Solving large structured Markov Decision Problems for perishable inventory management and traffic control

Haijema, R.

Publication date
2008

Chapter 8

Networks of intersections

8.1 Introduction

When intersections are part of a network, departures from one intersection may be arrivals to another intersection. Typically the departure process is far from uniform, as the process is controlled by an on/off server: the traffic lights that changes from green to yellow to red. When all cars leaving a queue proceed in the same direction and travel at constant speed, cars travel in platoons to a down-stream intersection.

Under *coordinated FC* a *network cycle* with cycle length D to all intersections is defined. The length and start of the green periods at the intersections can be set such that the long-run average waiting time is minimal. Therefore it may help to set so-called *green waves*: then cars leaving one intersection do not need to wait at the downstream intersection as its lights change just-in-time into green. In the practice, green waves may be hard to set and are easily lost as platoons get dispersed due to speed differences of cars and as some cars may turn left or right.

Coordinated FC through a network cycle is only possible for a limited number of cycle lengths D. Consequently coordinated FC may yield a higher average waiting time than uncoordinated FC, with locally optimal cycle lengths D_i for each intersection i. In this chapter we discuss how our decomposition approach applies to the control the lights in a network. We test by simulation whether coordinated and uncoordinated FC can be improved by dynamically adjusting traffic lights using information on the queue lengths and (if available) information on estimated arrival times.

8.2 One-step policy improvement for network control

8.2.1 Centralized coordinated control

In the Chapters 6 and 7 several policies are developed, that greatly improve FC by applying a policy improvement algorithm. For the control of network of intersection one may choose as initial policy any FC: coordinated or uncoordinated.

Coordinated FC requires a good network cycle. Therefore one needs to optimize over the cycle length D, the length of the effective green periods $d_{i,s}$ of each combination of flows s, and the start of a cycle for each intersection i, such that the long-run average waiting time per car is minimized. In general, finding an optimal cycle is a complicated task, since it depends, amongst other, on the distance between the intersections, on the (distribution of the) traveling speeds, on the routes selected by cars and on the arrival rate of each flow, as will be demonstrated in Section 8.3.

Once a network cycle is defined, a one-step policy improvement of coordinated FC can be done in a similar way as for an intersection in isolation. For each flow the slots of the network cycle are green, yellow, or red slots. The relative appreciation or relative value of a slot can be determined flow-by-flow by solving the underlying Markov chains. Every decision epoch one may select the best position in the cycle. However, there are two reasons why such an approach may not work well:

1. The network cycle can be interrupted only when at none of the intersections a light shows yellow; there is no choice when a switching phase is executed at one or more intersections, i.e. the lights of one (or more) combination(s) show yellow.

2. For coordinated control the implied decision at each intersection should be in line with historical decisions at its adjacent intersections, since it takes time for departing cars until they arrive at a downstream intersection.

Instead of improving the network cycle we propose a decentralized control of the traffic lights and try to achieve some synchronization by using arrival information.

8.2.2 Decentralized control

Under decentralized control the network is decomposed into its intersections at which the lights are controlled by FC with locally optimal cycle lengths. A locally optimal FC can be approximated by the Optimal-fixed-cycle algorithm in Appendix D by falsely modeling arrivals as Bernoulli experiments, instead of acknowledging that cars may arrive in platoons that are formed at upstream intersections. The resulting cycle lengths are not necessarily the same for all intersections. Although decentralized FC likely performs worse than coordinated FC, it may be a good initial policy for a one-step policy improvement. A motivation to take decentralized FC as the initial policy is that locally optimal FCs can be derived much more easily than a coordinated network FC.

The RV policies of the previous chapters are thus interesting to apply to a network of intersections. Once again, instead of considering the state of the entire network only local information for an intersection is used to adjust the traffic lights. There is no need to specify the offsets, since these are lost under dynamic control by the RV policies. By taking a number of slots of arrival information into account, one may obtain some degree of synchronization. By the nature of dynamic control, one should not expect to observe green waves under the RV policies. But, what we do hope is that the long-run average waiting time per car is much lower than under any of the other strategies, even lower than under coordinated FC.

8.2.3 Some modeling assumptions

In all cases that we will study, we presume that all cars flow through the network: all cars arrive from outside the network and do not have their destination in the network. In addition, we assume that the distance between the intersections is large enough to assume that under the tested policies the intersections will not be blocked by cars waiting at downstream queues.

Further, we assume that at each intersection the traffic flows are grouped such that non-conflicting opposite flows are served together. In the simulation model we do not allow cars to overtake slower traffic that proceeds along the same lane. (These assumption is used when specifying how opposite green waves are set.) We do not consider the optimization of the grouping of flows nor do we optimize the sequence in which the combinations are served. Principle assumptions regarding the test cases and the simulation model are discussed in Section 8.4.

8.2.4 Outline

As FC is an important benchmark to check the results of the new control policies, we discuss ideal and less-ideal circumstances to FC and how a good coordinated FC is set for specific cases in Section 8.3.

By simulating several network scenarios, the performance of the RV policies is compared against that of FC and exhaustive control. In Section 8.4, the test cases as well as a simulation model are presented.

In Section 8.5, we check for the performance of the rules under conditions that vary from utmost-ideal conditions for coordinated FC, to, what we consider, more realistic conditions. Therefore we illustrate the optimization of coordinated FC and demonstrate the impact on the average waiting time of the distance between intersections, the traveling speed of cars and the turning behavior of car drivers.

Although minimizing the overall average waiting time per car is our main objective, we study in Section 8.6 the progression of cars along a simple arterial. Next, in Section 8.7, we consider more complex arterials with more intersections and other more-complicated intersections. Finally, we consider, in Section 8.8, a network of nine intersection on a grid of three-by-three intersections.

8.3 Optimization of coordinated FC

To gain some insight, we explain the optimization of coordinated FC by means of an simple example under ideal conditions. Next less-ideal circumstances are discussed. Numerical results to compare different cycles of FC are reported in Section 8.5.2.

8.3.1 The I2F4C2 infrastructure: some notations, terminology and complications

Before we discuss how to set a good network cycle, we introduce an arterial with two simple intersections of the F4C2 type, as displayed in Figure 8.1. This simple infrastructure appears to be very insightful for optimizing the network cycle. We refer to this case as the I2F4C2 infrastructure, where I2 indicates the number of intersections along the arterial. The $I = 2$ intersections are numbered from left to right, or say from West to East: $i \in \{1, \ldots, I\}$. The distance between the intersection may be expressed in seconds or slots: $T_{1,i}$ is the time it takes to travel from intersection 1 to intersection i, when driving at constant speed equal to the speed limit and all lights are green. In all cases that we study $T_{1,i}$ is set to the distance in meters divided by a speed limit of 50km/h, which is $13\frac{8}{9}\text{ms}^{-1}$ or $27\frac{7}{9}$m. per slot.

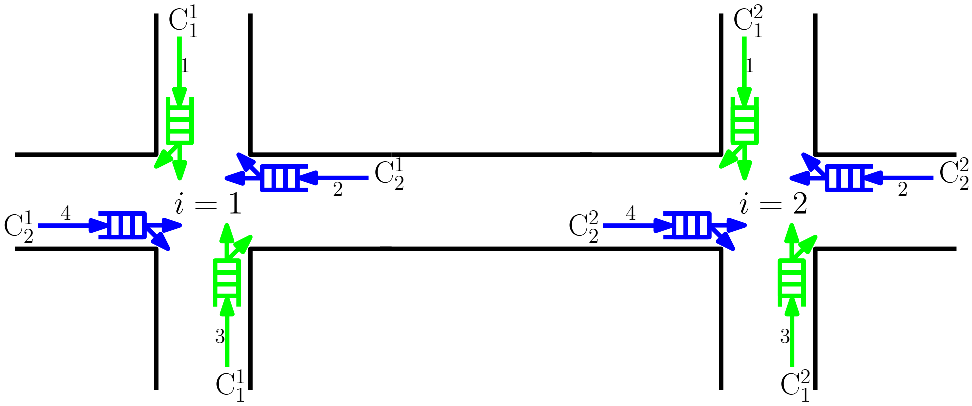


Figure 8.1: Arterial with 2 simple intersections: I2F4C2.

The West-to-East and East-to-West approaches are called the arterial. The traffic flows between the two intersection are the *internal flows*. The flows entering the network from the outside are called the *external flows*. Again, flows are numbered clockwise for each intersection. Combination s of intersection i is in short notation C_s^i; queue f at intersection i is referred to by Q_f^i.

Cars that leave Q_4^1 and Q_2^2 travel in two groups, called *platoons*, from one intersection to the other. When we assume that cars do not turn left or right, all cars in the platoons that travel along the arterial originate from one of these queues. When, in addition all cars travel at identical speed, the platoons remain intact. Under these assumptions a green wave under FC is easily accomplished.

For coordinated FC a network cycle applies. A cycle at intersection i starts ψ_i slots after a cycle starts at intersection 1. To find the best coordinated FC, one should integrally optimize over the effective green times $d_{i,s}$, the resulting cycle lengths D_i and the start ψ_i of a cycle for both intersections. Note that when the cycle length differs between the intersections, the lights cannot be unsynchronized and thus a (static) green wave cannot be set. Therefore the search is limited to FCs with identical cycle lengths $D_i = D$. Moreover, we assume that the infrastructures of the intersections are identical and that the arrival rate of a flow is the same at every intersection. Hence, the durations of the effective green times $d_{i,s}$ for every combination s is the same at all intersections: $d_{i,s} = d_s$.

In the next two subsections we focus on setting green waves for the I2F4C2 infrastructure, since green waves may reduce the long-run average waiting time per car.

8.3.2 A green wave in one direction

By setting the offset equal to the traveling time between the two intersections, one sets a green wave in one direction, from West to East, but not necessarily in the opposite direction. Furthermore, when cars do not travel at a constant speed equal to the speed limit, some cars may not experience a green wave. Slow moving cars may cause a big delay to cars directly behind them: when the lights at the downstream intersection have just turned into red upon the arrival of a car, it has to wait for the next green period.

Even when cars travel at a constant speed equal to the speed limit, not all cars may experience a green wave when some other cars are still queued upon the arrival at the downstream traffic light. These queued cars may originate from another queue: for example, cars that have turned right at Q_3^1 may still be present at Q_4^2 upon the arrival of cars originating from Q_4^1. To clear the queues, the lights should be switch to green before a new platoon of cars arrive. Consequently, the tail of the platoon may not experience a green wave, when the green periods of the downstream and upstream intersection have identical length.

8.3.3 A green wave in two directions

When the average number of cars traveling from East to West differs not much from the opposite direction one may prefer a green wave in both directions. This might also be desired in the light of minimizing the long-run average waiting time per car. Green waves can be set in both directions for any cycle length D, when one could control the speed at which cars travel. In our study, we do not control car speeds.

To ease the discussion at this point, we focus on what we call *to-FC-ideal cases* in which we assume that:

- all cars drive precisely at the speed limit, and
- no cars turn left or right.

In the next section, we discuss the impact of less-ideal conditions.

Network cycle length

For coordinated control the signals at the intersections should be phased and thus the cycle lengths should be identical. Since we study the problem under the *to-FC-ideal conditions*, the length of the green periods to C_2^1 and C_2^2 are equal. One condition to obtain a green wave is that a multiple of the cycle length D equals two times the traveling time $T_{1,2}$, according to Equation (8.1):

$$n \cdot D = 2 \cdot T_{1,2}, \qquad n \in \mathbb{N}. \tag{8.1}$$

The traveling time $T_{1,2}$ and the cycle length D are measured either in slots or in seconds. The equation is explained through the following simple example.

Example – Consider an arterial with two signalized intersection of the F4C2 type. All traffic is thru traffic and cars drive at constant speed such that the distance between the intersections is $T_{1,2} = 22$ slots. Suppose at time 0 a cycle of FC starts by changing the lights of C_2^1 into green. Then cars leaving Q_4^1 will arrive at Q_4^2 after $T_{1,2} = 22$ slots. When no cars are waiting at Q_4^2 and the lights of C_2^2 change into green at time 22, then all cars traveling from West to East experience a green wave as long as the green periods at the two intersections are of equal length.

In the opposite direction cars pull up from Q_2^2 at time $T_{1,2} = 22$ and travel to intersection 1 where they arrive at time $2 \cdot T_{1,2} = 44$. To impose a green wave in two directions, a new cycle should start (at least) every $2 \cdot T_{1,2} = 44$ slots. The cycle length should thus be $D = 2 \cdot T_{1,2} = 44$, $D = T_{1,2} = 22$ or $D = T_{1,2} \,/\, 2 = 11$ slots. (In this example $D = 2 \cdot T_{1,2} \,/\, 5$ is not considered as it does not result in an integer cycle length. Rounding D is possible but will imply that the average waiting time in the two opposite direction will differ. FC with $D = 2 \cdot T_{1,2} \,/\, 5$ and lower cycle lengths are not possible as a cycle last at least 8 slot: switching takes 3 slots and the minimum green time is 1 slot for each combination.)

The best choice of D depends on the load of the intersection, as set by the arrival rates.

Optimal offset

A second condition that needs to be satisfied to achieve a green wave in two directions concerns the *offset*: the time between the start of a cycle at intersection 1 and the start of a cycle at intersection 2. When cars travel at constant speed and do not turn left or right, an offset ψ_2 equal to the traveling time between the intersections is optimal given D is set according to Equation (8.1). Multiple values of the offsets may yield a green wave, we therefore define the offset according to Equation (8.2).

$$\psi_i = T_{1,i} \mod D. \tag{8.2}$$

When D and $T_{1,i}$ are in slots, then ψ_i is also in slots. (Note that by definition $\psi_1 = 0$.)

