Live fast and die young
_Evolution and fate of massive stars_
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_Citation for published version (APA):
Gravitational-wave detections are now probing the black hole (BH) mass distribution including the predicted pair-instability mass gap. These data require robust quantitative predictions, which are challenging to obtain. The BH progenitors experience episodic mass ejections on timescales shorter than the convective turn-over timescale. This invalidates the steady-state assumption on which the classic mixing-length theory for convection relies.

We compare the final BH masses computed with two different versions of the stellar evolutionary code MESA: (1) using the default by Paxton et al. (2018) and (2) solving an additional equation accounting for the timescale for convective deceleration. We find weaker pulses in the second grid, where stronger convection develops during the pulses and carries part of the energy. This leads to lower amounts of mass being ejected and thus higher final BH masses of up to \( \sim 5 \, M_\odot \). The differences are much smaller for the progenitors which determine the maximum mass for BHs below the gap. This prediction is robust at \( M_{\text{BH,max}} \approx 48 \, M_\odot \), at least within the context of this study.

This is an encouragement that current models are robust enough for comparison with the early gravitational-wave detections. However, the large differences between individual models emphasize the importance of improving the treatment of convection in stellar models, especially in the light of the data anticipated from the third generation of detectors.
8.1 Introduction

One of the most challenging aspects of simulating the interior evolution of stars is the treatment of convection (e.g., Renzini 1987; Arnett et al. 2018a; Buldgen 2019). The development of convective motion in highly stratified media is an inherently multidimensional problem, which involves turbulence. Spherically symmetric stellar models typically rely on the Mixing Length Theory (MLT, Böhm-Vitense 1958), which provides an averaged description of subsonic, steady-state convection. Albeit with many well-known caveats, MLT is often a sufficient description for the energy transportation and chemical mixing provided by convection. This is because the evolutionary timescale of a star is typically much longer than the convective turnover timescale: within a single timestep it is typically reasonable to assume that the steady state described by MLT can be achieved in the convective layers of the model. However, stars can experience dynamical phases of evolution which are too short for convection to achieve the steady state described by MLT. In this situation, the time-dependence of convection can become important, and stellar evolution models typically lack a first principles model for the convective acceleration. Sometimes, the convective acceleration due to buoyancy is limited to a fraction of the local gravitational acceleration to prevent unphysically large accelerations (e.g., Arnett 1969; Wood 1974).

Here, we focus on a timely example of a situation in which the time dependence of convection can be important: the evolution of very massive stars experiencing pulsational-pair instability (PPI, Fowler & Hoyle 1964; Barkat et al. 1967). Because of the large mass required to encounter this instability, it is a rare phenomenon in nature, but the recent detection of black holes (BH) with masses $30 \, M_\odot \lesssim M_{BH} \lesssim 50 \, M_\odot$ (LVC 2018b) has driven the interest in understanding the evolution of the most massive (stellar) BH progenitors. To fully harvest the information carried by gravitational waves (GW) and use it to constrain stellar evolution, we need to have robust stellar models and characterize their sensitivity to uncertain ingredients (e.g., Farmer et al. 2016; Renzo et al. 2017; Davis et al. 2019).

Stars that develop helium (He) core masses exceeding $M_{He} \gtrsim 30 \, M_\odot$ are predicted to encounter the PPI and shed significant amounts of mass in subsequent pulsation episodes (e.g., Yoshida et al. 2016; Woosley 2017; Takahashi 2018; Marchant et al. 2018; Leung et al. 2019; Woosley 2019). The amount of mass lost in these pulses, together with the previous wind mass loss, determines the mass distribution of BHs formed. Increasing further to $M_{He} \gtrsim 60 \, M_\odot$, the instability becomes so violent that the entire star is disrupted in a pair-instability supernova (PISN, Barkat et al. 1967; Fraley 1968), without leaving any compact remnant. For $M_{He} \gtrsim 135 \, M_\odot$, all the energy released by the thermonuclear explosion is used to photodisintegrate the newly formed nuclei, instead to accelerate the stellar gas, and BH formation resumes (e.g., Bond et al. 1984). Thus, PISNe are expected to carve a gap in the BH mass distribution.

