Fiscal policy under rules and restrictions

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In the last fifteen years, several countries have adopted fiscal rules and restrictions in order to guarantee fiscal discipline. This thesis discusses some of the main motivations, forms, and effects of such restrictions. Restrictions on public debt are compared with restrictions on primary deficit, showing that economies with the former type of sanctions feature higher social welfare than economies with the latter type. The incentives of a government facing electoral uncertainty to implement structural reforms in the presence of a deficit restriction are also explored. In designing a reform package, the government faces a trade-off between enhancing its electoral chances by compensating individuals and the cost of violating the restriction. Regarding Europe's Maastricht Treaty and Stability and Growth Pact, we find that they have been relevant to reduce deficits, although none of them have altered the cyclical behaviour of fiscal authorities in the Eurozone. We also conclude that the recent reform of the SGP has given more room for the implementation of necessary structural reforms in Europe. Overall, this thesis demonstrates that rules and restrictions can be effective in curbing excessive deficits and to guarantee fiscal sustainability. Nevertheless, their design deserves a careful attention in order to avoid providing the wrong incentives to policymakers and negative economic spillovers.

Marcos Poplawski Ribeiro (1977) graduated in Economics from the University of São Paulo, Brazil (1999), and obtained a M.Sc. in Economics from the Fundação Getúlio Vargas - SP (2002) in Brazil. In 2002 he joined the Tinbergen Institute and the University of Amsterdam (UvA) to pursue his PhD study. During his Ph.D., he received his M.Phil. from the Tinbergen Institute (2004) and did consultancies and internships at the United Nations, the International Monetary Fund and the European Central Bank. Currently, he is an economist at the Centre d'Etudes Prospectives et d'Informations Internationales (CEPII) in Paris and continues his collaboration with UvA.

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Fiscal Policy under Rules and Restrictions

Marcos Poplawski Ribeiro

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Fiscal Policy under Rules and Restrictions

Marcos Poplawski Ribeiro

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This PhD thesis is the result of almost five and a half years of work at the University of Amsterdam (UvA) and Tinbergen Institute (TI) in the Netherlands. Its completion has been a dream since the beginning of my studies in Economics twelve years ago in São Paulo. The move from Brazil to Europe and everything that I learned and experienced during this time is impossible to describe in full. Writing a PhD thesis in a foreign country; indeed, another continent, is a process that requires not only hard work but a lot of support from family, friends, and colleagues. Therefore, I would like to thank some of those who helped me and contributed directly or indirectly to the accomplishment of this degree.

I would like to start from the beginning and thank Jolanda Baptista, who was the first lecturer in my Master in São Paulo, and a fundamental personage in the history of this thesis. In June 2000, Jolanda brought Professor Rob Alessie from TI to teach a winter course in my Master program. During that course, he made me an invitation that culminated in my first visit to the TI in Amsterdam in September 2001, for which I would like to thank him here.

During that visit, I came to understand and appreciate the standard of academic quality that the UvA and TI represent. I also made a lot of friends there. Among them were Aljaz, Bang, Lev, Jos, Johan, Martijn, Mathijs, Robert, Silvia, and Simona; to all of whom I am grateful for their kindness and friendship. A special thank goes to Carla, Miguel and Mauro, who helped me a lot in my first time living abroad, far away from Brazil.

Following my first, positive experience of TI, I decided to apply for a Ph.D position in Amsterdam. That decision was particularly encouraged by the DGE of TI at that period, Professor Maarten Lindeboom. He suggested some PhD proposals to me, among them that of Professor Frank de Jong. Eventually, our research interests did not match, but I am thankful for his suggestions and introduction to my future supervisor, Professor Roel Beetsma.

In Brazil, while I was waiting for the final decision about my PhD candidature at UvA, several people supported my application and helped me in finishing my Masters. In particular, I would like to thank Professor Gesner Oliveira, Professor Willian Eid Jr., and Ricardo Rochman for all their support and the projects that we did together. Professor Carolina Leme was fundamental in supervising my MSc. thesis and encouraging me to pursue a PhD abroad.

On September 20th of 2002, I started simultaneously my PhD at UvA and the Mphil program at TI. In the following two years, I worked very hard and learned a lot, but also managed to enjoy Amsterdam. During that time, I made several new friends at TI, who remained close during all my PhD years. My thanks go to Ana, Carlos, Dessi, Dinara, Harris, José, Razvan, Rute, Seble, Valentin, and Yin Yen, among others. They made my life more pleasant during the Mphil. Moreover, I am grateful to Sara and Brigitte, who introduced me to the Dutch culture and with whom I learned so much and had so much fun. Further, my international friends Alberto, Fred, Irfan, Isabel, and Mischa helped me a lot to feel at home during this initial period in Amsterdam.

During my MPhil, I was also fortunate enough to be able to participate in the creation of the TI Student Council, which organizes a lot of activities for TI students and established itself as the main channel of communication between the TI management and students. As a member of the Council, I shared very nice time with the other fellows: Ernesto, Marco, Maria, Sandra, Sebi, Vali, and Wendy; who became very good friends. The TI staff also deserves a special mention due to their attention and assistance through all these years. My special thanks go to Marian, Carine, Professor Jaap Abbring, and Nora.

My stay in Amsterdam was not always only fun and hard work though. In the beginning of 2004, I had a terrible fire in my apartment, in which some neighbours were seriously injured, and where I lost all my belongings. That maybe was the toughest time of my life and I managed to quickly recover from it due to the help of my family, friends, and colleagues. Especially, I would like to acknowledge the exceptional support received from the staff of the Personnel Department of the Faculty of Economics from UvA. Bernadette Clemens, in particular, was like an angel to me. I also would like to include here my thoughts to all the victims of the fire in Marnixtraat, to whom I also dedicate this thesis and years of work.

In the past three and a half years, I have worked almost full time at my office at UvA. There, the environment in the TOE research group was always stimulating and cordial. I am grateful to: Professor Henk Jager, who guided me in preparing my first course taught in International Economics at UvA; Dirk Veestraeten, who advised me in supervising Bachelors theses; and Koen Vermeylen, who supervised my tutorials of the Macroeconomics course. Recently, Professor Wouter den Haan and Philip Jung joined TOE, making the quality and the environment of the group even better. Further, I would like to thank Naomi and Tijmen for their friendship and company during these years sharing the same floor and situation in UvA. The staff of the *Algemene Economie Afdeling* should also be acknowledged. In particular, Jolanda, Robert and Wilma were very helpful and caring with me. Bas Bouten from Resam deserves a special mention for his friendship and support with several UvA-bureaucracy issues.

Living in Amsterdam is per se a fantastic experience. In this city, my second home, I did various extra activities in which I met a lot of people. The first and most important was, as usual, Karate. Since my childhood this sport has provided me with a lot, and in Amsterdam that was no different. I am grateful for the friendship of Alex, Almir, Andrew, Daniel, Hein, Hogni, Katel, Shireen, and Sureshi. A special thanks go to my Senseis, Bregje, Carl, and Tom; who accepted me in the SKCA and made my life in Amsterdam more pleasant. Thanks also go to Markus, who became a close friend and my favorite DJ.

Throughout these years, I also managed to learn Dutch, a language whose degree of difficulty of its courses is comparable to any TI MPhil course. In learning such hard but *gezellig* language, I created a nice group of friends. Claudia, Daniel, Fabrizio, Janelle, Joana, Kateryna, Nicholas, Maya, and others, *dank jullie wel voor alles*! I should also thank my very good friends Pietro and Srdjan. Pietro was a great colleague in UvA. Further, Mischa, Paul and Joost were nice shareholders of our enormous boat...

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During my PhD, I did not stay the entire time in Amsterdam. I was very fortunate to do internships in some of the main international organizations around the world. My first, and therefore, most remarkable experience was at the Office of the Secretary General of the United Nations in New York in 2006. The view of the Hudson river from the UN building and the meetings on the floor of the then Secretary General, Kofi Anan, are unforgettable. In New York, I had a fantastic time and made good friends, David, Maria, Michael, Ribu, Sam and my bosses Silva, Minh-Thu, Mark, who I miss a lot! In particular, I am grateful to my friend Neeltje, with whom I always had fun during my visits to the US.

In the winter of 2007, I joined the IMF in Washington DC for another internship. That was also a fascinating experience academically and personally speaking. There, I entered into contact with the topic of the African economy, which became part of my current research agenda. DC is also a nice city where I made new contacts. Flavia and Claudia, and my roommate Kathleen deserve a special acknowledge for having received me in their place in DC. The other IMF interns, besides their quality, formed a very nice group of friends. My thoughts go to Alberto, Ernesto, Gunes, Giovanna, Sofia, and in particular to Luciana and Rodriguinho who became close friends. Finally, I am very grateful for all confidence deposited on me from my friend and co-author Alfredo Baldini, from whom I learned so much there in the Fund.

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Brazilian background. I was very luck to meet the two of them. Bas, Els, Karen, Marcelo, Marco, Wouter, and Yinske are other friends from Amsterdam that I must mention.

If Sandra and Débora were my sisters, Massimo was definitely my brother in Amsterdam. In the last 4 years, it is difficult to remember a day in Amsterdam that I did not speak with him or ask his advice. Massimo would always be there helping and giving me good advice on what to do. He also contributed substantially to my PhD thesis, particularly in Chapter 4, in which I benefited from his knowledge on Econometrics and Applied Macroeconomics. Thank you, Massimo, Holly and Rose for everything! Very recently, another nice Italian friend has been Jacopo, who warmly welcomed me in Paris, and who has helped me to survive in this city without speaking a word of French.

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Amsterdam, December 2007.

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Chapter 1

Introduction

In the last fifteen years several countries around the globe have adopted stringent fiscal measures, in particular fiscal rules and restrictions, in order to guarantee (or at least *signal*) fiscal discipline.

Fiscal rules and restrictions are not a recent phenomenon. As Basseto and Sargent (2006) mention, types of golden rules were followed by several national governments already in the eighteenth and nineteenth centuries. However, in recent years probably more fiscal restrictions have been adopted than ever before.

Fiscal restrictions have been adopted for a wide variety of reasons, for example: (i) to ensure macroeconomic stability; (ii) to enhance the credibility of fiscal discipline and aid in deficit elimination; (c) to ensure long-term sustainability of fiscal policy, especially in light of population ageing; or (d) to minimize negative externalities within a federation or international arrangement. Underlying most fiscal rules and restrictions is a sense that present or future governments may not be willing or able to implement optimal fiscal policy measures without external pressure (Kennedy and Robbins, 2001).

Perhaps the most prominent example of this new generation of fiscal restrictions are the fiscal rules imposed by the Treaty on the European Union (the "Maastricht Treaty" – MT), ratified in 1992, and specified in more detail in the Stability Growth Pact (SGP) ratified in 1998. However, several countries outside the EU have adopted rules that mimic the EU rules.

The introduction of such rules and restrictions can have substantial macroeconomic effects. This thesis studies some of those effects. Chapter 2 compares from a theoretical perspective restrictions on the public debt versus restrictions on the primary deficit. This analysis is motivated by proposals to give a more prominent role to debt rather than deficits in the SGP. There is no consensus in the literature what is the most desirable type of restriction. Proponents of debt-based restrictions claim that debt levels give a more accurate picture of the sustainability of a government's finance than deficit levels (see, for example, Wyplosz, 2005). Critics, on the other hand, argue that incumbent governments should not be penalized for the overaccumulation of debt by previous administrations. Hence, restrictions on the primary deficit would be fairer while still guaranteeing fiscal sustainability. Chapter 2, therefore, explores these issues by analyzing differences in the economy's responses and in the welfare consequences under the two types of fiscal restrictions.

Chapter 3 contains a political-economy analysis of the consequences of fiscal restrictions for structural reforms. Many countries are currently confronted with the simultaneous need to pursue fiscal discipline and to implement structural reforms, such as making their labor and product markets more flexible and reforming their welfare and pension systems. Those needs should not be seen in isolation. In fact, the two are tightly related to each other, as structural reforms are conducive to maintaining the long-term sustainability of the public finances. Nevertheless, it has been argued that fiscal restrictions, while in principle beneficial for fiscal discipline, may in the short run conflict with the willingness to conduct structural reforms. Therefore, Chapter 3 analyzes in the context of a political-economy framework the incentives for a government to implement structural reforms that yield long-run benefits in the presence of electoral uncertainty and a deficit restriction that reduces the scope for providing short-run compensation to the losers from the reform. The analysis is closely related to the debate about the recent reform of Europe's SGP, which now takes more explicit account of the short-run costs of certain structural reforms.

In Chapter 4 we investigate how effective Europe's fiscal restrictions have been in disciplining fiscal policy. The analysis separates the MT-period and the SGP-period, disentangling the effects of the Treaty provisions on the fiscal efforts of the Euro zone candidate members from the effects of the SGP once countries had made it into the Eurozone. We estimate fiscal reaction functions for a panel of 11 members of the Euro zone (except Luxembourg) using the cyclically adjusted primary deficit (CAPD) as the dependent variable. Controlling for fixed effects and relevant economic and political variables, we examine (i) whether the average level of the CAPD and (ii) its response to the output gap have changed between the MT- and SGP-periods, and (iii) how the CAPD reacted in cases when the reference deficit level was exceeded. Chapter 4 also makes similar comparisons between fiscal behavior in the EU and that in a number of other OECD countries.

Finally, Chapter 5 concludes this thesis summarizing the main results and policy implications derived from the previous chapters. That chapter also discusses some of the main gaps in the literature on fiscal rules and restrictions, and suggests potential directions for future research in this field.

Before proceeding to those analyses, this introduction presents a brief survey of the literature on why, how and where fiscal rules and restrictions are implemented.

1.1 Why are fiscal rules and restrictions implemented?

Wyplosz (2005) claims that the role of fiscal rules and restrictions is to promote fiscal discipline by ensuring fiscal solvency. In a simple set up, this can be represented by a government that issues bonds (B_t), raises taxes revenues (T_t), and purchases goods from private agents (G_t) in each period t. The government's period-t flow budget constraint reads:

$$B_{t+1} = (1+r)\, B_t + G_t - T_t,$$

where B_{t+1} is the stock of bonds (public debt) in period $t+1$ and r is the debt interest rate which, for convenience, we assume constant. Finally, and for simplicity, we assume that G_t and T_t are deterministic.

Independent of the government's objectives, the role of fiscal restrictions would be to ensure fiscal solvency, i.e. that the government's intertemporal budget constraint is always fulfilled:

$$(1+r)B_t = \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (T_s - G_s) \Leftrightarrow \lim_{s\to\infty} \frac{B_{s+1}}{(1+r)^s} = 0. \tag{1.1}$$

However, fiscal rules or restrictions may neither be necessary nor sufficient to guarantee that (2.11) is satisfied. For example, the constraint can be compatible with a succession of deficits ($T - G < 0$) followed by a series of surpluses. Imposing restrictions on the deficits would therefore be unnecessary. So, why are fiscal restrictions imposed in practice? This section tries to answer this question by discussing other main arguments in the literature on fiscal restrictions.

1.1.1 Deficit biases and other motivations on why to implement fiscal rules and restrictions

Another main argument for placing limits on the degree of fiscal policy discretion stems from the fact that democratically elected governments seem to have biases toward excessive deficits and debt. Fatás (2005), for instance, identifies four main biases in fiscal policy discussed in the literature, which are:

(a) Volatile fiscal policy. Discretionary changes in fiscal policy (changes in taxes or spending around election times) can have an effect on macroeconomic outcomes, and as a result, can bring undesirable volatility to the economy. Incompetent or greedy politicians can also generate substantial volatility in fiscal policy instruments.

(b) Procyclical fiscal policy. In response to economic fluctuations, fiscal policy should be countercyclical, such that the budget surplus should increase in booms and decrease in recessions in order to smooth out fluctuations in income. There is evidence, however, that in many cases spending increases in excess of the increase in taxes in good times (procyclical fiscal policy). One reason can be the misinterpretation by politicians of cyclical increases in revenue as structural, which would lead them to mistakenly cut tax rates or increase spending.

(c) Excessive deficits and unsustainable budgetary plans. Governments may not fully internalize the costs of additional debt. One main reason for that is described via *common-pool* models (see, for example, Drazen, 2004; Coeuré and Pisani-Ferry, 2005; Fatás, 2005; Anett, 2006; and Krogstrup and Wyplosz, 2006). This class of models stresses that politicians, who represent different groups and vested interests, have no incentive to constrain their spending demands given that the costs are shared by the population as a whole. So,

the common pool problem appears whenever there is more than one decision maker involved in setting the budget (Krogstrup and Wyplosz, 2006). Thus, when decision makers compete for their preferred public goods, they fail to internalize the cost of their choices on current and future cost in terms of higher taxes needed for debt service and repayment, leading to a deficit bias. Another political argument for excessive deficits concerns bureaucratic behavior, or *budget maximization*. Bureaucrats try to maximize their budgets since a higher budget translates into both higher salaries and more power in a typical principal-agent problem with asymmetric information (Drazen, 2004). A third argument stressing political factors is that deficits are used to constrain successor governments who may have different spending preferences (Alesina and Tabellini). Finally, a transitory accumulation of debt can also be caused by the postponement of fiscal adjustment after a cyclical downturn.

The last type of bias highlighted by that Fatás (2005) is *(d) intergenerational unfairness/inequity*. Children and the unborn do not have lobbying power in that they cannot vote, and are thus underrepresented in the political process. Hence, the short horizons under which governments and politicians operate, and that fact that future generations cannot participate in the political process, can result in excessive deficits in the present, which build up public debt and pass it on as a burden to those generations (see also Basseto and Sargent, 2006; and Morris et al., 2006).

Drazen (2004), Wyplosz (2005), and Morris et al. (2006) further mention that fiscal policy may also suffer from a deficit bias driven by time inconsistency. In parallel with monetary policy, fiscal policy has a long-run and a short-run objective. The long-run objective is debt sustainability. In the short run though, fiscal policy may also stabilize output over the business cycle. For instance, this is the only stabilization tool left at the national level in the European monetary union. Then, a standard argument for rules over discretion for a social welfare maximizing policymaker concerns cases where first, individual behavior depends on expectations about future policy, and second, the policymakers is limited in his choice of policy instruments. If he can change policies over time, he will often have the incentive to announce one policy for the future and then implement a different policy when it comes time to carry out his policy announcement. Rules, therefore, would aim at binding fiscal policy in the short run.

Related to the latter is also the bias due to coordination problems with the policy mix: fiscal and monetary policies (Wyplosz, 1999). Both governments and central banks are concerned with inflation and output gap. The outcome is the existence of two biases: an inflation bias and budget deficit bias. The central bank is ultimately deciding on the inflation rate and is responsible for the inflation bias. Given the legislature nature of the budgetary process, governments act less frequently than central banks do. They must anticipate the central bank's ultimate choice of inflation, possibly influencing its choice by affecting output. Thus, governments set their budget with several motivations in mind: the output-inflation trade-off and the central bank's own perception of, and reaction to, this trade-off. The result is the perception by governments of a trade-off between inflation and the budget deficit. This makes then the two authorities strategic substitutes, the more inflation is tolerated by the central bank, the less incentive for the government to attempt to expand through a deficit. Conversely, the more expansionary is the deficit the lower is the central bank's incentive to inflate.

Finally, *fiscal illusion*, a notion already formalized by Buchanan and Wagner (1977), can be also considered as an origin of deficit bias. Faced with deficit-financed expenditure, voters overestimate the value of the expenditure and underestimate the future tax burden. Hence, popular policies such as higher spending and/or lower taxes, even if not sustainable, create incentives for politicians to behave myopically. This is especially true in periods preceding elections, giving rise to *electoral cycles*. Such behavior is also likely to lead to asymmetric stabilization with higher deficits during recessions and more limited or no surpluses in booms.

Even though all those possibilities of deficit biases, why should we care about them? As Fatás (2005) explains, they cause governments to set a fiscal stance that is not appropriate given the cyclical position of the economy. The accumulation of debt leads either to default or to a large fiscal adjustment. In both cases, negative effects on economic volatility and the business cycle might occur as a consequence of the crisis or the large fiscal adjustment. Thus, in many cases the build up of excessive deficits leads to scenarios that can be a source of business cycle volatility.

Moreover, those deficit biases may be even greater in a monetary union (see Tornell and Velasco, 2000; Coeuré and Pisani-Ferry, 2005; and Anett, 2006). With a common currency, the exchange rate risk and the related interest rate risk premium associated with bad fiscal policy takes a much longer time to show up, decreasing the potential disciplinary effect of fiscal policy. Price stability might also be jeopardized if the monetary policymaker faces pressure to lower interest rates and inflate away the debt. Moreover, a country running into fiscal difficulties could be bailed out by other countries or by the common central bank, especially if this path would stave off a banking system crisis, which could increase moral hazard (see, for instance, De Grauwe, 2003; Buti and Giudice, 2004; and De Grauwe, 2006).

However, a monetary union can also induce greater fiscal discipline, ruling out therefore the need for fiscal rules. A monetary union dilutes the strategic influence of any single government over the monetary authority, since it is only one of several governments in that position and all may not face the same circumstances (Masson and Patillo, 2001). Nevertheless, fiscal policy coordination may have the perverse effect of once again strengthening the hand of the governments over the central bank (Beetsma and Bovenberg, 1998). A second favorable effect of a monetary union is to provide an "agency of restraint" over macroeconomic policies (Collier, 1991). By joining the union, countries voluntarily sign on to conservative monetary and fiscal policies, and hence would be reinforcing domestic tendencies in that direction. This is more likely to be effective if there is some external link and external currency peg. For instance, the CFA franc zones in the Western African countries have a fixed peg to the French franc (now the euro), as well as a guarantee of convertibility of their currency from the French Treasury.

Therefore, the debate on the costs and benefits of fiscal policy rules and restrictions is a complex one, as it deals with a variety of economic issues for which there is no clear consensus. Fatás (2005) points at least two levels of the discussion where there is disagreement and often a lack of clarity. The first relates to what constitutes good (or bad) fiscal policy. For him, while there is consensus that, if left alone, policy makers will make fiscal policy decisions far from their optimal level, there is no consensus on what constitutes poor fiscal policy management, and, more importantly, what its costs are. The second issue that follows is how to build fiscal policy rules that are optimal.

1.2 How are fiscal rules and restrictions implemented?

One approach to ensuring fiscal discipline would be to rely financial markets (see, for example, Wyplosz, 1997, Morris et al., 2006, and De Grauwe, 2007). To the extent that markets price risks correctly, the demand for the public debt issued by the various governments could act as both a barometer and a constraint. However, historical experience suggests skepticism about the ability of markets to impose discipline in this way. As Morris et al. (2006) discuss, there is a widespread perception that, due to asymmetric information and incentive problems, the reactions of financial markets to fiscal developments can be deficient (i.e. they exhibit delayed, volatile and non-linear behavior).[1] Given the difficulties of markets in inducing fiscal discipline, this section describes some of the types of fiscal rules and restrictions imposed to ensure such discipline. It also describes some of the issues related to the design of those restrictions.

1.2.1 Classes of fiscal restrictions

Several different designs of fiscal restrictions are currently implemented in the world. These can be divided in different classes.

Numerical rules

Legislated quantitative fiscal constraints on fiscal policy (or *target-oriented* approach - von Hagen and Harden, 1995) are perhaps the most well known class of fiscal restrictions. These rules can be broadly defined as a permanent constraint on fiscal performance (Goldfaijn and Guardia, 2004), or more formally, as a statutory or constitutional restriction on fiscal policy that sets a specific limit (or an upper ceiling) on a fiscal indicator such as

[1] Balassone et al. (2004) suggest that the effectiveness of the markets in inducing fiscal discipline requires certain conditions: no government body should have privileged access to the market, the market should have access to all the information necessary to evaluate the financial conditions of each government, bailing out troubled governments should not be allowed, and the public authorities should react to market signals.

the budgetary balance, debt, spending or taxation (Kennedy and Robbins, 2001). As this last definition mentions, they may take several forms, for example: restrictions on deficit financing, including balanced-budget laws; expenditure ceilings; numerical targets for fiscal variables; borrowing rules; and restrictions on issuance of debt (Drazen, 2004). Their severity, moreover, depends on the degree of coverage of the government sector, the fiscal indicator chosen, and the threshold being targeted (Buti and Giudice, 2004).

Golden Rules Among the numerical fiscal rules, one that has been discussed extensively in the literature is the so-called *“”golden rule”.* (see Blanchard and Giavazzi, 2003; and Creel, 2003). This type of fiscal rule is contingent on the amount of public investment. It attenuates fiscal restrictions, when debt (deficit) is generated to boost public investment. The usual argument used is that fiscal constraints should not restrain public investment, which increases the stock of capital and therefore, output. Instead, it should reward government expenditures that promote a more productive economy, even though the possibility that public investment pays for itself do not receive strong empirical support (see Fatás, 2005).

Moreover, a “golden rule” provides a proper accounting framework. Capital expenditures are different from current expenditures. It also provides a framework to think about debt sustainability. Given that net investment represents an increase in the assets of the government, it should be removed from the borrowing constraints even if the goal is to keep net debt equal to zero. Golden rules help also to mitigate the strong contractionary effects on output of fiscal consolidations, facilitating the success and public (and political) survival of those consolidations. In this way, they also ensure popular support. Given the different nature of public investment and the fact that it is perceived by some as a productive investment, leaving it out of the strict constraints on fiscal policy rules might look more reasonable in the eye of the public and therefore, find less political opposition.

Another strong arguments in favor of a “golden rule” are those of transparency and intergenerational fairness (see Fatás, 2005; and Basseto and Sargent, 2006). Transparency follows immediately from the first two points. Given their different nature, only by separating current and capital expenditures governments can provide an accurate and transparent picture of the fiscal policy stance. Regarding intergenerational equity, there is a need to differentiate between expenditures that benefit the current generation and those whose benefits will be spread over the current and future generations. While some forms of expenditure should be financed by current revenues (so that they are paid by the current generation), there is no reason to force the current generation to pay for spending that will render services over a long horizon (see also Basseto and Sargent, 2006).

Nevertheless, Eichengreen (2003) points as the main problem of golden rules the identification of productive public investment. As the author comments, governments on the verge of violating a fiscal target or upper threshold would be tempted to relabel current spending as public investment. More fundamentally, not all public investments are equally productive; not all public construction projects have positive rates of return. In the same vein, Beetsma and van der Ploeg (2007) highlight the political bias that this type of rule may create for too much public investment and government borrowing. Those authors demonstrate that indeed in a partisan set up, governments have incentives to over-invest in ideologically motivated public investment projects.

Procedural restrictions

Restrictions imposed on the procedure by which fiscal decisions are taken (or *procedure-oriented* approach - von Hagen and Harden, 1995) are another type of fiscal constraint. These restrictions may concern fiscal policy formulation, as well as the actual execution of policy. They may involve institutional arrangements according to which government budgets are presented, adopted, and executed.

Buti and Giudice (2004), for instance, divide these procedures in "hierarchical" and "collegial". They claim that the former is more conducive to fiscal discipline than the latter. Hierarchical procedures attribute strong authority to the *prudent* finance minister to overrule spending ministers during the intragovernmental preparation of the budget, limiting in this way the ability of the parliament to amend the government's budget proposals (delegation) and avoiding the common-pool problem (van der Ploeg, 2007).[2]

Commitment, whereby the different parties negotiate a "fiscal contract" involving strict budget targets, is another type of procedure or budgetary coordination. Those authors claim that delegation may be more suited to single-party governments, or where there are few policy differences on the budget. Commitment is the more logical choice for diverse coalitions, where the threat of breaking up the government serves as the enforcement mechanism. Finally, the mixed system is the one whereby the finance minister is granted a strong role in setting the budget, and this is followed by a negotiated agreement with parliament. This system may work well with single-party minority governments, where the budget is set by one party and then negotiated with the opposition to secure passage through parliament.

In the comparison between the two classes of fiscal restrictions, von Hagen and Harden (1995) argue that a *target-oriented* approach is adequate when the externality problem is large but the private gains from spending are relatively small, while a *procedure-oriented* approach is needed when the private gains from spending are large. Those authors suggest that the choice between these two approaches depends on the political environment of the government under consideration. Specifically, multi-party coalition governments will find a target-oriented approach more adequate as it emphasizes collective decision making rather than dominance of one or a few leading cabinet members.

[2] Van der Ploeg (2007) shows that a *prudent* minister of finance is one who deliberately underestimates future national income and the tax base, and sets the tax rate accordingly higher and public spending lower in order to minimize common-pool distortions.

Nevertheless, Drazen (2004) suggests that a fiscal restriction is more likely to be effective the more it addresses the specific cause of the problem, which then would favor procedural- instead of numerical rules. In addition, Fatás (2005) surveys empirical studies and shows that the design of budget processes (*e.g.* the relative power assigned to the Finance Minister and the importance given to budgetary targets), or different degrees of political constraints (*e.g.* number of veto points in the budgetary decisions) can have a significant impact on the size of budget deficits, on the success of fiscal consolidations, and on the volatility of discretionary changes in the budget.

Given those last points in favor of procedural restrictions, Drazen (2004) asks why there exists often a preference for simple numerical rules? A key reason according to him may be that both policymakers and the public are not convinced that procedural reforms will yield as unambiguous a discipline as simple numerical targets. A drawback of numerical rules, however, is the incentive to creative accounting, which entails a loss of information about the government's true budgetary situation and may reduce the credibility of the commitment to fiscal discipline. Two points discussed later in this chapter.

Stabilization funds

An alternative class of fiscal restriction pursued in a number of resource-abundant economies consists in accumulating part of the resource revenues in a stabilization fund, for smoothing the impact of short-term volatility, or in an endowment fund, for promoting long-term sustainability. A fund can make the treatment of resource earnings more visible, if subject to strict transparency requirements and if transfers between the fund and the government budget are embedded in a coherent macroeconomic framework. Because a basic function of the fund is to cover the budget deficit (while reflecting changes in government net financial worth), the fund cannot be separated from budgetary operations. Thus, potentially, the fund becomes the centerpiece of any effort to establish fiscal rules to ensure stability in the short run and sustainability in the long run.

Fiscal policy committees

Another proposal that has attracted a lot of attention is that of creating independent Fiscal Policy Committees (FPC) responsible for fiscal stabilization policy to overcome the political bias problems in fiscal policy (see, for instance, Eichengreen et al., 1999; Jonung and Larch, 2004; Andersen, 2005; Buti et al., 2005; and Wyplosz, 2005).

Andersen (2005) and Buti et al. (2005) comment that various proposals have been made in the literature for the structure and mandate for such committee. The main idea is that politicians should decide on the overall structure of fiscal policy depending on the political preferences for public sector activities given the constraint of fiscal sustainability. The FPC is entrusted with the responsibility for short run changes in fiscal policy aiming at stabilizing the economy. A more soft version of the idea is that the FPC does not have any formal decision power, but could be entrusted with the task of monitoring and assessing policy proposals and decisions thereby improving visibility and transparency, raising the political cost of opportunistic policies. As such, however, they appear more as a complement than as a substitute for numerical rules.

Therefore, while intellectually appealing, several authors conclude that one problem with the proposal is that it implicitly relies on the perception that it is possible to separate the tasks of allocation distribution and stabilization for governments. However, fiscal policy to a larger extent relates to distributional issues. Hence, the separation between setting a target for the budget balance (to be entrusted to the FPC) and the allocative and distributive functions (to remain in the responsibility of government and parliament) may turn out to be difficult. Another point is that an attempt to separate stabilization and allocation may restrict the feasible policy options. It is therefore hardly realistic to foresee politicians accepting the idea of an independent FPC as easily as an independent central bank for monetary policy.

Eichengreen et al. (1999) advocate this type of proposal, in particular, to Latin American countries. Those authors mention that institutional reforms are often thought of as choices between rules and discretion. Rules strengthen the ability to commit at the cost of flexibility; they are appropriate for economies which place a premium on the credibility of the policy process and operate in a relatively stable environment. In turn, discretion maximizes the responsiveness of policy; it is attractive to economies that place a premium on flexibility in a volatile environment. But those authors claim that a credibility-flexibility trade-off is unattractive for Latin America, where a history of high inflation implies a high shadow price of credibility, but at the same time the volatility of the economic environment implies a high shadow price of flexibility. So, for them an alternative that combines credibility with flexibility is to delegate the decision over the maximum permissible change in the public debt - the debt change limit (DCL) - to an independent national authority (i.e. an FPC). They argue that an independent agency with monitoring capacity and enforcement powers would be able to provide the signaling functions for external creditors currently supplied by the IMF. The advantages of a FPC relative to relying on the IMF are two-fold. First, unlike IMF programs, which tend to be negotiated only in times of crisis, the FPC would monitor the budget continuously, making fiscal policy less disruptive and the commitment to sound public finances more credible. Second, any restriction placed on the fiscal prerogatives of the Congress would be domestic in origin rather than being imposed from outside.

Nevertheless, as already briefly mentioned, the enactment of any of those fiscal rules and restrictions raises a number of issues regarding flexibility, credibility, and transparency. So, the rest of this section is devoted to the analysis of those concerns.

1.2.2 Flexibility of fiscal restrictions and countercyclical fiscal policy

One of the main concerns about fiscal rules is that they may be overly restrictive and limit a government's ability to engage in legitimate countercyclical fiscal policy when required (Kennedy and Robbins, 2001). In many OECD countries, as Chapter 4 of this thesis evinces, fiscal policy has a positive bias in the sense that expenditures are raised in a recession, but are not lowered in an expansion to balance the budget over the cycle.

A formal theoretic motivation for why fiscal policy should be countercyclical comes from Andersen (2005), for instance. That author, as many others, defends that fiscal policy should then primarily be left to the automatic stabilizers, which can be interpreted as providers of social or implicit insurance. They can be seen as the response to aggregate shocks and also serving a purpose in addressing idiosyncratic shocks.

However, do fiscal rules really hinder countercyclical fiscal policy? The empirical evidence is mixed. Some studies claim that the Maastricht Treaty and the SGP made European policy makers give up on output stabilization, concentrating instead on not letting the primary deficit get too large during recessions (Hughes-Hallet and Lewis, 2005; Cimadomo, 2007; and Chapter 4 of this thesis). Other authors, however, suggest that fiscal rules do not hinder countercyclical fiscal policy (e.g. Galí and Perotti, 2003; and Canova and Pappa, 2004). Canova and Pappa (2004), in particular, claim that over the last two decades, fiscal policy in the US and Europe has hardly focused on macroeconomic stabilization in any case. That happens for two complementary reasons. First, given the lags in the legislative process, discretionary fiscal policy may be unable to counteract business cycle fluctuations. Second, since automatic stabilizers are roughly given at business cycle frequencies, and since their share in total expenditure is typically large, also the non-discretionary component of expenditure cannot vary substantially over the cycles.

Furthermore, supporters of fiscal restrictions suggest that the medium term benefits of limiting government actions dominate the short run costs incurred by the inability of fiscal policy to react to business cycle conditions. This argument is usually based on two principles. First, by limiting the ability of governments to run politically motivated deficits and unsustainable levels of debt, fiscal constraints make governments more credible, reduce the suboptimality of political games, and induce a smoother path for taxes, which is the optimal policy to follow in a number of theoretical models. Second, since fluctuations in expenditure may have been themselves a source of undesirable fluctuations, restraining fiscal policy may actually stabilize the economy.

1.2.3 Credibility and enforcement of fiscal rules and restrictions

Credibility is perhaps the main concern regarding the adoption of fiscal rules and restrictions. Drazen (2004), for instance, analyzes how legislated rules, especially on outcomes, can be effective and make policy more credible than simply an announced commitment to the same goal. For him, governments truly committed to fiscal discipline build a reputation for sound budget policy and hence make credible their announcements to that effect, while governments not committed to fiscal discipline find ways to get around fiscal rules and restrictions. So, rules matter when politicians are predisposed to act in a fiscally disciplined manner. They strengthen politicians who want to be fiscally prudent, but they do not stand in the way of those who are determined to spend more than the rules allow (Schick, 2004). In this way, rules are not necessarily self-enforcing; they have to be willed into existence and sustained by political commitment and broad public consensus.

Nevertheless, those two authors see important differences between promises that have no legal backing *per se* and laws. One primary difference is that laws have *penalties* attached to them, so that there are explicit costs to breaking them. They (and institutions in general) *raise the cost* and *lower the benefit* from deviating from a given policy. In this way, a balanced-budget restriction in the constitution sends the signal that a society attaches fundamental importance to it. However, since constitutional laws are meant as an extreme form of commitment (and hence loss of flexibility), Drazen (2004) suggests that this solution should be used for fiscal restraint when other solutions have been tried, but have repeatedly failed.

Another way in which fiscal rules and restrictions can make the commitment to fiscal discipline more credible than simply an announcement commitment to deficit is the *signal* itself that their adoption conveys (Drazen, 2004). This is particularly true in emerging markets (Kopits, 2004-a). However, the effectiveness of signaling in enhancing reputation and reducing vulnerability depends on public perception of the authorities' readiness to match policy announcement with commensurate action (Kopits, 2004-b).

Enforcement is another very difficult question that arise in designing balanced-budget rules (Poterba, 1996). Adequate enforcement requires that compliance with a restriction is assessed ex post and not only ex ante (Morris et al, 2006). Many fiscal rules and restrictions have both explicit monitoring of the fiscal authority by some other agency as well as explicit penalties. The formalization, and the consequent visibility, of a rule may create new mechanisms and reduce the costs to monitor compliance. Another type of penalty is that failure to meet a fiscal policy target triggers an automatic expenditure cut of some sort.

There are other penalizations as well. Rules and restrictions cannot force legislators to be fiscally responsible; however, they may significantly increase the public´s awareness of deviations from fiscal responsibility by means of negative publicity. Therefore, further compliance with those constraints should be open to scrutiny by individual citizens or groups, who should be able to request an investigation. However, the power to voters to influence policymakers's behavior depend on the degree of budget transparency and on the possibility that the deficit bias occurs due to electoral incentives and not because of the political institutions *per se* (Debrun and Kumar, 2007).

Further, penalties for non-compliance should be sufficiently large. It should also be difficult for politicians to change the rules themselves. In the same line, Schick (2004) argues that fiscal restrictions may be even more powerful if they are an embedded norm that have been upheld for generations. For that author, many contemporary budget innovations seek to alter the cost-benefit ration of budgetary politics. These restrictions changes are effective only when budgeting is transparent, the media and interest group are attentive, and citizens feel they can influence public policy.

Therefore, the influence that rules and restrictions have on fiscal behavior depends on their design and the way in which they are implemented. At least, one of three types of enforcement: 1) exogenous enforcement, 2) self-enforcement, 3) signaling to external players; must be in place (Braun and Tommasi, 2004). In particular, the restrictions and their rationale need to be understood and supported by all parties concerned (i.e. politicians, voters and markets) and credible enforcement mechanisms need to be in place (Morris et al., 2006). Thus, next section summarizes some of the principles gathered in the literature that should drive the design of fiscal restrictions.

1.2.4 Guidelines and optimal design of fiscal rules and restrictions

Fatás (2005) argues that even though fiscal constraints seem to restrain the behavior of the government, they can also have undesirable side effects. In the trade-off between the positive effects they have on budget outcomes and the costs they impose because of their restrictiveness, there are several optimality principles and guidelines for their design around which the literature seems to have found a consensus (see Kopits and Symanski, 1998; Buti et al., 2003, 2005; Perry, 2003; and Morris et al., 2006). Those are:

- *Well defined*: The indicators that serve as targets, their institutional coverage (e.g. general versus central government) and the specification of escape clauses should be clear in order to facilitate monitoring and prevent creative accounting.

- *Transparent*: The goal that fiscal rules are designed to achieve has to be clear. As such, they should not be overly complicated and should not be easy to manipulate. Accounting, forecasting and institutional arrangements should be clearly communicated, thereby reducing the scope for creative accounting or misrepresentation of facts.

- *Adequate*: Rules and restrictions should be geared to the corresponding policy objective.

- *Simple*: Rules should be simple in order to enhance their appeal to politicians and the public.

- *Flexible:* Not all circumstances that affect public finances can be anticipated, and some flexibility is desirable to accommodate exogenous shocks beyond the control of the authorities, for example, by allowing the operation of the automatic stabilizers.

- *Consistent*: Fiscal restrictions should be consistent both internally and with other macroeconomic policies or policy rules.

- *Enforceable*: For a rule or restriction to be effective, it needs to be easily enforced. Hence, they should be backed by appropriate constitutional or legal norms, and the consequence of non-compliance, whether in the form of financial, judicial or reputational sanctions, should be clearly agreed upon.

- *Efficient*: Fiscal rules and restrictions should be supported by efficient policy actions.

Kopits and Symanski (1998) and Morris et al. (2006) point out that no fiscal constraint can fully combine all those desirable attributes, so when designing a restriction the policymaker has to balance those principles and find the best design given its policy objectives.

In addition, for Eichengreen et al. (1999) and Beetsma and Debrun (2007), in theory, contingent rules (rules with escape clauses) combine the advantages of the rules- and discretion-based approaches. Policy is constrained by rules except in the event of an observable, independently verifiable contingency not of the authorities own making. A contingent rule could, for instance, require that the budget be balanced except in the event of an exogenous macroeconomic disturbance or to allow the implementation of structural reforms (see Chapter 3 of this thesis; and Ribeiro and Beetsma, forthcoming). However, in practice it is difficult to satisfy the prerequisites for the operation of contingent rules. If the contingency cannot be observed and verified, politicians will be inclined to invoke it even when it has not occurred. If the contingency is manipulable, the authorities may provoke its occurrence in order to relax the rules-based constraint. Under these circumstances, the existence of an escape clause may be destabilizing.

In the context of a monetary union, Coeuré and Pisani-Ferry (2005) emphasize three desirable properties of a fiscal framework. It should be conducive to public finance sustainability; it should leave room for stabilisation; it should not discourage, and possibly encourage, structural reform. Moreover, the use of expenditure rules in a supranational context should be disregarded (see Buti et al., 2005). First, uniform spending rules would *de facto* impose homogeneous social preferences to politically heterogeneous countries while country-specific rules would be difficult to enforce. Second, spending norms do not refer to the fiscal variables which can produce negative externalities: while a rising deficit or debt level in one country can create area-wide problems, a rising expenditure level as such does not have negative repercussions on other countries. Moreover, expenditure rules cannot prevent deficit and debt increases stemming from tax cuts. Therefore, they would have to be complemented by a debt rule, which would prevent countries to expose themselves and the others of the union to a debt default (De Grauwe, 2007). Third, since no uniform expenditure to GDP ration can be prescribed, countries would be required to indicate targets for the expenditure ratio consistent with the desired deficit ratio.

For economies with large reserves of nonrenewable resources Bjerkholt and Niculescu (2004) argue that fiscal policy must contend with the additional tasks of phasing in the resource revenue, reallocating over time the resource earnings relative to the depletion and earnings profile, and protecting against the destabilization and possible default due to unfulfilled expectations. Those authors suggest that a balanced-budget rule might not be a desirable option for two reasons. First, an economy drawing down natural resource wealth, taking the depletion rate as given, may have good reasons for intertemporal redistribution of the liquidated wealth, including to generations living beyond the terminal phase of exploitation. Second, even absent intertemporal concerns, a balanced budget rule would need to be modified (by specifying in terms of a structural balance) to avert procyclical impact of high volatility in resource earnings; in other words, revenue used would have to be decoupled from the current resource earnings.

For emerging markets, Perry (2003) suggests that structural balance rules should form the basis of future attempts to establish Fiscal Responsibility Laws (FRLs) and Stabilizing Transfers for subnationals. Like Bjerkholt and Niculescu (2004), that author assess that most recent designs of FRL rely excessively on rigid quantitative ceilings that do not take into account the effects of shocks or the economic cycle. They are thus likely to accentuate procyclicality of fiscal policies and to prove non sustainable in the end. Those countries that are not yet in the capacity of adopting credible structural balance frameworks and rules, may benefit from considering more simple rules that would limit (real) expenditure growth to a moving average of past (real) revenues increases. Structural goals would be set according to fiscal consolidation needs. Thus, a country that starts below a sustainable structural balance should set goals that permit a gradual approximation to the required level. Perry (2003) also believes that it would be useful if the IMF decide to use systematically a structural balance framework when examining and discussing the fiscal stance of all countries and set the goals of programs accordingly. Moreover Kopits (2004-b) claims that permanent balanced-budget requirements or limits on public debt, implemented convincingly with the aim of reducing the public debt ratio to a sustainable path, over time can confer considerable benefits, including a likely decline in risk premia and thus in interest rates. In turn, the falling cost of capital paves the way for increased private investment and growth.

On the other hand, Braun and Tommasi (2004), argue against what they call a simplistic view that sees the writing of numerical limits on fiscal variables as the solution to fundamental fiscal problems. They suggest that international organizations should take a more comprehensive approach when dealing with fiscal problems of developing countries. Such an approach requires a deep understanding of the determinants of undesirable fiscal outcomes in each particular case, which in turn requires some explicit political analysis.

The gist of their argument is that the root of fiscal problems lies in politico-institutional factors, such as the incentives for fiscal profligacy at the local level caused by inadequate federal tax-sharing schemes, or the incentives for public spending caused by principal-agent type of problems. But for them, fiscal rules that do not address these underlying issues have a limited capacity to solve fiscal problems, and might even be counterproductive.

1.2.5 Subnational fiscal policy restrictions

In the context of subnational fiscal restrictions, Braun and Tommasi (2004) and Gonzalez et al. (2004) diagnose that the poor subnational fiscal behavior comes from large vertical fiscal imbalances, tax-sharing regimes with little incentives to raise taxes, several political motives by which the federal government would be willing to bail out provinces, and lack of enforcement of intergovernmental agreements.

Therefore, Braun and Tommasi (2004) suggest a general strategy to improve that behavior that should include: (a) reforms to the electoral mechanisms to lower the dependency of national legislators on local party elites; (b) reforms to the instruments of legislative interaction between the President and Congress; (c) reforms to the budget to curtail some Executive discretion, limiting the ability to perform bailouts; (d) institutional reforms to intergovernmental relations; (e) reforms to the tax-sharing agreement to improve the connection between the taxes raised and the public goods consumed within each jurisdiction; (f) macro-fiscal rules to guide through this transition towards a more cooperative fiscal stance.

Moreover, in order to improve fiscal performance in a sustainable and efficient manner, those authors highlight the need to focus on the determinants of fiscal behavior. In each country, the main deficiencies need to be identified, together with a complete diagnostic of their fiscal and political determinants. For them, fiscal rules and restrictions can be a useful instrument to accompany the transition path. On the other hand, an excessive focus on rules might lead to a risky sense of security when one sees such rules in place.

Regarding stabilization rules for national-to-subnational fiscal transfers, Gonzalez et al. (2004) argue that the following conditions should be in place before establishing them: (i) subnational governments should be credit constrained; (ii) the national government should have stable access to credit and quality debt management; (iii) there should not exist severe structural fiscal imbalances, either within or across levels of government. In other words, neither level of government faces unsustainable cyclically adjusted fiscal deficits. In addition, subnational governments do not spend excessively with financing by automatic transfers.

For that author, the federal level should play the predominant role in macroeconomic stabilization. In these circumstances, it would be efficient for the federal level to stabilize federal transfers during the downturn, so that it would be the one to borrow. However, if the federal government is on the brink of insolvency, or if there are long-term structural imbalances at the subnational level being financed by the current system of transfers, the stabilizing transfers may only complicate or even worse the initial problems.

Moreover, there are at least four types of arrangements for subnational governments that protect themselves with a cushion against macroeconomic shocks. First, subnational governments may maintain some margin to increase their revenue. Second, they may keep spending flexibility, especially with a deferrable investment program, without allowing close to 99 percent of revenue to finance wages and debt service. Third, they may build a state reserve to pursue other alternatives. Fourth, they may establish a secure credit line, available in times of fiscal distress.

All in all, the design of fiscal rules and restrictions must carefully balance incentives and constraints and include intratemporal and intertemporal considerations. As Canova and Pappa (2004) comment, critics and supporters of fiscal constraints do agree on one fact: deficits and debts have distributional effects which may have long lasting repercussions and, therefore, when designing the fiscal restriction these repercussions should be taken into account.

1.3 Where are fiscal rules implemented?

This section summarizes some of the most important experiences with fiscal rules around the world recently. We start by discussing the Maastricht Treaty and Stability and Growth Pact in the European Union. Then, we describe some other recent experiences with fiscal rules both in developing and developed countries.[3]

1.3.1 The Stability and Growth Pact

Historical background: the last sixty years of fiscal policy in Europe

Buti and Sapir (2006) makes a historic review of fiscal policy in Europe since the end of the Second World War. They divide those sixty years of public finance in Europe in two main periods: the 'Golden Age' between 1945 to 1973, and the most recent thirty years.

As those authors describe, in the first thirty years, Europe witnessed a 'Golden Age' of growth, stability and social cohesion, created by the post-war economic and social environment. Throughout that period, the size of welfare state increased considerably in Europe. By 1970 the share of total government expenditure in GDP reached between 35 and 40 percent in most countries - not considerably higher than in the United States, where total government expenditure stood at 32 percent of GDP.

Those authors claim that most of the economists in that period followed Musgrave (1959) and perceived the government as having three major economic roles: the provision of public goods and other measures to correct for market failures and improve the allocation of resources; the redistribution of income to achieve equitable distribution of income among households; and the stabilization of economic activity to attain high levels of employment

[3] One should be careful on comparing those rules though. In particular, the European with other national fiscal rules, since as Wyplosz (1997) well highlights, in true federations, the central government is as large as the lowest-level governments, and is in charge of macroeconomic stabilization. In Europe, in contrast, the equivalent of a central government is the European Commission, which is not allowed to run deficits and whose spending is irrelevant in terms of the Europe Union's gross domestic product.

with reasonable stability of prices. Higher taxes and social expenditures were seen as means not only to improve the distribution of income, but also to improve the allocation of resources and growth (by correcting market failures in the labour market) and to promote the stabilization of output. Moreover, the high growth rates helped to keep debt under control thereby ensuring the sustainability of public finances. There appeared to be no trade-off between allocative efficiency, redistribution and stabilization.

Buti and Sapir (2006) show, however, that the favorable development with respect to debt sustainability during that period was due mainly to a 'snow ball' effect rather than to underlying public finance decisions. The combination of high GDP growth rates and low real interest rates produced negative 'snow ball' effects that led to substantial declines in public debts. In that period, public finance also tended to play a stabilization role, in particular after 1961. Public debt decreased in 'good times' and increased in 'bad times' in Germany and Italy, for instance.

Turning to the most recent thirty years, the period from the mid-1970s to the mid-1980s was characterized by a combination of slow growth and high unemployment, resulting in increased demands for social protection and leading to severe consequence for public finances. The share of total government expenditure in GDP grew rapidly after 1973, reaching 45-50 percent in many European countries in 1985 - an increase of more than 10 percentage points compared to 1970.

By the mid-1980s, as those authors comment, Europe was stuck in a negative spiral: lower GDP growth and employment rates meant increasing public expenditure, which required increasing public revenue, which in turn implied higher social contributions and higher direct taxes, thereby reducing the incentive to work and to invest, hence further reducing the prospects for output and employment growth.

For them, the reason why Europe seemed durably trapped in this spiral was twofold. First, the shocks to the system were long-lasting. The slowdown of growth was initially not perceived as permanent which led policy makers to bet on stabilization rather than adjustment. After the two oil shocks of 1973 and 1979, Europe was confronted then with population ageing, the information technology revolution and globalization, all of which substantially increased the demand for social protection. Second, the system seemed politically unable to reform itself and to establish a new social contract aimed at increasing growth and preserving social welfare.

During that period, public debt increased during both good and bad times in France, Germany and Italy. By contrast, in the United Kingdom, public debt continued to decrease during both good and bad times. In other words, from 1973 till the mid-1990s there was an emergence of trade-offs between allocative efficiency and redistribution.

The combination of high levels of public expenditure and a high share of it devoted to social spending is therefore the main feature of European public finance for the past 30 years. It contrasts sharply with the situation that prevailed during the 30 years after World War Two, when public expenditure was much lower and the share of it devoted to social spending was also lower.

This high level trend of public expenditure was already identified in the beginning of the discussion regarding the formation of the European Union. The Delors' report, a document prepared in 1989 under the chairmanship of the then President of the European Commission (Jacques Delors), already outlined as necessary institutional arrangements to guarantee fiscal stability. This report portrayed the path towards economic and monetary union and culminated in the famous Maastricht Treaty, signed by the finance and foreign ministers of the European Union on 7 February 1992 in the Dutch city which gives its name.

The design of the Maastricht Treaty and the SGP[4]

The Maastricht Treaty (MT) updates and incorporates the 1957 Treaty of Rome, the founding act of the European Community, and incorporates the Single European Act implemented in 1992 (free movements of goods, people and capital). It is based on a strict stability orientation. The founders of the European Union were aware that only a monetary union based on growth and price stability could be successful and that for this it would be necessary to acquire the unqualified confidence of citizens and markets alike in the single European currency (Stark, 2001). In terms of fiscal policy, its main goal was to ensure that only countries with a sufficiently good fiscal track record could enter the euro area in order to prevent fiscal crises that would negatively affect other countries (see Articles 101 to 104 of that Treaty).

The Treaty sets out convergence criteria that must be satisfied in order for countries to participate in the European Economic and Monetary Union (EMU). It is formally structured around three stages. The first stage began in 1992 with the formal ratification of the treaty. During the second stage, started in January 1994, national central banks were given formal independence and ceased to grant direct loans to their nation's treasuries. The shift to the second stage also coincided with the establishment of the European Monetary Institute (EMI), with two main functions. One was to prepare the creation of the European Central Bank, whose statutes and mission are actually laid out in the Maastricht Treaty. The other function of the EMI was to oversee the 'convergence criteria' which is used to decide which countries are ready to enter the monetary union, marking the beginning of Stage III. This happened in January 1, 1999 when a sufficient number of countries met the convergence criteria.

Under the Treaty, fiscal discipline was to be judged on the basis of two main criteria: (1) whether the government deficit as a percentage of GDP exceeds the reference value of 3 per cent of GDP; and (2) whether the ratio of gross government debt to GDP exceeds the reference value of 60 per cent of GDP. Exceptions could be made with respect to the deficit criterion if the ratio of the deficit to GDP has declined significantly and was close

[4]For more information on the creation of the SGP and how that pact works see, for example, Stark, 2001; Costello, 2001; and Cabral, 2001.

to the reference value, or if the excess was only temporary and the ratio remained close to the reference value. Exceptions could be made with respect to the debt criterion if the debt-to-GDP ratio was diminishing at an acceptable pace.

The MT provisions were strengthened by the Stability and Growth Pact (SGP), which ensures that countries sustain their commitment to fiscal prudence once they have joined the EMU. The SGP and its "excessive deficit procedure" were settled in June 1997 and took effect when the euro was launched on January 1, 1999; even though the first thoughts on them were first presented during a meeting of the ECOFIN in Brussels on September 1995 (Stark, 2001). The "excessive deficit procedure" makes permanent one of the entry convergence criteria, the 3 percent deficit/GDP ceiling. It defines the 'exceptional conditions' underwhich a country may be temporarily allowed to breach the ceiling, and it specifies how noncompliant countries will face first private, and then public reprimands, before being fined. Moreover, the SGP also requires that member states set medium-term objectives of budgetary positions close to balance or in surplus, in order to provide sufficient flexibility to allow the operation of automatic fiscal stabilizers while remaining within the 3 per cent deficit limit. This last point was considered to be especially important since member countries can no longer rely on the exchange rate instrument to dampen economic shocks.

The SGP also provided for increased monitoring, with an annual review of the stability programmes of countries participating in the euro area (and convergence programmes of those not participating in the euro area). Each country must submit every year to the EU Commission its budget forecasts for the three following years. In the case of an excessive deficit in a country participating in the euro area, a course of remedial action would be proposed, which should be implemented within ten months. Otherwise, the country could be subject to sanctions in the form of a mandatory non-interest bearing deposit, which could vary in size with the magnitude of the excessive deficit, up to a maximum of 0.5 per cent of GDP. If the excessive deficit was eliminated within two years, the deposit would be returned to the country. If it is not eliminated within that time frame, the deposit would become a fine.

Reform of the SGP

The SGP nevertheless proved to be a hard policy to be implemented and enforced. Therefore, on 22 and 23 March 2005, the EU finance ministers reached a deal on reforms to the pact at an extraordinary meeting in advance of the EU summit of heads of state and government in June of the same year. This reform changed several items of the previous pact in its both preventive and corrective arm.

Under the preventive arm, Morris et al. (2006) discuss that the reform introduced various refinements to the earlier provisions concerning the setting of an progress towards sound medium-term budgetary positions and to the elements that are to be taken into account when assessing Member States' fiscal positions. These include:

- *The definition of the medium-term budgetary objective and the adjustment path toward it*: Rather than being required to target "close to balance or in surplus" budgetary positions, each Member State now presents its own country-specific medium-term objective (MTO) and its stability and convergence program on the basis of debt ratios and potential growth rates, which is then assessed by the Council. Targets are specified in structural terms, i.e. cyclically-adjusted and net of the effects of temporary measures, and range between a deficit of 1% of GDP and a small surplus. The latter applies to high-debt, slow-growth countries. A more articulated set of provisions concerns also the path towards the medium term objectives, though a minimum annual adjustment of 0,5% of GDP has to be ensured (Buti et al., 2005; and Morris et al., 2006).

- *Taking into account structural reforms*: Member states may be allowed to deviate from the MTO or the adjustment path towards it if they undertake structural reforms in a move closer to that advocated in Chapter 3 of this thesis. In this context, special attention is paid to pension reforms which introduce multi-pillar systems that include a mandatory, fully funded pillar. However, "only reforms which have direct long-term cost-saving effects, including by raising potential growth, and therefore a verifiable positive impact on the long-term sustainability of public finances, will be taken into account. Implicit liabilities will also be taken into account in the future, once further technical innovations allow the Council to agree on criteria and methodological aspects.

Regarding the changes in the corrective arm, they go in direction of introducing more flexibility into the EDP, in particular by relaxing, adding specificity, or clarifying the availability of various escape clauses. The changes include:

- *The definition of "severe economic downturn"*: It is now a negative annual real GDP growth rate or an accumulated loss of output during a protracted period of very low annual real GDP growth relative to potential growth.

- *Specification of the "other relevant factors"*: When evaluating deficits exceeding the 3% limit, the Commission will take into account a number of factors as: cyclical conditions to the implementation of the Lisbon agenda and policies to foster R&D and innovation; debt sustainability and the overall quality of public finances; financial contributions to international solidarity and fiscal burdens related to European unification.

- *Extension of procedural deadlines*: A number of procedural deadlines have been extended. In particular, in the event that a country is found to have an excessive deficit, the deadline to correct it has been extended from one year to two, and this period can be extended further in the event of "unexpected adverse economic events with major unfavorable budgetary effects during the excessive deficit procedure". So, countries could have as long as five years to correct their deficit (Chang, 2006).

- *Unexpected Adverse events and repeated recommendations*: The original SGP did not explicitly provide for the reissuance of Council recommendations or for the extension of deadlines for the correction of excessive deficits as those discussed above.

- *Increase the focus on debt sustainability*: Nevertheless, no agreement could be reached on a quantitative definition of the satisfactory pace of debt reduction.

Thus, while the basic rules, notably the 3% and 60% limits on deficit and debt in relation to GDP have remained in place, the reformed pact is more flexible, provides more explicit scope for exercising judgement and discretion than in its original version, and puts a new emphasis on public finance sustainability.

Among those changes, perhaps the most important was in the governance of the Pact (Coeuré and Pisani-Ferry, 2005). First, a consensus has emerged to give to the Commission the right to send an early warning to member countries without the approval of the Council. Second, with the reformed pact, the eurozone has moved away from its initial emphasis on governance by fixed rules and has reintroduced discretion. Moreover, the new Pact also acknowledges the importance of quality, timeliness and reliability of fiscal statistics and pledges to ensure the independence, integrity and accountability of both national statistics offices and Eurostat (Buti et al., 2005).

Overall, Morris et al. (2006) point that reactions to the reformed Pact have been mixed. Proponents of the reform consider that better adaptation of the rules to differing economic circumstances and needs will enhance commitment to them and thereby facilitate their enforcement. Opponents, by contrast, have criticized the changes as representing a watering-down of the rules, making them more complex and less transparent, and as a sign of a lack of commitment to fiscal discipline on the part of the Member States of the European Union. It has also not addressed the essential problem of weak enforcement provisions, considered by many to be the main shortcoming of the Pact.

1.3.2 Other fiscal constraints in the world

Besides the European Union, a number of other countries and regions have also introduced fiscal rules and restrictions. In most cases, these constraints impose limits on public spending and are multiannual, so that they cannot be circumvented by shifting some specific items, especially public investment, to further years. Others concern specific budgetary items or stipulate, as in the US, that additional spending must be matched by additional revenues (pay-as-you-go). Such rules do not directly concern fiscal discipline. However, since fiscal discipline is often the consequence of increased spending not matched by increased revenues, these rules may also have a disciplinary effect (Wyplosz, 2005).

In emerging market economies, the adoption of fiscal constraints has been much more recent and limited mainly to Latin America (see Kopits, 2004-a). In some cases rules were introduced following a financial crisis; in others they were adopted to reduce vulnerability to a potential crisis. Often the immediate motivation has been to reverse the buildup of public debt, to restore fiscal sustainability and, more generally, to enhance the credibility of macroeconomic management. In addition, in some regions, mainly Central and Eastern Europe, rules and restrictions are increasingly viewed as an anchor in the convergence to a broader monetary union. (Kopits, 2004-a).

In practically all these countries, fiscal policy constraints have been embedded in a rules-based monetary framework. The latter includes an inflation-targeting regime (Brazil, Chile, Colombia, Mexico, Peru, Poland), a currency board arrangement (Estonia, and until recently Argentina), or a dollarized regime (Ecuador). In this sense, fiscal rules can be viewed as means to reduce or eliminate fiscal dominance in macroeconomic policy.

Several countries have established targets for the phased reduction of the debt-GDP or debt-revenue ratio (Brazil), or limits on the debt-GDP ratio (Poland). The debt-ration target or ceiling usually presupposes, either implicitly or explicitly (Brazil), an annual operational target in terms of a minimum primary surplus.

Generally, the institutional coverage of the restrictions depends on the degree of fiscal decentralization and autonomy of various levels of government. Fiscal constraints can also be specified in a constitutional provision (Mexico, Poland), high-level legislation (Brazil), or ordinary legislation (India) that applies to governments over successive electoral cycles. Alternatively, they may merely consist of a policy guideline declared by a given government and not necessarily binding on future governments (Chile, Estonia, earlier in Indonesia).

In terms of contents, the statute may be very detailed (Brazil), specifying not only the nature of the restrictions, but also detailed procedural rules governing compliance. At the other end of the spectrum, it may define a broad framework (India). Rarely, in the case of top-down subnational government rules (Colombia), deviation from the rule is subject to financial penalties. However, in most countries, noncompliance, especially by the national government is punished with loss of reputation toward the electorate or financial markets (self-enforcement of the rules).

The rest of this section brings a small summary of some of the recent experiences with fiscal rules in the international scenario.

Argentina

In September 1999, the Argentine Congress passed the Fiscal Responsibility Law (see, for example, Kennedy and Robbins, 2001; Braun and Tommasi, 2004; and Cooper et al., 2005). This law set a ceiling for the non-financial public sector deficit between 1999 and 2002 and required its decline such that balance would be achieved in 2003. It also established a Fiscal Stabilization Fund, financed through tax revenues, to dampen the impact of cyclical fluctuations and external shocks on government revenues. In addition, the law prohibited the creation of off-budget items and set out new reporting requirements, limiting the growth of expenditures. Penalties for civil servants who do not implement the budget were provided and transparency measures to increase the availability of information regarding the state of public finances were included. This Fiscal Responsibility Law was modified by the 2001 Budgetary Law, which relaxed the deficit ceilings and extended the date at which budget balance should be achieved until 2005

At the subnational level, several governments followed the national example and passed fiscal solvency rules. These rules differ across provinces in some of their characteristics, as well as in the degree to which they have been adhered to; but most of them include limits on government debt and requirements regarding the timely and accurate publication of fiscal information.

Australia

The Charter of Budget Honesty, passed in 1998, introduced a fiscal framework in Australia that requires governments to set out their medium-term fiscal strategy in each budget as well as their short-term fiscal objectives and targets, although it does not place any constraints on the nature of the targets (see Kennedy and Robbins, 2001).

The governments original debt target was to reduce the Commonwealth general government debt-to-GDP ratio to half of its 1995-96 level by the turn of the century, which was comfortably met, with net debt falling from a peak of almost 20 per cent of GDP in 1995-96 to around 7 per cent in 2000-01. The government's current medium-term objective is to balance the budget over the economic cycle. As a supplementary objective, the government also aims to improve its net worth (a measure that includes physical as well as financial assets).

Brazil

After the stabilization of inflation following the "Plano Real" in 1994 and the resultant abrupt reduction of Seigniorage revenues, several states and municipia became highly indebted in Brazil.[5] To correct this effect of the stabilization, and given the balance of payments crisis in 1998/9, the "Lei de Responsabilidade Fiscal" (Fiscal Responsibility Law - FRL) was enacted in May 2000 (see, for example, Braun and Tommasi, 2004; Goldfaijn and Guardia, 2004; and Fioravante et al., 2006).

Generally speaking, the FRL - considered by some the most comprehensive fiscal responsibility law introduced in Latin America (Buti and Giudice, 2004) - sets a framework for the conduct of fiscal policy, including budget planning, execution, and reporting re-

[5]The Brazilian federal constitution guarantees financial and administrative autonomy for subnational governments, assigns spending responsibilities to them, and clearly define their tax base and legal transfers from the federal government. Under this high degree of fiscal decentralization, 27 state and over 5500 local governments are responsible for approximately one-half of public expenditure, concentrated mainly in the provision of basic education, health, and public security. However, subnational government borrowing is directly controlled by the Senate. In addition, the Central Bank of Brazil sets limits on domestic bank credit to subnational governments. As in most countries, the Central Bank is forbidden to finance the nonfinancial public sector; it is not authorized to extend loans to any public sector entity or to purchase primary issues of government securities.

quirements. In contrast to the legislation in Argentina and Peru, Brazilian's law applies to all levels of government. It is intended to sustain the structural adjustment of public finances (allowing borrowing only to finance investment projects - golden rule), to constrain public indebtedness, and to limit payroll expenditures. To this end, the law establishes policy rules consisting of limits and targets for selected fiscal indicators over a three-year horizon, procedural rules (including transparency requirements), and corrective steps and legal sanctions for noncompliance. It also includes stricter limitations for the final year in office for politicians in an effort to limit the political business cycle. In addition, the government is required to meet the pre-announced operational target for primary balance, set in the Annual Budget Guidelines Law (LDO) in accordance with the debt limit.

In this regard, the most important innovation introduced by the FRL is the formulation of the debt ceiling for each level of government, which imposes that "net consolidated debt over the net current revenue" must not surpass the 1,2 limit. Moreover, the limit on payroll expenditure, a binding fiscal rule aimed at reversing the sharp growth of the public payroll in the 1990s, restricts payroll spending (defined as wages, salaries, and pensions) at 50 percent of net revenue for the federal government and at 60 percent for all subnational governments. Fioravante et al. (2006) argue, however, that numerical targets in the indicator of expenses with public servants payroll over the net current revenue, however, underweights other types of expenditures. Therefore, such limit do not imply a complete control of public expenditure. Besides, the expenditure ceiling stipulated ad hoc by the law can provide perverse incentives that harms the allocative efficiency of public resources.

Goldfaijn and Guardia (2004) suggest that the new institutional framework contained in the FRL improves the transparency of fiscal activities, especially through comprehensive, timely, frequent, and detailed reporting at all levels of government. Budget documentation includes estimates of tax expenditures. In addition, the authorities are required to present a four-year medium-term macroeconomic budgetary framework (MBF), along with a clear statement of underlying macroeconomic assumptions, which then serves as the basis for the annual budget proposal.

Noncompliance with the rules is subject to corrective action and possible sanctions. Any excess over the debt limit prescribed for a given level of government has to be eliminated within one year. While the excess persists, new financing and discretionary transfers from the federal governments are prohibited; in addition, noncompliance may result in the banning of new debt and denial of credit guarantees. A list of governments that have exceeded the limit has to be published by the finance ministry on a monthly basis. Public officials in noncomplying governments are liable to criminal prosecution. The Fiscal Crime Law details penalties for mismanagement, ranging from fines to loss of job and ineligibility for public office for a maximum of five years, to imprisonment.

Canada

At the federal level in Canada, the Federal Spending Control Act set limits on program spending from 1991-92 to 1995-96 (see Kennedy and Robbins, 2001). It covered all program spending, with the exception of that under major self-financing programs, such as expenditure under the Unemployment Insurance Act.

More importantly, the government introduced a number of non-legislated policy rules that contributed significantly to the dramatic improvement in Canada's federal finances in the 1990s. In 1994, the government adopted the practice of basing budget planning on economic assumptions below the middle of the range of private sector forecasts to minimize the risk of taking inappropriate policy actions as a result of fiscal forecasts based on overly-optimistic economic assumptions. In addition, the government began setting two-year rolling deficit targets, with an ultimate goal of a balanced budget.

On the subnational level, nine provinces and territories have enacted or tabled fiscal rules. Each fiscal rule requires balanced budgets, except in Yukon, where deficits are permitted as long as no net debt is accumulated. Fiscal rules cover the consolidated budget in every jurisdiction, except Saskatchewan and the Yukon, where it is the general revenue fund in the former case and the non-consolidated Public Accounts in the latter case. Most provinces require a balanced budget on an annual basis and several of them have chosen to also target debt reduction and elimination.

Chile

The current Chile's fiscal rule, incepted in 2000 (see, for example, Perry, 2003; Fatás, 2005; and Wyplosz, 2005), consists of an explicit commitment to keep the structural surplus in one per cent of GDP each year. The structural balance is estimated by removing the effects of variations in copper prices and the economic cycle on revenues, therefore it allows the balance to fluctuate during the cycle. This rule forces the government to high surpluses during booms and high copper prices and allows to run moderate deficits during downturns and low copper prices.

Before that, Chile had long experience with Stabilization Funds, via the Chilean Copper Stabilization Fund (CPF), created in 1985. That fund was idealized to help stabilize fiscal revenues from the volatility of copper receipts. It was instrumental in facilitating fiscal surplus during the second half of the 1980s when copper prices were high. It specially helped to contain pressures of the new Democratic period, by keeping surplus out of reach of the political pressures inherent in the normal budgetary process (see Perry, 2003).

The fund's saving rules were automatic, based on a moving average reference of copper price. Unfortunately, the divestiture rules were not automatic. Authorities found in the downturn of 1998 and 1999 that previous surpluses and the savings in the CSF were not enough to facilitate a counter cyclical fiscal policy, leading the new government to search for the new rule.

Colombia

Colombia has also long experience with stabilization funds. Perry (2003) shows that the Colombian Coffee Fund was instituted with the objective of permitting some stabilization in the incomes of coffee growers along the cycle of international prices, as well as to enforce commitments under the International Coffee Quota arrangements. But it also had, as a side effect, significant fiscal stabilization properties that became major objective of government policy in the eighties and early nineties. Given the importance of coffee in the Colombian economy, booms and busts were closely associated with cycles in international coffee prices. As a para-fiscal institution, the Coffee Fund was not included in the Budget and was kept out of the normal fiscal political process, so their surpluses were kept from political eyes. The Fund lost importance during the nineties, as coffee became economically less important and oil took largely its place, so the attention changed towards the establishment of an Oil Stabilization Fund.

The Colombian Oil Stabilization Fund (OSF) instituted in 1995 uses automatic rules for both savings and retirement: it requires saving in excess revenues over the past moving averages and permits retirements up to the shortfalls of actual revenues from such previous averages. It was designed to tie the hand of authorities and "hide" from political view the expected surpluses during the impeding increase of oil production and revenues from new discoveries, as excess revenues to be deposited in the Fund are not included in the budget. The OSF did accumulate significant amounts. However, it turned out to be less important that expected and did not avoid increased overall public expenditures during the boom in non-oil tax revenues, which led to significant increase in the deficit in the downturn (Perry, 2003).

At the subnational level, in 1997 the so-called Traffic Light Law was created (see, Braun and Tommasi, 2004). This law brought into effect a rating system for territorial governments, based on the ratios of interest to operational savings and of debt to current revenues. Highly indebted local governments (red light) were prohibited from borrowing, and intermediate cases (yellow light) were required to obtain permission from the Ministry of Finance. By this law, the central government aimed to limit the growth of subnational debt. However, as those authors point out the indebtedness law has not been fully effective.

Therefore, in a new attempt to implement fiscal rules to stabilize subnational finances, Colombia passed Law 617 in the year 2000, the so-called Subnational Fiscal Responsibility Law. The main features of this law are: (i) primary current expenditure must be exclusively financed by non earnmarked current revenues, and should not exceed a fixed percentage, depending on the state or municipality category; (ii) expenditure for state legislatures is limited; (iii) across the board cuts should be put in place whenever effective non-earnmarked current revenues are lower than budgeted, and state and municipal central administrators are not allowed to make transfers to their public entities; (iv) there are strict limits to municipality creation, proven non-viable municipalities have to merge; (v) when subnational governments do not comply with the limits imposed by the Law, they have to adopt a fiscal rescue program to regain viability within the next two years; (vi) to promote transparency, there is an extensive list of characteristics and requirements for the election of governors, majors, legislators and their relatives.

