Live fast and die young
Evolution and fate of massive stars
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CIRCUMSTELLAR MATERIAL
FROM PULSATIONAL PAIR INSTABILITY


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Abstract

Present and upcoming large time-domain surveys and gravitational-wave followup efforts will unravel the variety of rare explosive transients in the sky. We focus here on pulsational pair-instability evolution, which can produce observable signatures both in gravitational and electromagnetic waves. Using the MESA stellar evolution code, we simulate grids of bare helium stars to characterize jointly the resulting black hole (BH) masses and ejecta composition, velocity, and thermal state. We find that the stars do not react “elastically” to the thermonuclear ignition in the core: there is not a one-to-one correspondence between pair-instability driven thermonuclear explosions and mass ejections, which causes ambiguity in what is an observable pair-instability pulse. At the lower mass end ($37.5 M_\odot \lesssim M_{\text{He,init}} \lesssim 41 M_\odot$) the explosions are not strong enough to affect the surface. With increasing helium core mass, they become progressively stronger causing first large radial expansion ($41 M_\odot \lesssim M_{\text{He,init}} \lesssim 42 M_\odot$), and finally also mass ejection episodes ($M_{\text{He,init}} \gtrsim 42.5 M_\odot$). The lowest mass helium core to be fully disrupted in a pair-instability supernova is $M_{\text{He,init}} \approx 80 M_\odot$, corresponding to a pre-explosion carbon oxygen core mass of $M_{\text{CO}} \approx 57 M_\odot$. Models with $M_{\text{He,init}} \gtrsim 200 M_\odot$ reach the photodisintegration regime, resulting in BHs with masses $M_{\text{BH}} \gtrsim 125 M_\odot$. If the pulsating models produce BHs via (weak) explosions, the previously-ejected material might be hit by the blast wave. This might convert kinetic energy into observable electromagnetic radiation because of circumstellar interactions. We characterize the hydrogen-free circumstellar material from the pulsational pair-instability of helium cores assuming simplistically that the ejecta maintain a constant velocity. We provide detailed output that can be used to improve on this. We find that our models produce helium-rich ejecta with mass $10^{-3} M_\odot \lesssim M_{\text{CSM}} \lesssim 40 M_\odot$, the larger values corresponding to the more massive progenitor stars. These ejecta are typically launched at a few thousands km s$^{-1}$, and reach distances $\sim 10^{12} - 10^{15}$ cm before the core-collapse of the star. The delays between mass ejection events and the final collapse span a wide and mass-dependent range (from sub-hour to $10^4$ years), and the shells ejected can also collide with each other, powering supernova impostor events. The range of properties we find suggests a possible connection with (some) type Ibn supernovae.
7.1 Introduction

Very massive stars are mainly supported by radiation pressure. Those that end their main sequence with a helium (He) core exceeding $M_{\text{He,init}} \gtrsim 80 \, M_\odot$ (which decreases to $M_{\text{He}} \approx 60 \, M_\odot$ accounting for wind mass loss) are predicted to end their evolution as pair-instability supernovae (PISN, Fowler & Hoyle 1964; Rakavy & Shaviv 1967). They evolve in hydrostatic equilibrium until they develop a carbon-oxygen (CO) core. Soon after, the conversion of photons into electron-positron ($e^\pm$) pairs softens the equation of state initiating the collapse of the star. This increases the inner temperature until explosive thermonuclear oxygen burning reverts the collapse and fully disrupts the star (e.g., Barkat et al. 1967; Fraley 1968; Kasen et al. 2011; Yoshida et al. 2016; Woosley 2017, 2019). Such stars do not leave any compact remnant at the end of their evolution. For initial $M_{\text{He,init}} \gtrsim 200 \, M_\odot$, stars also experience explosive thermonuclear oxygen burning but, owing to energy loss due to photo-disintegration of heavy nuclei, the explosion is not energetic enough to reverse the collapse into an explosion and disrupt the star, (e.g., Bond et al. 1984; Fryer et al. 2001; Heger et al. 2003). In these cases, the final fate is core collapse (CC), forming a massive black hole (BH). Therefore, if these stellar explosions do occur in nature, a “PISN black hole mass gap” (also called 2nd mass gap\(^1\)) is expected between the most massive BH that can be formed without encountering PISN fate and the least massive BH formed because of the photodisintegration instability.

The most massive BHs below the gap result from the evolution of He cores with initial masses just below $\sim 80 \, M_\odot$ (e.g., Yoon et al. 2012; Woosley 2017; Farmer et al. submitted). In these stars, the explosive burning releases less energy and thus is only able to eject a fraction of the outer layers of the star. This produces a mass-loss pulse, without fully disrupting the star (Rakavy & Shaviv 1967; Fraley 1968; Woosley et al. 2007; Woosley 2017, 2019). This phenomenon is the lower-mass analogue of a PISN, a pulsational pair-instability (PPI). The star may undergo multiple such pulses until the combined effects of pulsational mass loss, entropy loss to neutrinos, and fuel consumption stabilizes the core (Woosley 2017; Marchant et al. 2018; Leung et al. 2019). Ultimately, this star is likely to collapse to a BH, possibly with an associated supernova (SN).

Given the impact on the distribution of BH masses (Belczynski et al. 2016b; Woosley 2017; Marchant et al. 2018; Stevenson et al. 2019), the recent direct detection of gravitational waves (LVC 2017b, 2018b) has revived the interest in PPI evolution. Moreover, the followup of gravitational wave merger events is driving large observational efforts in time-domain astronomy, with new and upcoming facility such as the Zwicky Transient Factory (Bellm 2014), Large Synoptic Supernova Survey (LSST Science Collaboration et al. 2009), etc. When not following up gravitational wave events, these instruments will perform surveys of various depth and cadence which will soon unveil the variety of electromagnetic transients

\(^1\)The “first gap” is the apparent lack of compact objects with masses between the maximum neutron star mass, $\max[M_{\text{NS}}] \approx 2 \, M_\odot$ and the least massive BH known $\min[M_{\text{BH}}] \approx 5 \, M_\odot$, (e.g., Farr et al. 2011, but see also Wyrzykowski et al. 2016).
possible in the sky. The James Webb Space Telescope will be able to probe the transients expected at the death of the first stars in the Universe, increasing the chance of a direct unambiguous detection of PISN or PPI+CC (Whalen et al. 2013).

Therefore, it is important to characterize the observable characteristics of PPI evolution, i.e. address the question “what are the observable signatures of a pulse?”. Of particular interest is the question of how much mass do the pulses eject and at which velocity is it launched (Leung et al. 2019), or in other words, what are the circumstellar material (CSM) structures that this process can produce.

Previous studies from Chatzopoulos & Wheeler (2012a,b) investigated the fate of H-rich stars with zero age main sequence (ZAMS) masses above 40 $M_\odot$, with and without rotation, and found PPI evolution in the mass range 40−65 $M_\odot$ (for high rotation rates) and 80−110 $M_\odot$ (without rotation). These ranges are also sensitive to the details of the nuclear physics (e.g. Takahashi 2018; Farmer et al. submitted).

Woosley (2017, 2019) presented the first grids of stellar evolution calculations for a wide mass range enclosing both PPI followed by a core collapse (PPI+CC) and PISN. The light curves of the former are expected to show a series of brightening events as the individual pulses collide with each other (Woosley 2017), which has been proposed to explain the extremely luminous light curve of SN2006gy (Woosley et al. 2007). More recently, Arcavi et al. (2017b); Woosley (2018) also proposed PPI as a way to explain the peculiar photometric and spectroscopic evolution of SN iPTF14hls, possibly coming from a merger progenitor (Vigna-Gómez et al. 2019).

Although PISN need not be extremely luminous (Woosley 2017), they are routinely considered in the context of super-luminous supernovae (e.g. Gal-Yam et al. 2009; Chatzopoulos et al. 2013). No unambiguous identification of an astrophysical transient with a PISN is available as-yet, however Kozyreva et al. (2018) proposed OGLE14-073 as the best candidate. For models evolving through PPI before their final collapse, the best observational candidates are iPTF16eh, a type I super-luminous SN showing signs of hydrogen-rich circumstellar material through the detection of a light echo (Lunnan et al. 2018) and SN2016iet, for which a dense, H- and He-free CSM at 10$^{15}$ cm of the star can be invoked to explain the light curve (Gomez et al. 2019). Another potential candidate is PTF12dam, a fast rising type I super-luminous SNe modeled by Tolstov et al. (2017) as combination of CSM-interaction and radioactive decay.

Here, we calculate the detailed evolution of massive He cores to characterize jointly the effect that PPI evolution has on the final BH masses and on the circumstellar material (CSM) structure. Our stellar evolution models provide input for the hydrodynamical evolution of the CSM. We also provide (i) a criterion to determine which He cores encounter a global instability, resulting in PPI-driven mass loss and BH formation, and which He cores instead are fully disrupted in a PISN and (ii) the bulk properties of the pulses and their distribution as a function of mass.