Space-time diagram

In Figure 8.2 we illustrate through a so-called *space-time diagram* that green waves are set in both directions. The horizontal bars show the signaling scheme with respect to the flows in C_2^1 and C_2^2: the horizontal flows. The green, yellow, and red periods are marked by different colors. At the two plots at the top, the cycle length equals $D = 22$ respectively $D = 11$ slots and the offset equals $\psi_2 = 0$. When the cycle length equals $D = 44$ slots, as in the last plot, the offset is $\psi_2 = 22$ slots.

In the first plot at time 0 a green period of 8 slots starts at both intersections, followed by 2 yellow slots: the effective green time is 10 slots. The arrows indicate the progression of the traffic between the intersections: to simplify the plots we have assumed cars to travel at a constant speed as if they do not require time to accelerate when leaving a queue or to slow down when approaching the end of a queue.

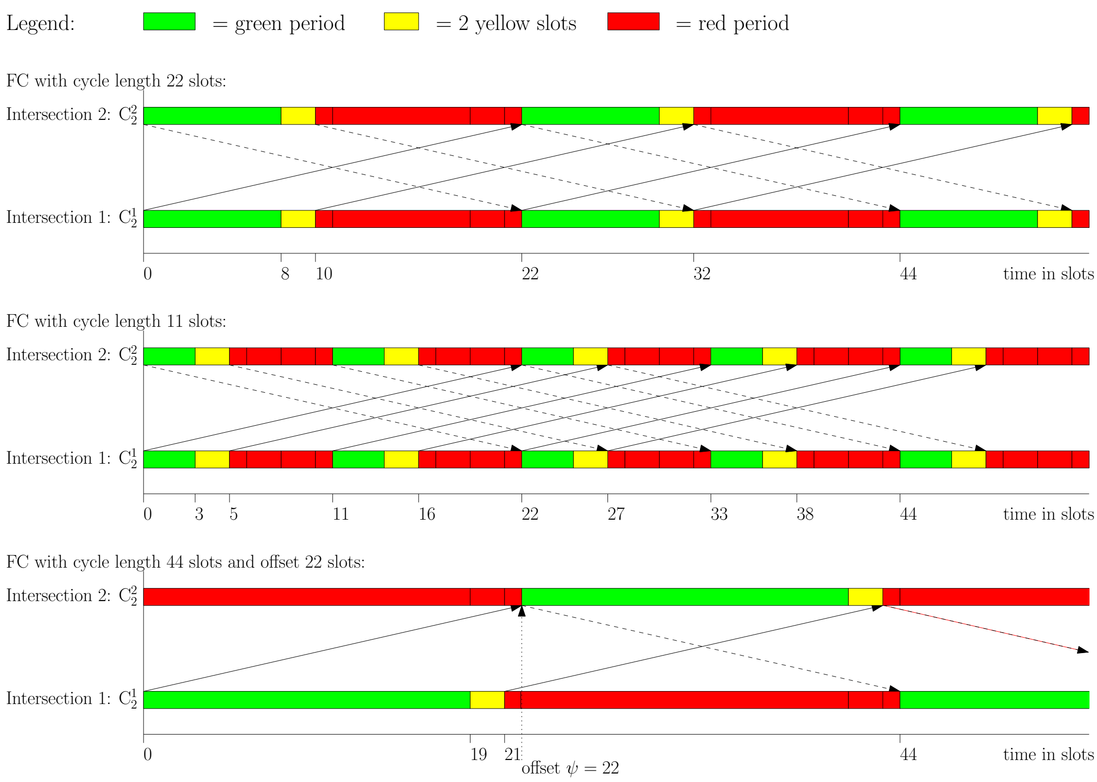


Figure 8.2: Synchronizing FC for a to-FC-ideal I2F4C2 case: green waves can be set in both direction when speeds are constant and identical; $\rho_1 = \rho_2 = 0.8$, $T_{1,2} = 22$ slots.

The plot shows that cars leaving Q_4^1 and Q_2^2 during the first $d_{i,2} = 8 + 2 = 10$ slots experience a green wave at the respective downstream intersection at slots $22 - 32$. In this case the green wave holds in two directions. Similarly green waves are plotted for cycle length $D = 11$ respectively $D = 44$ slots. Note that two opposite green waves at cycle length $D = 44$ slots require an offset of $\psi_2 = 22$ slots: hence at time 0 any cars present depart from Q_4^1, but no car leaves Q_2^2.

8.3.4 To-FC-ideal cases and less-ideal cases

The formulas in Equations (8.1) and (8.2) hold under the to-FC-ideal conditions, which we have formulated in Section 8.3.3. That is, cars travel at identical speeds equal to the speed limit and cars do not turn left or right. When in addition to these conditions the locally optimal cycle length found by the Optimal-fixed-cycle algorithm satisfies Equation (8.1), we deal with a what-we-call a *to-FC-utmost-ideal case*. In other words, in addition to the to-FC-ideal conditions, the traveling distance is thus ideal to set green waves while setting the locally optimal cycle lengths.

In reality the conditions may be not ideal:

1. cars may travel at different speeds,
2. cars are allowed to turn, and
3. the distance between intersections is fixed, while the locally optimal cycle length depends on the workload.

Then green waves may hold for only a fraction of the cars, but are easily lost, as we discuss below. In Section 8.5, we numerically show the impact when one or more of the conditions do not hold.

Green waves are distorted when speeds vary

When traveling speeds of cars differ, not all cars will experience a green wave. On the one hand fast moving cars, if not hampered by slow-moving traffic, may arrive at the downstream intersection before the green period starts. On the other hand, slow-moving cars may arrive (just) too late at the next intersection and have to wait for the next green period.

In the space-time diagram in Figure 8.3, the first two arrows show the progression of a fast moving car, respectively a slow moving car, that travel from intersection 1 to intersection 2. The third arrow has a bend and shows that a fast moving car gets delayed by a slow moving car. As we assume that cars are not allowed to overtake slow moving cars, platoons are formed. Consequently, the tail of a platoon may stay behind at the downstream intersection when the lights are red upon its arrival. These queued cars delay new arrivals.

Figure 8.3: Illustration that green waves under coordinated FC are lost when car speeds vary: the I2F4C2 case with $\rho_1 = \rho_2 = 0.8$ and ideal distance $T_{1,2} = 22$ slots (=611 meters).

Dealing with right turning traffic

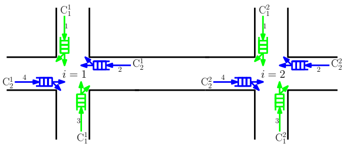
A green wave may be lost also when cars are (still) queued at the downstream intersection when a platoon of cars arrive at an internal queue. The queued cars may be the tail of a previous platoon or they may originate from another flow: e.g. cars that turn right when leaving Q_3^1 will accumulate at Q_4^2 and will thus delay thru traffic originating from Q_4^1.

When cars are allowed to turn right the green waves that are set under the to-FC-ideal conditions are easily lost. To reduce the waiting time one may adjust the cycle length and offset such that the internal queues are depleted before a new platoon arrives. Depending on the number of queued cars the head of a platoon may experience a green wave, but, depending on the length of the platoon, the tail of the platoon face a red light.

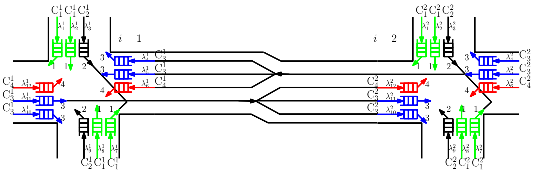
The length of a platoon is influenced by the duration of the green period at the upstream intersection. When the green time of the internal traffic flows is longer than that of the external flows, one may avoid that the tail has to stop while the head of a platoon experiences a green wave. Since in the infrastructures that we study, we have grouped opposite thru traffic in the same combination, we will not consider different green periods to internal and external flows. In general, setting a green wave in two directions is hard to settle when traffic leading to a queue originates from different combinations.

8.4 Simulation model for networks

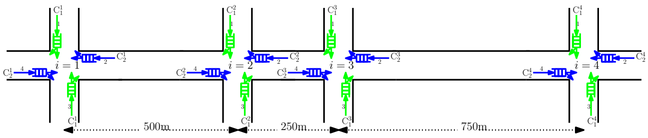
The RV policies developed in the previous chapters are applied to several network cases. Figure 8.4 shows a set of four infrastructures that are simulated in the next sections. The simulation model described in Section 7.3 is therefore extended, such that the outflow of one intersection is the inflow to a downstream intersection. The progression of cars along the lanes is tracked by monitoring the actual position of each car. In practice the positions are either known by cameras or induction loops or estimated based on the time elapsed since a car has left the upstream intersection ([93], [65]).



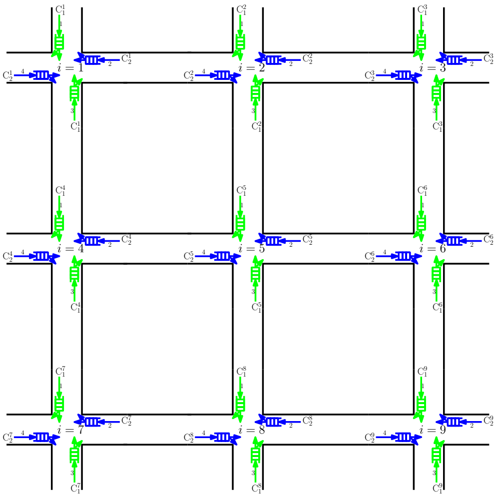
(a) I2F4C2: a simple arterial, see Sections 8.5 and 8.6.



(b) I2F12C4: an arterial with multiple lanes, see Section 8.7.



(c) I4F4C2: four unequally-spaced intersections, see Section 8.7



(d) I3x3F4C2: a network of nine intersections on a grid, see Section 8.8.

Figure 8.4: Outlook on test infrastructures and related sections.

We summarize the most relevant features and assumptions in the simulation model:

1. The simulation happens in discrete time with fixed time increments by 1 slot of 2 seconds.

2. Cars have their origin and destination outside the network.

3. External arrivals within a slot happen 500 meters upstream and are generated by a Bernoulli experiment for each flow f (with the probability of an arrival in a slot equal to λ_f^i).

4. The (desired) speed of a car is either 50 km/h for all cars, or is drawn from a Triangular distribution, Tri(40,50,60), with minimum, mean, and maximum car speed equal to respectively 40, 50 and 60km/h.

5. A car adjusts its speed to slow moving cars directly in front of it, since they are not allowed to overtake other cars. Cars keep a safe traveling distance of 2 seconds.

6. The time at which a car arrives at the Tail of the Queue (ToQ), is estimated based on the exact position of all cars, their traveling speed, and the position of the ToQ (=queue length).

7. As in most microscopic simulation programs, cars are queued horizontally: each queued car takes 7 meters.

8. When the lights show green or yellow the queued cars travel 7 meters per slot (=12.6km/h), since each slot 1 car leaves the queue. Cars that leave a queue accelerate in zero time to their desired speed (or less when hampered by slow moving cars). Similarly cars decelerate in zero time when joining a queue, or when running into slow moving traffic.

9. Cars are routed randomly based on fixed probabilities for turning left or right and lane selection, since they do not have specific destinations. Car drivers change lanes only when crossing an intersection. The routing model is explained in more detail in Sections 8.5.4 and 8.7.2.

The simulation model that we have developed, contains thus a number of realistic features that are not included in the decision model. Principal objective is to minimize the long-run average waiting time per car, which is defined as the integer number of slots a car spends in the queue. Despite the discrepancies between the simulation model and the Markov decision model, we expect the RV policies to perform well as will be shown in the next sections.

Outline of the simulation study

In Table 8.1 we show how the remainder of this chapter is organized. We consider five studies, I to V, based on four infrastructures. A number of scenarios will be related to each study; these are explained and motivated in the respective sections.

Table 8.1: Organization of the simulation study on controlling networks of intersections.

Study	Infrastructure	Motivation	Section
I	I2F4C2	Impact of distance, traveling speed and right turning traffic	8.5
II	I2F4C2	Progression on arterial	8.6
III	I4F4C2	Multiple intersection at unequal distances	8.7.1
IV	I2F12C4	More complex intersections	8.7.2
V	I3x3F4C2	Network of 9 intersections on a grid of 3-by-3	8.8

Regarding the test cases, we limit the number of test cases FC by focussing on the performance under a high workload: $\rho_i = 0.8$. In the previous chapters we have learned that the difference between the RV policies and the (uncoordinated) FC is bigger at lower workloads. In each of the five studies we are interested in the performance of the RV policies, compared to the 'best' FC. The choice of a best coordinated FC as a benchmark, is supported by reporting several FCs with different cycle lengths and offsets. To ease the analysis, most studies concern networks of F4C2 intersections with identical arrival rates. In Sections 8.6 and 8.7.2 we consider a few asymmetric cases. For the infrastructures with F4C2 intersections, we focus on cyclic RV1 policies. Acyclic control and RV2 policies are studied in Section 8.7.2 for the I2F12C4 case, which consists of more combinations per intersection.

Simulation horizon – All reported figures are averages over 100 replications of 72,000 slots, excluding a warming up period of 450 slots (=15 minutes) starting with an empty system. The figures are accurate up to at least 2 or 3 digits, hence we omit confidence intervals. Moreover, the relative performance of the different policies is the objective of the study rather than the absolute figures.