Germany

Germany has a history of fiscal restrictions dating back to 1969 (see Kennedy and Robbins, 2001). In that year, a constitutional rule was introduced which requires a balanced budget, but allows borrowing for investment expenditure (i.e. the golden rule). This rule applies to the federal government and the entirely of its budget - including consolidated federal enterprises and special funds. In addition, some states' constitutions include the golden rule.

The constitution specifies exceptions from a balanced budget during times of macroeconomic disequilibrium or war, and an important German policy mandate is that restrictive fiscal policy should not destabilize the economy or restrict growth and prosperity. Obviously, as a member of the European Union, Germany is also subject to the Maastricht Treaty and Stability and Growth Pact.

India

India has also moved towards a restrictions-based approach to fiscal adjustment. The Fiscal Responsibility and Budget Management Law (FRBM) was enacted at the central government in August 2003 level (see Kochar and Purfield, 2004).

This fiscal responsibility legislation (FRL) establishes a broad framework for the conduct of fiscal policy by setting a medium-term target to guide fiscal policy formulation. The framework places increased emphasis on transparency in budget formulation, implementation, and assessment. Moreover, the FRBM requires the central government to eliminate the "revenue deficit" (broadly equivalent to the current deficit), of 4.25 percent of GDP, by March 2008. In addition, by setting the current balance as the medium-term target, it effectively subjects fiscal policy to a golden rule.

Nevertheless, the FRBM does not impose any financial or judicial penalties from breaching the current balance target, requiring only that the government report to parliament the reasons for the overturn. Breaches of the ultimate medium-term target, or of the annual targets set under the supporting rules, are permitted for reasons of natural disaster, national security, or other exceptional circumstances specified by parliament. The minister of finance is only required to report to parliament on the extenuating circumstances after missing the targets. Therefore, enforcement of this fiscal restriction relies only on the loss of reputation that the government experiences from not implementing the framework.

Kochar and Purfield (2004) also simulates the effect of elimination of growth-interest rate differential on government debt under the FRBM. These simulations suggest that the exclusion of subnational governments from India's framework could complicate the achievement of fiscal sustainability.

Japan

Japan has had a legislated fiscal rule since 1947, which prescribes that bond issuance be limited to raising funds for financing public works (see Kennedy and Robbins, 2001). The rule covers only the general account budget of the central government, which represents only about 25 per cent of the central government's total budget. However, since 1975, the fiscal rule has not proven to be a binding constraint.

In order to address the deficit which had persisted through the early to mid-1990s, especially in light of future ageing-related pressures, the government engaged in fiscal tightening and passed the Fiscal Structural Reform Law in 1997 (see von Hagen, 2005). The legislation provided that the sum of the central and local government deficits as a percentage of GDP should be reduced to 3 per cent or less by the fiscal year of 2003 (from around 6 per cent in the fiscal year of 1997). Furthermore, it provided that the amount of deficit-financing bonds should be reduced every fiscal year and issuance of such bonds should cease by the fiscal year of 2003. The legislation also required that numerical limits be set for expenditures in each major programme from FY1998 to FY2000. Finally, it specified that the sum of taxes, payroll contributions and the deficit should not exceed 50 per cent of GDP.

However, the fiscal tightening initiated in 1997 was too much for the economy to bear. Under pressure from the Asian economic crisis and the failure of some major Japanese financial institutions, the economy fell into recession. In response, the government revised the Fiscal Structural Reform Law in May 1998 to introduce more flexibility, and then formally suspended its application in November 1998. Since that time, the government has followed expansionary policies and the general government gross debt-to-GDP ration has skyrocketed (see Chapter 4 of this thesis).

Mexico

Conesa et al. (2004) shows that in Mexico, fiscal consolidation and active debt management have been at the center of the authorities' economic strategy, especially since the 1995 crisis. Traditionally, fiscal policy management in that country has been guided by congressional authorization of yearly limits on net borrowing by the federal government and by Mexico City, based on the projected fiscal balance of the respective government. Similarly, each state congress must approve net borrowing by the state government. Under the Constitution, the federal and subnational governments are subject to the golden rule. Moreover, starting in 1998, contingent procedural restrictions have been introduced in the annual PEF (Federal Expenditure Budget) to absorb unexpected shocks, and to achieve fiscal targets. These restrictions have been changed over time as they are subjected to yearly revision by Congress.

Mexico has also made remarkable progress toward fiscal transparency in the recent years (Conesa et al., 2004). Starting in 1999, all information in the Federal Public Account and the PEF has been accessible electronically and a list of measures focusing on time limits, sequencing, and scope of the budget process to enhancing the efficiency of the budget process has been implemented.

At the subnational level, following several episodes of subnational bailouts in the aftermath of the 1995 financial crisis, Mexico has passed legislation to limit bailouts by the national government (see Braun and Tommasi, 2004). In 2000, the Zedillo administration established ex-ante, market based mechanisms in order to prevent excessive sub-national borrowing. At the same time, the administration attempted to convey a credible signal that it would no longer bail out local government debt. The new regulatory framework had four main components: (i) the president relinquished his power over discretionary transfers to states, thus limiting the ability of local government to "game" the federal government into bailing out them; (ii) the federal government gave up its role in securing debt with payments from the revenue sharing arrangement, which left the states and their creditors to assume the legal risks for the collaterization of debt; (iii) subnational debt was subjected to normal credit exposure ceilings, thus limiting the extent of financial-sector damage that one single state can cause and signaling that state debt must be evaluated on a basis similar with other debt; and (iv) bank's capital risk weighting of loans to subnational governments was linked to the international rating of the borrowing government's creditworthiness, making the pricing of bank loans a function of the underlying risk of the state government.

In addition, subnational governments are constitutionally barred from incurring liabilities, with foreign entities and from contracting liabilities in foreign currency. Mexico City may contract foreign debt, but only through the federal government. At a procedural level, borrowing by subnational governments (including Mexico City) against *participaciones* (federal transfers) as collateral must be registered with the purpose of monitoring the evolution of subnational public debt and guaranteeing the solvency of subnational governments.

As a further incentive to fiscal discipline at all levels of government, *participaciones* to states and municipalities are determined by collected federal revenue. This allows for a distribution of the risk associated with unexpected changes in revenue between the national and subnational governments, and accordingly, for sharing the burden of compensatory action at each government level.

New Zealand

In 1994, the Fiscal Responsibility Act (FRA) was enacted in New Zealand to improve the conduct of fiscal policy by setting out principles of responsible fiscal management and by promoting accountability and a long-term focus in fiscal planning (see Kennedy and Robbins, 2001). It requires that the government run annual operating surpluses in order to achieve unspecified "prudent" levels of Crown debt. Once these levels have been achieved, they must be maintained by, on average, avoiding an operating deficit. Temporary departures from these principles are allowed as long as the government specifies the reasons for the departure and sets out how and when it will return to the principles. Although the Act does not specify numerical debt targets, the government has defined its targets for fiscally prudent levels of debt.

Norway

In Norway, developed on the basis of a wide political consensus following three decades of oil production, the combination of the fiscal rule with an oil fund has become the cornerstone of fiscal policy (Bjerkholt and Niculescu, 2004).

Peru

Peru's Congress approved the Fiscal Transparency Law in December 1999, which sets limits on the deficit, the growth of government expenditure and the increase in public debt. Similar to Argentina's legislation, Peru's also established a fiscal stabilization fund to ensure savings in peak years that may be used in times of recession. Furthermore, it contains measures to encourage transparency and requires that the budget be prepared within a three-year macroeconomic framework (Braun and Tommasi, 2004).

Sweden

Sweden's Fiscal Budget Act of 1996 requires Parliament to set nominal expenditure limits for 27 expenditure ares of the central government, including transfers to other levels of government, for a three-year period (Kennedy and Robbins, 2001). Each year, Parliament sets new limits for the third year, and the ceilings are set so as to ensure that outlays fall as proportion of GDP. The measures were strengthened in 1999 through a prohibition on using allocations transferred from previous years, even though the spending caps are still not accompanied by sanctions.

United Kingdom

The UK has a long tradition on institutions to constrain the level of debt, dating back to the Seventeenth Century (North and Weingast, 1989). More recently, since May 1997, the British government has consistently stated that it will keep to two strict fiscal 'rules': the golden rule (described above), and the sustainable investment rule (see Kennedy and Robbins, 2001; and Emmerson et al., 2005). The last rule sets the ration of net public sector debt to GDP, over the economic cycle, at a 'stable and prudent level' defined as no more than 40% of GDP. For that, in 1998, the government adopted a similar legislation to Australia, setting out principles to guide its conduct of fiscal policy. In addition, according to the legislation, the government is required to table in Parliament its code for fiscal stability and fiscal strategy in accordance with these principles. UK, as a member of the European Union, is also subject to the Maastricht Treaty and Stability and Growth Pact.

United States

The US is another country with long tradition on fiscal institutions and constraints at the state government level. At the federal government level, more recently, deficit controls were introduced in 1985 through the Gramm-Rudman-Hollings (GRH) legislation - the Balanced Budget and Emergency Deficit Control Act - (see Kennedy and Robbins, 2001). That legislation imposed an annual deficit reduction schedule for a five-year period, with a balanced budget set for 1991, which was later revised to 1993. The Act covered on-budget items (i.e. excluding Social Security trust funds) and deficit objectives were to be accomplished mainly through spending cuts.

That legislation was replaced by the Budget Enforcement Act of 1990 (BEA), which shifted the focus away from deficits targets towards expenditure and revenue controls. The reason was the failure of the targets, in particular, in 1991 and 1993 due to overly optimistic economic and fiscal forecasts (Poterba, 1994).[6] Similar to the GRH legislation, the BEA applies only to the on-budget accounts. A sequestration procedure is triggered if aggregate discretionary appropriations enacted for a fiscal year exceeded that year's spending caps or if a fiscal year's aggregate mandatory spending and receipts legislation is considered to entail a net cost

In the subnational level, all states but Vermont have a fiscal constraint on their ability to borrow (see Poterba, 1994; Bohn and Inman, 1996; Levinson, 1998; and Cooper et al., 2005). Levinson (1998) shows that around 1980 many states began enacting budget stabilization funds, often called "rainy day funds", that allow them to save for unexpected revenues shortfalls. Prior to 1981, few states had such funds. By 1983, 19 states had rainy days funds in place and by 1994, 45 states had them. Also in the latter period many states' unemployment insurance trust funds borrowed from the federal treasury, repaying the loans from payroll taxes during more prosperous years. That author, in line with Bohn and Inman (1996) and Poterba (1994), classifies the balanced budget rules (BBRs) used by American States in five types: (1) the governor has to submit a balanced budget (44 states); (2) the legislature has to pass a balanced budget (37 states); (3) the state may carry over a deficit but must correct it in the next fiscal year; (4) the state may not carry over a deficit into the next budget period (often two years long); and (5) the state may not carry over a deficit into the next fiscal year. The last two and strictest types of balanced-budget rules are present in 24 of the 37 states of type (2).

The first two BBRs are *ex ante* and impose no constraint on what happens at the end of the year if expenditures exceed revenues. The third BBR is also virtually irrelevant,

[6] The targets were applied to the *projected*, rather than *actual* deficits. Therefore, overly optimistic budgetary projections easily met the targets, while the actual deficit exceeded the limit every year (Kennedy and Robbins, 2001).

because states with such requirements may continue to carry over deficits from year to year, as long as at the beginning of each year revenues are forecast to match expenditures. Some states with BBRs of this type systematically overestimate future revenues and underestimate future expenditures. The fourth and fifth BBRs are binding, at least in principle. They require that adjustments be made when taxes revenues fall short of expenditures. The fourth BBR binds every budget period, which can be two years long for states with biennial budget cycles. The fifth binds every year, regardless of the length of the state's budget cycle. As Poterba (1994) mentions, the last two anti-deficit rules are more common in small states than large ones; seven of the ten largest states allow deficits to be carried forward to subsequent years. Moreover, these state fiscal rules may place limitations on projected, or actual, deficits and, typically, carry no sanctions, being applied only to general funds, excluding separate accounts such as the capital account and accounts for social insurance and employee retirement. In addition to balanced budget rules, 27 states have tax and expenditure limitations, which set limits on annual revenue or expenditure increases.

Yet, enforcement of these balanced-budget rules is by the state's courts, with the state supreme court the ultimate arbiter. If the state supreme court is appointed by the governor or by the state legislature (i.e. by those accountable for the deficit), it is possible that enforcement of balanced-budget rules will be less strenuous (Bohn and Inman, 1996). Appointed supreme court may behave more like a government agency than a truly independent monitor of fiscal performance. As a consequence, the effectiveness of these constraints is disputed (Cooper et al., 2005).

Chapter 2

A comparison of debt versus primary-deficit constraints*

*This chapter is based on Ribeiro, Beetsma, and Schabert (2007).

2.1 Introduction

The recent reform of the European Stability and Growth Pact (SGP) has increased its emphasis on debt sustainability.[1] This way, the reform has broadened the Pact's previous narrow focus on public deficits. The reform ties in closely with the ongoing discussion whether it is preferable to impose constraints on the public debt or on the public deficit (see e.g. Fatás et al., 2003, and Wyplosz 2005).

The literature provides no clear answer to this question. Intuitively, relative advantages of debt-based constraints would be that (1) they reward fiscal discipline in good times, because there is a future benefit to reducing the debt when the economy is doing well; (2) they provide a more accurate picture of the sustainability of the government's finances; (3) as far as the incentive for surprise inflation is concerned, the stock of (nominal) debt is a better indicator than the deficit and (4) they avoid inefficient budget cuts and tax increases that would be sub-optimal from a longer-term perspective when the economy falls victim to sudden (unexpected) adverse developments.[2] However, relative disadvantages of constraints on the public debt would be that (1) the current government, who faces the constraint, is at best partly responsible for the current debt level; and (2) if the reference level of the debt substantially exceeds the actual debt level, there is sufficient room left before the constraint becomes binding that the government may be tempted to spend resources on wasteful projects (by contrast a deficit rule is more immediately binding).

This paper compares constraints on the public debt with constraints on the primary deficit.[3] Although the SGP imposes a constraint on the total deficit in European countries, we concentrate on the primary deficit, since this constraint is not affected by the level of the public debt in an economy, while the total deficit depends positively on the public debt via the interest payments on the debt.[4] In effect, a constraint on the total deficit would become tighter as the public debt increases. The focus of the analysis are the effects of the constraints on public borrowing, government spending and social welfare. The main novelty of this paper is that we take into account how an optimizing government reacts to different constraints when it decides on a time consistent spending/borrowing plan. We set up a simple open economy framework with income shocks and potentially myopic governments as a source of excessive spending. This provides a rationale for imposing fiscal constraints either on the debt or on the primary deficit.

A crucial aspect of our set-up is that the government internalizes the penalties that follow the violation of particular debt or primary-deficit limits, which leads to precautionary behavior. Thus, the constraints will not only affect fiscal policy in cases they are

[1] For a review of the reform of the SGP, see Buti et al. (2005) or Morris et al. (2006).

[2] The Netherlands and Germany provide examples in this respect. Not so long ago, the SGP forced these countries to make budget cuts, thereby reinforcing the adverse economic circumstances they were already facing.

[3] The paper limits itself to these two alternatives and does not aim at finding the generally optimal fiscal rule. It also does not analyze the optimal interaction between monetary policy and fiscal policy under constraints, such as in Lambertini (2006) and Pappa and Vassilatos (2007).

[4] Assuming a primary-deficit constraint is also more convenient analytically.

binding, but will also have an effect on average government expenditures and public debt when they are not binding. We find that the economy behaves in a very similar way under both types of constraint, although welfare is higher under the debt-based constraint. Debt-based constraints imply better smoothing of public spending after an income shock. Further, the appropriate debt constraint is more robust against changes in the interest rate than is the appropriate deficit constraint. This suggests that the former type of constraint might be easier to implement in practice, in view of the fact that it would be politically difficult to make frequent and large adjustments to the constraints. These results support the greater emphasis that the SGP puts on debt after its reform.

Variations in the model parameters yield interesting insights. In particular, an increase in the variance of the endowment shocks or an increase in their persistence produces lower debt on average under a debt constraint (in order to maintain a larger safety margin relative to the reference debt level), but higher average debt under a primary-deficit constraint. Higher average debt corresponds to a higher average primary surplus, which implies a larger safety margin to the reference deficit level, as desired by the government. For similar reasons, an increase in risk aversion produces lower average debt under a debt constraint, but higher average debt under a primary-deficit constraint. Finally, a more myopic government implies a rise in average debt under a primary-deficit constraint, but a fall in average debt under a debt constraint. This is the result of precautionary behavior: by being close to the reference debt level, the government would run the risk of front-loading severe spending cuts when income is hit by a bad shock. This consequence is worse for a more myopic government in view of its preferred time profile of public spending. Hence, in the presence of a debt constraint such a government accumulates *less* debt on average.

The rest of the paper is organized as follows. Section 3.2 describes the model and characterizes the equilibrium for the non-myopic government. In Section 2.3 we solve the model for the myopic government under each type of fiscal constraint. Section 2.4 provides the welfare derivation. Next, Section 2.5 performs a numerical evaluation of the model calibrated to the EU situation and checks the robustness of the results for changes in the parameters. Finally, Section 4.5 concludes the paper.

2.2 The model

In this section we present a model of a small open economy. The model is specified in real terms, such that neither money nor monetary policy is modelled here. Nevertheless, we interpret this country as a member of a monetary union, where its fiscal policy is constrained by unionwide constraints (as is the case for the countries that adopted the Euro). Further, we assume that the constraints are credibly enforced.

2.2.1 The private sector

There exists a continuum of infinitely lived and identical households of total mass one. Their utility increases in consumption C_t and government spending G_t. The objective of a representative household is given by

$$\mathrm{E}_t \sum_{s=t}^{\infty} \beta_w^{s-t} \left[v(C_s) + u(G_s) \right], \tag{2.1}$$

where $\beta_w \in (0,1)$ denotes the discount factor, and E_t the expectations operator conditional on information at time t. The functions u and v are further assumed to be increasing in C and G, strictly concave, twice continuously differentiable, and to satisfy the usual Inada conditions.

In each period households are endowed with a stochastic amount of goods Y_t and with wealth in form of one period risk-free government bonds, B_t, and foreign bonds, F_t. Households have to pay taxes on their endowment Y_t. As a result, the household flow budget constraint reads

$$C_t + B_t + F_t \leq (1 + r_{t-1}^H) B_{t-1} + (1 + r_{t-1}) F_{t-1} + (1 - \tau_t^y) Y_t. \tag{2.2}$$

where r_t denotes the exogenous real interest rate on foreign bonds, r_t^H the real interest rate on domestic bonds and τ_t^y the (income) tax rate. Maximizing (2.1) subject to no-Ponzi game conditions for domestic and foreign borrowing, $\lim_{t\to\infty} B_t \prod_{i=1}^{t}(1 + r_{i-1}^H)^{-1} \geq 0$ and $\lim_{t\to\infty} F_t \prod_{i=1}^{t}(1 + r_{i-1})^{-1} \geq 0$, and (2.2) leads to the first-order conditions

$$\frac{v'(C_t)}{\mathrm{E}_t v'(C_{t+1})} = \beta_w (1 + r_t), \tag{2.3}$$

$$r_t^H = r_t, \tag{2.4}$$

where (2.3) presupposes that the state(s) follow a Markov-process, and the transversality conditions $\lim_{s\to\infty} \mathrm{E}_t F_s \prod_{i=1}^{s}(1 + r_{i-1})^{-1} = 0$ and $\lim_{s\to\infty} \mathrm{E}_t B_s \prod_{i=1}^{s}(1 + r_{i-1}^H)^{-1} = 0$. We assume that the endowment follows an exogenous stochastic process

$$Y_t - \overline{Y} = \rho \left(Y_{t-1} - \overline{Y} \right) + \sigma_\varepsilon \varepsilon_t, \qquad \varepsilon_t \sim N(0,1), \tag{2.5}$$

where ε_t is independently, identically and normally distributed with mean zero and unitary variance ($\mathrm{E}_t \varepsilon_s = 0$ and $\mathrm{E}_t \varepsilon_s^2 = 1$ for $s > t$). In addition, ρ satisfies $0 \leq \rho \leq 1$ and $\sigma_\varepsilon \geq 0$ is a known parameter.

2.2.2 The government

The government issues bonds, raises tax revenues T_t and purchases goods G_t from the households. Without any further constraints, its period-by-period budget constraint reads $B_{t+1} = (1 + r_t) B_t + G_t - T_t$, where we have used that $r_t^H = r_t$. It chooses sequences for expenditures and debt until infinity. This assumption does not necessarily imply that the government stays in charge from today until infinity. We interpret the infinite planning horizon as an implication of the government's uncertainty about its term in office.

Thus the government chooses the sequences $\{G_s\}_{s=t}^{\infty}$ and $\{B_s\}_{s=t}^{\infty}$. Further, the government features a discount factor β_g that might deviate from the households' discount factor, $\beta_g \leq \beta_w$. The case of $\beta_g < \beta_w$ could be interpreted as corresponding to a situation in which there is a non-zero probability that the government will be removed from office in any future period. The chance of losing office drives the "effective" discount factor of the government below the social discount factor. The government's objective function then takes the form

$$\mathrm{E}_t \sum_{s=t}^{\infty} \beta_g^{s-t} \left[v(C_s) + u\left(G_s\right) \right]. \tag{2.6}$$

When $\beta_g < \beta_w$, the government will tend to frontload government expenditures. This implies excessive borrowing and, hence, a potential role for borrowing constraints. First, a stable equilibrium might not exist in this case, when government debt grows without bounds. Secondly, the equilibrium allocation might be inefficient compared to the case of an unbiased government (or a social planner) that shares the private sector's rate of time preference.

We consider two types of fiscal constraints that are intended to alleviate the incentive for excessive borrowing, namely constraints on public debt and constraints on primary deficits. While the SGP only penalizes excessive deficits, many have argued that a Pact that punishes excessive debt would be preferable. The stock of debt is considered a better (though imperfect) measure of fiscal sustainability. Moreover, one would expect that by putting a constraint on debt, governments are induced to follow prudent fiscal policies also during economic upturns. Below we shall see whether this is indeed the case.

However, such fiscal constraints do not come as a free lunch. Even if the government is not myopic, the constraints may sometimes lead to losses due to the possibility that unexpected, adverse macroeconomic shocks cause the constraints to be violated. In that case, it is simply the macroeconomic uncertainty rather than the opportunistic behavior of the government that gives rise to potential sanctions.

We assume that under a *debt-based* constraint, the government pays a fine if the stock of debt, B_t, exceeds some reference value, B^c, while no fine has to be paid otherwise. The government's period-t budget constraint is then modified into:

$$B_{t+1} = \left(1 + r_t\right) B_t + G_t - T_t + k^B \left(B_t - B^c\right) I_B \left[B_t; B^c\right], \tag{2.7}$$

where the parameter $k^B > 0$ captures the tightness of the constraint and $I_B \left[B_t; B^c\right]$ is an indicator function, such that $I_B \left[B_t; B^c\right] = 1$, if $B_t > B^c$, and $I_B \left[B_t; B^c\right] = 0$, otherwise.

The other possible constraint is expressed in terms of the primary deficit, which is defined as

$$D_{t+1} = B_{t+1} - \left(1 + r_t\right) B_t. \tag{2.8}$$

Under a *primary deficit-based* constraint, the government pays a fine if the primary deficit D_t exceeds some reference value D^c, while it pays no fine otherwise. The government's period-t budget constraint with this type of constraint becomes

$$B_{t+1} = \begin{cases} (1+r_t)\, B_t + G_t - T_t & if \quad D_t \leq D^c \\ (1+r_t)\, B_t + G_t - T_t + k^D\,(D_t - D^c) & if \quad D_t > D^c \end{cases},$$

Using the definition of a primary deficit (2.8) and defining the indicator function $I_D\,[D_t; D^c]$, such that $I_D\,[D_t; D^c] = 1$, if $D_t > D^c$, and $I_D\,[D_t; D^c] = 0$, otherwise, we can rewrite the budget constraint as

$$D_{t+1} = G_t - T_t + k^D\,(D_t - D^c)\, I_D\,[D_t; D^c]\,, \tag{2.9}$$

where the parameter $k^D > 0$ captures the tightness of the constraint when D_t exceeds D^c.

Finally, the government raises its tax revenues by taxing income at the constant and given rate τ^y, such that

$$T_t = \tau^y Y_t, \tag{2.10}$$

Hence, given that taxes are determined in this way, there is one degree of freedom left for the government, which is its choice of $\{G_s\}_{s=t}^{\infty}$. The debt path follows residually.

2.2.3 Equilibrium

We assume that the real interest rate on (foreign and domestic) bonds is exogenously determined and constant at a level r. Given that tax revenues are exogenous, the optimal decisions of the private sector and the government are determined independently of each other. Domestic demand consists of private sector consumption, C_t, and government consumption, G_t. Income can further be invested in internationally traded assets, F_t, such that the resource constraint reads $Y_t = C_t + G_t + F_t - (1+r)F_{t-1} + k_t$, where $k_t = k^B\,(B_t - B^c)\, I_B\,[B_t; B^c]$ or $k_t = k^D\,(D_t - D^c)\, I_D\,[D_t; D^c]$.

Definition 2.1 *For a given endowment sequence $\{Y_s\}_{s=t}^{\infty}$, constant tax rate τ^y, constant interest rate r, and initial values F_t and B_t, a rational expectations equilibrium consists of a set of sequences $\{C_s, G_s, B_s, T_s\}_{s=t}^{\infty}$ satisfying the utility-maximizing plan of the private sector, i.e. (2.3) and the transversality condition $\lim_{s\to\infty} E_t(1+r)^{s-t}B_s = 0$; the plan of the government that maximizes (2.6), subject to (2.7) or (2.9); and the intertemporal resource constraint $F_t = E_t \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (Y_s - C_s - G_s - k_s)$.*

To allow for the existence of a non-explosive equilibrium, we assume that the exogenous interest rate r satisfies $(1+r)\,\beta_w = 1$. We restrict our attention to rational expectations equilibria that are Markov perfect, such that equilibrium decision rules and, hence, outcomes depend on the current state of the economy only.

2.2.4 Equilibrium under a non-myopic government

To provide a benchmark for the subsequent analysis, we briefly examine the case, in which the government discounts future events at the same rate as households ($\beta_g = \beta_w$) and

$\beta_w (1+r) = 1$. The government's problem is then to maximize $\mathrm{E}_t \sum_{s=t}^{\infty} \beta_w^{s-t} \left[v(C_s) + u(G_s) \right]$ subject to the budget constraint and given tax revenues, T_s. We note that trades occur sequentially, in contrast to a framework where all trades are contracted in the initial period. The state of the economy is governed by a Markov process. Thus, the government's behavior in any period $s \geq t$ can be described by

$$\mathrm{E}_t u'(G_s) = \beta_w (1+r) \mathrm{E}_t u'(G_{s+1}).$$

Since $\beta_w = 1/(1+r)$, expenditures will therefore satisfy $u'(G_t) = \mathrm{E}_t u'(G_{t+j})$, $\forall\ j \geq 0$, implying a random walk behavior of government expenditures, G.[5] In equilibrium, the household first-order condition (2.3) must further be satisfied, which implies with $\beta_w = 1/(1+r)$ that $v'(C_t) = \mathrm{E}_t v'(C_{t+j})$, $\forall j \geq 0$. Thus, private consumption C also exhibits a unit root. Further, in equilibrium the intertemporal resource constraint, the intertemporal government budget constraint and the household's transversality condition must hold, implying

$$(1+r) B_t = \mathrm{E}_t \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} (T_s - G_s) \tag{2.11}$$

Given these conditions, one can easily solve for the set of equilibrium sequences $\{C_s, G_s, B_s, T_s\}_{s=t}^{\infty}$ given $\{Y_s\}_{s=t}^{\infty}$ and the initial values B_t and F_t (see Appendix 2.A). The state space solution form is linear and given by

$$B_{s+1} = B_s - \tau^y \frac{1-\rho}{1+r-\rho} \left(Y_s - \overline{Y} \right), \tag{2.12}$$

$$G_s = \tau^y Y_s \frac{r}{1+r-\rho} + \tau^y \overline{Y} \frac{1-\rho}{1+r-\rho} - r B_s, \tag{2.13}$$

$$F_{s+1} = F_s + \frac{1-\rho}{1-\rho+r} \left(Y_s - \overline{Y} \right), \tag{2.14}$$

$$C_s = (1-\tau^y) \frac{r}{r-\rho+1} Y_s + (1-\tau^y) \frac{1-\rho}{1+r-\rho} \overline{Y} + r (B_s + F_s). \tag{2.15}$$

Due to the unit root(s), the unconditional means are undetermined.

2.3 A myopic government

We maintain the assumption that $\beta_w = 1/(1+r)$, but now we allow for $\beta_g < \beta_w$. In this case, the government tends to pre-draw government expenditure and will start borrowing from the private sector. Below we solve the model under our two types of fiscal constraints, which are intended to limit public borrowing.

[5] If $G_t \neq \mathrm{E}_t G_{t+j}$, for $j > 0$, unconditional higher order moments of G_t and G_{t+j} must also differ to satisfy $u'(G_t) = \mathrm{E}_t u'(G_{t+j})$. This would introduce a non-recursive element in equilibrium, such that the state would not follow a Markov-process.

2.3.1 Debt constraints

We start with the case in which the government faces a debt constraint. Under this fiscal constraint, the government maximizes (2.6) subject to (2.7). To simplify the original problem, which exhibits a discontinuity, we approximate the indicator function $I\left[B_s; B^c\right]$ with a transition function, which allows us to apply standard local approximation (perturbation) methods. In particular, we apply the logistic function

$$L_s^B \equiv L\left(B_s; \gamma, B^c\right) = \frac{1}{1+\exp\left(-\gamma\left(B_s - B^c\right)\right)}, \quad \gamma > 0. \tag{2.16}$$

When $\gamma \longrightarrow \infty$, $L\left(B_s; \gamma, B^c\right) \longrightarrow I\left[B_s; B^c\right]$. Hence, for high values of γ, the logistic function will be a good approximation to the indicator function. This alters the intertemporal budget constraint of the government, since there are always fines to be paid (which become negative, but are close to zero when $B_s < B^c$).

$$B_{s+1} = (1+r)\,B_s + G_s - T_s + k^B\left(B_s - B^c\right) L_s^B. \tag{2.17}$$

The problem of the myopic government then reads

$$\max_{\{G_s, B_{s+1}\}} U_g = E_t\left\{\sum_{s=t}^{\infty} \beta_g^{s-t} u\left(G_s\right)\right\} \quad \text{s.t.} \quad (2.17)$$

Given that the approximated problem is now continuous and recursive, the first-order condition for B_{t+1} is given by

$$u'\left(G_t\right) = \beta_g E_t\left\{\left[1 + r + k^B L_{t+1}^B + \gamma k^B\left(B_{t+1} - B^c\right)\left(L_{t+1}^B - \left(L_{t+1}^B\right)^2\right)\right] u'\left(G_{t+1}\right)\right\}. \tag{2.18}$$

We see that the debt constraint affects the "effective" cost of issuing debt by the additional term $k^B L_{t+1}^B + \gamma k^B\left(B_{t+1} - B^c\right)\left(L_{t+1}^B - \left(L_{t+1}^B\right)^2\right)$. The Euler equation contains two unusual elements due to the fiscal constraint. The first element, $k^B L_{t+1}^B$, measures the additional marginal costs of each unit of debt that exceeds the reference value B^c. The second element, $\gamma k^B\left(B_{t+1} - B^c\right)\left(L_{t+1}^B - (L_{t+1}^B)^2\right)$, is the marginal effect of an increase in B_{t+1} on this marginal cost, multiplied by the factor $\left(B_{t+1} - B^c\right)$. In a sense, this term measures how an increase in debt raises the "probability" of hitting the constraint, reflecting the government's internalization of the fiscal constraint.

The set of conditions that characterize the equilibrium sequences $\{G_t, B_t\}_{t=0}^{\infty}$ under the debt constraint are given by (2.17), (2.18), and the terminal condition (see Appendix 2.B)

$$\left(1 + r + k^B L_t^B\right) B_t = \mathrm{E}_t \sum_{s=t}^{\infty} \frac{\tau^y Y_s - G_s + k^B B^c L_s^B}{\prod_{v=t+1}^{s}\left(1 + r + k^B L_v^B\right)}, \tag{2.19}$$

given an initial value B_t.

2.3.2 Deficit constraints

For the case of a primary deficit-based constraint, we apply an analogous approximation to the original indicator function using

$$L_s^D \equiv L\left(D_s;\gamma,D^c\right) = \frac{1}{1+\exp\left(-\gamma\left(D_s - D^c\right)\right)}, \quad \gamma > 0. \tag{2.20}$$

Rewriting the budget constraint in terms of the deficit, we can summarize the government's problem by

$$\max_{\{G_s D_{s+1}\}} U_g = E_t \left\{ \sum_{s=t}^{\infty} \beta_g^{s-t} u\left(G_s\right) \right\} \text{ s.t. } D_{s+1} = G_s - T_s + k^D \left(D_s - D^c\right) L_s^D, \tag{2.21}$$

Using that the problem is continuous and recursive, the government's choice for D_{s+1} is characterized by the following first-order condition

$$u'\left(G_t\right) = \beta_g k^D E_t \left\{ \left[L_{t+1}^D + \left(D_{t+1} - D^c\right)\gamma \left(L_{t+1}^D - \left(L_{t+1}^D\right)^2 \right) \right] u'\left(G_{t+1}\right) \right\}. \tag{2.22}$$

The set of conditions that characterize the equilibrium sequences $\{G_t, B_t\}_{t=0}^{\infty}$ under the deficit constraint are given by (2.21), (2.22), and the terminal condition (see Appendix 2.C)

$$\left(1+r\right) B_t = \mathrm{E}_t \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \frac{\tau^y Y_s - G_s + k^D D^c L_s^D}{1 - k^D L_s^D}, \tag{2.23}$$

given an initial value B_t.

2.3.3 The solution

In order to solve the model when fiscal constraints are imposed and to evaluate welfare departures from the solution under a non-myopic government, we use a perturbation method following Judd (1998), Collard and Juillard (2001), and Schmitt-Grohé and Uribe (2004). For a generic variable X_t, we define (1) its stochastic steady state value as its unconditional expectation: $\widehat{X} \equiv \mathrm{E}[X_t]$, and (2) deviations from a deterministic steady state fixed point, $\overline{X}$, as $\widetilde{X}_t \equiv X_t - \overline{X}$.

The values of government expenditures $\overline{G}$ and debt $\overline{B}$ (deficit $\overline{D}$) in a deterministic steady state under the two fiscal constraints are found by solving the systems formed by the government budget constraint and the Euler equation under the respective constraints (see Appendices 2.D and 2.E). The values $\overline{G}$ and $\overline{B}$ ($\overline{D}$) will generally differ, given the differences in the two systems. Nevertheless, as we shall discuss in Section 2.5, we can always calibrate the parameters k^B and k^D and the reference values under the fiscal constraints, B^c and D^c, so as to obtain the same deterministic steady states under the two constraints.

The solution under a debt-based constraint

For the case where the myopic government faces a debt-based constraint, the solution has to satisfy the set of conditions (2.17), (2.18), and (2.19). In order to solve that system, we apply a second order Taylor expansion at the deterministic steady state. To this end, we postulate the existence of two auxiliary functions that describe the system's evolution as a function of observable state variables $(B_t, Y_t, \sigma_\varepsilon)$ at period t. The general solution form for G_t and B_{t+1} is given by

$$G_t = f(B_t, Y_t, \sigma_\varepsilon), \tag{2.24}$$

$$B_{t+1} = h(B_t, Y_t, \sigma_\varepsilon). \tag{2.25}$$

Eliminating G_t with (2.17), (2.18) can be written as $0 = E_t[g(B_{t+2}, B_{t+1}, B_t, Y_{t+1}, Y_t, \sigma_\varepsilon)]$. The latter condition can, by using (2.25) and $B_{t+2} = h\left(h(B_t, Y_t, \sigma_\varepsilon), \rho Y_t + (1-\rho)\overline{Y} + \sigma_\varepsilon \varepsilon_{t+1}, \sigma_\varepsilon\right)$, by rewritten as a function of the state variables $B_t, Y_t, \sigma_\varepsilon, \varepsilon_{t+1}$ and the function $h(\cdot)$:

$$\mathrm{E}_t\left\{g\left(\begin{array}{c} h\left(h(B_t, Y_t, \sigma_\varepsilon), \rho Y_t + (1-\rho)\overline{Y} + \sigma_\varepsilon\varepsilon_{t+1}, \sigma_\varepsilon\right), \\ h(B_t, Y_t, \sigma_\varepsilon), B_t, \rho Y_t + (1-\rho)\overline{Y} + \sigma_\varepsilon\varepsilon_{t+1}, Y_t, \sigma_\varepsilon \end{array}\right)\right\} = 0. \tag{2.26}$$

We then identify the approximated solution for debt (2.25) by taking first- and second-order Taylor expansions of (2.26). Appendix 2.F describes in detail the derivation of the second-order approximation. The mean, i.e. the unconditional expectation, of debt under the debt constraint will be denoted by $\widehat{B^B}$ and the unconditional variance of B by $\mathrm{Var}(B_t) = \mathrm{E}\left(B_t - \widehat{B^B}\right)^2$.[6] The approximated solution of G_t is then derived using (2.17) and the solution for B_{t+1}. The unconditional expectation of government expenditure under the debt constraint will be denoted by $\widehat{G^B}$ (see Appendix 2.F).

The solution under primary deficit-based constraint

The same procedure is used under a primary deficit-based constraint to solve for sequences for D_t and G_t satisfying the conditions (2.21), (2.22), and (2.23). The general solution form for G_t and D_t is given by

$$G_t = j(D_t, Y_t, \sigma_\varepsilon), \tag{2.27}$$

$$D_{t+1} = l(D_t, Y_t, \sigma_\varepsilon), \tag{2.28}$$

Eliminating G_t in (2.22) with (2.21), we can write $0 = E_t[i(D_{t+2}, D_{t+1}, D_t, Y_{t+1}, Y_t, \sigma_\varepsilon)]$. Using (2.28) and $D_{t+2} = l\left(l(D_t, Y_t, \sigma_\varepsilon), \rho Y_t + (1-\rho)\overline{Y} + \sigma_\varepsilon\varepsilon_{t+1}, \sigma_\varepsilon\right)$, we get

$$\mathrm{E}_t\left[i\left(\begin{array}{c} l\left(l(D_t, Y_t, \sigma_\varepsilon), \rho Y_t + (1-\rho)\overline{Y} + \sigma_\varepsilon\varepsilon_{t+1}, \sigma_\varepsilon\right), \\ l(D_t, Y_t, \sigma_\varepsilon), D_t, \rho Y_t + (1-\rho)\overline{Y} + \sigma_\varepsilon\varepsilon_{t+1}, Y_t, \sigma_\varepsilon \end{array}\right)\right] = 0. \tag{2.29}$$

The approximated solution for the deficit (2.28) is found by taking first- and second-order Taylor expansions of (2.29) (derived analytically in Appendix 2.G). The means of D_t and G_t under the debt constraint will be denoted by $\widehat{D^D}$ and $\widehat{G^D}$.

[6] The variance is obtained by taking unconditional expectation of the square of the first-order approximation of B_{t+1}. See Appendix 2.F for the complete derivation of $\widehat{B^B}$.

2.4 Welfare

To assess the efficiency of different fiscal constraints, we refer to household welfare (2.1). Given that the consumption decision is independent of fiscal policy, we restrict our attention to the welfare effects of government expenditures. Specifically, we apply a second-order Taylor expansion of (2.1) at the deterministic steady state and use the solutions for the mean and the unconditional variance of government expenditures, $\widehat{G}$ and Var(G), respectively, to compute household welfare under both types of constraints. Our welfare measure is given by[7]:

$$\begin{aligned} U_w &\approx \sum_{t=0}^{\infty} \beta_w^t \left[u\left(\overline{G}\right) + u'\left(\overline{G}\right) * \left(\widehat{G}^z - \overline{G}\right) + \frac{1}{2} u''\left(\overline{G}\right) * \mathrm{Var}\left(G^z\right) \right] \\ &\approx \frac{u\left(\overline{G}\right) + u'\left(\overline{G}\right) * \left(\widehat{G}^z - \overline{G}\right) + \frac{1}{2} u''\left(\overline{G}\right) * \mathrm{Var}\left(G^z\right)}{1-\beta_w}, \end{aligned} \tag{2.30}$$

where z denotes the type of fiscal constraint that is imposed, B (debt-based) or D (primary-deficit based). Based on our welfare measure (2.30) we compare welfare under both constraints and analyze which constraint is preferred from the households' perspective.

2.5 Numerical evaluation

In this section we perform a numerical evaluation of the model and study public debt, government expenditures and welfare under the two different types of fiscal constraints. We also analyze the consequences of changes in both the "deep" parameters and the policy parameters.

2.5.1 Calibration

We calibrate the model using average values for eleven members of the Eurozone, which we denote by "Euro-11".[8] The calibration is based on annual data from the OECD Economic Outlook database (OECD, 2006, n^o. 79) for 1970-2006.

Table 2.1 summarizes the calibration of our benchmark specification. It describes the parameters, lists the values chosen for them and provides the motivation for these choices. For simplicity, we normalize average income, $\overline{Y}$, to 100 and set the income tax rate, τ^y, to 1. This choice is immaterial given that the private sector behavior is not affected by public policy in any case. Initial public debt, B_0, is set at 63.14, which is the average

[7]Notice that, given our hypothesis about partisan government, the intertemporal discount rate of society can differ from that of the government, $\beta_g \leq \beta_w$. Thus, while the government maximizes its expenditure level using its intertemporal discount factor β_g, society's welfare is evaluated using the discount factor β_w (see Appendix 2.H for further details).

[8]These countries are Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, The Netherlands, Portugal and Spain.

debt/GDP ratio over all observations in our panel (the Euro-11 over 1970 to 2006). The initial amount of foreign debt, F_0, is set to 0. The real interest rate, r, is fixed at 0.0262, which is the average ex-post long-term real interest rate over all observations in our panel. The coefficient of income autocorrelation, ρ, is set at 0.264. It is obtained by taking log-deviations of real GDP from its trend for each country,[9] and estimating an AR-1 process on these constructed series for the Euro-11 area over the period 1970-2006. We then take the non-weighted average of the estimated AR-1 coefficients, which provides us with our choice of ρ. The variance of the income shocks, $\sigma_\varepsilon^2 = 38.821$, is the non-weighted average for the Euro-11 of the sum of the squared residuals of the estimated AR-1 income process.

Following Grossman and van Huyck (1988) and Kumhof and Yakadina (2007), the myopic government discount factor, β_g, is expressed as a fraction of the social discount factor β_w:

$$\beta_g = \alpha\beta_w, \qquad 0 < \alpha < 1.$$

As the benchmark value for α we choose 0.933.[10] Further, utility from government expenditure is specified as

$$u(G) \equiv \frac{G^{1-\frac{1}{\mu}}}{1-\frac{1}{\mu}}, \tag{2.31}$$

where μ is the constant elasticity of intertemporal substitution. For the benchmark parametrization we set μ equal to 0.7.[11] Finally, the smoothness parameter γ of the logistic function is set at 300. This value enables a good approximation of the indicator function in (2.7) and (2.9), as shown in Franses and Van Dijk (2004).

As regards to the policy parameters, we set k^B and k^D, which govern the tightness of the constraints, at very high values ($k^B = k^D = 100$) so that the constraints are almost strictly binding. Then, we search for those reference values (B^c and D^c) for the debt, respectively primary deficit, constraints that yield as the deterministic steady state the level of the public debt that satisfies the intertemporal government budget constraints (2.19) and (2.23), i.e. $\overline{B^B} = \overline{B^D} = B_0 = 63.140$. Specifically, this results in a value of $B^c = 63.144$ under our benchmark parameter setting. For the primary deficit-based constraint, under the baseline parameter setting, the intertemporal government budget constraint (2.23) holds only when the reference value D^c is not higher than -1.649, implying that the reference value corresponds to a surplus.

[9] We use the Hodrick-Prescott filter with smoothing parameter $\lambda = 100$.

[10] The term $1/(1-\alpha)$ can also be interpreted as a measure of the planning horizon of the politician (see Grossman and van Huyck, 1998, and Kumhof and Yakadina, 2007). Our benchmark value of α corresponds to a planning horizon of the government of 15 periods (the same value as in the calibration by Kumhof and Yakadina, 2007).

[11] There is no consensus on the most appropriate value of μ. The value chosen here provides a coefficient of constant relative risk aversion close to 1.43, which falls within the range of values used in the literature. In particular, our benchmark value for μ is close to the value of 1.391 estimated for the Euro Area by Smets and Wouters (2003) and to the value of 1.5 assumed for the US by Ayagari and McGrattan (1998). In our sensitivity analyses we vary μ to check the robustness of our results for different degrees of relative risk aversion $1/\mu$.

2.5.2 Results

We are now ready to discuss our numerical results. First, we present our findings for the benchmark case. Next, we perform a sensitivity analysis in which we investigate how changes in the parameters affect welfare and the dynamics of the economy.

The benchmark case

Stochastic steady state Table 2.2 provides the results for our benchmark calibration, under which the respective intertemporal government budget constraints are strictly satisfied. Conform our earlier definition, bars above variables denote deterministic steady state values, while hats denote the mean of a variable. Further, making use of (2.8), Table 2.2 also reports the value $D^{Bc} = -1.653$ of the primary deficit that corresponds to the reference value B^c, and the level of the debt $B^{Dc} = 62.98$ that corresponds to the reference deficit level D^c. We separate the outcomes in four blocks. The first block presents the outcomes for a non-myopic government (i.e. $\beta_g = \beta_w$). Even though we consider a tax rate of 1 and initial net foreign debt of zero, because the government starts with positive initial public debt, $\overline{G}$ is smaller than $\overline{Y}$ and the steady state level of private consumption is positive ($\overline{C} = 1.65$). The steady state primary surplus $\overline{D}$ is then equal to the value of consumption in absolute terms.

The second and third blocks of Table 2.2 report the outcomes under a myopic government (i.e., $\beta_g < \beta_w$) subject to a debt, respectively primary deficit, constraint.[12] By construction, the deterministic steady states of debt under both types of fiscal constraints ($\overline{B^B}$ and $\overline{B^D}$) are equal to the initial value of the public debt B_0. However, the deterministic steady state government expenditure level is marginally higher (approximately 0.09% of GDP) under the myopic than under the non-myopic government.

This is the result of the approximation of the indicator function by the logistic functions $\overline{L^B}$ and $\overline{L^D}$ in (2.83) and (2.143),[13] which for the benchmark specification are equal to 0.218 and 0.226 respectively. These equilibrium values of the logistic function L are positive and non-negligible (even for a very large γ) because of the marginal effect of the debt (primary deficit) on the logistic approximation $\overline{L^B}$ ($\overline{L^D}$) in the government's Euler equation under the debt constraint (2.18) (under the primary deficit constraint (2.22)). The government takes into account that the tightness of the constraint varies when the debt or the primary deficit are close to their respective reference values. On the one hand, these are not fully binding when they are marginally exceeded. On the other hand, they will already be felt even before they are hit. The government faces a continuous trade-off between higher fines and being able to spend more in the short run. The trade-off is reflected by the additional non-zero terms $-k^B \left(\overline{B} - B^c\right) \overline{L^B}$ and $-k^D \left(\overline{D} - D^c\right) \overline{L^D}$ in the government's budget constraint when determining the solutions for $\overline{G^B}$ and $\overline{G^D}$, respectively. These

[12] With the benchmark value of α and the value of β_w given in Table 2.2, the myopic government discount factor becomes $\beta_g = 0.91$.

[13] The respective expressions are given in (2.83) and (2.143) in the Appendix.

additional terms are positive, implying steady state levels of government spending under the myopic government that slightly exceed those under the non-myopic government.[14]

Besides comparing steady state outcomes, we also investigate the responses of our variables to an income shock (i.e., we study the transition path to the steady state after an income shock). For the non-myopic government we do this using the benchmark specification and the system of linear dynamic equations (2.12) to (2.15). For the myopic government, we employ our second-order approximations.

We calculate the impulse responses to a transitory negative income shock of one percent of GDP in period t ($\sigma_\varepsilon \varepsilon_t = -1$), assuming that the economy is initially in its deterministic steady state and assuming no further shocks. Hence, $Y_t = 99$. The impulse responses over 10 periods for the non-myopic government, the myopic government under a debt constraint, and the myopic government under a primary deficit constraint are displayed in Figures 2.1, 2.2 and 2.3, respectively. In those figures, we plot the dynamics of income, public debt, government expenditure and the primary deficit. The dashed lines represent the deterministic steady states of the variables, whereas the solid lines show the impulse responses to the income shock. For the case of a non-myopic government we plot in addition the dynamics of the net foreign debt and of private consumption.

Under the non-myopic government (Figure 2.1), the negative income shock causes an immediate fall in the level of government expenditure in period t, which remains constant from then onwards. The fall in government spending is smaller than the initial drop in tax revenues and additional debt is accumulated until it reaches a new steady state level. This debt build up is consistent with the temporary decline in the primary surplus. However, the primary surplus converges to a new steady state level that is higher than before, consistent with government solvency (2.11). Private consumption remains constant throughout at its original steady state level. That is because the government and households can freely borrow or lend on the international capital market at a given interest rate, so that C_t is chosen independently of G_t to satisfy the household's first-order conditions. Therefore, the additional issuance of government bonds due to the shock in tax revenues combined with the constant level of private consumption implies an accumulation of foreign debt (negative impulse response of F_t in Figure 2.1).[15]

For the myopic government facing a debt constraint (Figure 2.2), the negative income shock leads to a sharper decrease in government expenditure in period t than for the non-myopic government. Under the debt constraint, the government abstains from excessive borrowing and will rather reduce its expenditures to meet the budget constraint. Over time, tax revenue and government expenditure converge back to the stochastic steady state level, which deviates only marginally from the initial, deterministic steady state. Hence, in the long run public debt and the primary deficit are almost unaffected by the

[14] Under the benchmark calibration the two terms are virtually equal (0.0928) and, therefore, neutral in terms of the difference between $\overline{G^B}$ and $\overline{G^D}$.

[15] Since all private income is taxed away and taxes fall below government spending in the short run, foreign debt must rise.

shock.

For the myopic government under the primary deficit constraint (Figure 2.3), the negative income shock leads to a pattern of government expenditure that is similar to that under the debt constraint but with a marginally stronger contraction in period t. The primary deficit remains virtually constant and close to D^c.

The results described so far provide the benchmark for the sensitivity analysis that we perform next. The benchmark setting is virtually neutral in terms of welfare for the two types of fiscal constraint as we can observe from the fourth block of Table 2.2. There, we also report social welfare U_w^B and U_w^D associated with public spending under the debt, respectively primary-deficit constraint, as well as the difference $W = U_w^B - U_w^D$. Because the difference in utility levels is uninformative about the true size of the welfare impact of different fiscal constraints, we also express it in terms of the permanent difference in government spending, G_{dif}, that generates this utility difference. To this end, we use the inverse function of (2.31) to compute the permanent constant government spending streams corresponding to U_w^B and U_w^D, respectively, and then take the difference between these two streams.[16] This results in a very small permanent spending difference $G_{dif} = 8.864E-05$. The final line of Table 2.2 reports the difference $Err_{dif} = k^D\left(\overline{D} - D^c\right)\overline{L^D} - k^B\left(\overline{B} - B^c\right)\overline{L^B}$ in the approximation error, which is virtually zero.

Sensitivity analysis

First, we investigate how changes in the deep parameters affect welfare and the impulse responses to income shocks under the two types of fiscal constraints. We adjust B^c, D^c, and k^D to keep the deterministic steady state of debt, $\overline{B}$, and the approximation error, Err_{dif}, always equal to their benchmark values.[17]

The government's discount factor (β_g) Because the introduction of our fiscal constraints is motivated by government myopia and the consequent lack of fiscal discipline, we start the sensitivity analysis by investigating the consequences of an increase in the degree of government myopia (i.e., β_g falls), while keeping all the other parameters at their benchmark values.

Table 2.3 reports the outcomes under the two fiscal constraints for different values of β_g. Under a debt constraint, if the government becomes more myopic, government spending $\widehat{G^B}$ rises, while the public debt $\widehat{B^B}$ falls. Also under the primary deficit constraint, government spending $\widehat{G^D}$ rises as β_g falls. However, public debt $\widehat{B^D}$ now increases. This

[16] More precisely, we compute the permanent government spending streams G_w^B and G_w^D from $\left(G_w^B\right)^{1-\frac{1}{\mu}} / \left[\left(1-\frac{1}{\mu}\right)(1-\beta_w)\right] = U_w^B$, respectively $\left(G_w^D\right)^{1-\frac{1}{\mu}} / \left[\left(1-\frac{1}{\mu}\right)(1-\beta_w)\right] = U_w^D$. Then, $G_{dif} = G_w^B - G_w^D$. For similar transformations of utility differences into permanent consumption equivalents, see e.g. Jensen (2002) and Beetsma and Jensen (2005).

[17] By adjusting B^c, D^c, and k^D, we keep Err_{dif} constant so that the difference in welfare under the two fiscal constraints is not affected by changes in the approximation errors when the deep parameters are varied.

occurs in spite of the tightening of the penalty parameter k^D needed to keep Err_{dif} constant. Otherwise, $\widehat{B^D}$ would increase even more as β_g falls, implying lower government spending. The intuition behind these outcomes is the following. Consider first the primary deficit constraint. More myopia provides the government with a stronger incentive to shift spending from the future to the present, implying a higher primary deficit for given reference value for the primary deficit and, hence, more borrowing. This implies that *in the steady state* debt and the primary surplus will be higher. Under a debt constraint a similar mechanism implies an increase in the steady state level of the debt. However, there is a mechanism pushing steady state debt in the opposite direction. With a direct constraint on the debt, a more myopic government has a stronger incentive to limit indebtedness to protect itself against bad income shocks that would force it to cut current spending in the very short run. Hence, a more myopic government engages in more precautionary saving. Notice that welfare in terms of permanent government consumption rises more under the debt- than under the primary deficit constraint (G_{dif} rises for a constant Err_{dif}) as we make the government more short-sighted.

The outcomes for this case can be summarized as:

Result 2.1 *Ceteris paribus, an increase in government myopia (a fall in β_g) leads to a fall (rise) in the stochastic steady state level of the public debt under a debt (primary deficit) constraint. Welfare in terms of permanent government consumption under a debt constraint rises relative to welfare under a primary-deficit constraint.*

The interest rate (r) Next, we investigate the welfare consequences of a change in the interest rate r. Changes in the interest rate affect the steady state outcomes under both the non-myopic and myopic government.

Table 2.4 reports the outcomes. In all cases, the lower is the interest rate, the lower is the steady state level of consumption $\overline{C}$ and higher is the steady state of government spending $\overline{G}$. A lower interest rate reduces interest income earned by the private sector on its assets, which thus reduces its private consumption. Lower interest spending also allows the government to have a higher amount of debt outstanding without violating the public budget constraint. Hence, the mean of debt under both constraints, $\widehat{B^B}$ and $\widehat{B^D}$, increases when r is smaller.

In order to keep the deterministic steady state value of the public debt at B_0 when the interest rate falls, the reference value D^c under the primary deficit constraint has to rise substantially (i.e., amount to a lower primary surplus), while the reference value B^c under the debt constraint needs to change only slightly. To see the intuition, notice that the deterministic steady state debt and primary deficit are linked as $\overline{B} = -(1/r)\,\overline{D}$ ($-\overline{D}$ is the constant primary surplus that pays off an initial debt $\overline{B}$). A given fall in r thus requires a relatively large increase in $\overline{D}$ to maintain $\overline{B} = B_0$. In turn, this rather large shift in $\overline{D}$ translates into a large shift in the reference value D^c.

Result 2.2 *In contrast to the reference value B^c under the debt-based constraint, the reference value D^c under the primary deficit-based constraint is relatively sensitive to changes in the interest rate. Further, a lower interest rate reduces the social welfare difference in terms of permanent government consumption under the debt based constraint relative to that under the primary-deficit based constraint.*

Because making frequent and large changes in reference values of fiscal constraints may be politically problematic (every change in the fiscal constraint is likely to re-open a political debate on what should be the appropriate design of the constraint).[18] Result 2.2 suggests that with frequent and large changes in the interest rate, a debt-based fiscal constraint becomes more attractive relative to a deficit-based constraint. In addition, Table 2.4 shows that as the interest rate falls, the relative disadvantage of the deficit-based constraint decreases. A lower interest rate reduces the cost of debt service, which, in particular, reduces the penalty parameter k^D associated with the primary deficit constraint that is needed to keep the steady state debt level and Err_{dif} constant.

The variance (σ_ε^2) and persistence (ρ) of the income shock To see how the relative desirability of the two fiscal constraints depends on the volatility of the business cycle, we vary σ_ε^2, keeping all other parameters at their benchmark values. Table 2.5 shows the outcomes of this numerical variation. Of course, changes in the variance of the income shock do not affect the deterministic steady state outcomes under both fiscal constraints. However, they do affect the stochastic steady state.

Under the debt constraint, a rise in uncertainty induces the government to reduce debt on average, thereby being able to absorb shocks that have become larger on average without hitting the reference debt level. By contrast, under the primary deficit constraint, a higher σ_ε^2 implies an increase in steady state public debt. The reason is that the latter corresponds to a higher stochastic steady state surplus, which is the result of a desire to maintain a larger safety margin relative to its reference level. Table 2.5 further shows that welfare falls as a result of the higher ex ante uncertainty about the available resources. The rise in the average surplus for higher σ_ε^2 reduces steady state government expenditure under the primary deficit constraint more than under the debt constraint.

Table 2.6 shows that welfare falls with the persistence of the income process. Higher income persistence implies a larger unconditional variance of income, for given σ_ε^2. The intuitions for the consequences of a rise in ρ are, therefore, very similar to the intuitions behind the effects of an increase in σ_ε^2.

Result 2.3 *Ceteris paribus, both an increase in income uncertainty (a higher σ_ε^2) and an increase in income persistence (a higher ρ), lower (raise) the expected debt level under the debt (primary-deficit) constraint. Under both constraints, social welfare falls in terms of*

[18]For a more formal discussion of the political issues involved in the implementation of fiscal constraints, see Krogstrup and Wyplosz (2006), Debrun and Kumar (2007), and Ribeiro and Beetsma (forthcoming).

permanent government consumption equivalents, but this fall is relatively larger under a primary deficit-based constraint than under a debt-based constraint.

The elasticity of intertemporal substitution (μ): Table 2.7 shows that the average debt level $\widehat{B^B}$ under the debt constraint increases when the relative risk aversion $1/\mu$ falls. The myopic government then prefers to keep a smaller margin relative to the reference debt level, because it is less bothered by the higher likelihood that the debt constraint becomes binding and that it has to implement an unexpected spending cut. Obviously, the higher average debt level is accompanied by lower average public spending. By contrast, the average debt level $\widehat{B^D}$ under the primary-deficit constraint falls as risk aversion falls, the reason being that the government now prefers to maintain a smaller safety margin to the reference deficit level. Hence, the average primary surplus falls, thereby also supporting a lower average debt level. The results and intuitions behind the increase in μ are very similar to those behind a fall in σ_ε^2. They are summarized as:

Result 2.4 *Ceteris paribus, a fall in relative risk aversion (a rise in μ), raises (lowers) the average public debt level under the debt (primary-deficit) constraint. As a result the difference between average public spending under the debt constraint and that under the primary-deficit constraint shrinks.*

Notice that changes in μ change the utility function itself, which makes it impossible to make a comparison of the change in the welfare difference as we increase μ.

2.6 Conclusion

This paper has compared primary-deficit constraints with debt constraints in a simple macroeconomic model with myopic governments. In our framework and based on our calibration, debt constraints yield higher welfare than primary deficit-based constraints. This finding is reinforced when (i) government myopia is stronger, (ii) the interest rate is higher and (iii) the income shocks have higher variances and are more persistent. Moreover, we also find that the debt constraint is more likely to be politically feasible.

Our analysis is of particular relevance for the design of the Stability and Growth Pact, which was reformed in July 2005. Our results support the shift from the narrow focus on the public deficit to the enhanced emphasis on the public debt in the Pact.

Our analysis also opens several avenues for further research. Throughout the paper we have assumed an exogenous and constant interest rate. Hence, neither the possibility of a myopic government, nor the presence of fiscal constraints, affect the risk premium on the interest rate as we would expect to observe it in reality. An interesting extension would be to allow for an endogenous interest rate in the presence of debt default risk. We have also not included public capital in our model. This would allow us to study golden rules as a third alternative fiscal constraint.

2.7 Tables and figures

Table 2.1: Calibration of the model

Par.	Value	Description and computation
Deep parameters		
$\overline{Y}$	100	*Average Income*: normalized value.
τ^y	1	*Income tax*: equal to 1 to focus on the utility of the public good.
B_0	63.14	*Initial domestic debt*: average $B_0/\overline{Y}$ for Euro-11 countries between 1970-2006.
F_0	0	*Initial foreign debt*: calibrated to zero.
r	0.02617585	*Long-term real interest rate*: average real interest rate for Euro-11 panel.
ρ	0.264	*Income autocorrelation*: average coefficient of an AR-1 estimation of log-deviations of the income trend for the Euro-11 countries between 1970-2006.
σ_ϵ^2	38.821	*Variance of the income shock*: average (over all observations) of the squared residuals of the above income regression.
α	0.933	Parameter that determines the ratio between the myopic government discount factor β_g and the social discount factor β_w, so that $\beta_g = \alpha\beta_w$.
μ	0.7	*Constant elasticity of intertemporal substitution*: provides a coefficient of relative risk aversion close to Ayagari and McGrattan (1998) and Smets and Wouters (2003).
γ	300	*Smoothness of the logistic function*: value that sufficiently approximates the indicator function.
Policy parameters		
k^B	100	*Severity of the debt-based sanction*: implies a very tight restriction.
k^D	100	*Severity of the primary deficit-based sanction*: implies a very tight restriction.
B^c	63.14425	*Reference value in the debt constraint*: provides a deterministic steady state of debt equal to B_0 and satisfies the transversality condition (tvc).
D^c	-1.64864435	*Reference value in the primary deficit constraint*: provides a deterministic steady state of debt equal to B_0 and satisfies the tvc.

Source: OECD (2006) and own calculations.

Table 2.2: Results of the model with benchmark calibration

Variables	Result	Description
Non-Myopic Government		
β_w	0.974492	Social discount factor.
$\overline{G}$	98.347257	Deterministic steady state of government spending.
$\overline{C}$	1.652743	Deterministic steady state of private consumption.
$\overline{D}$	-1.652743	Deterministic steady state of primary deficit.
Myopic Government with Debt Constraint		
B^c	63.144250	Reference value of debt constraint.
D^{Bc}	-1.652854	Primary deficit corresponding to reference value of debt under debt constraint.
$\overline{G^B}$	98.440078	Deterministic steady state of government spending.
$\overline{D^B}$	-1.652743	Deterministic steady state of primary deficit.
$\overline{B^B}$	63.140000	Deterministic steady state of public debt.
$\widehat{G^B}$	98.440082	Stochastic steady state of government spending.
$\widehat{D^B}$	-1.652743	Stochastic steady state of primary deficit.
$\widehat{B^B}$	63.139955	Stochastic steady state of public debt.
Myopic Government with Primary Deficit Constraint		
B^{Dc}	62.983415	Debt level corresponding to reference value of primary deficit.
D^c	-1.648644	Reference value of primary deficit constraint.
$\overline{G^D}$	98.439990	Deterministic steady state of government spending.
$\overline{D^D}$	-1.652743	Deterministic steady state of primary deficit.
$\overline{B^D}$	63.140000	Deterministic steady state of public debt.
$\widehat{G^D}$	98.439994	Stochastic steady state of government spending.
$\widehat{D^D}$	-1.652785	Stochastic steady state of primary deficit.
$\widehat{B^D}$	63.141595	Stochastic steady state of public debt.
Welfare		
U_w^B	-12.8130749	Welfare (utility of government spending) with debt constraint.
U_w^D	-12.81307986	Welfare (utility of government spending) with primary deficit constraint.
W	4.960E-06	Welfare difference under the two types of constraints ($U_w^B - U_w^D$).
G_{dif}	8.864E-05	Difference between the level of public consumption that provides U_w^B and U_w^D.
Err_{dif}	8.808E-05	Difference in the "approximation error" under the debt and primary deficit rule.

Table 2.3: Results of the model with different β_g

Variables[a,b]	$\alpha = 0.667$	$\alpha = 0.833$	$\alpha = 0.889$	$\alpha = 0.916$	Benchmark
β_g *	0.65	0.81	0.86	0.89	0.91
B^c	63.144185	63.144230	63.144241	63.144247	63.144250
D^{Bc}	-1.652853	-1.652854	-1.652854	-1.652854	-1.652854
k^B	100.000000	100.000000	100.000000	100.000000	100.000000
$\overline{G^B}$	98.440059	98.440075	98.440077	98.440078	98.440078
$\overline{D^B}$	-1.652743	-1.652743	-1.652743	-1.652743	-1.652743
$\overline{B^B}$	63.140000	63.140000	63.140000	63.140000	63.140000
$\widehat{G^B}$	98.440090	98.440086	98.440084	98.440083	98.440082
$\widehat{D^B}$	-1.652743	-1.652743	-1.652743	-1.652743	-1.652743
$\widehat{B^B}$	63.139940	63.139951	63.139954	63.139955	63.139955
B^{Dc}	62.985790	62.984154	62.983764	62.983550	62.983415
D^c	-1.648707	-1.648664	-1.648653	-1.648648	-1.648644
k^D	100.067622	100.021245	100.010050	100.003897	100.000000
$\overline{G^D}$	98.439971	98.439987	98.439989	98.439990	98.439990
$\overline{D^D}$	-1.652743	-1.652743	-1.652743	-1.652743	-1.652743
$\overline{B^D}$	63.140000	63.140000	63.140000	63.140001	63.140000
$\widehat{G^D}$	98.440001	98.439997	98.439996	98.439994	98.439994
$\widehat{D^D}$	-1.652800	-1.652790	-1.652787	-1.652786	-1.652785
$\widehat{B^D}$	63.142179	63.141778	63.141682	63.141629	63.141595
U_w^B	-12.813074	-12.813075	-12.813075	-12.813075	-12.813075
U_w^D	-12.813079	-12.813080	-12.813080	-12.813080	-12.813080
W	5.046E-06	4.982E-06	4.969E-06	4.963E-06	4.960E-06
G_{dif}	9.018E-05	8.903E-05	8.880E-05	8.870E-05	8.864E-05
Err_{dif}	8.808E-05	8.808E-05	8.808E-05	8.808E-05	8.808E-05

Notes: [a] The variables of the first column are defined as in Table 2.2. [b] In the other columns, in addition, we vary the value of β_g while keeping the other parameters at their benchmark values indicated in Table 2.1. * This line indicates the value of the myopic government discount factor, which is computed via $\beta_g = \alpha\beta_w$.

Table 2.4: Results of the model with different interest rates (r)

Var.[a,b]	$r = 0.002$	$r = 0.014$	Benchmark	$r = 0.041$	$r = 0.056$	$r = 0.071$	$r = 0.09$
β_w	0.998004	0.986193	0.974492	0.960615	0.946970	0.933707	0.917431
$\overline{G}$	99.873720	99.116040	98.347257	97.411260	96.464160	95.517060	94.317400
$\overline{C}$	0.126280	0.883960	1.652743	2.588740	3.535840	4.482940	5.682600
$\overline{D}$	-0.126280	-0.883960	-1.652743	-2.588740	-3.535840	-4.482940	-5.682600
B^c	63.144247	63.144249	63.144250	63.144253	63.144255	63.144257	63.144260
D^{Bc}	-0.126288	-0.884019	-1.652854	-2.588914	-3.536078	-4.483242	-5.682983
k^B	100.000000	100.000000	100.000000	100.000000	100.000000	100.000000	100.000000
$\overline{G^B}$	99.966541	99.208861	98.440078	97.504081	96.556981	95.609881	94.410222
$\overline{D^B}$	-0.126280	-0.883960	-1.652743	-2.588740	-3.535840	-4.482940	-5.682600
$\overline{B^B}$	63.140000	63.140000	63.140000	63.140000	63.140000	63.140000	63.140000
$\widehat{G^B}$	99.966545	99.208865	98.440082	97.504085	96.556985	95.609886	94.410226
$\widehat{D^B}$	-0.126280	-0.883960	-1.652743	-2.588740	-3.535840	-4.482940	-5.682600
$\widehat{B^B}$	63.139957	63.139957	63.139955	63.139955	63.139954	63.139953	63.139952
B^{Dc}	61.090621	62.847231	62.983415	63.040030	63.066808	63.082271	63.094458
D^c	-0.122181	-0.879861	-1.648644	-2.584641	-3.531741	-4.478841	-5.678501
k^D	99.999664	99.999843	100.000000	100.000159	100.000283	100.000370	100.000428
$\overline{G^D}$	99.966453	99.208773	98.439990	97.503993	96.556893	95.609793	94.410133
$\overline{D^D}$	-0.126280	-0.883960	-1.652743	-2.588740	-3.535840	-4.482940	-5.682600
$\overline{B^D}$	63.140000	63.140000	63.140000	63.140000	63.140000	63.140000	63.140000
$\widehat{G^D}$	99.966456	99.208777	98.439994	97.503997	96.556897	95.609797	94.410137
$\widehat{D^D}$	-0.12632	-0.88400	-1.65278	-2.58878	-3.53588	-4.48298	-5.68265
$\widehat{B^D}$	63.16024	63.14294	63.14159	63.14104	63.14077	63.14062	63.14050
U_w^B	-162.66298	-23.59313	-12.81307	-8.33275	-6.21478	-4.99261	-4.03038
U_w^D	-162.66304	-23.59314	-12.81308	-8.33276	-6.21478	-4.99261	-4.03038
W	6.197E-05	9.058E-06	4.960E-06	3.257E-06	2.454E-06	1.991E-06	1.628E-06
G_{dif}	8.860E-05	8.861E-05	8.864E-05	8.864E-05	8.867E-05	8.868E-05	8.869E-05
Err_{dif}	8.808E-05	8.808E-05	8.808E-05	8.808E-05	8.808E-05	8.808E-05	8.808E-05

Notes: [a] The variables in the first column are defined in Table 2.2. [b] In the other lines, in addition, we vary the value of the interest rate (r) while keeping the other parameters at their benchmark values indicated in Table 2.1.

Table 2.5: Results of the model with different σ_ϵ^2

Var.[a,b]	σ_ϵ^2 =1	$\sigma_\epsilon^2 = 12$	$\sigma_\epsilon^2 = 24$	Benchmark	$\sigma_\epsilon^2 = 50$	$\sigma_\epsilon^2 = 75$	$\sigma_\epsilon^2 = 100$
B^c	63.144250	63.144250	63.144250	63.144250	63.144250	63.144250	63.144250
D^{Bc}	-1.652854	-1.652854	-1.652854	-1.652854	-1.652854	-1.652854	-1.652854
k^B	100.000000	100.000000	100.000000	100.000000	100.000000	100.000000	100.000000
$\overline{G^B}$	98.440078	98.440078	98.440078	98.440078	98.440078	98.440078	98.440078
$\overline{D^B}$	-1.652743	-1.652743	-1.652743	-1.652743	-1.652743	-1.652743	-1.652743
$\overline{B^B}$	63.140000	63.140000	63.140000	63.140000	63.140000	63.140000	63.140000
$\widehat{G^B}$	98.440078	98.440078	98.440079	98.440082	98.440085	98.440094	98.440106
$\widehat{D^B}$	-1.652743	-1.652743	-1.652743	-1.652743	-1.652743	-1.652743	-1.652743
$\widehat{B^B}$	63.140000	63.139996	63.139983	63.139955	63.139926	63.139833	63.139702
B^{Dc}	62.983415	62.983415	62.983415	62.983415	62.983415	62.983415	62.983415
D^c	-1.648644	-1.648644	-1.648644	-1.648644	-1.648644	-1.648644	-1.648644
k^D	100.000000	100.000000	100.000000	100.000000	100.000000	100.000000	100.000000
$\overline{G^D}$	98.439990	98.439990	98.439990	98.439990	98.439990	98.439990	98.439990
$\overline{D^D}$	-1.652743	-1.652743	-1.652743	-1.652743	-1.652743	-1.652743	-1.652743
$\overline{B^D}$	63.140000	63.140000	63.140000	63.140000	63.140000	63.140000	63.140000
$\widehat{G^D}$	98.439990	98.439990	98.439991	98.439994	98.439996	98.440005	98.440017
$\widehat{D^D}$	-1.652743	-1.652747	-1.652759	-1.652785	-1.652813	-1.652900	-1.653023
$\widehat{B^D}$	63.140000	63.140146	63.140602	63.141595	63.142657	63.146008	63.150706
U_w^B	-12.796641	-12.801421	-12.806635	-12.813075	-12.817932	-12.828795	-12.839657
U_w^D	-12.796646	-12.801426	-12.806640	-12.813080	-12.817937	-12.828800	-12.839662
W	4.909E-06	4.921E-06	4.937E-06	4.960E-06	4.980E-06	5.034E-06	5.100E-06
G_{dif}	8.811E-05	8.821E-05	8.837E-05	8.864E-05	8.889E-05	8.960E-05	9.052E-05
Err_{dif}	8.808E-05	8.808E-05	8.808E-05	8.808E-05	8.808E-05	8.808E-05	8.808E-05

Notes: [a] The variables of the first column are defined as in Table 2.2. [b] In the other columns, in addition, we vary the value of σ_ϵ^2 while keeping the other parameters at their benchmark values indicated in Table 2.1.