In Sec. 7.2 we describe our calculations, before giving an overview of the evolutionary outcome of our models in Sec. 7.3 and of the resulting BH masses in Sec. 7.4. We focus
on the PPI models in Sec. 7.5, where we describe three physically motivated possible definitions of a “pulse”. While the basic ideas on how the evolution of these models proceeds are well established from the theoretical side, there is some ambiguity in the literature on what is called a pulse. We discuss the CSM that our models can produce with a toy-model assuming propagation of the ejecta at constant velocity in Sec. 7.6, and provide input files for a more sophisticated modeling of the CSM at http://10.5281/zenodo.3406357. In Sec. 7.7 we discuss whether the final core-collapse after the PPI evolution would produce an associated SN explosion, which would generate ejecta to interact with the previously ejected stellar layers. We compare our results to a few observational transients that have been interpreted as pulsational pair-instability events in Sec. 7.8. We define a criterion to distinguish pulsational evolution from full disruption without going through the hydrodynamic calculations in Sec. 7.9, before highlighting the main limitations of this study. Sec. 7.10 summarizes our main conclusions. Appendix E.1 presents a resolution study of one of our models, and Appendix E.2 compares the evolution of a naked He core to a hydrogen rich star with a similar He core mass.

### 7.2 Pulsational Pair-Instability evolution with MESA

We model the evolution of bare He cores because stars massive enough to encounter the PPI are likely to have lost their hydrogen-rich envelop beforehand. This could happen either because of the presence of a binary companion (e.g., Kobulnicky & Fryer 2007; Sana et al. 2012; Almeida et al. 2017), strong wind mass loss (e.g., Vink & de Koter 2005), or because of rotational mixing preventing the formation of a core-envelope structure (Maeder & Meynet 2000; Yoon et al. 2006; de Mink et al. 2009; Mandel & de Mink 2016; Marchant et al. 2016).

Another way to form very massive stars which are expected to produce the most massive (stellar mass) BHs is through runaway collisions in a dynamically excited environment (e.g., van den Heuvel & Portegies Zwart 2013), or binary mergers (e.g., de Mink et al. 2014; Vigna-Gómez et al. 2019). Either might result in the loss from the system of some hydrogen-rich material. Even if a binary merges before the onset of pulsations and retains a significant amount of hydrogen (e.g., Vigna-Gómez et al. 2019), the merger is expected to undergo asteroseismologic (non-PPI) pulsations enhancing wind mass loss removing of the remaining envelope (Moriya & Langer 2015). Finally, any remaining hydrogen-rich envelope is likely to be loosely bound and easily removed during the first PPI pulse (Fraley 1968; Leung et al. 2019, see also Appendix E.2).

We employ the open-source stellar evolution code MESA (release 11 701, Paxton et al. 2011, 2013, 2015, 2018, 2019) to evolve a grid of He stars in the mass range $35 M_\odot \lesssim M_{\text{He,init}} \lesssim 250 M_\odot$. Throughout this study, we define the He core mass as the total mass of our models, and the carbon-oxygen (CO) core boundary as the outermost location where the mass fraction of $^4\text{He}$ drops below 0.01. We adopt an initial metallicity of $Z = 0.001$, and we also ran a limited sample of models with $Z = 0.00198$. Both these values are below the upper limit for the occurrence of PISN of $Z_\odot/3 \approx 0.006$ obtained from single star models.
(Langer et al. 2007). Pair-instability evolution might even occur at higher metallicity because of late stellar mergers in a binary (Vigna-Gómez et al. 2019), or if magnetic fields funnel the mass lost to winds back to the star (Georgy et al. 2017). We do not study here the impact of binarity, rotation, and magnetic fields. The impact of rotation was investigated previously with 1D simulations by Chatzopoulos & Wheeler (2012a). We include wind mass loss as in Marchant et al. (2018) and in the fiducial model of Farmer et al. (submitted), that is we use the rate from Hamann et al. (1995); Hamann & Koesterke (1998) reduced by a factor of 10. For effective temperatures $T_{\text{eff}} < 23,300$ K, which can be achieved in between pulses due to the expansion of the star, we employ the maximum between the Vink et al. (2000, 2001) and Nieuwenhuijzen & de Jager (1990) wind mass loss rates. We turn off wind mass loss during the (physically brief) dynamical phases of evolution: PPI-driven dynamical mass ejections are the only source of mass loss in these phases. Uncertainties in the wind mass loss rate can have an impact on the core structure (Renzo et al. 2017), possibly influencing the CSM structure produced in PPI-driven mass loss events. However, Farmer et al. (submitted) showed that uncertainties in the wind mass loss do not significantly influence the range of possible BH remnant masses.

To follow the dynamical evolution of the pulses when they occur, we use MESA’s Riemann HLLC solver (Toro et al. 1994; Paxton et al. 2018; Marchant et al. 2018). We determine the dynamical stability of the star based on the adiabatic index $\Gamma_1 = \frac{\partial \log(P)}{\partial \log(\rho)}|_s$, (e.g., Kippenhahn et al. 2013). This is however a local quantity. To create a global metric descriptive of the entire star, we follow Stothers 1999 in defining a volumetric pressure-weighted average adiabatic index

$$\langle \Gamma_1 \rangle \overset{\text{def}}{=} \frac{\int \Gamma_1 P \, d^3r}{\int P \, d^3r} \equiv \frac{\int \Gamma_1 \frac{P}{\rho} \, dm}{\int \frac{P}{\rho} \, dm}, \quad (7.1)$$

where $P$, $\rho$ are the local pressure and density, and we used the continuity equation to transform the volumetric integral into and integral over the mass domain. Weighting the local $\Gamma_1$ with $P$ makes the average $\langle \Gamma_1 \rangle$ a dynamically relevant quantity, and guarantees that the inner regions contribute more to the average. Whenever $\langle \Gamma_1 \rangle = 4/3 + 0.01$, i.e. slightly before the stellar structure becomes formally unstable, we switch to a hydrodynamical treatment of the evolution and turn off the stellar winds (see also Marchant et al. 2018).

After a pulse, if the internal structure of the star meets the set of criteria specified in Marchant et al. (2018) to conservatively ensure hydrostatic equilibrium has been recovered, we excise the material moving faster than the local escape velocity and create a new star with the entropy, chemical composition, and mass of the layers remaining bound. Even for non-pulsating models, we turn on the hydrodynamics to follow the onset of core-collapse, when the core temperature rises above $T_c \gtrsim 10^{9.6}$ K.

We adopt a 22-isotope nuclear reaction network (approx21_plus_co56.net), which is sufficient to trace the energy output during the relevant burning phases but not the detailed nucleosynthesis (e.g., Farmer et al. 2016, submitted).

We assess convective stability using the Ledoux criterion, and adopt a mixing length
parameter of $\alpha_{\text{MLT}} = 2.0$. We consider semi-convective mixing with an efficiency $\alpha_s = 1.0$, whilst neglecting thermohaline mixing. We assume an exponential under/overshooting with parameters $^2 (f, f_0) = (0.01, 0.005)$ for all convective regions. We follow the approach of Marchant et al. (2018) based on Arnett (1969); Wood (1974) for the time-dependence of the convective velocity. This is required to compute dynamical phases of the evolution with timesteps shorter than the convective turnover timescale (Renzo et al. submitted).

We refer the reader to Appendix E.1 for the description of the numerical resolution and a quantitative assessment of its impact on our results.

We stop our evolution either at the onset of CC or at the onset of a PISN. We define the former as when the infall velocity anywhere in the model exceeds 1000 km s$^{-1}$ (Woosley et al. 2002). For the latter we check that the total energy (including the kinetic term) of the star is positive, that the minimum radial velocity is non-negative, and that the total energy is positive. These conditions guarantee that the star is unbound and there is an outflow of matter.

The input files (inlists) and customized routines added to the code (run_star_extras.f) needed to reproduce our results are available at http://10.5281/zenodo.3406357. We also provide our numerical results for the evolution and final structure of each of our models, including a customized output file storing averaged information for each layer moving beyond its local escape velocity during PPI-driven mass loss episodes$^3$. Such files can be used as inputs for hydrodynamic studies of the CSM structure produced by these stellar models.

### 7.3 Overview of progenitors evolution

Fig. 7.1 shows the BH masses resulting from our grid as a function of the initial He core mass ($M_{\text{He,init}}$, bottom axis) and approximate maximum CO core mass reached during the evolution ($M_{\text{CO}}$, top axis). Both can decrease because of PPI mass loss episodes towards the end of the evolution. We estimate the BH mass as the mass coordinate where the binding energy of the collapsing star reaches $10^{48}$ ergs, to allow for the possibility of mass loss during the CC from, either a weak explosion (Ott et al. 2018; Kuroda et al. 2018), or ejection of a fraction of the envelope (Nadezhin 1980; Lovegrove & Woosley 2013). This estimate is extremely close to the total final mass of the He star. We do not account for other energy loss terms during the core collapse, such as neutrinos which might carry away (part of) the core binding energy. This effect is typically estimated to be of the order of $\sim 10\%$ of the pre-collapse core rest mass energy (e.g., O’Connor & Ott 2011; Belczynski et al. 2016b; Spera & Mapelli 2017), and can shift our BH mass estimates further downwards.

