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Francke, M.K.

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The Hierarchical Trend Model

Marc K. Francke

Introduction

This chapter presents a time-series model for the selling prices of houses, called the hierarchical trend model. In The Netherlands, this model has been used to value approximately one million houses for property tax purposes without any significant problems. In the city of Amsterdam this model has been operational for more than ten years.

The size of the tax depends on the value of the property. Municipalities are allowed to raise part of their income by levying a tax on property within their boundary. Moreover, the law WOZ (Waardering Onroerende Zaken)[1] requires that the value thus determined be used for other legal purposes, such as the levy that the water board can raise and income taxes levied by central government. The municipalities have to determine the property market value, which has been defined as 'the value when full and unencumbered ownership is transferred and the buyer can take possession of that immediately and completely'. The value has to be determined every year, and for this reason mass appraisal techniques have become quite popular in the Netherlands.

Another application of the hierarchical trend model (which will be referred to as HTM) is to determine constant quality local price indices. The HTM is very useful in determining price indices for thin markets where relatively few observations are available. The price indices produced by the HTM measure the price developments of a standardized house of constant quality over time. Therefore the HTM corrects for differences in characteristics of the sold houses.

The HTM is a statistical model. A statistical model imposes *a priori* a structure on the data and requires distributional assumptions. It is sometimes claimed that other methods, such as neural networks, do not have distributional assumptions, but this is in general not true. The statistical framework enables testing for model assumptions and comparison of competing statistical models. Moreover, the strength of a statistical model is that it offers a coherent method for inference. Model results, for example, the implied relation by the model between house size and selling prices, can easily be explained to a non-statistical public, such as real estate valuers and tax payers.

An objection to statistical models, like the linear regression model, could be that these models are too rigid. However, within the statistical approach there are a large number of methods that can be used for flexible non-linear modelling, for example, state–space models. The HTM is an example of a state–space model.

The main strengths of the HTM are its modelling of the time-dependence of the selling prices and its sophisticated way of modelling the housing characteristics. The HTM also addresses the problem of spatial dependence of the selling prices, but in a rather straightforward way. In the HTM some parameters vary over time and other parameters are constant over time.

The time-independent part of the HTM concerns the specification of the housing characteristics, for example, the size of the house, the lot size, the year of construction and the condition of its maintenance. A striking feature in the many years that the model has been used is that it remains relatively simple without interaction terms and characteristics of minor importance. This would make the model too sensitive for incorrect or lacking data. 'Keeps it sophisticatedly simple' (KISS) appears to be a valuable motto. The time-independent part is a non-linear specification that enables separation of the value of the building and the value of the land. The assumption that these variables are constant over time can be relaxed.

One may see this part of the model as a way to adjust selling prices for differences in characteristics, thus giving standardized prices. These are used in the superimposed time-series model.

The time-varying part of the HTM consists of a general trend and other cluster aspects that evolve over time. Examples of the latter are districts and house types. The general trend has a more advanced time-series model specification. The cluster evolving processes are modelled as random walks in deviation from the general trend. The model does not use time dummies, because this would make forecasting impossible.

Together with the influence of the individual characteristics, the general trend, the district and house type time components are estimated within the HTM. The price development for a specific market segment, a combination of a district (j) and house type (k), is the sum of the general trend, the district (j) and the house type (k) time component. The trends are measured on a monthly basis.

The HTM is an example of a state–space model. State–space models are common in time-series econometrics, but are rarely used in applied real estate research. The main attractiveness of the Kalman filter recursions in state–space models is that they produce recursive predictions of the next period's observations based on information up until the present, so real predictions. The predicted values are compared to the actual values. The reliability of the predictions is provided by the log-likelihood, the ultimate measure of quality of forecasts.

A special feature of the Kalman filter is that it not only provides prediction, but also optimal revision of the estimates of the state (the trend, etc.), as time proceeds. In The Netherlands houses are valued at the price level from two years ago, so this is an important feature.

Besides the temporal dependence, the spatial dependence of selling prices plays a role in the HTM. In spatial econometrics two notions play a role, spatial heterogeneity and spatial dependence, see for example, Anselin (1988). Spatial heterogeneity says that functional forms and parameters vary with location and are not homogeneous throughout the data set. This is, for example, the case in the district time component. Spatial dependence says that the variation is a function of the distance. Spatial models for housing prices can be specified on an individual level (by using (x, y)-coordinates) and on a cluster level, for example, neighbourhood, or city level. Examples of spatial models on an individual level are given by Can (1992), Dubin (1992, 1998), and Wolverton and Senteza (2000). An example of a spatio-temporal model is given by Pace and colleagues (1998). These models are difficult to evaluate and are not considered here. In the HTM, the spatial dependence is modelled on a cluster level basis and by specific locational characteristics. Every cluster has an individual price trend. Within clusters, the price levels may vary over different neighbourhoods. The price levels are modelled as random effects within a cluster. Disadvantages in the use of cluster levels include the fact that there might be undesirable discontinuities at the borders, and that it requires knowledge of the spatial structure, which might be different from available administrative clusters.