Table 2.6: Results of the model with different ρ

Var.[a,b]	$\rho = 0.01$	Benchmark	ρ= 0.4	$\rho = 0.6$	$\rho = 0.8$	$\rho = 0.99$
B^c	63.144250	63.144250	63.144250	63.144250	63.144250	63.144250
D^{Bc}	-1.652854	-1.652854	-1.652854	-1.652854	-1.652854	-1.652854
k^B	100.00000	100.00000	100.00000	100.00000	100.00000	100.00000
$\overline{G^B}$	98.440078	98.440078	98.440078	98.440078	98.440078	98.440078
$\overline{D^B}$	-1.652743	-1.652743	-1.652743	-1.652743	-1.652743	-1.652743
$\overline{B^B}$	63.140000	63.140000	63.140000	63.140000	63.140000	63.140000
$\widehat{G^B}$	98.440082	98.440082	98.440082	98.440082	98.440082	98.440082
$\widehat{D^B}$	-1.652743	-1.652743	-1.652743	-1.652743	-1.652743	-1.652743
$\widehat{B^B}$	63.1399555	63.1399552	63.1399551	63.1399550	63.1399549	63.1399549
B^{Dc}	62.983415	62.983415	62.983415	62.983415	62.983415	62.983415
D^c	-1.648644	-1.648644	-1.648644	-1.648644	-1.648644	-1.648644
k^D	100.00000	100.00000	100.00000	100.00000	100.00000	100.00000
$\overline{G^D}$	98.439990	98.439990	98.439990	98.439990	98.439990	98.439990
$\overline{D^D}$	-1.652743	-1.652743	-1.652743	-1.652743	-1.652743	-1.652743
$\overline{B^D}$	63.140000	63.140000	63.140000	63.140000	63.140000	63.140000
$\widehat{G^D}$	98.439993	98.439994	98.439994	98.439994	98.439994	98.439994
$\widehat{D^D}$	-1.652785	-1.652785	-1.652785	-1.652785	-1.652785	-1.652785
$\widehat{B^D}$	63.141586	63.141595	63.141598	63.141603	63.141606	63.141608
U_w^B	-12.811901	-12.813075	-12.814888	-12.820726	-12.839797	-13.584787
U_w^D	-12.811906	-12.813080	-12.814893	-12.820731	-12.839803	-13.584794
W	4.958E-06	4.960E-06	4.964E-06	4.976E-06	5.017E-06	6.636E-06
G_{dif}	8.864E-05	8.864E-05	8.866E-05	8.875E-05	8.904E-05	9.759E-05
Err_{dif}	8.808E-05	8.808E-05	8.808E-05	8.808E-05	8.808E-05	8.808E-05

Notes: [a] The variables of the first column are defined as in Table 2.2. [b] In the other columns, in addition, we vary the value of ρ while keeping the other parameters at their benchmark values indicated in Table 2.1.

Table 2.7: Results of the model with different μ

Var.[a,b]	$\mu = 0.3$	$\mu = 0.5$	Benchmark	$\mu = 1.01$	$\mu = 2$	$\mu = 3$	$\mu = 4$
B^c	63.144250	63.144250	63.144250	63.144250	63.144250	63.144250	63.144250
D^{Bc}	-1.652854	-1.652854	-1.652854	-1.652854	-1.652854	-1.652854	-1.652854
k^B	100.00000	100.00000	100.00000	100.00000	100.00000	100.00000	100.00000
$\overline{G^B}$	98.440078	98.440078	98.440078	98.440078	98.440078	98.440078	98.440078
$\overline{D^B}$	-1.652743	-1.652743	-1.652743	-1.652743	-1.652743	-1.652743	-1.652743
$\overline{B^B}$	63.140000	63.140000	63.140000	63.140000	63.140000	63.140000	63.140000
$\widehat{G^B}$	98.440094	98.440085	98.440082	98.440080	98.440079	98.440079	98.440078
$\widehat{D^B}$	-1.652743	-1.652743	-1.652743	-1.652743	-1.652743	-1.652743	-1.652743
$\widehat{B^B}$	63.139815	63.139923	63.139955	63.139974	63.139990	63.139994	63.139996
B^{Dc}	62.983415	62.983415	62.983415	62.983415	62.983415	62.983415	62.983415
D^c	-1.648644	-1.648644	-1.648644	-1.648644	-1.648644	-1.648644	-1.648644
k^D	100.00000	100.00000	100.00000	100.00000	100.00000	100.00000	100.00000
$\overline{G^D}$	98.439990	98.439990	98.439990	98.439990	98.439990	98.439990	98.439990
$\overline{D^D}$	-1.652743	-1.652743	-1.652743	-1.652743	-1.652743	-1.652743	-1.652743
$\overline{B^D}$	63.140000	63.140000	63.140000	63.140000	63.140000	63.140000	63.140000
$\widehat{G^D}$	98.440005	98.439996	98.439994	98.439992	98.439991	98.439990	98.439990
$\widehat{D^D}$	-1.652917	-1.652815	-1.652785	-1.652767	-1.652752	-1.652748	-1.652747
$\widehat{B^D}$	63.146654	63.142761	63.141595	63.140905	63.140344	63.140204	63.140143
U_w^B	-0.000382	-0.399959	-12.813075	4143.502	777.505	1253.100	1632.914
U_w^D	-0.000382	-0.399959	-12.813080	4143.502	777.505	1253.099	1632.913
W	8.189E-10	3.629E-07	4.960E-06	3.694E-05	3.489E-04	7.490E-04	1.098E-03
G_{dif}	8.985E-05	8.895E-05	8.864E-05	8.844E-05	8.825E-05	8.820E-05	8.817E-05
Err_{dif}	8.808E-05	8.808E-05	8.808E-05	8.808E-05	8.808E-05	8.808E-05	8.808E-05

Notes: [a] The variables of the first column are defined as in Table 2.2. [b] In the other columns, in addition, we vary the value of μ while keeping the other parameters at their benchmark values indicated in Table 2.1.

Figure 2.1: Impulse responses for non-myopic government with benchmark parameters

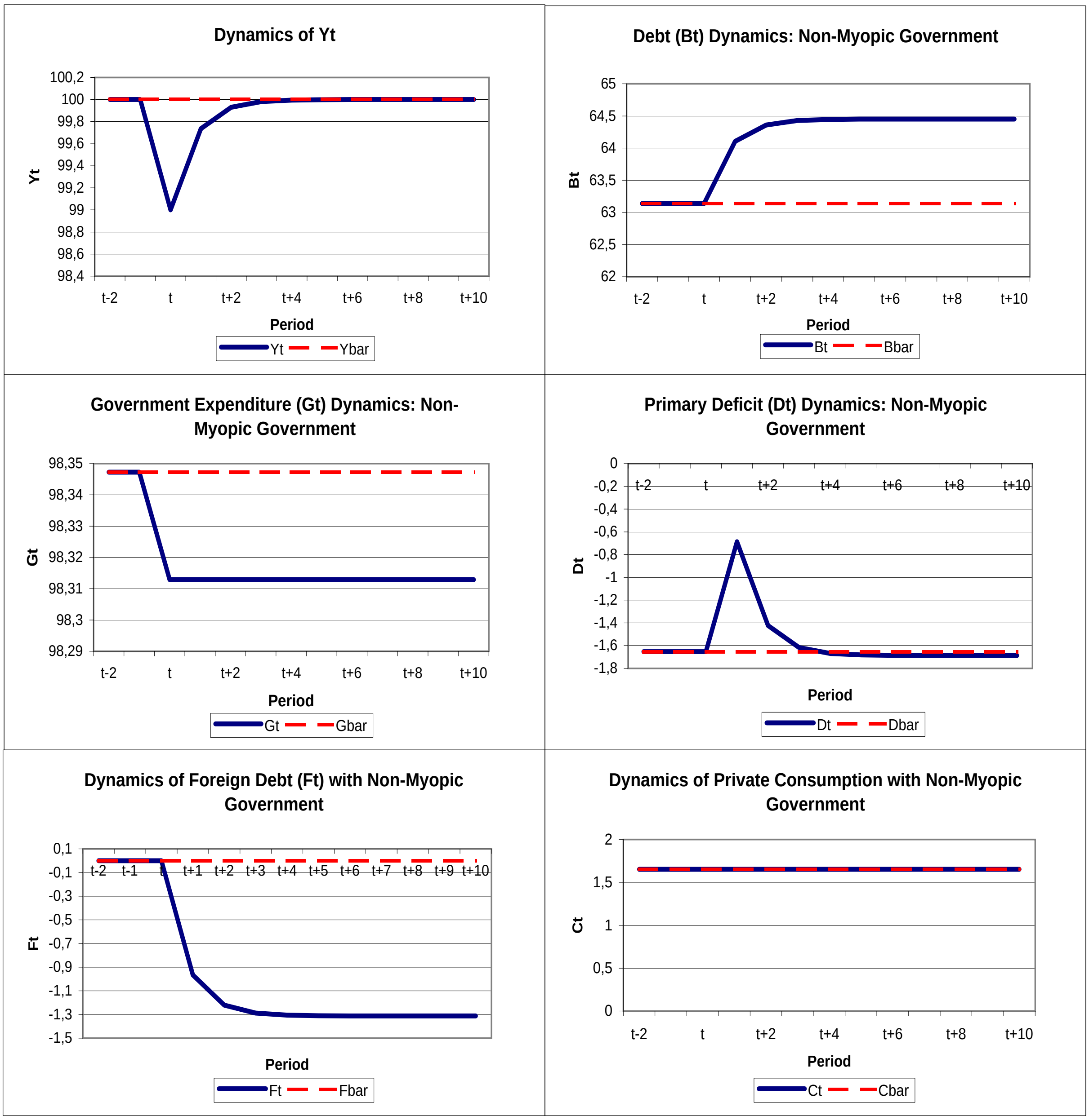


Figure 2.2: Impulse responses for myopic government under debt constraint with benchmark parameters

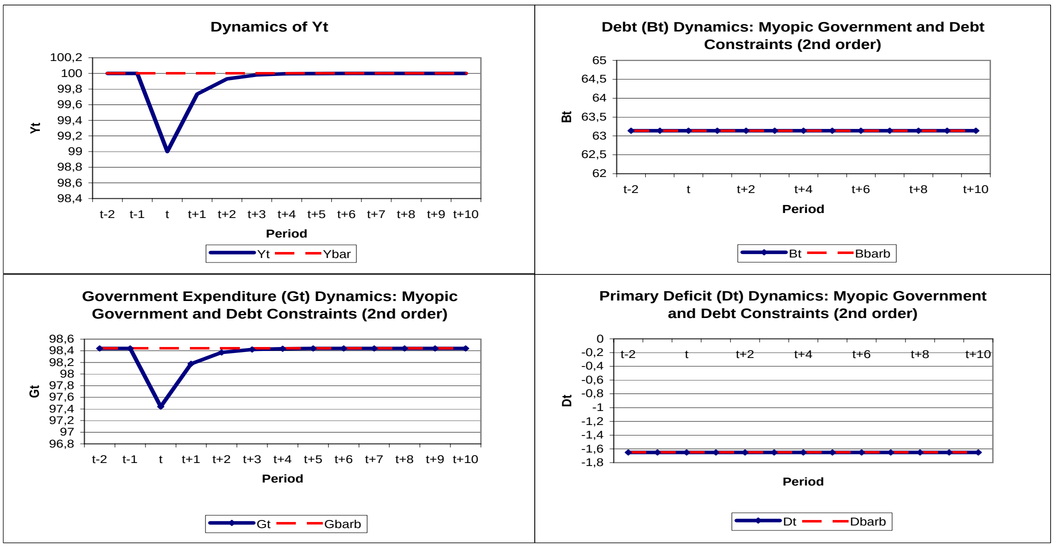
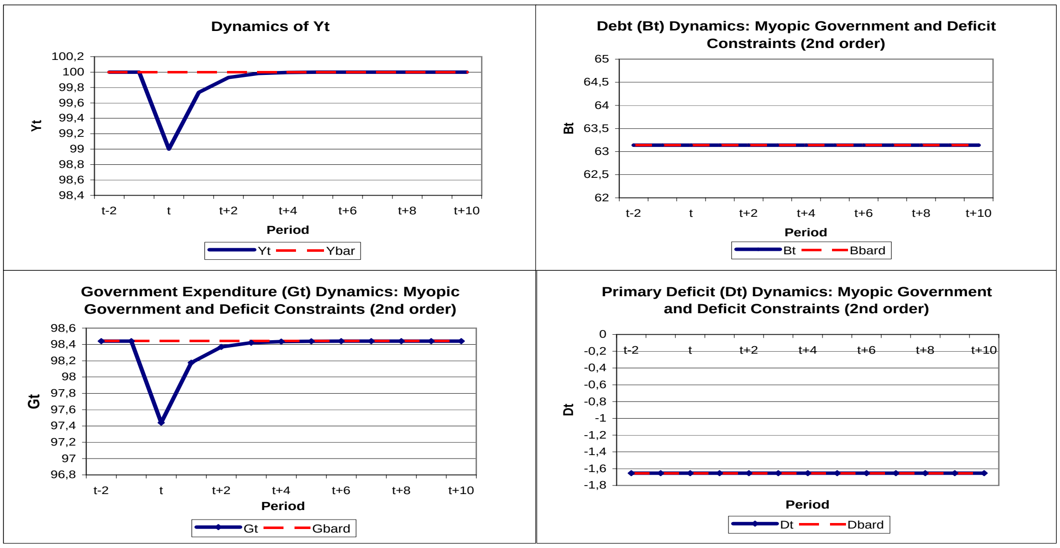


Figure 2.3: Impulse responses for myopic government under primary deficit constraint with benchmark parameters



Appendices to Chapter 2

2.A Solution for the non-myopic government

For a given endowment sequence $\{Y_t\}_{t=0}^{\infty}$, constant tax rate τ^y, and initial values F_0 and B_0, a rational expectations equilibrium under a non-myopic government consists of a set of sequences $\{C_t, G_t, B_{t+1}, F_{t+1}\}_{t=0}^{\infty}$ satisfying

$$\begin{aligned} u'(G_t) &= \mathrm{E}_t u'(G_{t+1}), \\ (1+r)B_t &= \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (\tau^y Y_s - G_s), \\ v'(C_t) &= \mathrm{E}_t v'(C_{t+1}), \\ -(1+r)F_t &= \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (Y_s - C_s - G_s). \end{aligned}$$

The first two conditions feature two unknowns, G_t and B_{t+1}. Hence, one can in principle separately solve for G_t and B_{t+1} and subsequently for C_t and F_{t+1}, using the last two conditions and the solution for G_t.

Since we restrict the equilibrium to be recursive, we know that the condition $u'(G_t) = \mathrm{E}_t u'(G_{t+1})$ implies that the conditional expectations of $E_t G_{t+j}$ are constant for all $j \geq 0$ and thus equal to G_t, $E_t G_{t+j} = G_t$, $\forall j \geq 0$. Similarly, $v'(C_t) = \mathrm{E}_t v'(C_{t+1})$, implies that $E_t C_{t+j} = C_t$, $\forall j \geq 0$.

We, firstly, aim to derive solutions for G_t and B_{t+1}, i.e., to identify the state space representation of G_t and B_{t+1}. We start with the subset of equilibrium conditions

$$\begin{aligned} u'(G_t) &= \mathrm{E}_t u'(G_{t+1}) \\ (1+r)B_t &= \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (\tau^y Y_s - G_s), \end{aligned}$$

given B_0. Taking expectations conditional on the information in period t, the intertemporal budget constraint implies

$$\mathrm{E}_t \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} G_s = \tau^y Y_t + \mathrm{E}_t \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \tau^y Y_s - (1+r)B_t.$$

Using $\mathrm{E}_t G_{t+j} = G_t$, $\forall j > 0$, and $\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} = 1/\left(1 - \frac{1}{1+r}\right) = \frac{r+1}{r}$ given that $1 + r > 1$, leads to

$$\frac{r+1}{r} G_t = \tau^y Y_t + \mathrm{E}_t \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \tau^y Y_s - (1+r)B_t.$$

To determine the period t expectation of Y_s for $s \geq t$, we use that Y_t follows the stochastic process

$$Y_t - \overline{Y} = \rho \left(Y_{t-1} - \overline{Y}\right) + \sigma_\varepsilon \varepsilon_t.$$

Expectations conditional on the information in period t are thus

$$\mathrm{E}_t Y_s = (1-\rho)\overline{Y} + \rho E_t Y_{s-1}.$$

To write $\mathrm{E}_t Y_s$ as a function of Y_t, we iterate backwards to get $E_t Y_s = (1-\rho)\overline{Y} + \rho(1-\rho)\overline{Y} + ... + \rho^{(s-t)-1}(1-\rho)\overline{Y} + \rho^{(s-t)} E_t Y_t$, or, concisely

$$\mathrm{E}_t Y_s = (1-\rho)\overline{Y} \sum_{k=0}^{(s-t)-1} \rho^k + \rho^{(s-t)} Y_t.$$

Eliminating $E_t Y_s$ in $\frac{r+1}{r} G_t = \tau^y Y_t + \tau^y \mathrm{E}_t \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} Y_s - (1+r)B_t$ then gives

$$\begin{aligned}
\frac{r+1}{r} G_t &= \tau^y Y_t + \tau^y \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left((1-\rho)\overline{Y} \sum_{k=0}^{(s-t)-1} \rho^k + \rho^{(s-t)} Y_t \right) - (1+r)B_t \\
&= \left\{ \begin{array}{c} \tau^y Y_t + \tau^y \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (1-\rho)\overline{Y} \sum_{k=0}^{(s-t)-1} \rho^k + \tau^y \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \rho^{(s-t)} Y_t \\ -(1+r)B_t \end{array} \right\} \\
&= \tau^y Y_t \left(1 + \frac{\rho}{r-\rho+1}\right) + \left[(1-\rho)\overline{Y}\tau^y \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \sum_{k=0}^{(s-t)-1} \rho^k \right] - (1+r)B_t.
\end{aligned}$$

Using $\sum_{k=0}^{(s-t)-1} \rho^k = \left(\frac{1}{1-\rho} - \frac{\rho^{s-t}}{1-\rho}\right)$ for any $\rho \neq 1$, the term in the square bracket can further be simplified to

$$\begin{aligned}
& \tau^y (1-\rho)\overline{Y} \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \sum_{k=0}^{(s-t)-1} \rho^k \\
= {} & \tau^y (1-\rho)\overline{Y} \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left(\frac{1}{1-\rho} - \frac{\rho^{s-t}}{1-\rho}\right) \\
= {} & \tau^y \overline{Y} \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left(1 - \rho^{s-t}\right).
\end{aligned}$$

By using $\sum_{k=0}^{\infty} a^{k+1} = a/(1-a)$ for any $|a| < 1$, such that $\sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} = 1/r$ and $\sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \rho^{s-t} = \frac{\rho}{r-\rho+1}$, the last expression becomes

$$\begin{aligned}
& \tau^y \overline{Y} \left(\frac{1}{r} - \frac{\rho}{r-\rho+1}\right) \\
= {} & \tau^y \overline{Y} \frac{(1-\rho)(1+r)}{r(1+r-\rho)}.
\end{aligned}$$

Hence, we end up with the following state space solution for G_t :

$$\begin{aligned}
\frac{r+1}{r} G_t &= \tau^y Y_t \left(1 + \frac{\rho}{r-\rho+1}\right) + \tau^y \overline{Y} \frac{(1-\rho)(1+r)}{r(1+r-\rho)} - (1+r)B_t \\
&= \tau^y Y_t \frac{r+1}{r-\rho+1} + \tau^y \overline{Y} \frac{(1-\rho)(1+r)}{r(1+r-\rho)} - (1+r)B_t \\
&\Rightarrow G_t = \tau^y Y_t \frac{r}{1+r-\rho} + \tau^y \overline{Y} \frac{1-\rho}{1+r-\rho} - rB_t.
\end{aligned}$$

The solution for B_{t+1} can then simply be found by eliminating G_t in the period-by-period budget constraint

$$\begin{aligned} B_{t+1} &= (1+r)\,B_t + G_t - \tau^y Y_t \\ &= (1+r)\,B_t + \left(\tau^y Y_t \frac{r}{1+r-\rho} + \tau^y \overline{Y} \frac{1-\rho}{1+r-\rho} - rB_t\right) - \tau^y Y_t \\ &= B_t - \tau^y \frac{1-\rho}{1+r-\rho}\left(Y_t - \overline{Y}\right), \end{aligned}$$

which discloses the unit root in B_t and thus in G_t. Hence, for a given initial value B_0 and a given endowment sequence $\{Y_t\}_{t=0}^{\infty}$ we obtain a unique set of sequences $\{G_t, B_{t+1}\}_{t=0}^{\infty}$. Due to the unit root, there are infinitely many solutions for the unconditional expectations for G and B, which satisfy

$$\overline{G} = \tau^y \overline{Y} - r\overline{B}.$$

We can now derive the solutions for C_t and F_{t+1}, for which we use the remaining conditions (taking the solution for G_t as given)

$$\begin{aligned} v'(C_t) &= \mathrm{E}_t v'(C_{t+1}) \\ -(1+r)F_t &= \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - C_s - G_s), \end{aligned}$$

given $F_0 = 0$. Taking expectations conditional on the information in period t, the intertemporal resource constraint leads to

$$\begin{aligned} \mathrm{E}_t \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} C_s &= \mathrm{E}_t \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - G_s) + (1+r)F_t \\ \Rightarrow \frac{1+r}{r} C_t &= \mathrm{E}_t \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} Y_s - \frac{1+r}{r} G_t + (1+r)F_t \end{aligned}$$

where we used $\mathrm{E}_t C_{t+j} = C_t$, $\forall j > 0$, and $\mathrm{E}_t G_{t+j} = G_t$, $\forall j > 0$. Using that

$$\mathrm{E}_t \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} Y_s = Y_t \frac{r+1}{r-\rho+1} + \overline{Y}\frac{(1-\rho)(1+r)}{r(1+r-\rho)},$$

holds (see above), consumption C_t can be written as a function of Y_t, G_t, and F_t :

$$\begin{aligned} \frac{1+r}{r} C_t &= Y_t \frac{r+1}{r-\rho+1} + \overline{Y}\frac{(1-\rho)(1+r)}{r(1+r-\rho)} - \frac{1+r}{r} G_t + (1+r)F_t \\ &\Rightarrow\ C_t = Y_t \frac{r}{r-\rho+1} + \overline{Y}\frac{1-\rho}{1+r-\rho} - G_t + rF_t. \end{aligned}$$

Plugging in the solution for G_t, $G_t = \tau^y Y_t \frac{r}{1+r-\rho} + \tau^y \overline{Y} \frac{1-\rho}{1+r-\rho} - rB_t$, we get the following state space solution for consumption

$$\begin{aligned} C_t &= Y_t \frac{r}{r-\rho+1} + \overline{Y}\frac{(1-\rho)}{(1+r-\rho)} - \left(\tau^y Y_t \frac{r}{1+r-\rho} + \tau^y \overline{Y}\frac{1-\rho}{1+r-\rho} - rB_t\right) + rF_t \\ &= (1-\tau^y)\frac{r}{r-\rho+1} Y_t + (1-\tau^y)\frac{1-\rho}{1+r-\rho}\overline{Y} + rB_t + rF_t. \end{aligned}$$

Using the period-by-period resource constraint, $F_{t+1} = (1+r)F_t + Y_t - C_t - G_t$, and the solutions for C_t and G_t, we can compute the solution for F_t with

$$\begin{aligned} F_{t+1} &= (1+r)F_t + Y_t \\ &\quad - \left((1-\tau^y)\frac{r}{r-\rho+1}Y_t + (1-\tau^y)\frac{1-\rho}{1+r-\rho}\overline{Y} + rB_t + rF_t\right) \\ &\quad - \left(\tau^y Y_t \frac{r}{1+r-\rho} + \tau^y \overline{Y}\frac{1-\rho}{1+r-\rho} - rB_t\right) \\ &= F_t + \frac{1-\rho}{1-\rho+r}\left(Y_t - \overline{Y}\right), \end{aligned}$$

and the initial value $F_0 = 0$. The unconditional means of F and C, which satisfy $-r\overline{F} = \overline{Y} - \overline{C} - \overline{G}$ are not determined.

To summarize, the solution $X_t = \Gamma_X(B_t, F_t, Y_t)$ for $X_t \in (G_t, C_t, B_{t+1}, F_{t+1})$ is represented by (2.12) to (2.15) given B_0 and F_0, with unconditional means satisfying

$$\overline{G} = \tau^y \overline{Y} - r\overline{B} \quad \text{and} \quad \overline{C} = (1-\tau^y)\overline{Y} + r\overline{F} + r\overline{B}$$

2.B Intertemporal budget constraint under debt sanction

Substitute (2.10) and $I\left[B_s; \overline{B}\right]$ by L_s^B in (2.7) and rewrite it for $s = t$:

$$\begin{aligned} B_{t+1} &= (1+r)B_t + G_t - \tau^y Y_t + k^B(B_t - B^c)L_t^B \Leftrightarrow \\ \left(1+r+k^B L_t^B\right)B_t &= \tau^y Y_t - G_t + k^B B^c L_t^B + B_{t+1}. \end{aligned}$$

Iterating this process forward one period, we can jot down this last expression like:

$$B_{t+1} = \frac{\tau^y Y_{t+1} - G_{t+1} + k^B B^c L_{t+1}^B}{\left(1+r+k^B L_{t+1}^B\right)} + \frac{B_{t+2}}{\left(1+r+k^B L_{t+1}^B\right)}.$$

So, the intertemporal budget constraint with debt sanction is equal to:

$$\left(1+r+k^B L_t^B\right)B_t = \left\{\begin{array}{c} \tau^y Y_t - G_t + k^B B^c L_t^B + \frac{\tau^y Y_{t+1} - G_{t+1} + k^B B^c L_{t+1}^B}{\left(1+r+k^B L_{t+1}^B\right)} \\ + \frac{\tau^y Y_{t+2} - G_{t+2} + k^B B^c L_{t+2}^B}{\left(1+r+k^B L_{t+1}^B\right)\left(1+r+k^B L_{t+2}^B\right)} + \ldots \end{array}\right\},$$

that can be represented by:

$$\begin{aligned} \left(1+r+k^B L_t^B\right)B_t &= \left[\begin{array}{c} \tau^y Y_t - G_t + k^B B^c L_t^B + \frac{\tau^y Y_{t+1} - G_{t+1} + k^B B^c L_{t+1}^B}{\left(1+r+k^B L_{t+1}^B\right)} \\ + \frac{\tau^y Y_{t+2} - G_{t+2} + k^B B^c L_{t+2}^B}{\prod_{v=t+1}^{t+2}\left(1+r+k^B L_v^B\right)} + \ldots + \frac{\tau^y Y_{t+T} - G_{t+T} + k^B B^c L_{t+T}^B}{\prod_{v=t+1}^{t+T}\left(1+r+k^B L_v^B\right)} \\ + \lim_{T\to\infty} \frac{B_{t+T+1}}{\prod_{v=t+1}^{t+T}\left(1+r+k^B L_v^B\right)} \end{array}\right] \Leftrightarrow \\ \left(1+r+k^B L_t^B\right)B_t &= \sum_{s=t}^{\infty} \frac{\tau^y Y_s - G_s + k^B B^c L_s^B}{\prod_{v=t+1}^{s}\left(1+r+k^B L_v^B\right)} + \lim_{T\to\infty} \frac{B_{t+T+1}}{\prod_{v=t+1}^{t+T}\left(1+r+k^B L_v^B\right)}, \end{aligned}$$

where we define $\prod_{v=t+1}^{t}\left(1+r+k^{B}L_{v}^{B}\right)\equiv 1$. In equilibrium that constraint is then equal to (2.19). Moreover, for the deterministic steady state, the transversality condition imposes:

$$\left(1+r+k^{B}L_{0}^{B}\right)B_{0}-\frac{1+r+k^{B}\overline{L^{B}}}{r+k^{B}\overline{L^{B}}}\left(\tau^{y}\overline{Y}-\overline{G}+k^{B}B^{c}\overline{L^{B}}\right)=0.$$

Note that if $B_0=\overline{B}$, as we assume in our numerical analysis, then the last equation simplifies as follows:

$$\begin{aligned}\left(1+r+k^{B}\overline{L^{B}}\right)\overline{B}-\frac{1+r+k^{B}\overline{L^{B}}}{r+k^{B}\overline{L^{B}}}\left(\tau^{y}\overline{Y}-\overline{G}+k^{B}B^{c}\overline{L^{B}}\right) &= 0\Leftrightarrow\\ \left(r+k^{B}\overline{L^{B}}\right)\overline{B} &= \tau^{y}\overline{Y}-\overline{G}+k^{B}B^{c}\overline{L^{B}}\Leftrightarrow\\ r\overline{B}+k^{B}\left(\overline{B}-B^{c}\right)\overline{L^{B}} &= \tau^{y}\overline{Y}-\overline{G}.\end{aligned}$$

For large values of γ and thus good approximations of the indicator function, this simplifies to:

$$r\overline{B}=\tau^{y}\overline{Y}-\overline{G},$$

which is the usual constraint that should hold in steady state.

2.C Intertemporal budget constraint under deficit sanction

Substituting (2.8), $I\left[D_t>D^c\right]$ by L_t^D, and (2.10) in (2.21):

$$\begin{aligned}B_{t+1} &= \left(1+r\right)B_t+G_t-\tau^{y}Y_t+k^{D}\left(D_t-D^c\right)L_t^D\Leftrightarrow\\ B_{t+1} &= \left(1+r\right)B_t+G_t-\tau^{y}Y_t+k^{D}\left(B_{t+1}-\left(1+r\right)B_t-D^c\right)L_t^D\Leftrightarrow\\ \left(1+r\right)\left(1-k^{D}L_t^D\right)B_t &= \tau^{y}Y_t-G_t+k^{D}D^{c}L_t^D+\left(1-k^{D}L_t^D\right)B_{t+1}\Leftrightarrow\\ \left(1+r\right)B_t &= \frac{\tau^{y}Y_t-G_t+k^{D}D^{c}L_t^D}{1-k^{D}L_t^D}+B_{t+1}.\end{aligned}$$

Iterating this process forward one period, we can rewrite this last expression like:

$$B_{t+1}=\frac{\tau^{y}Y_{t+1}-G_{t+1}+k^{D}D^{c}L_{t+1}^D}{\left(1+r\right)\left(1-k^{D}L_{t+1}^D\right)}+\frac{B_{t+2}}{1+r}.$$

So, the intertemporal budget constraint with debt sanction is equal to:

$$\left(1+r\right)B_t=\left\{\begin{array}{c}\frac{\tau^{y}Y_t-G_t+k^{D}D^{c}L_t^D}{1-k^{D}L_t^D}+\frac{\tau^{y}Y_{t+1}-G_{t+1}+k^{D}D^{c}L_{t+1}^D}{(1+r)\left(1-k^{D}L_{t+1}^D\right)}+\\ \frac{\tau^{y}Y_{t+2}-G_{t+2}+k^{D}D^{c}L_{t+2}^D}{(1+r)^2\left(1-k^{D}L_{t+2}^D\right)}+\frac{\tau^{y}Y_{t+3}-G_{t+3}+k^{D}D^{c}L_{t+3}^D}{(1+r)^3\left(1-k^{D}L_{t+3}^D\right)}+\ldots+\\ \frac{\tau^{y}Y_{t+T}-G_{t+T}+k^{D}D^{c}L_{t+T}^D}{(1+r)^T\left(1-k^{D}L_{t+T}^D\right)}+\lim_{T\to\infty}\frac{B_{t+T+1}}{(1+r)^{T+1}}\end{array}\right\},$$

which in equilibrium becomes (2.23). Finally, for the deterministic steady state, the transversality condition imposes:

$$(1+r)\,B_0 - \frac{1+r}{r}\left(\frac{\tau^y\overline{Y} - \overline{G} + k^D D^c \overline{L^D}}{1 - k^D \overline{L^D}}\right) = 0.$$

Note that if $B_0 = \overline{B}$, as we assume in our numerical analysis, then the last equation simplifies as follows:

$$\begin{aligned}
(1+r)\,\overline{B} - \frac{1+r}{r}\left(\frac{\tau^y\overline{Y} - \overline{G} + k^D D^c \overline{L^D}}{1 - k^D \overline{L^D}}\right) &= 0 \Leftrightarrow \\
r\left(1 - k^D \overline{L^D}\right)\overline{B} &= \tau^y\overline{Y} - \overline{G} + k^D D^c \overline{L^D}.
\end{aligned}$$

For large values of γ and thus good approximations of the indicator function, this simplifies to:

$$r\overline{B} = \tau^y\overline{Y} - \overline{G},$$

which is the usual constraint that should hold in steady state.

2.D Deterministic steady state under debt-based sanction

Under the debt constraint the equilibrium conditions are given by (2.17) and (2.18). Hence, a deterministic steady state has to satisfy the combination

$$\begin{aligned}
1 &= \beta_g\left[1 + r + k^B\overline{L^B} + \gamma k^B\left(\overline{B} - B^c\right)\overline{L^B}(1 - \overline{L^B})\right], \\
0 &= r\overline{B} + \overline{G} - \tau^y\overline{Y} + k^B\left(\overline{B} - B^c\right)\overline{L^B}
\end{aligned}$$

and thus

$$\begin{aligned}
\beta_g^{-1} - (1+r) &= k^B\overline{L^B}\left(1 + \gamma\left(\overline{B} - B^c\right)(1 - \overline{L^B})\right), \\
\overline{L^B} &= \frac{\tau^y\overline{Y} - r\overline{B} - \overline{G}}{k^B\left(\overline{B} - B^c\right)}.
\end{aligned} \tag{2.32}$$

Eliminating $\overline{L^B}$ gives

$$\begin{aligned}
\beta_g^{-1} - (1+r) &= k^B\frac{\tau^y\overline{Y} - r\overline{B} - \overline{G}}{k^B\left(\overline{B} - B^c\right)}\left(1 + \gamma\left(\overline{B} - B^c\right)\left(1 - \frac{\tau^y\overline{Y} - r\overline{B} - \overline{G}}{k^B\left(\overline{B} - B^c\right)}\right)\right) \\
&= \frac{\tau^y\overline{Y} - r\overline{B} - \overline{G}}{\overline{B} - B^c}\left(1 + \gamma\left(\overline{B} - B^c\right) - \gamma\frac{\tau^y\overline{Y} - r\overline{B} - \overline{G}}{k^B}\right).
\end{aligned} \tag{2.33}$$

Or, instead, using (2.16) to eliminate $\overline{G}$, in the previous system of equations, we obtain:

$$\beta_g^{-1} - (1+r) = \frac{k^B \left\{ \left[1+\exp\left(-\gamma\left(\overline{B}-B^c\right)\right)\right] * \left[1+\gamma\left(\overline{B}-B^c\right)\right] - \gamma\left(\overline{B}-B^c\right) \right\}}{\left[1+\exp\left(-\gamma\left(\overline{B}-B^c\right)\right)\right]^2}.$$

This equation determines $\overline{B}$. We can substitute it into (2.32) to find then $\overline{G}$. Moreover, if we pass the denominator multiplying the left-hand side of the previous equation we get:

$$\left[\beta_g^{-1} - (1+r)\right] * \left[1+\exp\left(-\gamma\left(\overline{B}-B^c\right)\right)\right]^2 = k^B \left\{1+\exp\left(-\gamma\left(\overline{B}-B^c\right)\right) * \left[1+\gamma\left(\overline{B}-B^c\right)\right]\right\}.$$

The derivative of the left-hand side of this expression is:

$$\left[\beta_g^{-1} - (1+r)\right] * 2\left[1+\exp\left(-\gamma\left(\overline{B}-B^c\right)\right)\right] * \left[-\gamma\exp\left(-\gamma\left(\overline{B}-B^c\right)\right)\right] < 0,$$

if $\beta_g < \beta_w$, $\gamma >> 0$ and $\overline{B} < B^c$. Further, the derivative of the right-hand side is:

$$k^B \left\{-\gamma\exp\left(-\gamma\left(\overline{B}-B^c\right)\right) * \left[1+\gamma\left(\overline{B}-B^c\right)\right] + \gamma\exp\left(-\gamma\left(\overline{B}-B^c\right)\right)\right\} > 0,$$

using the same assumptions as before. Therefore, this steady state is unique.

2.E Deterministic steady state under deficit-based sanction

Under the primary deficit-based constraint the equilibrium conditions are given by (2.21) and (2.22). Hence, a deterministic steady state has to satisfy

$$\begin{aligned} 1 &= \beta_g k^D \left[\overline{L^D} + \gamma\left(\overline{D}-D^c\right)\overline{L^D}(1-\overline{L^D})\right], \\ \overline{D} &= \overline{G} - \tau^y\overline{Y} + k^D\left(\overline{D}-D^c\right)\overline{L^D} \end{aligned}$$

and thus

$$\beta_g^{-1} = k^D\overline{L^D}\left(1+\gamma\left(\overline{D}-D^c\right)(1-\overline{L^D})\right), \quad (2.34)$$

$$\overline{L^D} = \frac{\tau^y\overline{Y}+\overline{D}-\overline{G}}{k^D\left(\overline{D}-D^c\right)}. \quad (2.35)$$

Eliminating $\overline{L^D}$ gives

$$\beta_g^{-1} = k^D\frac{\tau^y\overline{Y}+\overline{D}-\overline{G}}{k^D\left(\overline{D}-D^c\right)}\left(1+\gamma\left(\overline{D}-D^c\right)\left(1-\frac{\tau^y\overline{Y}+\overline{D}-\overline{G}}{k^D\left(\overline{D}-D^c\right)}\right)\right) \Leftrightarrow \quad (2.36)$$

$$\beta_g^{-1} = \frac{\tau^y\overline{Y}+\overline{D}-\overline{G}}{\overline{D}-D^c}\left(1+\gamma\left(\overline{D}-D^c\right)-\gamma\frac{\tau^y\overline{Y}+\overline{D}-\overline{G}}{k^D}\right). \quad (2.37)$$

Instead, if we use (2.20) in (2.35) and eliminate $\overline{G}$ in that system of equations, we arrive at:

$$\beta_g^{-1} = \frac{k^D \left\{ \left[1+\exp\left(-\gamma\left(\overline{D}-D^c\right)\right)\right] * \left[1+\gamma\left(\overline{D}-D^c\right)\right] - \gamma\left(\overline{D}-D^c\right) \right\}}{\left[1+\exp\left(-\gamma\left(\overline{D}-D^c\right)\right)\right]^2}.$$

This determines $\overline{D}$. We can substitute the solution into (2.35) to find $\overline{G}$. Moreover, If we pass the denominator multiplying the left-hand side of the previous equation we get:

$$\beta_g^{-1}\left[1+\exp\left(-\gamma\left(\overline{D}-D^c\right)\right)\right]^2 = k^D\left\{1+\exp\left(-\gamma\left(\overline{D}-D^c\right)\right)*\left[1+\gamma\left(\overline{D}-D^c\right)\right]\right\}.$$

The derivative of the left-hand side of this expression is:

$$\beta_g^{-1}*2\left[1+\exp\left(-\gamma\left(\overline{D}-D^c\right)\right)\right]*\left[-\gamma\exp\left(-\gamma\left(\overline{D}-D^c\right)\right)\right]<0,$$

if $\gamma >> 0$ and $\overline{D} < D^c$. Further, the derivative of the right-hand side is:

$$k^D\left\{-\gamma\exp\left(-\gamma\left(\overline{D}-D^c\right)\right)*\left[1+\gamma\left(\overline{D}-D^c\right)\right]+\gamma\exp\left(-\gamma\left(\overline{D}-D^c\right)\right)\right\}>0,$$

using the same assumptions as before. Therefore, this steady state is unique.

2.F Stochastic steady state with debt-based sanction

For the partisan government that faces debt-based sanction, the solution has to fulfill the system of equations given by the Euler equation (2.18) when $\gamma < \infty$, and the budget constraint (2.7) approximated by (2.16):

$$G_t = B_{t+1}-(1+r)B_t+\tau^y Y_t - k^B\left(B_t-B^c\right)L_t^B. \tag{2.38}$$

Hence, substituting (2.5) and (2.38) into the Euler equation (2.18) for $s=t$ implies

$$\mathrm{E}_t\left\{\begin{array}{c} u'\left(B_{t+1}-(1+r)B_t+\tau^y Y_t-k^B\left(B_t-B^c\right)L_t^B\right)- \\ \beta_g\left[\begin{array}{c} 1+r+k^B L_{t+1}^B+ \\ \gamma k^B\left(B_{t+1}-B^c\right)\left(L_{t+1}^B-\left(L_{t+1}^B\right)^2\right)\end{array}\right]u'\left[\begin{array}{c} B_{t+2}-(1+r)B_{t+1}+ \\ \tau^y Y_{t+1}-k^B\left(B_{t+1}-B^c\right)L_{t+1}^B\end{array}\right]\end{array}\right\}=0, \tag{2.39}$$

where we have used that

$$\partial L_{t+1}^B/\partial B_{t+1} = \gamma\left(L_{t+1}^B-\left(L_{t+1}^B\right)^2\right). \tag{2.40}$$

Thus (2.39) is of the format:

$$\mathrm{E}_t\left[g\left(B_t,B_{t+1},B_{t+2},Y_t,Y_{t+1}\right)\right]=0,$$

where $g(.)$ is a function implicitly defined by (2.39). Hence, by using (2.25) the unknown function $h(\cdot)$ satisfies

$$\begin{aligned} \mathrm{E}_t\left[q\left(B_t,Y_t,\sigma_\varepsilon\right)\right] &= \mathrm{E}_t\left\{g\left(B_t,h\left(B_t,Y_t,\sigma_\varepsilon\right),h\left(h\left(B_t,Y_t,\sigma_\varepsilon\right),Y_{t+1},\sigma_\varepsilon\right),Y_t,Y_{t+1}\right)\right\}=0 \Rightarrow \\ \mathrm{E}_t\left\{q\left(B_t,Y_t,\sigma_\varepsilon\right)\right] &= \mathrm{E}_t\left\{g\left(\begin{array}{c} B_t,h\left(B_t,Y_t,\sigma_\varepsilon\right), \\ h\left(h\left(B_t,Y_t,\sigma_\varepsilon\right),\rho Y_t+(1-\rho)\overline{Y}+\sigma_\varepsilon\varepsilon_{t+1},\sigma_\varepsilon\right), \\ Y_t,\rho Y_t+(1-\rho)\overline{Y}+\sigma_\varepsilon\varepsilon_{t+1}\end{array}\right)\right\}=0, \end{aligned}$$

where we have defined the new function $q\left(B_t,Y_t,\sigma_\varepsilon\right)$.

For $\sigma_\varepsilon = 0$, $B_{t+2} = B_{t+1} = B_t = \overline{B}$ and $Y_{t+1} = Y_t = \overline{Y}$, (2.26) becomes:

$$\mathrm{E}_t\left[q\left(\overline{B},\overline{Y},0\right)\right] = \mathrm{E}_t\left[g\left(\overline{B},\overline{B},\overline{B},\overline{Y},\overline{Y}\right)\right] = 0, \tag{2.41}$$

where we have used that $h\left(\overline{B},\overline{Y},0\right) = \overline{B}$ and $h\left(h\left(\overline{B},\overline{Y},0\right),\overline{Y},0\right) = h\left(\overline{B},\overline{Y},0\right) = \overline{B}$. Because $u' > 0$, $\mathrm{E}_t\left[q\left(\overline{B},\overline{Y},0\right)\right] = 0$ is equivalent to

$$\beta_g\left[1 + r + k^B\overline{L^B} + \gamma k^B\left(\overline{B} - B^c\right)\left(\overline{L^B} - \overline{L^B}^2\right)\right] = 1, \tag{2.42}$$

where $\overline{L^B}$ and u' are evaluated at the point $\left(\overline{B},\overline{Y},0\right)$. This expression yields a solution for $\overline{B}$. $\overline{B}$ and $\overline{Y}$ correspond to the average value of B_t and Y_t, since they are computed when $\sigma_\varepsilon = 0$. Thus, using (2.5) we know that $E_0\{Y_t\} = \widehat{Y} = \overline{Y}$ and $E_0\left\{\left(Y_t - \overline{Y}\right)^2\right\}$ =Var$(Y_t) = \frac{\sigma_\varepsilon^2}{1-\rho^2}$.

We find an approximate solution of (2.25) by taking first- and second-order Taylor expansions of (2.26)

2.F.1 First-order approximation of (2.26)

A first-order Taylor expansion of (2.26) yields:

$$\mathrm{E}_t\left[q\left(B_t,Y_t,\sigma_\varepsilon\right)\right] \simeq \mathrm{E}_t\left[q\left(\overline{B},\overline{Y},0\right) + q_B\left(\overline{B},\overline{Y},0\right)\widetilde{B}_t + q_Y\left(\overline{B},\overline{Y},0\right)\widetilde{Y}_t + q_{\sigma_\varepsilon}\left(\overline{B},\overline{Y},0\right)\sigma_\varepsilon\right] = 0, \tag{2.43}$$

where $q_B \equiv \frac{\partial q}{\partial B_t}$, $q_Y \equiv \frac{\partial q}{\partial Y_t}$, and $q_{\sigma_\varepsilon} \equiv \frac{\partial q}{\partial \sigma_\varepsilon}$.

From (2.41), we already know that $\mathrm{E}_t\left[q\left(\overline{B},\overline{Y},0\right)\right] = 0$. Thus, since we consider approximations of (2.43) around the point $\left(\overline{B},\overline{Y},0\right)$ the remaining unknown coefficients of that first-order approximation are found by solving

$$\mathrm{E}_t\left[q_B\left(\overline{B},\overline{Y},0\right)\widetilde{B}_t + q_{Y_t}\left(\overline{B},\overline{Y},0\right)\widetilde{Y}_t + q_{\sigma_\varepsilon}\left(\overline{B},\overline{Y},0\right)\sigma_\varepsilon\right] = 0.$$

This equation implies a system of three equations with three unknown coefficients related to the effects of B_t, Y_t and σ_ε in (2.25).

Derivation of the first order partial derivatives q_B, q_{Y_t}, and q_{σ_ε}

In this section, we compute q_B, q_{Y_t}, and q_{σ_ε}. We start by computing the partial derivatives of (2.25) with respect to $B_t, Y_t, \sigma_\varepsilon$:

First-order partial derivatives of (2.25) Differentiate B_{t+1} with respect to B_t:

$$\frac{\partial B_{t+1}}{\partial B_t} = \frac{\partial h\left(B_t,Y_t,\sigma_\varepsilon\right)}{\partial B_t} = \frac{\partial h_t}{\partial B_t} \equiv h_1^B, \tag{2.44}$$

where $h_t \equiv h\left(B_t,Y_t,\sigma_\varepsilon\right)$. Further, the derivative of B_{t+1} with respect to Y_t is

$$\frac{\partial B_{t+1}}{\partial Y_t} = \frac{\partial h\left(B_t,Y_t,\sigma_\varepsilon\right)}{\partial Y_t} = \frac{\partial h_t}{\partial Y_t} \equiv h_1^Y. \tag{2.45}$$

Finally, the derivative of B_{t+1} with respect to σ_ε is

$$\frac{\partial B_{t+1}}{\partial \sigma_\varepsilon} = \frac{\partial h\left(B_t,Y_t,\sigma_\varepsilon\right)}{\partial \sigma_\varepsilon} = \frac{\partial h_t}{\partial \sigma_\varepsilon} \equiv h_1^\sigma. \tag{2.46}$$

Derivation of the first-order partial derivative of B_{t+2} with respect to B_t, Y_t and σ_ε From 2.25, we can write B_{t+2} as

$$B_{t+2} = h\left(B_{t+1}, Y_{t+1}, \sigma_\varepsilon\right) = h\left(h\left(B_t, Y_t, \sigma_\varepsilon\right), \rho Y_t + (1-\rho)\overline{Y} + \sigma_\varepsilon \varepsilon_{t+1}, \sigma_\varepsilon\right). \tag{2.47}$$

Hence, differentiating (2.47) with respect to B_t, we obtain:

$$\begin{aligned}
\frac{\partial B_{t+2}}{\partial B_t} &= \frac{\partial h\left(B_{t+1}, Y_{t+1}, \sigma_\varepsilon\right)}{\partial B_t} = \frac{\partial h_{t+1}}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial B_t} + \frac{\partial h_{t+1}}{\partial Y_{t+1}}\frac{\partial Y_{t+1}}{\partial B_t} + \frac{\partial h_{t+1}}{\partial \sigma_\varepsilon}\frac{\partial \sigma_\varepsilon}{\partial B_t} \\
&= \frac{\partial h_{t+1}}{\partial B_{t+1}}\frac{\partial h_t}{\partial B_t} = \left(\frac{\partial h_t}{\partial B_t}\right)^2 = \left(h_1^B\right)^2,
\end{aligned}$$

where we have used (2.44), and the facts that $(i)\frac{\partial Y_{t+1}}{\partial B_t} = \frac{\partial \sigma_\varepsilon}{\partial B_t} = 0$, and (ii) the partials $\frac{\partial h_{t+1}}{\partial B_{t+1}}$ and $\frac{\partial h_t}{\partial B_t}$ are evaluated at the same point $\left(\overline{B}, \overline{Y}, 0\right)$.

Differentiate of B_{t+2} with respect to Y_t:

$$\begin{aligned}
\frac{\partial B_{t+2}}{\partial Y_t} &= \frac{\partial h\left(B_{t+1}, Y_{t+1}, \sigma_\varepsilon\right)}{\partial Y_t} = \frac{\partial h_{t+1}}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial Y_t} + \frac{\partial h_{t+1}}{\partial Y_{t+1}}\frac{\partial Y_{t+1}}{\partial Y_t} + \frac{\partial h_{t+1}}{\partial \sigma_\varepsilon}\frac{\partial \sigma_\varepsilon}{\partial Y_t} \\
&= \frac{\partial h_{t+1}}{\partial B_{t+1}} * \frac{\partial h_t}{\partial Y_t} + \frac{\partial h_{t+1}}{\partial Y_{t+1}}\rho = h_1^B h_1^Y + h_1^Y \rho,
\end{aligned}$$

where we have used (2.5), (2.44) and (2.45).

Finally, differentiate B_{t+2} with respect to σ_ε:

$$\begin{aligned}
\frac{\partial B_{t+2}}{\partial \sigma_\varepsilon} &= \frac{\partial h\left(B_{t+1}, Y_{t+1}, \sigma_\varepsilon\right)}{\partial \sigma_\varepsilon} = \frac{\partial h_{t+1}}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial \sigma_\varepsilon} + \frac{\partial h_{t+1}}{\partial Y_{t+1}}\frac{\partial Y_{t+1}}{\partial \sigma_\varepsilon} + \frac{\partial h_{t+1}}{\partial \sigma_\varepsilon} \\
&= h_1^B \frac{\partial h_t}{\partial \sigma_\varepsilon} + h_1^Y \varepsilon_{t+1} + h_1^\sigma = h_1^B h_1^\sigma + h_1^Y \varepsilon_{t+1} + h_1^\sigma.
\end{aligned}$$

Derivation of q_B Taking the partial derivative of (2.39) with respect to B_t at the point $\left(\overline{B}, \overline{Y}, 0\right)$, and multiplying it by $\widetilde{B}_t$, yields:

$$\mathrm{E}_t\left[q_B \widetilde{B}_t\right] \simeq E_t \left\{ \left[\begin{array}{c}
u''\left(G_t\right) * \left(\frac{\partial B_{t+1}}{\partial B_t} - (1+r) - k^B \overline{L^B} - k^B\left(\overline{B} - B^c\right)\frac{\partial L_t^B}{\partial B_t}\right) - \\
\beta_g k^B \left[\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial B_t} + \frac{\partial B_{t+1}}{\partial B_t}\frac{\partial L_{t+1}^B}{\partial B_{t+1}} + \left(\overline{B} - B^c\right)\frac{\partial^2 L_{t+1}^B}{\partial^2 B_{t+1}}\frac{\partial B_{t+1}}{\partial B_t}\right] u'\left(G_{t+1}\right) \\
-\beta_g \left[\begin{array}{c} 1 + r + k^B \overline{L^B} + \\ \gamma k^B \left(\overline{B} - B^c\right)\overline{L^B}(1 - \overline{L^B}) \end{array} \right] u''\left(G_{t+1}\right) * \\
\left(\begin{array}{c} \frac{\partial B_{t+2}}{\partial B_t} - (1+r)\frac{\partial B_{t+1}}{\partial B_t} - k^B \overline{L^B}\frac{\partial B_{t+1}}{\partial B_t} \\ -k^B\left(\overline{B} - B^c\right)\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial B_t} \end{array} \right)
\end{array} \right] \widetilde{B}_t \right\},$$

where u' and $u''$are the first- and second-order derivatives of the utility function evaluated at the non-stochastic steady state. Now, we need to compute the terms $\frac{\partial L_t^B}{\partial B_t}$, $\frac{\partial L_{t+1}^B}{\partial B_{t+1}}$ and $\frac{\partial^2 L_{t+1}^B}{\partial^2 B_{t+1}}$.

Then, differentiating (2.40) once more with respect to B_{t+1} we have that

$$\frac{\partial^2 L_{t+1}^B}{\partial^2 B_{t+1}} = \gamma \left[\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\left(1 - L_{t+1}^B\right) - L_{t+1}^B \frac{\partial L_{t+1}^B}{\partial B_{t+1}}\right].$$

Substituting (2.40) in the last equation:

$$\begin{aligned}\frac{\partial^2 L^B_{t+1}}{\partial B^2_{t+1}} &= \gamma\left[\gamma L^B_{t+1}\left(1-L^B_{t+1}\right)\left(1-L^B_{t+1}\right)-L^B_{t+1}\gamma L^B_{t+1}\left(1-L^B_{t+1}\right)\right] \\ &= \gamma\left[\gamma L^B_{t+1}\left(1-L^B_{t+1}\right)\left(1-L^B_{t+1}-L^B_{t+1}\right)\right]=\gamma^2\left(L^B_{t+1}-3\left(L^B_{t+1}\right)^2+2\left(L^B_{t+1}\right)^3\right)\end{aligned} \quad (2.48)$$

Using the functional form of L^B, we can also represent (2.48) by

$$\frac{\partial^2 L^B_{t+1}}{\partial^2 B_{t+1}}=\frac{\gamma^2\exp\left(-\gamma\left(B_{t+1}-B^c\right)\right)*\left[-1+\exp\left(-\gamma\left(B_{t+1}-B^c\right)\right)\right]}{\left[1+\exp\left(-\gamma\left(B_{t+1}-B^c\right)\right)\right]^3}.$$

Therefore, using (2.40) and (2.48), combined with the expressions for $\frac{\partial B_{t+1}}{\partial B_t}$ and $\frac{\partial B_{t+2}}{\partial B_t}$, we write $\mathrm{E}_t\left[q_B\widetilde{B}_t\right]$ as

$$\mathrm{E}_t\left[q_B\widetilde{B}_t\right]\simeq\left[\begin{array}{c} u''*\left(h^B_1-\left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\overline{L^B}(1-\overline{L^B})\right)\right)- \\ \beta_g k^B\left[\begin{array}{c}2\gamma\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)h^B_1+ \\ \left(\overline{B}-B^c\right)\gamma^2\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right)h^B_1\end{array}\right]*u' \\ -\beta_g\left[1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\overline{L^B}(1-\overline{L^B})\right]* \\ u''\left(\left(h^B_1\right)^2-\left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\overline{L^B}(1-\overline{L^B})\right)h^B_1\right)\end{array}\right]\widetilde{B}_t,$$

where we exclude the expectations operator from the right-hand side of the previous equation, since all its elements are known in period t. Using (2.42), it follows that

$$\mathrm{E}_t\left[q_B\widetilde{B}_t\right]\simeq\left[\begin{array}{c} u''*\left(h^B_1-\left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)\right) \\ -\beta_g k^B\gamma\left[\begin{array}{c}2\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)+ \\ \gamma\left(\overline{B}-B^c\right)\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right)\end{array}\right]*u'h^B_1- \\ u''*\left(\left(h^B_1\right)^2-\left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)h^B_1\right)\end{array}\right]\widetilde{B}_t. \quad (2.49)$$

Derivation of q_{Y_t} Differentiating the partial derivative of (2.39) with respect to Y_t at the point $\left(\overline{B},\overline{Y},0\right)$ and multiplying it by $\widetilde{Y}_t$, yields:

$$\mathrm{E}_t\left[q_{Y_t}\widetilde{Y}_t\right]\simeq\mathrm{E}_t\left\{\left[\begin{array}{c}\left(\frac{\partial B_{t+1}}{\partial Y_t}+\tau^y\right)u''\left(G_t\right)- \\ \beta_g k^B\left[\frac{\partial L^B_{t+1}}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial Y_t}+\frac{\partial B_{t+1}}{\partial Y_t}\frac{\partial L^B_{t+1}}{\partial B_{t+1}}+\left(\overline{B}-B^c\right)\frac{\partial^2 L^B_{t+1}}{\partial^2 B_{t+1}}\frac{\partial B_{t+1}}{\partial Y_t}\right]u'\left(G_{t+1}\right) \\ -\beta_g\left[1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right]* \\ \left(\begin{array}{c}\frac{\partial B_{t+2}}{\partial Y_t}-\left(1+r+k^B\overline{L^B}\right)\frac{\partial B_{t+1}}{\partial Y_t} \\ +\tau^y\rho-k^B\left(\overline{B}-B^c\right)\frac{\partial L^B_{t+1}}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial Y_t}\end{array}\right)u''\left(G_{t+1}\right)\end{array}\right]\widetilde{Y}_t\right\},$$

since $\frac{\partial B_t}{\partial Y_t} = 0$ (see the government's budget constraint) and from (2.5), $\frac{\partial Y_{t+1}}{\partial Y_t} = \rho$. Using (2.40), (2.42), (2.48) and the expressions for $\frac{\partial B_{t+1}}{\partial Y_t}$ and $\frac{\partial B_{t+2}}{\partial Y_t}$, we write

$$\mathrm{E}_t\left[q_{Y_t}\widetilde{Y}_t\right] \simeq \left\{ \begin{array}{c} \left(h_1^Y + \tau^y\right) u'' - \\ \beta_g k^B \gamma \left[\begin{array}{c} 2\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right) + \\ \gamma\left(\overline{B} - B^c\right)\left(\overline{L^B} - 3\left(\overline{L^B}\right)^2 + 2\left(\overline{L^B}\right)^3\right) \end{array} \right] h_1^Y u' - \\ \left[\begin{array}{c} h_1^B h_1^Y + h_1^Y \rho + \tau^y \rho - \\ \left(1 + r + k^B\overline{L^B} + \gamma k^B\left(\overline{B} - B^c\right)\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right)\right) h_1^Y \end{array} \right] u'' \end{array} \right\} \widetilde{Y}_t. \tag{2.50}$$

Derivation of q_{σ_ε} Taking the derivative of (2.39) with respect to σ_ε at the point $\left(\overline{B}, \overline{Y}, 0\right)$ and multiplying it by σ_ε yields:

$$\mathrm{E}_t\left[q_{\sigma_\varepsilon}\sigma_\varepsilon\right] \simeq \mathrm{E}_t \left\{ \left[\begin{array}{c} \left(\frac{\partial B_{t+1}}{\partial \sigma_\varepsilon}\right) u''(G_t) - \\ \beta_g k^B \left[\frac{\partial L^B_{t+1}}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial \sigma_\varepsilon} + \frac{\partial B_{t+1}}{\partial \sigma_\varepsilon}\frac{\partial L^B_{t+1}}{\partial B_{t+1}} + \left(\overline{B} - B^c\right)\frac{\partial^2 L^B_{t+1}}{\partial^2 B_{t+1}}\frac{\partial B_{t+1}}{\partial \sigma_\varepsilon}\right] u'(G_{t+1}) \\ -\beta_g \left[1 + r + k^B\overline{L^B} + \gamma k^B\left(\overline{B} - B^c\right)\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right)\right] * \\ \left(\begin{array}{c} \frac{\partial B_{t+2}}{\partial \sigma_\varepsilon} - \left(1 + r + k^B\overline{L^B}\right)\frac{\partial B_{t+1}}{\partial \sigma_\varepsilon} - \\ k^B\left(\overline{B} - B^c\right)\frac{\partial L^B_{t+1}}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial \sigma_\varepsilon} + \tau^y \varepsilon_{t+1} \end{array} \right) u''(G_{t+1}) \end{array} \right] \sigma_\varepsilon \right\},$$

since $\frac{\partial B_t}{\partial \sigma_\varepsilon} = \frac{\partial Y_t}{\partial \sigma_\varepsilon} = 0$. Using (2.42), (2.46) and the expression for $\frac{\partial B_{t+2}}{\partial \sigma_\varepsilon}$, we can rewrite the last expression as

$$\mathrm{E}_t\left[q_{\sigma_\varepsilon}\sigma_\varepsilon\right] \simeq \mathrm{E}_t \left\{ \left[\begin{array}{c} h_1^\sigma u'' - \beta_g k^B \gamma \left[\begin{array}{c} 2\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right) + \\ \gamma\left(\overline{B} - B^c\right)\left(\overline{L^B} - 3\left(\overline{L^B}\right)^2 + 2\left(\overline{L^B}\right)^3\right) \end{array} \right] h_1^\sigma u' \\ - \left(\begin{array}{c} h_1^B h_1^\sigma + h_1^Y \varepsilon_{t+1} + h_1^\sigma + \tau^y \varepsilon_{t+1} - \\ \left[1 + r + k^B\overline{L^B} + \gamma k^B\left(\overline{B} - B^c\right)\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right)\right] h_1^\sigma \end{array} \right) u'' \end{array} \right] \sigma_\varepsilon \right\}. \tag{2.51}$$

First-order Taylor approximation of (2.43)

With $q\left(\overline{B}, \overline{Y}, 0\right) = 0$ and (2.49), (2.50), and (2.51), we can write (2.43) as

$$
\begin{aligned}
A_1 \equiv & \left\{ \left[\begin{array}{c} \left(h_1^B - \left(1 + r + k^B \overline{L^B} + \gamma k^B \left(\overline{B} - B^c\right)\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right)\right)\right) u'' \\ -\beta_g k^B \gamma \left[\begin{array}{c} 2\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right) + \\ \gamma \left(\overline{B} - B^c\right)\left(\overline{L^B} - 3\left(\overline{L^B}\right)^2 + 2\left(\overline{L^B}\right)^3\right) \end{array} \right] h_1^B u' - \\ \left(\left(h_1^B\right)^2 - \left(1 + r + k^B \overline{L^B} + \gamma k^B \left(\overline{B} - B^c\right)\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right)\right) h_1^B\right) u'' \end{array} \right] \widetilde{B}_t \right. \\
& + \left[\begin{array}{c} \left(h_1^Y + \tau^y\right) u'' - \\ \beta_g k^B \gamma \left[\begin{array}{c} 2\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right) + \\ \gamma \left(\overline{B} - B^c\right)\left(\overline{L^B} - 3\left(\overline{L^B}\right)^2 + 2\left(\overline{L^B}\right)^3\right) \end{array} \right] h_1^Y u' - \\ \left(\begin{array}{c} h_1^B h_1^Y + h_1^Y \rho + \tau^y \rho - \\ \left(1 + r + k^B \overline{L^B} + \gamma k^B \left(\overline{B} - B^c\right)\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right)\right) h_1^Y u'' \end{array} \right) \end{array} \right] \widetilde{Y}_t + \\
& \left. h_1^\sigma \left[\begin{array}{c} u'' - \beta_g k^B \gamma \left[\begin{array}{c} 2\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right) + \\ \gamma \left(\overline{B} - B^c\right)\left(\overline{L^B} - 3\left(\overline{L^B}\right)^2 + 2\left(\overline{L^B}\right)^3\right) \end{array} \right] u' \\ - \left[\begin{array}{c} h_1^B + 1 - \\ \left[1 + r + k^B \overline{L^B} + \gamma k^B \left(\overline{B} - B^c\right)\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right)\right] \end{array} \right] u'' \end{array} \right] \sigma_\varepsilon \right\}
\end{aligned} \tag{2.52}
$$

where we have lost the terms in ε_{t+1} by taking expectations.

Then, because (2.26) must be equal to zero for any potential combination $\{B_t, Y_t, \sigma_\varepsilon\}$, it must be the case that the derivatives of any order of that expression must also be equal to zero. Hence the coefficients of B_t, Y_t and σ_ε in (2.52) should be zero. This allows us to compute the three unknown variables of that equation, namely h_1^B, h_1^Y, and h_1^σ.

Computation of h_1^B The coefficient of $\widetilde{B}_t$ in (2.52) is zero, hence

$$
\left[\begin{array}{c} -u'' * \left(h_1^B\right)^2 - u'' * \left(1 + r + k^B \overline{L^B} + \gamma k^B \left(\overline{B} - B^c\right)\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right)\right) + \\ \left[\begin{array}{c} u'' + u'' * \left(1 + r + k^B \overline{L^B} + \gamma k^B \left(\overline{B} - B^c\right)\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right)\right) - \\ \beta_g k^B \gamma \left[2\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right) + \gamma \left(\overline{B} - B^c\right)\left(\overline{L^B} - 3\left(\overline{L^B}\right)^2 + 2\left(\overline{L^B}\right)^3\right)\right] u' \end{array} \right] \left(h_1^B\right) \end{array} \right] = 0.
$$

Multiplying the last equation by $-\frac{\beta_g}{u''}$ and using (2.42) twice, we simplify it to

$$
\left[\begin{array}{c} \beta_g \left(h_1^B\right)^2 + 1 + \\ \left[\begin{array}{c} -\beta_g - 1 + \\ \frac{u'}{u''} \beta_g^2 k^B \gamma \left[2\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right) + \gamma \left(\overline{B} - B^c\right)\left(\overline{L^B} - 3\left(\overline{L^B}\right)^2 + 2\left(\overline{L^B}\right)^3\right)\right] \end{array} \right] \left(h_1^B\right) \end{array} \right] = 0, \tag{2.53}
$$

since u'' is always different from zero. Thus, the equation above is quadratic in h_1^B and by solving it, we obtain

$$h_1^B = \frac{\left\{ \left(\beta_g + 1 - \frac{u'}{u''}\beta_g^2 k^B \gamma \left[\begin{array}{c} 2\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right) + \\ \gamma\left(\overline{B} - B^c\right)\left(\overline{L^B} - 3\left(\overline{L^B}\right)^2 + 2\left(\overline{L^B}\right)^3\right) \end{array} \right]\right) \pm \sqrt{\left(\beta_g + 1 - \frac{u'}{u''}\beta_g^2 k^B \gamma \left[\begin{array}{c} 2\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right) + \\ \gamma\left(\overline{B} - B^c\right)\left(\overline{L^B} - 3\left(\overline{L^B}\right)^2 + 2\left(\overline{L^B}\right)^3\right) \end{array} \right]\right)^2 - 4\beta_g} \right\}}{2\beta_g} \tag{2.54}$$

Equation (2.54) features two solutions (roots). To ensure a non-explosive path for the public debt, we need to pick the solution that is smaller than 1 in absolute value.

Computation of h_1^Y The coefficient of $\widetilde{Y}_t$ in (2.52) equals zero, hence

$$\left[h_1^Y \left[\begin{array}{c} \left(1 - \rho - h_1^B + 1 + r + k^B\overline{L^B} + \gamma k^B\left(\overline{B} - B^c\right)\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right)\right) u'' - \\ \beta_g k^B \gamma \left[2\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right) + \gamma\left(\overline{B} - B^c\right)\left(\overline{L^B} - 3\left(\overline{L^B}\right)^2 + 2\left(\overline{L^B}\right)^3\right)\right] u' \end{array} \right] \begin{array}{c} \\ \\ +\tau^y(1-\rho)u'' \end{array} \right] = 0.$$

Hence,

$$h_1^Y = \frac{\tau^y(\rho - 1)u''}{\left[\begin{array}{c} \left(2 + r + k^B\overline{L^B} + \gamma k^B\left(\overline{B} - B^c\right)\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right) - \rho - h_1^B\right) u'' - \\ \beta_g k^B \gamma \left[2\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right) + \gamma\left(\overline{B} - B^c\right)\left(\overline{L^B} - 3\left(\overline{L^B}\right)^2 + 2\left(\overline{L^B}\right)^3\right)\right] u' \end{array} \right]}. \tag{2.55}$$

By plugging in the stable root of h_1^B (2.54), we thus find the solution for h_1^Y.

Computation of h_1^σ Because the coefficient of σ_ε in (2.52) is also zero, we have

$$h_1^\sigma = 0. \tag{2.56}$$

First-order approximation of B_{t+1}

Using (2.54), (2.55), and (2.56), the first order approximation of the true non-linear solution of B_{t+1} around the point $\left(\overline{B}, \overline{Y}, 0\right)$ becomes

$$B_{t+1} = h\left(B_t, Y_t, \sigma_\varepsilon\right) \approx h\left(\overline{B}, \overline{Y}, 0\right) + h_1^B \widetilde{B}_t + h_1^Y \widetilde{Y}_t. \tag{2.57}$$

2.F.2 Second-order approximation of (2.26)

The Second-Order Taylor Expansion of (2.26) around the point $\left(\overline{B}, \overline{Y}, 0\right)$ is

$$\mathrm{E}_t\left\{q\left(B_t, Y_t, \sigma_\varepsilon\right)\right\} \simeq \tfrac{1}{2}\mathrm{E}_t\left[\begin{array}{c} q_{BB}\widetilde{B}_t^2 + q_{YY}\widetilde{Y}_t^2 + q_{\sigma_\varepsilon\sigma_\varepsilon}\sigma_\varepsilon^2 + \\ 2q_{BY}\widetilde{B}_t\widetilde{Y}_t + 2q_{B\sigma_\varepsilon}\widetilde{B}_t\sigma_\varepsilon + 2q_{Y\sigma_\varepsilon}\widetilde{Y}_t\sigma_{\varepsilon\varepsilon}^2 \end{array}\right] = 0, \tag{2.58}$$

where all partial derivatives are evaluated at $\left(\overline{B}, \overline{Y}, 0\right)$ and where we have used that $\mathrm{E}_t\left[q + q_B\widetilde{B}_t + q_Y\widetilde{Y}_t + q_{\sigma_\varepsilon}\sigma_\varepsilon\right] = 0$, where $q \equiv q\left(\overline{B}, \overline{Y}, 0\right)$.