The colored background in the left panel of Fig. 7.1 indicates approximately the evolutionary path for the corresponding mass range. The four possibilities are summarized as follows, in order of increasing initial He core mass:

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$^2$cf. Equation 2 in Paxton et al. 2011 and the MESA documentation for the definition of $f$ and $f_0$.

$^3$The possibility of fallback is neglected in these files, even though our MESA models allow for it.
CC: relatively low mass He cores end their lives in a core collapse (CC, blue on the left of Fig. 7.1) event without losing mass to pair-production driven pulses. For these models, the layers which are unstable to pair production (if any) are not massive enough to cause an episode of mass ejection. In this mass range, the outcome of core-collapse is most likely BH formation, possibly associated to a weak SN with large fallback (Ott et al. 2018; Kuroda et al. 2018). We return on the “explodability” of our grid of models in Sec. 7.7.

PPI+CC: with increasing $M_{\text{He,init}}$, the pair instability becomes progressively more violent. Thermonuclear explosions (of oxygen and silicon) deep inside cause significant radial expansion. Increasing further in mass, models experience one or more mass loss episodes, before the core is stabilized by the consumption of fuel and entropy losses to neutrinos, and the stars finally collapse (PPI+CC, green in Fig. 7.1).

PISN: for $80 \, M_{\odot} \lesssim M_{\text{He,init}} \lesssim 200 \, M_{\odot}$, our models are completely disrupted in a PISN, and produce no remnant (yellow vertical area in Fig. 7.1). Our lowest mass model going PISN and leaving no remnant has $M_{\text{He,init}} = 80.75 \, M_{\odot}$, corresponding to a maximum CO core mass of $\sim 57 \, M_{\odot}$, in good agreement with the threshold value found by Farmer et al. (submitted).

![Fig. 7.1: Final BH masses as a function of the initial He core mass. The scale in the horizontal direction is logarithmic. The colors in the background indicate the approximate range for each evolutionary path, see also Sec. 7.3. The right panel shows the masses inferred from the binary BH mergers detected by LIGO/Virgo, with a red shade to emphasize the overlap between PPI and CC, and green and blue hatches to indicate the fate of the progenitor in different BH mass ranges.](image-url)
CC: for extremely massive cores, $M_{\text{He,init}} \gtrsim 200 M_\odot$, the energy release by the explosive thermonuclear burning triggered by the pair instability is insufficient to fully disrupt the star. This happens because most of that energy is used to photodisintegrate the nuclear ashes and lost to neutrinos, instead of becoming kinetic energy of the stellar gas (Bond et al. 1984; Fryer et al. 2001). Therefore, models above a certain threshold reach CC without any PPI-driven mass loss (blue area on the right of the panel of Fig. 7.1).

In Fig. 7.1 the transition between the CC and PPI+CC is smooth, and we have avoided quantifying the boundary between the low mass CC and PPI+CC because of the subtleties in the definition of “pulse”. We outline three physically motivated definitions, each one shifting the CC/PPI+CC boundary, in Sec. 7.5.

### 7.4 Resulting BH masses

The PISN BH mass gap is denoted by the hatched region in the left panel of Fig. 7.1. The lower and upper edge of the gap can be read from the y-axis. With our numerical setup, we find a maximum BH mass below the PISN gap of $\max \{M_{\text{BH}}\} \approx 45 M_\odot$, in good agreement with the lower boundary of the gap from previous studies Woosley (2017); Marchant et al. (2018); Leung et al. (2019); Woosley (2019); Farmer et al. (submitted). At the upper-end, the PISN BH mass gap is closed by the photodisintegration instability causing the direct collapse of an initially $\sim 200 M_\odot$ He core to a BH of $125 M_\odot$ (the difference is caused mainly by wind mass loss). This upper boundary too is in good agreement with the results from Woosley et al. (2002); Woosley (2017), although it is sensitive to the metallicity and uncertainties in the wind mass loss rate.

We do not expect these boundaries would have varied if our models had a hydrogen-rich envelope (e.g., Woosley 2017). The combination of the mass loss (Ott et al. 2018; Kuroda et al. 2018; Chan et al. 2018) and energy loss to neutrinos at BH formation (e.g., Coughlin et al. 2018) should be sufficient to unbind the residual hydrogen envelope, unless the progenitor is a blue super giant with a large binding energy of the envelope (exceeding $\sim 10^{48}$ erg, Lovegrove & Woosley 2013). Such blue supergiant pre-PPI structures might arise from binary mergers (e.g., Spera et al. 2019, for a population synthesis study), however stellar structure calculations of post-primary-main-sequence mergers from Vigna-Gómez et al. (2019) show extended convective envelopes at the onset of the instability, which support our expectation that the envelope would easily be shed at the onset of the instability (see also Appendix E.2). Whether the H-rich envelope can contribute to the BH mass or not deserves further investigation.

The right panel of Fig. 7.1 shows for comparison the individual BH masses of the binary BH mergers detected to date by LIGO/Virgo\(^4\), with the 90% confidence level uncertainty

\(^4\)Other events have since been reported by an independent analysis of the first two observing runs, see Zackay et al. (2019) and references therein.
ranges. The masses of the two BHs in a merger event are not direct observables, they are instead inferred from the chirp mass and total mass of the binary. Hatched regions denote possible progenitor evolution (see also Sec. 7.3): the red area emphasize the range of BH masses that can be obtained by CC of a lower mass model, or by severe PPI mass loss of the most massive PPI+CC models. Most BH progenitors for the gravitational wave mergers events detected to date could be compatible with encountering the PPI, although we do not expect most of them to have gone through this evolution because progenitors with sufficient mass are disfavored by the initial mass function.

7.5 The physics of pulses: cores, radii, and mass ejections

While the nuclear and thermal processes governing the evolution of a star through pair instability are well understood, the characterization of the observable properties of such events are not yet as clear. The main reason for this is that stars do not react “elastically” to the pair-instability: instead the nuclear binding energy released by burning episodes at each pulse is stored and re-distributed throughout the stellar structure, and there is not a one-to-one correspondence between what happens in the core and what can be observed at the surface.

To clarify the distinction between core behavior and observable properties from the outermost layers of the star, in this section we describe the physical processes that can be used to give three different physically motivated definition of “pulse”, and illustrate them with an example $M_{\text{He,init}} = 50 M_\odot$ He core (for which we also present a resolution study in Appendix E.1). These definitions do not cover all the possible ways in which pulses can be defined and counted. For example, Woosley 2017 uses the core temperature $T_c$ while in Marchant et al. 2018 we adopted a criterion based on the maximum velocity in the stellar interior.

7.5.1 Thermonuclear ignition

Historically, studies on pair-instability evolution have focused on the core of stars. Indeed, the region that becomes unstable because of the runaway production of $e^\pm$ is typically deep in the star, and the subsequent evolution is driven by the explosive burning of oxygen and heavier fuel (e.g., Barkat et al. 1967; Rakavy & Shaviv 1967; Fraley 1968; Woosley 2017; Marchant et al. 2018; Leung et al. 2019). This allows for a definition of a pulse based on the behavior of the deep interior of the star.

Fig. 7.2 shows the evolution of the central temperature (bottom panel) and nuclear and neutrino luminosity (top panel) for a $M_{\text{He,init}} = 50 M_\odot$ He core. The horizontal axis shows the time to the onset of CC on a reversed logarithmic scale to magnify the late evolutionary phases, during which the PPI is encountered.

Initially, during the hydrostatic phase of evolution, the core temperature $T_c$ grows steadily following the contraction of the core. As the instability is encountered the core becomes thermally unstable, because of the softening of the equation of state (EOS), causing the collapse...
and rise in $T_c$ allowing for the explosive ignition of fuel. The latter can be seen as spikes in the nuclear luminosity $L_{\text{nuc}}$. The thermonuclear release of energy expands the core, cooling it adiabatically and causing a temperature drop. Eventually, the core is stabilized by the loss of entropy to neutrinos and the burning of nuclear fuel, and it ends its life steadily increasing its core temperature until the onset of CC.

If we define PPI pulses based on core temperature spikes, or equivalently spikes in nuclear and neutrino luminosity, then the lowest mass He core showing hints of pulsational behavior is $M_{\text{He,init}} \approx 37.5 M_\odot$ corresponding to a final $M_{\text{CO}} \approx 28 M_\odot$. Even if the local adiabatic index $\Gamma_1 < 4/3$ somewhere in this model, however the volumetric pressure-weighted averaged adiabatic index is always $\langle \Gamma_1 \rangle > 4/3$ for the entire evolution. The thermonuclear ignition in the core never results in a global instability of the star. We find that $\langle \Gamma_1 \rangle$ crosses the stability threshold of $4/3$ at some point in the evolution only for $M_{\text{He,init}} > 40.5 M_\odot$.

The inset plot of Fig. 7.2 shows that in a one-dimensional spherical MESA model the core “bounces” off itself, which was already noted in Paxton et al. (2018). These readjustments of the core cause secondary burning episodes (Renzo et al. submitted) which can release further energy and introduce some complications in counting the $T_c$ spikes (see also Marchant et al. 2018). While we resolve in time these bounces by taking timesteps shorter than the dynamical timescale of the core, it is likely that multi-dimensional effects would affect them significantly. Even counting the core oscillations as one individual pulse, our $50 M_\odot$ model exhibits tens of thermonuclear-ignition pulses.