Numerical simulations of the PPI evolution require following hydrodynamical phases in between phases of hydrostatic equilibrium. This can be done alternating the use of two different codes (e.g., Chatzopoulos & Wheeler 2012a; Chatzopoulos et al. 2013; Yoshida et al. 2016).
which however limits the number of pulsational event that can be followed. To the best of our knowledge, two hydrodynamic Lagrangian stellar evolution codes can now follow the evolution of such massive stars. Woosley (2017, 2019) presented the first grids of stellar models computed with the KEPLER code (Weaver et al. 1978), building upon pre-existing models computed with the same code (Woosley et al. 2002, 2007). Recently, Marchant et al. (2018) and Leung et al. (2019) used two different implementations of hydrodynamics in the open-source code MESA (Paxton et al. 2011, 2013, 2015, 2018, 2019) to simulate the evolution of PPI.

Several authors have noted that the amount of mass lost is sensitive to the treatment of convection, both before and during the pulses (e.g., Woosley 2017; Marchant et al. 2018; Leung et al. 2019). In this letter, we compare two grids of massive bare He core models to highlight the differences resulting from variations in the treatment of time-dependent convection. In one of our grids, convection is treated similarly to Paxton et al. 2018 and Leung et al. (2019), while the other grid follows the approach used in Marchant et al. (2018) (hereafter, M18). In Sec. 8.3.1, we present the BH masses from both grids. In Sec. 8.3.2, we illustrate the differences in internal structure using two pairs of example stellar models, before discussing the implications of our results in Sec. 8.4.

We do not aim at solving a problem that has remained in stellar astrophysics for several decades, but hope to stimulate improvements in stellar evolution models that also account for the time-dependent behavior of convective motion.

### 8.2 Methods

We use the open-source stellar evolution code MESA to simulate the evolution of bare He cores at metallicity $Z = 0.001$ with masses in the range $25 M_\odot \leq M_{\text{He}} \leq 70 M_\odot$. All our input files are available at [http://cococubed.asu.edu/mesa_market/inlists.html](http://cococubed.asu.edu/mesa_market/inlists.html), and our models are available at doi:10.5281/zenodo.3406320. We track the energy generation with the 22-isotope nuclear reaction network approx21_plus_co56.net. When the star becomes dynamically unstable (i.e., the pressure weighted volumetric averaged adiabatic index throughout the star drops below $4/3$, Stothers 1999), we employ the HLLC Riemann solver in MESA\(^1\) (Toro et al. 1994), without relying on artificial viscosity to capture shocks. After a dynamical pulse, if/once the core has recovered hydrostatic equilibrium, we create a new stellar model of reduced mass with the entropy and chemical profile of the bound material. We do not include any wind mass loss, although the treatment of winds is known to influence the core structure of massive stars (Renzo et al. 2017). Preliminary tests including wind mass loss showed the same trends discussed here. The impact of uncertainties related to winds and other input physics on our PPI models are studied in Farmer et al. (submitted). Tests to ensure the robustness of our models against spatial and temporal discretization are discussed in (M18, Farmer et al. submitted, Renzo et al. in prep.). We refer the interested readers to M18 for a

\(^1\)Conversely, Leung et al. (2019) used the MESA implementation of artificial viscosity (see also Paxton et al. 2015).
full description of our setup. Here, we focus only on the treatment of convection.

We adopt the Ledoux criterion for convective stability with a mixing length parameter \( \alpha_{\text{MLT}} = 2.0 \) and an exponential under/overshooting with \((f,f_0) = (0.01,0.005)\) (cf. Eq. 2 in Paxton et al. 2011).