Derivation of the second-order partial derivatives q_{BB}, q_{BY}, $q_{B\sigma_\varepsilon}$, q_{YY}, $q_{Y\sigma_\varepsilon}$, and $q_{\sigma_\varepsilon\sigma_\varepsilon}$

For future use, we first calculate the second-order partial derivatives of (2.25) and (2.47) with respect to B_t, Y_t and σ_ε:

Derivation of the second-order partial derivative of (2.25) with respect to $B_t, Y_t, \sigma_\varepsilon$ We have:

$$\frac{\partial^2 B_{t+1}}{\partial^2 B_t} = \frac{\partial\left(\partial h_t/\partial B_t\right)}{\partial B_t} = \frac{\partial^2 h_t}{\partial^2 B_t} \equiv h_{11}^B. \tag{2.59}$$

- Further,

$$\frac{\partial^2 B_{t+1}}{\partial B_t \partial Y_t} = \frac{\partial^2 h_t}{\partial B_t \partial Y_t} \equiv h_{11}^{BY}. \tag{2.60}$$

- Further,

$$\frac{\partial^2 B_{t+1}}{\partial Y_t^2} = \frac{\partial^2 h_t}{\partial^2 Y_t} \equiv h_{11}^{Y}. \tag{2.61}$$

- Further,

$$\frac{\partial^2 B_{t+1}}{\partial B_t \partial \sigma_\varepsilon} = \frac{\partial^2 h_t}{\partial B_t \partial \sigma_\varepsilon} \equiv h_{11}^{B\sigma}. \tag{2.62}$$

- Further,

$$\frac{\partial^2 B_{t+1}}{\partial Y_t \partial \sigma_\varepsilon} = \frac{\partial^2 h_t}{\partial Y_t \partial \sigma_\varepsilon} \equiv h_{11}^{Y\sigma}. \tag{2.63}$$

- Finally,

$$\frac{\partial^2 B_{t+1}}{\partial^2 \sigma_\varepsilon} = \frac{\partial^2 h_t}{\partial^2 \sigma_\varepsilon} \equiv h_{11}^{\sigma}. \tag{2.64}$$

Derivation of the second-order partial derivatives of (2.47) with respect to $B_t, Y_t, \sigma_\varepsilon$

- Computation of $\frac{\partial^2 B_{t+2}}{\partial^2 B_t}$:

$$\begin{aligned}\frac{\partial^2 B_{t+2}}{\partial^2 B_t} &= \frac{\partial\left(\frac{\partial B_{t+2}}{\partial B_t}\right)}{\partial B_t} = \frac{\partial\left(\frac{\partial B_{t+2}}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial B_t}\right)}{\partial B_t} = \frac{\partial\left(\frac{\partial h_{t+1}}{\partial B_{t+1}}\right)}{\partial B_t} * \frac{\partial B_{t+1}}{\partial B_t} + \frac{\partial h_{t+1}}{\partial B_{t+1}} * \frac{\partial\left(\frac{\partial B_{t+1}}{\partial B_t}\right)}{\partial B_t} \\ &= \left[\frac{\partial\left(\frac{\partial h_{t+1}}{\partial B_{t+1}}\right)}{\partial B_{t+1}} * \frac{\partial B_{t+1}}{\partial B_t}\right] * \frac{\partial B_{t+1}}{\partial B_t} + \frac{\partial h_{t+1}}{\partial B_{t+1}} * \frac{\partial^2 B_{t+1}}{\partial^2 B_t},\end{aligned}$$

since the first order derivative of Y_{t+1} and σ_ε with respect to B_t is equal to zero. The second equality maked use of the chain rule. Using (2.44) and (2.59), we can rewrite it as

$$\frac{\partial^2 B_{t+2}}{\partial^2 B_t} = \left[\frac{\partial^2 h_{t+1}}{\partial^2 B_{t+1}} * \frac{\partial B_{t+1}}{\partial B_t}\right] * \frac{\partial B_{t+1}}{\partial B_t} + h_1^B * h_{11}^B = h_{11}^B\left(h_1^B\right)^2 + h_1^B h_{11}^B.$$

- Computation of $\frac{\partial^2 B_{t+2}}{\partial B_t \partial Y_t}$:

$$\begin{aligned}\frac{\partial^2 B_{t+2}}{\partial B_t \partial Y_t} &= \frac{\partial}{\partial B_t}\left(\frac{\partial h\left(B_{t+1}, Y_{t+1}, \sigma_\varepsilon\right)}{\partial Y_t}\right) \Rightarrow \\ \frac{\partial^2 B_{t+2}}{\partial B_t \partial Y_t} &= \frac{\partial}{\partial B_t}\left(\frac{\partial h_{t+1}}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial Y_t} + \frac{\partial h_{t+1}}{\partial Y_{t+1}}\frac{\partial Y_{t+1}}{\partial Y_t} + \frac{\partial h_{t+1}}{\partial \sigma_\varepsilon}\frac{\partial \sigma_\varepsilon}{\partial Y_t}\right) \Rightarrow \\ &= \frac{\partial\left(\frac{\partial h_{t+1}}{\partial B_{t+1}}\right)}{\partial B_t}\frac{\partial B_{t+1}}{\partial Y_t} + \frac{\partial h_{t+1}}{\partial B_{t+1}}\frac{\partial\left(\frac{\partial B_{t+1}}{\partial Y_t}\right)}{\partial B_t} + \frac{\partial\left(\frac{\partial h_{t+1}}{\partial Y_{t+1}}\right)}{\partial B_t}\frac{\partial Y_{t+1}}{\partial Y_t} + \frac{\partial h_{t+1}}{\partial Y_{t+1}}\frac{\partial\left(\frac{\partial Y_{t+1}}{\partial Y_t}\right)}{\partial B_t} \\ &= \frac{\partial\left(\frac{\partial h_{t+1}}{\partial B_{t+1}}\right)}{\partial B_t}\frac{\partial B_{t+1}}{\partial Y_t} + \frac{\partial h_{t+1}}{\partial B_{t+1}}\frac{\partial\left(\frac{\partial B_{t+1}}{\partial Y_t}\right)}{\partial B_t} + \frac{\partial\left(\frac{\partial h_{t+1}}{\partial Y_{t+1}}\right)}{\partial B_t}\rho \\ &= \left[\frac{\partial^2 h_{t+1}}{\partial^2 B_{t+1}}\frac{\partial B_{t+1}}{\partial B_t} + \frac{\partial^2 h_{t+1}}{\partial B_{t+1}\partial Y_{t+1}}\frac{\partial Y_{t+1}}{\partial B_t} + \frac{\partial^2 h_{t+1}}{\partial B_{t+1}\partial \sigma_\varepsilon}\frac{\partial \sigma_\varepsilon}{\partial B_t}\right]\frac{\partial B_{t+1}}{\partial Y_t} + \\ &\quad h_1^B * \left[\frac{\partial^2 B_{t+1}}{\partial Y_t \partial B_t}\frac{\partial B_t}{\partial B_t} + \frac{\partial^2 B_{t+1}}{\partial^2 Y_t}\frac{\partial Y_t}{\partial B_t} + \frac{\partial^2 B_{t+1}}{\partial Y_t \partial \sigma_\varepsilon}\frac{\partial \sigma_\varepsilon}{\partial B_t}\right] + \\ &\quad \rho * \left[\frac{\partial^2 h_{t+1}}{\partial Y_{t+1}\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial B_t} + \frac{\partial^2 h_{t+1}}{\partial^2 Y_{t+1}}\frac{\partial Y_{t+1}}{\partial B_t} + \frac{\partial^2 h_{t+1}}{\partial Y_{t+1}\partial \sigma_\varepsilon}\frac{\partial \sigma_\varepsilon}{\partial B_t}\right] \\ &= h_{11}^B h_1^B h_1^Y + (1+\rho)\, h_1^B h_{11}^{BY},\end{aligned}$$

using $\frac{\partial \sigma_\varepsilon}{\partial Y_t} = 0$, $\frac{\partial Y_{t+1}}{\partial Y_t} = \rho$, (2.44), and (2.60).

- Computation of $\frac{\partial^2 B_{t+2}}{\partial B_t \partial \sigma_\varepsilon}$: knowing (2.44) and (2.62), we compute this derivative as

$$\begin{aligned}\frac{\partial^2 B_{t+2}}{\partial B_t \partial \sigma_\varepsilon} &= \frac{\partial}{\partial B_t}\left(\frac{\partial h\left(B_{t+1}, Y_{t+1}, \sigma_\varepsilon\right)}{\partial \sigma_\varepsilon}\right) = \frac{\partial}{\partial B_t}\left(\frac{\partial h_{t+1}}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial \sigma_\varepsilon} + \frac{\partial h_{t+1}}{\partial Y_{t+1}}\frac{\partial Y_{t+1}}{\partial \sigma_\varepsilon} + \frac{\partial h_{t+1}}{\partial \sigma_\varepsilon}\frac{\partial \sigma_\varepsilon}{\partial \sigma_\varepsilon}\right) \\ &= \left[\begin{array}{c}\frac{\partial\left(\frac{\partial h_{t+1}}{\partial B_{t+1}}\right)}{\partial B_t} * \frac{\partial B_{t+1}}{\partial \sigma_\varepsilon} + \frac{\partial h_{t+1}}{\partial B_{t+1}} * \frac{\partial\left(\frac{\partial B_{t+1}}{\partial \sigma_\varepsilon}\right)}{\partial B_t} + \frac{\partial\left(\frac{\partial h_{t+1}}{\partial Y_{t+1}}\right)}{\partial B_t} * \frac{\partial Y_{t+1}}{\partial \sigma_\varepsilon} \\ + \frac{\partial h_{t+1}}{\partial Y_{t+1}} * \frac{\partial\left(\frac{\partial Y_{t+1}}{\partial \sigma_\varepsilon}\right)}{\partial B_t} + \frac{\partial\left(\frac{\partial h_{t+1}}{\partial \sigma_\varepsilon}\right)}{\partial B_t} + \frac{\partial h_{t+1}}{\partial \sigma_\varepsilon} * \frac{\partial(1)}{\partial B_t}\end{array}\right] \\ &= \left[\begin{array}{c}\frac{\partial^2 h_{t+1}}{\partial^2 B_{t+1}} * \frac{\partial B_{t+1}}{\partial B_t} * \frac{\partial B_{t+1}}{\partial \sigma_\varepsilon} + \frac{\partial h_{t+1}}{\partial B_{t+1}} * \frac{\partial^2 B_{t+1}}{\partial B_t \partial \sigma_\varepsilon} + \frac{\partial^2 h_{t+1}}{\partial Y_{t+1}\partial B_{t+1}} * \frac{\partial B_{t+1}}{\partial B_t} * \frac{\partial Y_{t+1}}{\partial \sigma_\varepsilon} \\ + \frac{\partial h_{t+1}}{\partial Y_{t+1}} * \frac{\partial \varepsilon_{t+1}}{\partial B_t} + \frac{\partial^2 h_{t+1}}{\partial \sigma_\varepsilon \partial B_{t+1}} * \frac{\partial B_{t+1}}{\partial B_t}\end{array}\right] \\ &= h_{11}^B h_1^B h_1^\sigma + h_1^B h_{11}^{B\sigma} + h_1^B h_{11}^{BY}\varepsilon_{t+1} + h_1^B h_{11}^{B\sigma} = h_{11}^B h_1^B h_1^\sigma + 2h_1^B h_{11}^{B\sigma} + h_1^B h_{11}^{BY}\varepsilon_{t+1}.\end{aligned}$$

- Computation of $\frac{\partial^2 B_{t+2}}{\partial^2 Y_t}$:

$$\begin{aligned}
\frac{\partial^2 B_{t+2}}{\partial^2 Y_t} &= \frac{\partial}{\partial Y_t}\left(\frac{\partial h\left(B_{t+1}, Y_{t+1}, \sigma_\varepsilon\right)}{\partial Y_t}\right) = \frac{\partial}{\partial Y_t}\left(\frac{\partial h_{t+1}}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial Y_t} + \frac{\partial h_{t+1}}{\partial Y_{t+1}}\frac{\partial Y_{t+1}}{\partial Y_t} + \frac{\partial h_{t+1}}{\partial \sigma_\varepsilon}\frac{\partial \sigma_\varepsilon}{\partial Y_t}\right) \\
&= \frac{\partial\left(\frac{\partial h_{t+1}}{\partial B_{t+1}}\right)}{\partial Y_t}\frac{\partial B_{t+1}}{\partial Y_t} + \frac{\partial h_{t+1}}{\partial B_{t+1}}\frac{\partial\left(\frac{\partial B_{t+1}}{\partial Y_t}\right)}{\partial Y_t} + \frac{\partial\left(\frac{\partial h_{t+1}}{\partial Y_{t+1}}\right)}{\partial Y_t}\frac{\partial Y_{t+1}}{\partial Y_t} + \frac{\partial h_{t+1}}{\partial Y_{t+1}}\frac{\partial\left(\frac{\partial Y_{t+1}}{\partial Y_t}\right)}{\partial Y_t} \\
&= \left[\begin{array}{c} \left[\frac{\partial^2 h_{t+1}}{\partial^2 B_{t+1}}\frac{\partial B_{t+1}}{\partial Y_t} + \frac{\partial^2 h_{t+1}}{\partial B_{t+1}\partial Y_{t+1}}\frac{\partial Y_{t+1}}{\partial Y_t} + \frac{\partial^2 h_{t+1}}{\partial B_{t+1}\partial \sigma_\varepsilon}\frac{\partial \sigma_\varepsilon}{\partial Y_t}\right] * \frac{\partial B_{t+1}}{\partial Y_t} + \frac{\partial h_{t+1}}{\partial B_{t+1}} * \frac{\partial^2 B_{t+1}}{\partial^2 Y_t} \\ + \left[\frac{\partial^2 h_{t+1}}{\partial Y_{t+1}\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial Y_t} + \frac{\partial^2 h_{t+1}}{\partial^2 Y_{t+1}}\frac{\partial Y_{t+1}}{\partial Y_t} + \frac{\partial^2 h_{t+1}}{\partial Y_{t+1}\partial \sigma_\varepsilon}\frac{\partial \sigma_\varepsilon}{\partial Y_t}\right] * \frac{\partial Y_{t+1}}{\partial Y_t} + \frac{\partial h_{t+1}}{\partial Y_{t+1}} * \frac{\partial \rho}{\partial Y_t} \end{array}\right],
\end{aligned}$$

since $\frac{\partial \sigma_\varepsilon}{\partial Y_t} = 0$ and $\frac{\partial Y_{t+1}}{\partial Y_t} = \rho$. In addition, given (2.45), (2.60) and (2.61); we rewrite the last equation as

$$\begin{aligned}
\frac{\partial^2 B_{t+2}}{\partial^2 Y_t} &= h_{11}^B\left(h_1^Y\right)^2 + h_{11}^{BY}\rho h_1^Y + h_1^B h_{11}^Y + h_{11}^{BY} h_1^Y \rho + h_{11}^Y \rho^2 \\
&= h_{11}^B\left(h_1^Y\right)^2 + 2h_{11}^{BY}\rho h_1^Y + h_{11}^Y\left(h_1^B + \rho^2\right).
\end{aligned}$$

- Computation of $\frac{\partial^2 B_{t+2}}{\partial Y_t \partial \sigma_\varepsilon}$:

$$\begin{aligned}
\frac{\partial^2 B_{t+2}}{\partial Y_t \partial \sigma_\varepsilon} &= \frac{\partial}{\partial Y_t}\left(\frac{\partial h\left(B_{t+1}, Y_{t+1}, \sigma_\varepsilon\right)}{\partial \sigma_\varepsilon}\right) \\
&= \frac{\partial}{\partial Y_t}\left(\frac{\partial h_{t+1}}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial \sigma_\varepsilon} + \frac{\partial h_{t+1}}{\partial Y_{t+1}}\frac{\partial Y_{t+1}}{\partial \sigma_\varepsilon} + \frac{\partial h_{t+1}}{\partial \sigma_\varepsilon}\frac{\partial \sigma_\varepsilon}{\partial \sigma_\varepsilon}\right) \\
&= \left[\begin{array}{c} \frac{\partial\left(\frac{\partial h_{t+1}}{\partial B_{t+1}}\right)}{\partial Y_t} * \frac{\partial B_{t+1}}{\partial \sigma_\varepsilon} + \frac{\partial h_{t+1}}{\partial B_{t+1}} * \frac{\partial\left(\frac{\partial B_{t+1}}{\partial \sigma_\varepsilon}\right)}{\partial Y_t} + \frac{\partial\left(\frac{\partial h_{t+1}}{\partial Y_{t+1}}\right)}{\partial Y_t} * \varepsilon_{t+1} \\ + \frac{\partial h_{t+1}}{\partial Y_{t+1}} * \frac{\partial(\varepsilon_{t+1})}{\partial Y_t} + \frac{\partial\left(\frac{\partial h_{t+1}}{\partial \sigma_\varepsilon}\right)}{\partial Y_t} + \frac{\partial h_{t+1}}{\partial \sigma_\varepsilon} * \frac{\partial(1)}{\partial Y_t} \end{array}\right] \\
&= \left[\begin{array}{c} \left[\frac{\partial^2 h_{t+1}}{\partial^2 B_{t+1}}\frac{\partial B_{t+1}}{\partial Y_t} + \frac{\partial^2 h_{t+1}}{\partial B_{t+1}\partial Y_{t+1}}\frac{\partial Y_{t+1}}{\partial Y_t} + \frac{\partial^2 h_{t+1}}{\partial B_{t+1}\partial \sigma_\varepsilon}\frac{\partial \sigma_\varepsilon}{\partial Y_t}\right] * \frac{\partial B_{t+1}}{\partial \sigma_\varepsilon} + \frac{\partial h_{t+1}}{\partial B_{t+1}} * \frac{\partial^2 B_{t+1}}{\partial \sigma_\varepsilon \partial Y_t} \\ + \left[\frac{\partial^2 h_{t+1}}{\partial Y_{t+1}\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial Y_t} + \frac{\partial^2 h_{t+1}}{\partial^2 Y_{t+1}}\frac{\partial Y_{t+1}}{\partial Y_t} + \frac{\partial^2 h_{t+1}}{\partial Y_{t+1}\partial \sigma_\varepsilon}\frac{\partial \sigma_\varepsilon}{\partial Y_t}\right] * \varepsilon_{t+1} + \\ \frac{\partial^2 h_{t+1}}{\partial \sigma_\varepsilon \partial B_{t+1}}\frac{\partial B_{t+1}}{\partial Y_t} + \frac{\partial^2 h_{t+1}}{\partial \sigma_\varepsilon \partial Y_{t+1}}\frac{\partial Y_{t+1}}{\partial Y_t} + \frac{\partial^2 h_{t+1}}{\partial^2 \sigma_\varepsilon}\frac{\partial \sigma_\varepsilon}{\partial Y_t} \end{array}\right] \\
&= \left(h_{11}^B h_1^Y + h_{11}^{BY}\rho\right) h_1^\sigma + h_1^B h_{11}^{Y\sigma} + \left(h_{11}^{BY} h_1^Y + h_{11}^Y \rho\right)\varepsilon_{t+1} + h_{11}^{B\sigma} h_1^Y + h_{11}^{Y\sigma}\rho.
\end{aligned}$$

- Computation of $\frac{\partial^2 B_{t+2}}{\partial^2 \sigma_\varepsilon}$:

$$
\begin{aligned}
\frac{\partial^2 B_{t+2}}{\partial^2 \sigma_\varepsilon} &= \frac{\partial}{\partial \sigma_\varepsilon}\left(\frac{\partial h\left(B_{t+1}, Y_{t+1}, \sigma_\varepsilon\right)}{\partial \sigma_\varepsilon}\right) \\
&= \frac{\partial}{\partial \sigma_\varepsilon}\left(\frac{\partial h_{t+1}}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial \sigma_\varepsilon} + \frac{\partial h_{t+1}}{\partial Y_{t+1}}\frac{\partial Y_{t+1}}{\partial \sigma_\varepsilon} + \frac{\partial h_{t+1}}{\partial \sigma_\varepsilon}\frac{\partial \sigma_\varepsilon}{\partial \sigma_\varepsilon}\right) \\
&= \left[\begin{array}{c} \frac{\partial\left(\frac{\partial h_{t+1}}{\partial B_{t+1}}\right)}{\partial \sigma_\varepsilon} * \frac{\partial B_{t+1}}{\partial \sigma_\varepsilon} + \frac{\partial h_{t+1}}{\partial B_{t+1}} * \frac{\partial\left(\frac{\partial B_{t+1}}{\partial \sigma_\varepsilon}\right)}{\partial \sigma_\varepsilon} + \frac{\partial\left(\frac{\partial h_{t+1}}{\partial Y_{t+1}}\right)}{\partial \sigma_\varepsilon} * \varepsilon_{t+1} \\ + \frac{\partial h_{t+1}}{\partial Y_{t+1}} * \frac{\partial(\varepsilon_{t+1})}{\partial \sigma_\varepsilon} + \frac{\partial\left(\frac{\partial h_{t+1}}{\partial \sigma_\varepsilon}\right)}{\partial \sigma_\varepsilon} + \frac{\partial h_{t+1}}{\partial \sigma_\varepsilon} * \frac{\partial(1)}{\partial \sigma_\varepsilon} \end{array}\right] \\
&= \left[\begin{array}{c} \left[\frac{\partial^2 h_{t+1}}{\partial^2 B_{t+1}}\frac{\partial B_{t+1}}{\partial \sigma_\varepsilon} + \frac{\partial^2 h_{t+1}}{\partial B_{t+1}\partial Y_{t+1}}\frac{\partial Y_{t+1}}{\sigma_\varepsilon} + \frac{\partial^2 h_{t+1}}{\partial B_{t+1}\partial \sigma_\varepsilon}\frac{\partial \sigma_\varepsilon}{\partial \sigma_\varepsilon}\right] * \frac{\partial B_{t+1}}{\partial \sigma_\varepsilon} + \frac{\partial h_{t+1}}{\partial B_{t+1}} * \frac{\partial^2 B_{t+1}}{\partial^2 \sigma_\varepsilon} \\ + \left[\frac{\partial^2 h_{t+1}}{\partial Y_{t+1}\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial \sigma_\varepsilon} + \frac{\partial^2 h_{t+1}}{\partial^2 Y_{t+1}}\frac{\partial Y_{t+1}}{\partial \sigma_\varepsilon} + \frac{\partial^2 h_{t+1}}{\partial Y_{t+1}\partial \sigma_\varepsilon}\frac{\partial \sigma_\varepsilon}{\partial \sigma_\varepsilon}\right] * \varepsilon_{t+1} + \\ \frac{\partial^2 h_{t+1}}{\partial \sigma_\varepsilon \partial B_{t+1}}\frac{\partial B_{t+1}}{\partial \sigma_\varepsilon} + \frac{\partial^2 h_{t+1}}{\partial \sigma_\varepsilon \partial Y_{t+1}}\frac{\partial Y_{t+1}}{\partial \sigma_\varepsilon} + \frac{\partial^2 h_{t+1}}{\partial^2 \sigma_\varepsilon}\frac{\partial \sigma_\varepsilon}{\partial \sigma_\varepsilon} \end{array}\right] \\
&= \left[\begin{array}{c} \left(h_{11}^B h_1^\sigma + h_{11}^{BY}\varepsilon_{t+1} + h_{11}^{B\sigma}\right) h_1^\sigma + \left(h_{11}^{BY} h_1^\sigma + h_{11}^{Y\sigma}\right)\varepsilon_{t+1} \\ + h_{11}^Y \varepsilon_{t+1}^2 + h_{11}^{B\sigma} h_1^\sigma + h_{11}^{Y\sigma}\varepsilon_{t+1} + \left(1 + h_1^B\right) h_{11}^\sigma \end{array}\right].
\end{aligned}
$$

Derivation of q_{BB} That is easier found by taking the partial derivative of q_B with respect to B_t and approximating it around the point $\left(\overline{B}, \overline{Y}, 0\right)$:

$$
q_{BB} = \left[\begin{array}{c}
\frac{\partial}{\partial B_t}\left(u''\left(G_t\right)\right) * \left(\frac{\partial B_{t+1}}{\partial B_t} - \left(1 + r + k^B L_t^B\right) - k^B\left(B_t - B^c\right)\frac{\partial L_t^B}{\partial B_t}\right) \\
+ u''\left(G_t\right) * \frac{\partial}{\partial B_t}\left(\frac{\partial B_{t+1}}{\partial B_t} - \left(1 + r + k^B L_t^B\right) - k^B\left(B_t - B^c\right)\frac{\partial L_t^B}{\partial B_t}\right) - \\
\beta_g k^B \left[\begin{array}{c} \frac{\partial}{\partial B_t}\left(\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial B_t}\right) + \frac{\partial}{\partial B_t}\left(\frac{\partial B_{t+1}}{\partial B_t}\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\right) + \\ \frac{\partial}{\partial B_t}\left(\left(B_{t+1} - B^c\right)\frac{\partial^2 L_{t+1}^B}{\partial^2 B_{t+1}}\frac{\partial B_{t+1}}{\partial B_t}\right) \end{array}\right] u' \\
-\beta_g k^B \left[\begin{array}{c} \frac{\partial L_{t+1}^B}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial B_t} + \frac{\partial B_{t+1}}{\partial B_t}\frac{\partial L_{t+1}^B}{\partial B_{t+1}} + \\ \left(B_{t+1} - B^c\right)\frac{\partial^2 L_{t+1}^B}{\partial B_{t+1}^2}\frac{\partial B_{t+1}}{\partial B_t} \end{array}\right] * \frac{\partial}{\partial B_t}\left(u'\left(G_{t+1}\right)\right) - \\
\beta_g * \frac{\partial}{\partial B_t}\left(\begin{array}{c} 1 + r + k^B L_{t+1}^B + \\ k^B\left(B_{t+1} - B^c\right)\frac{\partial L_{t+1}^B}{\partial B_{t+1}} \end{array}\right) * u'' * \left(\begin{array}{c} \frac{\partial B_{t+2}}{\partial B_t} - \left(1 + r + k^B L_{t+1}^B\right)\frac{\partial B_{t+1}}{\partial B_t} \\ -k^B\left(B_{t+1} - B^c\right)\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial B_t} \end{array}\right) - \\
\beta_g \left[\begin{array}{c} 1 + r + k^B L_{t+1}^B + \\ k^B\left(B_{t+1} - B^c\right)\frac{\partial L_{t+1}^B}{\partial B_{t+1}} \end{array}\right] * \frac{\partial}{\partial B_t}\left(u''\left(G_{t+1}\right)\right) * \left(\begin{array}{c} \frac{\partial B_{t+2}}{\partial B_t} - \\ \left(1 + r + k^B L_{t+1}^B\right)\frac{\partial B_{t+1}}{\partial B_t} \\ -k^B\left(B_{t+1} - B^c\right)\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial B_t} \end{array}\right) \\
-\beta_g \left[\begin{array}{c} 1 + r + k^B L_{t+1}^B + \\ k^B\left(B_{t+1} - B^c\right)\frac{\partial L_{t+1}^B}{\partial B_{t+1}} \end{array}\right] * u'' * \frac{\partial}{\partial B_t}\left(\begin{array}{c} \frac{\partial B_{t+2}}{\partial B_t} - \left(1 + r + k^B L_{t+1}^B\right)\frac{\partial B_{t+1}}{\partial B_t} \\ -k^B\left(B_{t+1} - B^c\right)\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial B_t} \end{array}\right).
\end{array}\right].
$$

Then, using (2.40) and (2.48) we already simplify the last expression around the point

$\left(\overline{B},\overline{Y},0\right)$ to:

$$
q_{BB}=\left[\begin{array}{c}
\left[\frac{\partial B_{t+1}}{\partial B_t}-\left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)\right]^2 u'''+\\
\left[\frac{\partial^2 B_{t+1}}{\partial^2 B_t}-k^B\left(2\frac{\partial L_t^B}{\partial B_t}+\left(\overline{B}-B^c\right)\frac{\partial^2 L_t^B}{\partial^2 B_t}\right)\right]u''-\\
\beta_g k^B\left[\begin{array}{c}
\frac{\partial^2 L_{t+1}^B}{\partial^2 B_{t+1}}\left(\frac{\partial B_{t+1}}{\partial B_t}\right)^2+\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\frac{\partial^2 B_{t+1}}{\partial^2 B_t}+\frac{\partial^2 B_{t+1}}{\partial^2 B_t}\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\\
+\left(\frac{\partial B_{t+1}}{\partial B_t}\right)^2\frac{\partial^2 L_{t+1}^B}{\partial^2 B_{t+1}}+\frac{\partial^2 L_{t+1}^B}{\partial^2 B_{t+1}}\left(\frac{\partial B_{t+1}}{\partial B_t}\right)^2+\\
\left(\overline{B}-B^c\right)\frac{\partial^3 L_{t+1}^B}{\partial^3 B_{t+1}}\left(\frac{\partial B_{t+1}}{\partial B_t}\right)^2+\left(\overline{B}-B^c\right)\frac{\partial^2 L_{t+1}^B}{\partial^2 B_{t+1}}\frac{\partial^2 B_{t+1}}{\partial^2 B_t}
\end{array}\right]u'-\\
\beta_g k^B\gamma\left[2\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)+\gamma\left(\overline{B}-B^c\right)\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right)\right]\frac{\partial B_{t+1}}{\partial B_t}*\\
\left[\frac{\partial B_{t+2}}{\partial B_t}-\left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)\frac{\partial B_{t+1}}{\partial B_t}\right]u''-\\
\beta_g k^B\left[2\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial B_t}+\left(B_{t+1}-B^c\right)\frac{\partial^2 L_{t+1}^B}{\partial^2 B_{t+1}}\frac{\partial B_{t+1}}{\partial B_t}\right]*\\
\left[\frac{\partial B_{t+2}}{\partial B_t}-\left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)\frac{\partial B_{t+1}}{\partial B_t}\right]u''-\\
\beta_g\left[1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\overline{L^B}(1-\overline{L^B})\right]*\\
\left[\frac{\partial B_{t+2}}{\partial B_t}-\left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)\frac{\partial B_{t+1}}{\partial B_t}\right]^2 u'''-\\
\beta_g\left[1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\overline{L^B}(1-\overline{L^B})\right]*\\
\left[\begin{array}{c}
\frac{\partial^2 B_{t+2}}{\partial^2 B_t}-2k^B\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\left(\frac{\partial B_{t+1}}{\partial B_t}\right)^2-\\
\left(1+r+k^B\overline{L^B}+k^B\left(\overline{B}-B^c\right)\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\right)\frac{\partial^2 B_{t+1}}{\partial^2 B_t}\\
-k^B\left(\overline{B}-B^c\right)\frac{\partial^2 L_{t+1}^B}{\partial^2 B_{t+1}}\left(\frac{\partial B_{t+1}}{\partial B_t}\right)^2
\end{array}\right]u''
\end{array}\right],
$$

where all functions are evaluated at $\left(\overline{B},\overline{Y},0\right)$.

Using the derivatives of B_{t+1} and B_{t+2} computed above and again (2.40) and (2.48), we simplify the last equation once more to

$$q_{BB}=\left[\begin{array}{c}\left[h_1^B-\left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)\right]^2u'''+\\ \left[h_{11}^B-k^B\gamma\left(2\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)+\gamma\left(\overline{B}-B^c\right)\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right)\right)\right]u''-\\ \beta_g k^B\left[\begin{array}{c}\left[3\gamma^2\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right)+\left(\overline{B}-B^c\right)\frac{\partial^3 L_{t+1}^B}{\partial B_{t+1}^3}\right]\left(h_1^B\right)^2+\\ \left[2\gamma\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)+\gamma^2\left(\overline{B}-B^c\right)\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right)\right]h_{11}^B\end{array}\right]u'\\ -2\beta_g k^B\gamma\left[2\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)+\gamma\left(\overline{B}-B^c\right)\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right)\right]u''*\\ h_1^B\left[\left(h_1^B\right)^2-\left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)h_1^B\right]-\\ \beta_g\left[1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right]*\\ \left[\left(h_1^B\right)^2-\left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)h_1^B\right]^2u'''\\ -\beta_g\left[1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\overline{L^B}^2\right)\right]*\\ \left[\begin{array}{c}h_{11}^B\left(h_1^B\right)^2+h_1^Bh_{11}^B-\\ \left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)h_{11}^B-\\ k^B\left(\begin{array}{c}2\gamma\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)+\\ \gamma^2\left(\overline{B}-B^c\right)\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right)\end{array}\right)\left(h_1^B\right)^2\end{array}\right]u''\end{array}\right].$$

The only additional term that we have to calculate in the equation above is $\frac{\partial^3 L_{t+1}^B}{\partial^3 B_{t+1}}$, but from (2.48) we get

$$\begin{aligned}\frac{\partial^3 L_{t+1}^B}{\partial B_{t+1}^3}&=\frac{\partial}{\partial B_{t+1}}\left[\gamma^2\left(L_{t+1}^B-3\left(L_{t+1}^B\right)^2+2\left(L_{t+1}^B\right)^3\right)\right]\\&=\gamma^2\left(\begin{array}{c}\gamma(L_{t+1}^B-\left(L_{t+1}^B\right)^2)-6L_{t+1}^B\gamma(L_{t+1}^B-\left(L_{t+1}^B\right)^2)\\+6\left(L_{t+1}^B\right)^2\gamma(L_{t+1}^B-\left(L_{t+1}^B\right)^2)\end{array}\right)\\&=\gamma^3\left(L_{t+1}^B-7\left(L_{t+1}^B\right)^2+12\left(L_{t+1}^B\right)^3-6\left(L_{t+1}^B\right)^4\right).\qquad(2.65)\end{aligned}$$

Thus, using (2.65) in our equation of q_{BB} together with (2.42), we have that:

$$
q_{BB}=\begin{bmatrix}
\left[h_1^B-\left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)\right]^2 u'''+\\
\left[h_{11}^B-k^B\gamma\left(2\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)+\gamma\left(\overline{B}-B^c\right)\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right)\right)\right]u''-\\
\beta_g k^B\begin{bmatrix}\begin{pmatrix}3\gamma^2\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right)+\\ \left(\overline{B}-B^c\right)\gamma^3\left(\overline{L^B}-7\left(\overline{L^B}\right)^2+12\left(\overline{L^B}\right)^3-6\left(\overline{L^B}\right)^4\right)\end{pmatrix}\left(h_1^B\right)^2\\ +\begin{pmatrix}2\gamma\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)+\\ \gamma^2\left(\overline{B}-B^c\right)\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right)\end{pmatrix}h_{11}^B\end{bmatrix}u'\\
-2\beta_g k^B\gamma\left[2\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)+\gamma\left(\overline{B}-B^c\right)\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right)\right]*\\
\left[\left(h_1^B\right)^3-\left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)\left(h_1^B\right)^2\right]u''\\
-\left[\left(h_1^B\right)^2-\left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)h_1^B\right]^2 u'''-\\
\begin{bmatrix}\left(\left(h_1^B\right)^2+h_1^B\right)h_{11}^B-\\ \left[1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right]h_{11}^B-\\ k^B\begin{bmatrix}2\gamma\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)+\\ \gamma^2\left(\overline{B}-B^c\right)\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right)\end{bmatrix}\left(h_1^B\right)^2\end{bmatrix}u''
\end{bmatrix}.
$$

Finally, isolating the only unknown term h_{11}^B of the equation above and multiplying it to $\widetilde{B}_t^2$:

$$q_{BB}\widetilde{B}_t^2 = \left[\begin{array}{c} h_{11}^B \left[\begin{array}{c} \left(2+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)-\left(h_1^B\right)^2-h_1^B\right)u'' \\ -\beta_g k^B\gamma\left(\begin{array}{c} 2\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)+ \\ \gamma\left(\overline{B}-B^c\right)\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right)\end{array}\right)u' \end{array}\right] \\ +\left[h_1^B-\left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)\right]^2\left[1-\left(h_1^B\right)^2\right]u''' \\ -\beta_g k^B\gamma^2\left[\begin{array}{c} 3\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right)+ \\ \gamma\left(\overline{B}-B^c\right)\left(\overline{L^B}-7\overline{L^B}^2+12\overline{L^B}^3-6\overline{L^B}^4\right)\end{array}\right]\left(h_1^B\right)^2u'+ \\ k^B\gamma\left[2\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)+\gamma\left(\overline{B}-B^c\right)\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right)\right]* \\ \left(-1+3\left(h_1^B\right)^2-2\beta_g\left(h_1^B\right)^3\right)u'' \end{array}\right]\widetilde{B}_t^2$$

$$= \left[\begin{array}{c} h_{11}^B\left[\begin{array}{c}\left(1+\frac{1}{\beta_g}-\left(h_1^B\right)^2-h_1^B\right)u''- \\ \beta_g k^B\gamma z_1 u'\end{array}\right]+ \\ k^B\gamma z_1\left(-1+3\left(h_1^B\right)^2-2\beta_g\left(h_1^B\right)^3\right)u''+\left(h_1^B-\frac{1}{\beta_g}\right)^2\left[1-\left(h_1^B\right)^2\right]u'''- \\ \beta_g k^B\gamma^2\left[\begin{array}{c} 3\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right)+ \\ \gamma\left(\overline{B}-B^c\right)\left(\overline{L^B}-7\overline{L^B}^2+12\overline{L^B}^3-6\overline{L^B}^4\right)\end{array}\right]\left(h_1^B\right)^2u' \end{array}\right]\widetilde{B}_t^2$$

$$\equiv \left(Mh_{11}^B+N\right)\widetilde{B}_t^2. \tag{2.66}$$

where

$$z_1\equiv 2\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)+\gamma\left(\overline{B}-B^c\right)\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right).$$

Derivation of the q_{BY_t} Differentiating q_{Y_t} with respect to B_t at the point $\left(\overline{B},\overline{Y},0\right)$ yields:

$$q_{BY_t} = \left[\begin{array}{c} \frac{\partial}{\partial B_t}\left(u''\left(G_t\right)\right)*\left(\frac{\partial B_{t+1}}{\partial Y_t}+\tau^y\right)+u''*\frac{\partial}{\partial B_t}\left(\frac{\partial B_{t+1}}{\partial Y_t}+\tau^y\right) \\ -\beta_g k^B\left[2\frac{\partial}{\partial B_t}\left(\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial Y_t}\right)+\frac{\partial}{\partial B_t}\left(\left(B_{t+1}-B^c\right)\frac{\partial^2 L_{t+1}^B}{\partial^2 B_{t+1}}\frac{\partial B_{t+1}}{\partial Y_t}\right)\right]u'- \\ \beta_g k^B*\left[2\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial Y_t}+\left(B_{t+1}-B^c\right)\frac{\partial^2 L_{t+1}^B}{\partial^2 B_{t+1}}\frac{\partial B_{t+1}}{\partial Y_t}\right]*\frac{\partial}{\partial B_t}\left(u'\left(G_{t+1}\right)\right) \\ -\beta_g*\frac{\partial}{\partial B_t}\left(1+r+k^BL_{t+1}^B+k^B\left(B_{t+1}-B^c\right)\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\right)* \\ \left[\frac{\partial B_{t+2}}{\partial Y_t}+\tau^y\rho-\left(1+r+k^BL_{t+1}^B+k^B\left(B_{t+1}-B^c\right)\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\right)\frac{\partial B_{t+1}}{\partial Y_t}\right]u'' \\ -\beta_g*\left[1+r+k^BL_{t+1}^B+k^B\left(B_{t+1}-B^c\right)\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\right]*\frac{\partial}{\partial B_t}\left(u''\left(G_{t+1}\right)\right)* \\ \left[\frac{\partial B_{t+2}}{\partial Y_t}+\tau^y\rho-\left(1+r+k^BL_{t+1}^B+k^B\left(B_{t+1}-B^c\right)\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\right)\frac{\partial B_{t+1}}{\partial Y_t}\right] \\ -\beta_g*\left[1+r+k^BL_{t+1}^B+k^B\left(B_{t+1}-B^c\right)\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\right]*u''* \\ \frac{\partial}{\partial B_t}\left(\frac{\partial B_{t+2}}{\partial Y_t}+\tau^y\rho-\left(1+r+k^BL_{t+1}^B+k^B\left(B_{t+1}-B^c\right)\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\right)\frac{\partial B_{t+1}}{\partial Y_t}\right) \end{array}\right].$$

Then, using (2.40), (2.48) and the derivation of (2.66), we already simplify the last expression around the point $\left(\overline{B},\overline{Y},0\right)$ to:

$$
q_{BY_t}=\left[\begin{array}{c}
\left[\frac{\partial B_{t+1}}{\partial B_t}-\left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)\right]\left(\frac{\partial B_{t+1}}{\partial Y_t}+\tau^y\right)u'''\\
+u''\frac{\partial^2 B_{t+1}}{\partial B_t\partial Y_t}-\\
\beta_g k^B\left[\begin{array}{c}2\left(\frac{\partial^2 L^B_{t+1}}{\partial^2 B_{t+1}}\frac{\partial B_{t+1}}{\partial B_t}\frac{\partial B_{t+1}}{\partial Y_t}+\frac{\partial L^B_{t+1}}{\partial B_{t+1}}\frac{\partial^2 B_{t+1}}{\partial B_t\partial Y_t}\right)+\left(\frac{\partial B_{t+1}}{\partial B_t}\frac{\partial^2 L^B_{t+1}}{\partial^2 B_{t+1}}\frac{\partial B_{t+1}}{\partial Y_t}\right)\\
+\left(\overline{B}-B^c\right)\left(\frac{\partial^3 L^B_{t+1}}{\partial^3 B_{t+1}}\frac{\partial B_{t+1}}{\partial Y_t}\frac{\partial B_{t+1}}{\partial B_t}+\frac{\partial^2 L^B_{t+1}}{\partial^2 B_{t+1}}\frac{\partial^2 B_{t+1}}{\partial B_t\partial Y_t}\right)\end{array}\right]u'\\
-\beta_g k^B\gamma\left[2\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)+\gamma\left(\overline{B}-B^c\right)\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right)\right]*\\
h_1^Y\left[\frac{\partial B_{t+2}}{\partial B_t}-\left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)\frac{\partial B_{t+1}}{\partial B_t}\right]u''\\
-\beta_g k^B\left[2\frac{\partial L^B_{t+1}}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial B_t}+\left(B_{t+1}-B^c\right)\frac{\partial^2 L^B_{t+1}}{\partial^2 B_{t+1}}\frac{\partial B_{t+1}}{\partial B_t}\right]*\\
\left[h_1^B h_1^Y+h_1^Y\rho+\tau^y\rho-\left(\begin{array}{c}1+r+k^B\overline{L^B}+\\ \gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\end{array}\right)h_1^Y\right]u''\\
-\beta_g\left[1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right]*\\
\left\{\begin{array}{c}\left[\frac{\partial B_{t+2}}{\partial B_t}-\left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)\frac{\partial B_{t+1}}{\partial B_t}\right]\\
*\left[\begin{array}{c}h_1^B h_1^Y+h_1^Y\rho+\tau^y\rho-\\ \left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)h_1^Y\end{array}\right]\end{array}\right\}u'''\\
-\beta_g\left[1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right]*\\
\left[\begin{array}{c}\frac{\partial^2 B_{t+2}}{\partial B_t\partial Y_t}-\\
k^B\left(\frac{\partial L^B_{t+1}}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial B_t}+\frac{\partial B_{t+1}}{\partial B_t}\frac{\partial L^B_{t+1}}{\partial B_{t+1}}+\left(\overline{B}-B^c\right)\frac{\partial^2 L^B_{t+1}}{\partial^2 B_{t+1}}\frac{\partial B_{t+1}}{\partial B_t}\right)\frac{\partial B_{t+1}}{\partial Y_t}\\
-\left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)\frac{\partial^2 B_{t+1}}{\partial B_t\partial Y_t}\end{array}\right]u''.
\end{array}\right].
$$

Using the derivatives of B_{t+1} and B_{t+2}, (2.40), (2.48) and (2.65), and (2.42), we simplify the last equation once more to

$$q_{BY_t} = \left[\begin{array}{c} \left[h_1^B - \left(1 + r + k^B\overline{L^B} + \gamma k^B\left(\overline{B} - B^c\right)\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right)\right)\right]\left(h_1^Y + \tau^y\right)u''' \\ +h_{11}^{BY}u'' - \\ \beta_g k^B \left[\begin{array}{c} 3\gamma^2\left(\overline{L^B} - 3\left(\overline{L^B}\right)^2 + 2\left(\overline{L^B}\right)^3\right)h_1^B h_1^Y + 2\gamma\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right)h_{11}^{BY} + \\ +\left(\overline{B} - B^c\right)\gamma^3\left(\overline{L^B} - 7\left(\overline{L^B}\right)^2 + 12\left(\overline{L^B}\right)^3 - 6\left(\overline{L^B}\right)^4\right)h_1^B h_1^Y + \\ \left(\overline{B} - B^c\right)\gamma^2\left(\overline{L^B} - 3\left(\overline{L^B}\right)^2 + 2\left(\overline{L^B}\right)^3\right)h_{11}^{BY} \end{array}\right] u' \\ -\beta_g k^B\gamma\left[2\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right) + \gamma\left(\overline{B} - B^c\right)\left(\overline{L^B} - 3\left(\overline{L^B}\right)^2 + 2\left(\overline{L^B}\right)^3\right)\right] * \\ \left(\left(h_1^B\right)^2 h_1^Y - \left(1 + r + k^B\overline{L^B} + \gamma k^B\left(\overline{B} - B^c\right)\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right)\right)h_1^B h_1^Y\right] u'' - \\ \beta_g k^B\gamma\left[2\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right) + \gamma\left(\overline{B} - B^c\right)\left(\overline{L^B} - 3\left(\overline{L^B}\right)^2 + 2\left(\overline{L^B}\right)^3\right)\right] h_1^B * \\ \left[\begin{array}{c} h_1^B h_1^Y + h_1^Y\rho + \tau^y\rho - \\ \left(1 + r + k^B\overline{L^B} + \gamma k^B\left(\overline{B} - B^c\right)\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right)\right)h_1^Y \end{array}\right] u'' \\ -\left\{\begin{array}{c} \left(\left(h_1^B\right)^2 - \left(1 + r + k^B\overline{L^B} + \gamma k^B\left(\overline{B} - B^c\right)\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right)\right)h_1^B\right) * \\ \left(\begin{array}{c} h_1^B h_1^Y + h_1^Y\rho + \tau^y\rho - \\ \left(1 + r + k^B\overline{L^B} + \gamma k^B\left(\overline{B} - B^c\right)\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right)\right)h_1^Y \end{array}\right) \end{array}\right\} u''' \\ -\left(\begin{array}{c} h_{11}^B h_1^B h_1^Y + h_1^B h_{11}^{BY} + \rho h_1^B h_{11}^{BY} - \\ k^B\gamma\left[\begin{array}{c} 2\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right) + \\ \gamma\left(\overline{B} - B^c\right)\left(\overline{L^B} - 3\left(\overline{L^B}\right)^2 + 2\left(\overline{L^B}\right)^3\right) \end{array}\right] h_1^B h_1^Y \\ -\left(1 + r + k^B\overline{L^B} + \gamma k^B\left(\overline{B} - B^c\right)\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right)\right)h_{11}^{BY} \end{array}\right) u'' \end{array}\right].$$

Thus, using (2.42) again and isolating the two unknown coefficients h_{11}^{BY} and h_{11}^{B}, we can rewrite the equation above as:

$$
q_{BY_t} = \left[\begin{array}{c}
h_{11}^{BY} \left[\begin{array}{c}
u'' - \\
\beta_g k^B \gamma \left[2(\overline{L^B} - \overline{L^B}^2) + \left(\overline{B} - B^c\right) \gamma \left(\overline{L^B} - 3\overline{L^B}^2 + 2\overline{L^B}^3 \right) \right] u' \\
- \left[\begin{array}{c}
h_1^B (1+\rho) - \\
\left(1 + r + k^B \overline{L^B} + \gamma k^B \left(\overline{B} - B^c\right) \left(\overline{L^B} - \overline{L^B}^2\right) \right)
\end{array} \right] u''
\end{array} \right] \\
-h_{11}^B h_1^B h_1^Y u'' + \\
\left[h_1^B - \left(1 + r + k^B \overline{L^B} + \gamma k^B \left(\overline{B} - B^c\right) \left(\overline{L^B} - \overline{L^B}^2\right) \right) \right] \left(h_1^Y + \tau^y \right) u''' \\
-\beta_g k^B \gamma^2 \left[\begin{array}{c}
3 \left(\overline{L^B} - 3\overline{L^B}^2 + 2\overline{L^B}^3 \right) + \\
\left(\overline{B} - B^c\right) \gamma \left(\overline{L^B} - 7\overline{L^B}^2 + 12\overline{L^B}^3 - 6\overline{L^B}^4 \right)
\end{array} \right] h_1^B h_1^Y u' - \\
2\beta_g k^B \gamma \left[2(\overline{L^B} - \overline{L^B}^2) + \gamma \left(\overline{B} - B^c\right) \left(\overline{L^B} - 3\overline{L^B}^2 + 2\overline{L^B}^3 \right) \right] * \\
\left[\left(h_1^B\right)^2 h_1^Y - \left(1 + r + k^B \overline{L^B} + \gamma k^B \left(\overline{B} - B^c\right) \left(\overline{L^B} - \overline{L^B}^2\right) \right) h_1^B h_1^Y \right] u'' - \\
\beta_g k^B \gamma \left[2(\overline{L^B} - \overline{L^B}^2) + \gamma \left(\overline{B} - B^c\right) \left(\overline{L^B} - 3\overline{L^B}^2 + 2\overline{L^B}^3 \right) \right] \left(h_1^B h_1^Y \rho + h_1^B \tau^y \rho \right) u'' \\
-h_1^B \left[h_1^B - \left(1 + r + k^B \overline{L^B} + \gamma k^B \left(\overline{B} - B^c\right) \left(\overline{L^B} - \overline{L^B}^2\right) \right) \right] * \\
\left[h_1^B h_1^Y + h_1^Y \rho + \tau^y \rho - \left(\begin{array}{c}
1 + r + k^B \overline{L^B} + \\
\gamma k^B \left(\overline{B} - B^c\right) \left(\overline{L^B} - \overline{L^B}^2\right)
\end{array} \right) h_1^Y \right] u''' \\
+k^B \gamma \left[2(\overline{L^B} - \overline{L^B}^2) + \gamma \left(\overline{B} - B^c\right) \left(\overline{L^B} - 3\overline{L^B}^2 + 2\overline{L^B}^3 \right) \right] h_1^B h_1^Y u''
\end{array} \right],
$$

which can be simplified once more to

$$
q_{BY_t} = \left[\begin{array}{c}
h_{11}^{BY} \left\{ \begin{array}{c}
u'' * \left[2 + r + k^B \overline{L^B} + \gamma k^B \left(\overline{B} - B^c\right) \left(\overline{L^B} - \left(\overline{L^B}\right)^2 \right) - h_1^B (1+\rho) \right] \\
-u' \beta_g k^B \gamma \left[\begin{array}{c}
2 \left(\overline{L^B} - \left(\overline{L^B}\right)^2 \right) + \\
\left(\overline{B} - B^c\right) \gamma \left(\overline{L^B} - 3\left(\overline{L^B}\right)^2 + 2\left(\overline{L^B}\right)^3 \right)
\end{array} \right]
\end{array} \right\} \\
-h_{11}^B u'' h_1^B h_1^Y + \\
u''' * \left[h_1^B - \left(1 + r + k^B \overline{L^B} + \gamma k^B \left(\overline{B} - B^c\right) \left(\overline{L^B} - \left(\overline{L^B}\right)^2 \right) \right) \right] * \\
\left[\begin{array}{c}
h_1^Y + \tau^y - \left(h_1^B\right)^2 h_1^Y - h_1^B h_1^Y \rho - h_1^B \tau^y \rho + \\
\left(1 + r + k^B \overline{L^B} + \gamma k^B \left(\overline{B} - B^c\right) \left(\overline{L^B} - \left(\overline{L^B}\right)^2 \right) \right) h_1^B h_1^Y
\end{array} \right] - \\
u' \beta_g k^B \gamma^2 \left[\begin{array}{c}
3 \left(\overline{L^B} - 3\left(\overline{L^B}\right)^2 + 2\left(\overline{L^B}\right)^3 \right) + \\
\left(\overline{B} - B^c\right) \gamma \left(\overline{L^B} - 7\left(\overline{L^B}\right)^2 + 12\left(\overline{L^B}\right)^3 - 6\left(\overline{L^B}\right)^4 \right)
\end{array} \right] h_1^B h_1^Y + \\
u'' k^B \gamma \left\{ \begin{array}{c}
\left[2 \left(\overline{L^B} - \left(\overline{L^B}\right)^2 \right) + \gamma \left(\overline{B} - B^c\right) \left(\overline{L^B} - 3\left(\overline{L^B}\right)^2 + 2\left(\overline{L^B}\right)^3 \right) \right] * \\
\left\{ \begin{array}{c}
h_1^B h_1^Y - \\
2\beta_g \left[\left(h_1^B\right)^2 h_1^Y - \left(\begin{array}{c}
1 + r + k^B \overline{L^B} + \\
\gamma k^B \left(\overline{B} - B^c\right) \left(\overline{L^B} - \overline{L^B}^2\right)
\end{array} \right) h_1^B h_1^Y \right] \\
-\beta_g \left(h_1^B h_1^Y \rho + h_1^B \tau^y \rho \right)
\end{array} \right\}
\end{array} \right\}
\end{array} \right].
$$

Finally, multiplying the last equation by $\widetilde{B}_t\widetilde{Y}_t$ leads to

$$
\begin{aligned}
& q_{BY_t}\widetilde{B}_t\widetilde{Y}_t \\
= & \left[\begin{array}{c}
h_{11}^{BY}\left\{\begin{array}{c}
\left[2+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)-h_1^B\left(1+\rho\right)\right]u'' \\
-\beta_g k^B\gamma\left[\begin{array}{c}
2\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)+ \\
\left(\overline{B}-B^c\right)\gamma\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right)
\end{array}\right]u'
\end{array}\right\} \\
-h_{11}^B h_1^B h_1^Y u''+ \\
\left[h_1^B-\left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)\right]* \\
\left[\begin{array}{c}
h_1^Y+\tau^y-\left(h_1^B\right)^2h_1^Y-h_1^B\tau^y\rho+ \\
\left(1+r+k^B\overline{L^B}-\rho+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)h_1^Bh_1^Y
\end{array}\right]u'''- \\
\beta_g k^B\gamma^2\left[\begin{array}{c}
3\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right)+ \\
\left(\overline{B}-B^c\right)\gamma\left(\overline{L^B}-7\left(\overline{L^B}\right)^2+12\left(\overline{L^B}\right)^3-6\left(\overline{L^B}\right)^4\right)
\end{array}\right]h_1^Bh_1^Yu' \\
+k^B\gamma\left\{\begin{array}{c}
\left[\begin{array}{c}
2\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)+ \\
\gamma\left(\overline{B}-B^c\right)\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right)
\end{array}\right]* \\
\left(3h_1^Bh_1^Y-2\beta_g\left(h_1^B\right)^2h_1^Y-\beta_g h_1^B\tau^y\rho-\beta_g h_1^Bh_1^Y\rho\right)
\end{array}\right\}u''
\end{array}\right]\widetilde{B}_t\widetilde{Y}_t \\
= & \left(Oh_{11}^{BY}+Ph_{11}^B+Q\right)\widetilde{B}_t\widetilde{Y}_t. \qquad (2.67)
\end{aligned}
$$

Therefore, this equation has two linear unknown coefficients h_{11}^{BY} and h_{11}^B. All other terms are numbers already known.

Derivation of $q_{B\sigma_\varepsilon}$ Next, we compute $q_{B\sigma_\varepsilon}$. For that, we derive q_{σ_ε} with respect to B_t around the point $\left(\overline{B},\overline{Y},0\right)$:

$$
q_{B\sigma_\varepsilon}=\left[\begin{array}{c}
\frac{\partial}{\partial B_t}\left(u''\left(G_t\right)\right)*\left(\frac{\partial B_{t+1}}{\partial\sigma_\varepsilon}\right)+u''*\frac{\partial}{\partial B_t}\left(\frac{\partial B_{t+1}}{\partial\sigma_\varepsilon}\right)- \\
\beta_g k^B*\left[2\frac{\partial}{\partial B_t}\left(\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial\sigma_\varepsilon}\right)+\frac{\partial}{\partial B_t}\left(\left(B_{t+1}-B^c\right)\frac{\partial^2L_{t+1}^B}{\partial^2B_{t+1}}\frac{\partial B_{t+1}}{\partial\sigma_\varepsilon}\right)\right]*u' \\
-\beta_g k^B\left[2\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial\sigma_\varepsilon}+\left(\overline{B}-B^c\right)\frac{\partial^2L_{t+1}^B}{\partial^2B_{t+1}}\frac{\partial B_{t+1}}{\partial\sigma_\varepsilon}\right]*\frac{\partial}{\partial B_t}\left(u'\left(G_{t+1}\right)\right)- \\
\beta_g*\frac{\partial}{\partial B_t}\left(1+r+k^BL_{t+1}^B+k^B\left(B_{t+1}-B^c\right)\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\right)*u''* \\
\left(\frac{\partial B_{t+2}}{\partial\sigma_\varepsilon}-\left(1+r+k^BL_{t+1}^B\right)\frac{\partial B_{t+1}}{\partial\sigma_\varepsilon}-k^B\left(B_{t+1}-B^c\right)\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial\sigma_\varepsilon}+\varepsilon_{t+1}\right) \\
-\beta_g\left[1+r+k^BL_{t+1}^B+k^B\left(B_{t+1}-B^c\right)\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\right]*\frac{\partial}{\partial B_t}\left(u''\left(G_{t+1}\right)\right)* \\
\left(\frac{\partial B_{t+2}}{\partial\sigma_\varepsilon}-\left(1+r+k^BL_{t+1}^B\right)\frac{\partial B_{t+1}}{\partial\sigma_\varepsilon}-k^B\left(B_{t+1}-B^c\right)\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial\sigma_\varepsilon}+\varepsilon_{t+1}\right) \\
-\beta_g\left[1+r+k^BL_{t+1}^B+k^B\left(B_{t+1}-B^c\right)\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\right]*u''* \\
\frac{\partial}{\partial B_t}\left(\frac{\partial B_{t+2}}{\partial\sigma_\varepsilon}-\left(1+r+k^BL_{t+1}^B\right)\frac{\partial B_{t+1}}{\partial\sigma_\varepsilon}-k^B\left(B_{t+1}-B^c\right)\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial\sigma_\varepsilon}+\varepsilon_{t+1}\right)
\end{array}\right].
$$

Using our earlier expressions for the derivatives of B_{t+1} and B_{t+2} with respect to σ_ε, the last expression becomes

$$q_{B\sigma_\varepsilon} = \left[\begin{array}{c} \left[h_1^B - \left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)\right] h_1^\sigma u''' + u'' * \frac{\partial^2 B_{t+1}}{\partial B_t \partial \sigma_\varepsilon} \\ -\beta_g k^B \left[\begin{array}{c} 2\frac{\partial L^B_{t+1}}{\partial B_{t+1}}\frac{\partial^2 B_{t+1}}{\partial B_t\partial\sigma_\varepsilon} + 3\frac{\partial^2 L^B_{t+1}}{\partial^2 B_{t+1}}\frac{\partial B_{t+1}}{\partial B_t} h_1^\sigma + \\ \left(\overline{B}-B^c\right)\frac{\partial^3 L^B_{t+1}}{\partial^3 B_{t+1}}\frac{\partial B_{t+1}}{\partial B_t}h_1^\sigma + \left(\overline{B}-B^c\right)\frac{\partial^2 L^B_{t+1}}{\partial^2 B_{t+1}}\frac{\partial^2 B_{t+1}}{\partial B_t\partial\sigma_\varepsilon} \end{array}\right] u' \\ -\beta_g k^B\left[2\frac{\partial L^B_{t+1}}{\partial B_{t+1}}h_1^\sigma + \left(\overline{B}-B^c\right)\frac{\partial^2 L^B_{t+1}}{\partial^2 B_{t+1}}h_1^\sigma\right] * \\ \left[\frac{\partial B_{t+2}}{\partial B_t} - \left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)\frac{\partial B_{t+1}}{\partial B_t}\right] u'' - \\ \beta_g k^B\left[2\frac{\partial L^B_{t+1}}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial B_t} + \left(\overline{B}-B^c\right)\frac{\partial^2 L^B_{t+1}}{\partial^2 B_{t+1}}\frac{\partial B_{t+1}}{\partial B_t}\right] * \\ \left[\begin{array}{c} h_1^B h_1^\sigma + h_1^Y\varepsilon_{t+1} + h_1^\sigma + \varepsilon_{t+1} - \\ \left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right) h_1^\sigma \end{array}\right] u'' - \\ \beta_g\left[1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right] * \\ \left\{\begin{array}{c} \left[\frac{\partial B_{t+2}}{\partial B_t} - \left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)\frac{\partial B_{t+1}}{\partial B_t}\right] * \\ \left[h_1^B h_1^\sigma + h_1^Y\varepsilon_{t+1} + h_1^\sigma + \varepsilon_{t+1} - \left(\begin{array}{c} 1+r+k^B\overline{L^B}+ \\ \gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\end{array}\right) h_1^\sigma\right] \end{array}\right\} u''' \\ -\beta_g\left[1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right] * \\ \left[\begin{array}{c} \frac{\partial^2 B_{t+2}}{\partial B_t\partial\sigma_\varepsilon} - \left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)\frac{\partial B_{t+1}}{\partial B_t\partial\sigma_\varepsilon} \\ -\left[2\frac{\partial L^B_{t+1}}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial B_t} + \left(\overline{B}-B^c\right)\frac{\partial^2 L^B_{t+1}}{\partial^2 B_{t+1}}\frac{\partial B_{t+1}}{\partial B_t}\right] h_1^\sigma \end{array}\right] u'' \end{array}\right].$$

From (2.56), we know that $h_1^\sigma = 0$. Using, in addition, (2.42) and the derivatives of B_{t+1} and B_{t+2}, we can simplify this expression as:

$$q_{B\sigma_\varepsilon} = \left[\begin{array}{c} +h_{11}^{B\sigma}u'' - \\ \beta_g k^B\gamma\left[\begin{array}{c} 2\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)+ \\ \gamma\left(\overline{B}-B^c\right)\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right)\end{array}\right] h_{11}^{B\sigma}u' - \\ \beta_g k^B\gamma\left[\begin{array}{c} 2\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)+ \\ \gamma\left(\overline{B}-B^c\right)\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right)\end{array}\right] h_1^B\left(h_1^Y+1\right)\varepsilon_{t+1}u'' - \\ \left[\begin{array}{c} \left(h_1^B\right)^2 - \\ \left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right) h_1^B \end{array}\right]\left(h_1^Y+1\right)\varepsilon_{t+1}u''' \\ -\left[\begin{array}{c} 2h_1^B h_{11}^{B\sigma} + h_1^B h_{11}^{BY}\varepsilon_{t+1} - \\ \left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right) h_{11}^{B\sigma}\end{array}\right] u'' \end{array}\right].$$

Grouping terms and multiplying it by $\widetilde{B}_t\sigma_\varepsilon$, we have

$$
\begin{aligned}
& q_{B\sigma_\varepsilon}\widetilde{B}_t\sigma_\varepsilon \\
= & \left\{ \begin{array}{c}
h_{11}^{B\sigma}\left[\begin{array}{c}
\left[2+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)-2h_1^B\right]u''- \\
\beta_g k^B\gamma\left[\begin{array}{c} 2\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)+ \\ \gamma\left(\overline{B}-B^c\right)\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right)\end{array}\right]u'
\end{array}\right] \\
-u''h_{11}^{BY}h_1^B\varepsilon_{t+1}- \\
h_1^B\left(h_1^Y+1\right)\left[\begin{array}{c}
\beta_g k^B\gamma\left[\begin{array}{c} 2\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)+ \\ \gamma\left(\overline{B}-B^c\right)\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right)\end{array}\right]u'' \\
+\left[h_1^B-\left(\begin{array}{c} 1+r+k^B\overline{L^B}+ \\ \gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\end{array}\right)\right]u'''
\end{array}\right]\varepsilon_{t+1}
\end{array}\right\}\widetilde{B}_t\sigma_\varepsilon \\
\equiv & \left(Rh_{11}^{B\sigma}+S\varepsilon_{t+1}\right)\widetilde{B}_t\sigma_\varepsilon.
\end{aligned}
$$

Finally, applying expectations, it follows that

$$
\mathrm{E}_t\left[q_{B\sigma_\varepsilon}\widetilde{B}_t\sigma_\varepsilon\right]=\mathrm{E}_t\left[\left(Rh_{11}^{B\sigma}+S\varepsilon_{t+1}\right)\widetilde{B}_t\sigma_\varepsilon\right]=Rh_{11}^{B\sigma}\widetilde{B}_t\sigma_\varepsilon. \tag{2.68}
$$

Derivation of $q_{Y_tY_t}$ Differentitae q_{Y_t} at the point $\left(\overline{B},\overline{Y},0\right)$:

$$
q_{Y_tY_t}=\left[\begin{array}{c}
\frac{\partial}{\partial Y_t}\left(u''\left(G_t\right)\right)*\left(\frac{\partial B_{t+1}}{\partial Y_t}+\tau^y\right)+u''*\frac{\partial}{\partial Y_t}\left(\frac{\partial B_{t+1}}{\partial Y_t}+\tau^y\right)- \\
\beta_g k^B*\left[2\frac{\partial}{\partial Y_t}\left(\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial Y_t}\right)+\frac{\partial}{\partial Y_t}\left(\left(B_{t+1}-B^c\right)\frac{\partial^2 L_{t+1}^B}{\partial B_{t+1}^2}\frac{\partial B_{t+1}}{\partial Y_t}\right)\right]*u' \\
-\beta_g k^B*\left[2\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial Y_t}+\left(B_{t+1}-B^c\right)\frac{\partial^2 L_{t+1}^B}{\partial B_{t+1}^2}\frac{\partial B_{t+1}}{\partial Y_t}\right]*\frac{\partial}{\partial Y_t}\left(u'\left(G_{t+1}\right)\right) \\
-\beta_g*\frac{\partial}{\partial Y_t}\left(1+r+k^BL_{t+1}^B+k^B\left(B_{t+1}-B^c\right)\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\right)*u''* \\
\left[\frac{\partial B_{t+2}}{\partial Y_t}+\tau^y\rho-\left(1+r+k^BL_{t+1}^B+k^B\left(B_{t+1}-B^c\right)\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\right)\frac{\partial B_{t+1}}{\partial Y_t}\right] \\
-\beta_g\left[1+r+k^BL_{t+1}^B+k^B\left(B_{t+1}-B^c\right)\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\right]*\frac{\partial}{\partial Y_t}\left(u''\left(G_{t+1}\right)\right)* \\
\left[\frac{\partial B_{t+2}}{\partial Y_t}+\tau^y\rho-\left(1+r+k^BL_{t+1}^B+k^B\left(B_{t+1}-B^c\right)\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\right)\frac{\partial B_{t+1}}{\partial Y_t}\right] \\
-\beta_g\left[1+r+k^BL_{t+1}^B+k^B\left(B_{t+1}-B^c\right)\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\right]*u''* \\
\frac{\partial}{\partial Y_t}\left(\frac{\partial B_{t+2}}{\partial Y_t}+\tau^y\rho-\left(1+r+k^BL_{t+1}^B+k^B\left(B_{t+1}-B^c\right)\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\right)\frac{\partial B_{t+1}}{\partial Y_t}\right)
\end{array}\right].
$$

Hence,

$$
q_{Y_tY_t} = \left[\begin{array}{c}
\left(\frac{\partial B_{t+1}}{\partial Y_t} + \tau^y\right)^2 u''' + \frac{\partial^2 B_{t+1}}{\partial^2 Y_t} u'' \\
-\beta_g k^B \left[\begin{array}{c} 2\left(\frac{\partial^2 L^B_{t+1}}{\partial^2 B_{t+1}}\left(\frac{\partial B_{t+1}}{\partial Y_t}\right)^2 + \frac{\partial L^B_{t+1}}{\partial B_{t+1}}\frac{\partial^2 B_{t+1}}{\partial^2 Y_t}\right) + \left(\frac{\partial^2 L^B_{t+1}}{\partial^2 B_{t+1}}\left(\frac{\partial B_{t+1}}{\partial Y_t}\right)^2\right) \\ + \left(\overline{B} - B^c\right)\frac{\partial^3 L^B_{t+1}}{\partial^3 B_{t+1}}\left(\frac{\partial B_{t+1}}{\partial Y_t}\right)^2 + \left(\overline{B} - B^c\right)\frac{\partial^2 L^B_{t+1}}{\partial^2 B_{t+1}}\frac{\partial^2 B_{t+1}}{\partial^2 Y_t} \end{array}\right] u' \\
-\beta_g k^B \gamma \left[\begin{array}{c} 2\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right) + \\ \gamma\left(\overline{B} - B^c\right)\left(\overline{L^B} - 3\left(\overline{L^B}\right)^2 + 2\left(\overline{L^B}\right)^3\right) \end{array}\right] h_1^Y * \\
\left[\frac{\partial B_{t+2}}{\partial Y_t} + \tau^y \rho - \left(1 + r + k^B\overline{L^B} + \gamma k^B\left(\overline{B} - B^c\right)\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right)\right)\frac{\partial B_{t+1}}{\partial Y_t}\right] u'' \\
-\beta_g k^B\left[2\frac{\partial L^B_{t+1}}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial Y_t} + \left(B_{t+1} - B^c\right)\frac{\partial^2 L^B_{t+1}}{\partial^2 B_{t+1}}\frac{\partial B_{t+1}}{\partial Y_t}\right] * \\
\left[\frac{\partial B_{t+2}}{\partial Y_t} + \tau^y \rho - \left(1 + r + k^B\overline{L^B} + \gamma k^B\left(\overline{B} - B^c\right)\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right)\right)\frac{\partial B_{t+1}}{\partial Y_t}\right] u'' \\
-\beta_g\left[1 + r + k^B\overline{L^B} + \gamma k^B\left(\overline{B} - B^c\right)\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right)\right] * \\
\left[\frac{\partial B_{t+2}}{\partial Y_t} + \tau^y \rho - \left(1 + r + k^B\overline{L^B} + \gamma k^B\left(\overline{B} - B^c\right)\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right)\right)\frac{\partial B_{t+1}}{\partial Y_t}\right]^2 u''' \\
-\beta_g\left[1 + r + k^B\overline{L^B} + \gamma k^B\left(\overline{B} - B^c\right)\left(\overline{L^B} - \left(\overline{L^B}\right)^2\right)\right] * \\
\left[\begin{array}{c} \frac{\partial^2 B_{t+2}}{\partial^2 Y_t} - k^B\left[2\frac{\partial L^B_{t+1}}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial Y_t} + \left(B_{t+1} - B^c\right)\frac{\partial^2 L^B_{t+1}}{\partial^2 B_{t+1}}\frac{\partial B_{t+1}}{\partial Y_t}\right]\frac{\partial B_{t+1}}{\partial Y_t} \\ -\left(1 + r + k^B\overline{L^B} + \gamma k^B\left(\overline{B} - B^c\right)\left(\overline{L^B} - \overline{L^B}^2\right)\right)\frac{\partial^2 B_{t+1}}{\partial^2 Y_t} \end{array}\right] u''
\end{array}\right]
$$

Hence, also using (2.42)

$$
q_{Y_tY_t} = \left[\begin{array}{c}
\left(h_1^Y + \tau^y\right)^2 u''' + h_{11}^Y u'' - \\
\beta_g k^B \left[\begin{array}{c}
2 \left[\gamma^2 \left(\overline{L^B} - 3 \left(\overline{L^B}\right)^2 + 2 \left(\overline{L^B}\right)^3 \right) \left(h_1^Y\right)^2 + \gamma \left(\overline{L^B} - \left(\overline{L^B}\right)^2 \right) h_{11}^Y \right] \\
+\gamma^2 \left(\overline{L^B} - 3 \left(\overline{L^B}\right)^2 + 2 \left(\overline{L^B}\right)^3 \right) \left(h_1^Y\right)^2 + \\
\left(\overline{B} - B^c\right) \gamma^2 \left(\overline{L^B} - 3 \left(\overline{L^B}\right)^2 + 2 \left(\overline{L^B}\right)^3 \right) h_{11}^Y \\
+ \left(\overline{B} - B^c\right) \gamma^3 \left(\overline{L^B} - 7 \left(\overline{L^B}\right)^2 + 12 \left(\overline{L^B}\right)^3 - 6 \left(\overline{L^B}\right)^4 \right) \left(h_1^Y\right)^2
\end{array} \right] u' \\
-\beta_g k^B \gamma \left[2 \left(\overline{L^B} - \left(\overline{L^B}\right)^2 \right) + \gamma \left(\overline{B} - B^c\right) \left(\overline{L^B} - 3 \left(\overline{L^B}\right)^2 + 2 \left(\overline{L^B}\right)^3 \right) \right] h_1^Y * \\
u'' \left[h_1^B h_1^Y + h_1^Y \rho + \tau^y \rho - \left(1 + r + k^B \overline{L^B} + \gamma k^B \left(\overline{B} - B^c\right) \left(\overline{L^B} - \left(\overline{L^B}\right)^2 \right) \right) h_1^Y \right] \\
-\beta_g k^B \gamma \left[2 \left(\overline{L^B} - \left(\overline{L^B}\right)^2 \right) + \gamma \left(\overline{B} - B^c\right) \left(\overline{L^B} - 3 \left(\overline{L^B}\right)^2 + 2 \left(\overline{L^B}\right)^3 \right) \right] h_1^Y * \\
u'' \left[h_1^B h_1^Y + h_1^Y \rho + \tau^y \rho - \left(1 + r + k^B \overline{L^B} + \gamma k^B \left(\overline{B} - B^c\right) \left(\overline{L^B} - \left(\overline{L^B}\right)^2 \right) \right) h_1^Y \right] \\
-u''' \left[\begin{array}{c}
h_1^B h_1^Y + h_1^Y \rho + \tau^y \rho - \\
\left(1 + r + k^B \overline{L^B} + \gamma k^B \left(\overline{B} - B^c\right) \left(\overline{L^B} - \left(\overline{L^B}\right)^2 \right) \right) h_1^Y
\end{array} \right]^2 \\
-u'' \left[\begin{array}{c}
h_{11}^B \left(h_1^Y\right)^2 + 2 h_{11}^{BY} \rho h_1^Y + h_{11}^Y \left(h_1^B + \rho^2\right) - \\
k^B \gamma \left[\begin{array}{c}
2 \left(\overline{L^B} - \left(\overline{L^B}\right)^2 \right) + \\
\gamma \left(\overline{B} - B^c\right) \left(\overline{L^B} - 3 \left(\overline{L^B}\right)^2 + 2 \left(\overline{L^B}\right)^3 \right)
\end{array} \right] \left(h_1^Y\right)^2 \\
- \left(1 + r + k^B \overline{L^B} + \gamma k^B \left(\overline{B} - B^c\right) \left(\overline{L^B} - \left(\overline{L^B}\right)^2 \right) \right) h_{11}^Y
\end{array} \right]
\end{array} \right].
$$

Thus, using (2.42) again, and isolating the three unknown terms h_{11}^{Y}, h_{11}^{BY} and h_{11}^{B}; we obtain

$$
q_{Y_tY_t} = \left[\begin{array}{c}
h_{11}^{Y}\left[\begin{array}{c}
\left[2+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)-h_1^B-\rho^2\right]u''- \\
\beta_g k^B\gamma\left[\begin{array}{c}
2\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)+ \\
\gamma\left(\overline{B}-B^c\right)\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right)
\end{array}\right]u'
\end{array}\right] \\
-2\rho h_1^Y h_{11}^{BY}u''-\left(h_1^Y\right)^2 h_{11}^B u''- \\
\beta_g k^B\gamma^2\left[\begin{array}{c}
3\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right)+ \\
\left(\overline{B}-B^c\right)\gamma\left(\overline{L^B}-7\left(\overline{L^B}\right)^2+12\left(\overline{L^B}\right)^3-6\left(\overline{L^B}\right)^4\right)
\end{array}\right]\left(h_1^Y\right)^2 u' \\
+k^B\gamma\left[2\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)+\gamma\left(\overline{B}-B^c\right)\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right)\right]* \\
\left[3\left(h_1^Y\right)^2-2\beta_g h_1^B\left(h_1^Y\right)^2-2\beta_g\left(h_1^Y\right)^2\rho-2\beta_g h_1^Y\tau^y\rho\right]u''+ \\
\left\{\begin{array}{c}
\left(h_1^Y+\tau^y\right)^2- \\
\left[\begin{array}{c}
h_1^B h_1^Y+h_1^Y\rho+\tau^y\rho- \\
\left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)h_1^Y
\end{array}\right]^2
\end{array}\right\}u'''
\end{array}\right]
$$

$$
\Rightarrow\quad q_{Y_tY_t}\widetilde{Y}_t^2=\left(Vh_{11}^Y+Xh_{11}^{BY}+Th_{11}^B+Z\right)\widetilde{Y}_t^2. \tag{2.69}
$$

This equation contains three variables that need to be solved for (h_{11}^{Y}, h_{11}^{BY} and h_{11}^{B}).