This behavior of the core of very massive stars encountering the pair instability is well established (e.g., Barkat et al. 1967; Woosley 2017; Marchant et al. 2018; Leung et al. 2019). However, since these processes happens deep inside the optically thick layers of the star, their only direct observable is the rapid variation of orders of magnitude of the neutrino luminosity during each pulse (Fryer et al. 2001, and possibly during the post-pulse bounces). However, because of the rarity of such massive stars in the local Universe, such variations in the neutrino luminosity are unlikely to be easily observed.
7.5 The physics of pulses: cores, radii, and mass ejections

7.5.2 Radial expansion

In Sec. 7.5.1 we have discussed a definition of a PPI pulse based on the thermonuclear behavior deep inside the core. However, for $M_{\text{He,init}} > 41 M_\odot$, the nuclear binding energy released by the thermonuclear explosions deep down can have an observable impact on the surface of the star. Note that either because of other evolutionary processes, or because of a previous PPI mass-loss episode, the surface can be a He rich layer for these stars, and we can use our He core models to define a PPI pulse based on surface properties, assuming the hydrogen-rich layers have been lost before.

The thermonuclear burning injects energy into the core and drives a pulse wave, which propagates down the decreasing density profile of the star and eventually steepens into a shock. The core, post-explosion, readjusts and can contribute to driving secondary shocks. There is a small range in mass, $41 M_\odot < M_{\text{He,init}} < 42 M_\odot$ in which these shocks, which can often catch up with each other below the stellar surface, are not energetic enough to dynamically unbind any significant amount of matter (see also Sec. 7.5.3). Nevertheless, even in this mass range, they produce a potentially observable extreme radial expansion of the star. For models more massive than $M_{\text{He,init}} \gtrsim 42 M_\odot$, the energy released in the thermonuclear explosion also cause the ejection of material (see Sec. 7.5.3).

Fig. 7.3 shows the radial evolution of our 50 $M_\odot$ example. The orange line shows the photospheric radius (defined as the location where the optical depth is 2/3). Initially, at $t_{\text{CC}} - t \approx 10^{5.5}$ years, the core contracts in a phase of “pseudo-evolution” in which the model is relaxed to the desired initial conditions. Afterwards, for most of the evolution in hydrostatic equilibrium (until $t_{\text{CC}} - t \approx 10^{-2}$ years), the stellar radius is of the order of the solar radius ($R_\odot \approx 6.9 \cdot 10^{10}$ cm) or less. As the star contracts and approaches the instability, we switch to the HLLC solver at around $t_{\text{CC}} - t \lesssim 1$ year, when the thicker red line appears in Fig. 7.3. This line shows the radius of the bound material $R(v < v_{\text{esc}})$ and is plotted only when the hydrodynamics is on.

We follow the thermal contraction of the star due to the pair-instability, and at $t_{\text{CC}} - t \approx$
10^{-2} years a shock wave propagating from the core causes a huge radial expansion by two orders of magnitude on a dynamical timescale. The material remaining bound to the star extends to \( \sim 10^{13} \) cm. For our 50 \( M_{\odot} \) model, the pulse also ejects matter and the ejected layer extends beyond \( 10^{14} \) cm. For numerical stability reasons we cap the radii at \( 10^4 R_{\odot} \approx 6.9 \cdot 10^4 \) cm (horizontal dashed line in Fig. 7.3), and treat such limit as an open boundary. We discuss the ejected matter more extensively in Sec. 7.5.3.

The ejected layer can obscure the bound surface of the star: the location where the optical depth is \( 2/3 \) extends all the way to the outermost layers of our Lagrangian mesh, although the structure of the material moving faster than the escape velocity should be recomputed accounting for radiative losses for a better determination of the photosphere. It is possible that such stars would exhibit large radius differences at different wavelengths, with some that might even appear red during their maximal radial expansion. For this particular model, the photospheric radius does not have time to recover its pre-pulse value, since the radial expansion only started days before the final core-collapse. More massive models have more violent pulses that drive the inner core farther out of thermal equilibrium and for which it takes longer to recover the condition for further (explosive or stable) nuclear burning (see also Sec. 7.6): this can give time to the photospheric radius to decrease again.

The bound radius instead experiences large oscillations between \( 10^{11} \) cm and \( 10^{13} \) cm (the maximal expansion reached initially). In this case, these might not be directly observable since they are embedded within the pseudo photosphere of the ejecta. Models more massive than our example might have rather long lived phases with large radii, which might have implications for binary interactions (e.g., Marchant et al. 2018) and wind mass loss physics.

7.5.3 Ejection of material

Although the two definitions of a pulse we introduced in the previous sections (based on the core explosive behavior and on the radial expansion, respectively) might possibly give observable “pulses”, the processes they are based on do not leave a direct imprint neither on the CSM structure, nor on the remnant BH mass. Observational confirmation of the occurrence in nature of PPI+CC evolution (and possibly PISN) is most likely to come from observations of transients which can probe the CSM around the exploding star (e.g., Tolstov et al. 2017; Lunnan et al. 2018; Gomez et al. 2019; Wang & Li 2019) and/or the distribution of BH masses probed through gravitational waves (Fishbach & Holz 2017; Talbot & Thrane 2018; Stevenson et al. 2019; Mangiagli et al. 2019). It is therefore worth giving a definition of pulse based on the ejection of material from the stars: the ejecta carry away mass, decreasing the final BH mass and shaping the CSM structure.

Our simulations produce output for the ejecta at each timestep. The top panel of Fig. 7.4 shows the cumulative mass lost to PPI-driven pulses for our example \( M_{\text{He,init}} = 50 M_{\odot} \) He core. In this specific model, the total (H-free) ejecta mass is about \( 1.2 M_{\odot} \) by the end of the evolution. Had our star retained an H-rich envelope until the onset of the first pulse, the

\[\text{In none of our models is such an upper limit in radius reached by the bound material}\]
remaining H-rich envelope at the onset of the instability would likely add to the amount of mass in the CSM (see also Appendix E.2).

The bottom panel of Fig. 7.4 shows the density distribution around the star as a function of distance from the star (y-axis) and time until the final CC (x-axis). The cyan line shows the radius of the bound material (cf. the red curve in Fig. 7.3), which we assume to be the initial radius from which the ejecta are launched. To compute the CSM density we assume propagation of the ejecta at constant velocity. We use the velocity at the time the material first exceeds the local escape velocity as computed by MESA, and it is typically of a few thousand km s\(^{-1}\).

We return to the ejecta velocity in Sec. 7.6. Assuming a constant velocity for the propagation of the ejecta corresponds to neglecting radiative cooling, internal collisions of the ejecta, and multi-dimensional effects (e.g., Chen \& Woosley 2019), and we discuss it here only for illustration purposes. Our output files\(^6\) contain the amount of mass ejected, its center of mass velocity, chemical composition and thermal state (averaged by mass over all the mesh points that exceed the local escape velocity in the current timestep), which could be used as input for more sophisticated simulations to predict the details of the CSM structure around PPI+CC models. This ejecta output neglects the possibility of fallback, which however could be implemented when using these files as input for hydrodynamical simulations of the CSM.

The CSM structure shown in Fig. 7.4 for our example model shows H-poor/He-rich CSM starting from \(\sim 10^{13}\) cm and extending out to \(\sim 10^{14}\) cm. The CSM densities reach \(10^{-5} - 10^{-4}\) g cm\(^{-3}\). These values are typical for the models in our grid, and fall in the range of CSM distances and densities inferred from transient observations (e.g., Gomez et al. 2019).

However, mass ejections that happen in subsequent timesteps (possibly with no mass ejected in between) in MESA might not be physically distinct events. To count the mass ejection events, we need to group mass ejections in timesteps separated by less than a dynamical timescale in one individual event. Moreover, one mass ejection event can last several dynam-

CIRCUMSTELLAR MATERIAL FROM PULSATIONAL PAIR-INSTABILITY

The dynamical timescales, for example the final mass ejection and full disruption of a PISN is expected to produce a long transient (e.g., Gal-Yam et al. 2009). We estimate the dynamical timescale as the free-fall timescale $\tau_{\text{ff}} = 2\pi \sqrt{\frac{G R_{\text{photo}}^3}{M_{\text{bound}}}}$, where $G$ is Newton’s constant, and $R_{\text{photo}}$ and $M_{\text{bound}}$ are the (time-dependent) photospheric radius and mass gravitationally bound to the star. We define the beginning of a mass loss event as the timestep during which at least $10^{-6} M_\odot$ has been removed from the star since the last mass loss event (either in one timestep, or cumulatively). We require each mass ejection event to last at least one free fall timescale (calculated at the beginning of the pulse), and define its end as soon as the amount of mass to be ejected in the following $100 \tau_{\text{ff}}$ (now calculated at the end of the pulse) is less than $10^{-7} M_\odot$. This last condition allows us to count as a single event mass ejection episodes that last longer than a dynamical timescale. All together, these requirements enforce that ejections which numerically happen in different timesteps separated by less than a free fall timescale are not counted as separate events. Adopting this criterion to count the mass ejection events, our $M_{\text{He,init}} = 50 M_\odot$ model only has one mass-ejection episode (cf. tens of core ignitions, see Sec. 7.5.1), which starts roughly $\sim 0.015$ years $\approx 130$ hours before CC. The duration of the PISN events in our grid, assuming these threshold to define the beginning of the explosion, exceeds months even for the least massive PISN model with $M_{\text{He,init}} = 80.75 M_\odot$ in agreement with previous studies. Note that our stopping conditions do not allow models to reach what would look as the observational end of a PISN.