To test the sensitivity of our results to the treatment of time-dependent convection, we compute two grids of models using two different MESA versions. Other differences between the two code versions might contribute to the variations described here. Our first grid of models, which we refer to as the “classical MLT” grid, is computed using MESA release 10108. For this grid, the convective velocity \( v_c \) is obtained from MLT under the steady-state assumption, similarly to Paxton et al. (2018) and Leung et al. (2019), although the latter authors turn off convection during hydrodynamical phases of evolution. For this grid we employ MLT++, which is an enhancement of the convective flux in superadiabatic radiation pressure dominated regions prone to developing density inversions (Paxton et al. 2013; Jiang et al. 2018). After the onset of the hydrodynamic phase of evolution we enforce short timesteps, therefore we apply a limit to the convective acceleration based on Wood (1974) to avoid unphysical infinite convective acceleration. This approach still allows for infinite convective deceleration: if a stellar layer becomes radiatively stable, the convective velocity is instantaneously set to zero.

We compute our second grid, which we refer to as the “time dependent deceleration” grid, using MESA version 11123. In this case, we obtain \( v_c \) solving, together with the stellar structure and composition equations, an equation designed to asymptotically give the MLT value of \( v_c \) over long timescales, and to damp \( v_c \) in radiative regions over a characteristic buoyancy timescale. The equation we solve reads (cf. Eq. A1 and A2 in M18 and Eq. 11 in Arnett 1969):

\[
\frac{\partial v_c}{\partial t} = \begin{cases} 
\frac{(v_{\text{MLT}}^2 - v_c^2)}{\lambda} & \text{for convectively unstable regions} \\
-\frac{v_c^2}{\lambda} - Nv_c & \text{for convectively stable regions}
\end{cases}, \tag{8.1}
\]

where \( \lambda = \alpha_{\text{MLT}} H_p \) is the mixing length, assumed to be proportional through a free parameter \( \alpha_{\text{MLT}} \) to the local pressure scale height \( H_p \), \( N \) is the Brunt-Väisälä frequency, and \( v_{\text{MLT}} \) is the MLT steady state convective velocity.

Our main parameter of interest is the resulting BH mass, which we estimate using the mass coordinate where the binding energy reaches \( 10^{48} \) ergs. This allows for the possibility of mass loss during the final core-collapse from either a weak explosion (Ott et al. 2018; Kuroda et al. 2018), ejection of a fraction of the envelope (e.g., Lovegrove & Woosley 2013), or energy loss to neutrinos. This typically gives estimated BH masses within a few \( 0.01 M_\odot \) of the total baryonic mass slower than the escape velocity at the onset of core collapse.
8.3 Results

8.3.1 Impact on the BH masses

Figure 8.1 shows the BH masses resulting from our numerical experiment. Dots show models from our “classical MLT” grid, where increases in $v_c$ are limited following Wood (1974) and the decreases in $v_c$ are unlimited, while crosses mark the BH masses for models in our “time dependent deceleration” grid. The two inset panels emphasize the main differences found, which could affect both the the BH mass function and their detection rate in GW events.

The colors in Fig. 8.1 emphasize in blue the range of $M_{\text{He}}$ that collapse without any PPI-driven mass ejection (CC) and in green the PPI range, which we define here requiring that PPI remove at least\(^2\) 1 $M_\odot$. The yellow region shows models fully disrupted in a PISN. The boundary mass between PPI+CC behavior and full disruption only shifts by $\sim 2 M_\odot$ between our two grids. This is smaller than variations induced by other uncertainties (e.g., nuclear reaction rates, metallicity, Farmer et al. submitted).

The inset (a) of Fig. 8.1 magnifies the range at which PPI starts, around $M_{\text{He}} \approx 32 M_\odot$. This mass threshold for the occurrence of thermonuclear explosions driven by the pair instability is in very good agreement with Woosley (2017, 2019). The models from our “time dependent convective deceleration” grid show, in this mass range, a one-to-one linear correspondence between $M_{\text{He}}$ and the BH mass. The occurrence of weak pulses does not drive significant mass loss, blurring the boundary between CC and PPI+CC evolution. Instead, the approach used in our “classical MLT” grid produces stronger pulses at the low mass end, resulting in a turn-over in $M_{\text{BH}} \equiv M_{\text{BH}}(M_{\text{He}})$. Since lower mass He cores are expected to be more common, if the pulses of the least massive stars experiencing PPI can remove a significant amount of mass, then it might be possible to detect an overabundance of BHs of mass corresponding roughly to the minimum $M_{\text{He}}$ for PPI.