8.5 Study I – I2F4C2 arterial

We put the RV1 policies to the test by simulation of traffic along the simple arterial depicted in Figure 8.4(a): I2F4C2. This simple infrastructure illustrates that green waves can be set under ideal circumstances, but as we will see these are easily lost. The purpose of the simulation study in this section, is to investigate the sensitivity of FC and to check whether the RV policies perform better than coordinated FC.

All policies are simulated, under varying assumptions concerning the car speed, distance between the intersections and the fraction of cars that turn right. Table 8.2 shows an overview of the four scenarios, numbered I-A to I-D, that are studied in the next four sections. In all simulations, we assume identical arrival rates equal to 0.4, and thus the workload of each intersection is 0.8.

Table 8.2: Overview of 4 scenarios for fully-symmetric I2F4C2 ($\rho_1 = \rho_2 = 0.8$; $\lambda_f^i = 0.4$).

Scenario	Short description	Travel distance in meters	Travel distance in seconds	Distribution car speeds (km/h)	% cars that turn right	Section
I-A	To-FC-utmost-ideal	611m	$T_{1,2} = 44$s.	Det(50)	0%	8.5.1
I-B	To-FC-ideal case	500m	$T_{1,2} = 36$s.	Det(50)	0%	8.5.2
I-C	Speed differences	500m	$T_{1,2} = 36$s.	Tri(40,50,60)	0%	8.5.3
I-D	Speed differences and right turning cars	500m	$T_{1,2} = 36$s.	Tri(40,50,60)	25%	8.5.4

A summary of the results and a list of major conclusions is provided in Section 8.5.5.

8.5.1 Scenario I-A – An utmost-ideal case for FC (I2F4C2)

In the to-FC-ideal case, cars travel at identical speed (50 km/h) and do not turn left or right. In an utmost-ideal case, the traveling time $T_{1,2}$ between the intersections equals the locally optimal cycle length D. Then one can impose green waves in both directions as discussed in Section 8.3 with a network cycle identical to the locally optimal cycle.

The locally optimal cycle length of a fully-symmetric F4C2 intersection at a load of 80% is 22 slots (44 seconds), as we have seen in Table 6.2. An utmost ideal distance between the intersections is thus $T_{1,2} = 44$ seconds; or 611 meters as cars travel 50 km/h. When $T_{1,2}$ is 44 seconds, the best network FC is most likely the one with $D = 44$, $d_{i,s} = 20$ and $\psi_2 = 0$ seconds.

Table 8.3: Mean waiting times (in sec.) for I2F4C2 when $\rho = 0.8$ ($\lambda_f^i = 0.4$): a to-FC-utmost-ideal case with distance 611 meters ($T_{1,2} = 44$s.), all cars speeds are 50km/h and all traffic is thru traffic.

Policy	EW	% above RV1(5)	Internal flows EW_2^1, EW_4^2	External flows EW_4^1, EW_2^2	EW_1^i, EW_3^i
RV1(5)[a]	12.7		10.9	13.7	13.1
RV1[a]	15.9	+25.4%	17.0	14.9	16.0
FC[a]	13.7	+8.1%	0	18.3	18.2
XC	24.9	+96.0%	25.2	23.5	25.4
XC-1	19.4	+52.8%	19.3	18.3	20.0
XC-2	16.1	+27.2%	15.4	15.7	16.8

[a]Based on FC with $D = 44$, $\psi_2 = 0$, and $d_{i,s} = 20$.

In Table 8.3 we report on the long-run average waiting times EW_f^i (in seconds) at all queues as obtained through simulation. The overall expected waiting time (EW) is reported in the second column. Clearly, RV1(5), the RV1 policy with 5 slots of arrival information, performs best.

Coordinated FC yields an average waiting time per car that is 8.1% above that of RV1(5), although in this case FC implies a green wave at the internal queues Q_2^1 and Q_4^2. Under dynamic control, e.g. under the XC and RV1 policies, green waves are lost. The RV1 policy with no arrival information yields a waiting time that is 25% above that for RV1(5).

Even for this simple symmetric infrastructure with an identical number of flows in each combination, the exhaustive control policies perform bad compared to FC and RV1(5): the overall average waiting time is 27 to 96% higher than that under RV1(5). This shows that RV1(5) is a relatively simple policy that is superior to both FC and XC policies.

Other ideal distances between the intersections

For the given workload $\rho = 0.8$, the locally optimal FC with cycle length 44 seconds was argued to be optimal FC for the I2F4C2 infrastructure when $T_{1,2} = 44$ seconds (or 611 meters). In fact, according to Equation (8.1) $D = 44$ is also optimal when the traveling time traveling time is any multiple of 22 seconds. Hence when traveling at at 50 km/h one experiences a green wave whenever the distance between the two intersections is 306m, 611m, 917m, 1222m, etc. For any other distance the length of a cycle must be adjusted to keep a green wave. The FC that minimizes the overall long-run average waiting time is then no longer trivial.

8.5.2 Scenario I-B – Non-ideal distance: 500 meters (I2F4C2)

When the distance between the two intersections is only 500 meters, then the traveling time equals $T_{1,2} = 36$ seconds when traveling 50 km/h. According to Equation (8.1), a green wave is possible only under cycle lengths 72, 36, 24, 18 seconds. Since the locally optimal cycle equals 44 seconds, a promising network cycle has cycle length D equal to 36, 44 or 72.

In Table 8.4, we present the long-run average waiting times under RV1(5), FC and XC-2. Note that, here and in the remainder, we omit the results for XC and XC-1, since these policies perform much worse than XC-2 as demonstrated already in Table 8.3.

The waiting times under FC are significantly higher, even if one sets green waves in both directions. The best FC has cycle length 36 seconds and shows green waves in both directions. But, the implied long-run average waiting time is now 11.6% above that under RV1(5). Still, coordinated FC performs better than the best XC-2.

Coordinated control with $D = 72$s. and offset $\psi_2 = 36$s., yields a much higher average waiting time than FC with $D = 36$ and $\psi_2 = 0$. Under non-coordinated FC, the cycle length is set to the locally optimal one (44 seconds) and the offset is set to zero, such that cars traveling from west to east experiences the same average waiting as cars traveling in the opposite direction. As expected, green waves are then lost: cars arriving at the intersection have to wait at least 8 seconds, since the head of a platoon arrives at the downstream intersection ($44 - 36 =$) 8 seconds before a green period starts. Since we assume queued cars to take 7 meters of space per car, the average waiting time (=the average time to spend in each queue) is higher than 8 seconds.

Table 8.4: Mean waiting times (in sec.) for I2F4C2 at $\rho = 0.8$ ($\lambda_f^i = 0.4$): the to-FC-ideal case (cars speed = 50km/h, no cars turn, and distance = 500 m.).

Policy	EW	% above RV1(5)	Internal flows EW_2^1, EW_4^2	External flows EW_4^1, EW_2^2	EW_1^i, EW_3^i
RV1(5)[a]	13.0		12.1	13.6	13.1
RV1[a]	15.3	+17.7%	15.0	15.3	15.5
FC($D = 36, \psi_2 = 0, d_{i,s} = 16$)	14.5	+11.6%	0	19.3	19.4
FC ($D = 44, \psi_2 = 0, d_{i,s} = 20$)	16.2	+24.5%	9.8	18.3	18.3
FC ($D = 72, \psi_2 = 36, d_{i,s} = 34$)	16.1	+23.9%	0	21.5	21.4
XC-2	16.2	+24.4%	15.4	15.7	16.8

[a]Based on FC with $D = 44$, $\psi_2 = 0$, and $d_{i,s} = 20$.

Conclusions on I2F4C2 under to-FC-ideal conditions, but non-ideal distance

Under to-FC-ideal conditions, cars do not turn left or right and do travel at constant speed equal to the speed limit (50km/h).

- In the case under consideration with $\rho = 0.8$ the best FC is coordinated FC with $D = T_{1,2} = 36$ and $\psi_2 = 0$ seconds.
- Compared to the case with ideal distance between the intersections, of the previous subsection, the waiting time under FC has increased by about 6 percent, from 13.7 to 14.5 seconds.
- The waiting time under the RV1 policies are more robust to changes in the distance between the intersections.
- RV1(5) performs better than the best coordinated FC, which in turn is even better than the best exhaustive control policy (XC-2).
- Apparently, the RV policies can easily cope with batch arrivals, although in the computation of the relative values, arrivals occur uniformly over time.

8.5.3 Scenario I-C – Different speeds (I2F4C2)

When cars travel at different speeds, the platoon originating from an upstream intersection falls apart into smaller platoons. Sub-platoons are formed by slow moving traffic that delays fast moving traffic. The time between the first car and the last car arriving at the downstream intersection may exceed the length of the effective green time under coordinated FC. Consequently, some cars will not experience a green wave. Under FC one has to make a trade-off between giving all cars in the (first) platoon a small delay or giving a larger delay to a few cars at the tail of a platoon.

The individual preferred car speeds are drawn from a triangular distribution with mean 50 km/h and minimum and maximum, 40 respectively 60 km/h. Traveling 500 meters to an intersection, will take a car 30 to 45 seconds. Since cars are not allowed to overtake, the average traveling speed will be below 50 km/h.

As the average speed is lower, the best cycle length under coordinated FC is most likely higher than 36 seconds, which was optimal in the previous section. Finding the best cycle length and offset is now considerably more complicated. By a partly enumerate search we investigate a number of configurations of FC.

Search for the best coordinated FC

By simulation of a number of combinations (D, ψ_2), we search for a good cycle length and offset. The results are reported in Figure 8.5. Figure 8.5(a) considers the case where the duration of the green periods for the two combinations are identical. Starting with a short cycle of 12 slots (24 seconds), under which the queues are just stable, both green periods are lengthened by one slot resulting in cycle length 28 seconds. The cycle length is increased by increments of 4 seconds until $D = 92$ seconds. We limit the choices for the offset to $\psi_2 = 0$ and $\psi_2 = D/2$, such that the waiting times in both (horizontal) directions are identical. The best coordinated FC seems to have $D = 44$s. and $\psi_2 = 0$.

In Figure 8.5(b) we have refined the search, by considering cycle lengths that imply green periods that differ in length by one slot. Since the arrival processes at the internal and external queues are not the same, unequal green periods could give a slight improvement even when the arrival rates are identical. In the case under consideration, an improvement stays out: all the green periods are of identical length: $d_{i,s} = 20$ seconds.

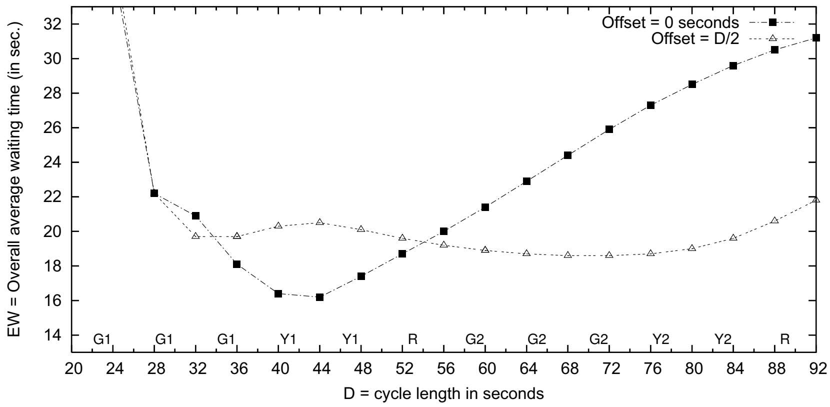
Finally, we consider alternative offsets in Figure 8.5(c), with D fixed to 44 seconds. Since speeds are identical and all traffic is thru traffic, offset 0 is indeed optimal. (The worst choice of ψ_2 is 22 seconds, since $\psi_2 = 22$ implies all cars in the horizontal directions to wait for at least $D - \psi_2 = 18$ seconds.)

The 'best' network FC seems to be the locally optimal cycle: $D = 44$ and $\psi_2 = 0$ seconds.

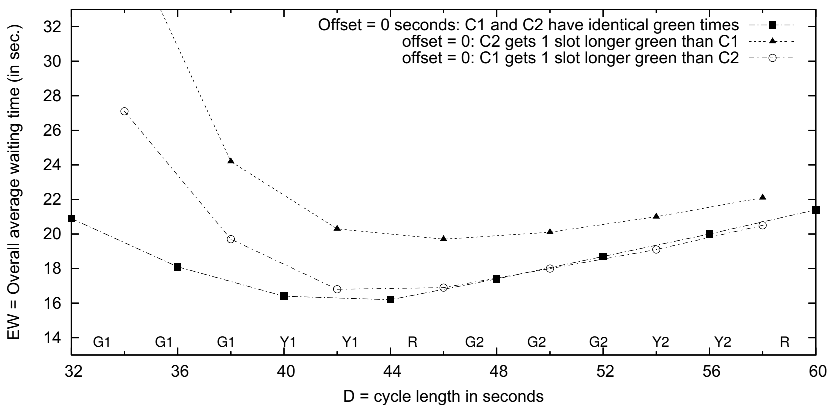
Comparison of RV1 policies and coordinated FC

In Table 8.5 we report the overall long-run average waiting time when speeds do vary. The average waiting time under all policies are higher than when travel speeds were constant and identical, because of the increased uncertainty in car arrivals.

