A new approach to distributed data fusion

de Waard, H.W.

Citation for published version (APA):

General rights
It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

Disclaimer/Complaints regulations
If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: http://uba.uva.nl/en/contact, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.
Chapter 8

A new TBM tracking method

The primary contribution of this chapter is the introduction of a new method to provide fast convergence to very accurate TBM position and velocity estimates and to demonstrate robustness against sensor error biases. The new method only uses the very accurate Doppler and range information provided by a distributed network of radar sensors. A comparison is made with single sensor and multisensor measurement tracking, which also use the inaccurate bearing and elevation information provided by the network of radar sensors.

8.1 Introduction

The Persian Gulf War in 1991, with Iraq’s use of Scud missiles, proved that the growing threat posed by ballistic missiles had to be addressed. Since that war, the threat from missiles has grown steadily as sophisticated missile technology becomes available on a wide scale. Former Warsaw Pact countries and nations such as Egypt, Iran, Libya, North Korea and Syria possess these weapons. It is not only a matter of short-range projectiles that can cover 600 to 1000 km, but also of medium-range (up to 3000 km), intermediate-range (up to 5500 km) and even intercontinental ballistic missiles which reach still further.

The larger the range of the ballistic missiles, the higher their maximum altitude will be. This in turn influences the rate of descent. The greater the velocity in coming down, which can easily reach $2500 \text{m/s}$, the harder it is to intercept them. The maximum altitude is approximately one-third of the range of the ballistic missile, but this depends on its trajectory. While trying to intercept a ballistic missile in flight, it is necessary to limit the consequences, such as a shower of debris, as much as possible. If the ballistic missile is successfully intercepted at extra-atmospheric altitudes, the debris are burnt during the re-entry.

\footnote{Patent pending.}
8.1.1 TBM aspects

For this section the terminology of Bloemen [28] is used. Single-stage TBMs have one stage that contains both the warhead and the propulsion system. Multiple-stage rockets use parts of the rocket sequentially for optimal propulsion with respect to the weight and velocity of the vehicle. The stages that are burnt out are detached from the missile to lower its dead weight and thus increase its velocity in a more energy-efficient manner. It should be noted that not every single or multiple-stage TBM will separate the warhead from the rest of the missile at the point of burn-out. Multiple-stage missiles, with each stage having its own independent propulsion system, are more efficient for longer-range missions.

Figure 8.1: An example of a TBM trajectory. Normally, burn-out and re-entry are not at the same altitude.

Generally, a TBM trajectory is divided into a number of phases and events (Fig. 8.1):

- **Boost phase**
  The boost phase begins with ignition of the missile and ends with the burn-out event, which occurs when the rocket propellant is depleted or after engine cut-off. In multiple-stage TBMs, a booster is ejected and a following stage is ignited.

- **Mid-course phase**
  The mid-course (or ballistic) part of a TBM trajectory is determined by the speed and angle at burn-out, and by gravity. Roughly 75 percent of the total duration of the TBM trajectory is spent in the mid-course phase. Maximum altitude is, by definition, reached at apogee.

- **Re-entry phase**
  The re-entry phase has no distinct starting point; it begins when the earth’s atmosphere begins to influence the TBM trajectory (at approximately 100 km altitude).

8.2 Literature overview

In literature two approaches are found to determine the position of a moving object. The first approach is based on receiving the signals of at least four
8.2. LITERATURE OVERVIEW

global positioning system (GPS) satellites and the second approach is based on applying triangulation on the range measurements simultaneously made by three stations at known sites.

Assume that we have a global positioning system (GPS) receiver which has a number of satellites in view [119]. When at least signals from four satellites are received, four pseudoranges are derived which can be used to determine the position of the receiver. From the four pseudoranges, four nonlinear equations of the form

\[(x - x_k)^2 + (y - y_k)^2 + (x - z_k)^2 + (c \times \Delta t)^2 = (\rho_k)^2\]  

(8.1)
can be derived, where \(c\) is the velocity of light, \((x_k, y_k, z_k)\) is the known position of satellite \(k\) \((k \in [1, 4])\), \(\rho_k\) is the measured pseudorange between the receiver and satellite \(k\), \(\vec{x} = (x, y, z)\) is the unknown position of the receiver and \(\Delta t\) is the unknown synchronization error between the synchronized satellite clocks and the receiver clock. The corresponding geometrical model is based on the difference of two pseudoranges \(d = \rho_i - \rho_j\) \((i \neq j, i, j \in [1, 4])\), which does not depend on \(\Delta t\). This means that the receiver must lie on a two-sheeted hyperboloid, with the two satellites as the foci [119]. An example of a two-sheeted hyperboloid is shown in Fig. 8.2. This is the graph of all points in space whose distances from

Figure 8.2: To create the two-sheeted hyperboloid, the example hyperbola has been rotated around the line connecting both loci.

the satellites, located in the foci, differ by \(\sqrt{d}\). The third pseudorange locates the receiver on another independent two-sheeted hyperboloid. It intersects the first in a curve. The fourth pseudorange contributes a third independent hyperboloid, which normally cuts the curve in two points, provided the four satellites are not in the same plane. Bancroft [9] has derived an analytical expression for the two solutions, using the set of four nonlinear equations (eq. 8.1). When the number of hyperboloids is larger than 4, an analytical solution does not exist anymore. Using a least-squares approach [37], it is possible to determine a numerical solution when more than 4 pseudoranges are measured [109, 119].

\[\text{A pseudorange is a measured range which includes a contribution from the unknown clock synchronization error.}\]

\[\text{A two-sheeted hyperboloid is a surface of revolution, obtained by rotating a hyperbola about the line joining the foci [128].}\]
Trilateration is a method to determine the position of an object based on simultaneous range measurements from three stations at known sites. Using the three range measurements, it is possible to derive three nonlinear spherical equations of the form

\[(x - x_k)^2 + (y - y_k)^2 + (z - z_k)^2 = R_k^2\]  \hspace{1cm} (8.2)

where \((x_k, y_k, z_k)\) represents the position of station \(k\), \(R_k\) is the range measured by station \(k\) and \(\vec{x} = (x, y, z)\) represents the unknown position of the object.

