Fine aspects of pluripotential theory

El Marzguioui, S.

Citation for published version (APA):
covered that the theory of finely holomorphic functions can be fruitfully applied to explain the behavior of pluripolar hulls of graphs of holomorphic functions. This unexpected connection, which actually was implicit in earlier work of Edgarian and Wiegerinck [39], raised a whole series of interesting problems, which are studied in this thesis.

The important role which fine potential theory, and the theory of finely holomorphic function play in the study of pluripolar hulls, has led us to introduce and study the concept of finely plurisubharmonic functions. These functions are the analogues of plurisubharmonic functions in the so-called pluri-finely open sets. A first attempt in this direction was made by El Kadiri [47], see also Fuglede [65, 69]. But it was necessary to first overcome some problems about the pluri-fine topology before developing “fine pluripotential theory”, or the theory of finely holomorphic functions of several variables.

Therefore, we will start in this thesis with a thorough study of the pluri-fine topology, with much focus on connectedness properties. After establishing pleasant properties of this topology in chapter 3, we turn to study finely plurisubharmonic functions. This is done in the same spirit as Fuglede’s finely subharmonic functions. See Chapter 4. It turns out that a rich theory of finely plurisubharmonic functions can be developed. In fact it will be proved that most fundamental results on finely subharmonic functions have a counterpart in the finely plurisubharmonic setting. These results will subsequently yield precise information about pluripolar hulls. See Section 1.5 for a complete description of the contents of this thesis.

Acknowledgments

I am greatly indebted to my supervisor (and advisor) Professor Jan Wiegerinck for his collaboration and excellent guidance. Despite many responsibilities, under which the task to run the Korteweg-de Vries institute (KDVI), he was/is always there for discussions and advice. When new ideas were discovered, there was contact even during his vacations or the short stays of one of us at other universities.

When I discussed the possibilities of a PhD project with Jan in 2004, he suggested to study the pluri-fine topology and to develop the theory of finely plurisubharmonic functions. This sounded to me hard and perhaps unworkable project. But what was even inconceivable is that this would help us to understand pluripolar hulls. Gradually, the plan turned out to be promising, and was eventually successfully completed. I immensely admire Jan’s strategic approach to problems and his deep mathematical insight.

Professor Tom Koornwinder was very kind to act as my supervisor during the first year. I would like to express my sincere gratitude to him too and thank him for taking part in the committee.

Next I would like to express my admiration and deep gratitude to Professor Ahmed Zeriahi who initiated me to pluripotential theory. The courses I took from him at the university of Rabat (Morocco) were fundamental for my mathematical career. I am also grateful for his invitations, supervision and the support he gave me during my stay at Laboratoire Emile Picard (Université Paul Sabatier) in the
period 2001-2002. The hospitality of this institution and the kindness of its staff have made my time enjoyable.

Also I would like to thank Professor Bent Fuglede for the invitation to the mathematics department at Copenhagen university. During a tow-weeks stay we had very interesting discussions, and he explained to me several results in fine potential theory. I am also indebted to him for the illuminating and fruitful correspondences, and for keeping his interest in my work.

Professor Armen Edigarian was very kind to invite me to the institute of Mathematics of Jagiellonian University. A visit which was very fruitful. I also would like to thank him for his collaboration which resulted in writing the joint paper [41].

Furthermore I thank my colleague Tomas Edlund. The collaboration with him has not only resulted in writing a joint paper (Chapter 6), but it was also the beginning of an excellent friendship.

Thanks also to Guus Balkema for an illuminating discussion concerning Lemma 3.4.5.

Also I would like to thank all the promotion committee members for the careful reading of the manuscript.

The excellent working conditions and the nice atmosphere at the KDV institute have made the last four years an enjoyable experience. Of course this would not have been possible without very kind friends, colleagues and staff. Thanks to everybody, but in particular Abdelghafour, Evelien, Hicham, Michel, Phyllis, Ramon, René and Sjors.

Finally, I am indebted to my parents and brother Hossein for their support in many ways. My beloved wife Ouarda and my dear children Redouan and Leila have being so patient when yet another weekend was spent doing mathematics. This is more than enough reason to dedicate this thesis to them.