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DEVELOPMENTS in QCD

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I provide a cursory review of recent developments in perturbative QCD, for both calculational methods and results.

1 Introduction

To appreciate the impressive progress made in enabling perturbative QCD for collider physics, let us, glancingly, compare the proceedings of DIS 1999 (Zeuthen) and the present contribution. Apart from many other interesting topics such as small-x physics and diffraction, the focus in fixed order perturbation theory was on NLO calculations for $2 \rightarrow 2$ processes, together with ever more accurate NLO PDF sets and splitting functions. Two processes were known to NNLO (DIS and DY). Higher-multiplicity LO matrix-element calculations, based on Berends-Giele recursion relations, were available for a few processes. The general purpose parton-shower Monte Carlo programs ARIADNE, ISAJET, HERWIG, PYTHIA were being updated and extended, albeit in F77, to include processes from HERA and other colliders. As this review will show, remarkable progress has been made in all these topics, as well as in connecting them. This holds true both for conceptual development and range of application. The challenges posed by HERA and Tevatron analyses and the promise of the LHC have provided a challenge to which theorists that provide (tools for) QCD predictions have indeed stepped up. An added bonus is that compared to 1999, sociologically, it is now cool to be a phenomenologist.

Impressive progress has also been made in developing and applying the framework of k_T factorization, and in the area of small-x physics. Lack of expertise, and space, compel me to skip these issues in what follows.

2 Early-LHC QCD

The LHC will provide proton collisions at an energy 7 times that of the Tevatron. Parton collisions will inherit much of this energy increase, and produce final states with high multiplicity. A huge fraction of these events will be pure QCD reactions, thus to find signals for Higgs, top, etc production will require, besides excellent understanding of the detectors, accurate predictions for signals and backgrounds in order to separate them.

The first mission should be to rediscover the Standard Model. For this, a few reactions will be important early on. One will be top quark production, with a quite generous event rate of about 50 events/day even at low luminosity. With its rich decay modes involving leptons, neutrinos, jets with and without b-tags, the reaction will be very helpful to calibrate the detectors. Event samples in the lepton+jets channel, after b-tagging, will be quite pure.

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Order	$2\to 1$	$2\to 2$	$2\to 3$	$2\to 4$	$2\to 5$	$2\to 6$
1	LO					
α_s	NLO	LO				
α_s^2	NNLO	NLO	LO			
α_s^3	NNNLO	NNLO	NLO	LO		
α_s^4					LO	
α_s^5						LO

Table 1: Status of higher order calculations.

By first reconstructing the W-mass a good calibration of the jet-energy scale is possible. Because a top sample can even be assembled without b-tagging, it is possible to study b-tagging. Then, a first measurement of the inclusive top quark cross section will give a top mass determination. More differential cross sections can help understand how the detectors perform in different regions of phase space.

Another important process will the inclusive production of vector bosons. This process is important for determining the parton distributions functions from LHC data, and later on for determining the parton luminosity [1]. To this end precise knowledge of parton distribution functions, and higher orders in the partonic cross section will be very important.

Higher order corrections are also important for calculating the background to the early-LHC Higgs-discovery mode $pp \to H \to W^*W^* \to l\bar{\nu}\bar{l}'\nu'$, for the Higgs mass range between 140 and 180 GeV. The presence of the two neutrinos prevents the construction of a mass peak and thereby side-band subtraction of backgrounds. The higher order corrections to the various subprocesses for the background process $pp \to WW$ [2, 3, 4, 5] are now bringing this under control.