The geometrical model is shown in Fig. 8.3. The position of station \(k\) \((k \in [1, 3])\) is indicated by \(S_k\). The information of only one station indicates that the target is located somewhere on the sphere, centered at the position of the station. Two measured ranges provide the information that the target is located on the circle (dashed circle), resulting from the intersection of both spheres (e.g. the blue and the green sphere). The additional information from the third range (orange sphere) produces an intersection of the blue sphere and the orange sphere, which is also a circle. Finally, the intersection of both circles produces two possible solutions. The other circles indicate the horizontal plane of the different stations. Manolakis [79] has derived an analytic expression for both solutions. One of the solutions is discarded, because it represents a solution falling inside the earth.

8.3 Sensor error biases

In distributed multisensor systems, it is necessary to estimate and correct for sensor measurement biases so that the multiple sensor measurements and/or tracks can be referenced to a common tracking coordinate system [25]. In this section, three different kinds of error biases are considered.

The first and probably the most significant error source is the misalignment between each local sensor reference system and the global reference system in which the single integrated air picture (SIAP) results are represented. Sensor measurement error biases form a second error source. A third general source of error biases is the sensor location error. However, since electronic position
location systems, such as GPS, can locate a position on the Earth’s surface with very high accuracy, this source of error is of less significance for most modern multiple sensor surveillance systems.

It is foreseen, that a network of distributed radar sensors will be used to track a TBM. Since misalignment biases and sensor measurement error biases have to be corrected in a different way, it is important to estimate both type of biases. However, the estimation problem can become very complex if it is attempted to simultaneously estimate the different error biases [25].

To estimate the different type of biases, targets, common to at least two radars, or stationary (artificial) objects with an accurately known position have to be available. Due to the large distances between the deployed radar systems and the curvature of the earth, the probability of sufficient common targets will be low. Furthermore, in the foreseen operation areas the height of stationary (artificial) objects will be rather limited. Probably, most of the objects will have to be located within the radar horizon of the deployed radar systems.

In this chapter a new method to determine a TBM position and velocity estimate is proposed, using only the very accurate Doppler and range information provided by a distributed sensor network of at least 3 radar sensors. The method only assumes that the accurate position of the sensors is known in the frame of reference, which is used to present the SIAP. This is possible because the method is not sensitive for the enumerated error biases and the actual TBM tracking can be carried out in the SIAP reference frame.

8.4 A new approach

The main characteristics of TBMs are that they have high velocities and follow a ballistic trajectory, reaching extra-atmospheric altitudes during mid-course flight (section 8.1). The objective is to engage a TBM as soon as possible during mid-course flight. Assuming that a TBM track is maintained, the actual estimated TBM position, velocity and accuracy information is regularly used to predict the intercept volume at the calculated intercept point. Normally, the missile to intercept the TBM has very limited manoeuvre-capability at extra-atmospheric altitudes, requiring an intercept volume with a very small radius (e.g. 1000 m). If the predicted intercept volume is small enough, a missile will be fired to intercept the TBM. If the ballistic missile is successfully intercepted at extra-atmospheric altitude, the debris are burnt during atmospheric re-entry. Compared with classical methods to track a TBM, the proposed method has to provide:

• faster convergence to very accurate TBM position and velocity estimates;
• robustness against sensor error biases.

8.5 Analytical solution for three sensors

A network of three radar sensors at known sites is assumed. Each of the sensors provides very accurate range and Doppler velocity information. The position of sensor $k$ is given by $(x_k, y_k, z_k)$ ($k \in [1, 3]$). Furthermore, it is assumed that

\[4\] It is assumed that the sensor location error, using GPS, is so small that it can be neglected.
a TBM is detected by each of the sensors. Each measurement has a different
time $t_k$ ($t_1 \leq t_2 \leq t_3$) and contains the information $[R_k, \dot{R}_k]$, where $R_k$ is the
range measured by sensor $k$ and $\dot{R}_k$ is the Doppler velocity measured by the
same sensor. Using the Doppler velocity, it is possible to predict the different
measured ranges to the same time $t_3$ with the equation

$$\dot{R}_k = R_k + \dot{R}_k \times (t_3 - t_k)$$  \hspace{1cm} (8.3)$$

where $k \in [1, 2]$. Each of the time-aligned ranges represents a sphere, centered
at the position of the sensor which measured the original range. Using the time-
aligned range measurements, three nonlinear spherical equations of the form

$$(x - x_k)^2 + (y - y_k)^2 + (z - z_k)^2 = \bar{R}_k^2$$  \hspace{1cm} (8.4)$$

are derived, where $(x_k, y_k, z_k)$ represent the position of sensor $k$ and $\bar{x} = (x, y, z)$
is the unknown position of the TBM which has to be estimated. The position
of the TBM relative to the position of sensor $k$ is given by $\bar{p}_k = \bar{x} - \bar{x}_k$. The complete derivation of the analytical expressions for both possible solutions for
the nonlinear spherical equations (eq. 8.4) is given in appendix J.1. One of the
solutions is discarded, because it represents a solution falling inside the earth.