The set-up of this chapter is as follows. In the next section the specification of the hierarchical trend model is provided. The third section concerns the estimation of the HTM. The fourth section provides some estimation results. The fifth section concludes.

Model description

The dependent variable

The dependent variable in the HTM is the natural logarithm of the selling price. Of course, other transformations of the selling price are possible and can be tested for, for example, by the **Box–Cox** transformation, see Halvorsen and Pollakowski (1981). There are several reasons to use the natural logarithm of the selling price. The first is that we assume that the time components for district and house type work in a multiplicative way. Taking the logarithm of a multiplicative model results in an additive model that can easily be dealt with by standard statistical methods such as linear regression. Another reason is that the goal is to minimize the relative standard deviations (in percentages) instead of the absolute standard deviation. In the standard linear model, the residual sum of squares, and hence the standard deviation, is minimized. This means that by taking the logarithm of the selling price (Y) the relative errors $(Y - M)/M$ are approximately minimized, where M is the model value. If the dependent variable is not the logarithm of the selling price, but the selling price itself, the absolute errors are minimized. In the first case an error of €5000 on a selling price of €50 000 has a greater impact on the standard deviation than an error of €5000 on a selling price of €200 000. In the last, both errors have the same impact on the standard deviation of the residuals. An additional assumption is that the error terms have a log-normal density, which can be checked for by evaluating the residuals.

Specification of the time-independent part of the model

This section concerns the specification of the influence of the individual characteristics of the house, such as the house and lot size, on the selling price. The value of a house (V) can be written as the value of the land (L) plus the value of the buildings (B),

$$V = B + L \tag{8.1}$$

Although in practice it rarely occurs that undeveloped land is sold, one of the demands of real estate valuers is that the model should separate the value of the land and the value of the building. Another constraint is that characteristics, like the year of construction and the maintenance condition, should only influence the value of building, and not the value of the land. Variables such as the selling date, the location and the house type, influence both the building and the land value. The specification of these variables will be dealt with in the next section.

Another demand on the model is that, for the house and lot size, the law of diminishing returns must hold.

Let us first consider the specification of the house size (x_1). In the log-linear specification the selling price depends on the house size in the following way,

$$y = \beta_1 \ln x_1 + Z\delta + \varepsilon \tag{8.2}$$

where y is the natural logarithm of the selling price, Z contains other housing characteristics and a constant, and ε is the error term. In this specification an increase of 1% in house size will result in approximately β_1% increase in value. It is expected that the coefficient $\beta_1 < 1$, so the value of the house increase is less than proportional to the house size. This is another reason to use the natural logarithm of the selling price as the dependent variable.

If we added the lot size (x_2) to equation (8.2) in the following way,

$$y = \beta_1 \ln x_1 + \beta_2 \ln x_2 + Z\delta + \varepsilon \tag{8.3}$$

then (a power of) the house size is multiplied by (a power of) the lot size, as can be seen from

$$Y = x_1^{\beta_1} x_2^{\beta_2} \exp(Z\delta) \exp(\varepsilon) \tag{8.4}$$

where Y is the selling price. It is clear that specification (8.3) makes no sense.

Another option is to model house and lot size like

$$\begin{aligned} y &= \ln(x_1^{\beta_1} \exp(Z\delta) + \beta_2 x_2) + \varepsilon \\ Y &= (x_1^{\beta_1} \exp(Z\delta) + \beta_2 x_2) \exp(\varepsilon) \end{aligned} \tag{8.5}$$

In this specification the value of the lot size is added to the value of the house, and the value of the house is less than proportional to the house size if $\beta_1 < 1$.

For moderate lot sizes, specification (8.5) works quite well in practice. However, the assumption that the value is proportional to the lot size is not valid for large lot sizes. In practice, real estate agents often use a step function for the valuation of the lot, as shown in Figure 8.1. The first 300 m^2 counts for 100%, from 300 m^2 till 500 m^2 counts for 53%, and so on. The choice of the borders and percentages are both arbitrarily chosen.