Derivation of $q_{Y_t\sigma_\varepsilon}$ Next, we compute $q_{Y\sigma_\varepsilon}$. For that, we derive q_{σ_ε} with respect to Y_t around the point $\left(\overline{B},\overline{Y},0\right)$:

$$
q_{Y\sigma_\varepsilon}=\left[\begin{array}{c}
\frac{\partial}{\partial Y_t}\left(u''\left(G_t\right)\right)*\left(\frac{\partial B_{t+1}}{\partial\sigma_\varepsilon}\right)+u''*\frac{\partial}{\partial Y_t}\left(\frac{\partial B_{t+1}}{\partial\sigma_\varepsilon}\right)- \\
\beta_g k^B*\left[2\frac{\partial}{\partial Y_t}\left(\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial\sigma_\varepsilon}\right)+\frac{\partial}{\partial Y_t}\left(\left(B_{t+1}-B^c\right)\frac{\partial^2 L_{t+1}^B}{\partial^2 B_{t+1}}\frac{\partial B_{t+1}}{\partial\sigma_\varepsilon}\right)\right]*u' \\
-\beta_g k^B\left[2\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial\sigma_\varepsilon}+\left(\overline{B}-B^c\right)\frac{\partial^2 L_{t+1}^B}{\partial^2 B_{t+1}}\frac{\partial B_{t+1}}{\partial\sigma_\varepsilon}\right]*\frac{\partial}{\partial Y_t}\left(u'\left(G_{t+1}\right)\right)- \\
\beta_g*\frac{\partial}{\partial Y_t}\left(1+r+k^B L_{t+1}^B+k^B\left(B_{t+1}-B^c\right)\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\right)*u''* \\
\left(\frac{\partial B_{t+2}}{\partial\sigma_\varepsilon}-\left(1+r+k^B L_{t+1}^B\right)\frac{\partial B_{t+1}}{\partial\sigma_\varepsilon}-k^B\left(B_{t+1}-B^c\right)\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial\sigma_\varepsilon}+\tau^y\varepsilon_{t+1}\right) \\
-\beta_g\left[1+r+k^B L_{t+1}^B+k^B\left(B_{t+1}-B^c\right)\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\right]*\frac{\partial}{\partial Y_t}\left(u''\left(G_{t+1}\right)\right)* \\
\left(\frac{\partial B_{t+2}}{\partial\sigma_\varepsilon}-\left(1+r+k^B L_{t+1}^B\right)\frac{\partial B_{t+1}}{\partial\sigma_\varepsilon}-k^B\left(B_{t+1}-B^c\right)\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial\sigma_\varepsilon}+\tau^y\varepsilon_{t+1}\right) \\
-\beta_g\left[1+r+k^B L_{t+1}^B+k^B\left(B_{t+1}-B^c\right)\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\right]*u''* \\
\frac{\partial}{\partial Y_t}\left(\begin{array}{c}
\frac{\partial B_{t+2}}{\partial\sigma_\varepsilon}-\left(1+r+k^B L_{t+1}^B\right)\frac{\partial B_{t+1}}{\partial\sigma_\varepsilon} \\
-k^B\left(B_{t+1}-B^c\right)\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial\sigma_\varepsilon}+\tau^y\varepsilon_{t+1}
\end{array}\right)
\end{array}\right].
$$

Hence, also using (2.42),

$$q_{Y\sigma_\varepsilon} = \begin{bmatrix}
\left(\frac{\partial B_{t+1}}{\partial Y_t}+\tau^y\right)h_1^\sigma u''' + u''\frac{\partial^2 B_{t+1}}{\partial Y_t\partial\sigma_\varepsilon} - \\
\beta_g k^B \begin{bmatrix} 2\frac{\partial L^B_{t+1}}{\partial B_{t+1}}\frac{\partial^2 B_{t+1}}{\partial Y_t \partial\sigma_\varepsilon} + 3\frac{\partial^2 L^B_{t+1}}{\partial^2 B_{t+1}}\frac{\partial B_{t+1}}{\partial Y_t}h_1^\sigma + \\ \left(\overline{B}-B^c\right)\frac{\partial^3 L^B_{t+1}}{\partial^3 B_{t+1}}\frac{\partial B_{t+1}}{\partial Y_t}h_1^\sigma + \left(\overline{B}-B^c\right)\frac{\partial^2 L^B_{t+1}}{\partial^2 B_{t+1}}\frac{\partial^2 B_{t+1}}{\partial Y_t\partial\sigma_\varepsilon}\end{bmatrix} u' \\
-\beta_g k^B \left[2\frac{\partial L^B_{t+1}}{\partial B_{t+1}} + \left(\overline{B}-B^c\right)\frac{\partial^2 L^B_{t+1}}{\partial^2 B_{t+1}}\right] h_1^\sigma * \\
\left[\frac{\partial B_{t+2}}{\partial Y_t} + \tau^y\rho - \left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)\frac{\partial B_{t+1}}{\partial Y_t}\right] u'' \\
-\beta_g k^B\left[2\frac{\partial L^B_{t+1}}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial Y_t} + \left(\overline{B}-B^c\right)\frac{\partial^2 L^B_{t+1}}{\partial^2 B_{t+1}}\frac{\partial B_{t+1}}{\partial Y_t}\right] * \\
\begin{bmatrix} h_1^B h_1^\sigma + h_1^Y\varepsilon_{t+1} + h_1^\sigma + \tau^y\varepsilon_{t+1} - \\ \left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)h_1^\sigma\end{bmatrix} u'' \\
- \begin{bmatrix} \frac{\partial B_{t+2}}{\partial Y_t}+\tau^y\rho - \\ \left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right) \frac{\partial B_{t+1}}{\partial Y_t}\end{bmatrix} * \\
\begin{bmatrix} h_1^B h_1^\sigma + h_1^Y \varepsilon_{t+1} + h_1^\sigma + \tau^y\varepsilon_{t+1} - \\ \left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right) h_1^\sigma \end{bmatrix} u''' \\
- \begin{bmatrix} \frac{\partial^2 B_{t+2}}{\partial Y_t\partial\sigma_\varepsilon} - \left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)\frac{\partial^2 B_{t+1}}{\partial Y_t\partial\sigma_\varepsilon} \\ -\left[2\frac{\partial L^B_{t+1}}{\partial B_{t+1}}\frac{\partial B_{t+1}}{\partial Y_t} + \left(\overline{B}-B^c\right)\frac{\partial^2 L^B_{t+1}}{\partial^2 B_{t+1}}\frac{\partial B_{t+1}}{\partial Y_t}\right] h_1^\sigma \end{bmatrix} u''
\end{bmatrix}.$$

From (2.56), we know that $h_1^\sigma = 0$. Using this, and working out, yields:

$$q_{Y\sigma_\varepsilon} = \begin{bmatrix}
+h_{11}^{Y\sigma}u'' - \\
\beta_g k^B\gamma\left[2\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)+\gamma\left(\overline{B}-B^c\right)\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right)\right]h_{11}^{Y\sigma}u' \\
-\beta_g k^B\gamma \begin{bmatrix} 2\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)+ \\ \gamma\left(\overline{B}-B^c\right)\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right) \end{bmatrix} h_1^Y\left(h_1^Y+\tau^y\right)\varepsilon_{t+1}u'' - \\
\begin{bmatrix} h_1^B h_1^Y + h_1^Y\rho+\tau^y\rho - \\ \left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right) h_1^Y\end{bmatrix}\left(h_1^Y+\tau^y\right)\varepsilon_{t+1}u''' - \\
\begin{bmatrix} \left(h_{11}^B h_1^Y + h_{11}^{BY}\rho\right)h_1^\sigma + h_1^B h_{11}^{Y\sigma} + \left(h_{11}^{BY}h_1^Y + h_{11}^Y\rho\right)\varepsilon_{t+1} + h_{11}^{B\sigma}h_1^Y + h_{11}^{Y\sigma}\rho \\ - \left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right) h_{11}^{Y\sigma}\end{bmatrix} u''
\end{bmatrix}.$$

Then, grouping terms, again using $h_1^{\sigma} = 0$ and multiplying by $\widetilde{Y}_t \sigma_{\varepsilon}$, we obtain:

$$
\begin{aligned}
& q_{Y\sigma_{\varepsilon}} \widetilde{Y}_t \sigma_{\varepsilon} \\
= & \left\{ \begin{array}{c}
h_{11}^{Y\sigma} \left[\begin{array}{c}
u'' \left(2 + r + k^B \overline{L^B} + \gamma k^B \left(\overline{B} - B^c \right) \left(\overline{L^B} - \left(\overline{L^B} \right)^2 \right) - h_1^B - \rho \right) \\
-u' \beta_g k^B \gamma \left[\begin{array}{c}
2 \left(\overline{L^B} - \left(\overline{L^B} \right)^2 \right) + \\
\gamma \left(\overline{B} - B^c \right) \left(\overline{L^B} - 3 \left(\overline{L^B} \right)^2 + 2 \left(\overline{L^B} \right)^3 \right)
\end{array} \right]
\end{array} \right] \\
-h_{11}^{B\sigma} u'' h_1^Y - u'' \left(h_{11}^Y \rho + h_{11}^{BY} h_1^Y \right) \varepsilon_{t+1} - \\
\left(h_1^Y + \tau^y \right) * \\
\left[\begin{array}{c}
u'' \beta_g k^B \gamma \left[\begin{array}{c}
2 \left(\overline{L^B} - \left(\overline{L^B} \right)^2 \right) + \\
\gamma \left(\overline{B} - B^c \right) \left(\overline{L^B} - 3 \left(\overline{L^B} \right)^2 + 2 \left(\overline{L^B} \right)^3 \right)
\end{array} \right] h_1^Y + \\
u''' \left[\begin{array}{c}
h_1^B h_1^Y + h_1^Y \rho + \tau^y \rho - \\
\left(1 + r + k^B \overline{L^B} + \gamma k^B \left(\overline{B} - B^c \right) \left(\overline{L^B} - \left(\overline{L^B} \right)^2 \right) \right) h_1^Y
\end{array} \right]
\end{array} \right] \varepsilon_{t+1}
\end{array} \right\} \widetilde{Y}_t \sigma_{\varepsilon} \\
= & \left(O_1 h_{11}^{Y\sigma} - W h_{11}^{B\sigma} - \Delta \varepsilon_{t+1} \right) \widetilde{Y}_t \sigma_{\varepsilon}.
\end{aligned}
$$

Applying expectations yields

$$
\mathrm{E}_t \left[q_{Y\sigma_{\varepsilon}} \widetilde{Y}_t \sigma_{\varepsilon} \right] = \left(O_1 h_{11}^{Y\sigma} - W h_{11}^{B\sigma} \right) \widetilde{Y}_t \sigma_{\varepsilon}. \tag{2.70}
$$

Derivation of $q_{\sigma_{\varepsilon}\sigma_{\varepsilon}}$ Differentiating $q_{\sigma_{\varepsilon}}$ with respect to σ_{ε} at the point $\left(\overline{B}, \overline{Y}, 0 \right)$:

$$
q_{\sigma_{\varepsilon}\sigma_{\varepsilon}} = \left[\begin{array}{c}
\frac{\partial}{\partial \sigma_{\varepsilon}} \left(u'' \left(G_t \right) \right) * \left(\frac{\partial B_{t+1}}{\partial \sigma_{\varepsilon}} \right) + u'' * \frac{\partial}{\partial \sigma_{\varepsilon}} \left(\frac{\partial B_{t+1}}{\partial \sigma_{\varepsilon}} \right) - \\
\beta_g k^B * \left[2 \frac{\partial}{\partial \sigma_{\varepsilon}} \left(\frac{\partial L_{t+1}^B}{\partial B_{t+1}} \frac{\partial B_{t+1}}{\partial \sigma_{\varepsilon}} \right) + \frac{\partial}{\partial \sigma_{\varepsilon}} \left(\left(B_{t+1} - B^c \right) \frac{\partial^2 L_{t+1}^B}{\partial^2 B_{t+1}} \frac{\partial B_{t+1}}{\partial \sigma_{\varepsilon}} \right) \right] * u' \\
-\beta_g k^B \left[2 \frac{\partial L_{t+1}^B}{\partial B_{t+1}} \frac{\partial B_{t+1}}{\partial \sigma_{\varepsilon}} + \left(\overline{B} - B^c \right) \frac{\partial^2 L_{t+1}^B}{\partial^2 B_{t+1}} \frac{\partial B_{t+1}}{\partial \sigma_{\varepsilon}} \right] * \frac{\partial}{\partial \sigma_{\varepsilon}} \left(u' \left(G_{t+1} \right) \right) - \\
\beta_g * \frac{\partial}{\partial \sigma_{\varepsilon}} \left(1 + r + k^B L_{t+1}^B + k^B \left(B_{t+1} - B^c \right) \frac{\partial L_{t+1}^B}{\partial B_{t+1}} \right) * u'' * \\
\left(\frac{\partial B_{t+2}}{\partial \sigma_{\varepsilon}} - \left(1 + r + k^B L_{t+1}^B \right) \frac{\partial B_{t+1}}{\partial \sigma_{\varepsilon}} - k^B \left(B_{t+1} - B^c \right) \frac{\partial L_{t+1}^B}{\partial B_{t+1}} \frac{\partial B_{t+1}}{\partial \sigma_{\varepsilon}} + \tau^y \varepsilon_{t+1} \right) \\
-\beta_g \left[1 + r + k^B L_{t+1}^B + k^B \left(B_{t+1} - B^c \right) \frac{\partial L_{t+1}^B}{\partial B_{t+1}} \right] * \frac{\partial}{\partial \sigma_{\varepsilon}} \left(u'' \left(G_{t+1} \right) \right) * \\
\left(\frac{\partial B_{t+2}}{\partial \sigma_{\varepsilon}} - \left(1 + r + k^B L_{t+1}^B \right) \frac{\partial B_{t+1}}{\partial \sigma_{\varepsilon}} - k^B \left(B_{t+1} - B^c \right) \frac{\partial L_{t+1}^B}{\partial B_{t+1}} \frac{\partial B_{t+1}}{\partial \sigma_{\varepsilon}} + \tau^y \varepsilon_{t+1} \right) \\
-\beta_g \left[1 + r + k^B L_{t+1}^B + k^B \left(B_{t+1} - B^c \right) \frac{\partial L_{t+1}^B}{\partial B_{t+1}} \right] * u'' * \\
\frac{\partial}{\partial \sigma_{\varepsilon}} \left(\frac{\partial B_{t+2}}{\partial \sigma_{\varepsilon}} - \left(1 + r + k^B L_{t+1}^B \right) \frac{\partial B_{t+1}}{\partial \sigma_{\varepsilon}} - k^B \left(B_{t+1} - B^c \right) \frac{\partial L_{t+1}^B}{\partial B_{t+1}} \frac{\partial B_{t+1}}{\partial \sigma_{\varepsilon}} + \tau^y \varepsilon_{t+1} \right)
\end{array} \right].
$$

Working out and using (2.42)

$$q_{\sigma_\varepsilon\sigma_\varepsilon}=\left[\begin{array}{c} \beta_g k^B\left[\begin{array}{c} u'''\left(h_1^\sigma\right)^2+u''\frac{\partial^2 B_{t+1}}{\partial\sigma_\varepsilon^2}- \\ 2\frac{\partial L_{t+1}^B}{\partial B_{t+1}}\frac{\partial^2 B_{t+1}}{\partial\sigma_\varepsilon^2}+3\frac{\partial^2 L_{t+1}^B}{\partial B_{t+1}^2}\left(h_1^\sigma\right)^2+ \\ \left(\overline{B}-B^c\right)\frac{\partial^3 L_{t+1}^B}{\partial B_{t+1}^3}\left(h_1^\sigma\right)^2+\left(\overline{B}-B^c\right)\frac{\partial^2 L_{t+1}^B}{\partial B_{t+1}^2}\frac{\partial^2 B_{t+1}}{\partial\sigma_\varepsilon^2}\end{array}\right]u' \\ -\beta_g k^B\left[2\frac{\partial L_{t+1}^B}{\partial B_{t+1}}+\left(\overline{B}-B^c\right)\frac{\partial^2 L_{t+1}^B}{\partial B_{t+1}^2}\right]h_1^\sigma * \\ \left[\begin{array}{c} h_1^B h_1^\sigma+h_1^Y\varepsilon_{t+1}+h_1^\sigma+\tau^y\varepsilon_{t+1}- \\ \left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)h_1^\sigma\end{array}\right]u'' \\ -\beta_g k^B\left[2\frac{\partial L_{t+1}^B}{\partial B_{t+1}}+\left(\overline{B}-B^c\right)\frac{\partial^2 L_{t+1}^B}{\partial B_{t+1}^2}\right]h_1^\sigma * \\ \left[\begin{array}{c} h_1^B h_1^\sigma+h_1^Y\varepsilon_{t+1}+h_1^\sigma+\tau^y\varepsilon_{t+1}- \\ \left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)h_1^\sigma\end{array}\right]u''- \\ \left(\begin{array}{c} h_1^B h_1^\sigma+h_1^Y\varepsilon_{t+1}+h_1^\sigma+\tau^y\varepsilon_{t+1}- \\ \left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)h_1^\sigma\end{array}\right)^2 u'''- \\ \left[\begin{array}{c} \frac{\partial^2 B_{t+2}}{\partial\sigma_\varepsilon^2}-\left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)\frac{\partial^2 B_{t+1}}{\partial\sigma_\varepsilon^2} \\ -\left[2\frac{\partial L_{t+1}^B}{\partial B_{t+1}}+\left(\overline{B}-B^c\right)\frac{\partial^2 L_{t+1}^B}{\partial B_{t+1}^2}\right]\left(h_1^\sigma\right)^2\end{array}\right]u''\end{array}\right].$$

Using $h_1^\sigma=0$ and working out further, we obtain

$$q_{\sigma_\varepsilon\sigma_\varepsilon}=\left[\begin{array}{c} +u''h_{11}^\sigma-u'''\left(\left(h_1^Y+\tau^y\right)\varepsilon_{t+1}\right)^2- \\ u'\beta_g k^B\gamma\left[2\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)+\gamma\left(\overline{B}-B^c\right)\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right)\right]h_{11}^\sigma \\ -u''\left[\begin{array}{c} \left(h_{11}^B h_1^\sigma+h_{11}^{BY}\varepsilon_{t+1}+h_{11}^{B\sigma}\right)h_1^\sigma+\left(h_{11}^{BY}h_1^\sigma+h_{11}^{Y\sigma}\right)\varepsilon_{t+1} \\ +h_{11}^Y\varepsilon_{t+1}^2+h_{11}^{B\sigma}h_1^\sigma+h_{11}^{Y\sigma}\varepsilon_{t+1}+\left(1+h_1^B\right)h_{11}^\sigma- \\ \left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)\right)h_{11}^\sigma\end{array}\right]\end{array}\right].$$

Next, grouping terms, using again $h_1^\sigma=0$, and multiplying by σ_ε^2:

$$\begin{aligned} & q_{\sigma_\varepsilon\sigma_\varepsilon}\sigma_\varepsilon^2 \\ = & \left\{\begin{array}{c} h_{11}^\sigma\left[\begin{array}{c} u''\left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)-h_1^B\right) \\ -u'\beta_g k^B\gamma\left[\begin{array}{c} 2\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)+ \\ \gamma\left(\overline{B}-B^c\right)\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right)\end{array}\right]\end{array}\right] \\ -u''\left(h_{11}^{BY}h_1^\sigma+2h_{11}^{Y\sigma}\right)\varepsilon_{t+1}-\left[u'''\left(h_1^Y+\tau^y\right)^2+u''h_{11}^Y\right]\varepsilon_{t+1}^2\end{array}\right\}\sigma_\varepsilon^2 \\ = & \left\{\Theta h_{11}^\sigma-2u''h_{11}^{Y\sigma}\varepsilon_{t+1}-\left[u'''\left(h_1^Y+\tau^y\right)^2+u''h_{11}^Y\right]\varepsilon_{t+1}^2\right\}\sigma_\varepsilon^2,\end{aligned}$$

since again $h_1^\sigma=0$. Finally, applying expectations:

$$\begin{aligned} \mathrm{E}_t\left[q_{\sigma_\varepsilon\sigma_\varepsilon}\sigma_\varepsilon^2\right] &= \mathrm{E}_t\left\{\left\{\Theta h_{11}^\sigma-2u''h_{11}^{Y\sigma}\varepsilon_{t+1}-\left[u'''\left(h_1^Y+\tau^y\right)^2+u''h_{11}^Y\right]\varepsilon_{t+1}^2\right\}\sigma_\varepsilon^2\right\} \\ &= \left\{\Theta h_{11}^\sigma-\left[u'''\left(h_1^Y+\tau^y\right)^2+u''h_{11}^Y\right]\sigma_\varepsilon^2\right\}\sigma_\varepsilon^2. \qquad (2.71)\end{aligned}$$

Working out the second-order Taylor expansion

Substituting (2.66), (2.67), (2.68), (2.69), (2.70) and (2.71) into (2.58):

$$\mathrm{E}_t\left[q\left(B_t,Y_t,\sigma_\varepsilon\right)\right]\simeq\frac{1}{2}\left\{\left[\begin{array}{c}\left(Mh_{11}^B+N\right)\widetilde{B}_t^2+2\left(Oh_{11}^{BY}+Ph_{11}^B+Q\right)\widetilde{B}_t\widetilde{Y}_t\\ +2Rh_{11}^{B\sigma}\widetilde{B}_t\sigma_\varepsilon+\left(Vh_{11}^Y+Xh_{11}^{BY}+Th_{11}^B+Z\right)\widetilde{Y}_t^2\\ +2\left(O_1h_{11}^{Y\sigma}-Wh_{11}^{B\sigma}\right)\widetilde{Y}_t\sigma_\varepsilon+\\ \left\{\Theta h_{11}^\sigma-\left[u'''\left(h_1^Y+\tau^y\right)^2+u''h_{11}^Y\right]\sigma_\varepsilon^2\right\}\sigma_\varepsilon^2\end{array}\right]\right\}=0. \tag{2.72}$$

This must hold for all possible 6-tuples $\left\{\widetilde{B}_t^2,\widetilde{B}_t\widetilde{Y}_t,\widetilde{B}_t\sigma_\varepsilon,\widetilde{Y}_t^2,\widetilde{Y}_t\sigma_\varepsilon,\sigma_\varepsilon^2\right\}$, implying six equations in six unknowns. First,

$$Mh_{11}^B+N=0\Leftrightarrow h_{11}^B=-N/M,$$

since $M\neq 0$. So, $h_{11}^B\equiv\frac{\partial^2 h}{\partial B_t^2}$ is equal to:

$$h_{11}^B=\frac{\left[\begin{array}{c}\beta_g k^B\gamma^2\left[\begin{array}{c}3\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right)+\\ \gamma\left(\overline{B}-B^c\right)\left(\overline{L^B}-7\overline{L^B}^2+12\overline{L^B}^3-6\overline{L^B}^4\right)\end{array}\right]\left(h_1^B\right)^2u'-\\ k^B\gamma z_8\left(-1+3\left(h_1^B\right)^2-2\beta_g\left(h_1^B\right)^3\right)u''-\left(h_1^B-\frac{1}{\beta_g}\right)^2\left[1-\left(h_1^B\right)^2\right]u'''\end{array}\right]}{\left[\left(1+\frac{1}{\beta_g}-\left(h_1^B\right)^2-h_1^B\right)u''-\beta_g k^B\gamma z_8 u'\right]}, \tag{2.73}$$

where

$$z_8\equiv 2\left(\overline{L^B}-\left(\overline{L^B}\right)^2\right)+\gamma\left(\overline{B}-B^c\right)\left(\overline{L^B}-3\left(\overline{L^B}\right)^2+2\left(\overline{L^B}\right)^3\right).$$

Using h_{11}^B, we can solve for h_{11}^{BY} as:

$$Oh_{11}^{BY}+Ph_{11}^B+Q=0\Leftrightarrow h_{11}^{BY}=-\frac{Ph_{11}^B+Q}{O},$$

or

$$h_{11}^{BY}=\frac{\left[\begin{array}{c}h_{11}^B h_1^B h_1^Y u''-\\ \left[h_1^B-\left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\overline{L^B}\left(1-\overline{L^B}\right)\right)\right]u'''*\\ \left[\begin{array}{c}h_1^Y+\tau^y-\left(h_1^B\right)^2h_1^Y-h_1^B\tau^y\rho+\\ \left(1+r+k^B\overline{L^B}-\rho+\gamma k^B\left(\overline{B}-B^c\right)\overline{L^B}\left(1-\overline{L^B}\right)\right)h_1^Bh_1^Y\end{array}\right]+\\ \beta_g k^B\gamma^2\overline{L^B}\left(1-\overline{L^B}\right)\left[3\left(1-2\overline{L^B}\right)+\gamma\left(\overline{B}-B^c\right)\left(1-6\overline{L^B}+6\left(\overline{L^B}\right)^2\right)\right]h_1^Bh_1^Yu'\\ -k^B\gamma z_8\left(3h_1^Bh_1^Y-2\beta_g\left(h_1^B\right)^2h_1^Y-\beta_g h_1^B\tau^y\rho-\beta_g h_1^Bh_1^Y\rho\right)u''\end{array}\right]}{\left[2+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\overline{L^B}\left(1-\overline{L^B}\right)-h_1^B\left(1+\rho\right)\right]u''-\beta_g k^B\gamma z_8 u'}. \tag{2.74}$$

Next, we find h_{11}^Y as:

$$Vh_{11}^Y+Xh_{11}^{BY}+Th_{11}^B+Z=0\Leftrightarrow h_{11}^Y=-\frac{Xh_{11}^{BY}+Th_{11}^B+Z}{V},$$

or

$$h_{11}^{Y}=\frac{\left[\begin{array}{c}2\rho h_1^Y h_{11}^{BY}u''+\left(h_1^Y\right)^2 h_{11}^B u''+\\ \beta_g k^B\gamma^2\overline{L^B}\left(1-\overline{L^B}\right)\left[\begin{array}{c}3\left(1-2\overline{L^B}\right)+\\ \gamma\left(\overline{B}-B^c\right)\left(1-6\overline{L^B}+6\left(\overline{L^B}\right)^2\right)\end{array}\right]\left(h_1^Y\right)^2u'-\\ k^B\gamma\overline{L^B}\left(1-\overline{L^B}\right)\left[\begin{array}{c}2+\\ \gamma\left(\overline{B}-B^c\right)\left(1-2\overline{L^B}\right)\end{array}\right]\left[\begin{array}{c}3\left(h_1^Y\right)^2-2\beta_g h_1^B\left(h_1^Y\right)^2-\\ 2\beta_g\left(h_1^Y\right)^2\rho-2\beta_g h_1^Y\tau^y\rho\end{array}\right]u''\\ -\left\{\begin{array}{c}\left(h_1^Y+\tau^y\right)^2-\\ \left[\begin{array}{c}h_1^B h_1^Y+h_1^Y\rho+\tau^y\rho-\\ \left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\overline{L^B}\left(1-\overline{L^B}\right)\right)h_1^Y\end{array}\right]^2\end{array}\right\}u'''\end{array}\right]}{\left[\begin{array}{c}\left[2+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\overline{L^B}\left(1-\overline{L^B}\right)-h_1^B-\rho^2\right]u''\\ -\beta_g k^B\gamma\overline{L^B}\left(1-\overline{L^B}\right)\left[2+\gamma\left(\overline{B}-B^c\right)\left(1-2\overline{L^B}\right)\right]u'\end{array}\right]}. \tag{2.75}$$

Further,

$$h_{11}^{B\sigma}=0. \tag{2.76}$$

Hence,

$$h_{11}^{Y\sigma}=0. \tag{2.77}$$

Finally, given (2.55) and (2.75), we obtain:

$$\Theta h_{11}^{\sigma}-\left[u'''\left(h_1^Y+\tau^y\right)^2+u''h_{11}^Y\right]\sigma_\varepsilon^2=0\Leftrightarrow h_{11}^{\sigma}=\frac{\left[u'''\left(h_1^Y+\tau^y\right)^2+u''h_{11}^Y\right]\sigma_\varepsilon^2}{\Theta},$$

or

$$h_{11}^{\sigma}=\frac{\left[\left(h_1^Y+\tau^y\right)^2u'''+h_{11}^Y u''\right]\sigma_\varepsilon^2}{\left\{\begin{array}{c}\left(1+r+k^B\overline{L^B}+\gamma k^B\left(\overline{B}-B^c\right)\overline{L^B}\left(1-\overline{L^B}\right)-h_1^B\right)u''\\ -\beta_g k^B\gamma\overline{L^B}\left(1-\overline{L^B}\right)\left[2+\gamma\left(\overline{B}-B^c\right)\left(1-2\overline{L^B}\right)\right]u'\end{array}\right\}}. \tag{2.78}$$

Second-order approximation of B_{t+1}

The second-order approximation of the true non-linear solution of B_{t+1} around the point $\left(\overline{B},\overline{Y},0\right)$ can be written as

$$B_{t+1}\approx\left\{\begin{array}{c}h\left(\overline{B},\overline{Y},0\right)+h_1^B\widetilde{B}_t+h_1^Y\widetilde{Y}_t+h_1^\sigma\sigma_\varepsilon+\\ \frac{1}{2}\left[h_{11}^B\widetilde{B}_t^2+2h_{11}^{BY}\widetilde{B}_t\widetilde{Y}_t+2h_{11}^{B\sigma}\widetilde{B}_t\sigma_\varepsilon+h_{11}^Y\widetilde{Y}_t^2+2h_{11}^{Y\sigma}\widetilde{Y}_t\sigma_\varepsilon+h_{11}^\sigma\sigma_\varepsilon^2\right]\end{array}\right\}.$$

Hence, using $h_1^\sigma=h_{11}^{B\sigma}=h_{11}^{Y\sigma}=0$,

$$B_{t+1}\approx\left\{\begin{array}{c}h\left(\overline{B},\overline{Y},0\right)+h_1^B\widetilde{B}_t+h_1^Y\widetilde{Y}_t+\\ \frac{1}{2}\left[h_{11}^B\widetilde{B}_t^2+2h_{11}^{BY}\widetilde{B}_t\widetilde{Y}_t+h_{11}^Y\widetilde{Y}_t^2+h_{11}^\sigma\sigma_\varepsilon^2\right]\end{array}\right\}. \tag{2.79}$$

2.F.3 Stochastic steady state

We take the unconditional expectation of debt in (2.79) and use the definition of the variables with a tilde:

$$\mathrm{E}\left[B_{t+1}\right] \approx \left\{ \begin{array}{c} \overline{B} + h_1^B \left[\mathrm{E}\left(B_t\right) - \overline{B}\right] + h_1^Y \left[\mathrm{E}\left(Y_t\right) - \overline{Y}\right] + \\ \frac{1}{2}\left[\begin{array}{c} h_{11}^B \mathrm{E}\left[\left(B_t - \overline{B}\right)^2\right] + 2h_{11}^{BY}\mathrm{E}\left[\left(B_t - \overline{B}\right)\left(Y_t - \overline{Y}\right)\right] \\ + h_{11}^Y \mathrm{E}\left[\left(Y_t - \overline{Y}\right)^2\right] + h_{11}^{\sigma}\sigma_{\varepsilon}^2 \end{array} \right] \end{array} \right\}.$$

We have that $\mathrm{E}(Y_t) = \overline{Y}$ and $\mathrm{Var}(Y) = \frac{\sigma_\varepsilon^2}{1-\rho^2}$. Define

$$\widehat{B} \equiv \mathrm{E}\left(B_t\right) = \mathrm{E}\left[B_{t+1}\right].$$

Let's work out the terms in the expression for $\mathrm{E}[B_{t+1}]$:

$$\mathrm{E}\left(Y_t\right) - \overline{Y} = 0,$$

$$\mathrm{E}\left[\left(Y_t - \overline{Y}\right)^2\right] = \mathrm{Var}\left(Y_t\right) = \frac{\sigma_\varepsilon^2}{1-\rho^2},$$

$$\begin{aligned} \mathrm{E}\left[\left(B_t - \overline{B}\right)^2\right] &= \mathrm{E}\left\{\left[\left(B_t - \widehat{B}\right) + \left(\widehat{B} - \overline{B}\right)\right]^2\right\} \\ &= \mathrm{E}\left[\left(B_t - \widehat{B}\right)^2\right] + 2\mathrm{E}\left[\left(B_t - \widehat{B}\right)\left(\widehat{B} - \overline{B}\right)\right] + \mathrm{E}\left[\left(\widehat{B} - \overline{B}\right)^2\right] \\ &= \mathrm{E}\left[\left(B_t - \widehat{B}\right)^2\right] + \left(\widehat{B} - \overline{B}\right)^2 = \mathrm{Var}\left(B_t\right) + \left(\widehat{B} - \overline{B}\right)^2, \end{aligned}$$

$$\begin{aligned} \mathrm{E}\left[\left(B_t - \overline{B}\right)\left(Y_t - \overline{Y}\right)\right] &= \mathrm{E}\left\{\left[\left(B_t - \widehat{B}\right) + \left(\widehat{B} - \overline{B}\right)\right]\left(Y_t - \overline{Y}\right)\right\} \\ &= \mathrm{E}\left[\left(B_t - \widehat{B}\right)\left(Y_t - \overline{Y}\right)\right] + \mathrm{E}\left[\left(\widehat{B} - \overline{B}\right)\left(Y_t - \overline{Y}\right)\right] \\ &= \mathrm{E}\left[\left(B_t - \widehat{B}\right)\left(Y_t - \overline{Y}\right)\right] = \mathrm{Cov}\left(B_t, Y_t\right). \end{aligned}$$

Substitute these terms into the expression for $\mathrm{E}[B_{t+1}]$, to give:

$$\widehat{B} - \overline{B} \approx \frac{1}{1 - h_1^B}\left\{\left[\begin{array}{c} \frac{1}{2}h_{11}^B\left[\mathrm{E}\left(B_t - \widehat{B}\right)^2 + \left(\widehat{B} - \overline{B}\right)^2\right] + \\ h_{11}^{BY}\mathrm{E}\left[\left(B_t - \widehat{B}\right)\left(Y_t - \overline{Y}\right)\right] + \frac{1}{2}h_{11}^Y\frac{\sigma_\varepsilon^2}{1-\rho^2} + \frac{1}{2}h_{11}^{\sigma}\sigma_\varepsilon^2 \end{array} \right]\right\}.$$

Notice that the difference between $\widehat{B}$ and $\overline{B}$ is of second-order magnitude. Approximating terms of higher than second order by zero, we have:

$$\mathrm{E}\left(\widehat{B} - \overline{B}\right)^2 \simeq 0,$$

and

$$\begin{aligned}
\mathrm{E}\left[\left(B_t - \widehat{B}\right)\widetilde{Y}_t\right] &= \mathrm{E}\left[\left(B_{t+1} - \widehat{B}\right)\widetilde{Y}_{t+1}\right] = \mathrm{E}\left[\left(\overline{B} + h_1^B\widetilde{B}_t + h_1^Y\widetilde{Y}_t - \widehat{B}\right)\left(\rho\widetilde{Y}_t + \sigma_\varepsilon\varepsilon_{t+1}\right)\right], \\
&= \mathrm{E}\left[\left(\rho h_1^B\widetilde{B}_t\widetilde{Y}_t + \rho h_1^Y\widetilde{Y}_t^2\right)\right].
\end{aligned}$$

Hence,

$$\mathrm{E}\left[\widetilde{B}_t\widetilde{Y}_t\right] \simeq \left(\frac{\rho h_1^Y}{1-\rho h_1^B}\right)\frac{\sigma_\varepsilon^2}{1-\rho^2},$$

Further,

$$\begin{aligned}
\mathrm{E}\left(B_t - \widehat{B}\right)^2 &= \mathrm{E}\left(B_{t+1} - \widehat{B}\right)^2 \\
&\simeq \mathrm{E}\left[\left(\overline{B} + h_1^B\widetilde{B}_t + h_1^Y\widetilde{Y}_t - \widehat{B}\right)^2\right] \simeq \mathrm{E}\left[\left(h_1^B\widetilde{B}_t + h_1^Y\widetilde{Y}_t\right)^2\right] \\
&= \mathrm{E}\left[\left(h_1^B\right)^2\left(B_t - \overline{B}\right)^2 + 2h_1^B h_1^Y\left(B_t - \overline{B}\right)\left(Y_t - \overline{Y}\right) + \left(h_1^Y\right)^2\left(Y_t - \overline{Y}\right)^2\right] \\
&\simeq \mathrm{E}\left[\left(h_1^B\right)^2\left(B_t - \overline{B}\right)^2\right] + 2\left(\frac{\rho h_1^B\left(h_1^Y\right)^2}{1-\rho h_1^B}\right)\frac{\sigma_\varepsilon^2}{1-\rho^2} + \left(h_1^Y\right)^2\mathrm{E}\left(Y_t - \overline{Y}\right)^2 \\
&= \left(h_1^B\right)^2\mathrm{E}\left[\left(B_t - \overline{B}\right)^2\right] + \left(\frac{2\rho h_1^B}{1-\rho h_1^B} + 1\right)\left(h_1^Y\right)^2\tfrac{\sigma_\varepsilon^2}{1-\rho^2} \\
&= \left(h_1^B\right)^2\mathrm{E}\left[\left(B_t - \overline{B}\right)^2\right] + \left(\frac{1+\rho h_1^B}{1-\rho h_1^B}\right)\left(h_1^Y\right)^2\tfrac{\sigma_\varepsilon^2}{1-\rho^2}.
\end{aligned}$$

Hence,

$$\mathrm{Var}\left(B_t\right) = \mathrm{E}\left(B_t - \widehat{B}\right)^2 \simeq \left(\frac{1+\rho h_1^B}{1-\rho h_1^B}\right)\frac{\left(h_1^Y\right)^2}{1-\left(h_1^B\right)^2}\frac{\sigma_\varepsilon^2}{1-\rho^2}. \tag{2.80}$$

Hence, we obtain:

Solution 2.1 *The unconditional expectation or "stochastic steady state" value of the debt in the debt-based sanction case is given by*

$$\widehat{B} \simeq \overline{B} + \frac{1}{2}\frac{\sigma_\varepsilon^2}{1-h_1^B}\left[\left(\frac{h_{11}^B\left(h_1^Y\right)^2}{1-\left(h_1^B\right)^2}\left(\frac{1+\rho h_1^B}{1-\rho h_1^B}\right) + 2\frac{\rho h_{11}^{BY}h_1^Y}{1-\rho h_1^B} + h_{11}^Y\right)\frac{1}{1-\rho^2} + h_{11}^\sigma\right]. \tag{2.81}$$

2.F.4 Stochastic steady state of government expenditure with debt-based sanction

In (2.38) the only unknown variable is B_{t+1}. However, (2.79) provides a second-order approximation of that variable. Hence, plugging this into (2.38), we can approximate G_t as

$$G_t \approx \left[\begin{array}{c} \overline{B} + h_1^B \left(B_t - \overline{B}\right) + h_1^Y \left(Y_t - \overline{Y}\right) + \\ \frac{1}{2} \left[h_{11}^B \left(B_t - \overline{B}\right)^2 + 2h_{11}^{BY} \left(B_t - \overline{B}\right) \left(Y_t - \overline{Y}\right) + h_{11}^Y \left(Y_t - \overline{Y}\right)^2 + h_{11}^\sigma \sigma_\varepsilon^2 \right] \\ - (1+r) B_t + \tau^y Y_t - k^B \left(B_t - B^c\right) L_t^B \end{array} \right]. \quad (2.82)$$

Next, from (2.38) we know that

$$\overline{G} \equiv f\left(\overline{B}, \overline{Y}, 0\right) = \tau^y \overline{Y} - r\overline{B} - k^B \left(\overline{B} - B^c\right) \overline{L^B}. \quad (2.83)$$

First-order approximation of G_t

Analogous to what we did for B_{t+1}, we take a first-order Taylor expansion of (2.24) around the point $\left(\overline{B}, \overline{Y}, 0\right)$:

$$G_t \approx f\left(\overline{B}, \overline{Y}, 0\right) + f_1^B \widetilde{B}_t + f_1^Y \widetilde{Y}_t + f_1^\sigma \sigma_\varepsilon, \quad (2.84)$$

where $f_1^B \equiv \frac{\partial G_t}{\partial B_t}$, $f_1^Y \equiv \frac{\partial G_t}{\partial Y_t}$ and $f_1^\sigma \equiv \frac{\partial G_t}{\partial \sigma_\varepsilon}$ are evaluated at the point $\left(\overline{B}, \overline{Y}, 0\right)$. Hence, using (2.82) we can calculate these derivatives as follows:

- First order partial derivative of (2.82) with respect to B_t:

$$\frac{\partial G_t}{\partial B_t} = h_1^B + \frac{1}{2}\left[2h_{11}^B \left(B_t - \overline{B}\right) + 2h_{11}^{BY} \left(Y_t - \overline{Y}\right)\right] - (1+r) - k^B L_t^B - k^B \left(B_t - B^c\right) \frac{\partial L_t^B}{\partial B_t}.$$

  If we evaluate this derivative at the point $(B_t, Y_t, \sigma_\varepsilon) = \left(\overline{B}, \overline{Y}, 0\right)$, use (2.40) and (2.42), we arrive at

$$f_1^B = h_1^B - 1/\beta_g. \quad (2.85)$$

- First order partial derivative of (2.82) with respect to Y_t:

$$\frac{\partial G_t}{\partial Y_t} = h_1^Y + \frac{1}{2}\left[2h_{11}^{BY} \left(B_t - \overline{B}\right) + 2h_{11}^Y \left(Y_t - \overline{Y}\right)\right] + \tau^y.$$

  If we evaluate this derivative at the point $(B_t, Y_t, \sigma_\varepsilon) = \left(\overline{B}, \overline{Y}, 0\right)$, we arrive at

$$f_1^Y = h_1^Y + \tau^y. \quad (2.86)$$

- First order partial derivative of (2.82) with respect to σ_ε:

$$\frac{\partial G_t}{\partial \sigma_\varepsilon} = \frac{1}{2}\left[2h_{11}^\sigma \sigma_\varepsilon\right].$$

  Since we evaluate at $(B_t, Y_t, \sigma_\varepsilon) = \left(\overline{B}, \overline{Y}, 0\right)$, we obtain

$$f_1^\sigma = 0. \quad (2.87)$$

- Evaluating the right-hand side of (2.82) at $(B_t, Y_t, \sigma_\varepsilon) = \left(\overline{B}, \overline{Y}, 0\right)$ yields:

$$\overline{G} \equiv f\left(\overline{B}, \overline{Y}, 0\right) = \tau^y \overline{Y} - r\overline{B} - k^B \left(\overline{B} - B^c\right) \overline{L^B}. \quad (2.88)$$

- Hence

$$G_t \approx \overline{G} + \left[h_1^B - (1/\beta_g)\right] \widetilde{B}_t + \left(h_1^Y + \tau^y\right) \widetilde{Y}_t. \quad (2.89)$$

Second-order approximation of G_t

The second-order approximation of (2.24) around the point $\left(\overline{B},\overline{Y},0\right)$ is:

$$G_t \approx \left\{ \begin{array}{c} f\left(\overline{B},\overline{Y},0\right) + f_1^B\widetilde{B}_t + f_1^Y\widetilde{Y}_t + f_1^\sigma\sigma_\varepsilon + \\ \frac{1}{2}\left[f_{11}^B\widetilde{B}_t^2 + 2f_{11}^{BY}\widetilde{B}_t\widetilde{Y}_t + 2f_{11}^{B\sigma}\widetilde{B}_t\sigma_\varepsilon + f_{11}^Y\widetilde{Y}_t^2 + 2f_{11}^{Y\sigma}\widetilde{Y}_t\sigma_\varepsilon + f_{11}^\sigma\sigma_\varepsilon^2\right] \end{array} \right\},$$

where for generic variables X and Z, $f_{11}^X \equiv \frac{\partial^2 G_t}{\partial^2 X_t}$ and $f_{11}^{XZ} \equiv \frac{\partial^2 G_t}{\partial X_t \partial Z_t}$.

- Computation of f_{11}^B: Differentiating $\frac{\partial G_t}{\partial B_t}$ with respect to B_t, we obtain:

$$\frac{\partial^2 G_t}{\partial^2 B_t} = \frac{1}{2}\left[2h_{11}^B\right] - k^B\frac{\partial L_t^B}{\partial B_t} - k^B\frac{\partial L_t^B}{\partial B_t} - k^B\left(B_t - B^c\right)\frac{\partial^2 L_t^B}{\partial B_t^2}.$$

If we evaluate this derivative at the point $\left(\overline{B},\overline{Y},0\right)$ and use (2.48), we arrive to

$$f_{11}^B = h_{11}^B - k^B\gamma\overline{L^B}\left(1-\overline{L^B}\right)\left[2 + \left(\overline{B} - B^c\right)\gamma\left(1 - 2\overline{L^B}\right)\right]. \tag{2.90}$$

- Computation of f_{11}^{BY}: Differentiating $\frac{\partial G_t}{\partial Y_t}$ with respect to B_t, we obtain:

$$\frac{\partial^2 G_t}{\partial B_t \partial Y_t} = \frac{1}{2}\left[2h_{11}^{BY}\right] \Longrightarrow f_{11}^{BY} = h_{11}^{BY}. \tag{2.91}$$

- Computation of $f_{11}^{B\sigma}$ and $f_{11}^{Y\sigma}$: Differentiating $\frac{\partial G_t}{\partial B_t}$ and $\frac{\partial G_t}{\partial Y_t}$ with respect to σ_ε, we obtain:

$$f_{11}^{B\sigma} = f_{11}^{Y\sigma} = \frac{\partial^2 G_t}{\partial\sigma_\varepsilon\partial B_t} = \frac{\partial^2 G_t}{\partial\sigma_\varepsilon\partial Y_t} = 0. \tag{2.92}$$

- Computation of f_{11}^Y: Differentiating $\frac{\partial G_t}{\partial Y_t}$ with respect to Y_t, we obtain:

$$\frac{\partial^2 G_t}{\partial Y_t^2} = \frac{1}{2}\left[2h_{11}^Y\right] \Longrightarrow f_{11}^Y = h_{11}^Y. \tag{2.93}$$

- Computation of f_{11}^σ: Differentiating $\frac{\partial G_t}{\partial \sigma_\varepsilon}$ with respect to σ_ε, we obtain:

$$\frac{\partial^2 G_t}{\partial \sigma_\varepsilon^2} = \frac{1}{2}\left[2h_{11}^\sigma\right] \Longrightarrow f_{11}^\sigma = h_{11}^\sigma. \tag{2.94}$$

With all those derivatives, we write down the second-order approximation for G_t as

$$G_t \approx \left\{ \begin{array}{c} \overline{G} + \left[h_1^B - (1/\beta_g)\right]\widetilde{B}_t + \left(h_1^Y + \tau^y\right)\widetilde{Y}_t + \\ \frac{1}{2}\left\{ \begin{array}{c} \left[h_{11}^B - k^B\gamma\overline{L^B}\left(1-\overline{L^B}\right)\left(2 + \left(\overline{B}-B^c\right)\gamma\left(1-2\overline{L^B}\right)\right)\right]\widetilde{B}_t^2 \\ +2h_{11}^{BY}\widetilde{B}_t\widetilde{Y}_t + h_{11}^Y\widetilde{Y}_t^2 + h_{11}^\sigma\sigma_\varepsilon^2 \end{array} \right\} \end{array} \right\}. \tag{2.95}$$

Unconditional expectation of government expenditure with a debt-based sanction

Next, using that

$$\mathrm{E}\left[B_t\right]=\widehat{B},\quad \mathrm{E}\left[\widetilde{B}_t^2\right]\simeq \mathrm{Var}\left(B_t\right),\quad \mathrm{E}\left[\widetilde{B}_t\widetilde{Y}_t\right]\simeq\left(\frac{\rho h_1^Y}{1-\rho h_1^B}\right)\frac{\sigma_\varepsilon^2}{1-\rho^2},$$

the unconditional expectation of (2.95) can be written as:

$$\widehat{G}\approx\left\{\begin{array}{c}\overline{G}+\left[h_1^B-(1/\beta_g)\right]\left(\widehat{B}-\overline{B}\right)+\left(\frac{\rho h_1^Y h_{11}^{BY}}{1-\rho h_1^B}\right)\frac{\sigma_\varepsilon^2}{1-\rho^2}+\\ \frac{1}{2}\left\{\begin{array}{c}\left[h_{11}^B-k^B\gamma\overline{L^B}\left(1-\overline{L^B}\right)\left(2+\left(\overline{B}-B^c\right)\gamma\left(1-2\overline{L^B}\right)\right)\right]\mathrm{Var}\left(B_t\right)\\ +\left(\frac{h_{11}^Y}{1-\rho^2}+h_{11}^\sigma\right)\sigma_\varepsilon^2\end{array}\right\}\end{array}\right\}.$$

Then, substituting (2.80), (2.81) and (2.88), it follows that

$$\widehat{G}\approx\left\{\begin{array}{c}\overline{G}+\frac{\left[h_1^B-(1/\beta_g)\right]\sigma_\varepsilon^2}{2\left(1-h_1^B\right)}\left[\frac{h_{11}^B\left(h_1^Y\right)^2}{\left(1-\left(h_1^B\right)^2\right)(1-\rho^2)}\left(\frac{1+\rho h_1^B}{1-\rho h_1^B}\right)+2\frac{\rho h_{11}^{BY}h_1^Y}{1-\rho h_1^B}\frac{1}{1-\rho^2}+\frac{h_{11}^Y}{1-\rho^2}+h_{11}^\sigma\right]\\ +\frac{1}{2}\left[\begin{array}{c}\frac{\left[h_{11}^B-k^B\gamma\overline{L^B}\left(1-\overline{L^B}\right)\left(2+\left(\overline{B}-B^c\right)\gamma\left(1-2\overline{L^B}\right)\right)\right]\left(h_1^Y\right)^2\sigma_\varepsilon^2}{\left(1-\left(h_1^B\right)^2\right)(1-\rho^2)}\left(\frac{1+\rho h_1^B}{1-\rho h_1^B}\right)\\ +2\left(\frac{\rho h_1^Y h_{11}^{BY}}{1-\rho h_1^B}\right)\frac{\sigma_\varepsilon^2}{1-\rho^2}+\left(\frac{h_{11}^Y}{1-\rho^2}+h_{11}^\sigma\right)\sigma_\varepsilon^2\end{array}\right]\end{array}\right\},$$

or

Solution 2.2 *The unconditional expectation or "stochastic steady state" value of the government expenditures in the debt-based sanction case is given by*

$$\widehat{G}\approx\left\{\begin{array}{c}\tau^y\overline{Y}-r\overline{B}-k^B\left(\overline{B}-B^c\right)\overline{L^B}+\\ \frac{\sigma_\varepsilon^2}{2}\left\{\begin{array}{c}\frac{h_1^B-(1/\beta_g)}{1-h_1^B}\left[\frac{h_{11}^B\left(h_1^Y\right)^2}{\left(1-\left(h_1^B\right)^2\right)(1-\rho^2)}\left(\frac{1+\rho h_1^B}{1-\rho h_1^B}\right)+2\frac{\rho h_{11}^{BY}h_1^Y}{1-\rho h_1^B}\frac{1}{1-\rho^2}+\frac{h_{11}^Y}{1-\rho^2}+h_{11}^\sigma\right]\\ +\left[\begin{array}{c}\frac{\left[h_{11}^B-k^B\gamma\overline{L^B}\left(1-\overline{L^B}\right)\left(2+\left(\overline{B}-B^c\right)\gamma\left(1-2\overline{L^B}\right)\right)\right]\left(h_1^Y\right)^2}{\left(1-\left(h_1^B\right)^2\right)(1-\rho^2)}\left(\frac{1+\rho h_1^B}{1-\rho h_1^B}\right)\\ +2\left(\frac{\rho h_1^Y h_{11}^{BY}}{1-\rho h_1^B}\right)\frac{1}{1-\rho^2}+\frac{h_{11}^Y}{1-\rho^2}+h_{11}^\sigma\end{array}\right]\end{array}\right\}\end{array}\right\}. \tag{2.96}$$

2.G Stochastic steady state with primary deficit-based sanction

The same procedure is used to obtain the stochastic steady state of debt with a primary deficit-based sanction. First, we isolate G_s in (2.21) for $s=t$ and $t+1$, and substitute into (2.22), to give

$$\mathrm{E}_t\left\{\begin{array}{c}u'\left(D_{t+1}+Y_t-k^D\left(D_t-D^c\right)L_t^D\right)\\ -\beta_g k^D\left[L_{t+1}^D+\left(D_{t+1}-D^c\right)\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\right]u'\left(\begin{array}{c}D_{t+2}+\rho Y_t+(1-\rho)\overline{Y}+\sigma_\varepsilon\varepsilon_{t+1}\\ -k^D\left(D_{t+1}-D^c\right)L_{t+1}^D\end{array}\right)\end{array}\right\}=0, \tag{2.97}$$

Using that

$$\frac{\partial L_{t+1}^D}{\partial D_{t+1}} = \frac{\gamma \exp\left(-\gamma\left(D_{t+1}-D^c\right)\right)}{\left(1+\exp\left(-\gamma\left(D_{t+1}-D^c\right)\right)\right)^2} = \gamma L_{t+1}^D\left(1-L_{t+1}^D\right), \tag{2.98}$$

this becomes

$$\mathrm{E}_t\left\{\begin{array}{c} u'\left(D_{t+1}+Y_t-k^D\left(D_t-D^c\right)L_t^D\right)- \\ \beta_g k^D\left[\begin{array}{c} L_{t+1}^D+ \\ \left(D_{t+1}-D^c\right)\gamma\left(L_{t+1}^D-\left(L_{t+1}^D\right)^2\right)\end{array}\right]u'\left(\begin{array}{c} D_{t+2}+\rho Y_t+(1-\rho)\overline{Y}+ \\ \sigma_\varepsilon\varepsilon_{t+1}-k^D\left(D_{t+1}-D^c\right)L_{t+1}^D\end{array}\right)\end{array}\right\}=0. \tag{2.99}$$

Thus, (2.99) has the format

$$\mathrm{E}_t\left\{i\left(D_t,D_{t+1},D_{t+2},Y_t,Y_{t+1}\right)\right\}=0.$$

Hence, using (2.5) and (2.28) the unknown $l(\cdot)$ satisfies

$$\begin{aligned}
\mathrm{E}_t\left[p\left(D_t,Y_t,\sigma_\varepsilon\right)\right] &= \mathrm{E}_t\left[i\left(D_t,l\left(D_t,Y_t,\sigma_\varepsilon\right),l\left(l\left(D_t,Y_t,\sigma_\varepsilon\right),Y_{t+1},\sigma_\varepsilon\right),Y_t,Y_{t+1}\right)\right]=0 \Rightarrow \\
\mathrm{E}_t\left[p\left(D_t,Y_t,\sigma_\varepsilon\right)\right] &= \mathrm{E}_t\left[i\left(\begin{array}{c} D_t,l\left(D_t,Y_t,\sigma_\varepsilon\right), \\ l\left(l\left(D_t,Y_t,\sigma_\varepsilon\right),\rho Y_t+(1-\rho)\overline{Y}+\sigma_\varepsilon\varepsilon_{t+1},\sigma_\varepsilon\right), \\ Y_t,\rho Y_t+(1-\rho)\overline{Y}+\sigma_\varepsilon\varepsilon_{t+1}\end{array}\right)\right]=0.
\end{aligned}$$

For $\sigma_\varepsilon=0$, $D_{t+2}=D_{t+1}=D_t=\overline{D}$ and $Y_{t+1}=Y_t=\overline{Y}$, (2.29) becomes:

$$\mathrm{E}_t\left[p\left(\overline{B},\overline{Y},0\right)\right]=\mathrm{E}_t\left[i\left(\overline{D},\overline{D},\overline{D},\overline{Y},\overline{Y}\right)\right]=0,$$

where we have used that $l\left(\overline{D},\overline{Y},0\right)=\overline{D}$ and $l\left(l\left(\overline{D},\overline{Y},0\right),\overline{Y},0\right)=l\left(\overline{D},\overline{Y},0\right)=\overline{D}$. Because $u'>0$, $\mathrm{E}_t\left[p\left(\overline{D},\overline{Y},0\right)\right]=0$ is equivalent to

$$\beta_g k^D\overline{L^D}\left[1+\left(\overline{D}-D^c\right)\gamma\left(1-\overline{L^D}\right)\right]=1, \tag{2.100}$$

where $\overline{L^D}$ and u' are evaluated at the point $\left(\overline{D},\overline{Y},0\right)$. This expression determines $\overline{D}$.

2.G.1 First-order approximation of (2.29)

Derivation of p_D, p_{Y_t} and p_{σ_ε}

In this section, we compute p_D, p_{Y_t} and p_{σ_ε}. We start by computing the partial derivatives of (2.28) with respect to B_t, Y_t and σ_ε.

First-order partial derivatives of (2.28) Derivative of D_{t+1} with respect to D_t:

$$\frac{\partial D_{t+1}}{\partial D_t}=\frac{\partial l\left(D_t,Y_t,\sigma_\varepsilon\right)}{\partial D_t}=\frac{\partial l_t}{\partial D_t}\equiv l_1^D, \tag{2.101}$$

where l_t represents $l\left(D_t,Y_t,\sigma_\varepsilon\right)$, and since $\frac{\partial Y_t}{\partial D_t}=\frac{\partial \sigma_\varepsilon}{\partial D_t}=0$. Further, the derivative of D_{t+1} with respect to Y_t is

$$\frac{\partial D_{t+1}}{\partial Y_t}=\frac{\partial l\left(D_t,Y_t,\sigma_\varepsilon\right)}{\partial Y_t}=\frac{\partial l_t}{\partial Y_t}\equiv l_1^Y, \tag{2.102}$$

since at the initial period $\frac{\partial D_t}{\partial Y_t} = 0$. Finally, the derivative of D_{t+1} with respect to σ_ε is

$$\frac{\partial D_{t+1}}{\partial \sigma_\varepsilon} = \frac{\partial l\left(D_t, Y_t, \sigma_\varepsilon\right)}{\partial \sigma_\varepsilon} = \frac{\partial l_t}{\partial \sigma_\varepsilon} = l_1^\sigma, \tag{2.103}$$

since $\frac{\partial D_t}{\partial \sigma_\varepsilon} = 0$ and in the initial value of Y_t is also not correlated with σ_ε, causing $\frac{\partial Y_t}{\partial \sigma_\varepsilon} = 0$.

Derivation of the first order partial derivative of D_{t+2} with respect to $D_t, Y_t, \sigma_\varepsilon$ Using 2.28, we can write D_{t+2} as

$$D_{t+2} = l\left(l\left(D_t, Y_t, \sigma_\varepsilon\right), \rho Y_t + (1-\rho)\overline{Y} + \sigma_\varepsilon \varepsilon_{t+1}, \sigma_\varepsilon\right). \tag{2.104}$$

Hence, differentiating D_{t+2} with respect to D_t:

$$\frac{\partial D_{t+2}}{\partial D_t} = \frac{\partial l\left(D_{t+1}, Y_{t+1}, \sigma_\varepsilon\right)}{\partial D_t} = \frac{\partial l_{t+1}}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial D_t} + \frac{\partial l_{t+1}}{\partial Y_{t+1}}\frac{\partial Y_{t+1}}{\partial D_t} + \frac{\partial l_{t+1}}{\partial \sigma_\varepsilon}\frac{\partial \sigma_\varepsilon}{\partial D_t}.$$

Because $\frac{\partial Y_{t+1}}{\partial D_t} = 0$ and $\frac{\partial \sigma_\varepsilon}{\partial D_t} = 0$, and using (2.101), we have:

$$\frac{\partial D_{t+2}}{\partial D_t} = \frac{\partial l_{t+1}}{\partial D_{t+1}}\frac{\partial l_t}{\partial D_t} = \left(\frac{\partial l}{\partial D_t}\right)^2 = \left(l_1^D\right)^2,$$

where we use that $\frac{\partial l_{t+1}}{\partial D_{t+1}}$ and $\frac{\partial l_t}{\partial D_t}$ are evaluated at the same point. Further,

$$\frac{\partial D_{t+2}}{\partial Y_t} = \frac{\partial l_{t+1}}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial Y_t} + \frac{\partial l_{t+1}}{\partial Y_{t+1}}\frac{\partial Y_{t+1}}{\partial Y_t} + \frac{\partial l_{t+1}}{\partial \sigma_\varepsilon}\frac{\partial \sigma_\varepsilon}{\partial Y_t} = l_1^D l_1^Y + l_1^Y \rho.$$

Finally,

$$\begin{aligned}\frac{\partial D_{t+2}}{\partial \sigma_\varepsilon} &= \frac{\partial l_{t+1}}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial \sigma_\varepsilon} + \frac{\partial l_{t+1}}{\partial Y_{t+1}}\frac{\partial Y_{t+1}}{\partial \sigma_\varepsilon} + \frac{\partial l_{t+1}}{\partial \sigma_\varepsilon}\frac{\partial \sigma_\varepsilon}{\partial \sigma_\varepsilon} \\ &= l_1^D l_1^\sigma + l_1^Y \varepsilon_{t+1} + l_1^\sigma.\end{aligned}$$

Derivation of p_D Differentiate (2.97) with respect to D_t around the point $\left(\overline{D}, \overline{Y}, 0\right)$ and multiply by $\widetilde{D}_t$:

$$\mathrm{E}_t\left[p_D \widetilde{D}_t\right] = E_t\left\{\left[\begin{array}{c} u''\left(G_t\right) * \left(\frac{\partial D_{t+1}}{\partial D_t} - k^D\overline{L^D} - k^D\left(\overline{D} - D^c\right)\frac{\partial L_t^D}{\partial D_t}\right) - \\ \beta_g k^D * \left[2\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial D_t} + \left(\overline{D} - D^c\right)\frac{\partial^2 L_{t+1}^D}{\partial^2 D_{t+1}}\frac{\partial D_{t+1}}{\partial D_t}\right] * u'\left(G_{t+1}\right) \\ -\beta_g k^D\left[\overline{L^D} + \gamma\left(\overline{D} - D^c\right)\overline{L^D}(1 - \overline{L^D})\right] * \\ \left(\frac{\partial D_{t+2}}{\partial D_t} - k^D\overline{L^D}\frac{\partial D_{t+1}}{\partial D_t} - k^D\left(\overline{D} - D^c\right)\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial D_t}\right) u''\left(G_{t+1}\right) \end{array}\right]\widetilde{D}_t\right\}.$$

Notice that:

$$\frac{\partial^2 L_{t+1}^D}{\partial^2 D_{t+1}} = \gamma^2 L_{t+1}^D\left(1 - L_{t+1}^D\right)\left(1 - 2L_{t+1}^D\right) = \gamma^2\left(L_{t+1}^D - 3\left(L_{t+1}^D\right)^2 + 2\left(L_{t+1}^D\right)^3\right). \tag{2.105}$$

Using (2.105), the expressions for $\frac{\partial D_{t+1}}{\partial D_t}$ and $\frac{\partial D_{t+2}}{\partial D_t}$, and (2.100), we have

$$\mathrm{E}_t\left[p_D\widetilde{D}_t\right]=\left[\begin{array}{c} u''*\left(l_1^D-k^D\overline{L^D}-k^D\left(\overline{D}-D^c\right)\gamma\overline{L^D}(1-\overline{L^D})\right)- \\ \beta_g\gamma k^D l_1^D*\left[2\overline{L^D}(1-\overline{L^D})+\left(\overline{D}-D^c\right)\gamma^2\overline{L^D}(1-\overline{L^D})(1-2\overline{L^D})\right]*u' \\ -u''*\left(\left(l_1^D\right)^2-k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)l_1^D\right) \end{array}\right]\widetilde{D}_t,$$

where we exclude the expectations operator on the right-hand side, because all the values are known in period t. In addition, given (2.100), it follows that

$$\mathrm{E}_t\left[p_D\widetilde{D}_t\right]=\left[\begin{array}{c} -\left(l_1^D\right)^2u''+ \\ l_1^D\left(\begin{array}{c} u''+u''*k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)- \\ u'\beta_g k^D\gamma\overline{L^D}(1-\overline{L^D})*\left[2+\left(\overline{D}-D^c\right)\gamma\left(1-2\overline{L^D}\right)\right] \end{array}\right) \\ -u''*k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right) \end{array}\right]\widetilde{D}_t. \tag{2.106}$$

Derivation of p_Y Differentiate (2.97) with respect to Y_t around the point $\left(\overline{D},\overline{Y},0\right)$:

$$p_Y=\left[\begin{array}{c} \left(\frac{\partial D_{t+1}}{\partial Y_t}+\tau^y\right)u''\left(G_t\right)- \\ \beta_g k^D\left[\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial Y_t}+\frac{\partial D_{t+1}}{\partial Y_t}\frac{\partial L_{t+1}^D}{\partial D_{t+1}}+\left(\overline{D}-D^c\right)\frac{\partial^2 L_{t+1}^D}{\partial^2 D_{t+1}}\frac{\partial D_{t+1}}{\partial Y_t}\right]u'\left(G_{t+1}\right) \\ -u''\left(G_{t+1}\right)*\left(\frac{\partial D_{t+2}}{\partial Y_t}-k^D\overline{L^D}\frac{\partial D_{t+1}}{\partial Y_t}+\tau^y\rho-k^D\left(\overline{D}-D^c\right)\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial Y_t}\right) \end{array}\right].$$

Hence,

$$\mathrm{E}_t\left[p_Y\widetilde{Y}_t\right]=\left[\begin{array}{c} \left(l_1^Y+\tau^y\right)u''- \\ \beta_g k^D\gamma\overline{L^D}(1-\overline{L^D})\left[2+\gamma\left(\overline{D}-D^c\right)\left(1-2\overline{L^D}\right)\right]l_1^Y u'- \\ \left[l_1^D l_1^Y+l_1^Y\rho+\tau^y\rho-k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)l_1^Y\right]u'' \end{array}\right]\widetilde{Y}_t. \tag{2.107}$$

Again, we exclude the expectations operator on the right-hand side since all values are known in period t.

Derivation of p_{σ_ε} Differentiate (2.97) with respect to σ_ε around the point $\left(\overline{D},\overline{Y},0\right)$, multiply by σ_ε, take expectations and use (2.100), to yield:

$$\mathrm{E}_t\left[p_{\sigma_\varepsilon}\sigma_\varepsilon\right]=E_t\left\{\left[\begin{array}{c} \left(\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon}\right)u''\left(G_t\right)- \\ \beta_g k^D\left[2\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon}\frac{\partial L_{t+1}^D}{\partial D_{t+1}}+\left(\overline{D}-D^c\right)\frac{\partial^2 L_{t+1}^D}{\partial^2 D_{t+1}}\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon}\right]u'\left(G_{t+1}\right) \\ -\left[\begin{array}{c} \frac{\partial D_{t+2}}{\partial\sigma_\varepsilon}+\tau^y\varepsilon_{t+1}-k^D\overline{L^D}\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon} \\ -k^D\left(\overline{D}-D^c\right)\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon} \end{array}\right]u''\left(G_{t+1}\right) \end{array}\right]\sigma_\varepsilon\right\}.$$

Hence,

$$\mathrm{E}_t\left[p_{\sigma_\varepsilon}\sigma_\varepsilon\right]=E_t\left\{\left[\begin{array}{c} l_1^\sigma u''- \\ \beta_g k^D\gamma\overline{L^D}(1-\overline{L^D})\left[2+\gamma\left(\overline{D}-D^c\right)\left(1-2\overline{L^D}\right)\right]l_1^\sigma u' \\ -\left[\begin{array}{c} l_1^D l_1^\sigma+l_1^Y\varepsilon_{t+1}+l_1^\sigma+\tau^y\varepsilon_{t+1}- \\ k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)l_1^\sigma \end{array}\right]u'' \end{array}\right]\sigma_\varepsilon\right\}.$$

Hence,

$$\mathrm{E}_t\left[p_{\sigma_\varepsilon}\sigma_\varepsilon\right]=E_t\left\{\left[\begin{array}{c} u''*l_1^\sigma-\beta_g k^D\gamma\overline{L^D}(1-\overline{L^D})\left[2+\gamma\left(\overline{D}-D^c\right)\left(1-2\overline{L^D}\right)\right]*l_1^\sigma u' \\ -u''*\left[\left(1+l_1^D\right)l_1^\sigma-k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)l_1^\sigma\right]\end{array}\right]\sigma_\varepsilon\right\}. \tag{2.108}$$

Hence,

$$l_1^\sigma=0. \tag{2.109}$$

First-order Taylor expansion

Substituting (2.106), (2.107), $\mathrm{E}_t\left[p\left(\overline{B},\overline{Y},0\right)\right]=0$ and $\mathrm{E}_t\left[p_{\sigma_\varepsilon}\sigma_\varepsilon\right]=0$, we can write the first-order Taylor expansion of 2.29 as

$$\left\{\begin{array}{c}\left[\begin{array}{c} -\left(l_1^D\right)^2u''+ \\ l_1^D\left(\begin{array}{c} u''+k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)u''- \\ \beta_g k^D\gamma\overline{L^D}(1-\overline{L^D})\left[2+\left(\overline{D}-D^c\right)\gamma\left(1-2\overline{L^D}\right)\right]u'\end{array}\right) \\ -k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)u''\end{array}\right]\widetilde{D}_t \\ +\left[\begin{array}{c}\left(l_1^Y+\tau^y\right)u''- \\ \beta_g k^D\gamma\overline{L^D}(1-\overline{L^D})\left[2+\gamma\left(\overline{D}-D^c\right)\left(1-2\overline{L^D}\right)\right]l_1^Y u'- \\ \left[l_1^Dl_1^Y+l_1^Y\rho+\tau^y\rho-k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)l_1^Y\right]u''\end{array}\right]\widetilde{Y}_t\end{array}\right\}=0. \tag{2.110}$$

This expression must hold for $\left(\widetilde{D}_t,\widetilde{Y}_t\right)$, which allows us to solve for l_1^D and l_1^Y.

Computation of l_1^D Multiply the coefficient of $\widetilde{D}_t$ in (2.110) by $-\frac{\beta_g}{u''}$ and set the result equal to zero:

$$\left[\beta_g\left(l_1^D\right)^2-l_1^D\left(\begin{array}{c}(\beta_g+1)-\frac{u'}{u''}\beta_g^2\gamma k^D\overline{L^D}(1-\overline{L^D})* \\ \left[2+\left(\overline{D}-D^c\right)\gamma\left(1-2\overline{L^D}\right)\right]\end{array}\right)+1\right]=0.$$

where we have used (2.100). This a quadratic equation in l_1^D, with solutions:

$$l_1^D=\frac{\begin{array}{c}\beta_g+1-\frac{u'}{u''}\beta_g^2\gamma k^D\overline{L^D}(1-\overline{L^D})\left[2+\left(\overline{D}-D^c\right)\gamma\left(1-2\overline{L^D}\right)\right]\pm \\ \sqrt{\left(\beta_g+1-\frac{u'}{u''}\beta_g^2\gamma k^D\overline{L^D}(1-\overline{L^D})\left[2+\left(\overline{D}-D^c\right)\gamma\left(1-2\overline{L^D}\right)\right]\right)^2-4\beta_g}\end{array}}{2\beta_g}. \tag{2.111}$$

To ensure a non-explosive path for the primary deficit, we need to pick the solution that is smaller than unity in absolute value.

