)	1.4299 (0.00073)	1.7726 (0.00011)	2.1106 (0.00037)	3.0749 (0.00095)

Table 3 continued

n	$D_{0.01}$	$D_{0.05}$	$D_{0.10}$	$D_{0.20}$	$D_{0.25}$	$D_{0.50}$	$D_{0.75}$	$D_{0.80}$	$D_{0.90}$	$D_{0.95}$	$D_{0.99}$
10	0.0603 (0.00046)	0.2580 (0.00022)	0.3633 (0.00038)	0.52005 (0.00064)	0.5816 (0.00013)	0.9033 (0.00016)	1.3224 (0.00059)	1.4234 (0.00038)	1.7919 (0.00066)	2.1422 (0.00093)	3.1778 (0.00073)
11	0.1017 (0.00096)	0.3020 (0.00099)	0.4017 (0.00046)	0.5438 (0.00072)	0.6013 (0.00024)	0.9170 (0.00058)	1.3178 (0.00013)	1.4026 (0.00066)	1.6811 (0.00089)	1.9825 (0.00046)	2.7959 (0.00019)
12	0.1660 (0.00073)	0.3270 (0.00046)	0.4388 (0.00057)	0.5610 (0.00038)	0.6264 (0.00035)	0.9240 (0.00043)	1.2911 (0.00022)	1.3916 (0.00046)	1.6527 (0.00038)	1.9414 (0.00064)	2.5976 (0.00035)
13	0.1929 (0.00050)	0.3521 (0.00061)	0.4537 (0.00089)	0.5807 (0.00091)	0.6458 (0.00046)	0.9280 (0.00029)	1.2810 (0.00043)	1.3822 (0.00050)	1.6266 (0.00097)	1.8757 (0.00091)	2.5326 (0.00049)
14	0.2300 (0.00067)	0.3847 (0.00028)	0.5007 (0.00073)	0.6090 (0.00018)	0.6730 (0.00057)	0.9423 (0.00066)	1.2704 (0.00073)	1.3715 (0.00066)	1.5921 (0.00067)	1.8212 (0.00073)	2.3472 (0.00038)
15	0.2489 (0.00049)	0.4079 (0.00084)	0.5042 (0.00041)	0.6212 (0.00067)	0.6824 (0.00068)	0.9525 (0.00073)	1.2616 (0.00051)	1.3427 (0.00067)	1.5680 (0.00097)	1.7691 (0.00044)	2.2842 (0.00024)
20	0.3339 (0.00061)	0.4782 (0.00073)	0.5491 (0.00037)	0.6788 (0.00043)	0.7197 (0.00079)	0.9658 (0.00081)	1.2420 (0.00051)	1.3039 (0.00073)	1.4832 (0.00031)	1.6530 (0.00043)	2.1021 (0.00094)
25	0.4075 (0.00029)	0.5498 (0.00013)	0.6154 (0.00061)	0.7112 (0.00037)	0.7558 (0.00022)	0.9672 (0.00098)	1.1997 (0.00018)	1.2620 (0.00094)	1.4201 (0.00025)	1.5652 (0.00085)	1.8819 (0.00046)
30	0.4606 (0.00051)	0.5801 (0.00041)	0.6500 (0.00057)	0.7417 (0.00029)	0.7857 (0.00011)	0.9774 (0.00063)	1.1959 (0.00026)	1.2482 (0.00046)	1.3900 (0.00077)	1.5113 (0.00038)	1.7819 (0.00081)
50	0.5802 (0.00088)	0.6735 (0.00078)	0.7400 (0.00064)	0.8165 (0.00077)	0.8464 (0.00071)	0.9862 (0.00088)	1.1399 (0.00076)	1.1798 (0.00017)	1.2717 (0.00038)	1.3621 (0.00075)	1.5398 (0.00074)
100	0.6829 (0.00019)	0.7613 (0.00092)	0.8118 (0.00029)	0.8651 (0.00038)	0.8836 (0.00088)	0.9921 (0.00040)	1.0998 (0.00051)	1.1264 (0.00088)	1.1924 (0.00016)	1.2558 (0.00063)	1.3599 (0.00068)

Table 4 Quantile points of the distribution of D (when $|\rho_{yx}| = 0.20$)

n	$D_{0.01}$	$D_{0.05}$	$D_{0.10}$	$D_{0.20}$	$D_{0.25}$	$D_{0.50}$	$D_{0.75}$	$D_{0.80}$	$D_{0.90}$	$D_{0.95}$	$D_{0.99}$
5	-3.1876 (0.00019)	-0.1750 (0.00037)	0.0845 (0.00077)	0.2569 (0.00018)	0.3473 (0.00081)	0.7878 (0.00097)	1.4103 (0.00079)	1.6012 (0.00059)	2.2439 (0.00018)	3.0581 (0.00084)	6.1006 (0.00058)
6	-1.7178 (0.00011)	0.0360 (0.00051)	0.1646 (0.00022)	0.3462 (0.00027)	0.4206 (0.00019)	0.8215 (0.00067)	1.4011 (0.00038)	1.5519 (0.00077)	2.0624 (0.00037)	2.6395 (0.00038)	4.8744 (0.00051)
7	-0.5996 (0.00037)	0.1222 (0.00017)	0.2345 (0.00062)	0.4052 (0.00089)	0.4771 (0.00021)	0.8550 (0.00079)	1.3753 (0.00051)	1.5048 (0.00053)	1.9507 (0.00083)	2.3921 (0.00074)	3.9364 (0.00067)
8	-0.2586 (0.00085)	0.1771 (0.00051)	0.2767 (0.00033)	0.4599 (0.00081)	0.5271 (0.00088)	0.8713 (0.00059)	1.3416 (0.00077)	1.4616 (0.00061)	1.8411 (0.00019)	2.2438 (0.00018)	3.3670 (0.00079)
9	-0.0363 (0.00074)	0.2491 (0.00079)	0.3575 (0.00061)	0.4941 (0.00019)	0.5526 (0.00033)	0.8897 (0.00037)	1.3245 (0.00043)	1.4264 (0.00080)	1.7819 (0.00081)	2.1339 (0.00033)	3.1629 (0.00077)
10	0.0720 (0.00056)	0.2660 (0.00018)	0.3789 (0.00037)	0.5257 (0.00067)	0.5993 (0.00051)	0.9061 (0.00043)	1.3128 (0.00019)	1.4195 (0.00097)	1.7215 (0.00011)	2.0589 (0.00037)	2.9902 (0.00088)
11	0.1100 (0.00018)	0.3105 (0.00061)	0.4119 (0.00019)	0.5539 (0.00070)	0.6129 (0.00074)	0.9188 (0.00088)	1.3081 (0.00015)	1.3936 (0.00028)	1.6723 (0.00081)	1.9738 (0.00012)	2.7417 (0.00093)
12	0.1765 (0.00023)	0.3349 (0.00027)	0.4460 (0.00051)	0.5760 (0.00028)	0.6354 (0.00079)	0.9258 (0.00057)	1.2812 (0.00037)	1.3810 (0.00033)	1.6406 (0.00036)	1.9310 (0.00027)	2.5687 (0.00019)
13	0.2075 (0.00081)	0.3619 (0.00068)	0.4635 (0.00019)	0.5951 (0.00037)	0.6519 (0.00027)	0.9263 (0.00074)	1.2791 (0.00019)	1.3726 (0.00024)	1.6187 (0.00073)	1.8629 (0.00047)	2.4825 (0.00037)