3 Higher-order cross sections

The framework for calculating higher order corrections is the factorization theorem (1), which expresses the multi-differential hadronic cross section for $2 \to n$ scattering as a weighted sum of multi-differential partonic cross sections, the sum including parton flavors a, b and momentum fractions x_i, and the weights made of parton distribution functions f_a (PDF's), which must be determined in other reactions calculated to the same order. The factorized structure holds up to power corrections, as indicated.

$$\frac{d\sigma^{pp\to X}}{d^3p_1\dots d^3p_n} = \sum_{a,b}\int dx_1 dx_2 f_a(x_1,\mu_F) f_b(x_2,\mu_F) \\ \times \hat{\sigma}_{ab}(p_a+p_b \to p_X, \alpha_s(\mu_R), \mu_R, \mu_F) + \mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right) . \quad (1)$$

Dominant uncertainties arise from varying the renormalization and factorization scale (μ_R,μ_F) and PDF uncertainties. The degree of difficulty in computing $\hat{\sigma}_{ab}$ increases rapidity with n, and/or with the perturbative order. The present status is summarized in Table 1. Let us now discuss these entries in more detail.

3.1 LO

Here the difficulty lies not in handling divergences, but in handling complexity. Consider gluon production $gg \to ng$. The number of diagrams grows from 4 for $n = 2$ to 10525900 for $n = 8$. Though daunting, such cases are now routinely handled, and made accessible to users via matrix element event generators such as MadGraph/MadEvent [6] (using helicity amplitudes), Sherpa/Amegic++ [7, 8] and Helac/Phegas [9] and Alpgen [10] (using recursion methods) and Comphep (using matrix elements) [11].

Key to taming the number of diagrams to calculate is to manage the external quantum numbers. An important first step is to separate the color structure from the rest, by expanding the amplitude into color-ordered subamplitudes

$$\mathcal{A}_n(1,\ldots,n) = \sum_\sigma \mathrm{Tr}(T^{a_{\sigma(1)}}\ldots T^{a_{\sigma(n)}}) A_n(\sigma(1),\ldots,\sigma(n))\,. \tag{2}$$

For $n = 6$ that reduces the number of diagrams from 34300 to 501. Next, one should specify the external helicities, $A_n(1^\pm,\ldots,n^\pm)$. This enables the use of efficient spinor techniques, with notation $u_+(p) \equiv |p+\rangle$, $\bar{u}_-(p) \equiv \langle p-|$, $\langle pq\rangle \equiv \langle p-|q+\rangle$, and $[pq] \equiv \langle p+|q-\rangle$. In these terms, a remarkable result was reached over 20 years ago [12]. Color-ordered helicity amplitudes are zero with all helicities identical, or only one different. For two different helicities, maximally helicity violating (MHV), the result is strikingly elegant

$$A_n(j^-,k^-,\text{rest plus}) = i\frac{\langle jk\rangle^4}{\langle 12\rangle\cdots\langle n1\rangle} \tag{3}$$

What about amplitudes with more than two minuses? These are, and have been for a while, numerically accessible through recursion relations among subamplitudes with one off-shell external particle [13]. These recursions are the heart of the VECBOS matrix element Monte Carlo [14], which played a key role in the search for the top quark.

In 2003 a remarkable proposal was made [15] to use these, rather than the interactions from the QCD lagrangian, together with propagators as building blocks for general tree-level amplitudes. Their combination is dictated by recursion relations [16, 17], which have the noteworthy property that the component amplitudes have all external momenta on-shell. These must be complex-valued, but that is in fact a feature rather than a bug. This reduced the number of diagrams for the six gluon amplitudes with 3 plusses and 3 minusses from 220 to 2, yielding a compact analytical expression.

Speed comparisons [18, 19] between the off-shell BG recursions and the on-shell (BCF) recursions reveal however that for larger n the BG relations still outperform the newer ones.

3.2 NLO

Before discussing recent developments in higher orders for certain amplitudes in more detail, let us review some universal aspects. The benefits of going beyond LO are well-known. Production rates are more accurately predicted due to reduced scale dependence (particularly important for "standard candle" processes), distributions are more realistically modelled because of the extra parton, the size of the corrections is a self-diagnosis of the perturbative approach, and new channels can open up beyond LO that are not necessarily small.