The Doppler velocity with respect to sensor $k$ is defined by

$$\dot{R}_k = \vec{v}^T \times \frac{\bar{p}_k}{|\bar{p}_k|}$$  \hspace{1cm} (8.5)$$

Assuming that the Doppler velocity $\dot{R}_k$ and the direction vector $\frac{\bar{p}_k}{|\bar{p}_k|}$ are approximately constant during the time interval in which the 3 Doppler measurements
are received, eq. 8.5 results in the following three linear equations

$$a_1 \times v_x + a_2 \times v_y + a_3 \times v_z = \dot{R}_1$$
$$b_1 \times v_x + b_2 \times v_y + b_3 \times v_z = \dot{R}_2$$
$$c_1 \times v_x + c_2 \times v_y + c_3 \times v_z = \dot{R}_3$$  \hspace{1cm} (8.6)$$

where $\vec{v} = (v_x, v_y, v_z)^T$ is the unknown velocity of the TBM and

$$\vec{a} = \frac{\bar{p}_1}{|\bar{p}_1|}$$
$$\vec{b} = \frac{\bar{p}_2}{|\bar{p}_2|}$$
$$\vec{c} = \frac{\bar{p}_3}{|\bar{p}_3|}$$  \hspace{1cm} (8.7)$$

5This condition can always be fulfilled by sorting the times and renumbering the sensors.
6Manolakis [79] assumed that the measurements were made by synchronized stations.
7The $\times$ symbolizes a row-column product.
8For the sensors considered experiments have shown that the Doppler velocity and the
direction vector are approximately constant during the sensor scan time for TBM distances
of 50 km and larger.
Solving these linear equations, produces the following solution:

\[
\begin{align*}
v_x &= \frac{1}{f_1} \times (\dot{R}_2 - \frac{b_3}{a_3} \times \dot{R}_1) - \frac{d_2}{e_2 \times f_1} \times (\dot{R}_3 - \frac{c_3}{a_3} \times \dot{R}_1) \\
v_y &= \frac{1}{d_2} \times (\dot{R}_2 - \frac{b_3}{a_3} \times \dot{R}_1) - \frac{d_1}{d_2} \times v_x \\
v_z &= \frac{\dot{R}_1 - a_1 \times v_x - a_2 \times v_y}{a_3}
\end{align*}
\]  

where

\[
\begin{align*}
d_1 &= b_1 - \frac{a_1 \times b_3}{a_3} \\
d_2 &= b_2 - \frac{a_2 \times b_3}{a_3} \\
e_1 &= c_1 - \frac{a_1 \times c_3}{a_3} \\
e_2 &= c_2 - \frac{a_2 \times c_3}{a_3} \\
f_1 &= d_1 - \frac{d_2 \times e_1}{e_2}
\end{align*}
\]

### 8.6 Error covariance matrix

Based on the \((R_k, \dot{R}_k)\) information \((k \in [1, 2, 3])\), it is possible to produce at time \(t_3\) the estimate \(\hat{s}\) for the unknown state vector \(\vec{s} = (\vec{x}, \vec{v})\) of the TBM, where \(t_3\) is the time of the last measurement. Using the analytical solutions for position and velocity, derived using the approach described in section 8.5, the estimated state vector is written as

\[
\hat{s} = \hat{f}(\bar{R}_1, \bar{R}_2, \bar{R}_3, \dot{\bar{R}}_1, \dot{\bar{R}}_2, \dot{\bar{R}}_3) \equiv \hat{f}(\bar{Q}(t)) \tag{8.10}
\]

Generalizing, it is possible to express the real, but unknown state vector as

\[
\vec{s} = \hat{f}(R'_1, R'_2, R'_3, \dot{R}'_1, \dot{R}'_2, \dot{R}'_3) \equiv \hat{f}(\vec{Q}(t)) \tag{8.11}
\]

where \(\dot{R}'_k\) is the real Doppler velocity and \(R'_k\) is the actual (unknown) range between the real position of the target and the position of sensor \(k\) at time \(t\). Assuming that the difference \(\bar{Q}(t) - \hat{Q}(t)\) is small enough, it is possible to linearize eq. 8.11 in the neighborhood of \(\hat{s}\), producing

\[
\vec{s} = \hat{s} + F \times (\bar{Q}(t) - \hat{Q}(t)) \tag{8.12}
\]

using the Taylor formula for functions of several variables ([111], page 9). The Jacobian \(F\) is defined by

\[
F = \begin{pmatrix}
\frac{\partial f_1(\bar{Q}(t))}{\partial R_1} & \ldots & \frac{\partial f_1(\bar{Q}(t))}{\partial R_3} \\
\ldots & \ldots & \ldots \\
\frac{\partial f_m(\bar{Q}(t))}{\partial R_1} & \ldots & \frac{\partial f_m(\bar{Q}(t))}{\partial R_3}
\end{pmatrix} \tag{8.13}
\]
Using eq. 8.12, the error covariance matrix is estimated by

\[
P = E[(\tilde{s} - \hat{s}) \times (\tilde{s} - \hat{s})^T] = F \times E[(\tilde{Q} - \hat{Q}) \times (\tilde{Q} - \hat{Q})^T] \times F^T
\]

\[
= F \times R \times F^T
\]  

(8.14)

Using eq. 8.3, it is possible to derive the elements of the covariance matrix \(R\). The first two range measurements are predicted from time \(t_k\) to time \(t_3\) \((k \in [1, 2])\). The variance for the predicted range \(\hat{R}_k\) is given by:

\[
\sigma^2_{\hat{R}_k} = E((R^*_k - \hat{R}_k)^2)
\]

\[
= E(R^*_k + \hat{R}_k \times \Delta t_k - (R_k + \hat{R}_k \times \Delta t_k))^2
\]

\[
= \sigma^2_{R_k} + \Delta^2 t_k \times \sigma^2_{\hat{R}_k}
\]  