Table 4 continued

n	$D_{0.01}$	$D_{0.05}$	$D_{0.10}$	$D_{0.20}$	$D_{0.25}$	$D_{0.50}$	$D_{0.75}$	$D_{0.80}$	$D_{0.90}$	$D_{0.95}$	$D_{0.99}$
14	0.2325 (0.00051)	0.3909 (0.00019)	0.5037 (0.00044)	0.6181 (0.00043)	0.6779 (0.00081)	0.9477 (0.00033)	1.2628 (0.00067)	1.3613 (0.00019)	1.5807 (0.00051)	1.8143 (0.00013)	2.3370 (0.00028)
15	0.2582 (0.00083)	0.4195 (0.00037)	0.5169 (0.00025)	0.6354 (0.00055)	0.6872 (0.00038)	0.9495 (0.00079)	1.2508 (0.00055)	1.3326 (0.00031)	1.5629 (0.00037)	1.7633 (0.00088)	2.2754 (0.00074)
20	0.3414 (0.00088)	0.4866 (0.00011)	0.5574 (0.00037)	0.6866 (0.00074)	0.7274 (0.00051)	0.9654 (0.00056)	1.2312 (0.00044)	1.2917 (0.00036)	1.4713 (0.00065)	1.6415 (0.00019)	2.0628 (0.00081)
25	0.4150 (0.00078)	0.5529 (0.00059)	0.6271 (0.00079)	0.7258 (0.00045)	0.7682 (0.00084)	0.9704 (0.00019)	1.1916 (0.00081)	1.2549 (0.00061)	1.4199 (0.00037)	1.5521 (0.00093)	1.8786 (0.00051)
30	0.4640 (0.00011)	0.5819 (0.00019)	0.6525 (0.00088)	0.7446 (0.00017)	0.7892 (0.00079)	0.9768 (0.00081)	1.1823 (0.00034)	1.2308 (0.00022)	1.3836 (0.00048)	1.5018 (0.00067)	1.7770 (0.00049)
50	0.5810 (0.00064)	0.6788 (0.00037)	0.7410 (0.00073)	0.8182 (0.00024)	0.8477 (0.00054)	0.9841 (0.00017)	1.1394 (0.00054)	1.1796 (0.00054)	1.2701 (0.00067)	1.3605 (0.00053)	1.5387 (0.00065)
100	0.6844 (0.00028)	0.7621 (0.00091)	0.8188 (0.00067)	0.8689 (0.00064)	0.8896 (0.00043)	0.9915 (0.00033)	1.0994 (0.00088)	1.1219 (0.00036)	1.1910 (0.00019)	1.2507 (0.00031)	1.3597 (0.00025)

Table 5 Quantile points of the distribution of D (when $|\rho_{yx}| = 0.30$)

n	$D_{0.01}$	$D_{0.05}$	$D_{0.10}$	$D_{0.20}$	$D_{0.25}$	$D_{0.50}$	$D_{0.75}$	$D_{0.80}$	$D_{0.90}$	$D_{0.95}$	$D_{0.99}$
5	-3.0162 (0.00088)	-0.1507 (0.00053)	0.0956 (0.00033)	0.2694 (0.00042)	0.3568 (0.00029)	0.7830 (0.00077)	1.4013 (0.00028)	1.5946 (0.00054)	2.2304 (0.00031)	3.0406 (0.00005)	5.9501 (0.00091)
6	-1.4037 (0.00005)	0.0418 (0.00011)	0.1795 (0.00055)	0.3507 (0.00083)	0.4367 (0.00028)	0.8372 (0.00080)	1.3945 (0.00095)	1.5473 (0.00025)	2.0568 (0.00010)	2.6254 (0.00068)	4.8445 (0.00009)
7	-0.5941 (0.00054)	0.1392 (0.00020)	0.2471 (0.00076)	0.4161 (0.00091)	0.4893 (0.00045)	0.8559 (0.00023)	1.3617 (0.00070)	1.4999 (0.00006)	1.9400 (0.00019)	2.3851 (0.00041)	3.8990 (0.00011)
8	-0.2390 (0.00022)	0.1836 (0.00008)	0.3045 (0.00031)	0.4694 (0.00066)	0.5367 (0.00175)	0.8732 (0.00066)	1.3365 (0.00163)	1.4509 (0.00076)	1.8347 (0.00016)	2.2388 (0.00028)	3.3494 (0.00033)
9	-0.0302 (0.00019)	0.2571 (0.00107)	0.3685 (0.00068)	0.5005 (0.00090)	0.5662 (0.00054)	0.8977 (0.00008)	1.3143 (0.00083)	1.4112 (0.00090)	1.7733 (0.00163)	2.1200 (0.00005)	3.1313 (0.00077)
10	0.0895 (0.00061)	0.2782 (0.00017)	0.3899 (0.00177)	0.5393 (0.00175)	0.6077 (0.00073)	0.8994 (0.00043)	1.3028 (0.00010)	1.4078 (0.00020)	1.7105 (0.00091)	2.0468 (0.00042)	2.9712 (0.00016)
11	0.1224 (0.00091)	0.3233 (0.00172)	0.4276 (0.00049)	0.5682 (0.00011)	0.6261 (0.00028)	0.9225 (0.00030)	1.2905 (0.00021)	1.3800 (0.00018)	1.6613 (0.00041)	1.9618 (0.00027)	2.7090 (0.00013)
12	0.1820 (0.00083)	0.3411 (0.00095)	0.4524 (0.00032)	0.5814 (0.00077)	0.6457 (0.00005)	0.9317 (0.00031)	1.2722 (0.00068)	1.3702 (0.00080)	1.6342 (0.00092)	1.9232 (0.00065)	2.5314 (0.00016)
13	0.2163 (0.00021)	0.3750 (0.00020)	0.4704 (0.00054)	0.6069 (0.00031)	0.6620 (0.00023)	0.9308 (0.00091)	1.2664 (0.00028)	1.3612 (0.00102)	1.6048 (0.00045)	1.8581 (0.00039)	2.4439 (0.00002)
14	0.2446 (0.00019)	0.4053 (0.00043)	0.5100 (0.00013)	0.6293 (0.00025)	0.6814 (0.00076)	0.9313 (0.00042)	1.2530 (0.00029)	1.3549 (0.00033)	1.5743 (0.00055)	1.8054 (0.00042)	2.3280 (0.00007)
15	0.2665 (0.00013)	0.4245 (0.00030)	0.5249 (0.00028)	0.6446 (0.00045)	0.6990 (0.00008)	0.9493 (0.00076)	1.2407 (0.00083)	1.3299 (0.00066)	1.5516 (0.00031)	1.7442 (0.00011)	2.2641 (0.00091)
20	0.3579 (0.00004)	0.4980 (0.00010)	0.5679 (0.00070)	0.6909 (0.00042)	0.7378 (0.00019)	0.9669 (0.00095)	1.2289 (0.00017)	1.2840 (0.00025)	1.4662 (0.00037)	1.6343 (0.00046)	2.0256 (0.00018)
25	0.4203 (0.00042)	0.5653 (0.00089)	0.6392 (0.00080)	0.7353 (0.00092)	0.7793 (0.00320)	0.9721 (0.00031)	1.1849 (0.00020)	1.2401 (0.00013)	1.4090 (0.00068)	1.5459 (0.00089)	1.8643 (0.00033)