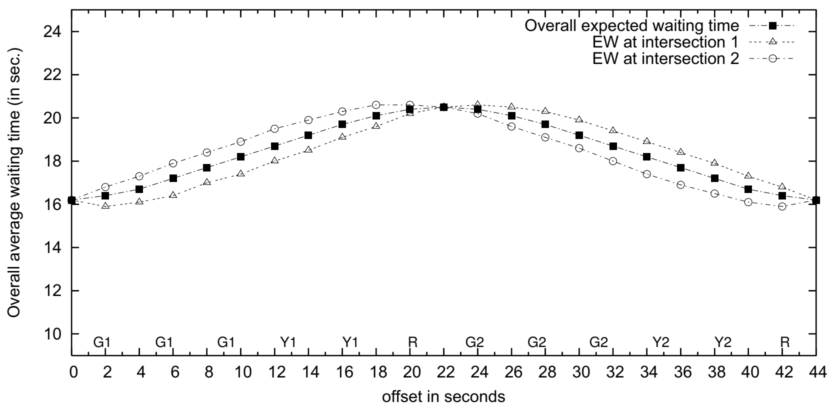
The difference between FC and RV1(5) is larger than when speeds were identical. This is partly due to the fact that the average waiting time under FC is no longer zero at the internal queues: green waves are lost due to the speed differences. The average waiting time at the internal queues is lowest when the cycle length is set higher than the average traveling time between the intersection. A longer cycle time (with offset zero), causes cars to accumulate at the downstream queue and thus 'repairs' the dispersion of the platoons. As a result the green periods at the downstream intersection are used more effectively.



(a) Optimization of FC over D (a multiple of 4 seconds) with $\psi_2 \in \{0, \frac{D}{2}\}$.



(b) Optimization of FC over D (a multiple of 2 seconds) with $\psi_2 = 0$.



(c) Optimization of FC over ψ_2 for $D = 44$.

Figure 8.5: Optimization of FC phased in two directions for I2F4C2 case with different car speeds and all thru traffic.

Table 8.5: Mean waiting times (in sec.) for I2F4C2 at $\rho = 0.8$ ($\lambda_f^i = 0.4$): all thru traffic and car speeds $\sim$ Tri(40,50,60).

Policy	EW	% above RV1(5)	Internal flows EW_2^1, EW_4^2	External flows EW_4^1, EW_2^2	External flows EW_1^i, EW_3^i
RV1(5)[a]	14.2		12.6	15.3	14.3
RV1[a]	16.3	+15.4%	16.0	16.4	16.4
FC ($D = 36, \psi_2 = 0, d_{i,s} = 16$)	18.1	+28.0%	10.2	20.8	20.8
FC ($D = 44, \psi_2 = 0, d_{i,s} = 20$)	16.2	+14.5%	6.1	19.5	19.5
FC ($D = 72, \psi_2 = 36, d_{i,s} = 34$)	18.6	+31.4%	6.5	22.6	22.6
XC-2	17.2	+21.8%	16.9	16.7	17.7

[a]Based on FC with $D = 44$, $\psi_2 = 0$, and $d_{i,s} = 20$.

Conclusions on I2F4C2 when cars do not turn and travel at different speeds

- When speeds vary, the best FC has a cycle length that is higher than the average travel time between the intersections.
- The best coordinated FC performs almost equally well as RV1, which does not use arrival information.
- Adding 5 slots of arrival information to RV1, makes a big difference, even when the information is not accurate due to speed difference.
- Although green waves are lost when speeds vary, the average waiting times under RV1(5) at the internal queues are lower than at the external queues.
- RV1(5) results in the lowest overall average waiting time: the best FC is still 14.5% off.

8.5.4 Scenario I-D – Right turning traffic (I2F4C2)

So far, we did not allow cars to turn right; this to ease the discussion on how to set green waves. In this section we investigate the impact of allowing cars to turn right. We introduce right turning traffic not only to make the problem more realistic but also to demonstrate the complexity of optimizing FC, as discussed already in Section 8.3. Left turns are prohibited, since this would be in conflict with the (opposite) thru traffic of the same combination.

Consider the I2F4C2 infrastructure. Upon passing the stopping line, 25% of the cars decide to turn right, rather than to go 'thru'.[1]

We simulate right turning traffic by means of a probability distribution: with probability $P_f^i(\mathrm{R}) = 0.25$ a car of flow f turns right at intersection i, and with probability $P_f^i(\mathrm{T}) = 0.75$ it goes thru, upon passing the stopping line. In the next sections we consider other probabilities and a case with separate lanes for left turning, right turning, and thru traffic.

Again, we evaluate several configurations of FC to find out which combination of D and ψ_2 is best. For setting good RV1 policies, any good FC satisfies. Therefore, we choose the cycle length (D_i) of intersection i equal to the locally optimal one, since the cycle lengths do not need to be identical for the RV1 policies. One may set the offsets to 0, since under dynamic control the offsets are irrelevant.

Search for the best coordinated FC

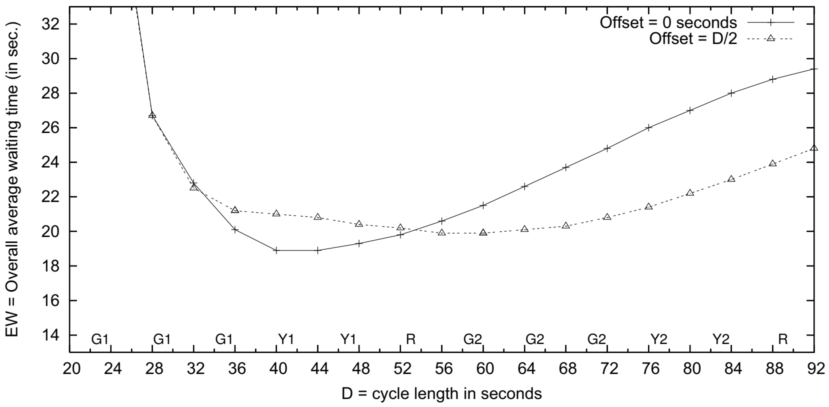
Figure 8.6 shows the search for an optimal FC when cars speeds are generated randomly from a triangular distribution, as in the previous section. The best cycle length that we have found is 40 or 44 seconds with offset 0. In comparison to Figure 8.5, the curves are more flat when cars are allowed to turn right. The upper two curves in Figure 8.6(b) almost coincide. Thus the arrival processes at the internal queues differ little from the arrival process at the external queues. Groups of cars leaving a queue do not form a platoon, since some cars leave the platoon by turning right at the stopping line. In addition, the arrival process is more uniformly distributed, or say more smooth, as the cars arriving at the internal queues originate from more queues, e.g. cars arriving at Q_4^2 originate from either Q_3^1 or Q_4^1.

Consequently, the minimal average waiting time under FC is somewhat higher and choosing a suboptimal value for the offset has less impact on the performance of FC.

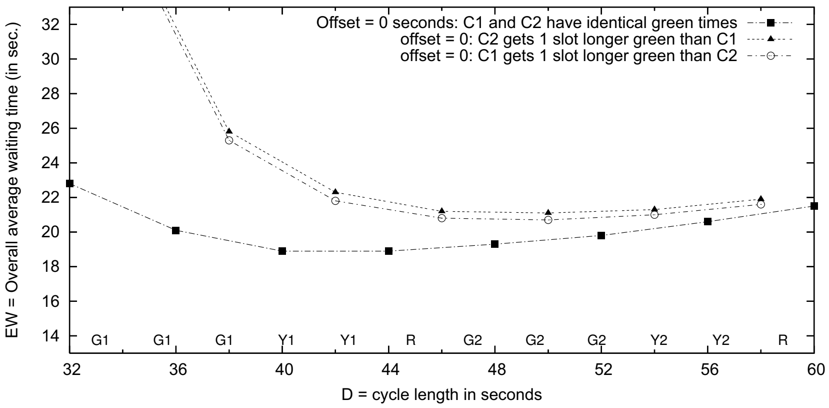
Comparison of RV1 policies and coordinated FC

In Table 8.6, we report the long-run average waiting times. Since the arrival of cars at the internal queues are more evenly spread over time, the average waiting time per car differs not much between the queues. Although the arrival process thus has a lower coefficient of variation, the average waiting times per car under FC and the RV1 policies are higher

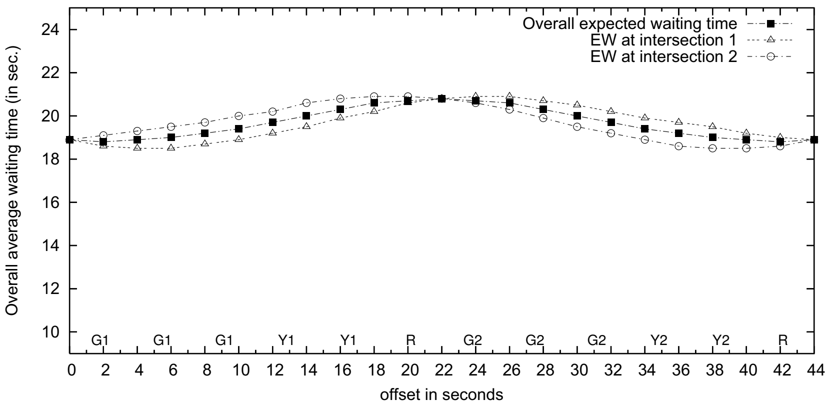
[1]The typical F4C2 intersection, at which left turns are forbidden, can be found in a number of American and Canadian cities, where (during rush hours) cars have to make three right turns around a block instead of making a left turn.



(a) Optimization of FC over D (a multiple of 4 seconds) with $\psi_2 \in \{0, \frac{D}{2}\}$.



(b) Optimization of FC over D (a multiple of 2 seconds) with $\psi_2 = 0$.



(c) Optimization of FC over ψ_2 for $D = 44$.

Figure 8.6: Optimization of FC phased in two directions for I2F4C2 case with different car speeds and 25% of cars turns right.

Table 8.6: Mean waiting times (in sec.) for I2F4C2 at $\rho = 0.8$ ($\lambda_f^i = 0.4$): 25% of traffic turns right and cars travel at different speeds.

Policy	EW	% above RV1(5)	Internal flows EW_2^1, EW_4^2	External flows EW_4^1, EW_2^2	EW_1^i, EW_3^i
RV1(5)[a]	14.7		14.4	14.8	14.8
RV1[a]	16.5	+12.2%	16.7	16.3	16.6
FC[a]	18.8	+27.8%	16.7	19.6	19.5
XC	25.4	+72.6%	25.4	24.9	25.7
XC-1	20.0	+36.0%	20.1	19.6	20.2
XC-2	16.7	+13.1%	16.6	16.4	16.8

[a]Based on FC with $D = 44$, $\psi_2 = 0$, and $d_{i,s} = 20$.

than those reported in Table 8.5, since it becomes harder to anticipate future arrivals. XC-2 yields higher waiting times when the arrival process is more evenly spread over time: XC-2 performs best when successive platoons are far apart.

Again we conclude that RV1(5) is superior to FC and XC-2. XC-2 is close to RV1, but yields an overall average waiting time that is 13% above that of RV1(5). The best FC is about 28% worse than RV1(5).

Remark – Although not reported we have seen similar results when speeds would be identical. The difference between RV1(5) and FC is then even bigger, since RV1(5) is more sensitive to speed differences than FC.

Table 8.7: Summary of results of all four scenarios for symmetric I2F4C2 with load 0.8.

Scenario	I-A	I-B	I-C	I-D
Section	8.5.1	8.5.2	8.5.3	8.5.4
Travel distance (in meters)	611	**500**	**500**	**500**
Distribution car speeds (km/h)	Det(50)	Det(50)	**Tri(40,50,60)**	**Tri(40,50,60)**
% cars that turn right	0%	0%	0%	**25%**
Best network FC: $(D, \psi_2, d_{i,s})$	(44, 0, 20)	(36, 0, 16)	(44, 0, 20)	(44, 0, 20)
Policy	Average waiting time per car			
RV1(5)	12.7	13.0	14.2	14.7
RV1	15.9	15.3	16.3	16.5
FC	13.7	14.5	16.2	18.8
XC-2	16.1	16.2	17.2	16.7

8.5.5 Conclusions for Scenario I-A to Scenario I-D

We summarize in Table 8.7 the main results for all four scenarios I-A to I-D.

Although the results relate to the symmetric I2F4C2 arterial at workload 0.8, we stipulate some conclusions that may hold in general:

1. RV1(5) is favored above the best coordinated FC, because the average waiting time per car is much lower under RV1(5) and the optimization of FC can be difficult:
   - Finding a locally-good FC that act as initial policy to the RV policies is much easier than finding a well-coordinated FC.
   - Imposing a green wave in two directions requires identical cycle lengths at all intersections and offsets to be equal to 0 or half the cycle length.
   - Green waves in two opposite directions exist only under ideal conditions, and are easily lost when car speeds vary or part of the traffic turns right.
2. Speed differences and right turning traffic smooth the arrival process at internal queues, and cause an increase of the average waiting time at the internal queues under both coordinated FC and RV1(5):
   - Platoons get dispersed, when cars travel at different speeds and when cars turn right at the upstream intersection.
   - When platoons fall apart into two (or more) sub-platoons, the best coordinated FC has a cycle length larger than average traveling time: the head of a platoon gets delayed to reduce a potentially excessive waiting time of cars at the very tail of the platoon.
   - RV1(5) is more sensitive to speed differences, since it uses the estimated arrival times of cars.
3. RV1 with a few slots arrival information yields a much lower average waiting time per car than the best FC and XC-2.

In the next section we investigate how we may steer the RV policies to satisfy secondary criteria such as reducing the waiting time at the internal queues. Further we investigate a skew case where most of traffic travels from west to east and one may set a green wave in 1 direction. In Sections 8.7 and 8.8 we consider more general infrastructures.

8.6 Study II – Progression along arterial

Although the principal criterion in this study is to minimize the long-run average waiting time per car, the progression along the arterial may be important. Therefore we investigate whether the RV1 policies allow to decrease the average waiting times at the internal queues. Three scenarios are considered as tabulated in Table 8.8.