Computation of l_1^Y The third term of (2.110) also has to be zero for any value of Y_t. This requires that:

$$l_1^Y \left\{ \begin{array}{l} \left[1-\rho-l_1^D+k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)\right]u'' \\ -\beta_g k^D \gamma \overline{L^D}(1-\overline{L^D})\left[2+\gamma\left(\overline{D}-D^c\right)\left(1-2\overline{L^D}\right)\right]u' \end{array} \right\} + \tau^y (1-\rho)\, u'' = 0.$$

Thus, plugging the stable root of l_1^D (2.111), we find that

$$l_1^Y = \frac{\tau^y (\rho-1)\, u''}{\left\{ \begin{array}{l} \left[1-\rho-l_1^D+k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)\right]u'' \\ -\beta_g k^D \gamma \overline{L^D}(1-\overline{L^D})\left[2+\gamma\left(\overline{D}-D^c\right)\left(1-2\overline{L^D}\right)\right]u' \end{array} \right\}}. \tag{2.112}$$

Finally, $l_1^\sigma = 0$.

2.G.2 Second-order approximation of (2.29)

Derivation of the second-order partial derivatives p_{DD}, p_{DY_t}, $p_{D\sigma_\varepsilon}$, $p_{Y_tY_t}$, $p_{Y_t\sigma_\varepsilon}$, and $p_{\sigma_\varepsilon\sigma_\varepsilon}$

For later use, we compute the second-order partial derivatives of (2.28) and (2.104) with respect to D_t, Y_t and σ_ε.

Derivation of the second-order partial derivative of (2.28) with respect to $D_t, Y_t, \sigma_\varepsilon$ With (2.101), we have:

$$\frac{\partial^2 D_{t+1}}{\partial^2 D_t} = \frac{\partial\left(\partial l_t/\partial D_t\right)}{\partial D_t} = \frac{\partial^2 l_t}{\partial^2 D_t} \equiv l_{11}^D. \tag{2.113}$$

- Further,

$$\frac{\partial^2 D_{t+1}}{\partial D_t \partial Y_t} = \frac{\partial^2 l_t}{\partial D_t \partial Y_t} \equiv l_{11}^{DY}. \tag{2.114}$$

- Further,

$$\frac{\partial^2 D_{t+1}}{\partial^2 Y_t} = \frac{\partial^2 l_t}{\partial^2 Y_t} \equiv l_{11}^{Y}. \tag{2.115}$$

- Further,

$$\frac{\partial^2 D_{t+1}}{\partial D_t \partial \sigma_\varepsilon} = \frac{\partial^2 l_t}{\partial D_t \partial \sigma_\varepsilon} \equiv l_{11}^{D\sigma}. \tag{2.116}$$

- Further,

$$\frac{\partial^2 D_{t+1}}{\partial Y_t \partial \sigma_\varepsilon} = \frac{\partial^2 l_t}{\partial Y_t \partial \sigma_\varepsilon} \equiv l_{11}^{Y\sigma}. \tag{2.117}$$

- Finally,

$$\frac{\partial^2 D_{t+1}}{\partial^2 \sigma_\varepsilon} = \frac{\partial^2 l_t}{\partial^2 \sigma_\varepsilon} \equiv l_{11}^{\sigma}. \tag{2.118}$$

Derivation of the second-order partial derivatives of (2.104) with respect to D_t, Y_t and σ_ε

- Computation of $\frac{\partial^2 D_{t+2}}{\partial^2 D_t}$:

$$\begin{aligned}
\frac{\partial^2 D_{t+2}}{\partial^2 D_t} &= \frac{\partial\left(\frac{\partial l_{t+1}}{\partial D_{t+1}}\right)}{\partial D_t}\frac{\partial D_{t+1}}{\partial D_t} + \frac{\partial l_{t+1}}{\partial D_{t+1}}\frac{\partial\left(\frac{\partial D_{t+1}}{\partial D_t}\right)}{\partial D_t} \\
&= \left[\frac{\partial\left(\frac{\partial l_{t+1}}{\partial D_{t+1}}\right)}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial D_t} + \frac{\partial\left(\frac{\partial l_{t+1}}{\partial D_{t+1}}\right)}{\partial Y_{t+1}}\frac{\partial Y_{t+1}}{\partial D_t} + \frac{\partial\left(\frac{\partial l_{t+1}}{\partial D_{t+1}}\right)}{\partial \sigma_\varepsilon}\frac{\partial \sigma_\varepsilon}{\partial D_t}\right]\frac{\partial D_{t+1}}{\partial D_t} + \frac{\partial l_{t+1}}{\partial D_{t+1}}\frac{\partial^2 D_{t+1}}{\partial^2 D_t} \\
&= \left[\frac{\partial\left(\frac{\partial l_{t+1}}{\partial D_{t+1}}\right)}{\partial D_{t+1}} * \frac{\partial D_{t+1}}{\partial D_t}\right]\frac{\partial D_{t+1}}{\partial D_t} + \frac{\partial l_{t+1}}{\partial D_{t+1}}\frac{\partial^2 D_{t+1}}{\partial D_t^2}.
\end{aligned}$$

Hence,

$$\frac{\partial^2 D_{t+2}}{\partial^2 D_t} = l_{11}^D\left(l_1^D\right)^2 + l_1^D l_{11}^D.$$

- Computation of $\frac{\partial^2 D_{t+2}}{\partial D_t \partial Y_t}$:

$$\begin{aligned}
\frac{\partial^2 D_{t+2}}{\partial D_t \partial Y_t} &= \frac{\partial}{\partial D_t}\left(\frac{\partial l_{t+1}}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial Y_t} + \frac{\partial l_{t+1}}{\partial Y_{t+1}}\frac{\partial Y_{t+1}}{\partial Y_t} + \frac{\partial l_{t+1}}{\partial \sigma_\varepsilon}\frac{\partial \sigma_\varepsilon}{\partial Y_t}\right) \Rightarrow \\
\frac{\partial^2 D_{t+2}}{\partial D_t \partial Y_t} &= \left[\frac{\partial\left(\frac{\partial l_{t+1}}{\partial D_{t+1}}\right)}{\partial D_t}\frac{\partial D_{t+1}}{\partial Y_t} + \frac{\partial l_{t+1}}{\partial D_{t+1}}\frac{\partial\left(\frac{\partial D_{t+1}}{\partial Y_t}\right)}{\partial D_t} + \frac{\partial\left(\frac{\partial l_{t+1}}{\partial Y_{t+1}}\right)}{\partial D_t}\rho + \frac{\partial l_{t+1}}{\partial Y_{t+1}}\frac{\partial(\rho)}{\partial D_t}\right] \\
&= \left[\begin{array}{c} \frac{\partial^2 l_{t+1}}{\partial^2 D_{t+1}}\frac{\partial D_{t+1}}{\partial D_t}\frac{\partial D_{t+1}}{\partial Y_t} + l_1^D * \left[\frac{\partial^2 D_{t+1}}{\partial Y_t \partial D_t} + \frac{\partial^2 D_{t+1}}{\partial^2 Y_t}\frac{\partial Y_t}{\partial D_t} + \frac{\partial^2 D_{t+1}}{\partial Y_t \partial \sigma_\varepsilon}\frac{\partial \sigma_\varepsilon}{\partial D_t}\right] \\ +\rho * \left[\frac{\partial^2 l_{t+1}}{\partial Y_{t+1}\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial D_t} + \frac{\partial^2 l_{t+1}}{\partial^2 Y_{t+1}}\frac{\partial Y_{t+1}}{\partial D_t} + \frac{\partial^2 l_{t+1}}{\partial Y_{t+1}\partial \sigma_\varepsilon}\frac{\partial \sigma_\varepsilon}{\partial D_t}\right] \end{array}\right] \\
&= l_{11}^D l_1^D l_1^Y + l_1^D l_{11}^{DY} + \rho l_1^D l_{11}^{DY},
\end{aligned}$$

using that $\frac{\partial \sigma_\varepsilon}{\partial Y_t} = 0$, $\frac{\partial Y_{t+1}}{\partial Y_t} = \rho$, (2.101), and (2.114).

- Computation of $\frac{\partial^2 D_{t+2}}{\partial D_t \partial \sigma_\varepsilon}$:

$$\begin{aligned}
\frac{\partial^2 D_{t+2}}{\partial D_t \partial \sigma_\varepsilon} &= \frac{\partial}{\partial D_t}\left(\frac{\partial l_{t+1}}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial \sigma_\varepsilon} + \frac{\partial l_{t+1}}{\partial Y_{t+1}}\frac{\partial Y_{t+1}}{\partial \sigma_\varepsilon} + \frac{\partial l_{t+1}}{\partial \sigma_\varepsilon}\frac{\partial \sigma_\varepsilon}{\partial \sigma_\varepsilon}\right) \\
&= \left[\begin{array}{c} \frac{\partial\left(\frac{\partial l_{t+1}}{\partial D_{t+1}}\right)}{\partial D_t} * \frac{\partial D_{t+1}}{\partial \sigma_\varepsilon} + \frac{\partial l_{t+1}}{\partial D_{t+1}} * \frac{\partial\left(\frac{\partial D_{t+1}}{\partial \sigma_\varepsilon}\right)}{\partial D_t} + \frac{\partial\left(\frac{\partial l_{t+1}}{\partial Y_{t+1}}\right)}{\partial D_t} * \varepsilon_{t+1} \\ + \frac{\partial l_{t+1}}{\partial Y_{t+1}} * \frac{\partial(\varepsilon_{t+1})}{\partial D_t} + \frac{\partial\left(\frac{\partial l_{t+1}}{\partial \sigma_\varepsilon}\right)}{\partial D_t} + \frac{\partial l_{t+1}}{\partial \sigma_\varepsilon} * \frac{\partial(1)}{\partial B_t} \end{array}\right] \\
&= \left[\begin{array}{c} \frac{\partial^2 l_{t+1}}{\partial D_{t+1}^2} * \frac{\partial D_{t+1}}{\partial D_t} * \frac{\partial D_{t+1}}{\partial \sigma_\varepsilon} + \frac{\partial l_{t+1}}{\partial D_{t+1}} * \frac{\partial^2 D_{t+1}}{\partial D_t \partial \sigma_\varepsilon} + \\ \frac{\partial^2 l_{t+1}}{\partial Y_{t+1}\partial D_{t+1}} * \frac{\partial D_{t+1}}{\partial D_t} * \varepsilon_{t+1} + \frac{\partial^2 l_{t+1}}{\partial \sigma_\varepsilon \partial D_{t+1}} * \frac{\partial D_{t+1}}{\partial D_t} \end{array}\right] \\
&= l_{11}^D l_1^D l_1^\sigma + l_1^D l_{11}^{D\sigma} + l_1^D l_{11}^{DY}\varepsilon_{t+1} + l_1^D l_{11}^{D\sigma} = l_{11}^D l_1^D l_1^\sigma + 2 l_1^D l_{11}^{D\sigma} + l_1^D l_{11}^{DY}\varepsilon_{t+1}.
\end{aligned}$$

- Computation of $\frac{\partial^2 D_{t+2}}{\partial^2 Y_t}$:

$$\begin{aligned}
\frac{\partial^2 D_{t+2}}{\partial^2 Y_t} &= \frac{\partial}{\partial Y_t}\left(\frac{\partial l_{t+1}}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial Y_t} + \frac{\partial l_{t+1}}{\partial Y_{t+1}}\frac{\partial Y_{t+1}}{\partial Y_t} + \frac{\partial l_{t+1}}{\partial \sigma_\varepsilon}\frac{\partial \sigma_\varepsilon}{\partial Y_t}\right) \\
&= \left[\frac{\partial\left(\frac{\partial l_{t+1}}{\partial D_{t+1}}\right)}{\partial Y_t} * \frac{\partial D_{t+1}}{\partial Y_t} + \frac{\partial l_{t+1}}{\partial D_{t+1}} * \frac{\partial\left(\frac{\partial D_{t+1}}{\partial Y_t}\right)}{\partial Y_t} + \frac{\partial\left(\frac{\partial l_{t+1}}{\partial Y_{t+1}}\right)}{\partial Y_t} * \rho + \frac{\partial l_{t+1}}{\partial Y_{t+1}} * \frac{\partial(\rho)}{\partial Y_t}\right] \\
&= \left[\begin{array}{c} \left[\frac{\partial^2 l_{t+1}}{\partial D_{t+1}^2}\frac{\partial D_{t+1}}{\partial Y_t} + \frac{\partial^2 l_{t+1}}{\partial D_{t+1}\partial Y_{t+1}}\frac{\partial Y_{t+1}}{\partial Y_t} + \frac{\partial^2 l_{t+1}}{\partial D_{t+1}\partial\sigma_\varepsilon}\frac{\partial\sigma_\varepsilon}{\partial Y_t}\right] * \frac{\partial D_{t+1}}{\partial Y_t} + \frac{\partial l_{t+1}}{\partial D_{t+1}} * \frac{\partial^2 D_{t+1}}{\partial Y_t^2} \\ + \left[\frac{\partial^2 l_{t+1}}{\partial Y_{t+1}\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial Y_t} + \frac{\partial^2 l_{t+1}}{\partial Y_{t+1}^2}\frac{\partial Y_{t+1}}{\partial Y_t} + \frac{\partial^2 l_{t+1}}{\partial Y_{t+1}\partial\sigma_\varepsilon}\frac{\partial\sigma_\varepsilon}{\partial Y_t}\right] * \rho \end{array}\right],
\end{aligned}$$

since $\frac{\partial\sigma_\varepsilon}{\partial Y_t} = 0$ and $\frac{\partial Y_{t+1}}{\partial Y_t} = \rho$. Using (2.102), (2.114) and (2.115), we can rewrite the last equation as

$$\frac{\partial^2 D_{t+2}}{\partial^2 Y_t} = l_{11}^D\left(l_1^Y\right)^2 + l_{11}^{DY} l_1^Y\rho + l_1^D l_{11}^Y + l_{11}^{DY} l_1^Y\rho + l_{11}^Y\rho^2 = l_{11}^D\left(l_1^Y\right)^2 + 2l_{11}^{DY} l_1^Y\rho + l_{11}^Y\left(l_1^D + \rho^2\right).$$

- Computation of $\frac{\partial^2 D_{t+2}}{\partial Y_t\partial\sigma_\varepsilon}$:

$$\begin{aligned}
\frac{\partial^2 D_{t+2}}{\partial Y_t\partial\sigma_\varepsilon} &= \frac{\partial}{\partial Y_t}\left(\frac{\partial l_{t+1}}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon} + \frac{\partial l_{t+1}}{\partial Y_{t+1}}\frac{\partial Y_{t+1}}{\partial\sigma_\varepsilon} + \frac{\partial l_{t+1}}{\partial\sigma_\varepsilon}\frac{\partial\sigma_\varepsilon}{\partial\sigma_\varepsilon}\right) \\
&= \left[\begin{array}{c} \frac{\partial\left(\frac{\partial l_{t+1}}{\partial D_{t+1}}\right)}{\partial Y_t} * \frac{\partial D_{t+1}}{\partial\sigma_\varepsilon} + \frac{\partial l_{t+1}}{\partial D_{t+1}} * \frac{\partial\left(\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon}\right)}{\partial Y_t} + \frac{\partial\left(\frac{\partial l_{t+1}}{\partial Y_{t+1}}\right)}{\partial Y_t} * \varepsilon_{t+1} \\ + \frac{\partial l_{t+1}}{\partial Y_{t+1}} * \frac{\partial(\varepsilon_{t+1})}{\partial Y_t} + \frac{\partial\left(\frac{\partial l_{t+1}}{\partial\sigma_\varepsilon}\right)}{\partial Y_t} + \frac{\partial l_{t+1}}{\partial\sigma_\varepsilon} * \frac{\partial(1)}{\partial Y_t} \end{array}\right] \\
&= \left[\begin{array}{c} \left[\frac{\partial^2 l_{t+1}}{\partial D_{t+1}^2}\frac{\partial D_{t+1}}{\partial Y_t} + \frac{\partial^2 l_{t+1}}{\partial D_{t+1}\partial Y_{t+1}}\frac{\partial Y_{t+1}}{\partial Y_t} + \frac{\partial^2 l_{t+1}}{\partial D_{t+1}\partial\sigma_\varepsilon}\frac{\partial\sigma_\varepsilon}{\partial Y_t}\right] * \frac{\partial D_{t+1}}{\partial\sigma_\varepsilon} + \frac{\partial l_{t+1}}{\partial D_{t+1}} * \frac{\partial^2 D_{t+1}}{\partial\sigma_\varepsilon\partial Y_t} \\ + \left[\frac{\partial^2 l_{t+1}}{\partial Y_{t+1}\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial Y_t} + \frac{\partial^2 l_{t+1}}{\partial Y_{t+1}^2}\frac{\partial Y_{t+1}}{\partial Y_t} + \frac{\partial^2 l_{t+1}}{\partial Y_{t+1}\partial\sigma_\varepsilon}\frac{\partial\sigma_\varepsilon}{\partial Y_t}\right] * \varepsilon_{t+1} + \\ \frac{\partial^2 l_{t+1}}{\partial\sigma_\varepsilon\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial Y_t} + \frac{\partial^2 l_{t+1}}{\partial\sigma_\varepsilon\partial Y_{t+1}}\frac{\partial Y_{t+1}}{\partial Y_t} + \frac{\partial^2 l_{t+1}}{\partial\sigma_\varepsilon^2}\frac{\partial\sigma_\varepsilon}{\partial Y_t} \end{array}\right] \\
&= \left(l_{11}^D l_1^Y + l_{11}^{DY}\rho\right) l_1^\sigma + l_1^D l_{11}^{Y\sigma} + \left(l_{11}^{DY} l_1^Y + l_{11}^Y\rho\right)\varepsilon_{t+1} + l_{11}^{D\sigma} l_1^Y + l_{11}^{Y\sigma}\rho.
\end{aligned}$$

- Computation of $\frac{\partial^2 D_{t+2}}{\partial\sigma_\varepsilon^2}$:

$$\begin{aligned}
\frac{\partial^2 D_{t+2}}{\partial\sigma_\varepsilon^2} &= \frac{\partial}{\partial\sigma_\varepsilon}\left(\frac{\partial l_{t+1}}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon} + \frac{\partial l_{t+1}}{\partial Y_{t+1}}\frac{\partial Y_{t+1}}{\partial\sigma_\varepsilon} + \frac{\partial l_{t+1}}{\partial\sigma_\varepsilon}\frac{\partial\sigma_\varepsilon}{\partial\sigma_\varepsilon}\right) \\
&= \left[\begin{array}{c} \frac{\partial\left(\frac{\partial l_{t+1}}{\partial D_{t+1}}\right)}{\partial\sigma_\varepsilon} * \frac{\partial D_{t+1}}{\partial\sigma_\varepsilon} + \frac{\partial l_{t+1}}{\partial D_{t+1}} * \frac{\partial\left(\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon}\right)}{\partial\sigma_\varepsilon} + \frac{\partial\left(\frac{\partial l_{t+1}}{\partial Y_{t+1}}\right)}{\partial\sigma_\varepsilon} * \varepsilon_{t+1} \\ + \frac{\partial l_{t+1}}{\partial Y_{t+1}} * \frac{\partial(\varepsilon_{t+1})}{\partial\sigma_\varepsilon} + \frac{\partial\left(\frac{\partial l_{t+1}}{\partial\sigma_\varepsilon}\right)}{\partial\sigma_\varepsilon} + \frac{\partial l_{t+1}}{\partial\sigma_\varepsilon} * \frac{\partial(1)}{\partial\sigma_\varepsilon} \end{array}\right] \\
&= \left[\begin{array}{c} \left[\frac{\partial^2 l_{t+1}}{\partial D_{t+1}^2}\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon} + \frac{\partial^2 l_{t+1}}{\partial D_{t+1}\partial Y_{t+1}}\frac{\partial Y_{t+1}}{\sigma_\varepsilon} + \frac{\partial^2 l_{t+1}}{\partial D_{t+1}\partial\sigma_\varepsilon}\frac{\partial\sigma_\varepsilon}{\partial\sigma_\varepsilon}\right] * \frac{\partial D_{t+1}}{\partial\sigma_\varepsilon} + \frac{\partial l_{t+1}}{\partial D_{t+1}} * \frac{\partial^2 D_{t+1}}{\partial\sigma_\varepsilon^2} \\ + \left[\frac{\partial^2 l_{t+1}}{\partial Y_{t+1}\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon} + \frac{\partial^2 l_{t+1}}{\partial Y_{t+1}^2}\frac{\partial Y_{t+1}}{\partial\sigma_\varepsilon} + \frac{\partial^2 l_{t+1}}{\partial Y_{t+1}\partial\sigma_\varepsilon}\frac{\partial\sigma_\varepsilon}{\partial\sigma_\varepsilon}\right] * \varepsilon_{t+1} + \\ \frac{\partial^2 l_{t+1}}{\partial\sigma_\varepsilon\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon} + \frac{\partial^2 l_{t+1}}{\partial\sigma_\varepsilon\partial Y_{t+1}}\frac{\partial Y_{t+1}}{\partial\sigma_\varepsilon} + \frac{\partial^2 l_{t+1}}{\partial\sigma_\varepsilon^2}\frac{\partial\sigma_\varepsilon}{\partial\sigma_\varepsilon} \end{array}\right] \\
&= \left(l_{11}^D l_1^\sigma + l_{11}^{DY}\varepsilon_{t+1} + 2l_{11}^{D\sigma}\right) l_1^\sigma + \left(l_{11}^{DY} l_1^\sigma + 2l_{11}^{Y\sigma}\right)\varepsilon_{t+1} + l_{11}^Y\varepsilon_{t+1}^2 + \left(1 + l_1^D\right) l_{11}^\sigma.
\end{aligned}$$

Derivation of p_{DD} Differentiate p_D with respect to D_t, and evaluate at $\left(\overline{D}, \overline{Y}, 0\right)$:

$$p_{DD} = \begin{bmatrix} \frac{\partial(u''(G_t))}{\partial D_t}\left(\frac{\partial D_{t+1}}{\partial D_t} - k^D\overline{L^D} - k^D\left(\overline{D} - D^c\right)\frac{\partial L_t^D}{\partial D_t}\right) + \\ \frac{\partial}{\partial D_t}\left(\frac{\partial D_{t+1}}{\partial D_t} - k^D L_t^D - k^D\left(D_t - D^c\right)\frac{\partial L_t^D}{\partial D_t}\right) u'' - \\ \left[2\frac{\partial}{\partial D_t}\left(\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial D_t}\right) + \frac{\partial}{\partial D_t}\left(\left(D_{t+1} - D^c\right)\frac{\partial^2 L_{t+1}^D}{\partial^2 D_{t+1}}\frac{\partial D_{t+1}}{\partial D_t}\right)\right]\beta_g k^D u' \\ -\left[2\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial D_t} + \left(\overline{D} - D^c\right)\frac{\partial^2 L_{t+1}^D}{\partial^2 D_{t+1}}\frac{\partial D_{t+1}}{\partial D_t}\right]\beta_g k^D \frac{\partial}{\partial D_t}\left(u'\left(G_{t+1}\right)\right) - \\ \left[\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial D_t} + \frac{\partial D_{t+1}}{\partial D_t}\frac{\partial L_{t+1}^D}{\partial D_{t+1}} + \left(\overline{D} - D^c\right)\frac{\partial^2 L_{t+1}^D}{\partial^2 D_{t+1}}\frac{\partial D_{t+1}}{\partial D_t}\right] * \beta_g k^D u'' * \\ \left(\frac{\partial D_{t+2}}{\partial D_t} - k^D\overline{L^D}\frac{\partial D_{t+1}}{\partial D_t} - k^D\left(\overline{D} - D^c\right)\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial D_t}\right) - \\ \beta_g k^D\left[\overline{L^D} + \gamma\left(\overline{D} - D^c\right)\overline{L^D}(1 - \overline{L^D})\right] * \frac{\partial}{\partial D_t}\left(u''\left(G_{t+1}\right)\right) * \\ \left(\frac{\partial D_{t+2}}{\partial D_t} - k^D\overline{L^D}\frac{\partial D_{t+1}}{\partial D_t} - k^D\left(\overline{D} - D^c\right)\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial D_t}\right) \\ -\beta_g k^D\left[\overline{L^D} + \gamma\left(\overline{D} - D^c\right)\overline{L^D}(1 - \overline{L^D})\right] * u'' * \\ \frac{\partial}{\partial D_t}\left(\frac{\partial D_{t+2}}{\partial D_t} - k^D L_{t+1}^D\frac{\partial D_{t+1}}{\partial D_t} - k^D\left(D_{t+1} - D^c\right)\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial D_t}\right) \end{bmatrix}.$$

Using (2.98) and (2.105), as well as (2.100), this becomes:

$$p_{DD} = \begin{bmatrix} \left[\frac{\partial D_{t+1}}{\partial D_t} - k^D\overline{L^D} - k^D\left(\overline{D} - D^c\right)\frac{\partial L_t^D}{\partial D_t}\right]^2 u''' + \\ \left[\frac{\partial^2 D_{t+1}}{\partial^2 D_t} - k^D\left(2\frac{\partial L_t^D}{\partial D_t} + \left(\overline{D} - D^c\right)\frac{\partial^2 L_t^D}{\partial^2 D_t}\right)\right] u'' - \\ \beta_g k^D \begin{bmatrix} 2\frac{\partial^2 L_{t+1}^D}{\partial^2 D_{t+1}}\left(\frac{\partial D_{t+1}}{\partial D_t}\right)^2 + 2\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial^2 D_{t+1}}{\partial^2 D_t} + \\ \frac{\partial^2 L_{t+1}^D}{\partial^2 D_{t+1}}\left(\frac{\partial D_{t+1}}{\partial D_t}\right)^2 + \left(\overline{D} - D^c\right)\frac{\partial^3 L_{t+1}^D}{\partial^3 D_{t+1}}\left(\frac{\partial D_{t+1}}{\partial D_t}\right)^2 \\ + \left(\overline{D} - D^c\right)\frac{\partial^2 L_{t+1}^D}{\partial^2 D_{t+1}}\frac{\partial^2 D_{t+1}}{\partial^2 D_t} \end{bmatrix} u' - \\ \beta_g k^D\left[2l_1^D\gamma\overline{L^D}(1 - \overline{L^D}) + l_1^D\left(\overline{D} - D^c\right)\gamma^2\overline{L^D}(1 - \overline{L^D})\left(1 - 2\overline{L^D}\right)\right] * u'' * \\ \left(\frac{\partial D_{t+2}}{\partial D_t} - k^D\overline{L^D}\frac{\partial D_{t+1}}{\partial D_t} - k^D\left(\overline{D} - D^c\right)\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial D_t}\right) - \\ \beta_g k^D\left[2l_1^D\gamma\overline{L^D}(1 - \overline{L^D}) + l_1^D\left(\overline{D} - D^c\right)\gamma^2\overline{L^D}(1 - \overline{L^D})\left(1 - 2\overline{L^D}\right)\right] * u'' * \\ \left(\frac{\partial D_{t+2}}{\partial D_t} - k^D\overline{L^D}\frac{\partial D_{t+1}}{\partial D_t} - k^D\left(\overline{D} - D^c\right)\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial D_t}\right) \\ -u'''\left(\frac{\partial D_{t+2}}{\partial D_t} - k^D\overline{L^D}\frac{\partial D_{t+1}}{\partial D_t} - k^D\left(\overline{D} - D^c\right)\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial D_t}\right)^2 - \\ \begin{bmatrix} \frac{\partial^2 D_{t+2}}{\partial^2 D_t} - k^D\left(2\frac{\partial L_{t+1}^D}{\partial D_{t+1}} + \left(\overline{D} - D^c\right)\frac{\partial^2 L_{t+1}^D}{\partial^2 D_{t+1}}\right)\left(\frac{\partial D_{t+1}}{\partial D_t}\right)^2 \\ -k^D\left(\overline{L^D} + \left(\overline{D} - D^c\right)\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\right)\frac{\partial^2 D_{t+1}}{\partial^2 D_t} \end{bmatrix} u'' \end{bmatrix}.$$

Using the derivatives of D_{t+1} and D_{t+2}, (2.98) and (2.105), we simplify the last equation once more to

$$p_{DD} = \left[\begin{array}{c} \left[l_1^D - k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)\right]^2 u'''+ \\ \left[l_{11}^D - k^D\gamma\overline{L^D}(1-\overline{L^D})\left(2+\gamma\left(\overline{D}-D^c\right)\left(1-2\overline{L^D}\right)\right)\right]u''- \\ \beta_g k^D \left[\begin{array}{c} \left[3\gamma^2\overline{L^D}(1-\overline{L^D})\left(1-2\overline{L^D}\right)+\left(\overline{D}-D^c\right)\frac{\partial^3 L_{t+1}^D}{\partial^3 D_{t+1}}\right]\left(l_1^D\right)^2+ \\ \left[2\gamma\overline{L^D}(1-\overline{L^D})+\left(\overline{D}-D^c\right)\gamma^2\overline{L^D}(1-\overline{L^D})\left(1-2\overline{L^D}\right)\right]l_{11}^D \end{array}\right]u'- \\ 2l_1^D\beta_g k^D\gamma\overline{L^D}(1-\overline{L^D})* \\ \left[2+\left(\overline{D}-D^c\right)\gamma\left(1-2\overline{L^D}\right)\right]\left[\left(l_1^D\right)^2 - k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)l_1^D\right]u'' \\ -\left[\left(l_1^D\right)^2 - k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)l_1^D\right]^2 u''' \\ -\left[\begin{array}{c} l_{11}^D\left(l_1^D\right)^2 + l_1^D l_{11}^D - k^D\gamma\overline{L^D}(1-\overline{L^D})\left(2+\left(\overline{D}-D^c\right)\gamma\left(1-2\overline{L^D}\right)\right)\left(l_1^D\right)^2 \\ -k^D\overline{L^D}\left[1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right]l_{11}^D \end{array}\right]u'' \end{array} \right].$$

The only additional term that we have to compute in the equation above is $\frac{\partial^3 L_{t+1}^D}{\partial^3 D_{t+1}}$, but from (2.105) we get

$$\begin{aligned} \frac{\partial^3 L_{t+1}^D}{\partial D_{t+1}^3} &= \frac{\partial}{\partial D_{t+1}}\left[\gamma^2 L_{t+1}^D\left(1-L_{t+1}^D\right)\left(1-2L_{t+1}^D\right)\right] \\ &= \gamma^2\left[\begin{array}{c} \frac{\partial L_{t+1}^D}{\partial D_{t+1}}\left(1-L_{t+1}^D\right)\left(1-2L_{t+1}^D\right) - L_{t+1}^D\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\left(1-2L_{t+1}^D\right) \\ -L_{t+1}^D\left(1-L_{t+1}^D\right)2\frac{\partial L_{t+1}^D}{\partial D_{t+1}} \end{array}\right] \\ &= \gamma^2\left[\begin{array}{c} \gamma L_{t+1}^D\left(1-L_{t+1}^D\right)^2\left(1-2L_{t+1}^D\right) - \gamma\left(L_{t+1}^D\right)^2\left(1-L_{t+1}^D\right)\left(1-2L_{t+1}^D\right) \\ -2\gamma\left(L_{t+1}^D\left(1-L_{t+1}^D\right)\right)^2 \end{array}\right] \\ &= \gamma^3 L_{t+1}^D\left(1-L_{t+1}^D\right)\left[\begin{array}{c} \left(1-L_{t+1}^D\right)\left(1-2L_{t+1}^D\right) - L_{t+1}^D\left(1-2L_{t+1}^D\right) \\ -2L_{t+1}^D\left(1-L_{t+1}^D\right) \end{array}\right] \qquad (2.119) \\ &= \gamma^3 L_{t+1}^D\left(1-L_{t+1}^D\right)\left[\left(1-2L_{t+1}^D\right)\left(1-2L_{t+1}^D\right) - 2L_{t+1}^D\left(1-L_{t+1}^D\right)\right] \\ &= \gamma^3 L_{t+1}^D\left(1-L_{t+1}^D\right)\left[1-6L_{t+1}^D+6\left(L_{t+1}^D\right)^2\right]. \end{aligned}$$

Hence,

$$p_{DD}=\left[\begin{array}{c}
\left[l_1^D-k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)\right]^2u'''+\\
\left[l_{11}^D-k^D\gamma\overline{L^D}(1-\overline{L^D})\left(2+\gamma\left(\overline{D}-D^c\right)\left(1-2\overline{L^D}\right)\right)\right]u''-\\
\beta_g k^D\gamma\overline{L^D}(1-\overline{L^D})\left[\begin{array}{c}\left[\begin{array}{c}3\gamma\left(1-2\overline{L^D}\right)+\\ \left(\overline{D}-D^c\right)\gamma^2\left(1-6\overline{L^D}+6\left(\overline{L^D}\right)^2\right)\end{array}\right]\left(l_1^D\right)^2\\ +\left[2+\left(\overline{D}-D^c\right)\gamma\left(1-2\overline{L^D}\right)\right]l_{11}^D\end{array}\right]u'\\
-2l_1^D\beta_g k^D\gamma\overline{L^D}(1-\overline{L^D})*\\
\left[2+\left(\overline{D}-D^c\right)\gamma\left(1-2\overline{L^D}\right)\right]\left[\left(l_1^D\right)^2-\frac{1}{\beta_g}l_1^D\right]u''\\
-\left[\left(l_1^D\right)^2-k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)l_1^D\right]^2u'''\\
-\left[\begin{array}{c}\left(\left(l_1^D\right)^2+l_1^D-k^D\overline{L^D}\left[1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right]\right)l_{11}^D-\\ k^D\gamma\overline{L^D}(1-\overline{L^D})\left(2+\left(\overline{D}-D^c\right)\gamma\left(1-2\overline{L^D}\right)\right)\left(l_1^D\right)^2\end{array}\right]u''
\end{array}\right].$$

Finally, we can isolate the only unknown term l_{11}^D of that equation and multiply it to $\widetilde{D}_t^2$:

$$\begin{aligned}
&p_{DD}\widetilde{D}_t^2\\
=&\left[\begin{array}{c}
l_{11}^D\left[\begin{array}{c}\left(1+k^D\overline{L^D}\left[1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right]-\left(l_1^D\right)^2-l_1^D\right)u''-\\ \beta_g k^D\gamma\overline{L^D}(1-\overline{L^D})\left[2+\left(\overline{D}-D^c\right)\gamma\left(1-2\overline{L^D}\right)\right]u'\end{array}\right]\\
+\left[l_1^D-k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)\right]^2\left(1-\left(l_1^D\right)^2\right)u'''-\\
\beta_g k^D\gamma^2\overline{L^D}(1-\overline{L^D})\left[\begin{array}{c}3\left(1-2\overline{L^D}\right)+\\ \left(\overline{D}-D^c\right)\gamma\left(1-6\overline{L^D}+6\left(\overline{L^D}\right)^2\right)\end{array}\right]\left(l_1^D\right)^2u'\\
+k^D\gamma\overline{L^D}(1-\overline{L^D})\left[2+\left(\overline{D}-D^c\right)\gamma\left(1-2\overline{L^D}\right)\right]\left[-1+3\left(l_1^D\right)^2-2\beta_g\left(l_1^D\right)^3\right]u''
\end{array}\right]\widetilde{D}_t^2\\
\equiv&\left(\Lambda l_{11}^D+\Xi\right)\widetilde{D}_t^2. \qquad (2.120)
\end{aligned}$$

Derivation of the p_{DY} Differentiate p_Y with respect to D_t around the point $\left(\overline{D},\overline{Y},0\right)$ and use (2.100) to give:

$$p_{DY}=\left[\begin{array}{c}
\frac{\partial}{\partial D_t}\left(u''\left(G_t\right)\right)*\left(\frac{\partial D_{t+1}}{\partial Y_t}+\tau^y\right)+u''*\frac{\partial}{\partial D_t}\left(\frac{\partial D_{t+1}}{\partial Y_t}+\tau^y\right)-\\
\beta_g k^D*\left[2\frac{\partial}{\partial D_t}\left(\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial Y_t}\right)+\frac{\partial}{\partial D_t}\left(\left(D_{t+1}-D^c\right)\frac{\partial^2L_{t+1}^D}{\partial^2D_{t+1}}\frac{\partial D_{t+1}}{\partial Y_t}\right)\right]*u'\\
-\beta_g k^D\left[2\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial Y_t}+\left(D_{t+1}-D^c\right)\frac{\partial^2L_{t+1}^D}{\partial^2D_{t+1}}\frac{\partial D_{t+1}}{\partial Y_t}\right]*\frac{\partial}{\partial D_t}\left(u'\left(G_{t+1}\right)\right)-\\
\beta_g k^D\left[\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial D_t}+\frac{\partial D_{t+1}}{\partial D_t}\frac{\partial L_{t+1}^D}{\partial D_{t+1}}+\left(\overline{D}-D^c\right)\frac{\partial^2L_{t+1}^D}{\partial D_{t+1}^2}\frac{\partial D_{t+1}}{\partial D_t}\right]*u''*\\
\left(\frac{\partial D_{t+2}}{\partial Y_t}-k^D\overline{L^D}\frac{\partial D_{t+1}}{\partial Y_t}+\tau^y\rho-k^D\left(\overline{D}-D^c\right)\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial Y_t}\right)-\\
\frac{\partial}{\partial D_t}\left(u''\left(G_{t+1}\right)\right)*\left(\frac{\partial D_{t+2}}{\partial Y_t}-k^D\overline{L^D}\frac{\partial D_{t+1}}{\partial Y_t}+\tau^y\rho-k^D\left(\overline{D}-D^c\right)\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial Y_t}\right)\\
-u''*\frac{\partial}{\partial D_t}\left(\frac{\partial D_{t+2}}{\partial Y_t}-k^DL_{t+1}^D\frac{\partial D_{t+1}}{\partial Y_t}+\tau^y\rho-k^D\left(D_{t+1}-D^c\right)\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial Y_t}\right)
\end{array}\right].$$

Then, using (2.98), (2.100) and (2.105), we can rewrite this expression as:

$$p_{DY} = \left[\begin{array}{c}
\left[\frac{\partial D_{t+1}}{\partial D_t} - k^D\overline{L^D} - k^D\left(\overline{D}-D^c\right)\frac{\partial L_t^D}{\partial D_t}\right]\left(\frac{\partial D_{t+1}}{\partial Y_t}+\tau^y\right)u''' + u''\frac{\partial^2 D_{t+1}}{\partial D_t\partial Y_t} - \\
\beta_g k^D * \left\{\begin{array}{c} 2\left[\frac{\partial^2 L^D_{t+1}}{\partial^2 D_{t+1}}\frac{\partial D_{t+1}}{\partial D_t}\frac{\partial D_{t+1}}{\partial Y_t} + \frac{\partial L^D_{t+1}}{\partial D_{t+1}}\frac{\partial^2 D_{t+1}}{\partial D_t\partial Y_t}\right] + \frac{\partial^2 L^D_{t+1}}{\partial^2 D_{t+1}}\frac{\partial D_{t+1}}{\partial Y_t}\frac{\partial D_{t+1}}{\partial D_t} \\ +\left(\overline{D}-D^c\right)\frac{\partial^2 D_{t+1}}{\partial D_t\partial Y_t}\frac{\partial^2 L^D_{t+1}}{\partial D^2_{t+1}} + \left(\overline{D}-D^c\right)\frac{\partial^3 L^D_{t+1}}{\partial^3 D_{t+1}}\frac{\partial D_{t+1}}{\partial D_t}\frac{\partial D_{t+1}}{\partial Y_t}\end{array}\right\} * u' \\
-\beta_g k^D\left[2l_1^Y\gamma\overline{L^D}(1-\overline{L^D}) + l_1^Y\left(\overline{D}-D^c\right)\gamma^2\overline{L^D}(1-\overline{L^D})\left(1-2\overline{L^D}\right)\right] * \\
\left[\frac{\partial D_{t+2}}{\partial D_t} - k^D\overline{L^D}\frac{\partial D_{t+1}}{\partial D_t} - k^D\left(\overline{D}-D^c\right)\frac{\partial L^D_{t+1}}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial D_t}\right]u'' - \\
\beta_g k^D\left[2l_1^D\gamma\overline{L^D}(1-\overline{L^D}) + l_1^D\left(\overline{D}-D^c\right)\gamma^2\overline{L^D}(1-\overline{L^D})\left(1-2\overline{L^D}\right)\right] * \\
\left[\frac{\partial D_{t+2}}{\partial Y_t} - k^D\overline{L^D}\frac{\partial D_{t+1}}{\partial Y_t} + \tau^y\rho - k^D\left(\overline{D}-D^c\right)\frac{\partial L^D_{t+1}}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial Y_t}\right]u'' - \\
u''' * \left[\begin{array}{c}\frac{\partial D_{t+2}}{\partial D_t} - k^D\overline{L^D}\frac{\partial D_{t+1}}{\partial D_t} - \\ k^D\left(\overline{D}-D^c\right)\frac{\partial L^D_{t+1}}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial D_t}\end{array}\right] * \left[\begin{array}{c}\frac{\partial D_{t+2}}{\partial Y_t} - k^D\overline{L^D}\frac{\partial D_{t+1}}{\partial Y_t} + \\ \tau^y\rho - k^D\left(\overline{D}-D^c\right)\frac{\partial L^D_{t+1}}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial Y_t}\end{array}\right] \\
-u'' * \left[\begin{array}{c}\frac{\partial^2 D_{t+2}}{\partial D_t\partial Y_t} - k^D\frac{\partial L^D_{t+1}}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial D_t}\frac{\partial D_{t+1}}{\partial Y_t} - k^D\overline{L^D}\frac{\partial^2 D_{t+1}}{\partial D_t\partial Y_t} - k^D\frac{\partial L^D_{t+1}}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial D_t}\frac{\partial D_{t+1}}{\partial Y_t} \\ -k^D\left(\overline{D}-D^c\right)\frac{\partial^2 L^D_{t+1}}{\partial^2 D_{t+1}}\frac{\partial D_{t+1}}{\partial D_t}\frac{\partial D_{t+1}}{\partial Y_t} - k^D\left(\overline{D}-D^c\right)\frac{\partial L^D_{t+1}}{\partial D_{t+1}}\frac{\partial^2 D_{t+1}}{\partial D_t\partial Y_t}\end{array}\right]
\end{array}\right].$$

Using again the derivatives of D_{t+1} and D_{t+2} computed above, (2.98), (2.105) and (2.119), we simplify the last equation once more to

$$p_{DY} = \left[\begin{array}{c}
\left[l_1^D - k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)\right]\left(l_1^Y+\tau^y\right)u''' + l_{11}^{DY}u'' - \\
\beta_g k^D * \left\{\begin{array}{c} \left[\begin{array}{c} 3\gamma^2\overline{L^D}(1-\overline{L^D})\left(1-2\overline{L^D}\right) + \\ \left(\overline{D}-D^c\right)\gamma^3\overline{L^D}(1-\overline{L^D})\left(1-6\overline{L^D}+6\left(\overline{L^D}\right)^2\right)\end{array}\right] l_1^D l_1^Y + \\ \left[2\gamma\overline{L^D}(1-\overline{L^D}) + \left(\overline{D}-D^c\right)\gamma^2\overline{L^D}(1-\overline{L^D})\left(1-2\overline{L^D}\right)\right] l_{11}^{DY}\end{array}\right\} * u' \\
-\beta_g k^D\gamma\overline{L^D}(1-\overline{L^D})\left[2+\left(\overline{D}-D^c\right)\gamma\left(1-2\overline{L^D}\right)\right] * \\
\left[\left(l_1^D\right)^2 l_1^Y - k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)l_1^D l_1^Y\right]u'' - \\
\beta_g k^D\gamma\overline{L^D}(1-\overline{L^D})\left[2+\left(\overline{D}-D^c\right)\gamma\left(1-2\overline{L^D}\right)\right] * \\
\left[\left(l_1^D\right)^2 l_1^Y + l_1^D l_1^Y\rho + l_1^D\tau^y\rho - k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)l_1^D l_1^Y\right]u'' - \\
u''' * \left[\left(l_1^D\right)^2 - k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)l_1^D\right] * \\
\left[l_1^D l_1^Y + l_1^Y\rho + \tau^y\rho - k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)l_1^Y\right] \\
-u'' * \left[\begin{array}{c} l_{11}^D l_1^D l_1^Y + (1+\rho)\, l_1^D l_{11}^{DY} - \\ k^D\gamma\overline{L^D}(1-\overline{L^D})\left[2+\left(\overline{D}-D^c\right)\gamma\left(1-2\overline{L^D}\right)\right] l_1^D l_1^Y \\ -k^D\overline{L^D}\left[1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right] l_{11}^{DY}\end{array}\right]
\end{array}\right].$$

Next, we isolate the two unknown variables, namely l_{11}^{DY} and l_{11}^D, and multiply p_{DY} by

$\widetilde{D}_t\widetilde{Y}_t$ to give:

$$
\begin{aligned}
& p_{DY}\widetilde{D}_t\widetilde{Y}_t \\
= & \left[\begin{array}{c}
l_{11}^{DY}\left\{\begin{array}{c}\left[1+k^D\overline{L^D}\left[1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right]-(1+\rho)\,l_1^D\right]u'' \\ -\beta_g k^D\gamma\overline{L^D}(1-\overline{L^D})\left[2+\left(\overline{D}-D^c\right)\gamma\left(1-2\overline{L^D}\right)\right]u'\end{array}\right\}- \\
l_{11}^D l_1^D l_1^Y u''+\left[l_1^D-k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)\right]* \\
\left[\begin{array}{c} l_1^Y+\tau^y-\left(l_1^D\right)^2 l_1^Y-l_1^D l_1^Y\rho-l_1^D\tau^y\rho+ \\ k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)l_1^D l_1^Y\end{array}\right]u'''- \\
\beta_g k^D\gamma^2\overline{L^D}(1-\overline{L^D})\left[\begin{array}{c}3\left(1-2\overline{L^D}\right)+ \\ \left(\overline{D}-D^c\right)\gamma\left(1-6\overline{L^D}+6\left(\overline{L^D}\right)^2\right)\end{array}\right]l_1^D l_1^Y u'- \\
k^D\gamma\overline{L^D}(1-\overline{L^D})\left[2+\left(\overline{D}-D^c\right)\gamma\left(1-2\overline{L^D}\right)\right]\left[\begin{array}{c}2\beta_g\left(l_1^D\right)^2 l_1^Y+\beta_g l_1^D l_1^Y\rho \\ +\beta_g l_1^D\tau^y\rho-3l_1^D l_1^Y\end{array}\right]u''
\end{array}\right]\widetilde{D}_t\widetilde{Y}_t \\
= & \left(\Pi l_{11}^{DY}+\Upsilon l_{11}^D+\Phi\right)\widetilde{D}_t\widetilde{Y}_t. \qquad (2.121)
\end{aligned}
$$

Derivation of $p_{D\sigma_\varepsilon}$ Differentiate p_{σ_ε} with respect to D_t around the point $\left(\overline{D},\overline{Y},0\right)$ and use

$$
p_{D\sigma_\varepsilon}=\left[\begin{array}{c}
\frac{\partial}{\partial D_t}\left(u''\left(G_t\right)\right)*\left(\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon}\right)+u''*\frac{\partial}{\partial D_t}\left(\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon}\right)- \\
\beta_g k^D*\left[2\frac{\partial}{\partial D_t}\left(\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon}\right)+\frac{\partial}{\partial D_t}\left(\left(D_{t+1}-D^c\right)\frac{\partial^2 L_{t+1}^D}{\partial^2 D_{t+1}}\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon}\right)\right]*u' \\
-\beta_g k^D\left[2\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon}+\left(\overline{D}-D^c\right)\frac{\partial^2 L_{t+1}^D}{\partial^2 D_{t+1}}\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon}\right]*\frac{\partial}{\partial D_t}\left(u'\left(G_{t+1}\right)\right)- \\
\beta_g k^D*\frac{\partial}{\partial D_t}\left(L_{t+1}^D+\left(D_{t+1}-D^c\right)\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\right)*u''* \\
\left(\frac{\partial D_{t+2}}{\partial\sigma_\varepsilon}+\varepsilon_{t+1}-k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon}\right)- \\
\frac{\partial}{\partial D_t}\left(u''\left(G_{t+1}\right)\right)*\left(\frac{\partial D_{t+2}}{\partial\sigma_\varepsilon}+\varepsilon_{t+1}-k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon}\right) \\
-u''*\frac{\partial}{\partial D_t}\left(\frac{\partial D_{t+2}}{\partial\sigma_\varepsilon}+\varepsilon_{t+1}-k^D L_{t+1}^D\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon}-k^D\left(D_{t+1}-D^c\right)\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon}\right)
\end{array}\right].
$$

Recall from (2.109) that $\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon}=l_1^\sigma=0$. In addition, using the first- and second-order derivatives of D_{t+1} and D_{t+2} with respect to σ_ε, we obtain

$$
p_{D\sigma_\varepsilon}=\left[\begin{array}{c}
\frac{\partial^2 D_{t+1}}{\partial D_t\partial\sigma_\varepsilon}u''- \\
\beta_g k^D\left[\begin{array}{c}2\left(\frac{\partial^2 L_{t+1}^D}{\partial D_{t+1}^2}\frac{\partial D_{t+1}}{\partial D_t}\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon}+\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial^2 D_{t+1}}{\partial D_t\partial\sigma_\varepsilon}\right)+\frac{\partial D_{t+1}}{\partial D_t}\frac{\partial^2 L_{t+1}^D}{\partial D_{t+1}^2}\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon}+ \\ \left(\overline{D}-D^c\right)\left(\frac{\partial^3 L_{t+1}^D}{\partial D_{t+1}^3}\frac{\partial D_{t+1}}{\partial D_t}\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon}+\frac{\partial^2 L_{t+1}^D}{\partial D_{t+1}^2}\frac{\partial^2 D_{t+1}}{\partial D_t\partial\sigma_\varepsilon}\right)\end{array}\right]u'- \\
\left[2\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial D_t}+\left(\overline{D}-D^c\right)\frac{\partial^2 L_{t+1}^D}{\partial D_{t+1}^2}\frac{\partial D_{t+1}}{\partial D_t}\right]\left(l_1^D l_1^\sigma+l_1^Y\varepsilon_{t+1}+l_1^\sigma+\varepsilon_{t+1}\right)\beta_g k^D u''- \\
\left[\frac{\partial D_{t+2}}{\partial D_t}-k^D\overline{L^D}\frac{\partial D_{t+1}}{\partial D_t}-k^D\left(\overline{D}-D^c\right)\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial D_t}\right]\left(l_1^D l_1^\sigma+l_1^Y\varepsilon_{t+1}+l_1^\sigma+\varepsilon_{t+1}\right)u''' \\
-u''*\left(\frac{\partial^2 D_{t+2}}{\partial D_t\partial\sigma_\varepsilon}-k^D\overline{L^D}\frac{\partial^2 D_{t+1}}{\partial D_t\partial\sigma_\varepsilon}-k^D\left(\overline{D}-D^c\right)\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial^2 D_{t+1}}{\partial D_t\partial\sigma_\varepsilon}\right)
\end{array}\right].
$$

We can rewrite the last equation further as

$$p_{D\sigma_\varepsilon} = \begin{bmatrix} u'' l_{11}^{D\sigma} - \\ \beta_g k^D * \left[2\gamma \overline{L^D}(1-\overline{L^D}) + \left(\overline{D}-D^c\right)\gamma^2 \overline{L^D}(1-\overline{L^D})\left(1-2\overline{L^D}\right)\right] * l_{11}^{D\sigma} u' \\ -\beta_g k^D * \begin{bmatrix} 2\gamma \overline{L^D}(1-\overline{L^D}) + \\ \left(\overline{D}-D^c\right)\gamma^2 \overline{L^D}(1-\overline{L^D})\left(1-2\overline{L^D}\right) \end{bmatrix} * l_1^D u'' \left(l_1^Y+1\right)\varepsilon_{t+1} \\ -u''' * \left[\left(l_1^D\right)^2 - k^D \overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right) l_1^D\right] * \left(l_1^Y+1\right)\varepsilon_{t+1} \\ -u'' * \left(2 l_1^D l_{11}^{D\sigma} + l_1^D l_{11}^{DY}\varepsilon_{t+1} - k^D \overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right) l_{11}^{D\sigma}\right) \end{bmatrix}.$$

Then, grouping the terms of the equation above and multiplying it by $\widetilde{D}_t \sigma_\varepsilon$, we have

$$\begin{aligned} & p_{D\sigma_\varepsilon}\widetilde{D}_t\sigma_\varepsilon \\ = & \left\{ \begin{array}{c} l_{11}^{D\sigma} \begin{bmatrix} u''\left[1+k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right) - 2l_1^D\right] - \\ u'\beta_g k^D \gamma \overline{L^D}(1-\overline{L^D})\left[2+\left(\overline{D}-D^c\right)\gamma\left(1-2\overline{L^D}\right)\right] \end{bmatrix} \\ -u'' l_{11}^{DY} l_1^D \varepsilon_{t+1} - \\ l_1^D\left(l_1^Y+1\right) \begin{bmatrix} u''\beta_g k^D \gamma \overline{L^D}(1-\overline{L^D})\left[2+\left(\overline{D}-D^c\right)\gamma\left(1-2\overline{L^D}\right)\right] + \\ u'''\left[l_1^D - k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)\right] \end{bmatrix} \varepsilon_{t+1} \end{array} \right\} \widetilde{D}_t\sigma_\varepsilon \\ = & \left(\Psi l_{11}^{D\sigma} + \Omega\varepsilon_{t+1}\right)\widetilde{D}_t\sigma_\varepsilon. \end{aligned}$$

Finally, applying expectations to the equation above, we have that

$$E_t\left[p_{D\sigma_\varepsilon}\widetilde{D}_t\sigma_\varepsilon\right] = E_t\left[\left(\Psi l_{11}^{D\sigma} + \Omega\varepsilon_{t+1}\right)\widetilde{D}_t\sigma_\varepsilon\right] = \Psi l_{11}^{D\sigma}\widetilde{D}_t\sigma_\varepsilon. \tag{2.122}$$

Derivation of p_{YY} Differentiate p_Y once more with respect to Y_t around the point $\left(\overline{D},\overline{Y},0\right)$ and use (2.100):

$$p_{YY} = \begin{bmatrix} \frac{\partial}{\partial Y_t}\left(u''\left(G_t\right)\right) * \left(\frac{\partial D_{t+1}}{\partial Y_t} + \tau^y\right) + u'' * \frac{\partial}{\partial Y_t}\left(\frac{\partial D_{t+1}}{\partial Y_t} + \tau^y\right) \\ -\beta_g k^D * \left[2\frac{\partial}{\partial Y_t}\left(\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial Y_t}\right) + \frac{\partial}{\partial Y_t}\left(\left(D_{t+1}-D^c\right)\frac{\partial^2 L_{t+1}^D}{\partial D_{t+1}^2}\frac{\partial D_{t+1}}{\partial Y_t}\right)\right] * u' \\ -\beta_g k^D\left[2\left(\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial Y_t}\right) + \left(D_{t+1}-D^c\right)\frac{\partial^2 L_{t+1}^D}{\partial D_{t+1}^2}\frac{\partial D_{t+1}}{\partial Y_t}\right] * \frac{\partial}{\partial Y_t}\left(u'\left(G_{t+1}\right)\right) - \\ \beta_g k^D \begin{bmatrix} \frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial Y_t} + \frac{\partial D_{t+1}}{\partial Y_t}\frac{\partial L_{t+1}^D}{\partial D_{t+1}} \\ +\left(\overline{D}-D^c\right)\frac{\partial^2 L_{t+1}^D}{\partial D_{t+1}^2}\frac{\partial D_{t+1}}{\partial Y_t} \end{bmatrix} * u'' * \begin{pmatrix} \frac{\partial D_{t+2}}{\partial Y_t} - k^D\overline{L^D}\frac{\partial D_{t+1}}{\partial Y_t} + \\ \tau^y\rho - k^D\left(\overline{D}-D^c\right)\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial Y_t} \end{pmatrix} \\ -\frac{\partial}{\partial Y_t}\left(u''\left(G_{t+1}\right)\right) * \left(\frac{\partial D_{t+2}}{\partial Y_t} - k^D\overline{L^D}\frac{\partial D_{t+1}}{\partial Y_t} + \tau^y\rho - k^D\left(\overline{D}-D^c\right)\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial Y_t}\right) \\ -u'' * \frac{\partial}{\partial Y_t}\left(\frac{\partial D_{t+2}}{\partial Y_t} - k^D L_{t+1}^D\frac{\partial D_{t+1}}{\partial Y_t} + \tau^y\rho - k^D\left(D_{t+1}-D^c\right)\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial Y_t}\right) \end{bmatrix}.$$

Hence, also using (2.100),

$$
p_{YY} = \left[\begin{array}{c}
\left(l_1^Y + \tau^y\right)^2 u''' + \frac{\partial^2 D_{t+1}}{\partial Y_t^2} u'' - \\
u'\beta_g k^D * \left[\begin{array}{c} 2\left[\frac{\partial^2 L^D_{t+1}}{\partial^2 D_{t+1}} \left(\frac{\partial D_{t+1}}{\partial Y_t}\right)^2 + \frac{\partial L^D_{t+1}}{\partial D_{t+1}} \frac{\partial^2 D_{t+1}}{\partial^2 Y_t} \right] + \frac{\partial^2 L^D_{t+1}}{\partial^2 D_{t+1}} \left(\frac{\partial D_{t+1}}{\partial Y_t}\right)^2 \\
+ \left(\overline{D} - D^c\right) \frac{\partial^3 L^D_{t+1}}{\partial^3 D_{t+1}} \left(\frac{\partial D_{t+1}}{\partial Y_t}\right)^2 + \left(\overline{D} - D^c\right) \frac{\partial^2 L^D_{t+1}}{\partial^2 D_{t+1}} \frac{\partial^2 D_{t+1}}{\partial^2 Y_t} \end{array} \right] \\
-2\beta_g k^D \left[2\gamma \overline{L^D}(1 - \overline{L^D}) + \left(\overline{D} - D^c\right) \gamma^2 \overline{L^D}(1 - \overline{L^D}) \left(1 - 2\overline{L^D}\right) \right] l_1^Y u'' * \\
\left[l_1^D l_1^Y + l_1^Y \rho + \tau^y \rho - k^D \overline{L^D} \left(1 + \left(\overline{D} - D^c\right)\gamma(1 - \overline{L^D})\right) l_1^Y \right] \\
-u''' * \left[l_1^D l_1^Y + l_1^Y \rho + \tau^y \rho - k^D \overline{L^D} \left(1 + \left(\overline{D} - D^c\right)\gamma(1 - \overline{L^D})\right) l_1^Y \right]^2 - \\
u'' * \left(\begin{array}{c} \frac{\partial^2 D_{t+2}}{\partial Y_t^2} - k^D \frac{\partial L^D_{t+1}}{\partial D_{t+1}} \left(\frac{\partial D_{t+1}}{\partial Y_t}\right)^2 - k^D \overline{L^D} \frac{\partial^2 D_{t+1}}{\partial Y_t^2} - k^D \frac{\partial L^D_{t+1}}{\partial D_{t+1}} \left(\frac{\partial D_{t+1}}{\partial Y_t}\right)^2 \\
-k^D \left(\overline{D} - D^c\right) \frac{\partial^2 L^D_{t+1}}{\partial D^2_{t+1}} \left(\frac{\partial D_{t+1}}{\partial Y_t}\right)^2 - k^D \left(\overline{D} - D^c\right) \frac{\partial L^D_{t+1}}{\partial D_{t+1}} \frac{\partial^2 D_{t+1}}{\partial Y_t^2} \end{array} \right)
\end{array} \right].
$$

Again, given (2.98), (2.105), (2.119) and first- and second-order derivatives of D_{t+1} and D_{t+2}, it follows that

$$
p_{YY} = \left[\begin{array}{c}
\left(l_1^Y + \tau^y\right)^2 u''' + l_{11}^Y u'' - \\
\beta_g k^D \left[\begin{array}{c} \left[2\gamma \overline{L^D}(1 - \overline{L^D}) + \left(\overline{D} - D^c\right) \gamma^2 \overline{L^D}(1 - \overline{L^D}) \left(1 - 2\overline{L^D}\right) \right] l_{11}^Y \\
+ \left[\begin{array}{c} 3\gamma^2 \overline{L^D}(1 - \overline{L^D}) \left(1 - 2\overline{L^D}\right) + \\
\left(\overline{D} - D^c\right) \gamma^3 \overline{L^D}(1 - \overline{L^D}) \left(1 - 6\overline{L^D} + 6\left(\overline{L^D}\right)^2\right) \end{array} \right] \left(l_1^Y\right)^2 \end{array} \right] u' \\
-2k^D \gamma \overline{L^D}(1 - \overline{L^D}) \left[2 + \left(\overline{D} - D^c\right)\gamma\left(1 - 2\overline{L^D}\right) \right] * \\
\left[\begin{array}{c} \beta_g l_1^D \left(l_1^Y\right)^2 + \beta_g \left(l_1^Y\right)^2 \rho + \beta_g l_1^Y \tau^y \rho - \\
\beta_g k^D \overline{L^D} \left(1 + \left(\overline{D} - D^c\right)\gamma(1 - \overline{L^D})\right) \left(l_1^Y\right)^2 \end{array} \right] u'' \\
- \left[l_1^D l_1^Y + l_1^Y \rho + \tau^y \rho - k^D \overline{L^D} \left(1 + \left(\overline{D} - D^c\right)\gamma(1 - \overline{L^D})\right) l_1^Y \right]^2 u''' - \\
\left(\begin{array}{c} l_{11}^D \left(l_1^Y\right)^2 + 2 l_{11}^{DY} l_1^Y \rho + l_{11}^Y \left(l_1^D + \rho^2\right) - \\
k^D \left[\begin{array}{c} \left[2\gamma \overline{L^D}(1 - \overline{L^D}) + \left(\overline{D} - D^c\right) \gamma^2 \overline{L^D}(1 - \overline{L^D}) \left(1 - 2\overline{L^D}\right) \right] \left(l_1^Y\right)^2 \\
+ \left(\overline{L^D} + \left(\overline{D} - D^c\right)\gamma \overline{L^D}(1 - \overline{L^D})\right) l_{11}^Y \end{array} \right] \end{array} \right) u''
\end{array} \right].
$$

Using (2.100) again and isolating the three unknown coefficients l_{11}^Y, l_{11}^{DY} and l_{11}^D, we obtain

$$p_{YY} = \left[\begin{array}{c} l_{11}^Y \left[\begin{array}{c} \left[1 + k^D\overline{L^D}\left(1 + \left(\overline{D} - D^c\right)\gamma(1-\overline{L^D})\right) - \left(l_1^D + \rho^2\right)\right] u'' - \\ \beta_g k^D \gamma \overline{L^D}(1-\overline{L^D})\left[2 + \left(\overline{D} - D^c\right)\gamma\left(1 - 2\overline{L^D}\right)\right] u' \end{array}\right] \\ -2\rho l_1^Y l_{11}^{DY} u'' - \left(l_1^Y\right)^2 l_{11}^D u'' - \\ \beta_g k^D \gamma^2 \overline{L^D}(1-\overline{L^D}) \left[\begin{array}{c} 3\left(1 - 2\overline{L^D}\right) + \\ \left(\overline{D} - D^c\right)\gamma\left(1 - 6\overline{L^D} + 6\left(\overline{L^D}\right)^2\right) \end{array}\right] \left(l_1^Y\right)^2 u' + \\ k^D\gamma\overline{L^D}(1-\overline{L^D})\left[2 + \left(\overline{D} - D^c\right)\gamma\left(1 - 2\overline{L^D}\right)\right] \left[\begin{array}{c} 3\left(l_1^Y\right)^2 - 2\beta_g l_1^D \left(l_1^Y\right)^2 - \\ 2\beta_g \left(l_1^Y\right)^2 \rho - 2\beta_g l_1^Y \tau^y \rho \end{array}\right] u'' \\ + \left\{\begin{array}{c} \left(l_1^Y + \tau^y\right)^2 - \\ \left[l_1^D l_1^Y + l_1^Y \rho + \tau^y \rho - k^D \overline{L^D}\left(1 + \left(\overline{D} - D^c\right)\gamma(1-\overline{L^D})\right) l_1^Y\right]^2 \end{array}\right\} u''' \end{array}\right]$$

$$\Rightarrow \quad p_{YY}\widetilde{Y}_t^2 = \left(\digamma l_{11}^Y - 2\rho l_1^Y l_{11}^{DY} u'' - \left(l_1^Y\right)^2 l_{11}^D u'' + \yen\right)\widetilde{Y}_t^2. \tag{2.123}$$

This equation contains three unknown variables (l_{11}^Y, l_{11}^{DY} and l_{11}^D).