Table 5 continued

n	$D_{0.01}$	$D_{0.05}$	$D_{0.10}$	$D_{0.20}$	$D_{0.25}$	$D_{0.50}$	$D_{0.75}$	$D_{0.80}$	$D_{0.90}$	$D_{0.95}$	$D_{0.99}$
30	0.4725 (0.00077)	0.5934 (0.00091)	0.6669 (0.00076)	0.7533 (0.00042)	0.7925 (0.00091)	0.9755 (0.00077)	1.1705 (0.00175)	1.2247 (0.00043)	1.3700 (0.00091)	1.4973 (0.00049)	1.7630 (0.00013)
50	0.5836 (0.00026)	0.6808 (0.00011)	0.7425 (0.00041)	0.8205 (0.00095)	0.8505 (0.00055)	0.9883 (0.00031)	1.1318 (0.00023)	1.1707 (0.00066)	1.2699 (0.00009)	1.3582 (0.00008)	1.5368 (0.00056)
100	0.6897 (0.00016)	0.7687 (0.00006)	0.8210 (0.00020)	0.8723 (0.00066)	0.8954 (0.00092)	0.9901 (0.00061)	1.0969 (0.00005)	1.1197 (0.00030)	1.1891 (0.00019)	1.2482 (0.00073)	1.3573 (0.00021)

Table 6 Quantile points of the distribution of D (when $|\rho_{yx}| = 0.40$)

n	$D_{0.01}$	$D_{0.05}$	$D_{0.10}$	$D_{0.20}$	$D_{0.25}$	$D_{0.50}$	$D_{0.75}$	$D_{0.80}$	$D_{0.90}$	$D_{0.95}$	$D_{0.99}$
5	-2.1369 (0.00088)	-0.1147 (0.00105)	0.1024 (0.00025)	0.2825 (0.00076)	0.3630 (0.00029)	0.7799 (0.00045)	1.3927 (0.00097)	1.5795 (0.00005)	2.2148 (0.00054)	2.9254 (0.00026)	5.6380 (0.00038)
6	-1.0454 (0.00021)	0.0568 (0.00055)	0.1966 (0.00028)	0.3701 (0.00042)	0.4433 (0.00036)	0.8147 (0.00090)	1.3778 (0.00083)	1.5259 (0.00076)	2.0315 (0.00011)	2.5895 (0.00027)	4.7015 (0.00009)
7	-0.4692 (0.00015)	0.1465 (0.00023)	0.2641 (0.00068)	0.4315 (0.00031)	0.5014 (0.00077)	0.8589 (0.00073)	1.3520 (0.00080)	1.4806 (0.00019)	1.9204 (0.00080)	2.3547 (0.00035)	3.8582 (0.00016)
8	-0.2116 (0.00005)	0.2093 (0.00043)	0.3232 (0.00175)	0.4797 (0.00030)	0.5454 (0.00013)	0.8744 (0.00017)	1.3258 (0.00042)	1.4431 (0.00177)	1.8143 (0.00025)	2.2119 (0.00028)	3.3371 (0.00023)
9	0.0338 (0.00026)	0.2628 (0.00033)	0.3787 (0.00163)	0.5265 (0.00045)	0.5807 (0.00031)	0.8974 (0.00028)	1.3017 (0.00009)	1.4026 (0.00042)	1.7585 (0.00054)	2.1025 (0.00025)	3.0929 (0.00019)
10	0.0931 (0.00027)	0.2927 (0.00041)	0.4028 (0.00016)	0.5525 (0.00028)	0.6131 (0.00163)	0.9163 (0.00070)	1.2917 (0.00031)	1.3920 (0.00105)	1.6926 (0.00085)	2.0212 (0.00056)	2.9398 (0.00006)
11	0.1354 (0.00027)	0.3328 (0.00080)	0.4300 (0.00020)	0.5709 (0.00026)	0.6319 (0.00076)	0.9111 (0.00105)	1.2813 (0.00021)	1.3707 (0.00068)	1.6469 (0.00077)	1.9002 (0.00130)	2.6710 (0.00017)
12	0.1964 (0.00013)	0.3625 (0.00008)	0.4615 (0.00049)	0.6081 (0.00076)	0.6531 (0.00029)	0.9287 (0.00042)	1.2604 (0.00066)	1.3626 (0.00011)	1.6209 (0.00033)	1.8871 (0.00026)	2.4855 (0.00013)
13	0.2268 (0.00018)	0.3874 (0.00026)	0.4813 (0.00019)	0.6158 (0.00090)	0.6780 (0.00092)	0.9405 (0.00066)	1.2581 (0.00005)	1.3540 (0.00020)	1.5878 (0.00155)	1.8193 (0.00022)	2.4198 (0.00008)
14	0.2537 (0.00011)	0.4142 (0.00031)	0.5175 (0.00320)	0.6399 (0.00077)	0.6953 (0.00008)	0.9379 (0.00102)	1.2429 (0.00023)	1.3405 (0.00041)	1.5679 (0.00095)	1.7780 (0.00035)	2.3123 (0.00012)
15	0.2777 (0.00013)	0.4355 (0.00011)	0.5330 (0.00016)	0.6511 (0.00025)	0.7014 (0.00019)	0.9471 (0.00031)	1.2379 (0.00054)	1.3159 (0.00172)	1.5469 (0.00036)	1.7322 (0.00015)	2.2506 (0.00021)
20	0.3690 (0.00054)	0.5027 (0.00042)	0.5731 (0.00061)	0.7064 (0.00013)	0.7428 (0.00045)	0.9536 (0.00175)	1.2180 (0.00026)	1.2795 (0.00065)	1.4587 (0.00028)	1.6248 (0.00083)	1.9850 (0.00008)
25	0.4229 (0.00042)	0.5704 (0.00003)	0.6464 (0.00008)	0.7413 (0.00045)	0.7850 (0.00068)	0.9639 (0.00320)	1.1721 (0.00055)	1.2353 (0.00070)	1.3987 (0.00089)	1.5374 (0.00010)	1.8591 (0.00027)
30	0.4776 (0.00105)	0.5982 (0.00124)	0.6718 (0.00070)	0.7694 (0.00080)	0.8086 (0.00320)	0.9731 (0.00177)	1.1687 (0.00083)	1.2166 (0.00077)	1.3577 (0.00030)	1.4784 (0.00011)	1.7435 (0.00034)
50	0.5877 (0.00006)	0.6881 (0.00030)	0.7498 (0.00175)	0.8286 (0.00020)	0.8569 (0.00066)	0.9865 (0.00008)	1.1293 (0.00042)	1.1666 (0.00080)	1.2691 (0.00049)	1.3541 (0.00012)	1.5321 (0.00086)
100	0.6963 (0.00016)	0.7754 (0.00031)	0.8222 (0.00077)	0.8781 (0.00090)	0.8995 (0.00023)	0.9919 (0.00013)	1.0914 (0.00055)	1.1172 (0.00026)	1.1836 (0.00013)	1.2416 (0.00005)	1.3558 (0.00055)