7.6 Pulsational Pair-Instability-generated CSM

We turn now to discussing the CSM that can be created by PPI evolution across our grid of models. In principle, this CSM can be probed by time-domain observations. Most of its properties do not depend on what criterion is used to define the beginning or end of the pulses, except the number of pulses and their duration. For these quantities, we adopt the definition of Sec. 7.5.3 based on the ejection of matter, which is the most relevant for discussing the CSM structure.

Tab. E.1 summarizes the time, duration, and amount of mass loss in each event for all the PPI+CC models in our grid, and Fig. 7.5 shows the number of pulses as mass-ejection events contributing to the CSM. Models evolving to CC without any mass ejection have zero pulses. We define full disruption in a PISN as a one-pulse event, although these would not contribute to the CSM itself. The color in the background emphasizes again the various evolutionary behaviors, but the mass threshold separating CC (blue) from PPI+CC (green) evolution is well defined at $M_{\text{He,init}} = 40.5 M_\odot$ with the definition from Sec. 7.5.3.

The number of pulses is zero at the lower end, and increases up to three distinct mass ejection events for the central part of the PPI+CC mass range. At even higher masses, approaching the PPI+CC/PISN boundary, the number of pulses decreases again, although the amount of mass ejected increases (see also Fig. 7.7): this is because pulses become more energetic and consume more nuclear fuel at once.
Fig. 7.5: Number of mass-ejection events caused by pair instability as a function of the initial He core mass. The color shading indicates the approximate range for each behavior: core collapse without experiencing PPI-driven mass loss (CC, blue), PPI-driven mass loss (PPI+CC, green), or full disruption in a PISN (yellow), which we define as one mass loss event. The noisiness is caused by the occurrence of mass loss event right at the time of the final core collapse.

The green region shows some noise in the number of pulses: the reason for this is illustrated in Fig. 7.6, which shows the velocity as a function of Lagrangian mass coordinate for our $M_{\text{He, init}} = 50 \, M_\odot$ core at the onset of core collapse. Many models exhibit a similar behavior, with an outgoing pulse wave at the onset of CC: this means that the PPI mass ejection is still going on while the Fe core starts collapsing.

Fig. 7.7 summarizes the amount of mass lost to PPI-driven pulses across our model grid. The bottom panel shows the amount of mass lost per individual pulse, the pulse number is represented by the color of the dots. The typical amount of mass lost varies from $\lesssim 10^{-3} \, M_\odot$ for the lowest-mass models ejecting some mass, up to $\simeq 20 \, M_\odot$ (of He-rich material) at the upper mass end, just below the minimum mass for PISN. For models producing more than one mass ejection event (i.e., the models for which also a pur-
ple and possibly a red dot are shown), the amount of mass lost per pulse does not behave monotonically with the pulse number. For $50 \, M_\odot \leq M_{\text{He,init}} \leq 62 \, M_\odot$ the second pulse (purple) ejects more mass than the first (blue), while for higher masses the first pulse removes more mass than the second.

The top panel of Fig. 7.7 shows the total amount of mass lost to PPI ejecta, i.e., the sum of the mass ejected in each individual pulse. A trend of more massive models producing more energetic pulses and driving more mass loss is evident. We provide a simple fitting formula for the total amount of He-rich mass lost in PPI-driven events for $45 \, M_\odot \leq M_{\text{He,init}} \leq 80 \, M_\odot$ which produce a CSM mass larger than $\sim 0.2 \, M_\odot$, shown as a dashed gray line in the top panel of Fig. 7.7:

$$\Delta M_{\text{tot}} = 0.00037 \cdot 10^{0.06234 \times M_{\text{He,init}}} \quad \text{for} \quad 45 \, M_\odot \leq M_{\text{He,init}} \leq 80 \, M_\odot \; ,$$  \hspace{1cm} (7.2)

where $\Delta M_{\text{tot}}$ and $M_{\text{He,init}}$ are in solar units. The total mass lost to pulses should be added to the amount of mass lost due to winds to calculate the total mass in the CSM. The density distribution of the CSM generated by PPI-driven pulses and wind mass loss rate are likely to be very different. In cases where stars retain a (loosely bound) H-rich envelope at the onset of the first pulse, the mass of such envelope at the start of the pulses should also be added to the total mass lost in the first mass loss event.

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**Fig. 7.7:** PPI-driven mass loss as a function of the initial He core mass. The top panel shows the total mass ejected in pulses, the bottom panel shows the mass lost in individual pulses. The amount of mass lost does not have a monotonic behavior with pulse number, and spans a wide range of values. The first pulse is shown in blue, and the second and third, if they occur, are shown in purple and red, respectively. Thin vertical lines connect multiple pulses for the same $M_{\text{He,init}}$.

**Fig. 7.8:** Top panel. Delay time between mass ejection episodes. Bottom panel. Delay time between each mass ejection episode and the final core-collapse. Typical delays are of the order of few months, but they increase steeply with the initial He core mass of the PPI progenitor, which produce fewer but more energetic pulses, up to $10^4$ years.
Fig. 7.8 shows the delay time between the end times of two subsequent mass ejections (top panel) and between each pulse and the final CC, as a function of the initial He core mass of our models. The timing of the mass-loss events also spans a large range, from zero (see also Fig. 7.6 for example) to $\sim 10^4$ years, corresponding roughly to the Kelvin-Helmholtz timescale of the most massive PPI+CC. We emphasize that many models in our grid show sub-month delays between the last pulse and the final CC, which makes them potential candidates to detect CSM interactions in early observations of the explosions.

The delay (in between pulses and between each pulse and CC) increases with the He core mass, because more massive models produce more energetic pulses that drive the star farther from gravo-thermal equilibrium, increasing the amount of time needed to return to equilibrium after a pulse and resume the final evolution. While typically for very massive stars the neutrino luminosity greatly exceeds their photon luminosity $L_\nu \gg L$ (they are “neutrino stars”, Fraley 1968), this is not always true for the most massive PPI+CC models. For these, the adiabatic expansion of the core can leads to central temperature and densities too low for significant neutrino cooling to occur. Thus, after a pulse begins, these models transition from evolving on a neutrino-mediated thermal timescale ($\propto GM^2/RL_\nu$) to a photon-mediated thermal timescale ($\propto GM^2/RL$) in between pulses, which increases their interpulse time.

Figure 7.9 shows the center of mass velocity of the layers ejected (which is calculated as the mass-weighted average of the center of mass velocity of the layers ejected at each timestep over the duration of the mass ejection event). Unlike the other quantities characterizing a pulse, the ejecta velocities we find do not span orders of magnitude, and are typically a few $\sim 1000$ km s$^{-1}$. This suggests that mass ejection during a PPI might explain the He-rich circumstellar material required to explain at least some of the spectra of SN Ibn showing narrow He lines (e.g. Pastorello et al. 2008), provided that the final CC results in a successful explosion (see also Sec. 7.7).

The first pulse (blue dots) are almost always faster than the later pulses, likely because after the first thermonuclear burning in the core, the nuclear fuel burned becomes progressively heavier, corresponding to a smaller energy release per nucleon. Nevertheless, assuming propagation at constant velocity, many models result in collisions in between the ejecta which can appear as SN impostors, as noted by Woosley (2017).
7.7 Explodability of CC and PPI+CC models

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Table 7.1: Iron core mass ($M_{\text{Fe}}$), mass location $M_4$ where the specific entropy decreases below $4k_B N_A$, and mass gradient $\mu_4 \equiv dM/dr_{r=4k_B N_A}$ for a representative subset of our models eventually forming a BH at $Z = 0.001$. All the models we computed are available at http://doi:10.5281/zenodo.3406357.

Does the final CC of a post-PPI star result in a successful explosion? This question is relevant in the context of SNe potentially powered by the interaction of the SN ejecta with previously-ejected shells. Tab. 7.1 lists, for a representative subset of models, quantities commonly used to determine the “explodability” of a stellar model. Specifically, we report the final iron core mass, defined as the outermost location where the mass fraction of $^{28}\text{Si} \leq 0.01$ and the mass fraction of Fe-group elements, i.e. with more than 46 nucleons, exceeds 0.1; and the two parameters proposed by Ertl et al. (2016). These are the mass coordinate $M_4$ at which the specific entropy decreases below $4k_B N_A$, where $k_B$ is Boltzmann’s constant and $N_A$ is Avogadro’s number, and the mass gradient at this location $\mu_4$ (Eq. 5 in Ertl et al. 2016). Since the nuclear reaction network we employ does not allow for detailed treatment of the electron captures and $\beta$ decays which determine the final electron-to-baryon ratio, we avoid listing the compactness parameter (e.g. O’Connor & Ott 2011), which is sensitive to these modeling assumptions (Farmer et al. 2016; Renzo et al. 2017). We caution that three-dimensional simulations (e.g. Ott et al. 2018; Kuroda et al. 2018) might give a different outcome than 1D parametric simulations used to assess the explodability of grids of models (e.g., O’Connor & Ott 2011; Ugliano et al. 2012; Müller 2019b; Couch et al. 2019).