The different amount of PPI mass loss for $M_{\text{He}} \lesssim 45 M_\odot$ results in a systematic offset in the final BH masses of $\sim 5 M_\odot$, shown in the inset (b) of Fig. 8.1, and highlighted by the gray background in both inset panels. Models in the “time dependent convective deceleration” grid generally produce more massive BHs, i.e., weaker pulses. This offset might affect the mass-dependent binary BH merger rate by changing which stars make BHs of a given mass. At $M_{\text{He}} \approx 45 M_\odot$, this grid shows hints of a turn-over qualitatively similar to the one at 32 $M_\odot$ for our “classical MLT” grid, cf. inset (a). This feature might produce a concentration of BHs at the corresponding mass $M_{\text{BH}} \approx 43 M_\odot$.

The PISN BH mass gap (the first part of which is shown by the hatched region in Fig. 8.1) starts above $M_{\text{BH}} \approx 48 M_\odot$ for both our grids, also in agreement with Woosley (2017, 2019). The different treatment of time-dependent convection in our two grids does not change the maximum BH mass below the PISN BH mass gap significantly (red dashed line in Fig. 8.1), corroborating the results of Farmer et al. (submitted). For $M_{\text{He}} \gtrsim 45 M_\odot$ the scatter in BH

\(^2\)Since we are concerned here with the features of the BH mass distribution, rather than all the potential observable signatures of a PPI, see also Renzo et al. (in prep.).
Sensitivity of the PISN BH mass gap to time dependent convection

Fig. 8.1: BH mass as a function of the He core mass for our two grids. The color shading indicates the approximate boundaries between evolution to core-collapse (CC, blue), pulsational pair-instability mass loss $\gtrsim 1 M_\odot$ or radial expansion beyond $R \gtrsim 2500 R_\odot$ (PPI+CC, green), and full disruption in a PISN (yellow). The gray area in the inset panels shows the systematic offset we find in the final BH masses from our two grids. The dashed red line indicates the maximum BH mass we find below the PISN BH mass gap (hatched area), which is not sensitive to the variations between our two grids of models.

masses increases, owing to the combination of more energetic pulses and the lack of wind mass loss in both our grids. The lack of winds produces structures with sharp density drops: these influence the propagation of shocks in the star and the amount of mass they remove. From a computational perspective they result in numerically less stable models. Wind mass loss (indirectly) and multi-dimensional effects are likely to smooth these boundaries in nature.

8.3.2 Illustrative examples

To illustrate the different internal evolution of the stars in our grids, we focus here on two pairs of models of $35 M_\odot$ and $54 M_\odot$. The former pair is representative of models in the inset panels of Fig. 8.1, while the latter pair is representative for the models producing the most massive BH below the PISN gap.

Figure 8.2 shows the specific entropy as a function of mass coordinate for these models in the conventional units of Boltzmann’s constant $k_B$ times Avogadro’s number $N_A$. The specific entropy characterizes the thermodynamic state of the gas, and it is therefore useful when discussing thermal instabilities such as convection. Flat entropy profiles are a signature of efficient convection.

The left panels in Fig. 8.2 show that the entropy profiles differ even at the onset of the
first pulse, which we define as the moment when we turn the HLLC solver on. The $35 M_\odot$ model from our “time dependent deceleration” grid (blue) shows a large convective shell (from $M \approx 16 M_\odot$ to $M \approx 32 M_\odot$) where helium, carbon, and neon are consumed. We find no clear trends in what determines whether the burning shell penetrates downwards (dashed blue, “time dependent deceleration”) or not (solid red, “classical MLT”). It is likely that this behavior depends on the sharp density drops due to the lack of wind. We have also found that turning off convective undershooting in the “time dependent deceleration” model produces a pre-pulse entropy profile which is closer to the red curve on the top left panel of Fig. 8.2, and ultimately results in BH masses in between the values shown in Fig. 8.1.

