Fine aspects of pluripotential theory
El Marzguioui, S.

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covered that the theory of finely holomorphic functions can be fruitfully applied to explain the behavior of pluripolar hulls of graphs of holomorphic functions. This unexpected connection, which actually was implicit in earlier work of Edigarian and Wiegerinck [39], raised a whole series of interesting problems, which are studied in this thesis.

The important role which fine potential theory, and the theory of finely holomorphic function play in the study of pluripolar hulls, has led us to introduce and study the concept of finely plurisubharmonic functions. These functions are the analogues of plurisubharmonic functions in the so-called pluri-finely open sets. A first attempt in this direction was made by El Kadiri [47], see also Fuglede [65, 69]. But it was necessary to first overcome some problems about the pluri-fine topology before developing “fine pluripotential theory”, or the theory of finely holomorphic functions of several variables.

Therefore, we will start in this thesis with a thorough study of the pluri-fine topology, with much focus on connectedness properties. After establishing pleasant properties of this topology in chapter 3, we turn to study finely plurisubharmonic functions. This is done in the same spirit as Fuglede’s finely subharmonic functions. See Chapter 4. It turns out that a rich theory of finely plurisubharmonic functions can be developed. In fact it will be proved that most fundamental results on finely subharmonic functions have a counterpart in the finely plurisubharmonic setting. These results will subsequently yield precise information about pluripolar hulls. See Section 1.5 for a complete description of the contents of this thesis.

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