Table 7 Quantile points of the distribution of D (when $|\rho_{yx}| = 0.50$)

n	$D_{0.01}$	$D_{0.05}$	$D_{0.10}$	$D_{0.20}$	$D_{0.25}$	$D_{0.50}$	$D_{0.75}$	$D_{0.80}$	$D_{0.90}$	$D_{0.95}$	$D_{0.99}$
5	-1.9481 (0.00088)	-0.0726 (0.00026)	0.1130 (0.00097)	0.2969 (0.00019)	0.3700 (0.00025)	0.7978 (0.00092)	1.3838 (0.00016)	1.5564 (0.00042)	2.1409 (0.00020)	2.8495 (0.00005)	5.48535 (0.00032)
6	-1.0253 (0.00047)	0.0871 (0.00023)	0.2188 (0.00017)	0.3838 (0.00031)	0.4667 (0.00045)	0.8403 (0.00028)	1.3669 (0.00076)	1.5165 (0.00019)	2.0155 (0.00068)	2.5507 (0.00021)	4.3806 (0.00105)
7	-0.4212 (0.00027)	0.1584 (0.00055)	0.2723 (0.00013)	0.4443 (0.00090)	0.5122 (0.00043)	0.8562 (0.00066)	1.3407 (0.00025)	1.4726 (0.00023)	1.8751 (0.00080)	2.3173 (0.00026)	3.7197 (0.00012)
8	-0.1964 (0.00005)	0.2174 (0.00095)	0.3344 (0.00008)	0.4891 (0.00061)	0.5557 (0.00013)	0.8674 (0.00054)	1.3112 (0.00041)	1.4388 (0.00092)	1.7890 (0.00005)	2.1931 (0.00016)	3.1767 (0.00058)
9	0.0586 (0.00025)	0.2721 (0.00026)	0.3808 (0.00031)	0.5333 (0.00076)	0.5964 (0.00090)	0.9026 (0.00006)	1.2985 (0.00090)	1.3965 (0.00041)	1.7463 (0.00163)	2.0948 (0.00019)	3.0207 (0.00054)
10	0.1100 (0.00019)	0.3045 (0.00022)	0.4165 (0.00105)	0.5622 (0.00320)	0.6237 (0.00045)	0.9061 (0.00008)	1.2887 (0.00095)	1.3860 (0.00045)	1.6887 (0.00033)	1.9980 (0.00042)	2.8962 (0.00015)
11	0.1440 (0.00107)	0.3468 (0.00077)	0.4430 (0.00029)	0.5872 (0.00080)	0.6495 (0.00175)	0.9203 (0.00028)	1.2734 (0.00019)	1.3608 (0.00030)	1.6263 (0.00036)	1.8926 (0.00083)	2.5854 (0.00011)
12	0.2068 (0.00042)	0.3776 (0.00102)	0.4748 (0.00092)	0.6120 (0.00105)	0.6656 (0.00049)	0.9320 (0.00023)	1.2586 (0.00031)	1.3528 (0.00077)	1.6114 (0.00073)	1.8505 (0.00113)	2.4260 (0.00013)
13	0.2353 (0.00008)	0.3998 (0.00042)	0.4971 (0.00066)	0.6257 (0.00026)	0.6863 (0.00175)	0.9372 (0.00092)	1.2501 (0.00070)	1.3459 (0.00163)	1.5762 (0.00005)	1.8081 (0.00085)	2.3882 (0.00044)
14	0.2688 (0.00013)	0.4299 (0.00019)	0.5239 (0.00070)	0.6523 (0.00054)	0.7034 (0.00011)	0.9458 (0.00068)	1.2399 (0.00029)	1.3327 (0.00177)	1.5508 (0.00055)	1.7648 (0.00023)	2.2954 (0.00068)
15	0.2857 (0.00056)	0.4447 (0.00005)	0.5401 (0.00066)	0.6642 (0.00025)	0.7159 (0.00009)	0.9531 (0.00016)	1.2329 (0.00042)	1.3112 (0.00031)	1.5313 (0.00077)	1.7290 (0.00028)	2.2203 (0.00080)
20	0.3750 (0.00089)	0.5140 (0.00023)	0.5869 (0.00028)	0.7105 (0.00043)	0.7545 (0.00083)	0.9603 (0.00025)	1.2024 (0.00097)	1.2673 (0.00010)	1.4430 (0.00083)	1.6028 (0.00175)	1.9755 (0.00021)
25	0.4302 (0.00016)	0.5645 (0.00080)	0.6444 (0.00175)	0.7476 (0.00076)	0.7884 (0.00010)	0.9704 (0.00019)	1.1943 (0.00105)	1.2457 (0.00055)	1.3960 (0.00102)	1.5311 (0.00192)	1.8384 (0.00012)
30	0.4867 (0.00105)	0.6068 (0.00163)	0.6779 (0.00023)	0.7724 (0.00009)	0.8116 (0.00042)	0.9738 (0.00030)	1.1651 (0.00045)	1.2090 (0.00002)	1.3443 (0.00026)	1.4683 (0.00085)	1.7180 (0.00039)
50	0.5908 (0.00011)	0.6937 (0.00041)	0.7527 (0.00077)	0.8317 (0.00010)	0.8602 (0.00055)	0.9835 (0.00008)	1.1258 (0.00021)	1.1609 (0.00027)	1.2673 (0.00065)	1.3601 (0.00016)	1.5312 (0.00009)
100	0.7037 (0.00021)	0.7812 (0.00015)	0.8265 (0.00008)	0.8805 (0.00036)	0.9019 (0.00013)	0.9930 (0.00177)	1.0859 (0.00097)	1.1125 (0.00035)	1.1763 (0.00042)	1.2368 (0.00012)	1.3522 (0.00011)

Table 8 Quantile points of the distribution of D (when $|\rho_{yx}| = 0.60$)

n	$D_{0.01}$	$D_{0.05}$	$D_{0.10}$	$D_{0.20}$	$D_{0.25}$	$D_{0.50}$	$D_{0.75}$	$D_{0.80}$	$D_{0.90}$	$D_{0.95}$	$D_{0.99}$
5	-1.7965 (0.00088)	-0.0500 (0.00025)	0.1299 (0.00068)	0.3075 (0.00026)	0.3878 (0.00054)	0.7796 (0.00073)	1.3585 (0.00092)	1.5290 (0.00005)	2.0852 (0.00042)	2.7292 (0.00019)	5.2449 (0.00008)
6	-0.7780 (0.00012)	0.0907 (0.00070)	0.2226 (0.00019)	0.3949 (0.00090)	0.4714 (0.00177)	0.8550 (0.00010)	1.3538 (0.00022)	1.5061 (0.00031)	1.9801 (0.00028)	2.5164 (0.00023)	4.2508 (0.00077)
7	-0.4123 (0.00042)	0.1642 (0.00005)	0.2885 (0.00029)	0.4588 (0.00095)	0.5288 (0.00028)	0.8754 (0.00043)	1.3333 (0.00026)	1.4613 (0.00162)	1.8698 (0.00076)	2.3045 (0.00120)	3.4546 (0.00013)
8	-0.1408 (0.00020)	0.2224 (0.00320)	0.3478 (0.00076)	0.5056 (0.00031)	0.5732 (0.00061)	0.8861 (0.00105)	1.3097 (0.00163)	1.4260 (0.00068)	1.7710 (0.00054)	2.1107 (0.00028)	3.1084 (0.00009)