(8.15)

where \(R^*_k = R_k^* + \hat{R}_k \times \Delta t_k\) is the real, but unknown range with respect to the sensor which produced the measurement \((R_k, \hat{R}_k)\), and \(\Delta t_k = t_3 - t_k\). Due to the fact that \(\hat{R}_k\) depends on \(R_k\), there is also a crosscorrelation between \(R^*_k - \hat{R}_k\), and \(\hat{R}_k - \hat{R}_k = R^*_k - \hat{R}_k\), given by

\[
\sigma_{R_k, \hat{R}_k} = E((R^*_k - \hat{R}_k) \times (R^*_k - \hat{R}_k))
\]

\[
= E(((R_k^* + \hat{R}_k \times \Delta t_k) - (R_k + \hat{R}_k \times \Delta t_k)) \times (R_k^* - \hat{R}_k)))
\]

\[
= E(R^*_k - \hat{R}_k)^2 \times \Delta t_k = \sigma^2_{\hat{R}_k} \times \Delta t_k
\]  

(8.16)

The symmetrical matrix \(R\) is defined by

\[
R = \begin{pmatrix}
\sigma^2_{R_1} & 0 & 0 & 0 & 0 \\
0 & \sigma^2_{R_2} & 0 & 0 & 0 \\
0 & 0 & \sigma^2_{R_3} & 0 & 0 \\
0 & 0 & 0 & \sigma^2_{R_4} & 0 \\
0 & 0 & 0 & 0 & \sigma^2_{R_5}
\end{pmatrix}
\]  

(8.17)

Substituting \(R\) in eq. 8.14 finally produces the error covariance matrix.

### 8.7 Generalization to \(N\) sensors

In section 8.4 the assumption was made, that the distributed sensor network consists of three radar sensors. In this section, the TBM-tracking problem is formulated for a distributed radar sensor network, where more than 2 sensors have detected the TBM. Each of the sensors is suitably located. Sensor \(i\) is positioned at location \(\vec{g}_i = (a_i, b_i, c_i)^T\), where \(i \in [1, \cdots, M]\) and \(M > 3\). Assume that a tactical ballistic missile is detected by \(N\) of the sensors during the pre-defined time interval \(\Delta t\), where \(3 \leq N \leq M\). Each of the \(N\) measurements has a time \(t_i\) \((t_1 \leq t_2 \leq \cdots \leq t_N)\). Trivial generalization to \(N\) sensors produces the following system of nonlinear spherical equations

\[
Sp_1 = (x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 = \hat{R}_1^2
\]

\[
\cdots
\]

\[
Sp_n = (x - x_N)^2 + (y - y_N)^2 + (z - z_N)^2 = \hat{R}_N^2
\]  

(8.18)
8.7. GENERALIZATION TO $N$ SENSORS

where $\vec{x} = (x, y, z)$ is the unknown position of the TBM and $\vec{g}_i = (x_i, y_i, z_i)$ is the position of sensor $i$. For the situation that the distributed sensor network contains only 3 sensors, it has been shown that there is a maximum of two intersection points (section 8.2). For $N > 3$, equation set 8.18 forms an over-determined system for the unknown TBM position parameters $(x, y, z)$, which does not usually have an exact solution because the predicted quantities $\vec{R}_i$ are perturbed by the measurement errors in $R_i$ and $\dot{R}_i$ ($i \in [1, N]$) [115].

Using the $N$ received Doppler measurements, it is trivial to derive the following system of $N$ linear equations

$$d_1 \times v_x + e_1 \times v_y + f_1 \times v_z = \dot{R}_1$$
$$\ldots$$
$$d_N \times v_x + e_N \times v_y + f_N \times v_z = \dot{R}_N \tag{8.19}$$

where

$$d_i = \frac{x - x_i}{\|r_i\|_2}$$
$$e_i = \frac{y - y_i}{\|r_i\|_2}$$
$$f_i = \frac{z - z_i}{\|r_i\|_2} \tag{8.20}$$

are assumed to depend only on the solution of equation set 8.18. The set of equations can be written in the form of $A \times \vec{v} = \vec{b}$. Due to the fact that $N > 3$, equation set 8.19 also forms an over-determined system for the parameters $(v_x, v_y, v_z)$, which does not normally have an exact solution.

Two different approximation methods to determine solutions for equation set 8.18 and 8.19 are discussed. The first method is based on determining solutions for each possible combination of three range and Doppler measurements. The second method is based on determining approximate least square solutions for both equation sets.

8.7.1 Permutation method

A possible method to solve equation set 8.18 is to determine every permutation of three spherical equations, based on the corresponding three range and Doppler measurements provided by three different sensors. Each permutation of three equations is solved by using the analytical solution for the position, which has been derived in section 8.5. For $N$ range measurements, the number of possible permutations is given by

$$U = \frac{N!}{3! \times (N-3)!} \tag{8.21}$$

Normally the number of sensors in the network will be limited, which guarantees that $N$ will be small. If the solution for the position, determined for permutation $i$ ($i \in [1, U]$), is given by $\vec{x}_i$, the TBM position at time $t_N$ is estimated by

$$\vec{x} = \frac{1}{U} \times \sum_{i=1}^{U} \vec{x}_i \tag{8.22}$$

It is assumed that the measurement errors in $\vec{g}_i$ can be neglected.
Using the estimated position, it is possible to fill in equation set 8.19. Similarly, for the Doppler measurements the solution for equation permutation $i$ is given by $\vec{v}_i$. It is assumed that permutation $i$ depends on $\vec{x}_i$. The TBM velocity at time $t_N$ is estimated by

$$\vec{v} = \frac{1}{U} \sum_{i=1}^{U} \vec{v}_i$$

(8.23)