The iron core mass is typically below $\sim 2.5 M_\odot$ below the PISN BH mass gap. The values for models above the PISN BH mass gap are sensitive to how much nuclear burning goes on before the stopping criterion based on the infall velocity is reached, and because of
their large mass, the entropy is larger than $4k_B N_A$ throughout these stars.

Because of the as-yet insufficient understanding of BH formation, it is hard to predict whether these models would give a successful, albeit possibly weak, explosion. However, the iron cores we find (cf. Tab. 7.1) might successfully explode even if forming a BH (e.g. Ott et al. 2018; Kuroda et al. 2018).

Fig. 7.10 shows the surface mass fractions of $^4\text{He}$, $^{12}\text{C}$, $^{16}\text{O}$ at the onset of core collapse for our CC and PPI+CC models. Should the CSM evolve to be (partially) optically thin at the time of the final explosion, the surface composition of the star would determine the spectral type of the SN. Most PPI+CC models experiencing a significant amount of mass loss show He-poor surfaces, and enhanced carbon (and to a smaller extent) oxygen mass fractions, corresponding to type Ic SNe. Because of the radial expansion caused by the pulses, some of these progenitors might look like extended and cool objects at the onset of collapse, rather than compact and hot progenitors. Conversely, if the He-rich CSM is optically thick, it might obscure the progenitor star and the embedded explosion, and we would expect that re-processing of the photons by the shell would produce He lines (possibly narrow and in emission) corresponding to a type Ib(n) SN.

We omit from Fig. 7.10 the PISN models because our stopping condition for these conservatively ensures the full star is unbound, but it might not correspond to the “beginning” of the explosion. Nevertheless, most PISN models start exploding while still retaining some He at their surface (corresponding again to type Ib SNe) with the wind mass-loss rate and metallicity adopted here.

Fig. 7.10 also shows a trend with the initial He core mass: the larger the initial $M_{\text{He,init}}$, the more mass is lost to winds and PPI, the lower the surface He mass fraction and the higher the carbon and oxygen mass fractions. In the mass range $50 M_\odot \lesssim M_{\text{He,init}} \lesssim 75 M_\odot$, corresponding roughly to the region where we find three distinct PPI driven mass loss events, the predicted surface abundances appear more noisy.
Finally, during the PPI evolution some $^{56}$Ni can be produced. We expect that a significant fraction of this $^{56}$Ni will fall back into the final BH. The energy release from its decay might pre-expand the innermost layers of the core, which could potentially help successful explosion of these stars at the final CC. Fig. 7.11 shows the amount of $^{56}$Ni present in our PPI+CC and PISN models at the end of the evolution: for PPI+CC models the total mass of $^{56}$Ni is typically $M_{\text{Ni}} \approx 0.2 - 0.4 M_\odot$, i.e. about one order of magnitude more than what is produced in typical core-collapse SNe (e.g., Wongwathanarat et al. 2013). This value increases steeply in the PISN range, reaching about $\sim 60 M_\odot$ at the upper end, in good agreement with Woosley et al. (2002) results. Note however that our calculations are based on a 22-isotope nuclear reaction network which is known to produce $M_{\text{Ni}}$ deviating by up to a factor of $\sim 1.5 \times$ in either directions from the results computed with larger nuclear reaction networks (regardless of the final fate of the models between CC, PPI+CC, or PISN).

7.8 Comparison to selected supernovae

Stars experiencing the PPI+CC evolution should intrinsically be rare because of the large initial mass necessary to build up a sufficiently massive core. To produce a significant amount of CSM via PPI-driven pulses, our results suggest that the He core mass needs to initially exceed $M_{\text{He,init}} \gtrsim 42 M_\odot$. Therefore, the rate of observed transients that can be interpreted as signatures of PPI evolution should be small. Possibly for this reason an unambiguous detection of PPI+CC/PISN in time-domain surveys is not yet available, although the physical mechanism underlying this phenomenon is well understood. We consider here a few notable and recent hydrogen-less type I SNe that have been proposed as PPI+CC candidates.

PTF12dam: Tolstov et al. (2017) modelled the H-less (type I) superluminous supernova PTF12dam as powered by the combination of $^{56}$Ni decay and CSM interaction. They proposed such a combination of energy sources invoking the following scenario: first the H-rich envelope is removed by stellar winds, then the PPI pulses produce $\sim 20 - 40 M_\odot$ of CSM before the final CC synthesizes and ejects $M_{\text{56Ni}} \approx 6 M_\odot$ of radioactive material. Our results, albeit computed with a small nuclear reaction network, never produce this combination
of CSM mass and $M_{^{56}\text{Ni}}$: PPI ejecta exceeding $20\,M_{\odot}$ are found only for $M_{\text{He,init}} \gtrsim 75\,M_{\odot}$ (cf. Fig. 7.7), but only about $0.2\,M_{\odot}$ of $^{56}\text{Ni}$ is synthesized for PPI+CC models. Assuming that the final CC proceeds similarly as for lower mass stars, we expect it would add $0.03$-$0.05\,M_{\odot}$ of $^{56}\text{Ni}$ (e.g., Wongwathanarat et al. 2013), which does not help to reach the high $M_{^{56}\text{Ni}}$ claimed. An initial He core mass exceeding $M_{\text{He,init}} \gtrsim 140\,M_{\odot}$ is required to reach the amount of radioactive material required by Tolstov et al. (2017), which would put the model in the PISN range where we do not expect CSM from PPI.

**iPTF16eh:** Lunnan et al. (2018) detected a time and frequency varying MgII line in the spectrum of the type I superluminous supernova iPTF16eh. They interpreted it as a light-echo of the explosion bouncing off a layer of CSM at $r \approx 3.5 \cdot 10^{17}\,\text{cm}$ moving at $\sim 3300\,\text{km s}^{-1}$, implying an ejection $\sim 30$ years before the final CC. Based on these CSM properties and the models from Woosley (2017), they inferred a progenitor with $M_{\text{He,init}} \approx 50$ -- $55\,M_{\odot}$ (or equivalently an initial total mass $\sim 115\,M_{\odot}$). Our models are in overall agreement with the results from Woosley (2017) used by Lunnan et al. (2018) to interpret iPTF16eh, although the delay time and ejecta velocity would agree better with slightly a more massive progenitor, with $M_{\text{He,init}} \approx 60$ -- $65\,M_{\odot}$.

**SN2016iet:** Gomez et al. (2019) analyzed the double-peaked peculiar type I SN 2016iet. They explored several scenarios (PISN, CSM interaction, and central engine) to power its light curve. This event showed an unusually high Ca/O ratio, and extreme offset from the nearest galaxy of $\sim 16\,\text{kpc}$, however H$\alpha$ lines appear in the spectra beyond 400 days, possibly indicating local star formation activity. They also detected a possible light-echo from a H- and He-poor shell moving at few thousand km s$^{-1}$. Regardless of the scenario assumed, they inferred a large progenitor mass with a CO core $55\,M_{\odot} \lesssim M_{\text{CO}} \lesssim 120\,M_{\odot}$. The model they favor to explain the light curve combines the signal from the shock cooling of the prompt explosion (first peak) and CSM interactions (second peak), but requires $\sim 35\,M_{\odot}$ of CSM. Both the possible presence of a shell of H- and He-poor material and the claimed progenitor and CSM masses suggest PPI+CC as a viable scenario for the progenitor of SN2016iet. Several models with initial He core mass $M_{\text{He,init}} \gtrsim 50\,M_{\odot}$ produce PPI-driven pulses with mass, timing, and velocity within a factor of about two from the values inferred by Gomez et al. (2019). However, reaching that total amount of CSM would either require the progenitor to be at the very edge of the PISN regime, for which we find long interpulse delays (cf. Fig. 7.8) and also the last pulse tends to produce little mass loss (cf. Fig. 7.7). Alternatively, allowing for a contribution of the stellar wind to the CSM mass budget (e.g., because of the wind in between pulses running into a slower moving previously ejected shell), models with $60\,M_{\odot} \lesssim M_{\text{He,init}} \lesssim 70\,M_{\odot}$ produce pulses removing larger amounts of mass in the final few years of the progenitor’s life. This might produce a better agreement with the observed features of SN2016iet. If that were the case, this event might be the birth of one of the most massive BHs predicted below the PISN mass gap, cf. Fig. 7.1. At http://10.5281/zenodo.3406357, we provide models computed at our fiducial metallicity.
value and at the metallicity of the galaxy at 16 kpc from SN2016iet ($Z = 0.00198 \simeq 0.14Z_\odot$). These can provide input for more detailed calculations of the CSM structure needed to compare with SN2016iet.