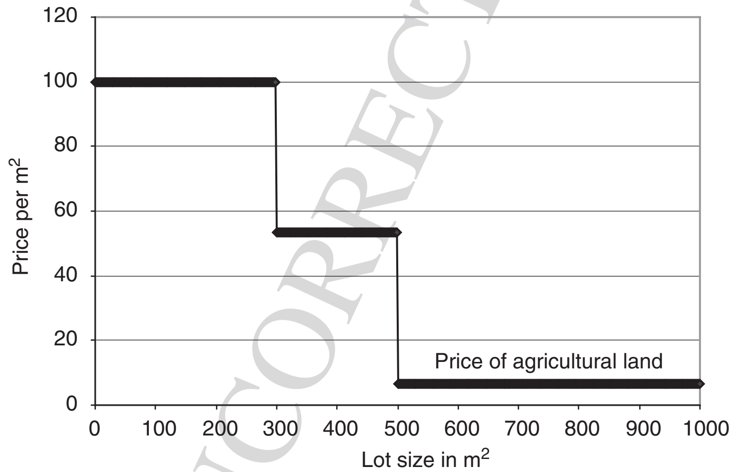

Figure 8.1 Step function of the price per square metre for lot size.

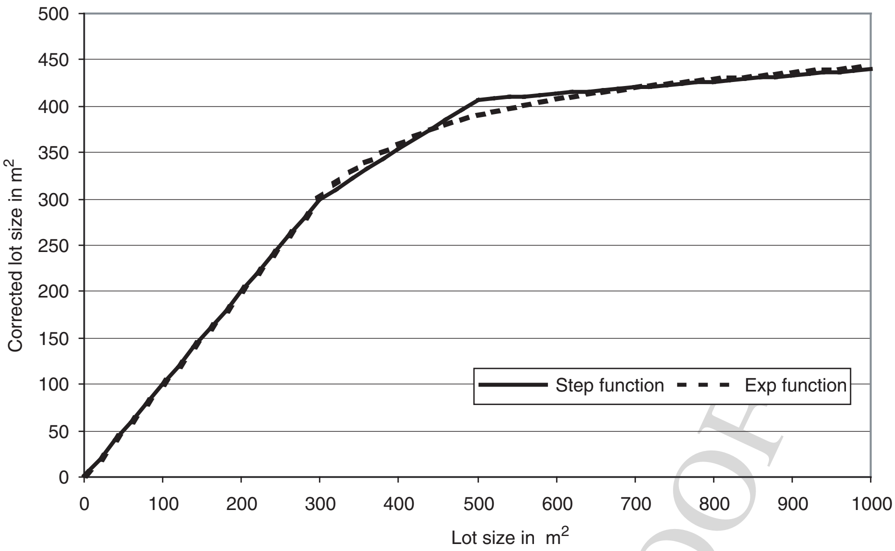

Figure 8.2 Step and exponential function for lot size.

The step function of the marginal price per square metre can be replaced by an exponential function

$$\alpha + \kappa \exp(-\gamma s) \tag{8.6}$$

where s is the number of square metres. For $s = 0$ the marginal price of the lot size has its maximum $\alpha + \kappa$. For s going to infinity the marginal price per square metre goes to α, a sort of agricultural land value. The ratio between the minimum and maximum marginal price per square metre is thus $1 + \kappa/\alpha$. The parameter γ determines the rate at which the marginal price goes to α. The value of a lot size of x_2 is the integral over (8.6) from 0 to x_2, and is given by

$$x_2^* = \alpha x_2 + \frac{\kappa}{\gamma}(1 - \exp(-\gamma x_2)). \tag{8.7}$$

It can also be seen as a corrected lot size. In Figure 8.2 corrected lot size by the step function and exponential function are shown. This correction is applied in order to obtain a more valid indicator of the lot size.

The parameters α, κ, γ may vary over different districts, for example, the city centre and outskirts. These parameters may be estimated by maximum likelihood. A more practical policy is to use some prior ideas from step functions and to choose the parameters α, κ, γ such that the corrected lot size by the step function almost coincides with the corrected lot size by the exponential function, as in Figure 8.2.

The specification that is used in the hierarchical trend model for the time-independent part is the combination of equations (8.5) and (8.7), resulting in

$$f(x,\beta) = \ln(x_1^{\beta_1}\exp(Z\delta) + \beta_2 x_2^* + \beta_3 x_3) + \beta_4 \ln x_4 \tag{8.8}$$

where the variable Z contains all variables relating to the house, such as year of construction and maintenance condition. The variable x_3 contains parts other than land and buildings, like garages and sheds. The variable x_4 concerns both the building and land value and the value of the other parts, for example, locational variables.

From equation (8.8) it is possible to derive the value of the land and the value of the building,

$$\begin{aligned} B &= x_1^{\beta_1} \times \exp(Z\delta) \times x_4^{\beta_4} \\ L &= \beta_2 \times x_2^* \times x_4^{\beta_4} \end{aligned} \tag{8.9}$$

Equation (8.8) is a non-linear specification that cannot be estimated by ordinary least squares (OLS). It is possible to linearize equation (8.8) by using the fact that, for small values of ξ, it holds that $\ln(1+\xi) \approx \xi$. A more general approach is provided by Gauss–Newton regression, as described by Davidson and MacKinnon (1993). It is a recursive procedure that uses the first derivative of the non-linear function.