Table 8.8: Scenarios with respect to the progression at I2F4C2 arterial.

Scenario	Description	Section
II-A	Internal queues have higher weight	8.6.1
II-B	Internal queues have more slots of arrival information	8.6.2
II-C	High arrival rates in the West-to-East direction	8.6.3

In all three scenarios, 25% of the cars turns right and individual (preferred) car speeds are drawn from a triangular distribution with mean 50 km/h and minimum and maximum 40 and 60 km/h respectively. In scenarios II-A and II-B, the arrival rates are identical ($\lambda_f^i = 0.4$), and we mainly investigate options to reduce the waiting time at the internal queues.

Scenario II-C is interesting for two reasons. Firstly, the asymmetry in the arrival rates may make a green wave in only one direction favorable, even though not all cars traveling in that direction will benefit. Secondly, non-coordinated FC, with locally optimal cycle lengths, may perform quite well, since the loads of the two intersections differ.

8.6.1 Scenario II-A – Higher waiting costs at internal queues (I2F4C2)

The RV policies are based on relative (cost) values of states under FC. In the evaluation of the Markov chain under FC, we incur 1 unit cost for each car that is queued at the start of a slot, independent of the queue at which the car is waiting. By accounting higher costs for cars waiting at the internal queues, one may reduce the waiting time at these queues. These costs per car per slot serve as weights of the flows.

In Table 8.9 we report on the waiting times, when the waiting costs at the internal queues, Q_2^1 and Q_4^2, are tripled. For easy comparison, we copy at the second line the results for RV1(5) of the previous section, in which all flows had identical weights. The exhaustive

Table 8.9: Mean waiting times (in sec.) for I2F4C2 at $\rho = 0.8$ ($\lambda_f^i = 0.4$): waiting costs at internal queues is tripled.

Policy	EW	% above RV1(5)	Internal flows EW_2^1, EW_4^2	External flows EW_4^1, EW_2^2	EW_1^i, EW_3^i
RV1(5)[a] – identical costs	14.7		14.4	14.8	14.8
RV1(5)[a]	15.4	+4.6%	9.3	15.9	18.2
RV1[a]	17.6	+19.3%	13.1	15.5	20.9
FC[a]	18.8	+27.8%	16.7	19.6	19.5

[a]Based on FC with $D = 44$, $\psi_2 = 0$, and $d_{i,s} = 20$.

control policies are not included, as the performance of these policies is not affected by changes in the cost structure.

In this case, the locally optimal cycle is also optimal for coordinated FC: the optimal cycle length is 44 seconds, just as in Section 8.5.4. The waiting time at the internal queues under RV1(5) is reduced from 14.4 to 9.3 seconds, since another set of relative values is used. The overall average waiting time per car under RV1(5) is only 4.6% higher than when all queues have identical weights, while the waiting time at the internal queues is reduced by 35%.

The best FC yields the same waiting time as before, since the length of the green periods have not changed. The average waiting time at the internal queues is $16.7 - 9.3 = 7.4$ seconds, or almost 80%, higher than under RV1(5) with the modified cost structure.

Conclusions

- RV1(5) is more reactive to changes in the cost structure than FC. The cost structure reflects the weights of each flow.
- By changing the weights of the flows, one can improve the progression along the arterial under RV1(5).
- RV1(5) yields a much lower average waiting time at the internal queues than under coordinated FC, which aims at minimizing the long-run average waiting time per car.

8.6.2 Scenario II-B – More slots of information on internal arrivals (I2F4C2)

Instead of changing the weights of the flows, one may question whether changing the number of slots of arrival information to be used at each flow affects the performance of the RV policies. In Section 7.5, we have demonstrated, for an isolated F12C4 intersection, that considering more than 5 slots of arrival information for all flows hardly gives any improvement. In a network setting, it may be favorable to extend the number of slots of arrival information for the internal flows, from say 5 to 10 slots. By simulation we check whether the resulting RV1(·) policies yield a reduction of the waiting time at the internal queues. . From the second line of Table 8.10, we conclude that more than 5 slots of arrival information does not necessarily yield better decisions under RV1(·).

Next, we test whether ignoring arrival information at the external queues helps to increase the progression of the internal flows. As reported on the third line of Table 8.10, the average waiting time at the internal queues has decreased slightly by 0.3 seconds, while at the external queues the waiting time has increased by more than 1.5 seconds.

For completeness, we have copied the results of the case where we use no arrival information at all. As we have seen before the average waiting time per car is then about 12% higher than when 5 slots of arrival information is used for all flows.

Conclusion – Although using a few slots of arrival information significantly reduces the overall average waiting time, arrival information is hardly an effective tool for reducing the average waiting time at the internal queues. Changing the cost parameters seems to be much more effective.

Table 8.10: Mean waiting times under RV1(·) (in sec.) for I2F4C2 at $\rho = 0.8$ ($\lambda_f^i = 0.4$): different amounts of arrival information.

# slots arrival information			% above	Internal flows	External flows	
internal	external	EW	RV1(5)	EW_2^1, EW_4^2	EW_4^1, EW_2^2	EW_1^i, EW_3^i
5	5	14.7		14.4	14.8	14.8
10	5	14.8	+0.5%	14.5	14.7	15.0
5	0	15.9	+7.8%	14.1	16.9	16.3
0	0	16.5	+12.2%	16.7	16.3	16.6

8.6.3 Scenario II-C – High arrival rates in the West-to-East direction (I2F4C2)

When investigating the progression along the arterial, an interesting scenario is when more cars travel from west to east than in the other directions. Coordinated control in this direction may then be of high importance to reduce the overall average waiting time per car. We study the I2F4C2 case with $\lambda_f^i = 0.2$ at the external, except at Q_4^1, at which three times as many cars arrive ($\lambda_4^1 = 0.6$). The load of intersection 1, ρ_1, equals thus 0.8. Since cars turn right with probability 0.25, the arrival rate at Q_4^2 equals $0.25 \cdot 0.2 + 0.75 \cdot 0.6 = 0.5$. The load of intersection 2, ρ_2, is thus only 0.7.

Since intersection 2 has a lower load than intersection 1, the locally optimal cycle length of the two intersections differ: $D_1 = 46$ and $D_2 = 28$ seconds. For coordinated FC the cycle lengths as well as the length of the green periods must be identical for the two intersection. Given the asymmetric arrival rates, one may favor a green wave primarily at Q_4^2, although a green wave cannot be obtained for all cars. Therefore we set the cycle length to $D_i = 46$ at both intersections. The duration of the effective green period is 12 and 30 seconds for respectively the combinations 1 and 2 of both intersections. The best offset, ψ_2, is 40 (or 38) seconds, as depicted in Figure 8.7.

This case is not only of interest to study the progression in one direction, but also because of the asymmetry in the workload of the two intersections.

In Table 8.11 we report the long-run average waiting time per car under the various

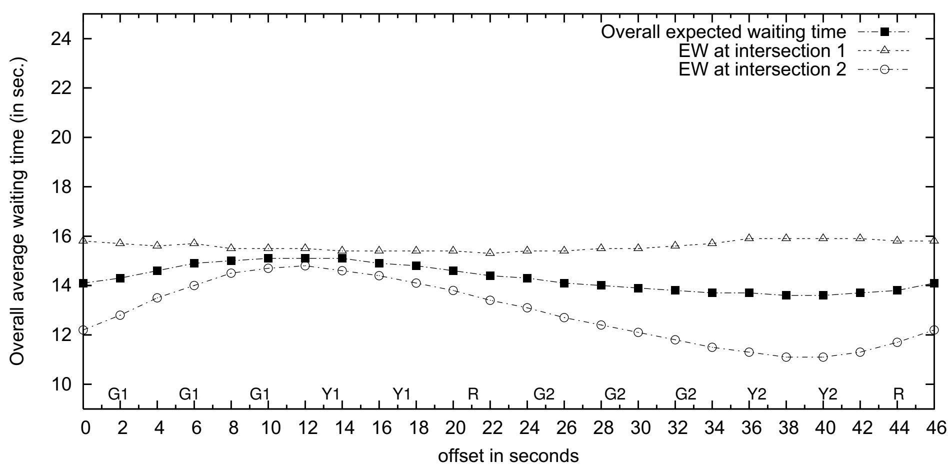


Figure 8.7: Optimization of FC over offset for $D = 46$: skew I2F4C2 case with different car speeds and 25% turns right.

Table 8.11: Mean waiting times (in sec.) for asymmetric I2F4C2 at $\rho_1 = 0.8$ and $\rho_2 = 0.7$ ($\lambda_f^i = 0.2$, except $\lambda_4^1 = 0.6$ and $\lambda_4^2 = 0.5$): different car speeds.

		% above			Internal flows		External flows			
Policy	EW	RV1(5)	EW_1	EW_2	EW_2^1	EW_4^2	EW_4^1	EW_2^2	EW_1^1, EW_3^1	EW_1^2, EW_3^2
RV1(5)[a]	8.7		10.2	7.1	4.0	5.5	7.9	4.1	16.7	10.5
RV1[a]	10.7	+23.0%	12.3	9.0	4.1	8.1	10.2	4.1	19.5	12.5
FC $D_1 \neq D_2$[a]	13.5	+55.2%	15.6	11.3	4.4	12.1	14.3	4.1	23.1	13.9
FC $D_i = 46$[b]	13.6	+56.3%	15.9	11.1	6.4	4.2	14.3	4.3	23.2	23.1
XC-2	10.8	+24.1%	12.2	9.2	4.4	10.1	12.5	4.6	15.7	10.4

[a](Relative values of) FC with $D_1 = 46$, $D_2 = 28$, and $d_{1,1} = 12$, $d_{1,2} = 30$, $d_{2,1} = 8$, $d_{2,2} = 16$.
[b]FC with $D_i = 46, d_{i,1} = 12, d_{i,2} = 30$, and $\psi_2 = 40$

policies. With respect to the overall average waiting time, coordinated FC hardly improves non-coordinated FC with locally optimal cycle lengths. However, the waiting time at Q_4^2 under coordinated FC is only a third of that under non-coordinated FC: 4.2 seconds versus 12.1 seconds.

RV1(5) is a very good dynamic alternative to FC. The overall average waiting time is only 8.7 seconds, which is a third (4.9 seconds) below that under coordinated FC. Under RV1(5), the waiting time at Q_4^2 is only 1.3 seconds higher, while the waiting time at all queues is much lower. Though the I2F4C2 infrastructure consists of simple intersections, XC-2 results in an average waiting time that is 24% above RV1(5).

8.6.4 Conclusions

RV1(5) results in a very low overall average waiting time per car, and allows to steer on a good progression in an arterial. The average waiting times at the internal queues can be reduced, by increasing the weights (=waiting costs per car) at these queues. Evaluating an horizon of future arrivals for more than 5 slots at these queues, does not improve the progression.

For an asymmetric arterial, where most traffic travels from west to east and the load of the two intersections differ, one may aim at setting a green wave under FC in the busiest direction. The best FC that we found appears to be more than 50% off from the overall average waiting time under RV1(5).

8.7 Study III and IV – More arterials

In this section, we report on Study III and Study IV, which relate to two different arterials:

Study III	–	I4F4C2	=	an arterial with four unequally spaced F4C2 intersections,
Study IV	–	I2F12C4	=	an arterial with two intersections of the F12C4-type.

The first arterial is of interest both for having more than two intersections and for having non-identical distances between the intersections. The second arterial is of interest for two reasons. Firstly, the number of flows in a combination is no longer identical and thus simple rules may not suit very well, as we have seen in the previous chapters. Secondly, through the arterial of F12C4 intersections one may test acyclic control policies in a network setting.

8.7.1 Study III - More intersections along an arterial (I4F4C2)

Thus far we have considered only arterials of 2 intersections. In practice one often deals with an arterial with multiple intersections at non-identical distance of each other. In Figure 8.8 we depict such an infrastructure with four intersections along an arterial.

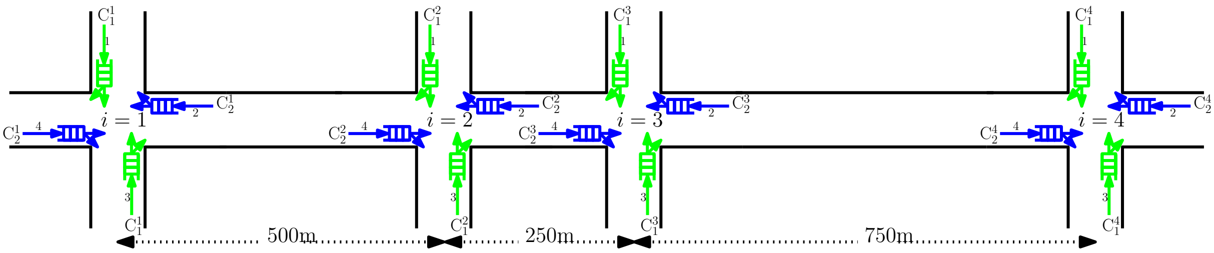


Figure 8.8: Arterial with four simple intersections: I4F4C2.

As in Scenario II-C, we assume non-identical arrival rates: much more cars travel from west to east than in the opposite direction. For the case that we study in this section, the arrival rates λ^i_f are 0.2, but the west-to-east traffic is three times as busy: $\lambda^i_4 = 0.6$. Cars in the busy traffic flow, from west to east, turn right with probability 0.2. To keep the load of all intersections identical, we assume that at the other queues 60% of the cars turn right. The load of each intersections equals thus 80%.