Solution of the combined set of equations for permutation $i$, results in the solution $\hat{s}_i = (\vec{x}_i, \vec{v}_i)$ for the TBM state at time $t_N$ with a corresponding covariance matrix $P_i = E((\vec{s} - \hat{s}) \times (\vec{s} - \hat{s})^T)$ (see section 8.6), which defines a (conditional) probability density function $f(\hat{s}_i, P_i)$, conditioned on the measurement set $Z_i$ containing the relevant range and Doppler measurements, representing the likely positions of the TBM, where $P_i$ is defined by eq. 8.14. Furthermore, taking the average of the calculated position and velocity estimates for the different combined equation permutation sets, results in the TBM state vector estimate $\hat{s} = (\vec{x}, \vec{v})$. The corresponding covariance matrix is defined by

$$E((\vec{s} - \hat{s}) \times (\vec{s} - \hat{s})^T) = \frac{1}{U} U \sum_{i=1}^{U} P_i$$

$$E((\hat{s}_i - \hat{s}) \times (\hat{s}_i - \hat{s})^T) = \frac{1}{U} U \sum_{i=1}^{U} (\hat{s}_i - \hat{s}) \times (\hat{s}_i - \hat{s})^T$$

(8.24)

where $\vec{s}$ is the real, but unknown state vector of the TBM. The different covariance matrices are estimated by

$$E((\vec{s} - \hat{s}) \times (\vec{s} - \hat{s})^T) = \frac{1}{U} U \sum_{i=1}^{U} P_i$$

$$E((\hat{s}_i - \hat{s}) \times (\hat{s}_i - \hat{s})^T) = \frac{1}{U} U \sum_{i=1}^{U} (\hat{s}_i - \hat{s}) \times (\hat{s}_i - \hat{s})^T$$

(8.25)

The first evaluation results for this method are presented in section 8.8.

### 8.7.2 Linearization method

Using $N$ range and Doppler measurements, the objective is to obtain an estimate for the TBM state. Normally, the derived information $\hat{s} = (\vec{x}, \vec{v})$, called a complete measurement, is processed by a TBM filter\(^\text{10}\) to refine the TBM state estimates. Assume that the filter produces a predicted TBM state at time $t_N$, which is given by $\bar{s} = (\bar{x}, \bar{v})$. The idea behind the second method is to linearize the nonlinear spherical equation, corresponding with the sphere centered in the position of sensor $i$, in the neighborhood of an appropriate point lying on the sphere. The center of sphere $i$ is defined by $\vec{g}_i = (a_i, b_i, c_i)^T$. The vector from the center of sphere $i$ to the predicted TBM position is given by $\vec{r}_i = \bar{x} - \vec{g}_i$. The unit vector in this direction is defined as $\vec{D} = \vec{r}_i / ||\vec{r}_i||^2$. The chosen approximation point is determined by the vector $\vec{o}_i = (x_i, y_i, z_i)^T = \vec{R}_i \times \vec{D} + \vec{g}_i$. The tangent

\(^{10}\)A TBM filter is a filter which assumes that the target follows a ballistic trajectory.
8.7. GENERALIZATION TO N SENSORS

plane to sphere \( i \), which touches the sphere in the point \( \vec{o}_i \) is defined by [75]

\[
\frac{\partial S_{p_i}}{\partial x} \vec{o}_i \times (x - x_i) + \frac{\partial S_{p_i}}{\partial y} \vec{o}_i \times (y - y_i) + \\
\frac{\partial S_{p_i}}{\partial z} \vec{o}_i \times (z - z_i) = 0 \tag{8.26}
\]

which produces the tangent plane equation

\[
2 \times (x_i - a_i) \times (x - x_i) + 2 \times (y_i - b_i) \times (y - y_i) + \\
2 \times (z_i - c_i) \times (z - z_i) = 0 \tag{8.27}
\]

The variables \( S_{p_i} \) are defined by equation set 8.18.

Sphere \( i \) is approximated by the tangent plane in the neighborhood of \( \vec{o}_i \).

Substitution of the approximations for the different spheres in equation set 8.18 produces the equation set

\[
2 \times (x_1 - a_1) \times (x - x_1) + 2 \times (y_1 - b_1) \times (y - y_1) + \\
2 \times (z_1 - c_1) \times (z - z_1) = 0 \\
\ldots \\
2 \times (x_N - a_N) \times (x - x_N) + 2 \times (y_N - b_N) \times (y - y_N) + \\
2 \times (z_N - c_N) \times (z - z_N) = 0 \tag{8.28}
\]

The resulting set of equations is linear and can be written in the form of \( A \times \vec{x} = \vec{b} \)

\[
(x_1 - a_1) \times x + (y_1 - b_1) \times y + (z_1 - c_1) \times z = \\
(x_1 - a_1) \times x_1 + (y_1 - b_1) \times y_1 + (z_1 - c_1) \times z_1 \\
\ldots \\
(x_N - a_N) \times x + (y_N - b_N) \times y + (z_N - c_N) \times z = \\
(x_N - a_N) \times x_N + (y_N - b_N) \times y_N + (z_N - c_N) \times z_N \tag{8.29}
\]

Due to the \( N > 3 \), the found approximation forms an overdetermined system, which does not usually have an exact solution. An approximate solution is obtained by determining the least squares solution of the system (see appendix K). This solution is defined by

\[
\vec{x}_{ls} = (A^T \times A)^{-1} \times A^T \times \vec{b} \tag{8.30}
\]

If \( A \) is (nearly) rank-deficient [11], the approximate solution is determined by using the method described in appendix K.3. A matrix is rank-deficient, when the number of independent columns (rows) \( n < \min(r,c) \), where \( r \) is the number of rows of \( A \) and \( c \) is the number of columns.