The hierarchical trend model

The temporal dependence of selling prices can be addressed in several ways. Fleming and Nellis (1992) propose repeated regression for every time period, for instance a year. In this way parameters can vary over time. There are two major drawbacks to this approach. The first is that this set-up is not parsimonious, because an extra parameter is added to the model for every time period per characteristic. Small numbers of observations, which are the case when, for example, short time periods like months are considered, may lead to unreliable estimates. The second disadvantage is that a parameter value in one period does not affect the parameter value in the next and previous period. So this is not a very efficient approach and, more importantly, it makes prediction impossible.

An alternative to this set-up is to assume that parameters may evolve over time by defining a stochastic structure on the parameter evolution as, for example, proposed by Schwann (1998). In this approach the parameter value in the current period is influenced by the parameter values in the previous and next periods. This can be done for all model parameters, or for a subset of the parameters. An example of a model for the parameter evolution is simply: 'the best prediction of the value in the next period is the value in the current period'. A structure is imposed on the parameters' evolution over time. This is not a deterministic pattern, but a stochastic structure, implying a flexible functional form.

In the hierarchical trend model a general trend, local and house type time components and specific housing characteristics play a role. It is a hierarchical model in which both a general trend, and cluster level aspects as deviations from the general trend, are specified. The clusters or market segments are combinations of districts and house types. The market segments are defined *a priori*. Every house in the same market segment is assumed to have the same price development.

Let us first assume a model where all selling prices follow a common trend, the general trend, which we can write as

$$y_t = \mathrm{i}\mu_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \tag{8.10}$$

where i is a vector of ones, y_t is the vector of selling prices at time t, μ_t is the general trend, and ε_t is the error term.

If we don't assume any structure on μ_t model (8.10) is simply a dummy variable model and the estimate of μ_t is the average of the selling prices at time t. Note that if no observations are available at time t no estimate of the price level at time t is available.

If we assume a stochastic structure on the general trend, μ_t can be specified in several ways, for example, as a random walk

$$\mu_{t+1} = \mu_t + \eta_t, \quad \eta_t \sim N(0, \sigma_\eta^2) \tag{8.11}$$

In this model the best prediction of the general trend level at time $t+1$ is the general trend level at time t. The estimate of μ_t based on all selling prices, is a weighted average of the mean of the selling prices at time t, and the means in previous and next periods, where the weights depend on the distance to time t and the ratio of the variances σ_ε^2 and σ_η^2. Note that if $\sigma_\eta^2 = 0$, $\mu_t = \mu$ is constant, and if $\sigma_\eta^2 = \infty$, the model reduces to the dummy variable model. In the case that $\sigma_\eta^2 < \infty$ we do have an estimate of μ_t in case no observations are available at time t, based on previous and next periods.

In the HTM we assume that the general trend follows a local linear trend model, which is given by

$$\begin{aligned} \mu_{t+1} &= \mu_t + \kappa_t + \eta_t, \quad && \eta_t \sim N(0, \sigma_\mu^2) \\ \kappa_{t+1} &= \kappa_t + \zeta_t, && \zeta_t \sim N(0, \sigma_\kappa^2) \end{aligned} \tag{8.12}$$

If $\sigma_\eta^2 = \sigma_\xi^2 = 0$ then $\kappa_{t+1} = \kappa_t = \kappa$, say, and $\mu_{t+1} = \mu_t + \kappa$ so the trend is exactly linear and equation (8.12) reduces to the deterministic linear trend plus noise model. The form (8.12) with $\sigma_\eta^2 > 0$ and $\sigma_\xi^2 > 0$ allows the trend level and slope to vary over time, see Durbin and Koopman (2001).

In the HTM district and house type time components are also distinguished as deviations from the general trend. The trend in district j for house type k is the sum of the general trend μ_t, the district time component ϑ_{jt} and the house type component λ_{kt}. The district time components ϑ_t and house type time components λ_t are specified as random walks, as in equation (8.11),

$$\begin{aligned} \vartheta_{t+1} &= \vartheta_t + \omega_t, \quad \omega_t \sim N(0, \sigma_\omega^2 I) \\ \lambda_{t+1} &= \lambda_t + \varsigma_t, \quad \varsigma_t \sim N(0, \sigma_\varsigma^2 I) \end{aligned} \tag{8.13}$$

where I is the identity matrix of appropriate dimension.