A LO calculation consists of those diagrams that contain the desired final state with a mininum of extra particles. NLO diagrams feature virtual corrections to those, as well as having one extra radiated parton, over (part of) whose phase space one integrates over. This continues, mutatis mutandis at NNLO and higher, see Fig. 1. Beyond LO, loop integrations and phase space integrations produce, besides finite terms, regularized infinities due to integration over UV or IR momenta in the loop or from emission. The former are straightforwardly handled using textbook renormalization methods. The latter in fact cancel between virtual and real graphs, not just for the inclusive cross section, but even when the momentum of the extra emitted parton is available. How to do this is not entirely straightforward, nor unique, and a number of such "subtraction schemes" have in recent times been fully developed. The generic form for a fully differential NLO cross section for n-particle production is

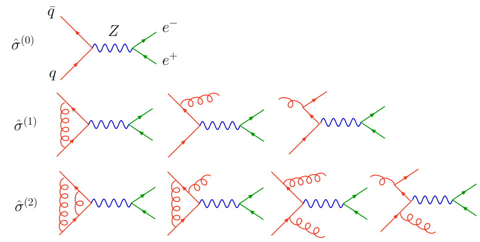

Figure 1: Representative diagrams for higher order contributions to Z production. LO only includes $\sigma^{(0)}$, NLO also $\sigma^{(1)}$, and NNLO also $\sigma^{(2)}$.

$$d\sigma_{NLO} = \int_{d\Phi_{n+1}} (d\sigma^R - d\sigma^S) + \Big[\int_{d\Phi_n} d\sigma^V + \int_{d\Phi_n} \Big(\int_{d\Phi_1} d\sigma^S \Big) \Big] . \qquad (4)$$

The "subtraction term" $d\sigma^S$ must (i) behave exactly like the bremstrahlung cross section in soft and collinear regions, thereby preventing the corresponding divergences, and (ii) must, when integrated over the extra parton phase space, cancel the divergences in the virtual calculations. Schemes vary by their methods for systematic construction of subtraction terms. The most common schemes in use are dipole subtraction [20], antenna subtraction [21], and FKS [22].

At DIS1999 these techniques were just coming in use, and were applied to $2 \to 2$ processes. Any worry that the added complexity of going to $2 \to 3$ processes would slow progress has been roundly confounded by the rapid, substantial progress in difficult calculations in recent years. A flood of new ideas has been brought to bear on dealing with such technical challenges, supported by intensive algebraic and numerical use of the computer. There are now many $2 \to 3$ NLO results, and the first $2 \to 4$ processes might be in reach. At the Les Houches 2005 workshop a wishlist for NLO differential cross section was composed, a reduced version of which is shown in Table 2 together with relevance and status. Clearly, in a short amount of time a lot has been achieved. A selection of $2 \to 3$ processes becoming available at NLO in recent times is $pp \to t\bar{t} + jet$ [23], $pp \to W + 2\ jets$ of which one has a b-tag [24], $pp \to Hb\bar{b}$ [25, 26], $pp \to H + 2\ jets$ via gluon fusion [27], $pp \to H + 2, 3\ jets$ via vector boson fusion [28, 29] $pp \to VVV$ [30, 31], etc. Some of these are available within the MCFM package [32]. Moreover, with the six-parton amplitudes becoming available at one-loop, the first NLO $2 \to 4$ predictions seem not too far off.

One-loop ideas

Process	Background to/relevant for	Status
$pp \to VV + \text{jet}$	$t\bar{t}H$, new physics	$W^+W^- + \text{jet}$
$pp \to H + 2\,\text{jets}$	H production via VBF	
$pp \to t\bar{t}b\bar{b}$	$t\bar{t}H$, new physics	
$pp \to t\bar{t} + 2\,\text{jets}$	$t\bar{t}H$, new physics	$t\bar{t} + \text{jet}$ ('07)
$pp \to VVb\bar{b}$	$VBF \to H \to VV$, $t\bar{t}H$, new physics	
$pp \to VV + 2\,\text{jets}$	$VBF \to H \to VV$	
$pp \to V + 3\,\text{jets}$	new physics	
$pp \to VVV$	new physics	ZZZ (07), WWZ ('07), VVV ('08)

Table 2: 2005 Les Houches wishlist of NLO calculations, what they are relevant for, and the present status.