**PS15dpn and other narrow line SNe:** Two out of the three SNe we considered above are super-luminous, however the final collapse of a PPI+CC progenitor or PISNe does not need to be superluminous (Woosley 2017). The PPI is just one possible mechanism to create CSM, which can produce extreme luminosities by tapping into the kinetic energy of the ejecta and/or narrow emission lines (even if the luminosity does not reach extreme values). The detection of narrow H lines determines the classification of a SN as a type IIn, while the detection of He emission lines determines the classification as type Ibn. Both kinds of event are too common to explain all of them with PPI+CC progenitors, and it is likely that both classes contain events with a diversity of physical mechanisms (e.g., Pastorello et al. 2008 but see also Hosseinzadeh et al. 2017). Nevertheless, it is possible that at least some of these events might correspond to the observational counterpart of the death of PPI+CC progenitors. In particular, our simulations can produce several solar masses of H-free CSM moving at a few thousand km s$^{-1}$, which correspond to the width of the He lines detected in some SN Ibn without any ad hoc fine-tuning required. A possible example of a SN Ibn light curve which can be fitted by combining CSM interaction and radioactive decay is PS15dpn, for which Wang & Li (2019) inferred CSM and $^{56}$Ni masses of $\sim 0.8 M_\odot$ and $\sim 0.1 M_\odot$, respectively.

### 7.9 Limitations and caveats

The stellar evolution simulations presented here require a large number of assumptions. Work to assess the robustness of these calculations has been carried out by Marchant et al. (2018); Farmer et al. (submitted); Renzo et al. (submitted) (see also Appendix E.1), to which we refer the readers for more details.

**Ignition location and spherical symmetry**

One of the key assumptions is that spherical symmetry is maintained during the evolution. Chen et al. (2014); Chen & Woosley (2019) showed that if a pulse starts symmetrically, hydrodynamic instabilities only weakly deform the pulse. However, the first stellar layers to become unstable due to pair-production in a pulsating model is not necessarily at the very center, especially at the lower mass end of the PPI+CC regime. The top panel of Fig. 7.12 shows the temperature and density profile of three examples with $M_{\text{He,init}} = 50, 81, 250 M_\odot$ representative for PPI+CC, PISN, and CC above the mass gap, respectively. The stellar tracks are plotted at the time when the volumetric pressure-weighted average $\langle \Gamma_1 \rangle$ first approaches the instability value $4/3$, i.e., $\langle \Gamma_1 \rangle - 4/3 = 0.01$. The red shade emphasizes the instability region (neglecting its weak dependence on the details of the chemical composition), and the
text annotations indicate the physical ingredients that stabilize the structure outside of this region (Zeldovich & Novikov 1999; Kippenhahn et al. 2013).

The bottom panel of Fig. 7.12 shows the local value of the adiabatic index in the center $\Gamma_{1,c}$ across the mass range we explore, also plotted when each model first reaches $\langle \Gamma_1 \rangle - 4/3 = 0.01$. The colors in the bottom panel have the same meaning as in Fig. 7.1.

Two example models that ultimately result in a core-collapse are shown in the top panel of Fig. 7.12. The green line corresponds to our $M_{\text{He,init}} = 50 M_\odot$ example for PPI+CC, which shows more features compared to the other models, because the chemical stratification is more important in lower mass models. The most central region of the 50 $M_\odot$ model has a local value of the adiabatic index in the center $\Gamma_{1,c} - 4/3 > 0.01$ when $\langle \Gamma_1 \rangle \approx 4/3$, i.e., the center is stable when the averaged $\langle \Gamma_1 \rangle$ approaches instability. The deepest interior is too dense to become unstable: $e^\pm$ pairs fill the available continuum energy levels, raising the Fermi energy $E_{\text{Fermi}}^{\pm}$ and consequently the minimum energy photons need to produce a pair, preventing layers from undergoing the runaway instability (e.g., Zeldovich & Novikov 1999).

Therefore, the instability starts off-center in our 50 $M_\odot$ example, and this is true for all our PPI+CC models: in the bottom panel of Fig. 7.12, all the green points corresponding to PPI+CC have central values $\Gamma_{1,c} - 4/3 > 0.01$ when $\langle \Gamma_1 \rangle - 4/3 = 0.01$. In these cases, because of the assumption of spherical symmetry, the off-center ignition of the unstable layers happens in a spherical shell, and the heat released by the burning typically triggers the ignition of all the stellar plasma below the unstable location. However, in nature the ignition might not happen simultaneously across the entire spherical shell, and this could possibly seed an asymmetric explosion. If asymmetries can build up rapidly during the pair-instability driven explosion (possibly aided by rotation), this could also lead to orbital “kicks” when PPI happens in a binary (Marchant et al. 2018).

The location of the unstable layer at the onset of the instability moves inward towards the center as $M_{\text{He,init}}$ increases. The least massive models to go PISN is characterize by having even its center close to the instability, i.e., $\Gamma_{1,c} - 4/3 \leq 0.01$, when $\langle \Gamma_1 \rangle - 4/3 = 0.01$ for the first time, as shown by the yellow dots in the bottom panel and the yellow solid line.
corresponding to \((M_{\text{He,init}} = 81 M_\odot)\) in the top panel of Fig. 7.12. Our least massive PISN model has \(M_{\text{He,init}} = 80.75 M_\odot\) and \(\Gamma_{1,c} - 4/3 = 0.01\) at this evolutionary stage.

Models forming a BH above the PISN mass gap (cf. blue line in the top panel of Fig. 7.12 for a \(M_{\text{He,init}} = 250 M_\odot\) He core) are also unstable in their very center when \(\langle \Gamma_1 \rangle\) reaches \(4/3\), but the ensuing thermonuclear explosion does not cause either pulses or full disruption. The different outcome is not caused by lack of energy released in the explosions, but rather by the inefficient use of that energy (Bond et al. 1984).

To summarize, the temperature, composition, and density profile of the star when it approaches the instability (i.e., \(\langle \Gamma_1 \rangle - 4/3 = 0.01\) for the first time) are indicative of its future evolution. In particular, the local value of the adiabatic index in the center \(\Gamma_{1,c}\) at this point can be used to approximately distinguish PISN evolution with no BH remnant (if also \(\Gamma_{1,c} - 4/3 \leq 0.01\)) or PPI+CC evolution with a BH remnant (if instead the center is safely stable with \(\Gamma_{1,c} - 4/3 > 0.01\) when the star as a whole is becoming unstable). This provides a criterion to decide the final fate of a star without having to compute the hydrodynamical phase.

### 7.9.1 CSM structure and composition

We have described in Sec. 7.6 the amount of mass ejected, its initial velocity, and the ejection timing resulting from our simulations. We typically keep the ejecta on our Lagrangian grid for several timesteps after ejection (until either the bound layers have recovered hydrostatic equilibrium or the onset of CC is reached). These ejected layers are moving significantly faster than the escape velocity and the sound speed, and our PPI+CC models exhibit an overall velocity gradient increasing outwards, so the ejecta do not cause any back-reaction on the inner layers that remain bound. Based on the initial mass, velocity, and time of the ejections, Sec. 7.6 illustrates the main features we expect in the CSM structure surrounding these stars with a toy-model assuming propagation at constant velocity of the ejecta. This is an oversimplification, since the low density ejecta are likely to be optically thin and thus can lose energy radiatively. Moreover, as previously noted by Woosley (2017), the ejected shells can in many cases collide with each other, and this could also significantly change the CSM structure at the end. Multidimensional radiation hydrodynamics calculations using our results as input for the mass, chemical composition, and thermal state of the ejecta could be used to predict more robustly the CSM structure around PPI+CC models for comparison with observed transients, and to address the question of how many progenitors might reach the final CC embedded in an optically-thick layer of previously ejected material.

Another assumption in our calculations is that the presence of a hydrogen-rich envelope can be neglected to study the dynamics of PPI (Woosley 2017, 2019) and that, even if present, such a envelope would be removed early in the evolution by winds or binary interactions. Should a star retain some hydrogen-rich material until the onset of the first pulse, we can estimate if it would be detectable in the CSM surrounding these stars assuming that (i) the PPI-driven mass loss timing is unaffected by the presence of the hydrogen-rich envelope and
(ii) the entire envelope is ejected in the first pulse (Woosley 2017). However, the presence of a hydrogen-rich envelope can significantly affect the interpulse time because of its impact on the evolution of the He core (e.g., Woosley 2019). In particular, the presence of a H-rich envelope makes the He core grow by mass between the end of the main sequence and the onset of the pair-instability, while naked He cores lose mass to winds, which can influence the interpulse time (see also Appendix E.2).

To estimate the ejection velocities and radii of the hydrogen-rich material, we ran a $150 \, M_\odot$ model with initial He abundance $Y = 0.27$ and metallicity $Z = 0.001$ with the same setup as in our grids. We compare this model to a He core of initial mass similar to the He core mass at the end of the main sequence of the hydrogen-rich star in Appendix E.2. This model reaches the onset of the PPI ($\langle \Gamma_1 \rangle - 4/3 = 0.01$) with $\Gamma_{1,c} - 4/3 = 0.04$, so we expect it to follow the PPI+CC evolutionary path based on Sec. 7.9. This expectation is confirmed by our results presented in Appendix E.2. At the onset of the instability and with our assumed wind mass loss, this model has a total mass of $M_{\text{tot}} = 89 \, M_\odot$, a He core of $M_{\text{He}} \approx 63 \, M_\odot$, and a CO core of $M_{\text{CO}} \approx 56 \, M_\odot$. The remaining envelope has a mass of $M_{\text{env}} \approx M_{\text{tot}} - M_{\text{He}} \approx 26 \, M_\odot$, however, the composition of this envelope is dominated by He, with a hydrogen mass fraction of $X \approx 0.15$, since the winds have carved out material down to the initial location of the main sequence core of the star. At this stage, the envelope spans from the He core edge at $R_{\text{He core}} = 0.61 \, R_\odot$ to $R_\ast = 2453 \, R_\odot$, so it is significantly extended.