A district is divided into a number of neighbourhoods, for which we assume separate levels, denoted by ϕ. The neighbourhood levels are modelled as random effects, so

$$\phi \sim N(0, \sigma_\phi^2 I) \tag{8.14}$$

The equation (8.8) and the equations (8.12–8.14) form the basis of the hierarchical trend model. The HTM can be summarized as

$$y_t = \mathrm{i}\mu_t + D_{\vartheta,t}\vartheta_t + D_{\lambda,t}\lambda_t + D_{\phi,t}\phi + f(X_t, \beta) + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma^2 I) \tag{8.15}$$

where

$$\begin{aligned} \mu_{t+1} &= \mu_t + \kappa_t + \eta_t, & \eta_t &\sim N(0, \sigma_\mu^2) \\ \kappa_{t+1} &= \kappa_t + \zeta_t, & \zeta_t &\sim N(0, \sigma_\kappa^2) \\ \vartheta_{t+1} &= \vartheta_t + \omega_t, & \omega_t &\sim N(0, \sigma_\omega^2 I) \\ \lambda_{t+1} &= \lambda_t + \varsigma_t, & \varsigma_t &\sim N(0, \sigma_\varsigma^2 I) \\ \phi &\sim N(0, \sigma_\phi^2 I) \end{aligned} \tag{8.16}$$

The matrices D are selection matrices, containing 0 and 1 to select the appropriate district, house type and neighbourhood. It is assumed that $\mu_1 = 0$, and $\lambda_{1,1} = 0$, to avoid identification problems.

Equation (8.15) can be paraphrased:

the natural logarithm of the selling price of house i in district j for house type k in neighbourhood l at time t is

the level of the general trend at time t +
the level of district time component j at time t +
the level of house type time component k at time t +
the neighbourhood level l +
the influence of the individual characteristics for house i +
an error term.

A detailed description of the HTM is provided by Francke and Vos (2004).

State–space models and estimation of the HTM

The HTM is a time-series model that can be characterized as a *structural time-series*, or *unobserved components*, model. In a structural time-series model the observations are a function of trends, cycles, the regression component and an error. Structural time-series models can be written in a state–space format.

State–space models are common in time-series econometrics, but are rarely used in real estate research. A few exceptions are Schwann (1998), Chen and colleagues (2004), Schulz and Werwatz (2004) and Hannonen (2005).

A state–space model consists of two equations. In the first equation the relation between the unobserved components, the state vector α_t, and the observations y_t is provided. This equation is called the measurement equation. The second equation describes the evolution of the unknown state vector in time. This equation is called the transition equation. The state vector at time $t+1$ can be predicted by the transition equation from the state vector at time t. The initial state is denoted by α_1 and is specified by a separate equation.

If both the transition and measurement equations are linear and the disturbances are assumed to be normally distributed, the state–space model is called linear Gaussian. The linear Gaussian state–space model is provided by

$$\begin{aligned} y_t &= Z_t\alpha_t + \varepsilon_t \\ \alpha_{t+1} &= T_t\alpha_t + R_t\eta_t \end{aligned} \tag{8.17}$$

where

$$\begin{aligned} \alpha_1 &= a_0 + A_0\beta + R_0\eta_0, \\ \beta &\sim N(\beta_0, \sigma^2\Sigma) \end{aligned} \tag{8.18}$$

The disturbances $\varepsilon_t \sim N(0, \sigma^2 H_t)$ for $t = 1, \ldots, T$, and $\eta_t \sim N(0, \sigma^2 Q_t)$ for $t = 0, \ldots, T$, are independently distributed. The matrices Z_t, T_t, R_t, H_t, Q_t and Σ may depend on unknown parameters.

In the HTM the state vector α_t is the stacked vector $(\mu_t, \kappa_t, \vartheta_t', \lambda_t', \phi_t', \beta_t')'$. A part of the state vector is time-independent. It holds that $\phi_{t+1} = \phi_t$, and $\beta_t + 1 = \beta$.

Models in state–space format can be estimated by the Kalman filter. The Kalman filter is usually applied to uni-variate observations, but is well suited to deal with multi-variate observations with an unequal number of observations over time, like in the HTM, even if there are missing observations for some time points. Conditional on the unknown parameters in the matrices

Z_t, T_t, R_t, H_t, Q_t and Σ and σ^2 the Kalman filter (and smoothing) recursions provide estimates of the unknown state vector α_t conditional on all observations. The Kalman filter also directly produces the likelihood function, which can be optimized to obtain estimates of the unknown parameters. A detailed description of state–space models and estimation procedures by the Kalman Filter is provided by Harvey (1989) and Durbin and Koopman (2001). An alternative Bayesian approach to state–space models (dynamic linear models) is provided by West and Harrison (1997).