Again it is assumed that the Doppler velocity \( \dot{R}_i \), measured by sensor \( i \), is approximately constant during the time interval \( \Delta t \) in which the \( N \) sensors collected the measurements. The unknown TBM velocity is given by \( \vec{v} = (v_x, v_y, v_z)^T \). Using the determined solution for the TBM position, the

\[\text{[11] A matrix can be nearly rank-deficient, because one of the columns (rows) is almost equal to a combination of two other columns (rows).}\]
TBM position relative to sensor \( i \) \((i \in [1, \cdots, N])\) is given by \( \vec{r}_i = \vec{x}_{ls} - \vec{g}_i \). The Doppler velocity with respect to sensor \( i \) is defined by

\[
\dot{R}_i = \vec{v}^T \times \frac{\vec{r}_i}{||\vec{r}_i||_2}
\]  
(8.31)

Assuming that the direction of the unit vector \( \frac{\vec{r}_i}{||\vec{r}_i||_2} \) is approximately constant during the time interval \( \Delta t \), eq. 8.31 results in the following \( N \) linear equations

\[
d_1 \times v_x + e_1 \times v_y + f_1 \times v_z = \dot{R}_1 \\
\vdots \\
d_N \times v_x + e_N \times v_y + f_N \times v_z = \dot{R}_N
\]  
(8.32)

where

\[
d_i = \frac{x - a_i}{||r_i||_2} \\
e_i = \frac{y - b_i}{||r_i||_2} \\
f_i = \frac{z - c_i}{||r_i||_2}
\]  
(8.33)

and \( \vec{x}_{ls} = (x, y, z) \). This linear set of equations also forms an overdetermined system. To obtain an approximate solution, the least square solution \( \vec{v}_{ls} \) is determined.

Using the \((\vec{R}_i, \dot{R}_i)\) information \((i \in [1, \cdots, N])\) which is available at the time of the last measurement, the complete measurement \( \hat{s} = (\vec{x}_{ls}, \vec{v}_{ls})^T \) has been determined. Using

\[
\hat{R}_i = \sqrt{(x - a_i)^2 + (y - b_i)^2 + (z - c_i)^2}
\]  
(8.34)

and eq. 8.31 for Doppler velocity \( \dot{R}_i \) it is possible to write a nonlinear analytical expression for

\[
\vec{z} = (R_1, \cdots, R_N, \dot{R}_1, \cdots, \dot{R}_N)^T
\]  
(8.35)

using the real, unknown TBM state vector \( \vec{s} \). \( R_i \) and \( \dot{R}_i \) are the real, but unknown range and Doppler velocity with respect to sensor \( i \). Using eq. 8.35 and the complete measurement \( \hat{s} = (\vec{x}_{ls}, \vec{v}_{ls})^T \), it is possible to determine \( \dot{\vec{z}} \).

Assuming that the difference \( \vec{z} - \dot{\vec{z}} \) is small, eq. 8.35 is linearized in the neighborhood of \( \hat{z} \), producing

\[
\vec{z} = \hat{z} + C \times (\vec{s} - \hat{s})
\]  
(8.36)

using the Taylor formula for functions of several variables ([111, 14]). The Jacobian \( C \) is defined by

\[
C = \begin{pmatrix}
\frac{\partial R_1}{\partial x} & \cdots & \frac{\partial R_1}{\partial v_z} \\
\vdots & \cdots & \vdots \\
\frac{\partial R_N}{\partial x} & \cdots & \frac{\partial R_N}{\partial v_z}
\end{pmatrix}
\]

\( \hat{s} \)  
(8.37)

\( ^{12} \)The \( \times \) symbolizes a row-column product.
Assuming full column rank for $C$ (see appendix K), we can write
\[ \vec{s} - \hat{\vec{s}} = (C^T \times C)^{-1} \times C^T \times (\vec{z} - \hat{\vec{z}}). \]
The result is that
\[
E((\vec{s} - \hat{\vec{s}}) \times (\vec{s} - \hat{\vec{s}})^T) = (C^T \times C)^{-1} \times C^T \times E((\vec{z} - \hat{\vec{z}}) \times (\vec{z} - \hat{\vec{z}})^T) \times C \times (C \times C^T)^{-1}
\] (8.38)

If $A$ does not have full column rank, the result is
\[
E((\vec{s} - \hat{\vec{s}}) \times (\vec{s} - \hat{\vec{s}})^T) = C^+ \times E((\vec{z} - \hat{\vec{z}}) \times (\vec{z} - \hat{\vec{z}})^T) \times (C^+)^T
\] (8.39)

The pseudo-inverse $C^+$ of the matrix $C$ is defined in appendix K.3.

In this section the set of equations, given by eq. 8.18, has been linearized, resulting in the set of equations given by eq. 8.29. Furthermore, for the two linear sets of equations, given by eq. 8.29 and eq. 8.32, least squares solutions and the corresponding covariance matrix have been determined which are used to form a complete measurement. The first evaluation results for this method are presented in section 8.8.

### 8.8 Analysis

A new method has been proposed to provide fast convergence to very accurate TBM state estimates, using only the very accurate Doppler and range measurements provided by a network of distributed sensors. Using the range and Doppler velocity information received from a network of three radar sensors, in section 8.5 an analytical solution has been derived to solve the set of range-based nonlinear spherical equations and the set of Doppler-based linear equations. In section 8.7, the problem has been generalized for a distributed network, where more than 2 sensors have detected the TBM. It has been shown that generalization to $N$ sensors ($N > 3$) results in overdetermined systems of $N$ range-based nonlinear spherical equations and $N$ Doppler-based linear equations. Two different approximation methods have been proposed to solve both sets of equations. In this section an analysis is carried out to determine the effectiveness of the proposed methods. During the analysis, the following methods are considered:

- the analytical solution (section 8.8.2), proposed in section 8.5 and 8.6;
- the first approximation method (section 8.8.3), proposed in section 8.7.1, which determines every permutation of three nonlinear spherical equations, based on three time-aligned ranges, and three linear equations, based on three Doppler velocities and uses the analytical solution for the position and the velocity. Using the result of the different permutations, an estimate for the TBM position and velocity is determined;
- the second approximation method (section 8.8.4), proposed in section 8.7.2, which is based on linearizing the set of nonlinear spherical equations, and determines the least squares solution for both sets of $N$ linear equations, providing an estimate for the TBM position and velocity.