While the occurrence of off-center convective shells appears not to be robust in our two grids, the impact on the entropy profile shows a systematic trend with mass. At the lower mass end, the occurrence of such a mixing episode effectively decreases the amount of mass at low specific entropy, and results in weaker pulses later on. These weak pulses do not eject any significant amount of mass resulting in the systematic offset shown in inset (b) of Fig. 8.1.

Conversely, the right column of Fig. 8.2 shows why the qualitative behavior of both grids is similar for large $M_{\text{He}} \gtrsim 45 M_\odot$ in Fig. 8.1, and why the resulting BH masses are roughly the same: we find only minor differences in the entropy profiles for the two $54 M_\odot$ models at the beginning of the hydrodynamic phase of evolution (top right). Such differences are smaller than the ones introduced by other physical uncertainties (e.g., nuclear reaction rates and overshooting, Farmer et al. submitted). The bottom right panel shows that, even after going through PPI, the entropy profiles at the onset of CC are very similar. Only in the outermost layers are there some differences, owing to a pulse wave still propagating outwards when the iron core becomes unstable.

Figure 8.3 shows the convective behavior during a pulse for the two $M_{\text{He}} = 54 M_\odot$ models in the right column of Fig. 8.2. These illustrate the differences in convective patterns between our two model grids, and are representative also for the convective behavior of lower mass models. The top axis of each panel gives the approximate time and duration of each pulse. The beginning time of the pulses differ by $\sim 3000$ years. The time range shown is larger
Fig. 8.3: Kippenhahn diagrams for the 54 $M_\odot$ helium cores in the right column of Fig. 8.2 around the first pulse. The left panel shows the evolution for our “classical MLT” approach, while the right panel shows the corresponding evolution for the “time dependent deceleration”. The red colors indicate net energy generation, the purple colors indicate net energy loss via neutrinos. The green hatching indicates convective layers. The solid black line indicates the total mass of the models. A fraction of it becomes unbound earlier than it is removed from the computational domain.

by a factor of $\sim 30$ in the model from the “time dependent deceleration” (right panel). This is because we can run with the hydrodynamics on for much longer thanks to the improved numerical stability obtained when solving Eq. 8.1.

The oxygen thermonuclear explosions (roughly between model number $\sim 4000 - 6000$) proceeds very differently in the two models. In the “time dependent deceleration” model (right), the oxygen ignition triggers convective mixing (green hatched areas) outwards of mass coordinate $\sim 10 M_\odot$. Convection remains in the intermediate layers of the star until and beyond the ejection of mass. We think that the presence of stronger convection during the pulse in the “time dependent deceleration” models results in the weaker pulses at the low mass end: convection carries out energy, so preventing it becoming kinetic energy of the stellar gas.

Conversely, the “classical MLT” model (left panel of Fig. 8.3) burns oxygen in a radiative layer, and convection turns on only at model number $\sim 7000$, after the main burning event is over. The absence of convection during the thermonuclear explosions in the “classical MLT” lower mass models might result in stronger pulses with more significant mass loss.

During the outward propagation of the pulse, both models show large convective regions, which are more extended in the “time dependent deceleration” model (right). This leads to the injection of helium into the hotter and deeper regions, and consequently to a large increase in nuclear energy generation rate within the convective region. However the evolutionary timescale is set by the dynamical propagation of the pulse, and it is much shorter than the
nuclear timescale. Therefore, while this burning changes the chemical profile inside the star, it does not release an amount of energy sufficient to modify significantly the dynamics of the pulse propagation. Indeed, the amount of mass lost by both these models in the first pulse is comparable, at about $4 \, M_\odot$.