Table 8 continued

n	$D_{0.01}$	$D_{0.05}$	$D_{0.10}$	$D_{0.20}$	$D_{0.25}$	$D_{0.50}$	$D_{0.75}$	$D_{0.80}$	$D_{0.90}$	$D_{0.95}$	$D_{0.99}$
9	0.0743 (0.00027)	0.2919 (0.00008)	0.3943 (0.00020)	0.5455 (0.00010)	0.6080 (0.00055)	0.9030 (0.00090)	1.2903 (0.00041)	1.3896 (0.00009)	1.7045 (0.00033)	2.0241 (0.00012)	2.8581 (0.00043)
10	0.1262 (0.00028)	0.3114 (0.00049)	0.4209 (0.00027)	0.5719 (0.00105)	0.6370 (0.00042)	0.9152 (0.00077)	1.2707 (0.00066)	1.3753 (0.00025)	1.6658 (0.00023)	1.9678 (0.00005)	2.7056 (0.00019)
11	0.1570 (0.00011)	0.3550 (0.00028)	0.4595 (0.00055)	0.5998 (0.00095)	0.6598 (0.00013)	0.9315 (0.00045)	1.2664 (0.00010)	1.3524 (0.00006)	1.6117 (0.00056)	1.8539 (0.00105)	2.5478 (0.00022)
12	0.2135 (0.00083)	0.3928 (0.00092)	0.4924 (0.00025)	0.6265 (0.00030)	0.6865 (0.00175)	0.9398 (0.00066)	1.2555 (0.00006)	1.3494 (0.00080)	1.5927 (0.00022)	1.8316 (0.00131)	2.4058 (0.00013)
13	0.2527 (0.00006)	0.4179 (0.00045)	0.5135 (0.00163)	0.6454 (0.00023)	0.6983 (0.00008)	0.9519 (0.00041)	1.2419 (0.00021)	1.3346 (0.00080)	1.5685 (0.00038)	1.7935 (0.00006)	2.3431 (0.00028)
14	0.2869 (0.00016)	0.4351 (0.00026)	0.5332 (0.00090)	0.6628 (0.00019)	0.7175 (0.00025)	0.9455 (0.00070)	1.2320 (0.00016)	1.3201 (0.00320)	1.5291 (0.00049)	1.7371 (0.00019)	2.2039 (0.00021)
15	0.2995 (0.00055)	0.4583 (0.00105)	0.5505 (0.00097)	0.6763 (0.00033)	0.7241 (0.00095)	0.9479 (0.00042)	1.2228 (0.00066)	1.3000 (0.00031)	1.5128 (0.00010)	1.7045 (0.00085)	2.1403 (0.00017)
20	0.3812 (0.00183)	0.5286 (0.00065)	0.6059 (0.00043)	0.7216 (0.00010)	0.7682 (0.00077)	0.9660 (0.00013)	1.1986 (0.00029)	1.2519 (0.00083)	1.4207 (0.00073)	1.5745 (0.00025)	1.90842 (0.00060)
25	0.4571 (0.00005)	0.5808 (0.00016)	0.6609 (0.00054)	0.7624 (0.00083)	0.8038 (0.00011)	0.9788 (0.00020)	1.1806 (0.00055)	1.2315 (0.00030)	1.3885 (0.00026)	1.5266 (0.00105)	1.8298 (0.00011)
30	0.4963 (0.00015)	0.6184 (0.00013)	0.6912 (0.00080)	0.7841 (0.00068)	0.8216 (0.00045)	0.9787 (0.00026)	1.1577 (0.00097)	1.2070 (0.00013)	1.3404 (0.00016)	1.4556 (0.00054)	1.6954 (0.00016)
50	0.6026 (0.00022)	0.6986 (0.00019)	0.7576 (0.00026)	0.8320 (0.00095)	0.8626 (0.00041)	0.9849 (0.00008)	1.1192 (0.00097)	1.1556 (0.00031)	1.2522 (0.00042)	1.3315 (0.00011)	1.5190 (0.00014)
100	0.7199 (0.00006)	0.7952 (0.00030)	0.8351 (0.00023)	0.8868 (0.00013)	0.9072 (0.00054)	0.9936 (0.00020)	1.0868 (0.00070)	1.1105 (0.00049)	1.1738 (0.00077)	1.2290 (0.00090)	1.3306 (0.00035)

Table 9 Quantile points of the distribution of D (when $|\rho_{yx}| = 0.70$)

n	$D_{0.01}$	$D_{0.05}$	$D_{0.10}$	$D_{0.20}$	$D_{0.25}$	$D_{0.50}$	$D_{0.75}$	$D_{0.80}$	$D_{0.90}$	$D_{0.95}$	$D_{0.99}$
5	-1.4808 (0.00088)	-0.0171 (0.00026)	0.1497 (0.00076)	0.3372 (0.00005)	0.4209 (0.00037)	0.8270 (0.00027)	1.3653 (0.00010)	1.5257 (0.00042)	2.0618 (0.00019)	2.6460 (0.00076)	5.1231 (0.00009)
6	-0.6964 (0.00034)	0.1040 (0.00061)	0.2368 (0.00031)	0.4229 (0.00066)	0.5022 (0.00008)	0.8683 (0.00023)	1.3347 (0.00092)	1.4661 (0.00068)	1.8987 (0.00054)	2.3601 (0.00026)	3.9949 (0.00014)
7	-0.2940 (0.00012)	0.1860 (0.00076)	0.3170 (0.00042)	0.4860 (0.00019)	0.5560 (0.00025)	0.8890 (0.00070)	1.3028 (0.00005)	1.4270 (0.00320)	1.7889 (0.00033)	2.1649 (0.00055)	3.3729 (0.00049)
8	-0.0168 (0.00079)	0.2579 (0.00054)	0.3706 (0.00083)	0.5328 (0.00010)	0.5993 (0.00043)	0.9035 (0.00030)	1.2929 (0.00090)	1.3870 (0.00039)	1.7236 (0.00019)	2.0894 (0.00006)	2.9749 (0.00054)
9	0.0849 (0.00005)	0.3043 (0.00011)	0.4215 (0.00055)	0.5736 (0.00090)	0.6396 (0.00055)	0.9223 (0.00028)	1.2822 (0.00163)	1.3794 (0.00070)	1.6604 (0.00016)	1.9503 (0.00029)	2.7168 (0.00008)
10	0.1395 (0.00026)	0.3423 (0.00102)	0.4507 (0.00092)	0.6016 (0.00020)	0.6665 (0.00016)	0.9332 (0.00005)	1.2711 (0.00066)	1.3609 (0.00097)	1.6292 (0.00036)	1.8959 (0.00055)	2.5926 (0.00024)
11	0.1639 (0.00105)	0.3702 (0.00025)	0.4762 (0.00077)	0.6141 (0.00010)	0.6732 (0.00020)	0.9308 (0.00175)	1.2633 (0.00002)	1.3566 (0.00042)	1.5683 (0.00095)	1.8076 (0.00123)	2.4014 (0.00023)
12	0.2318 (0.00035)	0.4118 (0.00073)	0.5160 (0.00027)	0.6559 (0.00045)	0.7098 (0.00025)	0.9569 (0.00021)	1.2497 (0.00080)	1.3304 (0.00105)	1.5540 (0.00089)	1.7809 (0.00022)	2.3517 (0.00030)