The Kalman filter assumes the first and second moment of the initial condition to be known, which is the case in equation (8.18) when $A_0 = 0$. In general this is not the case as (a part of) the initial state is unknown (diffuse), which is the case if the model includes non-stationary trends and regression coefficients, like in the HTM. Alternative estimation procedures to deal with a (partly) unknown initial condition are provided by the diffuse Kalman (de Jong, 1991) filter and the exact initial filter (Koopman, 1997).

An efficient procedure to estimate the HTM, a combination of ordinary least squares on the deviations on the means of selling prices and the diffuse Kalman filter on the means of selling prices, is given by Francke and de Vos (2000). This approach reduces the number of computations in the Kalman filter considerably by the reduced number of observations. All estimation procedures used in this chapter are written in GAUSS (Aptech Systems, Inc).

Estimation results

This section contains some estimation results of the hierarchical trend model for Heerlen, a city of approximately 91 000 inhabitants in the southeast of The Netherlands. The database contains 2658 selling prices of single-family houses in the period January 2001 until December 2004.The sold houses are situated in 52 different neighbourhoods, which are aggregated to 6 districts. The database contains 14 different house types, which are aggregated to 2 categories, as shown in Table 8.1. Price trends are estimated for 12 market segments. Time is measured in months.

In Table 8.2 the estimation results are provided for the time-invariant part of the HTM. The *house size*, age and maintenance variables only concern the building value (see equations (8.1) and (8.9)).

The coefficient for House size is 0.73: an increase of the house size of 10% gives an increase of value of approximately 7.3%. The variable *Age* indicates that the value of the building decreases with 0.75% a year.

The difference between the dummy variables *Age0020* and *Age2045* is negligible. The value of *Age00* is almost 0.15 higher than the other dummy age variables, meaning that the building value (B) of a house with a year of construction before 1900 is approximately 15% higher than a house with a

Table 8.1 House types.

House type	House type group	Number of observations
Detached house	1	159
Detached bungalow	1	12
Detached converted farmhouse	1	3
Semi-detached house	1	743
Semi-detached bungalow	1	16
Semi-detached converted farmhouse	1	1
Row house	2	1090
Row bungalow	2	17
Row drive-in	2	2
Corner house	2	506
Corner bungalow	2	7
Linked house	1	92
Linked bungalow	1	9
Linked converted farmhouse	1	1

Table 8.2 Estimation results for the hierarchical trend model.

Variable	Description	Coefficient	Standard error	*T*-value
HouseSize	House size in m^3	0.7339	0.0150	49.02
Age	selling year minus year of construction, if the year of construction >1945; 0 otherwise	−0.0075	0.0003	−22.69
Age20_45	Year of construction: 1920–1945	−0.3980	0.0154	−25.85
Age00_20	Year of construction: 1900–1920	−0.3971	0.0303	−13.11
Age00	Year of construction <1900	−0.2559	0.0589	−4.34
M1	Maintenance: poor	−0.3112	0.0598	−5.20
M2	Maintenance: below average	−0.1337	0.0305	−4.38
M4	Maintenance: good	0.0795	0.0230	3.45
LotSize	(Adjusted) Lot size in m^2	0.1230	0.0113	10.85
Dormers	Number of 'dormers'	3.7688	0.9724	3.88
Garage	Garage in m^3	0.1501	0.0191	7.87
Carport	Carport in m^2	0.2975	0.0955	3.12
Sunroom	Sun room in m^2	0.1315	0.0525	2.50
Cellar	Cellar in m^3	0.0913	0.0190	4.79
Detached	Detached house	0.1074	0.0121	8.90
Linked	Linked house	0.0597	0.0132	4.53
Corner	Corner house	0.0292	0.0070	4.18
Bungalow	Bungalow	0.1354	0.0168	8.08
	Number of observations	2658		
	Number of districts trends	6		
	Number of housing types trends	2		
	$\hat{\sigma}$	0.1200		
	$\hat{\sigma}_{\mu}$	0.0063		
	$\hat{\sigma}_{\kappa}$	0.0004		
	$\hat{\sigma}_{\vartheta}$	0.0041		
	$\hat{\sigma}_{\lambda}$	0.0003		
	$\hat{\sigma}_{\phi}$	0.1069		
	LL	1876.35		

year of construction between 1900 and 1945. An explanation for this is that some of these old buildings are listed buildings.

The difference in building value between poor and good maintenance is about 0.39, a difference of about 48% (= exp(0.39) −1).

The coefficient for the corrected lot size is 0.1230. The correction of the lot size is according to equation (8.7). The variables *Garage*, *Carport*, *Dormers*, *Sunroom* and *Cellar* are the variables x_3 in equation (8.8).