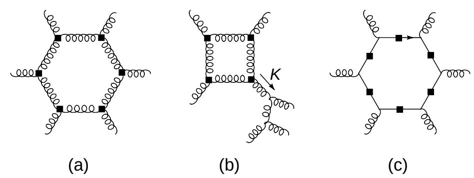

Figure 2: Color-ordered 6-point diagrams, having a ring-form with gluon propagators with on-shell (a), or non-ring form with off-shell external lines (b), or with quark propagators (c).

I will here briefly touch upon a selection of recent ideas that have spurred the NLO renaissance. A more extensive and clear review is Ref. [33]. One-loop diagrams become hard to calculate when the number of external lines is large, say 5,6 etc. Because of the arguments in Sect. 3.1 one can restrict oneself to a smaller set of diagrams having a particular color order. Generically, these take the form indicated in Fig. 2. After contracting all external lines with polarization vectors, the result will take the form of a numerator containing dot products among external momenta, polarization vectors, and the loop momentum. Such tensor integrals can be reduced to scalar integrals in a well-defined procedure that expresses external vectors in terms of a basis set of four. In this procedure denominators are cancelled, reducing the n-point function to lower-point ones. This leads to an expansion of the amplitude in terms of scalar functions down from n-point ones. Furthermore, up to (here irrelevant) $\mathcal{O}(\epsilon)$ terms, 5- and higher point functions can be expressed in terms of four-point functions and lower [34, 35, 36]. The price to pay is that for these lower point functions the external momenta are not subsets but rather combinations of the original, massless external momentum. These combinations then are not massless. Thus, we have

$$A_n^{\mathrm{one-loop}} = \sum_{j \in B} c_j \mathcal{I}_j \tag{5}$$

where the basis set B consists of a certain set of box-, triangle and bubble integrals with or without massive external legs [37], and the c_j are rational functions of dot products among external momenta and polarization vectors, see Fig. 3. With a generic representation (5) in hand, the task of calculating $A_n^{\mathrm{one-loop}}$ can therefore be cast as the task of find the c_j.

To this end, unitarity methods [38] may be used. In Eq. (5) the right hand side has branchcuts in the invariants on which the logarithms and dilogarithms in the $\mathcal{I}_j$ depend. One can also examine a particular discontinuity across a branch cut for a particular invariant, or channel, for the left-hand side in Eq. (5), which is done by cutting the amplitude and replacing cut propagators by delta functions.

$$\frac{1}{p^2 + i\epsilon} \rightarrow -i2\pi\,\delta(p^2) \qquad (6)$$

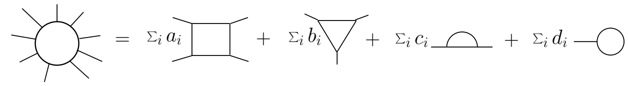

Figure 3: Expansion of n-leg one-loop amplitude in sum of tadpoles, bubbles, triangles and boxes.

This amounts to taking the imaginary part. From the comparison the coefficients c_j can in principle be determined. Essentially, one thus determines the function $A_n^{\text{one-loop}}$ from its poles and cuts. However, using four-dimensional momenta in the cuts, this leaves an ambiguity in the form of a rational function (using a $D = 4 - 2\epsilon$ version of the unitary method [39, 40] avoids this, but so far is somewhat more cumbersome to use). A number of methods have been devised to fix this ambiguity, such as using recursion relations [16, 17], or using D-dimensional unitarity [41, 42]. Particularly fruitful is the use of complex kinematics, which allows non-vanishing, non-trivial three-point amplitudes. This allows taking multiple cuts of a box integral, Fig. 4, which goes under the name "generalized unitarity". By so doing, one may determine the coefficients c_j purely algebraically [43], since the four delta-functions fix the loop momentum.

An effective way of solving Eq. (5) was proposed in Ref.[44]. Writing the equivalent of Eq. (5) at the integrand level, the coefficients of the box etc integral can be extracted by choosing different values of the loop momentum, and perform the inversion numerically. The method has been applied e.g. in [31].