The performance of both approximation methods is compared in section 8.8.4.
8.8.1 Introduction

To collect statistical information, a Monte Carlo approach has been taken. The performance of each proposed method is compared with:

1. single sensor measurement tracking, using the inaccurate bearing and elevation information;
2. multisensor measurement tracking, also using the inaccurate bearing and elevation information.

For single sensor and multisensor measurement tracking, range, bearing and elevation information is used to initiate and maintain the TBM track. Each proposed method produces complete measurements with corresponding accuracy covariance matrices. Each of the complete measurements contains an estimate of the TBM position and velocity. To filter the sensor and complete measurements, an extended first order Kalman filter is used, which assumes that the TBM follows a ballistic trajectory.

Before the simulation results can be evaluated, an appropriate measure for the TBM track accuracy has to be defined. The starting point is the volume of the hyperellipsoid, given by

$$(\vec{s} - \hat{s})^T \hat{P}^{-1} (\vec{s} - \hat{s}) = r^2$$

where $\vec{s}$ is the real, but unknown TBM state vector. The threshold $r^2$ determines the risk that the real TBM position lies outside the hyperellipsoid and is read from the $\chi^2$ table. Furthermore, $\hat{s}$ and $\hat{P}$ are the estimated state vector and error covariance matrix produced by the used TBM filter. The volume of the hyperellipsoid is given by [56]

$$V = V_d \times | \hat{P} |^{\frac{1}{2}} \times r^d$$

where $V_d$ is the volume of a $d$-dimensional unit hypersphere, defined by

$$V_d = \begin{cases} \frac{\pi^{\frac{d}{2}}}{\Gamma\left(\frac{d}{2}\right)} & d \text{ even} \\ 2^{\frac{d}{2}} \pi^{\frac{d-1}{2}} \times \left(\frac{d-1}{2}\right)! & d \text{ odd} \end{cases}$$

and $| \hat{P} |$ is the determinant of the matrix $\hat{P}$ \(^{13}\). For the pre-defined $r^2$-value, the value $r^2 = 2.3660$ has been chosen. This means that the probability is 50 percent that the position of the TBM lies within the calculated hyperellipsoid. The next step is to determine the radius of the accuracy sphere, which has the same volume of the hyperellipsoid and is defined as

$$r = \sqrt{\frac{3 \times V}{4 \times \pi}}$$

where $V$ is defined by eq. 8.41. The calculated radius is used as accuracy measure.

8.8.2 Performance analytical solution

In Fig. 8.4 an impression of the scenario is shown, where the TBM has an initial velocity $v_0 = 2500 \text{ m/s}$. The three dots represent the position of the three sensors.

\(^{13}\)Thus, for a given dimensionality, the scatter of the samples varies directly with $| \hat{P} |^{\frac{1}{2}}$. 
The sensor positions correspond with the positions of the first three sensors, specified in Table 8.3. For the analysis only a simplified TBM trajectory has been simulated, which does not take into account booster ejection, aerodynamic drag effects or break-up of the TBM during re-entry in the earth’s atmosphere. The process noise has been set to $0.1 \text{ m s}^{-1}$. The scan time of the different, non-synchronized sensors is $\Delta t = 5 \text{ sec}$. In both cases of single sensor tracking, using the measurements produced by sensor 2, and multisensor measurement tracking, each sensor produces range ($R$), bearing ($\beta$) and elevation ($\varepsilon$) measurements; in the new approach only range and Doppler ($\dot{R}$) measurements. The corresponding accuracies are: $\sigma_R = 30 \text{ m}$, $\sigma_\beta = 0.3^0$, $\sigma_\varepsilon = 0.3^0$ and $\sigma_{\dot{R}} = 5 \text{ m s}^{-1}$. For the analysis it is assumed, that the probability to detect the TBM during a sensor scan is $P_d = 1$, but other choices are possible.

Three different error biases have been taken into account during the simulation \(^{14}\). The first error bias is due to misalignment of each of the local sensor reference systems with the global reference system in which the SIAP results are represented. The misalignment is represented by the rotation angles $\delta_x$, $\delta_y$ and $\delta_z$, where $\delta_x$ indicates a rotation around the $x$-axis, etc. Table 8.1 presents an overview of the different reference frame error biases, used during the simulation. Sensor measurement biases form a second error source. For the simulation, the sensor measurement error biases, given in table 8.2, have been used where $\delta_\beta$ represents the bearing bias, $\delta_\varepsilon$ the elevation bias and $\delta_R$ the range bias. The last error bias which has been taken into account, is the tracking error bias.

\(^{14}\)It is assumed that the sensor location error, using GPS, is so small that it can be neglected.
bias, expressed as the difference between estimated target state vector and the ground truth state vector for the simulated target. This bias has been estimated for each of the TBM tracking approaches by using a parallel TBM filter, which was not influenced by the first two error biases. The statistical errors and error biases have been combined, using the mean squared error measure [24]. Due to the fact that the analytical solution approach uses only the Doppler and range information, it is not sensitive for the first two bias errors 15.