Table 9 continued

n	$D_{0.01}$	$D_{0.05}$	$D_{0.10}$	$D_{0.20}$	$D_{0.25}$	$D_{0.50}$	$D_{0.75}$	$D_{0.80}$	$D_{0.90}$	$D_{0.95}$	$D_{0.99}$
13	0.2685 (0.00013)	0.4343 (0.00065)	0.5354 (0.00013)	0.6685 (0.00041)	0.7116 (0.00068)	0.9460 (0.00041)	1.2308 (0.00083)	1.3062 (0.00056)	1.5238 (0.00010)	1.7321 (0.00034)	2.2339 (0.00009)
14	0.2914 (0.00021)	0.4516 (0.00070)	0.5486 (0.00320)	0.6795 (0.00026)	0.7226 (0.00019)	0.9561 (0.00076)	1.2283 (0.00177)	1.2993 (0.00031)	1.4982 (0.00077)	1.7014 (0.00028)	2.1682 (0.00062)
15	0.3036 (0.00017)	0.4781 (0.00006)	0.5697 (0.00008)	0.6934 (0.00090)	0.7394 (0.00027)	0.9620 (0.00054)	1.2124 (0.00097)	1.2868 (0.00043)	1.4830 (0.00026)	1.6522 (0.00029)	2.1114 (0.00005)
20	0.4179 (0.00013)	0.5576 (0.00036)	0.6452 (0.00055)	0.7532 (0.00026)	0.7948 (0.00061)	0.9732 (0.00028)	1.1850 (0.00031)	1.2432 (0.00042)	1.4081 (0.00183)	1.5505 (0.00005)	1.8939 (0.00034)
25	0.4707 (0.00010)	0.5943 (0.00029)	0.6759 (0.00042)	0.7749 (0.00163)	0.8141 (0.00020)	0.9791 (0.00092)	1.1590 (0.00175)	1.2128 (0.00066)	1.3517 (0.00022)	1.4798 (0.00011)	1.7331 (0.00032)
30	0.5123 (0.00019)	0.6299 (0.00033)	0.7024 (0.00105)	0.7930 (0.00049)	0.8299 (0.00066)	0.9841 (0.00023)	1.1497 (0.00042)	1.1931 (0.00015)	1.3136 (0.00068)	1.4177 (0.00105)	1.6545 (0.00019)
50	0.6221 (0.00023)	0.7190 (0.00083)	0.7753 (0.00054)	0.8417 (0.00077)	0.8699 (0.00122)	0.9877 (0.00080)	1.1148 (0.00055)	1.1482 (0.00081)	1.2409 (0.00085)	1.3142 (0.00030)	1.4747 (0.00022)
100	0.7255 (0.00009)	0.7997 (0.00080)	0.8416 (0.00061)	0.8920 (0.00063)	0.9124 (0.00020)	0.9947 (0.00105)	1.0818 (0.00016)	1.1047 (0.00320)	1.1620 (0.00026)	1.2129 (0.00077)	1.3117 (0.00009)

Table 10 Quantile points of the distribution of D (when $|\rho_{yx}| = 0.80$)

n	$D_{0.01}$	$D_{0.05}$	$D_{0.10}$	$D_{0.20}$	$D_{0.25}$	$D_{0.50}$	$D_{0.75}$	$D_{0.80}$	$D_{0.90}$	$D_{0.95}$	$D_{0.99}$
5	-0.8378 (0.00088)	0.0504 (0.00014)	0.2111 (0.00045)	0.4012 (0.00071)	0.4859 (0.00005)	0.8601 (0.00019)	1.3307 (0.00054)	1.4767 (0.00105)	1.9363 (0.00008)	2.4436 (0.00042)	4.2900 (0.00023)
6	-0.3362 (0.00062)	0.1399 (0.00043)	0.2893 (0.00119)	0.4758 (0.00055)	0.5576 (0.00023)	0.8993 (0.00092)	1.3110 (0.00049)	1.4293 (0.00076)	1.8097 (0.00010)	2.2034 (0.00027)	3.5384 (0.00083)
7	-0.1024 (0.00022)	0.2307 (0.00008)	0.3638 (0.00076)	0.5395 (0.00016)	0.6095 (0.00042)	0.9185 (0.00011)	1.2790 (0.00077)	1.3905 (0.00068)	1.7096 (0.00033)	2.0380 (0.00020)	2.9297 (0.00016)
8	0.0301 (0.00039)	0.2942 (0.00020)	0.4264 (0.00029)	0.5863 (0.00175)	0.6499 (0.00025)	0.9267 (0.00009)	1.2644 (0.00070)	1.3480 (0.00102)	1.6304 (0.00055)	1.8967 (0.00028)	2.7458 (0.00021)
9	0.1327 (0.00083)	0.3529 (0.00031)	0.4708 (0.00005)	0.6214 (0.00026)	0.6807 (0.00045)	0.9421 (0.00061)	1.2535 (0.00005)	1.3393 (0.00055)	1.5881 (0.00163)	1.8351 (0.00055)	2.4951 (0.00076)
10	0.1721 (0.00023)	0.3878 (0.00027)	0.5027 (0.00068)	0.6432 (0.00041)	0.6965 (0.00105)	0.9537 (0.00041)	1.2423 (0.00090)	1.3177 (0.00163)	1.5468 (0.00019)	1.7500 (0.00183)	2.3201 (0.00006)
11	0.2244 (0.00054)	0.4262 (0.00036)	0.5314 (0.00019)	0.6698 (0.00076)	0.7230 (0.00028)	0.9616 (0.00023)	1.2316 (0.00010)	1.3018 (0.00021)	1.5131 (0.00042)	1.7269 (0.00011)	2.2175 (0.00026)
12	0.2506 (0.00008)	0.4529 (0.00022)	0.5546 (0.00030)	0.6824 (0.00070)	0.7349 (0.00054)	0.9505 (0.00043)	1.2151 (0.00320)	1.2806 (0.00090)	1.4789 (0.00077)	1.6597 (0.00025)	2.1017 (0.00012)
13	0.3112 (0.00026)	0.4902 (0.00002)	0.5830 (0.00080)	0.7054 (0.00066)	0.7508 (0.00049)	0.9599 (0.00019)	1.2018 (0.00025)	1.2729 (0.00031)	1.4563 (0.00073)	1.6361 (0.00005)	2.0191 (0.00028)
14	0.3333 (0.00010)	0.4983 (0.00061)	0.5975 (0.00055)	0.7118 (0.00092)	0.7588 (0.00042)	0.9649 (0.00033)	1.1946 (0.00008)	1.2582 (0.00029)	1.4396 (0.00163)	1.6121 (0.00013)	1.9967 (0.00022)
15	0.3577 (0.00035)	0.5170 (0.00016)	0.6073 (0.00095)	0.7292 (0.00054)	0.7738 (0.00077)	0.9701 (0.00068)	1.1850 (0.00020)	1.2414 (0.00026)	1.4194 (0.00025)	1.5812 (0.00013)	1.9374 (0.00022)
20	0.4455 (0.00075)	0.5920 (0.00019)	0.6675 (0.00010)	0.7668 (0.00028)	0.8067 (0.00083)	0.9790 (0.00045)	1.1687 (0.00097)	1.2179 (0.00006)	1.3502 (0.00054)	1.4796 (0.00019)	1.7677 (0.00034)