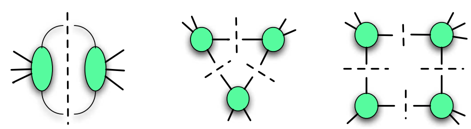

Figure 4: Generalized unitarity

New stable and efficient reduction techniques for tensor integrals have been proposed in Refs.[45, 46], and have found much use.

Furthermore, numerical [47, 48] and semi-numerical [49] techniques for loop integrals have progressed to the level where much work taken care of for the user through programs like Blackhat [50], Cuttools [51], or Rocket [52].

As this snaphot of a fast-developing field already makes clear, the area of NLO calculations has become a very lively marketplace of ideas and methods.

3.3 NNLO

NLO calculations may not always be sufficient. When uncertainties are still large, when the NLO corrections large, or when very precise parameter values need to be subtracted from data, a NNLO calculation may be called for. This is easier called for than done. Before sketching some technical points, let us recall some results thus far. For hadron colliders we have inclusive vector boson and Higgs boson cross sections [53, 54, 55, 56, 57] as well as differential distributions [58, 59, 60], and last but not least, the NNLO (3-loop) space-like splitting functions [61, 62]. Quite recently, a number of e^+e^- event shapes have been determined to NNLO [63]. This is very impressive feat, requiring appropiate extension of the antenna subtraction method. The result has very interesting features, shown for strong coupling extraction [64] in Fig. 5. One observes reduced scale uncertainty, better consistency and a lower central value.

Methods for calculating two-loop amplitudes, a key part of any NNLO calculation, have likewise seen fast-paced developments. The difficulty lies with performing the Feynman parameter integrals. Mellin-Barnes techniques, which, being in a sense an inverse Feynman parameter trick,

$$\frac{1}{(A+B)^{\nu}} = \frac{1}{\Gamma(\nu)}\frac{1}{2\pi i}\int_C dz \frac{A^z}{B^{\nu+z}}\Gamma(-z)\Gamma(\nu+z)$$

factorize the loop integrals into easier ones at the expense of multiple contour integrals [65, 66, 67], are now being automatized [68]. In sector decomposition [69, 70] the parameter space is deterministically divided into sectors containing only one (infrared or collinear) singularity. This can be automatized [71]. It yields analytic results for poles, numerical ones for residues. Another method, used with great success for (NNNLO) DIS structure functions and operator matrix elements, involves moments with respect to the scaling variable. In this language, efficient recursions, involving difference equations in the moment variable N, can be set up [72], that are solved using FORM [73]. The method is being extended for heavy quarks [74].

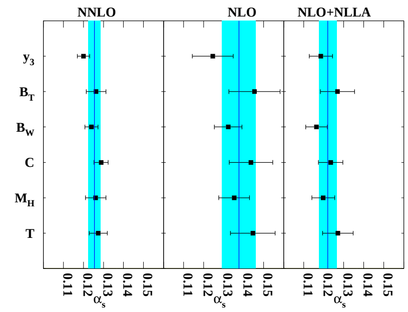

Figure 5: α_S extracted from different event shapes and approximations.

3.4 Resummation

In this brief discussion of resummation I will be restrict myself mostly to near-threshold kinematical situations. There, many cross sections take the generic form [75, 76]

$$\ln\sigma = Lg_1(\alpha L) + g_2(\alpha L) + \alpha g_3(\alpha L) + \dots\,, \tag{7}$$

where α is a coupling constant and L some large logarithm. The functions g_i are calculable via low order calculations, and their index marks the accuracy of the resummation (g_1 for LL, g_2 for NLL etc). This exponential form is very predictive (e.g. all $\alpha^n L^{2n}$ are predicted by the first order αL^2 term). There are various ways to derive it, using either evolution equations [75, 76, 77, 78, 79], or the more algebraic non-abelian exponentiation theorems [80, 81, 82], or using the language of soft-collinear effective theory [83]. For the inclusive electroweak boson production, the moments of the cross section take the form