Derivation of $p_{Y\sigma_\varepsilon}$ Differentiate p_{σ_ε} with respect to Y_t around the point $\left(\overline{D}, \overline{Y}, 0\right)$ and use (2.100)

$$p_{Y\sigma_\varepsilon} = \left[\begin{array}{c} \frac{\partial(u''(G_t))}{\partial Y_t} * \left(\frac{\partial D_{t+1}}{\partial \sigma_\varepsilon}\right) + u'' * \frac{\partial}{\partial Y_t}\left(\frac{\partial D_{t+1}}{\partial \sigma_\varepsilon}\right) - \\ \beta_g k^D * \left[2\frac{\partial}{\partial Y_t}\left(\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial \sigma_\varepsilon}\right) + \frac{\partial}{\partial Y_t}\left((D_{t+1} - D^c)\frac{\partial^2 L_{t+1}^D}{\partial^2 D_{t+1}}\frac{\partial D_{t+1}}{\partial \sigma_\varepsilon}\right)\right] * u' \\ -\beta_g k^D \left[2\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial \sigma_\varepsilon} + \left(\overline{D} - D^c\right)\frac{\partial^2 L_{t+1}^D}{\partial^2 D_{t+1}}\frac{\partial D_{t+1}}{\partial \sigma_\varepsilon}\right] * \frac{\partial(u'(G_{t+1}))}{\partial Y_t} - \\ \beta_g k^D * \frac{\partial}{\partial Y_t}\left(L_{t+1}^D + (D_{t+1} - D^c)\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\right) * u'' * \\ \left(\frac{\partial D_{t+2}}{\partial \sigma_\varepsilon} + \tau^y\varepsilon_{t+1} - k^D\overline{L^D}\left(1 + \left(\overline{D} - D^c\right)\gamma(1-\overline{L^D})\right)\frac{\partial D_{t+1}}{\partial \sigma_\varepsilon}\right) - \\ \frac{\partial}{\partial Y_t}\left(u''\left(G_{t+1}\right)\right) * \left(\frac{\partial D_{t+2}}{\partial \sigma_\varepsilon} + \tau^y\varepsilon_{t+1} - k^D\overline{L^D}\left(1 + \left(\overline{D} - D^c\right)\gamma(1-\overline{L^D})\right)\frac{\partial D_{t+1}}{\partial \sigma_\varepsilon}\right) \\ -u'' * \frac{\partial}{\partial Y_t}\left(\frac{\partial D_{t+2}}{\partial \sigma_\varepsilon} + \tau^y\varepsilon_{t+1} - k^D L_{t+1}^D\frac{\partial D_{t+1}}{\partial \sigma_\varepsilon} - k^D\left(D_{t+1} - D^c\right)\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial \sigma_\varepsilon}\right) \end{array}\right].$$

Using that $\frac{\partial D_{t+1}}{\partial \sigma_\varepsilon} = l_1^\sigma = 0$ and using the first- and second-order derivatives of D_{t+1} and D_{t+2} with respect to σ_ε, this expression becomes

$$p_{Y\sigma_\varepsilon} = \left[\begin{array}{c} u''\frac{\partial^2 D_{t+1}}{\partial Y_t\partial\sigma_\varepsilon} - \\ \beta_g k^D * \left[\begin{array}{c} 2\left(\frac{\partial^2 L_{t+1}^D}{\partial^2 D_{t+1}}\frac{\partial D_{t+1}}{\partial Y_t}\frac{\partial D_{t+1}}{\partial \sigma_\varepsilon} + \frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial^2 D_{t+1}}{\partial Y_t\partial\sigma_\varepsilon}\right) + \frac{\partial D_{t+1}}{\partial Y_t}\frac{\partial^2 L_{t+1}^D}{\partial^2 D_{t+1}}\frac{\partial D_{t+1}}{\partial \sigma_\varepsilon} \\ + \left(\overline{D} - D^c\right)\left(\frac{\partial^3 L_{t+1}^D}{\partial^3 D_{t+1}}\frac{\partial D_{t+1}}{\partial Y_t}\frac{\partial D_{t+1}}{\partial \sigma_\varepsilon} + \frac{\partial^2 L_{t+1}^D}{\partial^2 D_{t+1}}\frac{\partial^2 D_{t+1}}{\partial Y_t\partial\sigma_\varepsilon}\right) \end{array}\right] * u' - \\ \beta_g k^D * \left[\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial Y_t} + \frac{\partial D_{t+1}}{\partial Y_t}\frac{\partial L_{t+1}^D}{\partial D_{t+1}} + \left(\overline{D} - D^c\right)\frac{\partial^2 L_{t+1}^D}{\partial^2 D_{t+1}}\frac{\partial D_{t+1}}{\partial Y_t}\right] * u'' * \\ \left(l_1^D l_1^\sigma + l_1^Y\varepsilon_{t+1} + l_1^\sigma + \tau^y\varepsilon_{t+1}\right) \\ -u''' * \left[\frac{\partial D_{t+2}}{\partial Y_t} - k^D\overline{L^D}\frac{\partial D_{t+1}}{\partial Y_t} + \tau^y\rho - k^D\left(\overline{D} - D^c\right)\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial Y_t}\right] * \\ \left(l_1^D l_1^\sigma + l_1^Y\varepsilon_{t+1} + l_1^\sigma + \tau^y\varepsilon_{t+1}\right) - \\ u'' * \left(\begin{array}{c} \frac{\partial^2 D_{t+2}}{\partial Y_t\partial\sigma_\varepsilon} - k^D\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial Y_t}\frac{\partial D_{t+1}}{\partial \sigma_\varepsilon} - k^D\overline{L^D}\frac{\partial^2 D_{t+1}}{\partial Y_t\partial\sigma_\varepsilon} - k^D\frac{\partial D_{t+1}}{\partial Y_t}\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial \sigma_\varepsilon} \\ -k^D\left(\overline{D} - D^c\right)\frac{\partial^2 L_{t+1}^D}{\partial^2 D_{t+1}}\frac{\partial D_{t+1}}{\partial Y_t}\frac{\partial D_{t+1}}{\partial \sigma_\varepsilon} - k^D\left(\overline{D} - D^c\right)\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial^2 D_{t+1}}{\partial Y_t\partial\sigma_\varepsilon} \end{array}\right) \end{array}\right].$$

We can simplify the last equation once more to

$$p_{Y\sigma_\varepsilon} = \left[\begin{array}{c} l_{11}^{Y\sigma}u'' - \beta_g k^D * \left[2\gamma\overline{L^D}(1-\overline{L^D}) + \left(\overline{D}-D^c\right)\gamma^2\overline{L^D}(1-\overline{L^D})\left(1-2\overline{L^D}\right)\right] * l_{11}^{Y\sigma}u' \\ -\beta_g k^D * \left[\begin{array}{c} 2\gamma\overline{L^D}(1-\overline{L^D})+ \\ \left(\overline{D}-D^c\right)\gamma^2\overline{L^D}(1-\overline{L^D})\left(1-2\overline{L^D}\right) \end{array}\right] * l_1^Y\left(l_1^Y+\tau^y\right)\varepsilon_{t+1}u''- \\ u''' * \left[l_1^D l_1^Y + l_1^Y\rho + \tau^y\rho - k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)l_1^Y\right] * \left(l_1^Y+\tau^y\right)\varepsilon_{t+1} \\ -u'' * \left(\begin{array}{c} \left(l_{11}^D l_1^Y + l_{11}^{DY}\rho\right)l_1^\sigma + l_1^D l_{11}^{Y\sigma} + \left(l_{11}^{DY}l_1^Y + l_{11}^Y\rho\right)\varepsilon_{t+1} + l_{11}^{D\sigma}l_1^Y + l_{11}^{Y\sigma}\rho \\ -k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)l_{11}^{Y\sigma} \end{array}\right) \end{array}\right].$$

Then, grouping the terms of the equation above and multiplying it by $\widetilde{Y}_t\sigma_\varepsilon$, we have

$$\begin{aligned} & p_{Y\sigma_\varepsilon}\widetilde{Y}_t\sigma_\varepsilon \\ = & \left\{\begin{array}{c} l_{11}^{Y\sigma}\left[\begin{array}{c} \left[1+k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)-l_1^D-\rho\right]u''- \\ \beta_g k^D\gamma\overline{L^D}(1-\overline{L^D})\left[2+\left(\overline{D}-D^c\right)\gamma\left(1-2\overline{L^D}\right)\right]u' \end{array}\right] \\ -l_{11}^{D\sigma}l_1^Y u'' - l_{11}^{DY}l_1^Y\varepsilon_{t+1}u'' - l_{11}^Y\rho\varepsilon_{t+1}u''- \\ \left(l_1^Y+\tau^y\right)\left[\begin{array}{c} l_1^Y\beta_g k^D\gamma\overline{L^D}(1-\overline{L^D})\left[2+\left(\overline{D}-D^c\right)\gamma\left(1-2\overline{L^D}\right)\right]u'' \\ +\left[\begin{array}{c} l_1^D l_1^Y + l_1^Y\rho + \tau^y\rho- \\ k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)l_1^Y \end{array}\right]u''' \end{array}\right]\varepsilon_{t+1} \end{array}\right\}\widetilde{Y}_t\sigma_\varepsilon \\ = & \left(l_{11}^{Y\sigma} - l_{11}^{D\sigma}l_1^Y u'' + \varepsilon_{t+1}\right)\widetilde{Y}_t\sigma_\varepsilon. \end{aligned}$$

Finally, taking expectations

$$\mathrm{E}_t\left[p_{Y\sigma_\varepsilon}\widetilde{Y}_t\sigma_\varepsilon\right] = E_t\left[\left(l_{11}^{Y\sigma} - l_{11}^{D\sigma}l_1^Y u'' + \varepsilon_{t+1}\right)\widetilde{Y}_t\sigma_\varepsilon\right] = \left(l_{11}^{Y\sigma} - l_{11}^{D\sigma}l_1^Y u''\right)\widetilde{Y}_t\sigma_\varepsilon. \quad (2.124)$$

Derivation of $p_{\sigma_\varepsilon\sigma_\varepsilon}$ Differentiate p_{σ_ε} with respect to σ_ε around the point $\left(\overline{D},\overline{Y},0\right)$ and also use (2.100):

$$p_{\sigma_\varepsilon\sigma_\varepsilon} = \left[\begin{array}{c} \frac{\partial}{\partial\sigma_\varepsilon}\left(u''\left(G_t\right)\right) * \left(\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon}\right) + u'' * \frac{\partial}{\partial\sigma_\varepsilon}\left(\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon}\right) - \\ \beta_g k^D * \left[2\frac{\partial}{\partial\sigma_\varepsilon}\left(\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon}\right) + \frac{\partial}{\partial\sigma_\varepsilon}\left(\left(D_{t+1}-D^c\right)\frac{\partial^2 L_{t+1}^D}{\partial^2 D_{t+1}}\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon}\right)\right] * u' \\ -\beta_g k^D\left[2\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon} + \left(\overline{D}-D^c\right)\frac{\partial^2 L_{t+1}^D}{\partial^2 D_{t+1}}\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon}\right] * \frac{\partial(u'(G_{t+1}))}{\partial Y_t} \\ -\beta_g k^D * \frac{\partial}{\partial\sigma_\varepsilon}\left(L_{t+1}^D + \left(D_{t+1}-D^c\right)\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\right) * u''* \\ \left(\frac{\partial D_{t+2}}{\partial\sigma_\varepsilon} + \tau^y\varepsilon_{t+1} - k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon}\right) - \\ \frac{\partial}{\partial\sigma_\varepsilon}\left(u''\left(G_{t+1}\right)\right) * \left(\frac{\partial D_{t+2}}{\partial\sigma_\varepsilon} + \tau^y\varepsilon_{t+1} - k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon}\right) \\ -u'' * \frac{\partial}{\partial\sigma_\varepsilon}\left(\frac{\partial D_{t+2}}{\partial\sigma_\varepsilon} + \tau^y\varepsilon_{t+1} - k^D L_{t+1}^D\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon} - k^D\left(D_{t+1}-D^c\right)\frac{\partial L_{t+1}^D}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial\sigma_\varepsilon}\right) \end{array}\right].$$

Using that $\frac{\partial D_{t+1}}{\partial \sigma_\varepsilon} = l_1^\sigma = 0$ and the first- and second-order derivatives of D_{t+1} and D_{t+2} with respect to σ_ε, we write the last expression as

$$p_{\sigma_\varepsilon \sigma_\varepsilon} = \left[\begin{array}{c} u'' \frac{\partial^2 D_{t+1}}{\partial^2 \sigma_\varepsilon} - \\ \left[\begin{array}{c} 2\left(\frac{\partial^2 L_{t+1}^D}{\partial^2 D_{t+1}} \left(\frac{\partial D_{t+1}}{\partial \sigma_\varepsilon} \right)^2 + \frac{\partial L_{t+1}^D}{\partial D_{t+1}} \frac{\partial^2 D_{t+1}}{\partial^2 \sigma_\varepsilon} \right) + \frac{\partial^2 L_{t+1}^D}{\partial^2 D_{t+1}} \left(\frac{\partial D_{t+1}}{\partial \sigma_\varepsilon} \right)^2 + \\ \left(\overline{D} - D^c \right) \left(\frac{\partial^3 L_{t+1}^D}{\partial^3 D_{t+1}} \left(\frac{\partial D_{t+1}}{\partial \sigma_\varepsilon} \right)^2 + \frac{\partial^2 L_{t+1}^D}{\partial^2 D_{t+1}} \frac{\partial^2 D_{t+1}}{\partial^2 \sigma_\varepsilon} \right) \end{array} \right] \beta_g k^D u' \\ - \left[\frac{\partial L_{t+1}^D}{\partial D_{t+1}} \frac{\partial D_{t+1}}{\partial \sigma_\varepsilon} + \frac{\partial D_{t+1}}{\partial \sigma_\varepsilon} \frac{\partial L_{t+1}^D}{\partial D_{t+1}} + \left(\overline{D} - D^c \right) \frac{\partial^2 L_{t+1}^D}{\partial D_{t+1}^2} \frac{\partial D_{t+1}}{\partial \sigma_\varepsilon} \right] * \\ \left(l_1^D l_1^\sigma + l_1^Y \varepsilon_{t+1} + l_1^\sigma + \tau^y \varepsilon_{t+1} \right) \beta_g k^D u'' \\ -u''' * \left(l_1^D l_1^\sigma + l_1^Y \varepsilon_{t+1} + l_1^\sigma + \tau^y \varepsilon_{t+1} \right)^2 - \\ \left(\begin{array}{c} \frac{\partial^2 D_{t+2}}{\partial^2 \sigma_\varepsilon} - k^D \frac{\partial L_{t+1}^D}{\partial D_{t+1}} \left(\frac{\partial D_{t+1}}{\partial \sigma_\varepsilon} \right)^2 - k^D \overline{L^D} \frac{\partial^2 D_{t+1}}{\partial^2 \sigma_\varepsilon} - k^D \frac{\partial L_{t+1}^D}{\partial D_{t+1}} \left(\frac{\partial D_{t+1}}{\partial \sigma_\varepsilon} \right)^2 \\ -k^D \left(\overline{D} - D^c \right) \frac{\partial^2 L_{t+1}^D}{\partial^2 D_{t+1}} \left(\frac{\partial D_{t+1}}{\partial \sigma_\varepsilon} \right)^2 - k^D \left(\overline{D} - D^c \right) \frac{\partial L_{t+1}^D}{\partial D_{t+1}} \frac{\partial^2 D_{t+1}}{\partial^2 \sigma_\varepsilon} \end{array} \right) u'' \end{array} \right].$$

Simplifying once more, and using the first- and second-order derivatives of D_{t+1} and D_{t+2}, it follows that

$$p_{\sigma_\varepsilon \sigma_\varepsilon} = \left[\begin{array}{c} +u'' l_{11}^\sigma - u''' \left(\left(l_1^Y + \tau^y \right) \varepsilon_{t+1} \right)^2 - \\ \left[2\gamma \overline{L^D}(1 - \overline{L^D}) + \left(\overline{D} - D^c \right) \gamma^2 \overline{L^D}(1 - \overline{L^D}) \left(1 - 2\overline{L^D} \right) \right] l_{11}^\sigma \beta_g k^D u' - \\ \left(\begin{array}{c} \left(l_{11}^D l_1^\sigma + l_{11}^{DY} \varepsilon_{t+1} + 2 l_{11}^{D\sigma} \right) l_1^\sigma + \left(l_{11}^{DY} l_1^\sigma + 2 l_{11}^{Y\sigma} \right) \varepsilon_{t+1} + l_{11}^Y \varepsilon_{t+1}^2 + \left(1 + l_1^D \right) l_{11}^\sigma \\ -k^D \overline{L^D} \left(1 + \left(\overline{D} - D^c \right) \gamma (1 - \overline{L^D}) \right) l_{11}^\sigma \end{array} \right) u'' \end{array} \right].$$

Grouping terms and multiplying by σ_ε^2, we obtain

$$\begin{aligned} p_{\sigma_\varepsilon \sigma_\varepsilon} \sigma_\varepsilon^2 &= \left\{ \begin{array}{c} l_{11}^\sigma \left[\begin{array}{c} \left[1 + k^D \overline{L^D} \left(1 + \left(\overline{D} - D^c \right) \gamma (1 - \overline{L^D}) \right) - \left(1 + l_1^D \right) \right] u'' \\ -\beta_g k^D \gamma \overline{L^D} (1 - \overline{L^D}) \left[2 + \left(\overline{D} - D^c \right) \gamma \left(1 - 2\overline{L^D} \right) \right] u' \end{array} \right] \\ -2 l_{11}^{Y\sigma} \varepsilon_{t+1} u'' - \left[\left(l_1^Y + \tau^y \right)^2 u''' + l_{11}^Y u'' \right] \varepsilon_{t+1}^2 \end{array} \right\} \sigma_\varepsilon^2 \\ &= \left[e l_{11}^\sigma - 2 l_{11}^{Y\sigma} \varepsilon_{t+1} u'' - \left(\left(l_1^Y + \tau^y \right)^2 u''' + l_{11}^Y u'' \right) \varepsilon_{t+1}^2 \right] \sigma_\varepsilon^2. \end{aligned}$$

Finally, taking expectations, we obtain

$$\begin{aligned} \mathrm{E}_t \left[p_{\sigma_\varepsilon \sigma_\varepsilon} \sigma_\varepsilon^2 \right] &= \mathrm{E}_t \left\{ \left[e l_{11}^\sigma - 2 l_{11}^{Y\sigma} \varepsilon_{t+1} u'' - \left(\left(l_1^Y + \tau^y \right)^2 u''' + l_{11}^Y u'' \right) \varepsilon_{t+1}^2 \right] \sigma_\varepsilon^2 \right\} \\ &= \left[e l_{11}^\sigma - \left(\left(l_1^Y + \tau^y \right)^2 u''' + l_{11}^Y u'' \right) \sigma_\varepsilon^2 \right] \sigma_\varepsilon^2. \end{aligned} \quad (2.125)$$

Second-order Taylor expansion

Substituting (2.120), (2.121), (2.122), (2.123), (2.124) and (2.125), we can write the second-order Taylor expansion of 2.29 as

$$\mathrm{E}_t \left\{ p\left(D_t, Y_t, \sigma_\varepsilon \right) \right] \simeq$$

$$\frac{1}{2} \left\{ \left[\begin{array}{c} \left(\Lambda l_{11}^D + \Xi \right) \widetilde{D}_t^2 + 2 \left(\Pi l_{11}^{DY} + \Upsilon l_{11}^D + \Phi \right) \widetilde{D}_t \widetilde{Y}_t + \\ 2\Psi l_{11}^{D\sigma} \widetilde{D}_t \sigma_\varepsilon + \left(\digamma l_{11}^Y - 2\rho l_1^Y l_{11}^{DY} u'' - \left(l_1^Y \right)^2 l_{11}^D u'' + \yen \right) \widetilde{Y}_t^2 + \\ 2 \left(l_{11}^{Y\sigma} - l_{11}^{D\sigma} l_1^Y u'' \right) \widetilde{Y}_t \sigma_\varepsilon + \left\{ e l_{11}^\sigma - \left[\left(l_1^Y + \tau^y \right)^2 u''' + l_{11}^Y u'' \right] \sigma_\varepsilon^2 \right\} \sigma_\varepsilon^2 \end{array} \right] \right\} = (2.126)$$

Because this expression has to hold for all combinations $\left\{\widetilde{D}_t^2, \widetilde{D}_t\widetilde{Y}_t, \widetilde{D}_t\sigma_\varepsilon, \widetilde{Y}_t^2, \widetilde{Y}_t\sigma_\varepsilon, \sigma_\varepsilon^2\right\}$, we obtain six linear equations in six unknowns. Setting the coefficient of $\widetilde{D}_t^2$ to zero, we obtain

$$l_{11}^D = -\frac{\Xi}{\Lambda},$$

since $\Lambda \neq 0$. Then, l_{11}^D becomes

$$l_{11}^D = \frac{\left[\begin{array}{c} \left[l_1^D - k^D\overline{L^D}\left(1 + \left(\overline{D} - D^c\right)\gamma(1 - \overline{L^D})\right)\right]^2 \left(\left(l_1^D\right)^2 - 1\right) u''' + \\ \beta_g k^D \gamma^2 \overline{L^D}(1 - \overline{L^D}) \left[3\left(1 - 2\overline{L^D}\right) + \left(\overline{D} - D^c\right)\gamma\left(1 - 6\overline{L^D} + 6\left(\overline{L^D}\right)^2\right)\right] \left(l_1^D\right)^2 u' \\ -k^D\gamma\overline{L^D}(1 - \overline{L^D})\left[2 + \left(\overline{D} - D^c\right)\gamma\left(1 - 2\overline{L^D}\right)\right]\left[-1 + 3\left(l_1^D\right)^2 - 2\beta_g\left(l_1^D\right)^3\right] u'' \end{array}\right]}{\left[\begin{array}{c} \left(1 + k^D\overline{L^D}\left[1 + \left(\overline{D} - D^c\right)\gamma(1 - \overline{L^D})\right] - \left(l_1^D\right)^2 - l_1^D\right) u'' - \\ \beta_g k^D\gamma\overline{L^D}(1 - \overline{L^D})\left[2 + \left(\overline{D} - D^c\right)\gamma\left(1 - 2\overline{L^D}\right)\right] u' \end{array}\right]}. \tag{2.127}$$

With the value of l_{11}^D, we solve for l_{11}^{DY} by setting the coefficient of $\widetilde{D}_t\widetilde{Y}_t$ to zero:

$$l_{11}^{DY} = -\frac{\Upsilon l_{11}^D + \Phi}{\Pi},$$

or

$$l_{11}^{DY} = \frac{\left[\begin{array}{c} l_{11}^D l_1^D l_1^Y u'' + \\ \beta_g k^D\gamma^2\overline{L^D}(1 - \overline{L^D})\left[3\left(1 - 2\overline{L^D}\right) + \left(\overline{D} - D^c\right)\gamma\left(1 - 6\overline{L^D} + 6\left(\overline{L^D}\right)^2\right)\right] l_1^D l_1^Y u' \\ -\left[l_1^D - k^D\overline{L^D}\left(1 + \left(\overline{D} - D^c\right)\gamma(1 - \overline{L^D})\right)\right] * \\ \left[\begin{array}{c} l_1^Y + \tau^y - \left(l_1^D\right)^2 l_1^Y - l_1^D l_1^Y\rho - l_1^D\tau^y\rho + \\ k^D\overline{L^D}\left(1 + \left(\overline{D} - D^c\right)\gamma(1 - \overline{L^D})\right) l_1^D l_1^Y \end{array}\right] u''' + \\ k^D\gamma\overline{L^D}(1 - \overline{L^D})\left[2 + \left(\overline{D} - D^c\right)\gamma\left(1 - 2\overline{L^D}\right)\right]\left[\begin{array}{c} 2\beta_g\left(l_1^D\right)^2 l_1^Y + \beta_g l_1^D l_1^Y\rho \\ +\beta_g l_1^D\tau^y\rho - 3l_1^D l_1^Y \end{array}\right] u'' \end{array}\right]}{\left\{\begin{array}{c} \left[1 + k^D\overline{L^D}\left[1 + \left(\overline{D} - D^c\right)\gamma(1 - \overline{L^D})\right] - (1 + \rho)\, l_1^D\right] u'' \\ -\beta_g k^D\gamma\overline{L^D}(1 - \overline{L^D})\left[2 + \left(\overline{D} - D^c\right)\gamma\left(1 - 2\overline{L^D}\right)\right] u' \end{array}\right\}}, \tag{2.128}$$

Similarly, we obtain the solution of l_{11}^Y as

$$l_{11}^Y = \frac{2\rho l_1^Y l_{11}^{DY} u'' + \left(l_1^Y\right)^2 l_{11}^D u'' - \text{¥}}{\mathit{\Gamma}},$$

or

$$l_{11}^{Y}=\frac{\left[\begin{array}{c} 2u''\rho l_1^Y l_{11}^{DY}+u''\left(l_1^Y\right)^2 l_{11}^D+ \\ u'\beta_g k^D\gamma^2\overline{L^D}(1-\overline{L^D})\left[\begin{array}{c} 3\left(1-2\overline{L^D}\right)+ \\ \left(\overline{D}-D^c\right)\gamma\left(1-6\overline{L^D}+6\left(\overline{L^D}\right)^2\right) \end{array}\right]\left(l_1^Y\right)^2- \\ u''k^D\gamma\overline{L^D}(1-\overline{L^D})\left[2+\left(\overline{D}-D^c\right)\gamma\left(1-2\overline{L^D}\right)\right]\left[\begin{array}{c} 3\left(l_1^Y\right)^2-2\beta_g l_1^D\left(l_1^Y\right)^2 \\ -2\beta_g\left(l_1^Y\right)^2\rho-2\beta_g l_1^Y\tau^y\rho \end{array}\right] \\ -u'''\left\{\left(l_1^Y+\tau^y\right)^2-\left[l_1^D l_1^Y+l_1^Y\rho+\tau^y\rho-k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)l_1^Y\right]^2\right\} \end{array}\right]}{\left[\begin{array}{c} u''\left[1+k^D\overline{L^D}\left(1+\left(\overline{D}-D^c\right)\gamma(1-\overline{L^D})\right)-\left(l_1^D+\rho^2\right)\right] \\ -u'\beta_g k^D\gamma\overline{L^D}(1-\overline{L^D})\left[2+\left(\overline{D}-D^c\right)\gamma\left(1-2\overline{L^D}\right)\right] \end{array}\right]}. \tag{2.129}$$

In addition, (2.126) implies

$$l_{11}^{D\sigma} = 0 \Rightarrow \tag{2.130}$$
$$l_{11}^{Y\sigma} = 0. \tag{2.131}$$

Finally, given (2.112) and (2.129), we obtain the last unknown coefficient l_{11}^{σ}:

$$l_{11}^{\sigma}=\frac{\left[\left(l_1^Y+\tau^y\right)^2 u'''+l_{11}^Y u''\right]\sigma_\varepsilon^2}{e}. \tag{2.132}$$

Second-order approximation of D_{t+1}

The second-order approximation of the true non-linear solution of D_{t+1} around the point $\left(\overline{D},\overline{Y},0\right)$ can be written as

$$D_{t+1}\approx\left[\begin{array}{c} l\left(\overline{D},\overline{Y},0\right)+l_1^D\widetilde{D}_t+l_1^Y\widetilde{Y}_t+l_1^\sigma\sigma_\varepsilon+ \\ \frac{1}{2}\left[l_{11}^D\widetilde{D}_t^2+2l_{11}^{DY}\widetilde{D}_t\widetilde{Y}_t+2l_{11}^{D\sigma}\widetilde{D}_t\sigma_\varepsilon+l_{11}^Y\widetilde{Y}_t^2+2l_{11}^{Y\sigma}\widetilde{Y}_t\sigma_\varepsilon+l_{11}^\sigma\sigma_\varepsilon^2\right) \end{array}\right].$$

Using (2.111), (2.112), (2.109), (2.127), (2.128), (2.129), (2.130), (2.131) and (2.132), this becomes

$$D_{t+1}\approx\left\{\begin{array}{c} l\left(\overline{D},\overline{Y},0\right)+l_1^D\widetilde{D}_t+l_1^Y\widetilde{Y}_t+ \\ \frac{1}{2}\left[l_{11}^D\widetilde{D}_t^2+2l_{11}^{DY}\widetilde{D}_t\widetilde{Y}_t+l_{11}^Y\widetilde{Y}_t^2+l_{11}^\sigma\sigma_\varepsilon^2\right] \end{array}\right\}. \tag{2.133}$$

Using (2.8), we can rewrite (2.133) further as

$$B_{t+1}-(1+r)B_t\approx\left\{\begin{array}{c} l\left(\overline{D},\overline{Y},0\right)+l_1^D*\left[B_t-(1+r)B_{t-1}-\overline{D}\right]+l_1^Y*\left(Y_t-\overline{Y}\right) \\ +\frac{1}{2}\left[\begin{array}{c} l_{11}^D*\left[B_t-(1+r)B_{t-1}-\overline{D}\right]^2+ \\ 2l_{11}^{DY}*\left[B_t-(1+r)B_{t-1}-\overline{D}\right]*\left(Y_t-\overline{Y}\right) \\ +l_{11}^Y*\left(Y_t-\overline{Y}\right)^2+l_{11}^\sigma\sigma_\varepsilon^2 \end{array}\right] \end{array}\right\}. \tag{2.134}$$

2.G.3 Unconditional expectation of the primary deficit

The unconditional expectation of (2.133) is:

$$\mathrm{E}\left[D_{t+1}\right] \simeq \left\{ \begin{array}{c} \overline{D} + l_1^D \left(\mathrm{E}\left[D_t\right] - \overline{D}\right) + l_1^Y \left(\mathrm{E}\left[Y_t\right] - \overline{Y}\right) + \\ \frac{1}{2} \left[\begin{array}{c} l_{11}^D \mathrm{E}\left[\left(D_t - \overline{D}\right)^2\right] + 2 l_{11}^{DY} \mathrm{E}\left[\left(D_t - \overline{D}\right)\left(Y_t - \overline{Y}\right)\right] \\ + l_{11}^Y \mathrm{E}\left[\left(Y_t - \overline{Y}\right)^2\right] + l_{11}^{\sigma} \sigma_{\varepsilon}^2 \end{array} \right] \end{array} \right\}.$$

Define

$$\widehat{D} \equiv \mathrm{E}\left(D_t\right) = \mathrm{E}\left[D_{t+1}\right],$$

and work out the terms in the expression for $\mathrm{E}[D_{t+1}]$:

$$\mathrm{E}\left(Y_t\right) - \overline{Y} = 0,$$

$$\mathrm{E}\left[\left(Y_t - \overline{Y}\right)^2\right] = \mathrm{Var}\left(Y_t\right) = \frac{\sigma_{\varepsilon}^2}{1 - \rho^2},$$

$$\begin{aligned} \mathrm{E}\left[\left(D_t - \overline{D}\right)^2\right] &= \mathrm{E}\left\{\left[\left(D_t - \widehat{D}\right) + \left(\widehat{D} - \overline{D}\right)\right]^2\right\} \\ &= \mathrm{E}\left[\left(D_t - \widehat{D}\right)^2\right] + 2\mathrm{E}\left[\left(D_t - \widehat{D}\right)\left(\widehat{D} - \overline{D}\right)\right] + \mathrm{E}\left[\left(\widehat{D} - \overline{D}\right)^2\right] \\ &= \mathrm{E}\left[\left(D_t - \widehat{D}\right)^2\right] + \left(\widehat{D} - \overline{D}\right)^2 = \mathrm{Var}\left(D_t\right) + \left(\widehat{D} - \overline{D}\right)^2, \end{aligned}$$

$$\begin{aligned} \mathrm{E}\left[\left(D_t - \overline{D}\right)\left(Y_t - \overline{Y}\right)\right] &= \mathrm{E}\left\{\left[\left(D_t - \widehat{D}\right) + \left(\widehat{D} - \overline{D}\right)\right]\left(Y_t - \overline{Y}\right)\right\} \\ &= \mathrm{E}\left[\left(D_t - \widehat{D}\right)\left(Y_t - \overline{Y}\right)\right] + \mathrm{E}\left[\left(\widehat{D} - \overline{D}\right)\left(Y_t - \overline{Y}\right)\right] \\ &= \mathrm{E}\left[\left(D_t - \widehat{D}\right)\left(Y_t - \overline{Y}\right)\right] = \mathrm{Cov}\left(D_t, Y_t\right). \end{aligned}$$

Substitute these terms into the expression for $\mathrm{E}[D_{t+1}]$, to give:

$$\begin{aligned} \widehat{D} - \overline{D} &\simeq l_1^D \left(\widehat{D} - \overline{D}\right) + \frac{1}{2} \left[\begin{array}{c} l_{11}^D \left(\mathrm{Var}\left(D_t\right) + \left(\widehat{D} - \overline{D}\right)^2\right) + 2 l_{11}^{DY} \mathrm{Cov}\left(D_t, Y_t\right) \\ + l_{11}^Y \frac{\sigma_{\varepsilon}^2}{1-\rho^2} + l_{11}^{\sigma} \sigma_{\varepsilon}^2 \end{array} \right] \Rightarrow \\ \widehat{D} &\simeq \overline{D} + \frac{1}{2\left(1 - l_1^D\right)} \left[l_{11}^D \mathrm{Var}\left(D_t\right) + 2 l_{11}^{DY} \mathrm{Cov}\left(D_t, Y_t\right) + \left(\frac{l_{11}^Y}{1 - \rho^2} + l_{11}^{\sigma}\right) \sigma_{\varepsilon}^2 \right]. \end{aligned} \quad (2.135)$$

where we have made use of the fact that $\left(\widehat{D}-\overline{D}\right)^2$ is fourth-order, respectively. We have

$$\begin{aligned}
\mathrm{Cov}\left(D_t, Y_t\right) &= \mathrm{Cov}\left(D_{t+1}, Y_{t+1}\right) = \mathrm{E}\left[\left(D_{t+1}-\widehat{D}\right)\left(Y_{t+1}-\overline{Y}\right)\right] \\
&= \mathrm{E}\left[\left(l_1^D \widetilde{D}_t + l_1^Y \widetilde{Y}_t + \overline{D} - \widehat{D}\right)\left(\rho \widetilde{Y}_t + \sigma_\varepsilon \varepsilon_{t+1}\right)\right] \\
&= \rho l_1^D \mathrm{E}\left[\widetilde{D}_t \widetilde{Y}_t\right] + \rho l_1^Y \mathrm{E}\left[\widetilde{Y}_t^2\right] \\
&= \rho l_1^D \mathrm{E}\left[\widetilde{D}_t \widetilde{Y}_t\right] + \rho l_1^Y \frac{\sigma_\varepsilon^2}{1-\rho^2}.
\end{aligned}$$

Hence,

$$\mathrm{E}\left[\widetilde{D}_t \widetilde{Y}_t\right] = \frac{\rho l_1^Y}{1-\rho l_1^D} \frac{\sigma_\varepsilon^2}{1-\rho^2}.$$

Further, taking unconditional expectation of the square of the first-order approximation of D_t+1, or

$$\begin{aligned}
\mathrm{Var}\left(D\right) &= \mathrm{E}\left[\left(D_{t+1}-\widehat{D}\right)^2\right] \simeq \mathrm{E}\left[\left(l_1^D \widetilde{D}_t + l_1^Y \widetilde{Y}_t\right)^2\right] \\
&= \mathrm{E}\left[\left(l_1^D\right)^2\left(D_t-\overline{D}\right)^2 + 2 l_1^D l_1^Y\left(D_t-\overline{D}\right)\left(Y_t-\overline{Y}\right) + \left(l_1^Y\right)^2\left(Y_t-\overline{Y}\right)^2\right] \\
&= \left(l_1^D\right)^2 \mathrm{E}\left[\left(D_t-\overline{D}\right)^2\right] + 2 l_1^D l_1^Y \mathrm{E}\left[\left(D_t-\overline{D}\right)\left(Y_t-\overline{Y}\right)\right] + \left(l_1^Y\right)^2 \mathrm{E}\left[\left(Y_t-\overline{Y}\right)^2\right] \\
&= \left(l_1^D\right)^2 \mathrm{E}\left[\left(D_t-\overline{D}\right)^2\right] + 2 l_1^D l_1^Y \frac{\rho l_1^Y}{1-\rho l_1^D} \frac{\sigma_\varepsilon^2}{1-\rho^2} + \left(l_1^Y\right)^2 \frac{\sigma_\varepsilon^2}{1-\rho^2} \\
&= \left(l_1^D\right)^2 \mathrm{E}\left[\left(D_t-\overline{D}\right)^2\right] + \left(\frac{1+\rho l_1^D}{1-\rho l_1^D}\right)\left(l_1^Y\right)^2 \frac{\sigma_\varepsilon^2}{1-\rho^2},
\end{aligned}$$

where we have eliminated terms that are of higher order than two. Hence,

$$\mathrm{Var}\left(D\right) \simeq \left(\frac{1+\rho l_1^D}{1-\rho l_1^D}\right) \frac{\left(l_1^Y\right)^2}{\left(1-\left(l_1^D\right)^2\right)\left(1-\rho^2\right)} \sigma_\varepsilon^2. \tag{2.136}$$

Substitute into (2.135), to arrive at:

$$\widehat{D} \simeq \overline{D} + \frac{\sigma_\varepsilon^2}{2\left(1-l_1^D\right)}\left[\left(\frac{l_{11}^D\left(l_1^Y\right)^2}{1-\left(l_1^D\right)^2}\left(\frac{1+\rho l_1^D}{1-\rho l_1^D}\right) + 2\frac{\rho l_1^Y l_{11}^{DY}}{1-\rho l_1^D} + l_{11}^Y\right)\frac{1}{1-\rho^2} + l_{11}^\sigma\right]. \tag{2.137}$$

From $D_{t+1} = B_{t+1} - (1+r) B_t$, we have $\mathrm{E}[D_{t+1}]$ $=\mathrm{E}[B_{t+1}] - (1+r)\mathrm{E}[B_t]$. Hence,

Solution 2.3 *The unconditional expectation or "stochastic steady state" value of the debt in the primary deficit-based sanction case is given by*

$$\widehat{B} \simeq -\frac{1}{r}\widehat{D}. \tag{2.138}$$

2.G.4 Stochastic steady state of government expenditure with deficit-based sanction

Now, we approximate government expenditure G_t. For that, we use (2.21), where in period t the only unknown variable is D_{t+1}. Nevertheless, we have just obtained a second-order approximation of that variable (2.133). So, inserting it on (2.21) for $s = t$ and isolating G_t, allow us to approximate it as

$$G_t \simeq \left\{ \begin{array}{c} \overline{D} + l_1^D \widetilde{D}_t + l_1^Y \widetilde{Y}_t + \frac{1}{2}\left[l_{11}^D \widetilde{D}_t^2 + 2 l_{11}^{DY} \widetilde{D}_t \widetilde{Y}_t + l_{11}^Y \widetilde{Y}_t^2 + l_{11}^\sigma \sigma_\varepsilon^2 \right] \\ + \tau^y Y_t - k^D \left(D_t - D^c \right) L_t^D \end{array} \right\}. \tag{2.139}$$

First-order approximation of G_t

The first-order Taylor expansion of (2.27) around the point $\left(\overline{D}, \overline{Y}, 0\right)$ is

$$G_t \simeq j\left(\overline{D}, \overline{Y}, 0\right) + j_1^D \widetilde{D}_t + j_1^Y \widetilde{Y}_t + j_1^\sigma \sigma_\varepsilon,$$

where $j_1^D \equiv \frac{\partial G_t}{\partial D_t}$, $j_1^Y \equiv \frac{\partial G_t}{\partial Y_t}$ and $j_1^\sigma \equiv \frac{\partial G_t}{\partial \sigma_\varepsilon}$ evaluated at the point $\left(\overline{D}, \overline{Y}, 0\right)$. Hence, using (2.139) we can find these derivatives as:

- First order partial derivative of (2.139) with respect to D_t:

$$\frac{\partial G_t}{\partial D_t} = l_1^D + \frac{1}{2}\left[2 l_{11}^D \left(D_t - \overline{D} \right) + 2 l_{11}^{DY} \left(Y_t - \overline{Y} \right) \right] - k^D L_t^D - k^D \left(D_t - D^c \right) \frac{\partial L_t^D}{\partial D_t}.$$

  If we evaluate this derivative at the point $\left(\overline{D}, \overline{Y}, 0\right)$ and use (2.98), we arrive to

$$j_1^D = l_1^D - k^D \overline{L^D} \left(1 + \left(\overline{D} - D^c \right) \gamma \left(1 - \overline{L^D} \right) \right). \tag{2.140}$$

- First order partial derivative of (2.139) with respect to Y_t:

$$\frac{\partial G_t}{\partial Y_t} = l_1^Y + \frac{1}{2}\left[2 l_{11}^{DY} \left(D_t - \overline{D} \right) + 2 l_{11}^Y \left(Y_t - \overline{Y} \right) \right] + \tau^y \Rightarrow j_1^Y = l_1^Y + \tau^y, \tag{2.141}$$

  since we evaluate this derivative at the point $\left(\overline{D}, \overline{Y}, 0\right)$.

- First order partial derivative of (2.139) with respect to σ_ε:

$$\frac{\partial G_t}{\partial \sigma_\varepsilon} = \frac{1}{2}\left[2 l_{11}^\sigma \sigma_\varepsilon \right] \Rightarrow j_1^\sigma = 0, \tag{2.142}$$

  since in the point that we evaluate at $\left(D_t, Y_t, \sigma_\varepsilon\right) = \left(\overline{D}, \overline{Y}, 0\right)$.

Substituting the above derivatives as well as

$$\overline{G} \equiv f\left(\overline{D}, \overline{Y}, 0\right) = \tau^y \overline{Y} + \overline{D} - k^D \left(\overline{D} - D^c \right) \overline{L^D}, \tag{2.143}$$

into the first-order Taylor expansion of G_t yields:

$$G_t \simeq \overline{G} + \left[l_1^D - k^D \overline{L^D} \left(1 + \left(\overline{D} - D^c \right) \gamma \left(1 - \overline{L^D} \right) \right) \right] * \widetilde{D}_t + \left(l_1^Y + \tau^y \right) * \widetilde{Y}_t. \tag{2.144}$$

Second-order approximation of G_t

The second-order Taylor expansion of (2.27) around the point $\left(\overline{D},\overline{Y},0\right)$ is:

$$G_t \approx \left\{ \begin{array}{c} j\left(\overline{D},\overline{Y},0\right) + j_1^D \widetilde{D}_t + j_1^Y \widetilde{Y}_t + j_1^\sigma \sigma_\varepsilon + \\ \frac{1}{2}\left[j_{11}^D \widetilde{D}_t^2 + 2 j_{11}^{DY} \widetilde{D}_t \widetilde{Y}_t + 2 j_{11}^{D\sigma} \widetilde{D}_t \sigma_\varepsilon + j_{11}^Y \widetilde{Y}_t^2 + 2 j_{11}^{Y\sigma} \widetilde{Y}_t \sigma_\varepsilon + j_{11}^\sigma \sigma_\varepsilon^2 \right] \end{array} \right\},$$

where, for any generic variables X and Z, $j_{11}^X \equiv \frac{\partial^2 G_t}{\partial^2 X_t}$ and $j_{11}^{XZ} \equiv \frac{\partial^2 G_t}{\partial X_t \partial Z_t}$.

- Computation of j_{11}^D: Differentiating $\frac{\partial G_t}{\partial D_t}$ with respect to D_t, we obtain:

$$\frac{\partial^2 G_t}{\partial^2 D_t} = \frac{1}{2}\left[2 l_{11}^D\right] - k^D \frac{\partial L_t^D}{\partial D_t} - k^D \frac{\partial L_t^D}{\partial D_t} - k^D \left(D_t - D^c\right) \frac{\partial^2 L_t^D}{\partial^2 D_t}.$$

If we evaluate this derivative at the point $\left(\overline{D},\overline{Y},0\right)$ and use (2.105), we arrive at

$$j_{11}^D = l_{11}^D - k^D \gamma \overline{L^D}\left(1 - \overline{L^D}\right)\left(2 + \left(\overline{D} - D^c\right)\gamma\left(1 - 2\overline{L^D}\right)\right). \tag{2.145}$$

- Computation of j_{11}^{DY}: Differentiating $\frac{\partial G_t}{\partial Y_t}$ with respect to D_t, we obtain:

$$\frac{\partial^2 G_t}{\partial D_t \partial Y_t} = \frac{1}{2}\left[2 l_{11}^{DY}\right] \Longrightarrow j_{11}^{DY} = l_{11}^{DY}. \tag{2.146}$$

- Computation of $j_{11}^{D\sigma}$ and $j_{11}^{Y\sigma}$: Differentiating $\frac{\partial G_t}{\partial D_t}$ and $\frac{\partial G_t}{\partial Y_t}$ with respect to σ_ε, we obtain:

$$j_{11}^{D\sigma} = j_{11}^{Y\sigma} = \frac{\partial^2 G_t}{\partial \sigma_\varepsilon \partial D_t} = \frac{\partial^2 G_t}{\partial \sigma_\varepsilon \partial Y_t} = 0. \tag{2.147}$$

- Computation of j_{11}^Y: Differentiating $\frac{\partial G_t}{\partial Y_t}$ with respect to Y_t, we obtain:

$$\frac{\partial^2 G_t}{\partial Y_t^2} = \frac{1}{2}\left[2 l_{11}^Y\right] \Longrightarrow j_{11}^Y = l_{11}^Y. \tag{2.148}$$

- Computation of j_{11}^σ: Differentiating $\frac{\partial G_t}{\partial \sigma_\varepsilon}$ with respect to σ_ε, we obtain:

$$\frac{\partial^2 G_t}{\partial \sigma_\varepsilon^2} = \frac{1}{2}\left[2 l_{11}^\sigma\right] \Longrightarrow j_{11}^\sigma = l_{11}^\sigma. \tag{2.149}$$

Upon substitution, the second-order approximation for G_t becomes

$$G_t \simeq \left\{ \begin{array}{c} \overline{G} + \left[l_1^D - (1/\beta_g)\right] \widetilde{D}_t + \left(l_1^Y + \tau^y\right) \widetilde{Y}_t + \\ \frac{1}{2}\left\{ \begin{array}{c} \left[l_{11}^D - k^D \gamma \overline{L^D}\left(1 - \overline{L^D}\right)\left(2 + \left(\overline{D} - D^c\right)\gamma\left(1 - 2\overline{L^D}\right)\right)\right] \widetilde{D}_t^2 \\ +2 l_{11}^{DY} \widetilde{D}_t \widetilde{Y}_t + l_{11}^Y \widetilde{Y}_t^2 + l_{11}^\sigma \sigma_\varepsilon^2 \end{array} \right\} \end{array} \right\}, \tag{2.150}$$

where we have also used (2.100).

Unconditional expectation of government spending with primary-deficit-based sanction

Recall

$$\mathrm{E}\left[\widetilde{D}_t\widetilde{Y}_t\right] = \frac{\rho l_1^Y}{1-\rho l_1^D}\frac{\sigma_\varepsilon^2}{1-\rho^2}.$$

Further,

$$\begin{aligned}
\mathrm{E}\left[\widetilde{D}_t^2\right] &= \mathrm{E}\left[\left(D_t - \widehat{D} + \widehat{D} - \overline{D}\right)\left(D_t - \widehat{D} + \widehat{D} - \overline{D}\right)\right] \\
&= \mathrm{E}\left[\left(D_t - \widehat{D}\right)^2\right] + \left(\widehat{D} - \overline{D}\right)^2 \\
&\simeq \mathrm{E}\left[\left(D_t - \widehat{D}\right)^2\right] = \mathrm{Var}\left(D_t\right).
\end{aligned}$$

Hence, taking unconditional expectations of (2.150) and using (2.5), we obtain:

$$\widehat{G} \simeq \left\{ \begin{array}{c} \overline{G} + \left[l_1^D - (1/\beta_g)\right]\left(\widehat{D} - \overline{D}\right) + \frac{1}{2}\left(\frac{l_{11}^Y}{1-\rho^2} + l_{11}^\sigma\right)\sigma_\varepsilon^2 + \frac{\rho l_1^Y l_{11}^{DY}}{1-\rho l_1^D}\frac{\sigma_\varepsilon^2}{1-\rho^2} + \\ \frac{1}{2}\left\{\left[l_{11}^D - k^D\gamma\overline{L^D}\left(1-\overline{L^D}\right)\left(2 + \left(\overline{D} - D^c\right)\gamma\left(1 - 2\overline{L^D}\right)\right)\right]\mathrm{Var}\left(D_t\right)\right\} \end{array} \right\}.$$

Finally, substituting (2.136), and (2.137) into this expression, we obtain

Solution 2.4 *The unconditional expectation or "stochastic steady state" value of the government expenditures in the primary deficit-based sanction case is given by*

$$\begin{aligned}
\widehat{G} &\simeq \left\{ \begin{array}{c} \overline{G} + \frac{\left[l_1^D - (1/\beta_g)\right]\sigma_\varepsilon^2}{2\left(1-l_1^D\right)}\left[\left(\frac{l_{11}^D\left(l_1^Y\right)^2}{1-\left(l_1^D\right)^2}\left(\frac{1+\rho l_1^D}{1-\rho l_1^D}\right) + 2\frac{\rho l_1^Y l_{11}^{DY}}{1-\rho l_1^D} + l_{11}^Y\right)\frac{1}{1-\rho^2} + l_{11}^\sigma\right] + \frac{\rho l_1^Y l_{11}^{DY}}{1-\rho l_1^D}\frac{\sigma_\varepsilon^2}{1-\rho^2} \\ +\frac{1}{2}\left[\frac{\left[l_{11}^D - k^D\gamma\overline{L^D}\left(1-\overline{L^D}\right)\left(2+\left(\overline{D}-D^c\right)\gamma\left(1-2\overline{L^D}\right)\right)\right]\left(l_1^Y\right)^2\sigma_\varepsilon^2}{\left(1-\left(l_1^D\right)^2\right)\left(1-\rho^2\right)}\left(\frac{1+\rho l_1^D}{1-\rho l_1^D}\right) + \left(\frac{l_{11}^Y}{1-\rho^2} + l_{11}^\sigma\right)\sigma_\varepsilon^2\right] \end{array} \right\} \\
&\simeq \left\{ \begin{array}{c} \overline{D} + \tau^y\overline{Y} - k^D\left(\overline{D} - D^c\right)\overline{L^D} + \\ \frac{\sigma_\varepsilon^2}{2}\left\{ \begin{array}{c} \left[\frac{l_1^D - (1/\beta_g)}{1-l_1^D}\right]\left[\left(\frac{l_{11}^D\left(l_1^Y\right)^2}{1-\left(l_1^D\right)^2}\left(\frac{1+\rho l_1^D}{1-\rho l_1^D}\right) + 2\frac{\rho l_1^Y l_{11}^{DY}}{1-\rho l_1^D} + l_{11}^Y\right)\frac{1}{1-\rho^2} + l_{11}^\sigma\right] + \\ 2\frac{\rho l_1^Y l_{11}^{DY}}{1-\rho l_1^D}\frac{1}{1-\rho^2} + \frac{\left[l_{11}^D - k^D\gamma\overline{L^D}\left(1-\overline{L^D}\right)\left(2+\left(\overline{D}-D^c\right)\gamma\left(1-2\overline{L^D}\right)\right)\right]\left(l_1^Y\right)^2}{\left(1-\left(l_1^D\right)^2\right)\left(1-\rho^2\right)}\left(\frac{1+\rho l_1^D}{1-\rho l_1^D}\right) \\ +\frac{l_{11}^Y}{1-\rho^2} + l_{11}^\sigma \end{array} \right\} \end{array} \right\}
\end{aligned} \quad (2.151)$$

2.H Welfare analysis

Now we compute expected social welfare under both types of constraints. Taking a second-order Taylor expansion of $u\left(G_t\right)$, we obtain:

$$u\left(G_t\right) \simeq u\left(\overline{G}\right) + u'\left(\overline{G}\right) * \left(G_t - \overline{G}\right) + \frac{1}{2}u''\left(\overline{G}\right) * \left(G_t - \overline{G}\right)^2.$$

Hence,

$$\mathrm{E}\left[u\left(G_t\right)\right] \simeq u\left(\overline{G}\right) + u'\left(\overline{G}\right) * \left(\widehat{G} - \overline{G}\right) + \frac{1}{2}u''\left(\overline{G}\right) * \mathrm{Var}\left(G_t\right),$$

where $\widehat{G}$ has been computed above and we can compute Var(G) in a way analogous to our computation of Var(B) and Var(D). Hence, expected social welfare is equal to (2.30).

2.H.1 Welfare in the debt-based sanction case

Substituting (2.88) and (2.96) in (2.30), we obtain

$$U_w^B \approx \frac{\left[\begin{array}{c} u\left(\tau^y\overline{Y} - r\overline{B} - k^B\left(\overline{B} - B^c\right)\overline{L^B}\right) + \\ u'\left(\tau^y\overline{Y} - r\overline{B} - k^B\left(\overline{B} - B^c\right)\overline{L^B}\right) * \\ \frac{\sigma_\varepsilon^2}{2}\left\{\begin{array}{c} \frac{h_1^B-(1/\beta_g)}{1-h_1^B}\left[\frac{h_{11}^B\left(h_1^Y\right)^2}{\left(1-\left(h_1^B\right)^2\right)(1-\rho^2)}\left(\frac{1+\rho h_1^B}{1-\rho h_1^B}\right) + 2\frac{\rho h_{11}^{BY}h_1^Y}{1-\rho h_1^B}\frac{1}{1-\rho^2} + \frac{h_{11}^Y}{1-\rho^2} + h_{11}^\sigma\right] \\ + \left[\begin{array}{c} \frac{\left[h_{11}^B - k^B\gamma\overline{L^B}\left(1-\overline{L^B}\right)\left(2+\left(\overline{B}-B^c\right)\gamma\left(1-2\overline{L^B}\right)\right)\right]\left(h_1^Y\right)^2}{\left(1-\left(h_1^B\right)^2\right)(1-\rho^2)}\left(\frac{1+\rho h_1^B}{1-\rho h_1^B}\right) + \\ 2\left(\frac{\rho h_1^Y h_{11}^{BY}}{1-\rho h_1^B}\right)\frac{1}{1-\rho^2} + \frac{h_{11}^Y}{1-\rho^2} + h_{11}^\sigma \end{array}\right] \end{array}\right\} \\ +\frac{1}{2}u''\left(\tau^y\overline{Y} - r\overline{B} - k^B\left(\overline{B} - B^c\right)\overline{L^B}\right)\mathrm{Var}\left(G_t\right) \end{array}\right]}{1-\beta_w}, \tag{2.152}$$

where U_w^B is social welfare under the debt-based sanction. We still need to determine $\mathrm{Var}(G_t)$. Recall from (2.84) that $G_t \simeq \overline{G} + f_1^B\widetilde{B}_t + f_1^Y\widetilde{Y}_t$ (because $f_1^\sigma = 0$). Recall also $\mathrm{E}\left[\widetilde{B}_t\widetilde{Y}_t\right] \simeq \left(\frac{\rho h_1^Y}{1-\rho h_1^B}\right)\frac{\sigma_\varepsilon^2}{1-\rho^2}$. Hence,

$$\begin{aligned} \mathrm{Var}\left(G_t\right) &\simeq \mathrm{E}\left[\left(\overline{G} + f_1^B\widetilde{B}_t + f_1^Y\widetilde{Y}_t - \widehat{G}\right)^2\right] \\ &\simeq \mathrm{E}\left[\left(f_1^B\widetilde{B}_t + f_1^Y\widetilde{Y}_t\right)^2\right] + 2\left(\overline{G} - \widehat{G}\right)\mathrm{E}\left[f_1^B\widetilde{B}_t + f_1^Y\widetilde{Y}_t\right] + \mathrm{E}\left[\left(\overline{G} - \widehat{G}\right)^2\right] \\ &\simeq \mathrm{E}\left[\left(f_1^B\right)^2 * \widetilde{B}_t^2 + 2f_1^B f_1^Y\widetilde{B}_t\widetilde{Y}_t + \left(f_1^Y\right)^2 * \widetilde{Y}_t^2\right] \\ &\simeq \left(f_1^B\right)^2 * \mathrm{Var}\left(B_t\right) + \left[2f_1^B\left(\frac{\rho h_1^Y}{1-\rho h_1^B}\right) + f_1^Y\right] f_1^Y\frac{\sigma_\varepsilon^2}{1-\rho^2} \\ &\simeq \left\{\begin{array}{c} \left(h_1^B - 1/\beta_g\right)^2\mathrm{Var}\left(B_t\right) + \\ \left[2\left(h_1^B - 1/\beta_g\right)\left(\frac{\rho h_1^Y}{1-\rho h_1^B}\right) + \left(\tau^y + h_1^Y\right)\right]\left(\tau^y + h_1^Y\right)\frac{\sigma_\varepsilon^2}{1-\rho^2} \end{array}\right\}, \end{aligned}$$

where we have ignored higher-than-second-order terms. Using (2.80), we obtain:

$$\begin{aligned} \mathrm{Var}\left(G_t\right) &\approx \left\{\begin{array}{c} \left(\frac{1+\rho h_1^B}{1-\rho h_1^B}\right)\frac{\left(h_1^B-1/\beta_g\right)^2\left(h_1^Y\right)^2}{\left(1-\left(h_1^B\right)^2\right)(1-\rho^2)}\sigma_\varepsilon^2 + \\ \left[2\left(h_1^B - 1/\beta_g\right)\left(\frac{\rho h_1^Y}{1-\rho h_1^B}\right) + \left(1 + h_1^Y\right)\right]\frac{\left(1+h_1^Y\right)\sigma_\varepsilon^2}{1-\rho^2} \end{array}\right\} \Rightarrow \\ \mathrm{Var}\left(G_t\right) &\approx \left\{\frac{\sigma_\varepsilon^2}{(1-\rho^2)}\left\{\begin{array}{c} \left(\frac{1+\rho h_1^B}{1-\rho h_1^B}\right)\left(h_1^Y\right)^2\frac{\left(h_1^B-1/\beta_g\right)^2}{1-\left(h_1^B\right)^2} + \\ \left[2\left(\frac{\rho h_1^Y\left(h_1^B-1/\beta_g\right)}{1-\rho h_1^B}\right) + \left(1 + h_1^Y\right)\right]\left(1 + h_1^Y\right) \end{array}\right\}\right\}. \end{aligned} \tag{2.153}$$

Therefore, inserting (2.153) in (2.152) gives us welfare when the fiscal constraint is imposed on the debt:

$$U_w^B \approx \frac{\left[\begin{array}{c} u\left(\tau^y\overline{Y} - r\overline{B} - k^B\left(\overline{B} - B^c\right)\overline{L^B}\right) + \\ u'\left(\tau^y\overline{Y} - r\overline{B} - k^B\left(\overline{B} - B^c\right)\overline{L^B}\right) * \\ \frac{\sigma_\varepsilon^2}{2}\left\{\begin{array}{c} \frac{h_1^B-(1/\beta_g)}{1-h_1^B}\left[\frac{h_{11}^B\left(h_1^Y\right)^2}{\left(1-\left(h_1^B\right)^2\right)(1-\rho^2)}\left(\frac{1+\rho h_1^B}{1-\rho h_1^B}\right) + 2\frac{\rho h_{11}^{BY}h_1^Y}{1-\rho h_1^B}\frac{1}{1-\rho^2} + \frac{h_{11}^Y}{1-\rho^2} + h_{11}^\sigma\right] \\ + \left[\begin{array}{c} \frac{\left[h_{11}^B - k^B\gamma\overline{L^B}\left(1-\overline{L^B}\right)\left(2+\left(\overline{B}-B^c\right)\gamma\left(1-2\overline{L^B}\right)\right)\right]\left(h_1^Y\right)^2}{\left(1-\left(h_1^B\right)^2\right)(1-\rho^2)}\left(\frac{1+\rho h_1^B}{1-\rho h_1^B}\right) \\ +2\left(\frac{\rho h_1^Y h_{11}^{BY}}{1-\rho h_1^B}\right)\frac{1}{1-\rho^2} + \frac{h_{11}^Y}{1-\rho^2} + h_{11}^\sigma \end{array}\right] \end{array}\right\} \\ +\frac{1}{2}u''\left(\tau^y\overline{Y} - r\overline{B} - k^B\left(\overline{B} - B^c\right)\overline{L^B}\right) * \\ \frac{\sigma_\varepsilon^2}{(1-\rho^2)}\left\{\left(\frac{1+\rho h_1^B}{1-\rho h_1^B}\right)\left(h_1^Y\right)^2\frac{\left(h_1^B-1/\beta_g\right)^2}{1-\left(h_1^B\right)^2} + \left[2\left(\frac{\rho h_1^Y\left(h_1^B-1/\beta_g\right)}{1-\rho h_1^B}\right) + \left(\tau^y + h_1^Y\right)\right]\left(\tau^y + h_1^Y\right)\right\} \end{array}\right]}{1-\beta_w}. \tag{2.154}$$

2.H.2 Welfare in the primary deficit-based sanction case

Substituting from (2.151) into (2.30), we get welfare U_w^D under the primary deficit constraint

$$U_w^D \approx \frac{\left\{\begin{array}{c} u\left(\overline{D} + \tau^y\overline{Y} - k^D\left(\overline{D} - D^c\right)\overline{L^D}\right) + u'\left(\overline{D} + \tau^y\overline{Y} - k^D\left(\overline{D} - D^c\right)\overline{L^D}\right) * \\ \frac{\sigma_\varepsilon^2}{2}\left\{\begin{array}{c} \left[\frac{l_1^D-(1/\beta_g)}{1-l_1^D}\right]\left[\left(\frac{l_{11}^D\left(l_1^Y\right)^2}{1-\left(l_1^D\right)^2}\left(\frac{1+\rho l_1^D}{1-\rho l_1^D}\right) + 2\frac{\rho l_1^Y l_{11}^{DY}}{1-\rho l_1^D} + l_{11}^Y\right)\frac{1}{1-\rho^2} + l_{11}^\sigma\right] \\ +2\frac{\rho l_1^Y l_{11}^{DY}}{1-\rho l_1^D}\frac{1}{1-\rho^2} + \frac{\left[l_{11}^D - k^D\gamma\overline{L^D}\left(1-\overline{L^D}\right)\left(2+\left(\overline{D}-D^c\right)\gamma\left(1-2\overline{L^D}\right)\right)\right]\left(l_1^Y\right)^2}{\left(1-\left(l_1^D\right)^2\right)(1-\rho^2)}\left(\frac{1+\rho l_1^D}{1-\rho l_1^D}\right) \\ +\frac{l_{11}^Y}{1-\rho^2} + l_{11}^\sigma \end{array}\right\} \\ +\frac{1}{2}u''\left(\overline{D} + \overline{Y} - k^D\left(\overline{D} - D^c\right)\overline{L^D}\right) * \mathrm{Var}\left(G_t\right) \end{array}\right\}}{1-\beta_w}. \tag{2.155}$$

Using that $\mathrm{E}\left[\widetilde{D}_t\widetilde{Y}_t\right] = \frac{\rho l_1^Y}{1-\rho l_1^D}\frac{\sigma_\varepsilon^2}{1-\rho^2}$, we have

$$\begin{aligned} \mathrm{Var}\left(G_t\right) &= \mathrm{E}\left[\left(G_t - \overline{G}\right)^2\right] \simeq \mathrm{E}\left[\left(j_1^D\widetilde{D}_t + j_1^Y\widetilde{Y}_t\right)^2\right] \\ &\simeq \left(j_1^D\right)^2\mathrm{Var}\left(D_t\right) + 2j_1^D j_1^Y\mathrm{E}\left[\widetilde{D}_t\widetilde{Y}_t\right] + \left(j_1^Y\right)^2\mathrm{Var}\left(Y_t\right) \\ &\simeq \left\{\begin{array}{c} \left(l_1^D - 1/\beta_g\right)^2\left(\frac{1+\rho l_1^D}{1-\rho l_1^D}\right)\frac{\left(l_1^Y\right)^2}{1-\left(l_1^D\right)^2}\frac{\sigma_\varepsilon^2}{1-\rho^2} + \\ 2\left(l_1^D - 1/\beta_g\right)\frac{\rho l_1^Y\left(l_1^Y+\tau^y\right)}{1-\rho l_1^D}\frac{\sigma_\varepsilon^2}{1-\rho^2} + \left(l_1^Y + \tau^y\right)^2\frac{\sigma_\varepsilon^2}{1-\rho^2} \end{array}\right\}. \end{aligned}$$

Hence,

$$\mathrm{Var}\left(G_t\right) \simeq \left[\begin{array}{c} \left(\frac{1+\rho l_1^D}{1-\rho l_1^D}\right)\frac{\left(l_1^D-1/\beta_g\right)^2\left(l_1^Y\right)^2}{1-\left(l_1^D\right)^2} + \\ 2\frac{\rho l_1^Y\left(l_1^Y+\tau^y\right)\left(l_1^D-1/\beta_g\right)}{1-\rho l_1^D} + \left(l_1^Y + \tau^y\right)^2 \end{array}\right]\frac{\sigma_\varepsilon^2}{1-\rho^2}. \tag{2.156}$$

So, inserting (2.156) in (2.155) gives us the value of welfare when the fiscal constraint is imposed on the value of the primary deficit:

$$U_w^D \approx \frac{\left\{\begin{array}{c} u\left(\overline{D}+\tau^y\overline{Y}-k^D\left(\overline{D}-D^c\right)\overline{L^D}\right)+u'\left(\overline{D}+\tau^y\overline{Y}-k^D\left(\overline{D}-D^c\right)\overline{L^D}\right)* \\ \frac{\sigma_\varepsilon^2}{2}\left\{\begin{array}{c} \left[\frac{l_1^D-(1/\beta_g)}{1-l_1^D}\right]\left[\left(\frac{l_{11}^D\left(l_1^Y\right)^2}{1-\left(l_1^D\right)^2}\left(\frac{1+\rho l_1^D}{1-\rho l_1^D}\right)+2\frac{\rho l_1^Y l_{11}^{DY}}{1-\rho l_1^D}+l_{11}^Y\right)\frac{1}{1-\rho^2}+l_{11}^\sigma\right] \\ +2\frac{\rho l_1^Y l_{11}^{DY}}{1-\rho l_1^D}\frac{1}{1-\rho^2}+\frac{\left[l_{11}^D-k^D\gamma\overline{L^D}\left(1-\overline{L^D}\right)\left(2+\left(\overline{D}-D^c\right)\gamma\left(1-2\overline{L^D}\right)\right)\right]\left(l_1^Y\right)^2}{\left(1-\left(l_1^D\right)^2\right)(1-\rho^2)}\left(\frac{1+\rho l_1^D}{1-\rho l_1^D}\right) \\ +\frac{l_{11}^Y}{1-\rho^2}+l_{11}^\sigma \end{array}\right\} \\ +\frac{1}{2}u''\left(\overline{D}+\tau^y\overline{Y}-k^D\left(\overline{D}-D^c\right)\overline{L^D}\right)* \\ \left[\left(\frac{1+\rho l_1^D}{1-\rho l_1^D}\right)\frac{\left(l_1^D-1/\beta_g\right)^2\left(l_1^Y\right)^2}{1-\left(l_1^D\right)^2}+2\frac{\rho l_1^Y\left(l_1^Y+\tau^y\right)\left(l_1^D-1/\beta_g\right)}{1-\rho l_1^D}+\left(l_1^Y+\tau^y\right)^2\right]\frac{\sigma_\varepsilon^2}{1-\rho^2} \end{array}\right\}}{1-\beta_w}. \tag{2.157}$$

Chapter 3

The political economy of structural reforms under a deficit restriction*

*This chapter is based on Ribeiro and Beetsma, Journal of Macroeconomics, forthcoming.

"We must recognize that the goal of consolidating public budgets may well conflict in the short term with the goal of enhancing the potential for economic growth. A reformed pact must take this conflict into account, as well as the need to bring improved growth and employment opportunities into line in the long term with sound public budgets."

Gerhard Schröder
"A framework for a stable Europe," *Financial Times*, January 16, 2005.

3.1 Introduction

Many countries are now confronted with the simultaneous need to pursue fiscal discipline and to implement structural reforms, such as making their labor and product markets more flexible and reform of their welfare and pension systems. This is, for example, the case for Germany and France, but also for other countries. The needs for structural reform and fiscal discipline should not be seen in isolation. In fact, the two are tightly related to each other, as structural reforms are conducive to maintaining the long-term sustainability of the public finances.[1] Most directly this is probably the case for pension reform. However, also the supposedly larger growth potential and the greater shock-resilience of an economy with better functioning markets should reduce pressures for fiscal profligacy. Nevertheless, it has been argued that fiscal restrictions, while in principle beneficial for fiscal discipline, may in the short run conflict with the abovementioned structural reform measures.[2] In particular, implementing the necessary structural reforms may require substantial up-front spending, such as compensation for those who stand to lose from the reform.[3] In fact, the conflict between short-run and long-run economic objectives has been an important argument for the recent reform of Europe's Stability and Growth Pact (SGP) (see ECOFIN, 2005; European Commission, 2005b).[4]

This chapter analyses this conflict in the context of a model that combines the presence of a short-run deficit restriction with the possibility of carrying out structural reforms that yield long-run benefits, but impose upfront costs on private agents. Of course, not

[1] This is also recognized by the European Commision (2005a) in its three-pronged strategy for ensuring long-term sustainability and the quality of the public finances. The Commission evaluates what has been done and what is further required in terms of structural reforms. For most countries in the European Union, large parts of the necessary reforms still need to be completed.

[2] An early paper in this vein is Razin and Sadka (2002). They explore the political viability of a pay-as-you-go social security-system in an aging scenario and compare it to a funded system. Their work supports the idea that a deficit ceiling hampers the transition to a funded pension system.

[3] Beetsma and Debrun (2004, 2007) provide examples of such compensation costs (see also Saint-Paul, 2002, and Grüner, 2002).

[4] A large number of reform proposals have been made by academic researchers. See, for instance, Buiter (2003), Buti et al. (2003), Blanchard and Giavazzi (2004), Fatás et al. (2004) and Mathieu and Sterdyniak (2003).

all structural reforms fit this description. However, a number of important reforms in, for instance, labor and product markets do. Examples are the relaxation of firing restrictions or sectoral re-allocations leading to temporary unemployment and a loss in income. To enhance the support for structural reform and therefore its chance to be re-elected, the incumbent government can provide compensation to the voting population. However, such handouts come at the cost of a higher deficit and may thus lead to a violation of the fiscal restriction intended to restrain politically-motivated overspending. We build a political-economy framework for analyzing the feedback effects between fiscal restrictions and the incentives for structural reform in a setting where the policymaker is accountable to the voting population. Eventually, the desirability of a deficit restriction depends on the severity of the political distortion leading to the excessive deficit and the long-run benefit from the structural reform.

We demonstrate that a tighter deficit restriction limits the room for reform-related spending and, hence, reduces the likelihood that a reform will eventually be implemented. We also explore the effects of changes in income uncertainty and income inequality across the voters on the likelihood of structural reforms. In our model, an increase in income uncertainty reduces the likelihood of reforms, while more inequality makes reforms more likely. As we discuss, a more flexible implementation of the deficit restriction that takes explicit account of reform-related spending reduces the conflict between adhering to the restriction and increasing the likelihood of reform.

The remainder of the paper is organized as follows. Section 3.2 presents the basic model. In Section 3.3 we analyze the outcomes under a benevolent policymaker (a "social planner") who is not subject to electoral uncertainty. Then, in Section 3.4, we turn to the solution under a partisan government that is subject to electoral uncertainty. Section 3.5 provides a comparative static analysis of the model, exploring the effects of a tighter fiscal restrictions, changes in income uncertainty and income inequality. Section 3.6 explores how a tighter deficit restriction affects social welfare. Finally, Section 3.7 concludes the paper. Technicalities, proofs and some extensions of our analysis are relegated to the Appendix, which is available upon request or from the authors' websites.

3.2 The model

The model features three periods (0, 1 and 2) and focuses on the interaction between structural reform and the need to adhere to a restriction that limits public deficits. We shall describe an economy that can borrow and lend freely on the international capital market, while taking the real interest rate as given. The country operates as a democracy with two political parties in a set-up that follows Alesina and Tabellini (1990). Partisan politics leads to the emergence of a deficit bias which rationalizes the presence of a fiscal restriction.[5] As a benchmark to this case, we also study a social planner that makes all

[5]There exist a number of potential political mechanisms that all give rise to a deficit bias. An early paper that contains an overview is Roubini and Sachs (1989).

choices. The model in this paper is partially based on Beetsma and Debrun (2004, 2007). However, it differs in some crucial ways from these earlier models. Most importantly, while these earlier models assumed exogenous re-election chances for the incumbent party, in the current set-up the government internalizes a feedback from its structural reform proposal to its re-election chances.

3.2.1 Private agents

Consumption (private and public) takes only place in periods 1 and 2. The economy is populated by a continuum of private individuals with total mass normalized to unity. Individuals differ in terms of their income and in terms of their preferences for public goods. All individuals derive utility from private consumption, while the low-income individuals in addition derive utility from a public good G and high-income individuals in addition obtain utility from a public good F. The other, middle-income, individuals do not benefit from any of these two public goods. Empirical work shows that public spending on various categories can differ strongly across income groups. For example, for the U.S., Ruggles and O'Higgins (1981) found that relatively large shares of public spending on housing and health and hospitals went to the (very) poor, while most of the spending on safety (policy, firearms) went to the high-income groups. O'Higgins and Ruggles (1981) do a similar analysis for the U.K. with roughly similar findings as for the U.S.

Expected utility of agent i from the middle-income group is:

$$\mathrm{E}_0\left[u\left(c_{1i}\right)+u\left(c_{2i}\right)\right], \tag{3.1}$$

where c_{ti} is the amount of the private good consumed in period t (1 and 2), and $\mathrm{E}_0[\cdot]$ is the expectation conditional on information available at the start of the game, before any of the uncertainties have been resolved. We assume that $u' > 0$ and $u'' \leq 0$. We allow for the possibility that $u'' = 0$ for all consumption levels, in which case the utility from private consumption is linear. For convenience, we abstract from the discounting of the future. Expected utility of an agent i from the low-income, respectively high-income, group is:

$$\mathrm{E}_0\left[u\left(c_{1i}\right)+u\left(c_{2i}\right)+v\left(g_1\right)+v\left(g_2\right)\right],$$
$$\mathrm{E}_0\left[u\left(c_{1i}\right)+u\left(c_{2i}\right)+v\left(f_1\right)+v\left(f_2\right)\right],$$

where g_t and f_t are the amounts of public good G and F, respectively, consumed in period t (1 and 2). We assume that $v' > 0$ and $v'' < 0$. Moreover, we assume that $v(0) = 0$, $v'(0) \rightarrow \infty$ and $v'(\infty) \rightarrow 0$. We assume that the sizes of the low- and high-income groups are λ each. The sizes of these groups are sufficiently smaller than $\frac{1}{2}$, such that the outcome of the election discussed below is solely determined by individuals' income comparisons under the two parties.

Resources available for private consumption depend on whether a structural reform takes place. In the absence of the reform,

$$c_{1i} = (1-\theta)\, y_i(1+\varepsilon) + d, \quad c_{2i} = (1-\theta)\, y_i(1+\varepsilon) - d, \tag{3.2}$$

while with the reform,

$$c_{1i} = (1-\theta)\, y_i(1+\varepsilon) + (\eta - I)\,\gamma + d, \quad c_{2i} = (1-\theta)\, y_i(1+\varepsilon)\,(1+\Gamma) - d, \tag{3.3}$$

where θ is a given (and constant) tax rate, y_i is an exogenous income component specific to individual i, which is, for convenience, assumed to be equal in both periods, ε is a (common) macroeconomic income shock, and d is the amount that the individual borrows in period 1 and needs to pay off in period 2. Differences in income can result from differences in individual labor productivity. Further, $\gamma > 0$ is the size of the reform, $I > 0$ is a short-run (period-1) private cost associated with the reform, $\eta \geq 0$ is a compensation provided by the government to the individual and $\Gamma > 0$ is a boost to second period income resulting from the reform. Hence, while reforms produce costs in the short-run (period 1), they stimulate income in the longer run, for example because markets are made to work more efficiently. We model the total cost of the reform and the total compensation as proportional to the size of the reform. This seems more restrictive than it in fact is. Because the size of the reform is given, we could instead have used a formulation in which we introduce separate parameters for the total cost and the total compensation, without any implications for the results. The current formulation turns out to be slightly more appealing when we introduce functional specifications for a numerical analysis later on. Related to the previous point, one might expect that in reality the size of the future income boost generally depends on the size of the reform, γ. That is, the boost is a function $\Gamma(\gamma)$ of γ. However, because γ is fixed, we shall most of the time suppress the argument of Γ.

While there are many possible types of structural reforms, our analysis captures only those reforms that carry some upfront cost at the individual level before yielding its longer-run benefits. Obviously, this excludes a substantial number of reforms. However, relevant reforms in the context of the current framework, could be certain types of labor market or product market reform, or some types of social security or welfare reform. Key is the assumption that individuals bear a short-run cost of the reform. These costs include among other things the loss of rents, because reforms enhance competition in product and labor markets, thereby eroding wage premia, and salary losses due to temporary unemployment associated with the induced reallocation of resources across sectors or with reductions in employment protection (IMF, 2003). We summarize these costs as foregone private consumption. To focus on the relation between structural reforms and the presence of a deficit restriction, the model assumes that each individual experiences the same cost of the reform and also receives the same compensation in order to support the reforms. However, one should notice that, even if the short-run cost of reform would affect only a fraction of the population, the government might be strongly motivated to provide net transfers in order to prevent social unrest to undermine the broader support for the reform program. A good example was the recent proposal by the French government to introduce employment contracts that would allow employers to fire young workers at will. The proposal was withdrawn after massive popular protest, despite the fact that the group affected by the measure was only a minority of the population. For simplicity, in our model, there are no lobby groups or trade unions, so that people can only express their discontent through the ballot box.

As far as compensation is concerned, in practice it may range from direct monetary transfers to more indirect forms such as active labor market policies designed to enhance the employability of the individual and ease the matching between unemployed individuals and available vacancies. Some evidence of such costs is provided in Beetsma and Debrun (2004, 2007). We assume that the size of the reform is given and that, once it has been implemented, it cannot be reversed.[6] The former assumption contrasts with Beetsma and Debrun (2004, 2007), where the size of the reform follows from optimization on the side of the government. In reality, many (though by no means all) possible reforms are of a size that can only be varied to a limited extent.

Individuals face a very simple decision problem. They optimally choose their borrowing so as to equalize their consumption levels in the two periods. In the absence of reform, this leads to $d = 0$, implying an expected utility from private consumption of [7]

$$U_i^{NR} \equiv 2\mathrm{E}_0 \left[u \left((1-\theta)\, y_i(1+\varepsilon) \right) \right], \tag{3.4}$$

while with reform they set $d = \frac{1}{2}(1-\theta)\, y_i(1+\varepsilon)\Gamma + \frac{1}{2}(I-\eta)\,\gamma$, implying an expected utility from private consumption of

$$U_i^{R} \equiv 2\mathrm{E}_0 \left[u \left((1-\theta)\, y_i(1+\varepsilon)\left(1+\tfrac{1}{2}\Gamma\right) + \tfrac{1}{2}(\eta - I)\,\gamma \right) \right]. \tag{3.5}$$

3.2.2 The Parties

There are two political parties, F and G. These parties are exogenous. This assumption seems reasonable as in reality the party landscape usually changes only slowly. Party F (party G) only obtains utility from the provision of public good F (G). Hence, the expected utility of party F (and similar for party G) is:

$$\mathrm{E}_0 \left[v(f_1) + v(f_2) - \Delta_F k \left(b - \bar{b} \right) \right], \tag{3.6}$$

where b is the budget deficit in period 1 and $\bar{b}$ is a reference deficit level. Hence, when in power, party F (party G) will direct all its spending towards public good F (good G)

[6] For an analysis of economic reforms and dynamic political constraints, see Dewatripont and Roland (1992).

[7] In the case where function $u(.)$ is linear, any debt level d is optimal, as long as consumption does not become negative in one of the periods.

and spend nothing on the other public good. We assume that in period 1 the government is subject to some fiscal restriction intended to limit fiscal profligacy. In particular, the government incurs a utility cost from running an *excessive deficit,* that is, a deficit that exceeds its reference level ($b > \bar{b}$). For simplicity, we assume here that the costs associated with an excessive deficit are non-monetary. For example, they can arise from peer pressure by governments of other countries, tight monitoring of (or even control over) the country's budgetary policies by the enforcer of the deficit restriction and public embarrassment for the officials in charge. Appendix 3.G analyzes the case where the cost of the excessive deficit is tangible in the form of a fine, but this will have no implications for the results. In the case of the European Union, both mechanisms play a role. The enforcer of the pact, the Council of Economics and Finance Ministers (ECOFIN) with the help of the European Commission, exerts pressure on individual governments to keep their finances under control, while, in the case of persistently excessive deficits, they have the possibility to impose a fine on the violator of the Excessive Deficit Criterion. Parameter k captures the tightness of the deficit restriction or the severity of sanctions. Finally, Δ_F is an indicator function, taking a value of one when F is in power, and a value of zero, otherwise. In other words, party F only suffers from the implementation of the restriction when it is in office (and runs a too high deficit). It is important to realize that the expectations operator in (3.6) aggregates out all future uncertainties as seen from the start of the game. In particular, this is the case also for the electoral uncertainties discussed below.

