Fine aspects of pluripotential theory

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Preface

From its origins in 19th century physics, potential theory has developed into an extensive field of research encompassing concepts like harmonic and subharmonic functions, Dirichlet problem, Green functions, and capacities.

In the past century the subject has undergone a very rapid development, and important connections to many other mathematical branches were discovered. As examples, let us at least mention that potential theory is intimately related to complex analysis, probability theory, and the theory of partial differential equations.

Some of these interrelations were further elaborated and have gradually led to the emergence of new disciplines, largely inspired by potential theory. Examples are probabilistic potential theory, parabolic potential theory, axiomatic potential theory, pluripotential theory and fine potential theory, just to name a few.

Fine potential theory has its roots in the classical Cartan fine topology, and was mainly developed by Fuglede around 1972. It can be described as the theory of harmonic and subharmonic functions on open sets with respect to the fine topology. Very soon, this newly developed theory appeared to be useful and has brought more precisions and understanding of some classical results in potential theory. And more importantly, fine potential theory played a prominent role in the creation of the theory of finely holomorphic functions. This new concept of holomorphic functions turned out to be a quite natural generalization of the Borel’s old concept of monogenic functions, which was largely neglected for almost a century.

In this thesis fine potential theory will be used as a tool to treat several problems in pluripotential theory, which on the face have nothing to do with the fine potential theory. An introductory chapter (Chapter 2) is therefore devoted to a quite detailed exposition, though without proofs, of fine potential theory and the theory of finely holomorphic functions.

Pluripotential theory can be briefly described as the study of plurisubharmonic functions and their properties. It relates to holomorphic functions of several complex variables just as classical potential theory of $\mathbb{R}^2 = \mathbb{C}$ relates to holomorphic functions of one variable.

The study of pluripolar hulls, which will occupy a considerable part of this thesis, is an example of a new and rapidly developing part of pluripotential theory without classical counterpart. Only three years ago Edlund and Jörice [44] dis-
covered that the theory of finely holomorphic functions can be fruitfully applied to explain the behavior of pluripolar hulls of graphs of holomorphic functions. This unexpected connection, which actually was implicit in earlier work of Edigarian and Wiegerinck [39], raised a whole series of interesting problems, which are studied in this thesis.

The important role which fine potential theory, and the theory of finely holomorphic function play in the study of pluripolar hulls, has led us to introduce and study the concept of finely plurisubharmonic functions. These functions are the analogues of plurisubharmonic functions in the so-called pluri-finely open sets. A first attempt in this direction was made by El Kadiri [47], see also Fuglede [65, 69]. But it was necessary to first overcome some problems about the pluri-fine topology before developing “fine pluripotential theory”, or the theory of finely holomorphic functions of several variables.

Therefore, we will start in this thesis with a thorough study of the pluri-fine topology, with much focus on connectedness properties. After establishing pleasant properties of this topology in chapter 3, we turn to study finely plurisubharmonic functions. This is done in the same spirit as Fuglede’s finely subharmonic functions. See Chapter 4. It turns out that a rich theory of finely plurisubharmonic functions can be developed. In fact it will be proved that most fundamental results on finely subharmonic functions have a counterpart in the finely plurisubharmonic setting. These results will subsequently yield precise information about pluripolar hulls. See Section 1.5 for a complete description of the contents of this thesis.

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When I discussed the possibilities of a PhD project with Jan in 2004, he suggested to study the pluri-fine topology and to develop the theory of finely plurisubharmonic functions. This sounded to me hard and perhaps unworkable project. But what was even inconceivable is that this would help us to understand pluripolar hulls. Gradually, the plan turned out to be promising, and was eventually successfully completed. I immensely admire Jan’s strategic approach to problems and his deep mathematical insight.

Professor Tom Koornwinder was very kind to act as my supervisor during the first year. I would like to express my sincere gratitude to him too and thank him for taking part in the committee.

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