### 8.4 Summary & Discussion

The ongoing search for GWs is starting to provide direct constraints on the BH mass function and probe the theoretically predicted PISN BH mass gap (LVC 2018a; Fishbach & Holz 2017; Stevenson et al. 2019). This offers an unprecedented tool to understand the physics of their massive star progenitors. This requires quantitative predictions from stellar models robust enough for a sensible confrontation with the data. The variations in the model predictions resulting from algorithmic choices or simplifying assumptions should be small compared to the input physics that we wish to test and the observational uncertainties.

We have compared the predictions for the final BH masses at the lower edge of the predicted mass gap computed with two different versions and setups of the stellar evolutionary code MESA, which differ primarily in the treatment of convection. Our “classical MLT” grid adopts the defaults of Paxton et al. (2018), while in our “time dependent deceleration” grid we solve an additional equation incorporating the timescale for the damping of convection. Different groups have recently used setups very similar to the two options we compare here (M18, Leung et al. 2019, Farmer et al. submitted).

We find systematic differences when comparing individual models for the same initial mass. The final BH masses computed with our time-dependent treatment of convective deceleration are lower by up to $\sim 5 \, M_\odot$ than those in our grid computed adopting the classical mixing length theory. After inspection of the evolution of the internal structure, this seems to be a consequence of stronger convection during the propagation of a pulse. The convection carries out part of the energy, preventing it from being converted into bulk kinetic energy of ejecta. This results in weaker pulses and a lower amount of mass ejected. The differences are largest for models near the lower end of the mass range for pair pulsations to occur ($32 \, M_\odot \lesssim M_{\text{He}} \lesssim 45 \, M_\odot$), but are less important for the higher mass range ($45 \, M_\odot \lesssim M_{\text{He}} \lesssim 64 \, M_\odot$). Because of this, we find that the predicted maximum BH mass for BHs below the gap is robust at $\sim 48 \, M_\odot$.

We expect that the improvements in the treatment of convection are the main reason for the differences, but we caution that we cannot exclude that other minor differences contribute. We also stress that our calculations do not account for possible consequences of binary interactions.

For now, the robustness of the prediction for the location of the edge of the gap is encouraging. Even the variations we find for individual models are smaller than the typical uncertainties on the individual BH masses inferred from GW detections.

For the future, our results should be taken as a warning. The variations we find between
individual models are substantial. The constraints from GW events will become increasingly precise with more detections. Moreover, we can anticipate an increasing number of events detected with high signal-to-noise ratio and thus more accurately-determined parameters for the individual BHs. This will increase the robustness needed from the stellar model predictions.

The treatment of convection will likely remain a multifaceted challenge, of which the time-dependence is only one aspect, complimentary to other well known issues, but there are several ways forward. Multi-dimensional hydrodynamic simulations applicable to the stellar regime can be used to derive a more realistic expressions that can be included in stellar evolutionary codes (e.g., Meakin & Arnett 2007b; Couch & Ott 2013; Couch & O’Connor 2014; Arnett et al. 2018a,b; Yoshida et al. 2019). As a first step, a physically-motivated expression for the convective acceleration could be derived from the flow observed in multi-dimensional hydrodynamic simulations, instead of the ad hoc parametrizations presently used.

The increasing number gravitational-wave events detected will provide a major motivation for further improving the progenitor models. The anticipated capabilities of third-generation detectors are particularly promising. These should be able to detect massive binary BHs across all redshifts where significant star formation occurred in the Universe. They would enable us to probe the evolution of the BH mass distribution as a function of redshift and uncover possible detailed features in the shape of the mass distribution, which bears the imprints of the physical processes that govern the lives of their massive star progenitors.

MR, SJ, and SdM acknowledge funding by the European Union’s Horizon 2020 research and innovation programme from the European Research Council (ERC), Grant agreement No. 715063, and by the Netherlands Organisation for Scientific Research (NWO) as part of the Vidi research program BinWaves with project number 639.042.728. RF is supported by the Netherlands Organisation for Scientific Research (NWO) through a top module 2 grant with project number 614.001.501 (PI de Mink). Simulations were carried out on the Dutch national e-infrastructure (Cartesius, project number 16343) with the support of the SURF Cooperative.