Table 10 continued

n	$D_{0.01}$	$D_{0.05}$	$D_{0.10}$	$D_{0.20}$	$D_{0.25}$	$D_{0.50}$	$D_{0.75}$	$D_{0.80}$	$D_{0.90}$	$D_{0.95}$	$D_{0.99}$
25	0.5204 (0.00049)	0.6352 (0.00090)	0.7078 (0.00076)	0.8005 (0.00105)	0.8354 (0.00061)	0.9879 (0.00029)	1.1485 (0.00066)	1.1904 (0.00026)	1.3113 (0.00170)	1.4139 (0.00056)	1.6371 (0.00005)
30	0.5381 (0.00005)	0.6639 (0.00022)	0.7330 (0.00070)	0.8171 (0.00175)	0.8493 (0.00090)	0.9875 (0.00055)	1.1389 (0.00030)	1.1794 (0.00080)	1.2820 (0.00011)	1.3789 (0.00009)	1.5697 (0.00042)
50	0.6506 (0.00023)	0.7485 (0.00016)	0.8002 (0.00177)	0.8631 (0.00320)	0.8877 (0.00097)	0.9916 (0.00023)	1.1020 (0.00016)	1.1300 (0.00041)	1.2098 (0.00049)	1.2790 (0.00105)	1.4222 (0.00010)
100	0.7617 (0.00011)	0.8263 (0.00028)	0.8594 (0.00077)	0.9050 (0.00019)	0.9234 (0.00026)	0.9949 (0.00056)	1.0731 (0.00083)	1.0922 (0.00022)	1.1437 (0.00035)	1.1874 (0.00036)	1.2779 (0.00027)

Table 11 Quantile points of the distribution of D (when $|\rho_{yx}| = 0.90$)

n	$D_{0.01}$	$D_{0.05}$	$D_{0.10}$	$D_{0.20}$	$D_{0.25}$	$D_{0.50}$	$D_{0.75}$	$D_{0.80}$	$D_{0.90}$	$D_{0.95}$	$D_{0.99}$
5	-0.3556 (0.00088)	0.1710 (0.00028)	0.3500 (0.00054)	0.5504 (0.00011)	0.6216 (0.00026)	0.9278 (0.00019)	1.2784 (0.00076)	1.3850 (0.00045)	1.6871 (0.00068)	2.0380 (0.00029)	3.2179 (0.00076)
6	-0.1167 (0.00014)	0.2676 (0.00042)	0.4266 (0.00102)	0.6072 (0.00010)	0.6674 (0.00028)	0.9353 (0.00055)	1.2487 (0.00042)	1.3404 (0.00008)	1.6020 (0.00210)	1.8757 (0.00077)	2.7374 (0.00088)
7	0.0549 (0.00083)	0.3637 (0.00076)	0.5038 (0.00019)	0.6519 (0.00061)	0.7109 (0.00025)	0.9616 (0.00005)	1.2389 (0.00068)	1.3112 (0.00054)	1.5438 (0.00070)	1.7947 (0.00092)	2.4865 (0.00039)
8	0.1453 (0.00085)	0.4174 (0.00025)	0.5349 (0.00021)	0.6816 (0.00163)	0.7339 (0.00088)	0.9525 (0.00043)	1.2157 (0.00031)	1.2878 (0.00026)	1.4898 (0.00028)	1.6848 (0.00077)	2.2555 (0.00010)
9	0.2595 (0.00023)	0.4785 (0.00036)	0.5854 (0.00049)	0.7172 (0.00095)	0.7647 (0.00023)	0.9688 (0.00005)	1.2078 (0.00031)	1.2793 (0.00019)	1.4475 (0.00011)	1.6224 (0.00025)	2.1009 (0.00009)
10	0.2984 (0.00005)	0.5019 (0.00028)	0.6028 (0.00006)	0.7224 (0.00175)	0.7704 (0.00092)	0.9720 (0.00066)	1.1973 (0.00088)	1.2604 (0.00016)	1.4302 (0.00049)	1.5981 (0.00013)	2.0023 (0.00083)
11	0.3395 (0.00080)	0.5362 (0.00055)	0.6287 (0.00020)	0.7450 (0.00045)	0.7820 (0.00042)	0.9809 (0.00105)	1.1818 (0.00026)	1.2368 (0.00055)	1.3931 (0.00022)	1.5438 (0.00088)	1.9329 (0.00006)
12	0.3869 (0.00013)	0.5621 (0.00146)	0.6464 (0.00088)	0.7545 (0.00026)	0.7962 (0.00030)	0.9745 (0.00095)	1.1768 (0.00033)	1.2273 (0.00065)	1.3680 (0.00027)	1.5021 (0.00019)	1.8159 (0.00011)
13	0.4133 (0.00077)	0.5859 (0.00089)	0.6662 (0.00011)	0.7748 (0.00045)	0.8171 (0.00028)	0.9822 (0.00055)	1.1608 (0.00090)	1.2167 (0.00083)	1.3482 (0.00076)	1.4703 (0.00023)	1.7719 (0.00105)
14	0.4514 (0.00016)	0.6090 (0.00031)	0.6897 (0.00070)	0.7857 (0.00054)	0.8234 (0.00041)	0.9823 (0.00090)	1.1591 (0.00002)	1.2045 (0.00068)	1.3350 (0.00320)	1.4549 (0.00029)	1.7362 (0.00002)
15	0.4647 (0.00027)	0.6178 (0.00088)	0.6982 (0.00083)	0.7949 (0.00030)	0.8311 (0.00073)	0.9862 (0.00088)	1.1530 (0.00077)	1.1986 (0.00021)	1.3223 (0.00020)	1.4343 (0.00028)	1.6954 (0.00035)
20	0.5390 (0.00019)	0.6794 (0.00183)	0.7455 (0.00076)	0.8217 (0.00066)	0.8525 (0.00010)	0.9846 (0.00043)	1.1303 (0.00061)	1.1677 (0.00070)	1.2668 (0.00088)	1.3661 (0.00041)	1.5582 (0.00013)
25	0.6164 (0.00010)	0.7227 (0.00008)	0.7799 (0.00068)	0.8517 (0.00029)	0.8785 (0.00023)	0.9948 (0.00080)	1.1183 (0.00105)	1.1505 (0.00031)	1.2418 (0.00021)	1.3237 (0.00056)	1.4835 (0.00005)
30	0.6379 (0.00011)	0.7440 (0.00054)	0.7933 (0.00028)	0.8625 (0.00042)	0.8877 (0.00016)	0.9925 (0.00045)	1.1056 (0.00026)	1.1357 (0.00055)	1.2119 (0.00010)	1.2840 (0.00015)	1.4187 (0.00013)
50	0.7295 (0.00105)	0.8031 (0.00055)	0.8466 (0.00022)	0.8962 (0.00042)	0.9142 (0.00088)	0.9954 (0.00163)	1.0794 (0.00020)	1.1008 (0.00080)	1.1589 (0.00097)	1.2073 (0.00006)	1.3027 (0.00012)
100	0.8114 (0.00029)	0.8622 (0.00013)	0.8925 (0.00097)	0.9275 (0.00033)	0.9416 (0.00177)	0.9976 (0.00041)	1.0566 (0.00083)	1.0709 (0.00097)	1.1120 (0.00105)	1.1470 (0.00063)	1.2043 (0.00022)