The dummy variables of *Detached*, *Linked*, *Corner* and *Bungalow* concern both the land and building value. A detached house is approximately 11% more expensive than a semi-detached house (omitted variable), and a corner house is about 3% more expensive than a row house (omitted variable). Note that the dummy variable coefficients for a detached house and a row house cannot be compared directly, because they are members of different house type groups, see Table 8.1.

The standard deviation of the measurement equation (8.15) is 0.12, meaning that about 66% of the residuals are within one standard deviation, and 95% of the residuals are within two standard deviations.

The standard deviation of the trend level, $\hat{\sigma}_\mu$, is 0.0063 and the standard deviation of the slope, $\hat{\sigma}_\kappa$, is 0.0004, meaning that the slope hardly varies over time. The drift coefficient is 0. As a result the local linear trend model can be simplified to a random walk model. The general price deviation per year has a standard deviation of $\sqrt{12} \times 0.0063 \approx 2.2\%$, meaning that next years' price level is in 66% of the cases within one standard deviation. The general trend is provided in Figure 8.3.

The standard deviation of the district time component $\hat{\sigma}_\vartheta$ is 0.0041, a bit smaller than the standard deviation of the general trend. The standard deviation of the house type group time component $\hat{\sigma}_\lambda$ is 0.0003. This indicates that the price trends hardly vary over the different house type

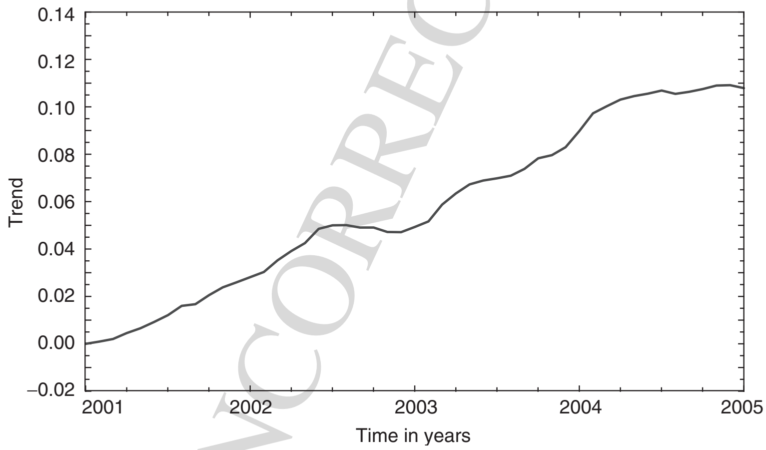

Figure 8.3 General trend estimated by the HTM.

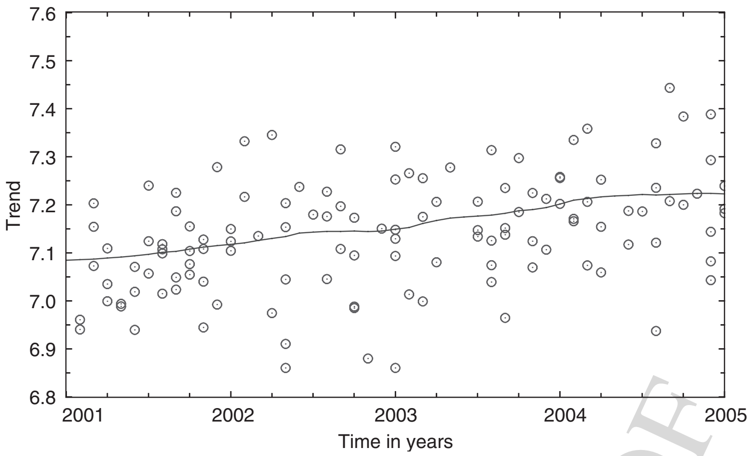

Figure 8.4 Trend for a specific market segment estimated by the HTM.

groups. An example of a trend for a specific market segment is shown in Figure 8.4.

The standard deviation of the random effects for the neighbourhoods $\hat{\sigma}_\phi$ is about 11%. This means that a neighbourhood level is in 66% of the cases within −11% and +11% from the district level. In Table 8.3 the neighbourhood levels are given. Note that the neighbourhood levels within a district sum up to zero.

The statistical approach has an important advantage when comparing different models. The likelihood of the model can be used as a formal criterion to compare competing models. The only problem is how to correct the log-likelihood for a different number of model parameters. Well known and often used tests are the Akaike information criterion and the Bayesian information criterion. In these measures the log-likelihood, evaluated in the estimated parameters, is corrected for the number of parameters in the model in order to have a fair comparison.