$$\hat{\sigma_i}(N) = C(\alpha_s)\,\exp\Big[\int_0^1 dz\frac{z^{N-1}-1}{1-z}\Big\{2\int_{\mu_F^2}^{(1-z)^2Q^2}\frac{d\mu^2}{\mu^2}A_i(\alpha_s(\mu^2)) + D_i(\alpha_s(1-z)Q^2)\Big\}\Big] \tag{8}$$

a very elegant summary of the logarithms in its perturbative QCD expansion in terms of functions that depend on α_s only. The function C matches the resummed to the fixed-order form. Similar expressions exist for more differential cross sections. The form may be used, upon an inverse moment transform, for numerical evaluation of the resummed cross section (after a suitable way to deal with the renormalon singularity in Eq. (8)), or, upon expanding in α_s, as a way to generate terms containing high powers of large logarithms in higher orders. Both the A and D function are now known to third order in QCD [61, 84, 85, 79] for all electroweak boson production partonic subprocesses. At this point, the best-determined cross section, theoretically, is the Higgs cross section in gluon fusion, which has been resummed to NNLL accuracy, and matched to a NNLO calculation [86]. Extensions to more differential cross sections involve more invariants, and more color structures, and are more difficult. An very interesting observation [87] was made that for the equivalent of the D function in $2 \to n$ processes, the two-loop contribution is simply proportional to the one-loop one, suggesting further insights to be gained.

4 Jets

I will select only one topic in the much larger field of jet research. It repairs a long-standing problem in comparing data and theory for jets constructed with iterative cone algorithms. These algorithms collect all radiation (hadrons) in a cone of radius R in the pseudo-rapidity (η) and azimuthal angle (ϕ) plane. Jets are ubiquitous in hadron colliders, and their importance can hardly be overestimated. However the jets are defined, they must be stable against collinear splittings of massless particles, and against the inclusion or not of very soft particles. Most current, practical implementations of the iterative cone algorithm, unfortunately, fail this infrared safety test, so that comparison with higher order theory becomes essentially meaningless. Cone algorithms search for stable cones pointing in the same directions as the total momentum of the partons inside, and come with splitting and merging procedures to handle overlapping cones. Usually, to save considerable time, the algorithms use "seeds", i.e. trial cones based on particles in the event. This is known to be infrared unsafe. The solution was known: don't use seeds, but sum over all possible directions (e.g. all calorimeter cells). This was however computationally impractical as it depended via $N2^N$ on the number of particles N. Recently [88] a practical seedless cone algorithms SIScone, which behaves as $N^2 \ln N$ has been devised. This makes jet cross sections using this algorithm legitimate to compare with the corresponding higher-order theoretical calculations, and constitutes important progress.

Another interesting suggestion in the field involves the definition of "jet areas" [89] that can help decontaminate measured jets from effects of pile-up and underlying events.

5 Monte Carlo

This area has seen enormous advances in recent years, in almost all its facettes. The latest incarnation of multipurpose parton-shower event generators as HERWIG++, PYTHIA 8 and SHERPA are all written in C++. Matrix element based generators such as Comphep, Helac, Alpgen, MadEvent have been extended in the number of final state partons they can handle and in the number of processes included. Models for underlying events and multiple interactions have been improved [90, 91].

Very important progress has been in made in matching procedures: NLO to parton shower-based Monte Carlo (MC@NLO [92] and POWHEG [93]). Matching is essentially an issue of avoiding double counting in the one-emission contribution, which can either come from NLO or from the PS, and in the virtual parts, between the virtual NLO part and the Sudakov form factors. MC@NLO matches, in practice, to HERWIG angular-ordered showers. A small percentage of the events it generates have a negative weight, reflecting virtual contributions and subtractions present in NLO and matching. POWHEG insists on having positive weights, and exponentiates the complete first order real matrix element to that end. Both these frameworks are growing in the list of processes, and realism (e.g. spin correlations [94]). Agreement is generally very good, see Fig. 6, also with PS-matched matrix-element generators [95], although interesting differences exist. Such differences reflect genuine ambiguities. Much has been gained in matching matrix element-based approaches to parton showers. The former should do well in final states with partons well-separated in angles, while at near-collinear angles the parton shower description should do better. Two matching procedures have cornered the market. CKKW [96] use k_T clustering to separate phase space into two regions in each of which one of the descriptions should hold. To match properly, the matrix elements are reweighted by Sudakov form factors and α_s factors at the scales correspond to the nodal branchings. On the PS side, the showers are vetoed to ensure that only emissions below the matching scale are included. MLM [97] also reweights the matrix elements, then showers them, but discards events where the shower generates emission harder than the matching scale. Both procedures have been implemented in a number of matrix-element event generators, and extensively compared [98]. Many theorists have recently entered the Monte Carlo field, to varying degrees, offering many new ideas and proposals (and code!), e.g. in novel NLO matching procedures (VINCIA [99], GenEvA [100]).