Table 12 Quantile points of the distribution of D (when $|\rho_{y,x}| = 0.99$)

n	$D_{0.01}$	$D_{0.05}$	$D_{0.10}$	$D_{0.20}$	$D_{0.25}$	$D_{0.50}$	$D_{0.75}$	$D_{0.80}$	$D_{0.90}$	$D_{0.95}$	$D_{0.99}$
5	0.4416 (0.00088)	0.6816 (0.00054)	0.7722 (0.00019)	0.8535 (0.00077)	0.8826 (0.00023)	0.9952 (0.00067)	1.1104 (0.00028)	1.1416 (0.00025)	1.2332 (0.00054)	1.3429 (0.00029)	1.6503 (0.00026)
6	0.5318 (0.00073)	0.7255 (0.00005)	0.7976 (0.00011)	0.8608 (0.00008)	0.8956 (0.00092)	0.9906 (0.00175)	1.1068 (0.00010)	1.1228 (0.00102)	1.2036 (0.00042)	1.2832 (0.00105)	1.5025 (0.00034)
7	0.6185 (0.00028)	0.7622 (0.00090)	0.8253 (0.00070)	0.8749 (0.00054)	0.9074 (0.00027)	0.9961 (0.00025)	1.0960 (0.00068)	1.1188 (0.00045)	1.1792 (0.00011)	1.2530 (0.00005)	1.4259 (0.00039)
8	0.6645 (0.00053)	0.7895 (0.00026)	0.8400 (0.00083)	0.8835 (0.00068)	0.9135 (0.00009)	0.9953 (0.00055)	1.0804 (0.00006)	1.1092 (0.00031)	1.1641 (0.00033)	1.2287 (0.00080)	1.3902 (0.00028)
9	0.6913 (0.00023)	0.8108 (0.00105)	0.8554 (0.00061)	0.8905 (0.00011)	0.9201 (0.00020)	0.9956 (0.00066)	1.0728 (0.00029)	1.0928 (0.00095)	1.1491 (0.00036)	1.2017 (0.00085)	1.3202 (0.00027)
10	0.7185 (0.00010)	0.8200 (0.00022)	0.8650 (0.00042)	0.9032 (0.00031)	0.9296 (0.00043)	0.9942 (0.00026)	1.0676 (0.00049)	1.0873 (0.00189)	1.1373 (0.00095)	1.1854 (0.00083)	1.2959 (0.00048)
11	0.7474 (0.00025)	0.8344 (0.00006)	0.8752 (0.00080)	0.9163 (0.00076)	0.9319 (0.00175)	0.9963 (0.00028)	1.0643 (0.00076)	1.0818 (0.00019)	1.1306 (0.00133)	1.1766 (0.00035)	1.2722 (0.00013)
12	0.7548 (0.00005)	0.8420 (0.00030)	0.8809 (0.00016)	0.9223 (0.00066)	0.9365 (0.00105)	0.9972 (0.00016)	1.0610 (0.00023)	1.0772 (0.00022)	1.1213 (0.00068)	1.1608 (0.00076)	1.2512 (0.00007)
13	0.7692 (0.00021)	0.8502 (0.00036)	0.8847 (0.00095)	0.9244 (0.00025)	0.9392 (0.00019)	0.9991 (0.00055)	1.0596 (0.00041)	1.0752 (0.00011)	1.1191 (0.00020)	1.1570 (0.00105)	1.2450 (0.00012)
14	0.7764 (0.00019)	0.8540 (0.00111)	0.8907 (0.00083)	0.9289 (0.00045)	0.9429 (0.00020)	0.9977 (0.00042)	1.0550 (0.00054)	1.0694 (0.00070)	1.1119 (0.00092)	1.1502 (0.00027)	1.2274 (0.00015)
15	0.8029 (0.00022)	0.8653 (0.00039)	0.8946 (0.00105)	0.9310 (0.00056)	0.9449 (0.00077)	0.9998 (0.00177)	1.0534 (0.00090)	1.0678 (0.00008)	1.1070 (0.00089)	1.1410 (0.00027)	1.2206 (0.00036)
20	0.8304 (0.00013)	0.8839 (0.00033)	0.9083 (0.00019)	0.9383 (0.00020)	0.9505 (0.00070)	0.9983 (0.00013)	1.0440 (0.00097)	1.0553 (0.00163)	1.0882 (0.00042)	1.1150 (0.00030)	1.1742 (0.00009)
25	0.8552 (0.00029)	0.9014 (0.00013)	0.9232 (0.00065)	0.9486 (0.00055)	0.9578 (0.00010)	0.9981 (0.00019)	1.0404 (0.00083)	1.0511 (0.00031)	1.0790 (0.00006)	1.1043 (0.00077)	1.1539 (0.00010)
30	0.8724 (0.00066)	0.9118 (0.00049)	0.9314 (0.00011)	0.9537 (0.00045)	0.9629 (0.00026)	0.9984 (0.00025)	1.0359 (0.00061)	1.0453 (0.00019)	1.0703 (0.00026)	1.0935 (0.00016)	1.1375 (0.00105)
50	0.9028 (0.00026)	0.9306 (0.00002)	0.9460 (0.00041)	0.9641 (0.00132)	0.9716 (0.00015)	0.9993 (0.00029)	1.0273 (0.00080)	1.0341 (0.00005)	1.0537 (0.00073)	1.0688 (0.00011)	1.1009 (0.00083)
100	0.9334 (0.00028)	0.9526 (0.00054)	0.9629 (0.00175)	0.9753 (0.00090)	0.9801 (0.00077)	0.9995 (0.00175)	1.0191 (0.00055)	1.0241 (0.00033)	1.0363 (0.00049)	1.0473 (0.00320)	1.0689 (0.00023)

Table 13 Control chart coefficient u_1 of S^2 chart

n	u_1
5	0.7071
6	0.6325
7	0.5774
8	0.5345
9	0.5000
10	0.4714
11	0.4472

Table 13 continued

n	u_1
12	0.4264
13	0.4082
14	0.3922
15	0.3780
20	0.3244
25	0.2887
30	0.2626
50	0.2020
100	0.1421

Table 14 Lowest $|\rho_{yx}|$ value l_n for superiority of V_r chart

n	l_n
5	0.9368 (0.00037)
6	0.8891 (0.00013)
7	0.8577 (0.00048)
8	0.8112 (0.00051)
9	0.7924 (0.00023)
10	0.7662 (0.00010)
11	0.7433 (0.00075)
12	0.7278 (0.00008)
13	0.7065 (0.00021)
14	0.6929 (0.00019)
15	0.6807 (0.00062)
20	0.6011 (0.00013)
25	0.5809 (0.00029)
30	0.5148 (0.00066)
50	0.4387 (0.00092)
100	0.2541 (0.00017)

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