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**Bose-Einstein condensates in radio-frequency-dressed potentials on an atom chip**

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# A The rf-dressed potential

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We calculate the potential that a neutral atom with a magnetic moment experiences in a time-dependent magnetic field  $\mathbf{B}(t)$ . The hamiltonian is

$$\hat{H} = -\hat{\boldsymbol{\mu}} \cdot \mathbf{B}(t), \quad (\text{A.1})$$

the same as in the static case. The magnetic dipole moment operator  $\hat{\boldsymbol{\mu}}$  is proportional to the angular momentum operator  $\hat{\mathbf{J}}$  with a proportionality factor  $\mu_s$

$$\hat{\boldsymbol{\mu}} = \mu_s \hat{\mathbf{J}}. \quad (\text{A.2})$$

At any fixed point in space the total field  $\mathbf{B}(t)$  can be divided in a static part  $\mathbf{B}_s$  and an oscillatory part  $\mathbf{B}_{\text{rf}}(t)$ . The  $z$  axis of the coordinate system we align with  $\mathbf{B}_s$  and take this as the quantization direction. The oscillatory field we assume has linear polarization and a sinusoidal time dependence with frequency  $\omega_{\text{rf}}$ . We align  $x$  along the oscillating field. We can thus write the field as

$$\mathbf{B} = \mathbf{B}_s + \mathbf{B}_{\text{rf}}(t) = (B_s + B_{\text{rf},z}(t)) \mathbf{e}_z + B_{\text{rf},x}(t) \mathbf{e}_x. \quad (\text{A.3})$$

The time dependence of the oscillatory field  $\mathbf{B}_{\text{rf}}(t) = \mathbf{b}_{\text{rf}} \cos(\omega_{\text{rf}} t)$  can also be written in terms of complex exponentials

$$\mathbf{B}_{\text{rf}}(t) = \frac{1}{2} \mathbf{b}_{\text{rf}} (e^{i\omega_{\text{rf}} t} + e^{-i\omega_{\text{rf}} t}). \quad (\text{A.4})$$

We write the wave function as  $|\psi\rangle = \sum_m c_m |m\rangle$  and take as orthonormal basis the eigenstates of  $\hat{J}_z$ :  $|m\rangle$  with eigenvalues:  $m = -J \dots J$ . We write the Schrödinger equation in matrix form

$$i\hbar \dot{c}_n(t) = \sum_m H_{nm} c_m(t). \quad (\text{A.5})$$

The matrix elements  $H_{nm}$  are

$$H_{nm} = [-\mu_s B_s - \mu_s B_{\text{rf},z}(t)] m \delta_{nm} - \mu_s B_{\text{rf},x}(t) \langle n | \hat{J}_x | m \rangle, \quad (\text{A.6})$$

with

$$\langle n | \hat{J}_x | m \rangle = \frac{1}{2} \sqrt{j(j+1) - m(m+1)} \delta_{n,m+1} + \frac{1}{2} \sqrt{j(j+1) - m(m-1)} \delta_{n,m-1}, \quad (\text{A.7})$$

which directly follows from the ladder operators  $\hat{J}_\pm = \hat{J}_x \pm i\hat{J}_y$  which have the eigenvalues [167]:

$$\begin{aligned}\hat{J}_+|j m\rangle &= \sqrt{j(j+1) - m(m+1)}|j m+1\rangle, \\ \hat{J}_-|j m\rangle &= \sqrt{j(j+1) - m(m-1)}|j m-1\rangle.\end{aligned}$$

In Eq. (A.6) and Eq. (A.7)  $\delta_{nm}$  is the Kronecker delta symbol which equals 1 for  $n = m$  and 0 in all other cases.