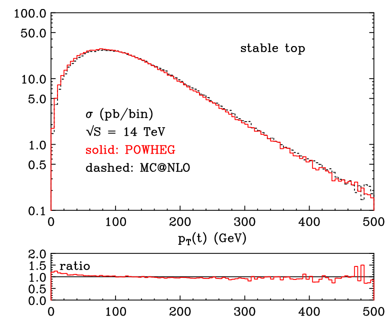

Figure 6: LHC transverse momentum distribution of top quarks according to MC@NLO and POWHEG.

6 Top Physics

The methods in section 3.3 have brought into view [101] the possibility of calculating the inclusive NNLO top quark cross section exactly. The logarithmic "edges" of the results having been chipped away using resummation insights [102, 103], updated predictions of this cross section, including a careful assessment of uncertainties, have appeared [104, 103, 105]. Another very important result has been the calculation of the NLO $t\bar{t}+$ `jet` process [23], which was performed (twice, independently) using more or less every tool available in the kit. This process helps unravel QCD production dynamics for top quarks, and is important in the search for new physics.

Top pair production has been been implemented in both MC@NLO and POWHEG.

6.1 Single top

Among recent inclusions into the MC@NLO framework has been the single-top process. Single tops are produced by the weak interaction, and are customarily categorized (Fig. 7) using Born kinematics.

A particularly interesting aspect of single-top production is the prospect of directly measuring V_{tb} and testing the chiral structure of the associ-

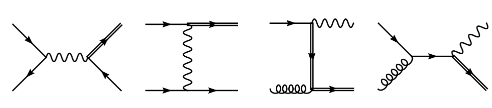

Figure 7: From left to right the s-channel, t-channel processes, and the Wt associated production channel.

ated vertex. It is is sensitive to new physics, different per channel. Thus, the s-channel will be sensitive to e.g. W' resonances, the t-channel to FCNC's. Experimentally, this process turns out to be very difficult to extract from backgrounds, and so far only evidence (albeit strong) has been found by the D0 and CDF collaborations. The NLO calculation [106, 107, 108, 109, 110] reveals that inclusive cross sections at the Tevatron are rather small, 0.9 (s) and 2 (t) pb, with the Wt channel negligible. At the LHC the numbers are, approximately, 10, 246 and 60 pb, respectively.

An interesting issue arises in the Wt mode. Some diagrams occurring at NLO contain an intermediate anti-top that can become resonant. These diagrams can be interpreted as LO $t\bar{t}$ "doubly resonant" production, with subsequent $\bar{t}$ decay. It thus becomes an issue to what extent the Wt and $t\bar{t}$ can be properly defined as individual processes. In Ref. [111] this was extensively addressed in the context of the NLO event generator MC@NLO. By defining two different procedures for subtracting the doubly-resonant contributions it was shown that, with suitable cuts, the interference terms are small, so that Wt and $t\bar{t}$ can be separatedly considered to NLO.

7 Conclusions

Recent years have seen a remarkable growth in quality of perturbative QCD predictions, to an extent that was quite unexpected. Smart methods and powerful computer tools have, before the LHC begins, brought processes until theoretical control that seemed out of reach. The pace continues in fact unabated, so that DIS2009 is likely to see yet many new results from rapid developments in QCD.

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