A more direct interpretable measure in Gaussian log-linear models is the standard deviation. In the estimated model the standard deviation in the measurement equation can be directly interpreted: 66% of the residuals are within one standard deviation (12%), and 95% of the residuals are within two standard deviations. Table 8.4 gives an overview of all kinds of more or less ad hoc criteria for model comparison.

Concluding remarks

In this chapter a structural time-series model for house prices is described that has proven its value for almost a decade. It is a model formulated in state–space format and it is estimated by the Kalman filter, which is optimal

Table 8.3 Neighbourhood levels.

District	Neighbourhood	Coefficient	Standard error	*T*-value	Number
1	60	−0.0420	0.0409	−1.03	73
1	62	−0.0187	0.0406	−0.46	86
1	63	−0.0299	0.0407	−0.74	90
1	64	−0.0239	0.0413	−0.58	59
1	65	−0.0199	0.0413	−0.48	59
1	67	0.0515	0.0408	1.26	82
1	70	0.0650	0.0709	0.92	2
1	80	0.0180	0.0399	0.45	160
2	45	0.0933	0.0498	1.88	7
2	47	0.0098	0.0381	0.26	26
2	48	−0.0566	0.0324	−1.75	226
2	49	−0.0607	0.0815	−0.74	1
2	50	0.1144	0.0402	2.84	18
2	51	0.0812	0.0341	2.38	90
2	53	−0.0683	0.0364	−1.88	35
2	54	−0.0648	0.0369	−1.76	32
2	55	−0.1687	0.0335	−5.04	99
2	56	0.1266	0.0694	1.82	2
2	57	−0.0578	0.0353	−1.64	49
2	58	−0.1402	0.0337	−4.17	86
2	59	0.1917	0.0329	5.82	128
3	18	−0.0820	0.0329	−2.49	87
3	20	−0.0888	0.0367	−2.42	31
3	30	−0.0328	0.0329	−1.00	120
3	34	0.0704	0.0340	2.07	98
3	36	0.0004	0.0347	0.01	45
3	37	0.1755	0.0704	2.49	2
3	38	0.1243	0.0346	3.59	48
3	39	−0.0377	0.0395	−0.96	19
3	40	0.0128	0.0398	0.32	19
3	41	0.0621	0.0358	1.74	38
3	42	−0.0419	0.0336	−1.25	64
3	43	−0.1731	0.0327	−5.29	102
3	44	0.0108	0.0334	0.32	77
4	12	−0.1142	0.0835	−1.37	1
4	14	0.0782	0.0591	1.32	9
4	15	0.0235	0.0538	0.44	41
4	16	−0.0109	0.0530	−0.21	115
4	17	0.0235	0.0551	0.43	24
5	21	0.0267	0.0415	0.64	18
5	22	0.0240	0.0371	0.65	43
5	23	0.0367	0.0374	0.98	40
5	24	−0.0171	0.0402	−0.43	23
5	25	0.0892	0.0386	2.31	30
5	26	−0.0487	0.0700	−0.70	2
5	27	−0.0446	0.0354	−1.26	85
5	28	−0.0996	0.0403	−2.47	22
5	29	−0.3079	0.0701	−4.39	2
5	31	0.1560	0.0444	3.52	13
5	32	0.2084	0.0406	5.14	23
5	33	−0.0232	0.0698	−0.33	2
6	11	0.0000	0.1069	0.00	5

Table 8.4 Model criteria.

Criterion	Percentage
Average error	0
Standard error	11.8
Mean absolute percentage error	9.3
Minimum percentage error	−42.6
Maximum percentage error	43.9
Percentage of absolute error <5%	33.3
Percentage of absolute error <10%	61.7
Percentage of absolute error <15%	80.4
Percentage of absolute error <20%	91.0
Percentage of absolute error <30%	98.6

when it is a Gaussian linear state–space model. This kind of model provides a direct interpretation of unobserved components, in the case of the hierarchical trend model, the trends and the coefficients for the time-invariant variables. It is a parametric model, but it allows for flexible forms by defining stochastic trends.

The hierarchical trend model in the form described here is the product of many decisions by the model developer. A few examples of the resulting (statistical) assumptions are given below:

- Some variables in the model are time and space invariant.
- The error term in the measurement equation is homoskedastic.
- The errors are assumed to be normal distributed.
- The *a priori* segmentation in district and house type groups.

There are many more. All these assumptions can be relaxed and tested for within the statistical context. The literature provides ample possibilities for generalizing the structure. The quality of any change in the model can be measured by the log-likelihood. This probably results in more complex models that are difficult to evaluate, and at present no standard estimation software is available. Our advice remains to keep it sophisticatedly simple (KISS).

Note

1. Valuation of Real Estate.

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AQ: Clarify if it is 2nd edition in the reference "West (1997)"