If we assume propagation at constant velocity of this envelope, we can estimate the minimum and maximum radii ($R_{\text{min}}$ and $R_{\text{max}}$, respectively) of this hydrogen-rich material at the time of the final CC and its average density ($\langle \rho \rangle$) with

$$R_{\text{min}} = R_{\text{He core}} + v_{\text{esc, in}} \times (t_{\text{CC}} - t_{\text{pulse end}})$$
$$R_{\text{max}} = R_\ast + v_{\text{esc, out}} \times (t_{\text{CC}} - t_{\text{pulse end}})$$
$$\langle \rho \rangle = \frac{3M_{\text{env}}}{4\pi(|R_{\text{max}}^3 - R_{\text{min}}^3|)} ,$$  \quad (7.3)
7.10 Conclusions

The theoretical understanding of the predicted pair-instability driven transients has been well established for several decades, however they remain somewhat elusive from an observational perspective. Recent developments in stellar evolution calculations allow for the exploration of synergies between gravitational waves and time-domain observations to better understand the formation process of the most massive stellar BHs.

We have computed a grid of naked He star models in the mass range $35 M_\odot \leq M_{\text{He,init}} \lesssim 250 M_\odot$ to investigate whether these would experience phases of global dynamical instability and pulsational mass loss due to the pair-production instability. We have computed grids at two different metallicities, $Z = 0.001$ and $Z = 0.00198 = 14\% Z_\odot$, although the main features we discuss are not significantly dependent on $Z$ (except for the wind mass loss rate, see also Farmer et al. submitted). All our input files and numerical results are available at http://10.5281/zenodo.3406357.

Fig. 7.13 summarizes our main results across the mass range considered. We find, in agreement with previous studies, that stars enter into the PPI regime progressively. The production of $e^\pm$ initially causes “oscillations” of the core temperature and nuclear luminosity at the lowest mass end. The least massive models experiencing an explosive thermonuclear ignition ($M_{\text{He,init}} \gtrsim 37.5 M_\odot$) do not suffer significant global consequences (“weak pulses”). Increasing further the initial He core mass, the pulses become progressively stronger, causing at first large radial expansions (for initial $41 \lesssim M_{\text{He,init}} \lesssim 42 M_\odot$), and finally (for initial $M_{\text{He,init}} \gtrsim 42 M_\odot$) also the ejection of matter. The values quoted here are for the initial He core mass of our models, which can be interpreted as the core mass at the end of the main sequence of the star. The mapping of these values to the final (pre-instability) He core mass is mass loss and metallicity dependent (see also Farmer et al. submitted).

The different effects of a pulsational pair instability event on the star allow for (at least) three different physically-motivated definitions of a “pulse”, depending on which observable is considered (Sec. 7.5). Depending which is adopted, the mass range where pulsations are observable, and the number of pulses, might shift significantly.

The first definition of pulse we consider (Sec. 7.5.1) is based on the core thermonuclear ignition, following the historical development of studies of pair-instability evolution. We find that in the lowest mass models the core ignition does not produce an observable electromagnetic signal or a significant impact on the final BH mass: the nuclear energy released in the burning is redistributed and stored in the star without affecting significantly the outermost layers. The most promising way to detect directly these core-ignition events is through the variations in the neutrino luminosity.
**Approximate supernova type**
(mass-loss dependent, Sec. 7)

**Pulse delay to core-collapse**
(Sec. 6)

**Thermonuclear ignition**
(Sec. 5.1)

**Radial expansion**
\[ \max R(\nu < \nu_{\text{esc}}) \] (Sec. 5.2)

**Number of mass ejections**
(Sec. 5.3)

**\( M_{\text{CISM}} \) He-rich**
(Sec. 6)

**Thermal stability**
(Sec. 5.1.1)

**BH remnant**
(Sec. 3)

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**Fig. 7.13:** Summary of the pair-instability driven behavior of models as a function of their initial He core mass \( M_{\text{He,init}} \) and maximum carbon-oxygen core mass \( M_{\text{CO}} \).
The second definition is based on the radial expansion of the models in response to the core ignition (Sec. 7.5.2): this definition shifts upwards in mass the lower edge of the pulsating regime. We note that for the most massive pulsating models (which also eject mass), the radial expansion itself might be hidden behind a pseudo-photosphere in the ejected layers. Because of the rarity of these stars in the local Universe, the most promising way to detect these radius variations is probably through their enhancement of the rate of binary interactions (e.g., Marchant et al. 2018).

The third definition is based on the ejection of mass (Sec. 7.5.3), which impacts both the circumstellar material around these stars and the final BH mass they produce. The ejected matter creates shells of ejecta surrounding the star. If the final collapse results in a successful (even if weak) explosion, the final SN ejecta can hit this PPI-produced CSM and convert kinetic energy into radiation.

The signature on the final BH masses is potentially detectable with a population of gravitational wave sources (Fishbach & Holz 2017; Talbot & Thrane 2018; Stevenson et al. 2019; Mangiagli et al. 2019). Only for $M_{\text{He,init}} \gtrsim 42 M_\odot$ is the stellar core mass significantly reduced and the circumstellar material significantly affected, as shown in Fig. 7.13.

The maximum BH mass below the PISN mass gap that we find is $\sim 45 M_\odot$ and it is formed by the collapse of an initially $M_{\text{He,init}} \approx 60 M_\odot$ He core that went through pulsational mass loss. More massive He cores also produce BHs, but because of the stronger mass loss due to winds and pair-instability driven pulses, the resulting BH masses are smaller.

In our grid, we find full disruption in a PISN for an initial He core mass of $M_{\text{He,init}} \approx 80 M_\odot$ corresponding to a final He core mass of $M_{\text{He}} \approx 60 M_\odot$ and $M_{\text{CO}} = 57 M_\odot$ after the wind mass loss. With our assumptions for the wind mass loss and metallicity, most PISN models would still retain He-rich material at their surface at the onset of the explosion. We propose a simplified criterion to distinguish full disruption in a PISN from pulsational behavior producing a final BH based on the adiabatic index at the center of the star $\Gamma_{1,c} \lesssim 0.01$ at the onset of the instability, defined as the first moment when the volumetric pressure-weighted average of the adiabatic index $\langle \Gamma_1 \rangle - 4/3 = 0.01$ (Sec. 7.9). While this threshold is arbitrarily chosen, it allows to approximately estimate the fate of a stellar model without the need to compute the hydrodynamical evolution.

Pair instability does not result in full disruption of the star for an initial He core mass of $M_{\text{He,init}} \approx 200 M_\odot$, which forms a BH of mass $M_{\text{BH}} = 125 M_\odot$ after wind mass loss. Above this He core mass, the photodisintegration of newly synthesized heavy elements during the thermonuclear explosion prevents the disruption of the entire star (e.g., Bond et al. 1984). The boundaries between PISN and BH formation we find are in very good agreement with previously published results.

We have characterized the CSM properties around the pulsating models resulting in mass ejections by assuming unperturbed propagation at constant velocity of the ejecta. Under this simplifying assumption, we find that the CSM mass grows almost monotonically with the initial $M_{\text{He,init}}$ from $\sim 10^{-6} M_\odot$ (for $M_{\text{He,init}} \approx 42 M_\odot$) to $\sim 20 M_\odot$ at the edge of the PISN range. For initial $M_{\text{He,init}} \gtrsim 50 M_\odot$, the combined ejection of matter and mixing during a
pulse propagation make He less abundant than C and O at the stellar surface at the onset of the final core-collapse. The stellar surface at the onset of core collapse might be obscured by the previously ejected layers.

The velocity of the ejecta is a few thousand km s$^{-1}$, with the first mass-loss event often resulting in larger velocities. Nevertheless, with our assumptions, we find numerous self-collisions of the ejecta with previously ejected layers in agreement with the predictions of Woosley (2017). This velocity range is close to the width of narrow He lines detected in some SN Ibn.

The timing of pair-instability driven mass ejections also spans a large range of values, with a systematic trend of longer delays between pulses for the more massive models. This is because more massive models produce more energetic pulses that require a longer time (up to $\sim 10^4$ years) for the star to recover its equilibrium.

With the velocity and timing of the ejecta produced by our models, we expect the PPI-produced circumstellar material to be at $\sim 10^{12} - 10^{16}$ cm away from the collapsing star at the end of its evolution. This range covers the distances inferred in observational candidates for pulsational pair instability evolution.

Upcoming gravitational and transient observations will soon shed light on the pair-instability evolution of the most massive stars and the BHs these produce.

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