Selection of coordinated FC

Given the insights from the previous sections, we synchronize FC by setting the cycle length and green times identical for all intersections, and carefully choosing the offsets ψ_i. The cycle length is set to the locally optimal one: $D = 46$ with $d_{i,1} = 12$ and $d_{i,2} = 30$ seconds. The distances between the intersections 1 and 2, 2 and 3, and, 3 and 4, is 500, 250, and 750 meters respectively. Optimal offsets under to-FC-ideal circumstances are $\psi_2 = 36$, $\psi_3 = 54$, and $\psi_4 = 108$ seconds. In fact, these offsets are the traveling times starting at intersection 1, when driving at constant speed of 50 km/h[2]. As a fraction of cars turns right, and as cars travel at different speeds, a perfect green wave cannot be set to all cars in the busy flow. Therefore we consider several sets of values for the offsets.

Average waiting time

In Table 8.12 we report several configurations of FC. The best FC yields an average waiting time that is 46% higher than that under RV1(5). We have seen similar results for the case of identical speeds. At the internal queues the waiting times are much lower than at the side streets. Compared to RV1(5), coordinated FC performs not so well, since the green periods are fixed, and are thus not adjusted to the actual number of cars queued. Furthermore, the coordination is complicated by the fact that, on average, a fifth of the cars arriving at Q_4^i (with $i \geq 2$), originate from Q_3^{i-1}.

Table 8.12: Mean waiting times (in sec.) for asymmetric I4F4C2 case ($\rho = 0.8$; for $f = 1, 2, 3$: $\lambda_f^i = 0.2$ and $P_f^i(\mathrm{R}) = 0.6$; $\lambda_4^i = 0.6$ and $P_4^i(\mathrm{R}) = 0.2$; and car speeds $\sim$ Tri(40,50,60).

Policy	EW	% above RV1(5)	West-to-East EW_4^1	EW_4^2	EW_4^3	EW_4^4	Side streets $EW_1^i = EW_3^i$
RV1(5)[a]	10.6		7.9	8.2	8.4	8.5	17.2
RV1[a]	12.9	+22.2 %	10.2	10.9	10.8	11.4	20.4
FC $\psi = (32, 46, 96)$[a]	15.6	+47.7 %	14.3	14.2	13.9	17.1	23.0
FC $\psi = (36, 54, 108)$[a]	15.5	+46.4 %	14.4	13.7	13.4	16.2	23.1
FC $\psi = (40, 62, 120)$[a]	15.6	+47.9 %	14.3	13.7	14.4	15.2	23.0
FC $\psi = (44, 70, 132)$[a]	16.0	+51.7 %	14.3	14.4	15.3	15.3	23.1
XC-2	12.7	+19.9 %	12.5	12.8	12.8	13.1	16.5

[a]Based on FC with $D = 46$, $d_{i,1} = 12$ and $d_{i,2} = 30$.

[2]According to the definition of ψ_i in Equation (8.2), the offsets should be $\psi_2 = 36$, $\psi_3 = 8$, and $\psi_4 = 16$ seconds, instead of $\psi_2 = 36$, $\psi_3 = 54$, and $\psi_4 = 108$. The latter choices give the same results, but emphasizes more the relation with the traveling time to intersection 1.

RV1(5) gives the lowest average waiting time. The average waiting time is lowest at the busiest queues. Apparently, these queues get high weights, likely because they contribute much to the total workload. The waiting time at the internal queues in the west-to-east direction is even more than 25% lower than under coordinated FC. Also the average waiting time per car at the side streets is lower under RV1(5) compared to the FC policies. Exhaustive control gives a slightly lower waiting time at the external queues, but at the expense of high average waiting times in the west-to-east direction. The overall average waiting time per car is about 20% above that of RV1(5).

8.7.2 Study IV – Arterial with more combinations (I2F12C4)

The I2F12C4 infrastructure, depicted in Figure 8.9, has separate lanes for right turning, left turning and thru traffic. At each internal queue, traffic may originate from three different queues of the upstream intersection. Thus, we cannot expect to set green waves to all cars arriving at the internal queue.

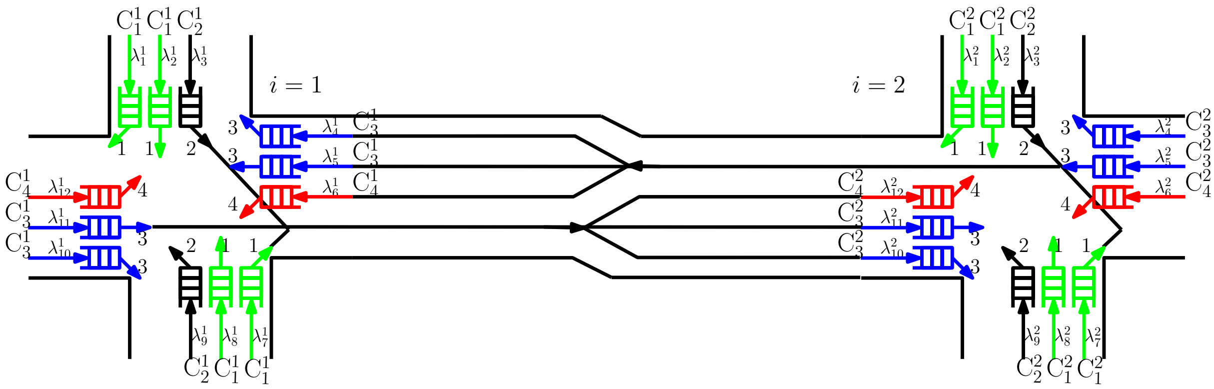


Figure 8.9: Typical example of arterial with two signalized intersections: I2F12C4.

The decision on when to switch from green to one combination, to green to another combination, is complicated by the asymmetry in the number of flows per combination. Further, under acyclic control, one should also decide which of the four combinations gets green next: thin combinations may get under-served, since serving a thick combination maybe more beneficial.

Some modeling assumptions and data

We adopt the modeling assumptions of the previous section(s), a.o.

- individual (desired) car speeds are drawn from Tri(40,50,60),
- the approach to and the distance between the intersections are 500 meters, and
- arrival information is available of cars that are up to 5 slots away from the end of the queue.

The arrival rates λ_f^i are set to 0.1 for all right and left turning flows. Thru traffic is three times as busy: $\lambda_2^i = \lambda_5^i = \lambda_8^i = \lambda_{11}^i = 0.3$ for each intersection i. The workload of each intersection is thus $\rho^i = 0.8$.

Upon leaving a queue a car chooses a lane according to fixed probabilities, as illustrated in Figure 8.10. Cars leaving Q_f^i proceed to a downstream approach and select with probability $P_f^i(\mathrm{L})$ the left lane, w.p. $P_f^i(\mathrm{T})$ the thru-lane and w.p. $P_f^i(\mathrm{R}) = 1 - P_f^i(\mathrm{L}) - P_f^i(\mathrm{T})$ the right lane. Cars stick to the lane that they have selected upon crossing the upstream intersection. Although the routing probabilities may depend on i and f, we choose them identical for all cars: $P_f^i(\mathrm{L}) = P_f^i(\mathrm{R}) = 0.2$, and $P_f^i(\mathrm{T}) = 0.6$.

Figure 8.11 shows how cars proceed from one intersection to another, and how the lane selection affects the dispersion of platoons. Therefore we have marked fictitious segments or cells, that cars can cross in one slot (when traveling 50km/h). A '1' in a cell indicates

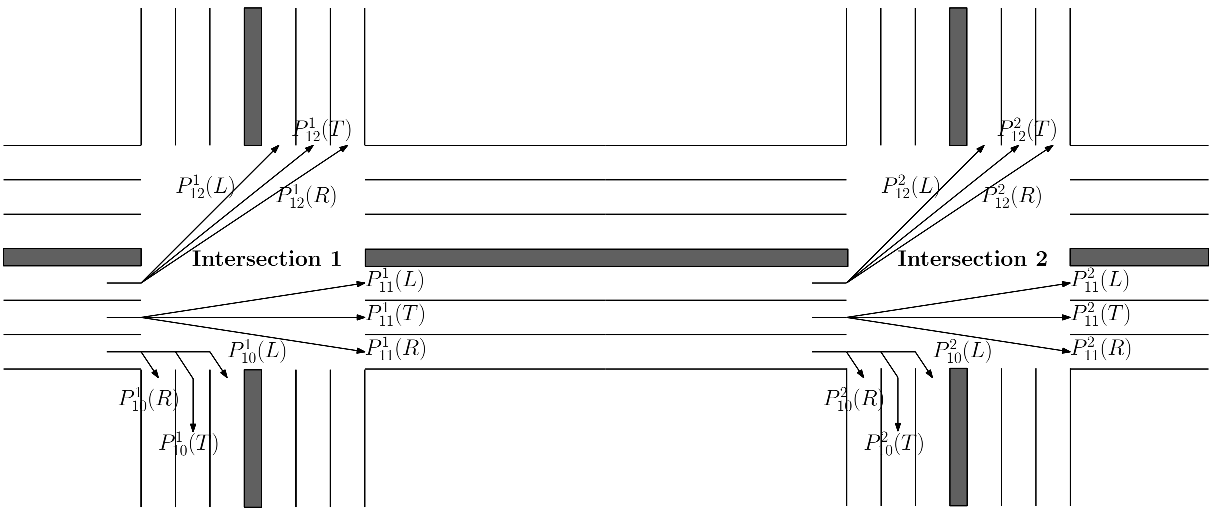


Figure 8.10: Routing probabilities in an arterial with two signalized intersections:

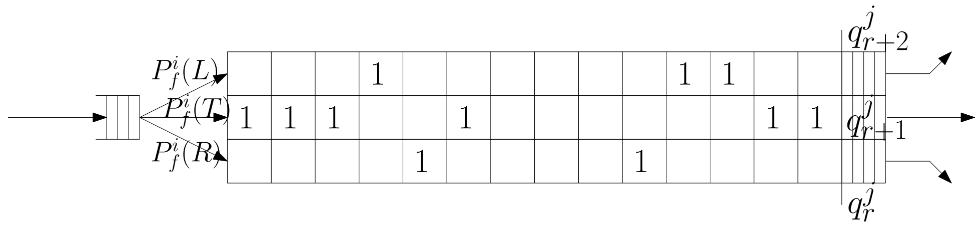


Figure 8.11: Routing probabilities and arrival information over three lanes.

that a car is present at that segment. The segments closest to the queues contains cars that are (at most) 1 slot away from the tail of the queue. Cars driving at the next-to closest segment, are still two slots away from the tail of queue. The segments not necessarily correspond to physical marked segments of the road, but are introduced to model the arrival times. Since cars leaving a single queue, have to choose between three lanes, cars do not travel in a single perfect platoon. When cars speeds differ, a platoon may be partly repaired by slow moving traffic as a car is not allowed to change lanes to overtake slow moving traffic on its lane.

Optimization of FC

The local optimization of FC for each intersection in isolation can be done by means of the Optimal-fixed-cycle algorithm (of Appendix D). In Figure 8.12 we show, for varying values of the cycle length D, the average waiting time per car under the best FC found by the algorithm. Apparently, the best FC for the F12C4 intersection, under the given arrival rates, is 84 seconds. The plot indicates also alternatives that are close to optimal. The local optima for a single intersection in isolation, are considered in finding a best coordinated FC.

For coordinated FC, one chooses both a cycle length D and an offset ψ_2, such that the overall average waiting time is minimal. According to the insights obtained in the previous sections, the offset should be zero or $D/2$, such that one experiences, in both horizontal directions, identical average waiting times per car. When the offset equals $D/2$, then the offset should be roughly equal to the average traveling time, between the two intersections. When driving at 50 km/h, the traveling time between the intersections is 36 seconds. According to Equations (8.1) and (8.2), $D = 72$ is a reasonable cycle length, since it is closest to the locally optimal FC (with cycle length 84 seconds). The FC with $D = 72$ and $\psi_2 = 36$, is, however, no local optimum in Figure 8.12.

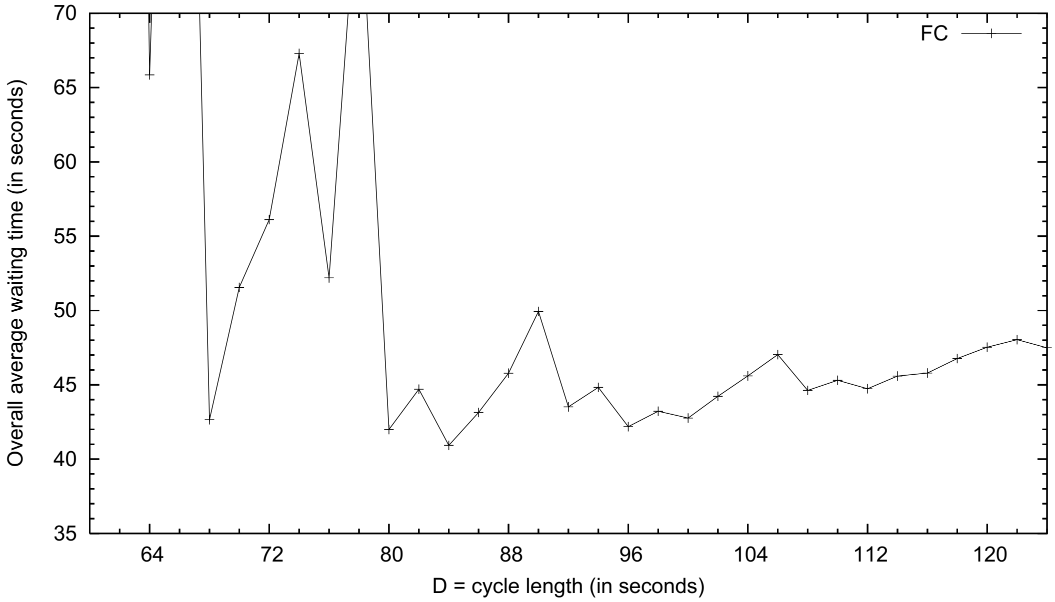


Figure 8.12: Application of the Optimal-fixed-cycle algorithm to find optimal value of the cycle length for F12C4 with non-identical arrival rates ($\lambda^i = \frac{1}{10}(1, 3, 1, 1, 3, 1, 1, 3, 1, 1, 3, 1)$).