We now apply the transformation

$$\tilde{c}_k(t) \equiv c_k(t)e^{-ik\omega_{\text{rf}}t}, \quad (\text{A.8})$$

which can be interpreted as the transformation to the rotating frame of the precessing magnetic moment. Taking the derivative with respect to  $t$  and inserting Eq. (A.5) we find

$$i\hbar\dot{\tilde{c}}_k(t) = \sum_m (H_{km}e^{i(m-k)\omega_{\text{rf}}t} + \delta_{km}m\hbar\omega_{\text{rf}}) \tilde{c}_m(t), \quad (\text{A.9})$$

which shows that the matrix elements in the rotating frame,  $\tilde{H}_{km}$ , are equal to

$$\tilde{H}_{km} = H_{km}e^{i(m-k)\omega_{\text{rf}}t} + \delta_{km}m\hbar\omega_{\text{rf}}. \quad (\text{A.10})$$

We now calculate the matrix elements after transformation by inserting Eq. (A.6) into Eq. (A.10). For the time dependence of the oscillating field we use Eq. (A.4).

$$\begin{aligned}\tilde{H}_{km} &= \mu_s m \left[ \frac{\hbar\omega_{\text{rf}}}{\mu_s} - B_s + \frac{b_{\text{rf},z}}{2} (e^{i\omega_{\text{rf}}t} + e^{-i\omega_{\text{rf}}t}) \right] \delta_{km} \\ &\quad - \mu_s \frac{b_{\text{rf},x}}{2} \left[ \frac{1}{2} \sqrt{j(j+1) - m(m+1)} (1 + e^{i2\omega_{\text{rf}}t}) \delta_{k,m+1} \right. \\ &\quad \left. + \frac{1}{2} \sqrt{j(j+1) - m(m-1)} (1 + e^{-i2\omega_{\text{rf}}t}) \delta_{k,m-1} \right].\end{aligned} \quad (\text{A.11})$$

Observe that this expression has static terms and terms that are oscillating at  $\pm\omega_{\text{rf}}t$  and at  $\pm 2\omega_{\text{rf}}t$ . We now apply the rotating wave approximation [108] which amounts to neglecting all time-dependent terms in Eq. (A.11), yielding

$$\begin{aligned}\tilde{H}_{km} &= \mu_s m \left( \frac{\hbar\omega_{\text{rf}}}{\mu_s} - B_s \right) \delta_{km} - \mu_s \frac{b_{\text{rf},x}}{2} \left( \frac{1}{2} \sqrt{j(j+1) - m(m+1)} \delta_{k,m+1} \right. \\ &\quad \left. + \frac{1}{2} \sqrt{j(j+1) - m(m-1)} \delta_{k,m-1} \right).\end{aligned} \quad (\text{A.12})$$

From comparison of Eq. (A.6) and Eq. (A.12) we see that Eq. (A.12) corresponds to a magnetic moment in a magnetic field

$$\tilde{\mathbf{B}} = \left( B_s - \frac{\hbar\omega_{\text{rf}}}{\mu_s} \right) \mathbf{e}_z + \frac{1}{2} b_{\text{rf},x} \mathbf{e}_x, \quad (\text{A.13})$$

and hence

$$|\tilde{\mathbf{B}}| = \sqrt{\left(B_s - \frac{\hbar\omega_{\text{rf}}}{\mu_s}\right)^2 + \left(\frac{b_{\text{rf},x}}{2}\right)^2}. \quad (\text{A.14})$$

Since  $\tilde{\mathbf{B}}$  is time independent it is easy to see that the magnetic moment in this field has eigenvalues  $\tilde{m}\mu_s|\tilde{\mathbf{B}}|$  with  $\tilde{m} = -J \dots J$ . If the atom moves slowly through the spatially-varying magnetic field, such that the adiabaticity criterion is satisfied we can write the position-dependent potential as

$$U = \tilde{m}\hbar\sqrt{\Delta^2 + \Omega^2}, \quad (\text{A.15})$$

where  $\Delta^2$  is called the resonance term with  $\Delta$  the local detuning of the rf frequency with respect to the Larmor frequency

$$\Delta = \omega_{\text{rf}} - \omega_L = \omega_{\text{rf}} - \frac{|g_F\mu_B|}{\hbar}|\mathbf{B}_s(\mathbf{r})|. \quad (\text{A.16})$$

The other term,  $\Omega^2$ , is referred to as the coupling term. The position-dependent Rabi frequency,  $\Omega$ , is given by the circularly polarized rf-field component referenced to the local direction of the static magnetic field. For a linearly polarized rf field the above shows that

$$\Omega = \frac{|g_F\mu_B|}{\hbar} \frac{|\mathbf{b}_{\text{rf}} \times \mathbf{B}|}{2|\mathbf{B}|}. \quad (\text{A.17})$$