The simulation results of the different tracking methods are shown in Fig. 8.5. In the upper figure, the accuracy of the estimated TBM position is presented as function of time and the lower figure shows the accuracy of the estimated velocity. The dotted lines represent the accuracy of the given estimated value, in terms of the estimated standard deviation. For this scenario the conclusion is, that the new method is significantly more accurate than the two other, classical methods to track a TBM. Compared with the two classical tracking methods, the new method provides significantly more accurate tracking results during track maintenance.

\[ \begin{array}{c|c|c|c}
\hline
& \text{sensor 1} & \text{sensor 2} & \text{sensor 3} \\
\hline
\delta_x & 0.5 & -0.5 & 0.15 \\
\delta_y & 0.5 & -0.5 & 0.15 \\
\delta_z & 0.5 & -0.5 & 0.15 \\
\hline
\end{array} \]

Table 8.1: Alignment errors

\[ \begin{array}{c|c|c|c}
\hline
& \text{sensor 1} & \text{sensor 2} & \text{sensor 3} \\
\hline
\delta_\beta & 1.0 & 0.5 & 2.0 \\
\delta_r & 1.0 & 0.5 & 0.5 \\
\delta_\epsilon & 1.0 & 1.0 & 1.0 \\
\hline
\end{array} \]

Table 8.2: Sensor measurement errors

15 This is also true for both approximation algorithms.
Figure 8.5: The accuracy obtained for the three different tracking methods.
8.8.3 Performance first approximation method

To analyze the performance of the first approximation method, described in section 8.7.2, two additional sensors are added to the sensor network, resulting in the scenario shown in Fig. 8.6. For the simulation, the same parameters for the sensors and the TBM as specified in section 8.8.2 have been used. The positions of the different sensors are specified in Table 8.3. Table 8.4 presents an overview

<table>
<thead>
<tr>
<th>x [m]</th>
<th>y [m]</th>
<th>z [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1500,000</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5,000</td>
<td>5,000</td>
<td>1,500</td>
</tr>
<tr>
<td>500,000</td>
<td>562,000</td>
<td>1,000</td>
</tr>
<tr>
<td>500,000</td>
<td>-562,000</td>
<td>200</td>
</tr>
<tr>
<td>200,000</td>
<td>400,000</td>
<td>3,000</td>
</tr>
</tbody>
</table>

Table 8.3: Sensor positions

<table>
<thead>
<tr>
<th>δx [mrad]</th>
<th>sensor 1</th>
<th>sensor 2</th>
<th>sensor 3</th>
<th>sensor 4</th>
<th>sensor 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>-0.5</td>
<td>0.15</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>δy [mrad]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td></td>
<td>-0.5</td>
<td>0.15</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>δz [mrad]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td></td>
<td>-0.5</td>
<td>0.15</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 8.4: Alignment errors

of the different reference frame error biases, used during the simulation. The used sensor measurement error biases are specified in Table 8.5. The simulation

<table>
<thead>
<tr>
<th>δβ [mrad]</th>
<th>sensor 1</th>
<th>sensor 2</th>
<th>sensor 3</th>
<th>sensor 4</th>
<th>sensor 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.5</td>
<td>2.0</td>
<td>1.0</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>δε [mrad]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>0.5</td>
<td>0.5</td>
<td>1.0</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>δr [m]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td></td>
</tr>
</tbody>
</table>

Table 8.5: Sensor measurement errors

results of the different tracking methods are shown in Fig. 8.7. In the upper figure, the accuracy of the estimated TBM position is shown as function of time and in the lower figure the accuracy of the estimated velocity is shown. For this scenario the conclusion is, that the approximation method is significantly more accurate than the two other, classical methods to track an TBM. Compared with the two classical tracking methods, the new method provides significantly more accurate tracking results.
Figure 8.6: The scenario, used to evaluate the performance of both approximation methods.
Figure 8.7: The accuracy obtained for the three different tracking methods.
8.8. ANALYSIS

8.8.4 Second approximation method

To analyze the performance of the second method, described in section 8.7.2, the scenario and the parameters from section 8.8.3 have been used. The simulation results of the different tracking methods are shown in Fig. 8.8. The upper figure shows the accuracy of the estimated TBM position as function of time and the lower figure the accuracy of the estimated velocity. This approximation method is also significantly more accurate than the two other tracking methods. Comparing the results of both approximation methods, we see that the position accuracy obtained by the second approximation method is most of
the time more accurate, but the variation in the different Monte Carlo runs is significantly higher. Furthermore, both methods produce a comparable velocity accuracy, but again the second method shows a significantly higher variation in the different Monte Carlo runs. This behavior could be due to the way in which the nonlinear spherical equations are approximated. More research is necessary to determine the exact cause for this behavior. Seen from an operational perspective, the first approximation method is the preferred method.

8.9 Conclusions

In this chapter a new method has been developed to provide fast convergence to very accurate TBM position and velocity estimates. The new method only uses the very accurate range and Doppler measurements, provided by a distributed network containing just three radar sensors. This means that the method is insensitive to reference frame misalignment errors and sensor measurement error biases in bearing and elevation (see section 8.3). The estimates, produced by the method, are used as input to a TBM filter. Exact analytical solutions have been derived to estimate the TBM position, velocity and the corresponding accuracy covariance matrix. To demonstrate the effectiveness of the new algorithm, an analysis has been carried out. For the scenario used, the results presented in section 8.8 show that TBM tracking using the new method is significantly more accurate than classical single sensor and multisensor measurement tracking, both of which are very sensitive to sensor error biases.

The TBM-tracking problem has been generalized for a distributed network, containing \( N \) radar sensors (\( N > 3 \)) which have detected a TBM. In section 8.7 two different approximation methods have been presented to determine approximate solutions for the TBM position, velocity and corresponding accuracy covariance matrix. For a certain scenario, an analysis has been carried out to determine and compare the performance of both approximation algorithms. Both algorithms are significantly more accurate than single sensor and multisensor measurement tracking. Comparing the results of both approximation methods, the second method demonstrates significantly higher performance variations in the different Monte Carlo runs. From an operational perspective, this is not acceptable. This means that the first approximation method is the preferred method.