Therefore, we report, in Table 8.13, the average waiting time under FC with different cycle lengths, ranging from $D = 68$ to 88 seconds with increments of 4 seconds. In all configurations of FC, the offset is set to $\psi_2 = D/2$. The effective green times are reported in the second and third column. It appears that in this case the best coordinated FC has a cycle length equal to the locally optimal one: $D = 84$ seconds. The reported overall average waiting times, does not differ much from those of a single intersection in isolation. Apparently, the arrival pattern at the internal queues, do differ not much from the Binomial arrival pattern. This is due to the fact that cars need to select a lane after crossing an intersection and to the fact that their traveling speeds differ, causing platoons to disperse.

Cyclic control

In Table 8.14 we report on the average waiting time both under cyclic and acyclic policies. We split the overall average waiting time in the average waiting time at the lanes of the internal and of the external queues.

We first discuss the top half of Table 8.14, which contains the results under cyclic control. RV1(5), which is reported in the first row, is selected as a base for comparing all other

Table 8.13: Mean waiting times ((in sec.)) under coordinated FCs for I2F12C4 at $\rho_i = 0.8$ and thru traffic three times as busy than left and right turning traffic (i.e. $\lambda^i_f = 0.1$, except $\lambda^i_2 = \lambda^i_5 = \lambda^i_8 = \lambda^i_{11} = 0.3$); car speeds $\sim$ Tri(40,50,60).

	Dep. times ($d_{i,f}$)		*EW*	Internal lanes			External lanes		
FC	C_1, C_3	C_2, C_4	overall	Left	Thru	Right	Left	Thru	Right
$D = 68, \psi_2 = 34$	22	8	43.6	62.8	39.2	10.6	65.8	46.8	18.6
$D = 72, \psi_2 = 36$	24	8	44.8	95.3	28.7	10.1	100.4	38.9	19.1
$D = 76, \psi_2 = 38$	26	8	50.8	45.4	55.2	11.2	47.0	65.7	21.1
$D = 80, \psi_2 = 40$	26	10	42.8	54.6	38.7	12.2	55.6	49.1	21.6
$D = 84, \psi_2 = 42$	*28*	*10*	*41.8*	*67.1*	*33.5*	*13.7*	*68.8*	*42.6*	*22.1*
$D = 88, \psi_2 = 44$	30	10	44.2	85.9	32.3	15.4	89.0	39.4	22.6
$D = 92, \psi_2 = 46$	30	12	45.2	51.3	45.4	17.7	53.9	51.5	24.6
$D = 96, \psi_2 = 48$	32	12	44.1	59.2	42.0	19.6	61.5	46.1	25.2

policies. RV1, which ignores any available arrival information, yields a somewhat higher overall average waiting time: +6.1%. Seemingly, considering arrival information is more important at the simple F4C2 intersections of the previous sections than at the more complex F12C4 intersections. The best coordinated FC results in an average waiting time per car that is 29% above that under RV1(5). The best cyclic exhaustive control policy (XC-2) yields a much higher average waiting time: 18.1% above RV1(5).

Table 8.14: Mean waiting times (in sec.) for I2F12C4 at $\rho_i = 0.8$ and thru traffic three times as busy than left and right turning traffic (i.e. $\lambda^i_f = 0.1$, except $\lambda^i_2 = \lambda^i_5 = \lambda^i_8 = \lambda^i_{11} = 0.3$); car speeds $\sim$ Tri(40,50,60).

		% above	Internal lanes			External lanes		
Policy	EW	RV1(5)	Left	Thru	Right	Left	Thru	Right
Cyclic policies:								
RV1(5)[a]	32.3		44.5	29.9	17.5	45.9	32.3	20.9
RV1[a]	34.3	+6.1%	47.1	33.9	20.8	47.5	34.4	21.4
FC ($D = 84, \psi_2 = 42$)	41.8	+29.3%	67.1	33.5	13.7	68.8	42.6	22.1
XC-2	38.4	+18.9%	49.2	38.3	27.0	48.6	38.9	27.5
Acyclic policies:								
RV1M(5)[a]	31.7	−1.7%	48.7	29.3	18.2	48.9	30.2	19.6
RV1M[a]	33.4	+3.3%	51.2	31.8	19.8	51.4	31.9	19.9
XA-2	37.7	+16.8%	73.8	29.4	21.2	74.7	31.3	22.8

[a]Based on FC with $D = 84$, $\psi_2 = 0$, $d_{i,1} = d_{i,3} = 28$ and $d_{i,2} = d_{i,4} = 10$.

Looking at the average waiting times at the different lanes, we observe that, on average, left turning cars are queued longest. Right turning traffic instead, enjoys a relatively long green period and thus has to wait shortest, since it is part of thick combinations. The waiting time for thru traffic is not very high, as the thru traffic plays a dominant role for having the highest arrival rate.

The highest average waiting times under cyclic control are under FC, in particular to left turning traffic. Under RV1(5) the average waiting times at all queues, except at Q_6^1 and Q_{12}^2, are much lower than under any of the other policies.

Acyclic control

When the control of the lights is no longer restricted to cyclic policies, one may select any combination to serve next after an all red phase. Under the acyclic exhaustive control policies that we have considered, one switches, after an all red slot, to the combination that takes longest until it is exhausted: the expected time to exhaustion is estimated based on the actual queue lengths and the constant arrival and departure rates.

Under exhaustive control we now observe much higher average waiting times to left turning traffic: at these queues XA2 results in an average waiting time that is 50% higher than under XC-2. Under the acyclic RV1 policies the average waiting times are much lower.

Regarding the overall average waiting time the difference between RV1(5) and the acyclic RV policies is only 1.7%. Since the difference is very small, cyclic policies may be preferred for psychological reasons. Although not reported, we have not seen a great improvement by RV2M and RV2M(5).

Conclusions

- Even for the more complex intersection with 4 combinations, cyclic control performs almost as good as an acyclic control policy.
- Adding arrival information to an RV policy gives a much smaller improvement for F12C4 intersections than for F4C2 intersections.
- RV1 and RV1(5) greatly improve the best FC: the waiting time at most internal and external queues are significantly lower under RV1(5).
- Under the RV1 policies, the difference between internal and external queues is very small.

8.8 Study V – Network of 9 intersections (I3x3F4C2)

8.8.1 Problem and data

In this final section, we demonstrate the application of the RV rules to a network of intersections on a grid. We consider the network depicted in Figure 8.13, which we refer to as the I3x3F4C2 infrastructure. The nine intersections of the F4C2 type are numbered 1 to 9 (row-by-row and from left to right). The distance between any two neighboring intersections is 500 meters.

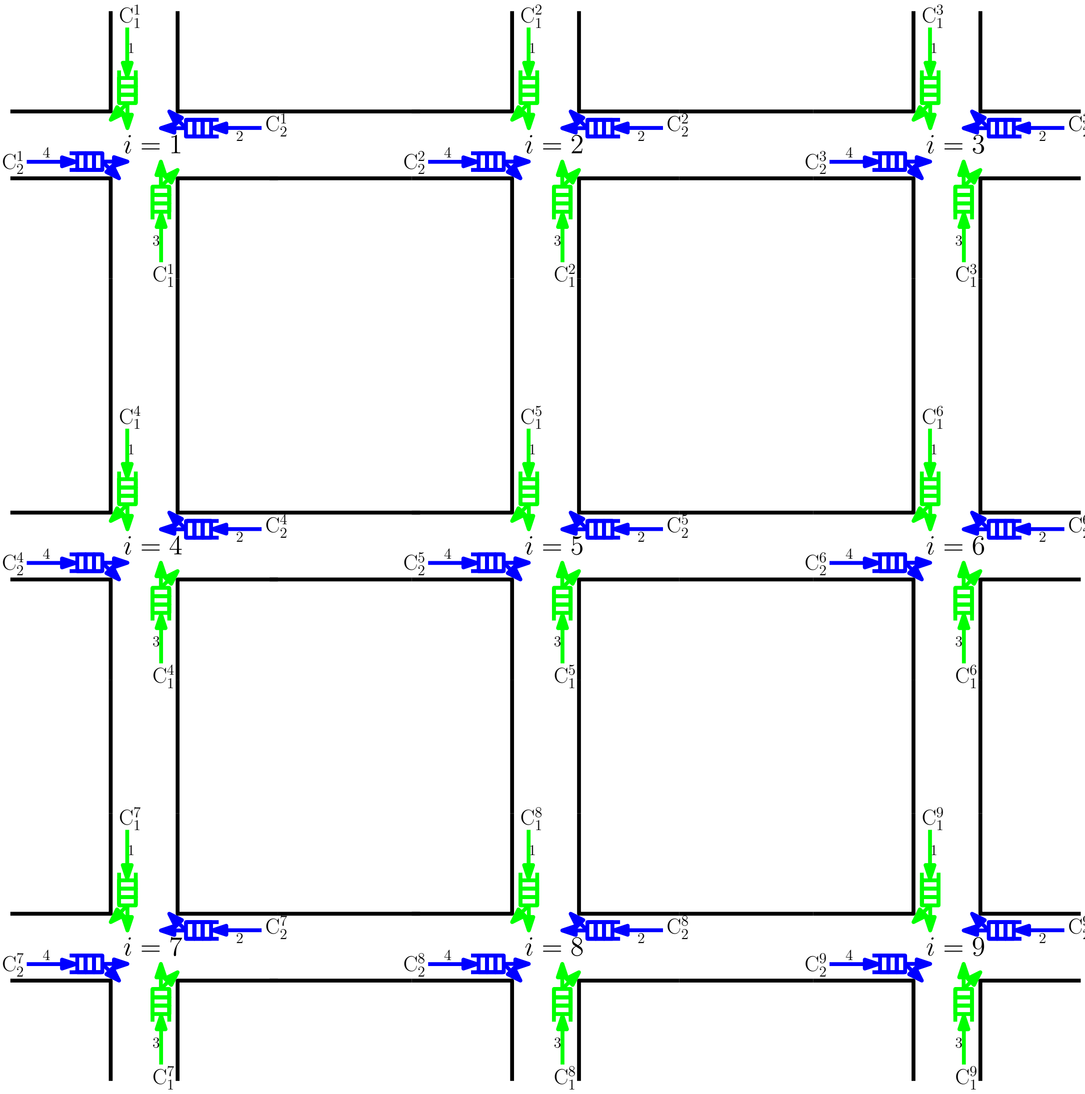


Figure 8.13: Network of 9 simple intersections: I3x3F4C2.

Under to-FC-ideal conditions (identical car speeds and thru traffic only), green waves may be set in both directions. Then the waiting time at intersection 5 can be zero under coordinated FC, as intersection 5 is the center of the network and is fed only by traffic leaving the neighboring intersections. However, we believe that such conditions are not very realistic. Therefore we assume desired car speeds to vary according to a Triangular distribution with mean 50 km/h and minimum and maximum speeds 40 and 60 km/h respectively.

We assume flow-dependent arrival rates that are identical to all intersections: $(\lambda_1^i, \lambda_2^i, \lambda_3^i, \lambda_4^i)$ $= (0.32, 0.16, 0.16, 0.48)$. The load of each intersection is thus 80%. At each intersection, the busiest flow is flow 4 that has on average 3 times as many cars as flows 2 and 3. The load of flow 1 is two times that of flows 2 and 3.

A fraction of the cars may turn right. For the given infrastructure there is no left turning traffic and thus $P_f^i(\mathrm{T}) = 1 - P_f^i(\mathrm{R})$ for all flows f. The flow-dependent routing probabilities are such that the arrival rates (λ_f^i) are preserved. Hence $P_f^i(\mathrm{R}) = P_f(\mathrm{R})$ must satisfy the set of balance equations (8.3) – (8.6) for flow 1 to flow 4 at all intersections.

$$0.32 = 0.32 \cdot (1 - P_1(\mathrm{R})) + 0.48 \cdot P_4(\mathrm{R}) \tag{8.3}$$

$$0.16 = 0.16 \cdot (1 - P_2(\mathrm{R})) + 0.32 \cdot P_1(\mathrm{R}) \tag{8.4}$$

$$0.16 = 0.16 \cdot (1 - P_3(\mathrm{R})) + 0.16 \cdot P_2(\mathrm{R}) \tag{8.5}$$

$$0.48 = 0.48 \cdot (1 - P_4(\mathrm{R})) + 0.16 \cdot P_3(\mathrm{R}) \tag{8.6}$$

$$P_1(\mathrm{R}),\ P_2(\mathrm{R}),\ P_3(\mathrm{R}),\ P_4(\mathrm{R}) \in [0,1] \tag{8.7}$$

One of the (infinitely many) solutions is $P_1(\mathrm{R}) = 0.15$, $P_2(\mathrm{R}) = 0.3$, $P_3(\mathrm{R}) = 0.3$, and $P_4(\mathrm{R}) = 0.1$. Then on average 17% of the cars turns right. This case is considered throughout the next subsections.

Since the routing probabilities are memoryless, a car may pass an intersection multiple times before actually leaving the network. Given the simulation model, we have to accept this phenomenon, but this can be avoided easily when the routes that cars will travel are set upon the generation of car arrivals at the borders of the network.

8.8.2 Optimization of coordinated FC

For the computation of the relative values required for the RV rules, we simply optimize FC for an intersection as if it is in isolation. A locally optimal uncoordinated FC can be found by applying the Optimal-fixed-cycle algorithm. Figure 8.14 shows the incremental search process: only the best increments of the cycle length by 1 slot are accepted and reported in the figure. Due to the asymmetry in the arrival rates, the local optima are unequally spread over the range of considered cycle lengths. The best cycle length seems to be $D = 44$ seconds. The duration of the effective green periods of the two combinations are then 16 and 24 seconds, for C_1 respectively C_2.

The best coordinated FC likely has a cycle length not too far from this optimum value. To be sure to compare the RV policies to the best possible coordinated FC, we check a whole range of cycle lengths D and offsets $(\psi_1, \psi_2, \psi_3, \ldots, \psi_9)$. The Optimal-fixed-cycle algorithm helps in finding a set of promising cycle lengths. Related to each cycle length D the algorithm gives for each combinations s a best effective green time $d_{i,s}$.

The number of choices for the offsets, can be limited since the infrastructure is well-structured: the traveling time between any two adjacent intersections is the same. Therefore we only consider offsets that satisfy: $(\psi_1, \psi_2, \ldots, \psi_9) = (0, 1, 2,\ 1, 2, 3,\ 2, 3, 4)\psi$. A

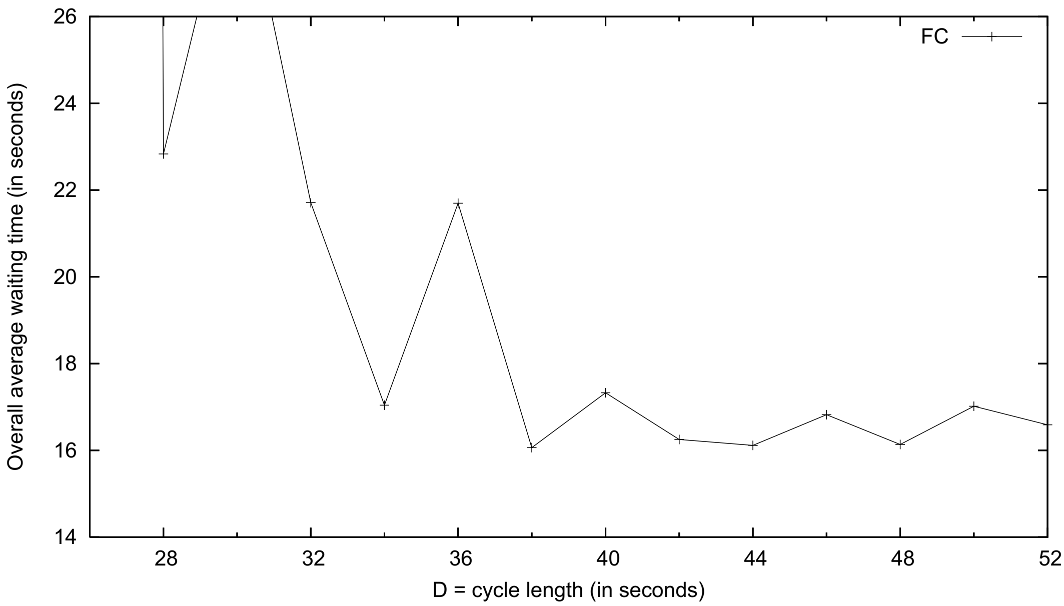


Figure 8.14: Application of the Optimal-fixed-cycle algorithm to determine (by simulation) optimal value of the cycle length D for F4C2 with non-identical arrival rates $(\lambda_1^i, \ldots, \lambda_4^i) = (0.32, 0.16, 0.16, 0.48))$; car speeds $\sim$ Tri(40,50,60).

wide range of values of ψ is considered: $\psi \in \{34, 36, \ldots, 48\}$ seconds. All pairs (D, ψ) are evaluated by simulating the entire network. The best pair (D, ψ) appears to be $(44, 42)$ seconds, with effective green periods of $d_{i,1} = 16$ and $d_{i,2} = 24$ seconds for C_1^i and C_2^i respectively.

8.8.3 Comparison of policies

In Table 8.15 we report the long-run average waiting times at an intersection per car under the various policies. To judge the quality of each policy, we primarily focus on the overall average waiting time, EW (which is the average waiting time that a car experiences at an (arbitrary) intersection). Since intersection 5 is fed by internal arrivals only, we also study the waiting time (EW^5) at this intersection. The progression at this central intersection is investigated by looking at the average waiting time at each queue of this central intersection: EW_1^5 to EW_4^5.

The overall average waiting time (per car per intersection) under the best coordinated FC appears to be about 29% above that under RV1(5). The best exhaustive control policy, XC-2, yields a lower waiting time than coordinated FC, but is still 21% above that of RV1(5). The use of arrival information turns out to be crucial to the RV policies: with no slots of arrival information RV1 performs a bit worse than coordinated FC.

In the case reported in Table 8.15, the control of the lights is dominated by the queue lengths of Q_1^i and Q_4^i. The system approximately behaves as a polling system, since each combination consists of only 1 thick flow next to a thin flow. For polling models with a similar cost structure, exhaustive control is optimal. It is thus not surprising that RV1

Table 8.15: Mean waiting times (in sec.) for skew I3x3F4C2 at $\rho_i = 0.8$ ($\lambda_1^i = 0.32$, $\lambda_2^i = \lambda_3^i = 0.16$ and $\lambda_4^i = 0.48$); 17% of cars turns right and car speeds $\sim$ Tri(40,50,60).

		% above	Waiting time at intersection 5				
Policy	EW	RV1(5)	EW^5	EW_1^5	EW_2^5	EW_3^5	EW_4^5
RV1(5)[a]	9.6		9.4	11.8	6.0	10.1	8.6
RV1[a]	12.7	+33%	13.1	17.2	6.1	11.9	13.2
FC, coordinated[b]	12.3	+29%	10.2	14.5	7.4	8.7	8.8
XC-2	11.6	+21%	11.7	14.0	6.0	10.2	12.6

[a]Based on FC with $D = 44$, $d_{i,1} = 16$ and $d_{i,2} = 24$ seconds.
[b]FC with $D_i = 44$, $\psi = 42$, $d_{i,1} = 16$ and $d_{i,2} = 24$ seconds.

does not beat XC-2. (As we have seen in Chapter 6, XC-2 performs well, in particular, for cases where a single flow dominates all other flows in the same combination. RV1 performs very well for both simple cases and more complex cases with multiple thick flows in the same combination or with identical arrival rates per flow.)

With respect to the progression at the internal lanes leading to intersection 5, RV1(5) performs better than coordinated FC, except at flow 3. C_1^5, which contains flow 3, is the thinnest combination of the two combinations. Since the thickest combination, C_2^5, dominates over C_1^5, the highest average waiting time per car is observed at the thickest flow of C_1^5: flow 1. Analogously, the lowest average waiting time is observed at the thinnest flow of the thickest combination: flow 2.

Conclusions for I3x3F4C2

For I3x3F4C2, with varying car speeds and with on average 17% of the cars turns right, we conclude that RV1(5) is favored above coordinated FC for three reasons:

1. RV1(5) yields the lowest overall average waiting time per car,
2. the average waiting time per car under RV1(5) is lower at most of the internal queues than under coordinated FC, and
3. setting an initial FC for RV1(5) is much easier than finding a good coordinated FC.

8.8.4 Sensitivity regarding the fraction of cars that turns right

We consider three different scenarios to investigate the impact of routing probabilities on the average waiting time per car at an arbitrary intersection under the different policies. The three scenarios, labeled V-A to V-C, are presented in Table 8.16. In Scenario V-A no cars turn right. Scenario V-B is the one studied in the previous sections. In Scenario V-C twice as many cars turn right as in Scenario V-B.

Table 8.16: Scenarios for fraction of cars that turns right in I3x3F4C2 network.

		Routing probabilities			
Scenario	Description	P_1(R)	P_2(R)	P_3(R)	P_4(R)
V-A	No cars turn right	0	0	0	0
V-B	About 17% of the cars turns right	0.15	0.3	0.3	0.1
V-C	About 34% of the cars turns right	0.30	0.6	0.6	0.2

Table 8.17: Mean waiting times (in sec.) for skew I3x3F4C2 at $\rho^i = 0.8$ ($\lambda_1^i = 0.32, \lambda_2^i = \lambda_3^i = 0.16$ and $\lambda_4^i = 0.48$): different percentages of right turning traffic.

	Scenario V-A		Scenario V-B		Scenario V-C	
Policy	No cars turn right		17% turns right		34% turns right	
RV1(5)[a]	8.9		9.6		10.0	
RV1[a]	12.9	+44%	12.7	+33%	12.6	+27%
FC, coordinated	8.8[b]	−0.9%	12.3[c]	+29%	14.4[d]	+44%
XC-2	11.6	+30%	11.6	+21%	11.6	+16%

[a] Based on FC with $D_i = 44$, $d_{i,1} = 16$ and $d_{i,2} = 24$ seconds.
[b] FC with $D_i = 42$, $\psi = 42$, $d_{i,1} = 16$ and $d_{i,2} = 22$ seconds.
[c] FC with $D_i = 44$, $\psi = 42$, $d_{i,1} = 16$ and $d_{i,2} = 24$ seconds.
[d] FC with $D_i = 48$, $\psi = 40$, $d_{i,1} = 18$ and $d_{i,2} = 26$ seconds.

In Table 8.17 we present the overall average waiting time per car for all three scenarios. As one may expect the waiting time is lowest when all traffic is thru traffic. Setting coordinated FC is then relatively easy, but since car speeds differ it is not possible to set a green wave to all cars arriving at the internal queues. When no cars turn right, the best coordinated FC performs slightly better than RV1(5), since platoons of cars are less dispersed and all cars arriving at an internal queue originate from the same queue.

When the percentage of right turning traffic increases, the impact of the arrival information reduces, as becomes clear from comparing RV1(5) to RV1. The gap between FC and RV1(5) grows when more traffic turns right, since it becomes much harder for FC to coordinate the lights. (This statement may no longer hold when more than half of the cars turn right, since then the optimal coordinated FC would have a different offset.) XC-2 is not sensitive to changes in the fraction of cars that turn right. The relative performance of XC-2, compared to RV1(5), improves when more cars turn right.

For the given network of nine F4C2 intersections, XC-2 may beat RV1 but not RV1(5). XC-2 performs quite well, since the arrival rates are very skew such that one flow dominates in each combination. The system then behaves like a polling system. However when the arrival rates of flows in the same combination would be close close to each other, or when the intersections are more complex, the difference between RV1(5) and XC-2 becomes bigger as we have seen before. We believe that RV1(5) is a promising policy that gives very good results and that is often superior to all other strategies.

8.9 Conclusions

In this chapter we have learned that controlling the traffic lights in a network of intersections is a challenging task. In general an optimal dynamic, state-dependent solution, which minimizes the average waiting time per car per intersection, cannot be obtained.

We have paid considerable attention to how to coordinate the traffic lights at different intersections, since coordinated control contributes to the minimization of the average waiting time per car. With respect to coordinated FC, we draw the following conclusions:

- Under FC so-called green waves can be set in multiple directions, but only under ideal conditions and for a limited number of cycle lengths.
- Green waves are easily lost when
  - platoons break due to different car speeds, and
  - cars may turn right, as internal arrivals originate then from different queues.
- Finding a well-coordinated FC requires an extensive search for a good network cycle $(D, d_{i,s}, \psi_i)$. This search can be streamlined by using experience, good intuition and insights for the specific infrastructure.

To reduce the long-run average waiting time per car, dynamic control based on local information may be preferred over coordinated FC. Centralized coordinated dynamic control may be quite complicated: when the traveling time between any two neighboring intersections is T seconds, it requires to coordinate the current decision with the decisions taken T seconds ago at the neighboring intersections. Even when the historic information on the state of neighboring lights is available to the controller, it might not be very helpful in predicting car arrivals, as cars speeds may vary. Then it might be better to use accurate information on the current state of the traffic lights and accurate estimates of arrival times at each intersection.

Therefore we propose the one-step policy improvement algorithm, RV1(5), which was already introduced in Chapter 7. Coordinated FC may outperform RV1(5) under (utmost) ideal conditions for FC. As often the conditions are not ideal, RV1(5) is preferred for three reasons:

- RV1(5) is easy to configure, since the optimization of the underlying FC is quite simple: the Optimal-fixed-cycle algorithm provides the locally optimal FCs for each intersection in isolation.

- RV1(5) looks ahead a number of slots for which estimated arrival times of cars are known, and thus considers the most accurate information available to synchronize the control of the lights.
- RV1(5) gives great improvements over coordinated FC and XC-2, especially for a number of complex cases where cars speeds may differ and where:
  - the intersections have unequal load, or
  - the intersections are unequally spaced, or
  - multiple flows are combined in the same combination, or
  - the arrivals at an internal queue originate from different upstream queues, or
  - all flows within the same combination have virtually identical arrival rates, such that none of the flows dominate.

For most of the network scenarios that we have studied, we have seen that even the best coordinated FC may yield an average waiting time per car that is 28–56% higher than that under RV1(5). In some cases XC-2 performs better than coordinated FC, but in most cases the waiting time is still about 20% above that under RV1(5).