Without loss of generality, we assume that at the start of the game, period 0, party F is in power and that this party presents a reform package to be implemented if it is re-elected into government in the first period. This reform package is a combination (γ, η). As mentioned above, the size γ of the reform is given and thus cannot be varied by party F. The amount of compensation is the only variable to be selected by party F at the start of the game. It is assumed to be continuous. As we shall see, it will involve a trade-off for F between enhancing its re-election chances and keeping the budgetary cost (and thus the conflict with the deficit restriction) limited. The other party, G, commits to a platform that does not involve reform (and, thus, also no compensation). We assume that the commitments made in period 0 are binding so that they are executed by the parties when they get to power. While this is not modeled explicitly, one can motivate this by assuming that concerns about their longer-run credibility prevents parties from reneging on their commitments. Since we focus on the consequences of introducing reform, we exclude the possibility that party F chooses a platform with no reform. This possibility would also lead to an indeterminacy because the parties would be preferred by equally-large groups of voters. We also exclude the possibility that party G chooses a reform platform with a certain amount of compensation. This would lead to complications that are not the focus of the current analysis. On the one hand, the parties could be drawn into a bidding contest for voters by promising higher compensation than the other party, while on the other hand, they would be restrained by the fact that higher levels of compensation reduce available resources for their preferred public good. We thus abstract from these complications. We believe that the current set up with only one party proposing a reform is reasonable, since the group of individuals that prefer this party because of the type of public good it provides, is also the group that will benefit most from the reform. Exactly the opposite will be the case for party G. This party provides the public good preferred by the low-income group, which will also be the group that is harmed most by the reform.

In the case where party G is elected to hold office in period 1 and no reform is undertaken, the first- and second-period government budget constraints thus read:

$$f_1 + g_1 = \theta\overline{y}(1+\varepsilon) + b, \quad f_2 + g_2 = \theta\overline{y}(1+\varepsilon) - b, \tag{3.7}$$

where $\overline{y}$ is the average over all y_i. The first term on the right-hand side of each budget constraint is the total tax revenue. In period 1, the government can issue public debt b, which has to be paid off in period 2. Given the absence of initial debt, the debt carried over from period 1 into period 2 is equal to the deficit in period 1. When party F is re-elected and reform is implemented in the first period, the government budget constraints become:

$$f_1 + g_1 = \theta\overline{y}(1+\varepsilon) - \eta\gamma + b, \quad f_2 + g_2 = \theta\overline{y}(1+\varepsilon)(1+\Gamma) - b. \tag{3.8}$$

In order to provide a rationale for the deficit restriction, the model needs to feature a deficit bias. Therefore, we assume that there is another election at the start of the second period. However, since the implementation (or not) of the reform package has already taken place and cannot be reversed, we assume that the incumbent in period 1 is re-elected into office in period 2 with an exogenous probability $0 < p < 1$. As we shall see, this electoral uncertainty, and thus the possibility of not being able to spend the remaining resources in period 2 on its own preferred public good, leads the period-1 government (whatever its identity is) to increase its deficit. The electoral uncertainty at the start of the second period may arise from various imperfections in the political process, such as uncertainty about the voter turnout, which can have a different impact on the two parties. It could also arise from uncertainty about the appeal of the parties' leaders to voters, the occurrence of scandals, and so on. Electoral uncertainty may thus reflect extra-economic events that are not explicitly modelled here, but that the electorate cares about and that may cause part of a party's natural constituency to vote for its opponent.

3.2.3 The timing

Figure 1 summarizes the timing of the model. In Stage 1, period 0, the incumbent government chooses η. In Stage 2, also period 0, the income shock ε materializes. Then, in Stage 3, beginning of period 1, the first election takes place. This is followed by the implementation (or not) of the reform and the compensation if the incumbent is re-elected (or not), as well as the selection of the period-1 deficit (Stage 4). At the beginning of period 2, new elections take place (Stage 5). Finally, the public debt is paid off (Stage 6).

3.3 The social Planner solution

To provide a benchmark for the solution under a partisan government, we consider a utilitarian social planner who is not subject to electoral uncertainty and whose utility is the average of the utilities of all individuals in society. The social planner chooses whether or not to implement the reform. When it decides to implement the reform, it chooses the level of compensation. In addition, the planner selects the public debt (= deficit) level to optimally shift resources between periods 1 and 2.

If the planner chooses not to implement the reform, he maximizes over b, f_1 and f_2:

$$\int U_i^{NR} di + \lambda \mathrm{E}_0 \left[\begin{array}{c} v(f_1) + v(\theta\overline{y}(1+\varepsilon) + b - f_1) + \\ v(f_2) + v(\theta\overline{y}(1+\varepsilon) - b - f_2) \end{array} \right]. \tag{3.9}$$

The first term in this objective function is the aggregate of the utilities from private consumption over periods 1 and 2. The second component is the aggregate of the utilities from public consumption. The first and third terms in this component are the aggregate utilities of the high-income individuals from public consumption in periods 1 and 2 (each F-type experiences equal utility from public consumption). Similarly, the other two terms are the corresponding aggregate utilities of the low-income individuals. The arguments in the public consumption utility functions follow directly from the public budget constraints (3.7) in the two periods, when reform is forsaken.

The planner makes his choices after ε has materialized. Hence, we find b, f_1 and f_2 by maximizing the term in square brackets in (3.9). The outcomes are $f_1 = g_1 = f_2 = g_2 = \frac{1}{2}\theta\overline{y}(1+\varepsilon)$ and $b = 0$. Within each period, the planner spreads the provision of public goods equally over the F- and G-type individuals. Moreover, given the absence of discounting and a zero interest rate on debt, the planner sets debt so as to smooth public goods consumption perfectly over time. Substituting the outcomes into (3.9), we obtain the planner's utility under the optimal solution in the absence of reform:

$$\int U_i^{NR} di + 4\lambda \mathrm{E}_0 \left[v\left(\tfrac{1}{2}\theta\overline{y}(1+\varepsilon)\right)\right]. \tag{3.10}$$

If the planner chooses to conduct reform, his objective is to maximize over η, b, f_1 and f_2:

$$\int U_i^{R} di + \lambda \mathrm{E}_0 \left[\begin{array}{c} v(f_1) + v(\theta\overline{y}(1+\varepsilon) - \eta\gamma + b - f_1) + \\ v(f_2) + v(\theta\overline{y}(1+\varepsilon)(1+\Gamma) - b - f_2) \end{array} \right], \tag{3.11}$$

Again, we solve the planner's problem by working backwards. The outcomes are:

$$f_1 = g_1 = f_2 = g_2 = \tfrac{1}{2}\left[\theta\overline{y}(1+\varepsilon)\left(1+\tfrac{1}{2}\Gamma\right) - \tfrac{1}{2}\eta\gamma\right], \tag{3.12}$$

and

$$b = \tfrac{1}{2}\eta\gamma + \tfrac{1}{2}\theta\overline{y}(1+\varepsilon)\Gamma. \tag{3.13}$$

Through its debt policy, the planner shifts half of the compensation cost to the second period and half of the income gain from reform to the first period. This way it can smooth public consumption perfectly over time. In the following sections, when we explore the partisan outcomes, the deficit will generally differ from the benchmark levels derived here. Then, in the absence of reform, when the deficit is above zero, or, in the presence of reform, when the deficit exceeds (3.13), we can meaningfully speak of a *deficit bias*.

When utility from private consumption is linear, one finds the following simple first-order condition for η:

$$1 = \lambda E_0 \left\{ v' \left[\tfrac{1}{2} \left(\theta \overline{y} (1+\varepsilon) \left(1 + \tfrac{1}{2}\Gamma \right) - \tfrac{1}{2}\eta\gamma \right) \right] \right\},$$

where we have used (3.8) and substituted away b with the help of (3.13). A higher future reform benefit Γ, higher average income $\overline{y}$ and a higher tax rate θ imply an increase in the optimal level of compensation. Using (3.12), the planner's utility then becomes:

$$\int U_i^R di + 4\lambda \mathrm{E}_0 \left\{ v \left[\tfrac{1}{2} \left(\theta \overline{y} (1+\varepsilon) \left(1 + \tfrac{1}{2}\Gamma \right) - \tfrac{1}{2}\eta\gamma \right) \right] \right\}. \tag{3.14}$$

With linear private consumption utility, one has $\int U_i^{NR} di = 2(1-\theta)\overline{y}$ and $\int U_i^R di = (1-\theta)\overline{y}(2+\Gamma) + (\eta - I)\gamma$. Then, if one assumes that $(1-\theta)\overline{y}\Gamma > I\gamma$, it is always optimal for the planner to conduct reform. Setting $\eta = 0$ would already produce higher social welfare under reform, as can be seen by comparing (3.10) with (3.14). The extra flexibility of choosing η allows for a possible further welfare gain.

3.4 Solution with a partisan government

We return now to the case of the partisan solution. According to the timing presented earlier, the only moments at which decisions are taken are Stages 1 and 4 when the level of compensation η and the deficit b, respectively, are chosen. We shall solve the model using backwards induction and thus first derive the optimal deficit level for given compensation and then derive the optimal amount of compensation (assuming that the deficit will be optimally chosen subsequently). Thoughout, we assume that the reference level $\overline{b}$ is set sufficiently low (and k is not too high), so that the equilibrium deficit level exceeds its reference level. This is the relevant case for our purposes.[8]

3.4.1 The deficit decision

The deficit is selected either by party F or party G, depending on which party is in government in period 1. Therefore, we compute the first-order conditions for the deficit selected in period 1 by party F (which implements a reform), respectively party G (which does not implement a reform):

[8] This case applies, for example, if $\overline{b} = 0$, which would be the social planner's debt choice in the absence of reform.

$$v'\left(f_1^R\right) = pv'\left(f_2^R\right) + k, \quad v'\left(g_1^{NR}\right) = pv'\left(g_2^{NR}\right) + k, \tag{3.15}$$

where

$$\begin{aligned} f_1^R &= \theta\overline{y}(1+\varepsilon) - \eta\gamma + b^R, \quad f_2^R = \theta\overline{y}(1+\varepsilon)(1+\Gamma) - b^R, \\ g_1^{NR} &= \theta\overline{y}(1+\varepsilon) + b^{NR}, \quad g_2^{NR} = \theta\overline{y}(1+\varepsilon) - b^{NR}, \end{aligned}$$

and where the superscript R (NR) indicates that reform has (has not) been implemented. The first-order conditions equate the marginal benefit of a higher deficit (in terms of more public good consumption in period 1) with the expected marginal cost in terms of foregone future public good consumption plus the marginal utility cost from the implementation of the deficit restriction. In deriving these first-order conditions, we have made use of the fact that the party in government optimally chooses to spend only on its own public good. Therefore, the period-1 government will rationally calculate that it only foregoes future consumption if it is re-elected in period 2. Hence, the marginal cost in terms of foregone future public spending should be weighted with the re-election probability p in (3.15). Effectively, the period-1 government discounts future utility more heavily than if it were certain to be re-elected.

The first-order conditions imply the following results:

Lemma 3.1 *(a) The deficit is higher under reform than under no reform, (b) holding everything else equal in the reform case, more compensation, η, and a larger reform package, γ, both imply a higher deficit, (c) a higher re-election probability p for period 2 reduces the deficit both under reform and under no reform, and (d) tighter sanctions (an increase in k) reduce the deficit both under reform and under no reform.*

Proof. See Appendices 3.A and 3.B. ■

The intuition for part (a) is that implementing structural reforms requires compensating transfers financed from the period-1 public budget, while, moreover, future tax revenues rise due to the income gain brought about by the reform. For both reasons, it is optimal to shift resources to period 1, so that $b^R > b^{NR}$. We can explain part (b) by noting that more compensation, given the reform level, and a larger reform package, given the amount of compensation, both reduce first-period resources for public consumption and thus lead the government to rebalance resources over time by issuing more debt.[9] Better re-election chances effectively make the government in period 1 less myopic and, thus, induce it to issue less public debt – see part (c). Finally, tighter sanctions raise the government's marginal utility cost of issuing public debt and induce the government to cut the deficit.

[9] If second-period resources also increase because Γ is positively related to the size of the reform package, then the effect of an increase in γ on debt will be strengthened.

3.4.2 The choice of compensation

We now move further back into the game and explore the choice of the optimal compensation level η. Before so doing, let us first look at the electoral preferences of the private agents (given the reform package and the shock ε), who have to cast their votes at the start of the first period. Each individual votes for the candidate that maximizes his utility. As the benefit from the structural reform is proportional to individual income, while the private cost is fixed over the individuals, the higher the income, the more likely it is that the individual will prefer reform to no reform, ceteris paribus. Therefore, low-income individuals would always vote for party G, while high-income individuals would always vote for party F. Assuming that the election outcome is based on a majority vote, the choice of the voter with the median income, which we denote by y^m, then becomes decisive.[10] Based on (3.4) and (3.5), it is easy to see that party F is re-elected when $(1-\theta)\, y^m(1+\varepsilon)\left(1+\frac{1}{2}\Gamma\right)+\frac{1}{2}\left(\eta-I\right)\gamma > (1-\theta)\, y^m\left(1+\varepsilon\right)$ or $(1-\theta)\, y^m(1+\varepsilon)\Gamma > (I-\eta)\,\gamma$. For a given level of compensation η, structural reform is more likely, the higher is the median income level, the more favorable is the income shock and the more effective (as measured Γ) is the structural reform.

To keep the algebra as simple as possible, we assume that the income shock has a uniform distribution on the interval $[-\overline{\varepsilon}, \overline{\varepsilon}]$. Thus, party F will be in government in period 1 if $\varepsilon \geq \varepsilon_L$, where

$$\varepsilon_L \equiv \frac{(I-\eta)\,\gamma}{(1-\theta)\, y^m\Gamma} - 1. \tag{3.16}$$

Therefore, the probability that party F is re-elected in period 1, and reform takes place, is:

$$\Pr(R) = \frac{1}{2\overline{\varepsilon}}\left[1+\overline{\varepsilon}+\frac{(\eta-I)\,\gamma}{(1-\theta)\, y^m\Gamma}\right]. \tag{3.17}$$

Not surprisingly, an increase in compensation raises the probability of re-election.

In Stage 1 of the game, party F faces the problem of maximizing over η:

[10] For the median voter theorem to hold, the richest among the individuals that still benefit from the public good provided by party G, must prefer party G to party F. That is, the following condition must hold:

$$\begin{aligned} & 2u\left((1-\theta)\, y_\lambda(1+\varepsilon)\right)+v\left(g_1^{NR}\right)+pv\left(g_2^{NR}\right) \\ > \; & 2u\left((1-\theta)\, y_\lambda(1+\varepsilon)\left(1+\tfrac{1}{2}\Gamma\right)+\tfrac{1}{2}\left(\eta-I\right)\gamma\right)+(1-p)\, v\left(g_2^{R}\right). \end{aligned}$$

where y_λ is the income level of individuals in the λ-th percentile of the income distribution. The first term on both sides of the inequality is the utility from private consumption over the two periods of such an individual under no reform and reform, respectively. If party G takes office in period 1, there is no reform and the individual benefits from the corresponding level of provision of the G-type public good in the first period and with probability p in the second period. If party F takes office in period 1, there is reform, but the individual does not benefit from public good provision in period 1 and benefits with probability $1-p$ from the provision of the G-type public good in period 2.

$$\max_{\eta}\left\{\begin{array}{c}\Pr(R)\,\mathrm{E}_{\varepsilon}\left[v\left(f_1^R\right)+pv\left(f_2^R\right)-k\left(b^R-\overline{b}\right)\mid\varepsilon\geqslant\varepsilon_L\right]+\\ \left[1-\Pr(R)\right]\mathrm{E}_{\varepsilon}\left[(1-p)\,v\left(f_2^{NR}\right)\mid\varepsilon<\varepsilon_L\right]\end{array}\right\}, \tag{3.18}$$

where the operator $\mathrm{E}_{\varepsilon}[.]$ indicates that expectations are only taken over ε (the electoral uncertainty at the start of the second period has been integrated out). In optimizing, the government takes account of the effect of a change in η on the probability of re-election. Applying Leibnitz' rule and using the first-order conditions for b^R and b^{NR}, (3.15), the first-order condition for η is:

$$-\frac{\partial\varepsilon_L}{\partial\eta}\left[v\left(f_{1L}^R\right)+pv\left(f_{2L}^R\right)-k\left(b_L^R-\overline{b}\right)-(1-p)\,v\left(f_{2L}^{NR}\right)\right]=\gamma\int_{\varepsilon_L}^{\overline{\varepsilon}}v'\left(f_1^R\right)d\varepsilon, \tag{3.19}$$

where

$$f_{1L}^R \equiv \theta\overline{y}(1+\varepsilon_L)-\eta\gamma+b_L^R, \tag{3.20}$$

$$f_{2L}^R \equiv \theta\overline{y}(1+\varepsilon_L)\,(1+\Gamma)-b_L^R, \tag{3.21}$$

$$f_{2L}^{NR} \equiv \theta\overline{y}(1+\varepsilon_L)-b_L^{NR}. \tag{3.22}$$

Here, b_L^R and b_L^{NR} are the deficit levels under reform, respectively no reform, when $\varepsilon=\varepsilon_L$. Condition (3.19) equates the marginal benefit of an increase in compensation – the left-hand side – with its marginal cost. The marginal benefit is (minus) the marginal reduction of the lower bound on the shock that admits F's re-election, multiplied by the utility difference to F of re-election versus no re-election. The marginal cost is the expected marginal utility loss in the first period from having to pay a higher compensation for any ε in the interval $[\varepsilon_L,\overline{\varepsilon}]$, for which re-election takes place. Substituting for $\partial\varepsilon_L/\partial\eta$, one can rewrite (3.19) as

$$v\left(f_{1L}^R\right)+pv\left(f_{2L}^R\right)-k\left(b_L^R-\overline{b}\right)-(1-p)\,v\left(f_{2L}^{NR}\right)=(1-\theta)\,y^m\Gamma(\gamma)\int_{\varepsilon_L}^{\overline{\varepsilon}}v'\left(f_1^R\right)d\varepsilon. \tag{3.23}$$

One can show that equation (3.23) has only one solution in η (when taking account of the subsequent effect of η on b_L^R).[11]

3.5 Comparative statics

Having derived the first-order conditions for the model, we can now turn to the comparative static analysis. In particular, we shall explore the implications of a tighter deficit restriction. We shall also study the consequences of more income uncertainty and less income inequality.

[11]See Appendix 3.C.1.

3.5.1 Tighter enforcement of the deficit restriction

We are particularly interested in how the enforcement of the deficit restriction influences the equilibrium deficit, compensation, and re-election (reform) chances. These effects are summarized in the following proposition:

Proposition 3.1 *If the re-election probability pat the start of the second period is sufficiently high (and, in particular, if $p = 1$), then tighter enforcement of the deficit restriction (i.e., a higher k) leads to (a) a lower level of reform compensation for private agents, (b) a lower deficit both under reform and (at least for quadratic utility of public consumption) under no reform, and (c) a lower probability of reform.*

Proof. (Summary) The proof of parts (a) and (b) is obtained by differentiating, with respect to k, the first-order conditions for the optimal deficits both under reform and no reform (evaluated at $\varepsilon = \varepsilon_L$), and the first-order condition for optimal compensation (3.23). Then, by solving the resulting system of equations, we obtain the reduced form effect of k on each one of the three endogenous variables. The quadratic specification mentioned under no reform in part (b) is discussed below. Finally, differentiating (3.17) with respect to k and using the result of part (a), one can prove part (c). See Appendices 3.D.1 and 3.D.2 for a formal proof of this proposition. ■

The intuition for part (a) is as follows. In the case of reform, the period-1 government issues additional debt in order to rebalance the compensation cost over the two periods. However, as tighter enforcement of the deficit restriction raises the cost of doing so, the government is forced to alter the time profile of public goods provision in a way it does not like. This exerts a negative effect on the utility it expects from providing the public good. To mitigate this effect, the period-1 government reduces the amount of compensation it offers. In so doing, it trades off an improvement in the time profile of its public spending against a reduction in its electoral chances. The case of no reform in part (b) was already covered by Lemma 3.1. Under reform, the effect of a tighter deficit restriction is now twofold, because the increase in enforcement also affects compensation, which exerts an indirect effect on the deficit. The overall effect of tighter enforcement is a reduction in the deficit. Finally, given that tightness k only affects the initial government's re-election chances through the choice of η, it follows immediately that tighter enforcement reduces the likelihood that the initial government is re-elected and thus that reforms will be implemented.[12]

In the recent past some countries have found it difficult to adhere to the 3% deficit ceiling of Europe's SGP. This may in part be explained by these countries' attempts to introduce reforms that require some compensation to render them politically acceptable. As discussed above, governments have an incentive to increase the deficit to spread the

[12]Given that we are only able to formally prove Proposition 3.1 for p sufficiently close to 1, we have performed a numerical check for a large set of parameter combinations of the results of the proposition for values of $p < 1$. The details are in Appendix 3.F.

compensation costs over time. This undermines the compliance with the deficit restriction. This may be particularly relevant for Germany, which is trying to implement several reforms (such as reforms in pensions and in the tax system).[13]

3.5.2 Increased income uncertainty

We can also explore the implications of an increase in income uncertainty. Income uncertainty can be summarized by the variance of the shock ε, which, in turn, is monotonically increasing in $\overline{\varepsilon}$. We formalize the implications of an increase in $\overline{\varepsilon}$ in the following proposition:

Proposition 3.2 *If the re-election probability pat the start of the second period is sufficiently high (and, in particular, if $p = 1$), an increase in income uncertainty (a rise in $\overline{\varepsilon}$), leads to less compensation η, a lower deficit after reform, and, if $(1-\theta)\, y^m \Gamma > (I-\eta)\,\gamma$, a lower probability that the initial incumbent is re-elected at the start of the first period.*

Proof. Differentiate the first-order conditions with respect to $\overline{\varepsilon}$. See Appendices 3.D.3 and 3.D.4 for the details. ■

The intuition for these, perhaps rather surprising, results is the following. An increase in $\overline{\varepsilon}$ does not directly affect ε_L, but increases the range of income shocks over which compensation must be paid. As a result, the government cuts back on compensation. This improves the public budget in period 1 for any shock that leads to the re-election of the incumbent and thus to reform. Then, less debt is issued in period 1, and the deficit will be lower. Of course, the reduction in compensation affects the re-election chances of the initial government negatively, and, hence, also reduces the chances that reforms will be implemented. Given that the results in the proposition are only formally confirmed for p sufficiently close to 1, for $p < 1$ we confirm them numerically for the setting and parameter combinations described in Appendix 3.F (for both $k = 0$ and $k = 0.2$).

3.5.3 Reduced income inequality

The degree of income inequality is often thought to affect the political acceptability of structural reforms. This may be relevant for the European Union, especially after the entry of the new members, which are characterized by relatively high income inequality, as measured by the ratio of median and average income (see Boix, 2004).[14] These countries are expected to become part of the EMU area in the foreseeable future and will then be subject to criteria of the SGP.[15]

The effects of a change in income inequality in the model are summarized in the following proposition:

[13]For two anecdotal examples for Germany that also allude to the political obstacles in the implementation of reforms, see Eichel (2004) and Schröder (2005).

[14]For a discussion of the increase in inequality in Eastern Europe, see Forster et al. (2002).

[15]In fact, the SGP requires the "pre-ins" already to submit Convergence Programs. These programs project the path for the government budget until the medium run.

Proposition 3.3 *Suppose that average income $\bar{y}$is held constant. Then, for p sufficiently large (and, in particular, if $p = 1$), a reduction in income inequality, as measured by an increase in median income, (a) reduces compensation for reform η, (b) reduces the deficit after reform and (c) has an ambiguous effect on the likelihood of re-election of the initial incumbent.*

Proof. Appendices 3.D.5 and 3.D.6 provide the proofs. ■

The intuition for parts (a) and (b) is as follows. For a richer median voter, reform becomes acceptable at a lower level of compensation. Hence, if reform takes place, the government offers less compensation, which improves the public budget in period 1 and, thus enables it to run a lower deficit. As far as part (c) is concerned, we notice that, on the one hand, an increase in the median voter's income renders reform more attractive to this voter, ceteris paribus. On the other hand, the reduced compensation given to the voters makes reform and thus re-election of the incumbent less attractive. The results in parts (a) and (b) of Proposition 3.3 are confirmed by a numerical evaluation based on the parameter combinations and setting described in Appendix 3.F. Moreover, this numerical evaluation shows that a reduction in income inequality reduces the likelihood that the initial incumbent is re-elected and that reform takes place.

3.6 Welfare effects

In this section we confine ourselves to linear private consumption utility and assume that $(1-\theta)\,\bar{y}\Gamma > I\gamma$, implying that reform is beneficial from a utilitarian social welfare perspective. By focussing on linear private consumption utility, we abstract from distributional issues and the concern is only with the "size of the cake" for private consumption. Distributional issues are beyond the scope of this paper in any case, so that with this specification not too much is lost in terms of the intuitions that we want to highlight. The concavity of public goods consumption *is* important here, though, because restrictions on deficits only make sense when deficits are harmful. That is, when the implied shift in the intertemporal public spending profile is harmful. For this to be the case, public consumption utility should be non-linear.

Taking a utilitarian perspective, hence each individual receives an equal weight, expected social welfare can be written as (see Appendix 3.E):

$$
\begin{aligned}
&2\,(1-\theta)\,\bar{y} + \left[\frac{\bar{\varepsilon}-\varepsilon_L}{2\bar{\varepsilon}}\right]\left[(\eta - I)\,\gamma + (1-\theta)\,\Gamma\bar{y}\right] + \frac{1}{4\bar{\varepsilon}}\left(\bar{\varepsilon}^2 - \varepsilon_L^2\right)(1-\theta)\,\Gamma\bar{y} + \\
&\frac{\lambda}{2\bar{\varepsilon}}\int_{-\bar{\varepsilon}}^{\varepsilon_L}\left[v\left(g_1^{NR}\right) + pv\left(g_2^{NR}\right) + (1-p)\,v\left(f_2^{NR}\right)\right]d\varepsilon + \\
&\frac{\lambda}{2\bar{\varepsilon}}\int_{\varepsilon_L}^{\bar{\varepsilon}}\left[v\left(f_1^{R}\right) + pv\left(f_2^{R}\right) + (1-p)\,v\left(g_2^{R}\right)\right]d\varepsilon. \qquad (3.24)
\end{aligned}
$$

The first line of this expression is expected private consumption over the two periods. The second line is society's expected utility from public goods consumption when party G comes to office and thus reforms do not take place. Party G stays in office in the second period with probability p and loses it again to F with probability $1-p$. The corresponding second period utilities from public consumption are given by the second and third term, respectively, in this line. Similarly, the third line of (3.24) is society's expected utility from public goods consumption when party F retains office after the first election.

We explore the consequences of tighter enforcement of the deficit restriction (a higher k) on social welfare. The social welfare effect can be separated into an effect on private consumption utility and an effect on public consumption utility. Crucial for the welfare effect is the effect of the aforementioned changes on the probability of re-election of the initial incumbent and thus the probability that reform takes place. This probability of re-election (reform) is increasing in the amount of compensation. Any exogenous change leading to an increase in compensation has a positive effect on the utility from private consumption for two reasons. First, an increase in compensation raises consumption when reform does actually take place. Second, average consumption is higher under reform than under no reform, while the increase in compensation raises the chance that reform indeed takes place (notice that the increase in η reduces ε_L, implying that both the second and the third term in the first line increase).[16]

A priori, the implications of an increase in compensation (and thus of the likelihood of reform) on the expected utility from public spending are ambiguous. On the one hand, more compensation for private agents means that there are less resources for public consumption when reform actually takes. On the other hand, if a reform leads to an increase in resources for public consumption,[17] then the higher likelihood of reform raises the expected utility from public consumption.

As we saw earlier from Proposition 3.1, a tighter deficit restriction implies a reduction in the likelihood of reform when p is sufficiently large, while this is confirmed numerically for a wide range of parameter combinations when $p < 1$. Hence, through the effect on the reform likelihood the increase in k impacts negatively on private consumption utility and in an ambiguous manner on the utility from public consumption. However, as is clear from (3.24), the change in k affects these welfare components not only through the probability of reform. Most importantly, a tighter deficit restriction can have a separate, beneficial effect on public consumption utility by reducing the deficit bias and ensuring a public consumption profile that is better balanced over time.

[16]Indeed, more formally, taking the derivative of the first line of (3.24) with respect to η yields:

$$-\frac{1}{2\overline{\varepsilon}}\left[(\eta - I)\,\gamma + (1-\theta)\,\Gamma\,(\gamma)\,\overline{y}\right]\frac{d\varepsilon_L}{d\eta} + \frac{\overline{\varepsilon}-\varepsilon_L}{2\overline{\varepsilon}}\gamma - \frac{1}{2\overline{\varepsilon}}\,(1-\theta)\,\Gamma\,(\gamma)\,\overline{y}\frac{d\varepsilon_L}{d\eta}.$$

Given that $d\varepsilon_L/d\eta < 0$ and that $(\eta - I)\,\gamma + (1-\theta)\,\Gamma\,(\gamma)\,\overline{y} > 0$, by assumption, for linear private consumption utility, this expression is positive.

[17]The present discounted values of public resources under no reform and reform are $2\theta\overline{y}\,(1+\varepsilon)$ and $2\theta\overline{y}\,(1+\varepsilon) + \theta\overline{y}\,(1+\varepsilon)\,\Gamma - \eta\gamma$, respectively. Hence, public resources benefit from reform if $\theta\overline{y}\,(1+\varepsilon)\,\Gamma > \eta\gamma$.

To assess the overall effect of changes in k on the social welfare components, we resort to a numerical analysis based on the following functional specifications for public consumption in both periods, respectively the additional income effect Γ in the second period:

$$v(x) = \omega\left[-(\xi-1)x^2/2+\xi x\right], \quad \xi>1 \text{ and } x<\xi/(\xi-1), \tag{3.25}$$
$$\Gamma(\gamma) = \gamma^q, \quad 0 \leqslant q \leqslant 1. \tag{3.26}$$

The restriction $x<\xi/(\xi-1)$ in (3.25) ensures that the marginal utilities of public consumption are always positive. Further, the parameter ω regulates the desirability of public spending relative to private consumption.[18] The baseline parameter combination is $\xi=1.03$, $\overline{y}=10$, $p=\theta=\gamma=0.5$, $I=2$, $q=0.25$, $\overline{\varepsilon}=1.2$, $y^m=0.8y$ and $\overline{b}=0.3$. It is selected to be roughly in line with the estimated benefits of structural reform in Europe (see IMF, 2003) and evidence on the magnitude of business cycle fluctuations for Europe (Artis et al., 2003, and Mitchell and Mouratidis, 2004). More detail is provided in Appendix 3.F.

Table 3.1 shows the effects of an increase in the tightness of the deficit restriction, both for the baseline parameter combination and for settings in which we vary each time one parameter away from its baseline value. We consider large perturbations of parameter values in order to cover a wide range of the parameter space.[19] As argued above, an increase in k in all instances leads to a reduction in the probability of re-election of the initial incumbent (and thus of reform) and a reduction in the utility from private consumption. We also see that, with one exception, in all instances, an increase in k has a positive effect on the utility from public consumption, indicating that the better intertemporal smoothing of public consumption (and the reduction in compensation) outweighs the potential reduction in available resources for public consumption associated with less frequent reform. We do not sum the effects on private and public consumption utility into an overall effect of a change in k on expected social welfare, because the relative weights of the two welfare components depend on λ and ω. Unless one could make specific assumptions about the values of these parameters, the resulting assessment would not be very meaningful.

Table 3.1 also shows whether the deficit in each case exceeds the corresponding deficit (3.13) under the social planner, given the outcome for compensation. We see that this is always the case, except when $p=0.8$ and $k=0.2$. With the re-election probability so high, the incentive for shifting spending from period 2 towards period 1 is already relatively weak. Adding to this the pressure of a sufficiently high value of k, the bias towards a too high deficit may actually turn into an underbias for the deficit.[20] An increase in k worsens

[18] For given values of the other parameters, ω can always be chosen such that the condition in Footnote 10 holds.

[19] As a robustness check, we repeated this numerical assessment for all possible *combinations* of parameter deviations and found that the effects of an increase in k generally correspond to those we report in Table 3.1.

[20] Although not reported in Table 3.1, the transition from $k=0$ to $k=0.2$ in all instances leads to a

the underbias and further distorts the time profile of public spending, thereby leading to a reduction in the utility from public consumption, as reported in Table 3.1.

3.7 Conclusion

This paper has explored the incentives for a government to implement structural reform in the presence of electoral uncertainty and a deficit restriction that reduces the scope for providing short-run compensation to the losers from the reform. Such compensation impacts on the government's budget and makes it more difficult to obey the deficit restriction. In solving for the optimal level of compensation and thus the likelihood of reform actually taking place, a trade-off arises between the marginal benefits of compensation in terms of better re-election chances for a government willing to execute reforms and the marginal costs associated with an increase in the deficit. While the deficit restriction is effective in restraining the actual public deficit, it reduces the likelihood of structural reform by forcing a reduction in compensation spending for the losers from the reform. As a result, social welfare may be negatively affected because the future reform benefits are more likely to be foregone. We also find that more individual income uncertainty reduces the likelihood of reform, because the range of shocks for which compensation payments need to be made becomes wider, forcing the initial incumbent to promise lower compensation for when it is actually re-elected.

This result suggests that the political feasibility of reforms is enhanced by making the deficit restriction contingent on the business cycle. In an extension which, for the sake of space, we did not analyze in the main text, we also explored an arrangement in which the reference deficit level was made contingent on both compensation spending and the business cycle shock.[21] We found that both types of contingency make reform more likely. Of course, such an arrangement would be more demanding in terms of the budgetary and macroeconomic information needed to take informed decisions for such a more flexible implementation of the deficit restriction. In addition, and in relation to this, mechanisms would need to be devised to ensure that governments do not abuse the flexibility of the fiscal restriction as an escape route for their lack of discipline (for example, by classifying government consumption as reform expenditure).

Appendix 3.G analyzes two additional extensions. In one of them, we make reform compensation income dependent and assume that it is targeted at those voters that as a result of the compensation are likely to switch from being opponents to being supporters of the reform. For given overall compensation costs, this would enable the incumbent to pursuade more voters to support its reforms. Our findings remain qualitatively unchanged. The other extension assumes that the deficit restriction had a direct budgetary effect rather than an effect on the government's utility. Also in this case, our main findings remain

reduction in the difference between the partisan deficit and the social planner's deficit, given the level of compensation.

[21] This, and other, extensions are analyzed in Appendix 3.G.

unchanged.

Our analysis is relevant for the current debate about the relation between structural reforms and Europe's SGP. While there is a consensus about the need to meet the objectives set by the Lisbon Agenda, most European countries fail in implementing the necessary structural reforms. Part of the blame is often put on the unpopularity of these structural reforms and the difficulty of carrying them out when at the same time countries are bound by the fiscal restrictions embedded in the SGP. After its reform in 2005, the preventive arm of the SGP now allows for deviations from the normal adjustment path to the medium term deficit objective (MTO). In the corrective arm (which specifies possible sanctions), structural reforms are explicitly mentioned as a relevant factor permitting a freeze or at least a slowdown of the Excessive Deficit Procedure for countries violating the deficit criterion.

Of course, one needs to be careful in translating the preceding analysis into the context of the SGP. The SGP is implemented in a setting in which countries have given up their monetary autonomy, but have retained their fiscal autonomy. Many experts view the SGP as an alternative for the missing formal fiscal coordination mechanism. The benefits and costs of relaxing Europe's SGP are more far-reaching than for the deficit restriction in our model.[22] While the latter was intended to reduce the costs of an intertemporal misallocation of public spending in a national context, the SGP also aims at limiting the consequences of negative fiscal spill-overs between the members of Europe's monetary union. The benefit of a such fiscal restriction is likely to be larger than in a national context. However, enforcement is also more complicated and typically involves peer pressure from other countries. Making such an international arrangement more flexible in the way suggested above is more complicated than in a purely national context, because the information requirements on the countries' economic situations become more stringent and countries have to trust each other not to abuse the more flexible arrangement. The need for structural reforms adds another dimension of complexity, because of the international consequences of those reforms. Europe's supranational institutions, such as the ECB and the European Commission repeatedly emphasize the need for reform. By reducing the incentives for fiscal profligacy, structural reform limits the potential negative fiscal policy spill-overs between countries and benefits the conduct of a credible monetary policy. In addition, the enhanced flexibility of the labor and product markets also reduces the pressure for protectionist stances of European governments, thereby contributing to further economic integration. Obviously, a fully-fledged analysis of the interaction between fiscal restrictions and structural reforms in a monetary union should take these external spill-over effects into account.

The current analysis offers various other possibilities for further research. First, we have assumed that the tax rate was exogenous and constant. While there is some merit

[22]See Chari and Kehoe (1997) and Beetsma and Uhlig (1999) for an analysis of fiscal restrictions in a monetary union.

in this assumption as the level of taxes captures the population's preference about the size of the public sector (which is beyond the scope of this analysis), obviously in reality governments possess some freedom to change the level taxes. Introducing this possibility into our current model would reduce the degree to which the deficit restriction is binding, because the first-period government could finance its overspending on public goods by raising taxes rather than issuing debt. However, a fully-fledged analysis along this line should also take account of the fact that varying taxes over time raises the income losses from distortions (Barro, 1979) and that, ceteris paribus, major tax hikes have adverse implications for the government's re-election chances. Indeed, in reality, we often see that governments finance additional expenditure with higher deficits. We conjecture that an analysis that takes proper account of these limitations to changing taxes, would in qualitative terms reproduce our main results. Second, in this paper, we have neglected issues pertaining to the credibility of the enforcement of the deficit restriction. The recent events with the SGP make clear that this would be an important matter to address. Also, it would be interesting to extend the model to a longer horizon, so that one can address the issue of the optimal timing of the implementation of the structural reform and explore how the dynamics of the debt are then affected by a deficit restriction. Such an extension could further take account of the fact that a very strict implementation of the restriction now might undermine its future credibility, if such a strict implementation discourages reforms and thus lowers future tax revenues in this way.

3.8 Table

Table 3.1: Marginal effects of an increase in the tightness of the deficit restriction

	$k=0$				$k=0.2$			
case[1]	$\frac{d\Pr(R)}{dk}$	$\frac{dVc}{dk}$	$\frac{dVp}{dk}$	Δb^R	$\frac{d\Pr(R)}{dk}$	$\frac{dVc}{dk}$	$\frac{dVp}{dk}$	Δb^R
Baseline	-	-	+	>0	-	-	+	>0
$p=0.35$	-	-	+	>0	-	-	+	>0
$p=0.80$	-	-	+	>0	-	-	-	<0
$\theta=0.25$	-	-	+	>0	-	-	+	>0
$\theta=0.7$	-	-	+	>0	-	-	+	>0
$\gamma=0.1$	-	-	+	>0	-	-	+	>0
γ=2	-	-	+	>0	-	-	+	>0
$I=0.5$	-	-	+	>0	-	-	+	>0
$I=4$	-	-	+	>0	-	-	+	>0
$q=0.1$	-	-	+	>0	-	-	+	>0
$q=0.5$	-	-	+	>0	-	-	+	>0
$y^m=0.5y$	-	-	+	>0	-	-	+	>0
$y^m=0.95y$	-	-	+	>0	-	-	+	>0
$\overline{\varepsilon}=0.8$	-	-	+	>0	-	-	+	>0
$\overline{\varepsilon}=1.6$	-	-	+	>0	-	-	+	>0
$b=0.1$	-	-	+	>0	-	-	+	>0
$b=1$	-	-	+	>0	-	-	+	>0

Notes: [1] We always vary one parameter (indicated in the first column), while keeping the others at their baseline values. $\frac{d\Pr(R)}{dk}$ = marginal effect of k on the probability of reform. $\frac{dVc}{dk}$ = marginal effect of k on private consumption utility. $\frac{dVp}{dk}$ = marginal effect of k on public consumption utility. Δb^R = difference under reform between deficit under partisan government and under social planner.

Appendices to Chapter 3

3.A Optimal deficit under re-election of initial incumbent in period 1

After the first election, if the initial incumbent remains in power, it chooses the public debt b^R in order to maximize:

$$\max_{b^R} v\left(f_1^R\right) + pv\left(f_2^R\right) + (1-p)\, v\,(0) - k\left(b^R - \overline{b}\right).$$

The first-order condition is written as follows:

$$v'\left(\theta\overline{y}(1+\varepsilon) - \eta\gamma + b^R\right) - k = pv'\left(\theta\overline{y}(1+\varepsilon)\left(1+\Gamma\left(\gamma\right)\right) - b^R\right). \tag{3.27}$$

By differentiating (3.27), we obtain:

$$\frac{\partial b^R}{\partial k} = \frac{1}{v"\left(f_1^R\right) + pv"\left(f_2^R\right)} < 0, \tag{3.28}$$

$$\frac{\partial b^R}{\partial \eta} = \frac{\gamma v"\left(f_1^R\right)}{v"\left(f_1^R\right) + pv"\left(f_2^R\right)} > 0, \tag{3.29}$$

$$\frac{\partial b^R}{\partial y^m}|_{y=\overline{y}} = 0, \tag{3.30}$$

$$\frac{\partial b^R}{\partial \gamma} = \frac{pv"\left(f_2^R\right)\left(\theta\overline{y}(1+\varepsilon)\Gamma'\left(\gamma\right)\right) + \eta v"\left(f_1^R\right)}{v"\left(f_1^R\right) + pv"\left(f_2^R\right)} > 0,$$

$$\frac{\partial b^R}{\partial p} = \frac{v'\left(f_2^R\right)}{v"\left(f_1^R\right) + pv"\left(f_2^R\right)} < 0.$$

3.B Optimal deficit under no re-election in period 1

In case of no re-election, the new party on power G chooses public debt b^{NR} in order to maximize:

$$\max_{b^{NR}} v\left(g_1^{NR}\right) + pv\left(g_2^{NR}\right) + (1-p)\, v\,(0) - k\left(b^{NR} - \overline{b}\right).$$

The first-order condition is written as follows:

$$v'\left(\theta\overline{y}(1+\varepsilon) + b^{NR}\right) - k = pv'\left(\theta\overline{y}(1+\varepsilon) - b^{NR}\right). \tag{3.31}$$

By differentiating (3.31), we obtain:

$$\frac{\partial b^{NR}}{\partial k} = \frac{1}{v"\left(g_1^{NR}\right) + pv"\left(g_2^{NR}\right)} < 0, \tag{3.32}$$

$$\frac{\partial b^{NR}}{\partial \eta} = 0, \tag{3.33}$$

$$\frac{\partial b^{NR}}{\partial y^m}|_{y=\overline{y}} = 0, \tag{3.34}$$

$$\frac{\partial b^{NR}}{\partial \gamma} = 0,$$

$$\frac{\partial b^{NR}}{\partial p} = \frac{v'\left(g_2^{NR}\right)}{v"\left(g_1^{NR}\right) + pv"\left(g_2^{NR}\right)} < 0.$$

3.C Choice of η

Before one starts to solve the maximization problem given in (3.18), it is useful to compute the conditional probability density functions for ε when $\varepsilon \geqslant \varepsilon_L$ or $\varepsilon < \varepsilon_L$. Since the shock ε is uniformly distributed, these functions are given by:

$$h_R(\varepsilon) = \begin{cases} \frac{1}{\overline{\varepsilon}-\varepsilon_L} & if \quad \varepsilon_L \leqslant \varepsilon \leqslant \overline{\varepsilon} \\ 0 & if \quad -\overline{\varepsilon} < \varepsilon \leqslant \varepsilon_L \end{cases},$$

and

$$h_{NR}(\varepsilon) = \begin{cases} 0 & if \quad \varepsilon_L < \varepsilon \leqslant \overline{\varepsilon} \\ \frac{1}{\varepsilon_L-(-\overline{\varepsilon})} & if \quad -\overline{\varepsilon} \leqslant \varepsilon \leqslant \varepsilon_L \end{cases}.$$

Using (3.16), the two expressions above can be rewritten respectively as:

$$h_R(\varepsilon) = \frac{1}{\overline{\varepsilon}-\varepsilon_L} = \frac{1}{\overline{\varepsilon} - \left(-\frac{(\eta-I)\gamma}{(1-\theta)y^m\Gamma} - 1\right)} = \frac{1}{1+\overline{\varepsilon}+\frac{(\eta-I)\gamma}{(1-\theta)y^m\Gamma}}, \tag{3.35}$$

and

$$h_{NR}(\varepsilon) = \frac{1}{\varepsilon_L-(-\overline{\varepsilon})} = \frac{1}{\left(-\frac{(\eta-I)\gamma}{(1-\theta)y^m\Gamma} - 1\right)+\overline{\varepsilon}} = \frac{1}{\overline{\varepsilon}-1-\frac{(\eta-I)\gamma}{(1-\theta)y^m\Gamma}}. \tag{3.36}$$

Using this, (3.18) can be rewritten as:

$$\max_{\eta} \left\{ \begin{array}{c} \Pr(R) \int\limits_{\varepsilon_L}^{\overline{\varepsilon}} \left\{ v\left(f_1^R\right) + pv\left(f_2^R\right) - k\left(b^R - \overline{b}\right)\right\} h_R(\varepsilon)\, d\varepsilon + \\ (1-\Pr(R)) \int\limits_{-\overline{\varepsilon}}^{\varepsilon_L} \left\{ v(0) + (1-p)\, v\left(f_2^{NR}\right)\right\} h_{NR}(\varepsilon)\, d\varepsilon \end{array} \right\}, \tag{3.37}$$

where ε_L, $h_R(\varepsilon)$, and $h_{NR}(\varepsilon)$ are given by (3.16), (3.35) and (3.36) respectively. Further, in (3.35) and (3.36) ε does not appear. So, we can take these two terms out of the integrand and compute $\Pr(Re) * h_R(\varepsilon)$ and $(1-\Pr(Re)) * h_{NR}(\varepsilon)$. Using (3.17), we have:

$$\begin{aligned}
\Pr(R)h_R(\varepsilon) &= \frac{1+\overline{\varepsilon}+\frac{(\eta-I)\gamma}{(1-\theta)y^m\Gamma}}{2\overline{\varepsilon}} * \frac{1}{1+\overline{\varepsilon}+\frac{(\eta-I)\gamma}{(1-\theta)y^m\Gamma}} = \frac{1}{2\overline{\varepsilon}} \\
& \text{and} \\
(1-\Pr(R))\,h_{NR}(\varepsilon) &= \left[1+\frac{1}{2\overline{\varepsilon}}\left(-\overline{\varepsilon}-1-\frac{(\eta-I)\,\gamma}{(1-\theta)\,y^m\Gamma}\right)\right]\left[\frac{1}{\overline{\varepsilon}-1-\frac{(\eta-I)\gamma}{(1-\theta)y^m\Gamma}}\right] \Leftrightarrow \\
(1-\Pr(R))\,h_{NR}(\varepsilon) &= \left[1+\frac{1}{2\overline{\varepsilon}}\left(\frac{(1-\theta)\,y^m\Gamma\,(-\overline{\varepsilon}-1)-(\eta-I)\,\gamma}{(1-\theta)\,y^m\Gamma}\right)\right]\left[\frac{1}{\overline{\varepsilon}-1-\frac{(\eta-I)\gamma}{(1-\theta)y^m\Gamma}}\right] \Leftrightarrow \\
(1-\Pr(R))\,h_{NR}(\varepsilon) &= \left[\frac{\begin{array}{c}2\overline{\varepsilon}\,(1-\theta)\,y^m\Gamma+ \\ (1-\theta)\,y^m\Gamma\,(-\overline{\varepsilon}-1)-(\eta-I)\,\gamma\end{array}}{2\overline{\varepsilon}\,(1-\theta)\,y^m\Gamma}\right]\left[\frac{1}{\frac{(1-\theta)y^m\Gamma(\overline{\varepsilon}-1)-(\eta-I)\gamma}{(1-\theta)y^m\Gamma}}\right] \Leftrightarrow \\
(1-\Pr(R))\,h_{NR}(\varepsilon) &= \left[\frac{(1-\theta)\,y^m\Gamma\,(2\overline{\varepsilon}-\overline{\varepsilon}-1)-(\eta-I)\,\gamma}{2\overline{\varepsilon}\,(1-\theta)\,y^m\Gamma}\right]\left[\frac{1}{\frac{(1-\theta)y^m\Gamma(\overline{\varepsilon}-1)-(\eta-I)\gamma}{(1-\theta)y^m\Gamma}}\right] \Leftrightarrow \\
(1-\Pr(R))\,h_{NR}(\varepsilon) &= \left[\frac{(1-\theta)\,y^m\Gamma\,(\overline{\varepsilon}-1)-(\eta-I)\,\gamma}{2\overline{\varepsilon}\,(1-\theta)\,y^m\Gamma}\right]\left[\frac{1}{\frac{(1-\theta)y^m\Gamma(\overline{\varepsilon}-1)-(\eta-I)\gamma}{(1-\theta)y^m\Gamma}}\right] = \frac{1}{2\overline{\varepsilon}} \\
& \Longrightarrow \\
\Pr(R)h_R(\varepsilon) &= (1-\Pr(R))\,h_{NR}(\varepsilon) = \frac{1}{2\overline{\varepsilon}}. \qquad (3.38)
\end{aligned}$$

With (3.38), we can rewrite (3.37):

$$\max_{\eta}\left\{\begin{array}{c}\frac{1}{2\overline{\varepsilon}}\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\left\{v\left(\theta\overline{y}(1+\varepsilon)-\eta\gamma+b^R\right)+pv\left(\theta\overline{y}(1+\varepsilon)\,(1+\Gamma)-b^R\right)-k\left(b^R-\overline{b}\right)\right\}d\varepsilon \\ +\frac{1}{2\overline{\varepsilon}}\int\limits_{-\overline{\varepsilon}}^{\varepsilon_L}\left\{v\,(0)+(1-p)\,v\left(\theta\overline{y}(1+\varepsilon)-b^{NR}\right)\right\}d\varepsilon\end{array}\right\}$$

$$\Leftrightarrow$$

$$\max_{\eta}\frac{1}{2\overline{\varepsilon}}\left\{\begin{array}{c}\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}v\left(\theta\overline{y}(1+\varepsilon)-\eta\gamma+b^R\right)d\varepsilon+p.\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}v\left(\theta\overline{y}(1+\varepsilon)\,(1+\Gamma)-b^R\right)d\varepsilon \\ +\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}-k\left(b^R-\overline{b}\right)d\varepsilon+\int\limits_{-\overline{\varepsilon}}^{\varepsilon_L}0d\varepsilon+(1-p)\,.\int\limits_{-\overline{\varepsilon}}^{\varepsilon_L}v\left(\theta\overline{y}(1+\varepsilon)-b^{NR}\right)d\varepsilon\end{array}\right\}.$$

Applying Leibnitz's rule[23] and maximizing this last expression on η:

$$\frac{1}{2\overline{\varepsilon}}\left\{\begin{array}{c} v\left(\theta\overline{y}(1+\overline{\varepsilon})-\eta\gamma+b^{R}\left(\overline{\varepsilon}\right)\right)\frac{\partial\overline{\varepsilon}}{\partial\eta}-v\left(\theta\overline{y}(1+\varepsilon_{L})-\eta\gamma+b^{R}\left(\varepsilon_{L}\right)\right)\frac{\partial\varepsilon_{L}}{\partial\eta}\\ +\int\limits_{\varepsilon_{L}}^{\overline{\varepsilon}}\frac{\partial v\left(\theta\overline{y}(1+\varepsilon)-\eta\gamma+b^{R}\right)}{\partial\eta}d\varepsilon+\\ p\left\{\begin{array}{c} v\left(\theta\overline{y}(1+\overline{\varepsilon})\left(1+\Gamma\right)-b^{R}\left(\overline{\varepsilon}\right)\right)\frac{\partial\overline{\varepsilon}}{\partial\eta}-v\left(\theta\overline{y}(1+\varepsilon_{L})\left(1+\Gamma\right)-b^{R}\left(\varepsilon_{L}\right)\right)\frac{\partial\varepsilon_{L}}{\partial\eta}\\ +\int\limits_{\varepsilon_{L}}^{\overline{\varepsilon}}\frac{\partial v\left(\theta\overline{y}(1+\varepsilon)(1+\Gamma)-b^{R}\right)}{\partial\eta}d\varepsilon \end{array}\right\}\\ -k\left(b^{R}\left(\overline{\varepsilon}\right)-\overline{b}\right)\frac{\partial\overline{\varepsilon}}{\partial\eta}+k\left(b^{R}\left(\varepsilon_{L}\right)-\overline{b}\right)\frac{\partial\varepsilon_{L}}{\partial\eta}+\int\limits_{\varepsilon_{L}}^{\overline{\varepsilon}}\frac{-\partial k\left(b^{R}-\overline{b}\right)}{\partial\eta}d\varepsilon+\\ (1-p)\left\{\begin{array}{c} v\left(\theta\overline{y}(1+\varepsilon_{L})-b^{NR}\left(\varepsilon_{L}\right)\right)\frac{\partial\varepsilon_{L}}{\partial\eta}-v\left(\theta\overline{y}(1-\overline{\varepsilon})-b^{NR}\left(-\overline{\varepsilon}\right)\right)\frac{\partial(-\overline{\varepsilon})}{\partial\eta}\\ +\int\limits_{-\overline{\varepsilon}}^{\varepsilon_{L}}\frac{\partial v\left(\theta\overline{y}(1+\varepsilon)-b^{NR}\right)}{\partial\eta}d\varepsilon \end{array}\right\} \end{array}\right\}=0.$$

Since $\frac{\partial\overline{\varepsilon}}{\partial\eta}=0$ and using the notation in (3.20) - (3.22), we have:

$$\left\{\begin{array}{c} -v\left(f_{1L}^{R}\right)\frac{\partial\varepsilon_{L}}{\partial\eta}+\int\limits_{\varepsilon_{L}}^{\overline{\varepsilon}}v'\left(f_{1}^{R}\right)\left(-\gamma+\frac{\partial b^{R}}{\partial\eta}\right)d\varepsilon+\\ p\left\{0-v\left(f_{2L}^{R}\right)\frac{\partial\varepsilon_{L}}{\partial\eta}+\int\limits_{\varepsilon_{L}}^{\overline{\varepsilon}}v'\left(f_{2}^{R}\right)\left(-\frac{\partial b^{R}}{\partial\eta}\right)d\varepsilon\right\}\\ -k\left(b^{R}\left(\overline{\varepsilon}\right)-\overline{b}\right).0-k\left(b_{L}^{R}-\overline{b}\right)\left(-\frac{\partial\varepsilon_{L}}{\partial\eta}\right)+\int\limits_{\varepsilon_{L}}^{\overline{\varepsilon}}-k\left(\frac{\partial b^{R}}{\partial\eta}\right)d\varepsilon\\ +(1-p)\left\{v\left(f_{2L}^{NR}\right)\frac{\partial\varepsilon_{L}}{\partial\eta}-0+\int\limits_{-\overline{\varepsilon}}^{\varepsilon_{L}}v'\left(f_{2}^{NR}\right)\left(-\frac{\partial b^{NR}}{\partial\eta}\right)d\varepsilon\right\} \end{array}\right\}=0$$

$$\Longrightarrow$$

$$\left\{\begin{array}{c} \frac{\partial\varepsilon_{L}}{\partial\eta}\left[-v\left(f_{1L}^{R}\right)-pv\left(f_{2L}^{R}\right)+k\left(b_{L}^{R}-\overline{b}\right)+(1-p)\,v\left(f_{2L}^{NR}\right)\right]\\ -\gamma\int\limits_{\varepsilon_{L}}^{\overline{\varepsilon}}v'\left(f_{1}^{R}\right)d\varepsilon+\int\limits_{\varepsilon_{L}}^{\overline{\varepsilon}}\frac{\partial b^{R}}{\partial\eta}\left[v'\left(f_{1}^{R}\right)-pv'\left(f_{2}^{R}\right)-k\right]d\varepsilon\\ +\int\limits_{-\overline{\varepsilon}}^{\varepsilon_{L}}\frac{\partial b^{NR}}{\partial\eta}\left[-(1-p)\,v'\left(f_{2}^{NR}\right)\right]d\varepsilon \end{array}\right\}=0.$$

[23]Leibnitz's rule: *Let $f(x,r)$ be continuous with respect to x for every value of r, with a continuous derivative $\frac{\partial f(x,r)}{\partial r}$ with respect to x and r in the rectangle $a\leq x\leq b$, $\underline{r}\leq r\leq\overline{r}$ of the $x-r$ plane. Let the functions $A(r)$ and $B(r)$ have continuous derivatives. If $V(r)=\int\limits_{A(r)}^{B(r)}f(x,r)\,dx$, then*

$$V'(r)=f(B(r),r)\,B'(r)-f(A(r),r)\,A'(r)+\int\limits_{A(r)}^{B(r)}\frac{\partial f(x,r)}{\partial r}dx.$$

For example,

$$\begin{aligned} V(r) &= \int\limits_{r^{2}}^{r}e^{-rs}P(s)\,ds\Longrightarrow\\ \frac{dV(r)}{dr} &= P(r)\,e^{-r^{2}}-2P\left(r^{2}\right)re^{-r^{3}}-\int\limits_{r^{2}}^{r}se^{-rs}P(s)\,ds. \end{aligned}$$

From (3.27) we know that $v'\left(f_1^R\right) - pv'\left(f_2^R\right) - k = 0$. Further, from (3.33), $\frac{\partial b^{NR}}{\partial \eta} = 0$. So, the last expression is rewritten as (3.19). Next, we can substitute

$$\frac{\partial \varepsilon_L}{\partial \eta} = \frac{\partial\left(-\frac{(\eta - I)\gamma}{(1-\theta)y^m\Gamma} - 1\right)}{\partial \eta} = -\frac{\gamma}{(1-\theta)\,y^m\Gamma} < 0, \tag{3.39}$$

into (3.19):

$$\frac{\gamma}{(1-\theta)\,y^m\Gamma}\left[v\left(f_{1L}^R\right) + pv\left(f_{2L}^R\right) - k\left(b_L^R - \bar{b}\right) - (1-p)\,v\left(f_{2L}^{NR}\right)\right] = \gamma\int_{\varepsilon_L}^{\bar{\varepsilon}} v'\left(f_1^R\right)d\varepsilon,$$

which results in (3.23).

In (3.23) both sides must have the same sign for an internal solution to exist. Clearly, the right-hand side of that expression is positive. The left-hand side is positive, since we have the deficit bias in the first period $b^R > \bar{b}$. Further, the probability of being in power in the second period p is the same for both parties and assumed to be sufficiently different from 0.

3.C.1 Proof that (3.23) has at most one solution

The derivative of the left-hand side of (3.23) with respect to η is:

$$\left\{\begin{array}{c} v'\left(f_{1L}^R\right)\left(\theta\overline{y}\frac{\partial\varepsilon_L}{\partial\eta} - \gamma + \frac{\partial b_L^R}{\partial\eta}\right) + pv'\left(f_{2L}^R\right)\left(\theta\overline{y}\left(1+\Gamma\right)\frac{\partial\varepsilon_L}{\partial\eta} - \frac{\partial b_L^R}{\partial\eta}\right) \\ -k\left(\frac{\partial b_L^R}{\partial\eta}\right) - (1-p)\,v'\left(f_{2L}^{NR}\right)\left(\theta\overline{y}\frac{\partial\varepsilon_L}{\partial\eta} - \frac{\partial b_L^{NR}}{\partial\eta}\right)\end{array}\right\}$$

$$= \left\{\begin{array}{c} \frac{\partial b_L^R}{\partial\eta}\left[v'\left(f_{1L}^R\right) - pv'\left(f_{2L}^R\right) - k\right] + \frac{\partial b_L^{NR}}{\partial\eta}\left[(1-p)\,v'\left(f_2^{NR}\right)\right] - \gamma v'\left(f_{1L}^R\right) \\ +\theta\overline{y}\frac{\partial\varepsilon_L}{\partial\eta}\left[v'\left(f_{1L}^R\right) + (1+\Gamma)\,pv'\left(f_{2L}^R\right) - (1-p)\,v'\left(f_{2L}^{NR}\right)\right]\end{array}\right\}.$$

We know that $v'\left(f_{1L}^R\right) - pv'\left(f_{2L}^R\right) - k = 0$ since (3.27) is valid for any realization of ε in the interval $(\varepsilon_L, \bar{\varepsilon})$. Further, by (3.39) and if we assume that p is sufficiently different from 0, we can rewrite the last expression as:

$$\left\{\begin{array}{c} +(1-p)\,v'\left(f_2^{NR}\right)\frac{\partial b_L^{NR}}{\partial\eta} - \gamma v'\left(f_{1L}^R\right) \\ -\frac{\theta\overline{y}\gamma}{(1-\theta)y^m\Gamma}\left[v'\left(f_{1L}^R\right) + (1+\Gamma)\,pv'\left(f_{2L}^R\right) - (1-p)\,v'\left(f_{2L}^{NR}\right)\right]\end{array}\right\} < 0.$$

Hence, an increase in η reduces the left-hand side of (3.23). The derivative of the right side of (3.23) with respect to η is:

$$(1-\theta)\,y^m\Gamma\frac{\partial}{\partial\eta}\left[\int_{\varepsilon_L}^{\bar{\varepsilon}} v'\left(\theta\overline{y}(1+\varepsilon) - \eta\gamma + b^R\right)d\varepsilon\right].$$

Using again Leibnitz's rule, we can write this as:

$$(1-\theta)\,y^m\Gamma\left[\begin{array}{c} v'\left(\theta\overline{y}(1+\bar{\varepsilon}) - \eta\gamma + b^R\left(\bar{\varepsilon}\right)\right)\frac{\partial\bar{\varepsilon}}{\partial\eta} - v'\left(\theta\overline{y}(1+\varepsilon_L) - \eta\gamma + b^R\left(\varepsilon_L\right)\right)\frac{\partial\varepsilon_L}{\partial\eta} \\ +\int_{\varepsilon_L}^{\bar{\varepsilon}} v''\left(\theta\overline{y}(1+\varepsilon) - \eta\gamma + b^R\right)\left(-\gamma + \frac{\partial b^R}{\partial\eta}\right)d\varepsilon\end{array}\right]$$

$$\Leftrightarrow\;(1-\theta)\,y^m\Gamma\left[-v'\left(f_{1L}^R\right)\frac{\partial\varepsilon_L}{\partial\eta} + \int_{\varepsilon_L}^{\bar{\varepsilon}}\left(-\gamma + \frac{\partial b^R}{\partial\eta}\right)v''\left(f_1^R\right)d\varepsilon\right].$$

Substituting (3.29) and (3.39) to this last expression, we get:

$$(1-\theta)\,y^m\Gamma\left\{-v'\left(f_{1L}^R\right)\left[-\frac{\gamma}{(1-\theta)\,y^m\Gamma}\right]+\int_{\varepsilon_L}^{\overline{\varepsilon}}\left[-\gamma+\frac{\gamma.v"\left(f_1^R\right)}{v"\left(f_1^R\right)+pv"\left(f_2^R\right)}\right]v"\left(f_1^R\right)d\varepsilon\right\}\Leftrightarrow$$

$$\gamma\left\{v\prime\left(f_{1L}^R\right)+(1-\theta)\,y^m\Gamma\int_{\varepsilon_L}^{\overline{\varepsilon}}\left[\frac{-pv"\left(f_2^R\right)}{v"\left(f_1^R\right)+pv"\left(f_2^R\right)}\right]v"\left(f_1^R\right)d\varepsilon\right\}\quad>\quad 0.$$

Hence, the right-hand side of (3.23) is positive. This shows that η has an maximum value.

3.D Comparative statics

3.D.1 Effect of k on η, b^R and b^{NR}

FOC of η

We differentiate (3.23) with respect to k using Leibnitz's rule and the notations (3.20) to (3.22):

$$\begin{aligned}
&v'\left(f_{1L}^R\right)\left[\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial k}+\frac{\partial\varepsilon_L}{\partial k}\right)-\gamma\frac{\partial\eta}{\partial k}+\frac{\partial b_L^R}{\partial k}\right]+\\
&pv'\left(f_{2L}^R\right)\left[\theta\overline{y}\,(1+\Gamma)\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial k}+\frac{\partial\varepsilon_L}{\partial k}\right)-\frac{\partial b_L^R}{\partial k}\right]-\left(b_L^R-\overline{b}\right)\\
&-k\left(\frac{\partial b_L^R}{\partial k}\right)-(1-p)\,v'\left(f_{2L}^{NR}\right)\left[\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial k}+\frac{\partial\varepsilon_L}{\partial k}\right)-\frac{\partial b_L^{NR}}{\partial k}\right]-\\
&(1-\theta)\,y^m\Gamma\left[-v'\left(f_{1L}^R\right)\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial k}+\frac{\partial\varepsilon_L}{\partial k}\right)+\int_{\varepsilon_L}^{\overline{\varepsilon}}v"\left(f_1^R\right)\left(-\gamma\frac{\partial\eta}{\partial k}+\frac{\partial b^R}{\partial k}\right)d\varepsilon\right]=0.
\end{aligned}$$

This expression can be written as:

$$\begin{aligned}
&\frac{\partial\eta}{\partial k}\left\{\theta\overline{y}\frac{\partial\varepsilon_L}{\partial\eta}\left[\begin{array}{c}v'\left(f_{1L}^R\right)+(1+\Gamma)\,pv'\left(f_{2L}^R\right)\\-(1-p)\,v'\left(f_{2L}^{NR}\right)\end{array}\right]-\gamma v'\left(f_{1L}^R\right)\right\}+\frac{\partial b_L^{NR}}{\partial k}v'\left(f_{2L}^{NR}\right)(1-p)\\
&+\theta\overline{y}\frac{\partial\varepsilon_L}{\partial k}\left[v'\left(f_{1L}^R\right)+(1+\Gamma)\,pv'\left(f_{2L}^R\right)-(1-p)\,v'\left(f_{2L}^{NR}\right)\right]+\frac{\partial b_L^R}{\partial k}\left[v'\left(f_{1L}^R\right)-pv'\left(f_{2L}^R\right)-k\right]\\
&-(1-\theta)\,y^m\Gamma\left[-v'\left(f_{1L}^R\right)\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial k}+\frac{\partial\varepsilon_L}{\partial k}\right)+\int_{\varepsilon_L}^{\overline{\varepsilon}}v"\left[f_1^R\right]\left(-\gamma\frac{\partial\eta}{\partial k}+\frac{\partial b^R}{\partial k}\right)d\varepsilon\right]=\left(b_L^R-\overline{b}\right).
\end{aligned}$$

The first and second terms on the previous expression show the effect of k on the variables η and b^{NR} respectively, evaluated at $\varepsilon=\varepsilon_L$. The second line shows the direct effect of k on ε_L and b^R. This line disappears since $\frac{\partial\varepsilon_L}{\partial k}=0$ and (3.27) is valid for any realization of ε in the interval $(\varepsilon_L,\overline{\varepsilon})$. Therefore, using (3.39), and if we assume that p is sufficiently different from 0, we can simplify this last expression to:

$$\left\{\begin{array}{c} \frac{\partial\eta}{\partial k}\left\{\theta\overline{y}\left(-\frac{\gamma}{(1-\theta)y^m\Gamma}\right)\left[\begin{array}{c} v'\left(f_{1L}^R\right)+(1+\Gamma)\,pv'\left(f_{2L}^R\right) \\ -(1-p)\,v'\left(f_{2L}^{NR}\right)\end{array}\right]-\gamma v'\left(f_{1L}^R\right)\right\} \\ +\frac{\partial b_L^{NR}}{\partial k}\left[v'\left(f_{2L}^{NR}\right)(1-p)\right]-(1-\theta)\,y^m\Gamma\left[\begin{array}{c} -v'\left(f_{1L}^R\right)\left(-\frac{\gamma}{(1-\theta)y^m\Gamma}\right)\frac{\partial\eta}{\partial k} \\ +\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma\frac{\partial\eta}{\partial k}+\frac{\partial b^R}{\partial k}\right)v"\left(f_1^R\right)d\varepsilon\end{array}\right]\end{array}\right\} = \left(b_L^R-\overline{b}\right)$$

$$\Leftrightarrow$$

$$\left\{\begin{array}{c} \frac{\partial\eta}{\partial k}\left\{-\frac{\theta\overline{y}\gamma}{(1-\theta)y^m\Gamma}\left[\begin{array}{c} v'\left(f_{1L}^R\right)+(1+\Gamma)\,pv'\left(f_{2L}^R\right) \\ -(1-p)\,v'\left(f_{2L}^{NR}\right)\end{array}\right]-2\gamma v'\left(f_{1L}^R\right)\right\} \\ -(1-\theta)\,y^m\Gamma\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma\frac{\partial\eta}{\partial k}+\frac{\partial b^R}{\partial k}\right)v"\left(f_1^R\right)d\varepsilon+\frac{\partial b_L^{NR}}{\partial k}\left[v'\left(f_{2L}^{NR}\right)(1-p)\right]\end{array}\right\} = \left(b_L^R-\overline{b}\right),$$

which can be represented by:

$$A_{1L}\frac{\partial\eta}{\partial k}-A_2\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma\frac{\partial\eta}{\partial k}+\frac{\partial b^R}{\partial k}\right)v"\left(f_1^R\right)d\varepsilon+A_{3L}\frac{\partial b_L^{NR}}{\partial k}=D_{1L}, \tag{3.40}$$

where:[24]

$$A_{1L} = -\frac{\theta\overline{y}\gamma}{(1-\theta)\,y^m\Gamma}\left[\begin{array}{c} v'\left(f_{1L}^R\right)+(1+\Gamma)\,pv'\left(f_{2L}^R\right) \\ -(1-p)\,v'\left(f_{2L}^{NR}\right)\end{array}\right]-2\gamma v'\left(f_{1L}^R\right), \tag{3.41}$$

$$A_2 = (1-\theta)\,y^m\Gamma>0, \tag{3.42}$$

$$A_{3L} = v'\left(f_{2L}^{NR}\right)(1-p)>0, \tag{3.43}$$

$$D_{1L} = b_L^R-\overline{b}>0. \tag{3.44}$$

We use a subscript L to indicate that the evaluation takes place at $\varepsilon=\varepsilon_L$. The reason is that the effect of k on the endogenous variable η will affect the probability of re-election via shifts in the inferior support of the distribution, ε_L. The sign of A_{1L} is generally ambiguous. However, for p sufficiently large, this ambiguity vanishes and $A_{1L}<0$.

FOC of b^R

Differentiating (3.27) with respect to k, we have:

$$\begin{aligned} v"\left(f_1^R\right)\left(-\gamma\frac{\partial\eta}{\partial k}+\frac{\partial b^R}{\partial k}\right)-1-pv"\left(f_2^R\right)\left(-\frac{\partial b^R}{\partial k}\right) &= 0\Leftrightarrow \\ \frac{\partial\eta}{\partial k}\left[-\gamma v"\left(f_1^R\right)\right]+\frac{\partial b^R}{\partial k}\left[v"\left(f_1^R\right)+pv"\left(f_2^R\right)\right] &= 1. \end{aligned}$$

This last expression can be represented by:

$$B_1\frac{\partial\eta}{\partial k}+B_2\frac{\partial b^R}{\partial k}=D_2, \tag{3.45}$$

[24] If p is sufficiently different from 0.

where:

$$B_1 = -\gamma v"\left(f_1^R\right) > 0, \quad (3.46)$$
$$B_2 = v"\left(f_1^R\right) + pv"\left(f_2^R\right) < 0, \quad (3.47)$$
$$D_2 = 1 > 0. \quad (3.48)$$

FOC of b^{NR}

When the government chooses the optimal η, it takes into account b^{NR} evaluated at $\varepsilon = \varepsilon_L$. Therefore, (3.31) evaluated at $\varepsilon = \varepsilon_L$ yields:

$$v'\left(\theta\overline{y}(1+\varepsilon_L) + b_L^{NR}\right) - k = pv'\left(\theta\overline{y}(1+\varepsilon_L) - b_L^{NR}\right). \quad (3.49)$$

Differentiating (3.49) with respect to k:

$$v"\left(g_{1L}^{NR}\right)\left[\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial k} + \frac{\partial\varepsilon_L}{\partial k}\right) + \frac{\partial b_L^{NR}}{\partial k}\right] - 1 - pv"\left(g_{2L}^{NR}\right)\left[\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial k} + \frac{\partial\varepsilon_L}{\partial k}\right) - \frac{\partial b_L^{NR}}{\partial k}\right] = 0 \Leftrightarrow$$
$$\frac{\partial\eta}{\partial k}\left\{\theta\overline{y}\frac{\partial\varepsilon_L}{\partial\eta}\left[v"\left(g_{1L}^{NR}\right) - pv"\left(g_{2L}^{NR}\right)\right]\right\} + \frac{\partial b_L^{NR}}{\partial k}\left[v"\left(g_{1L}^{NR}\right) + pv"\left(g_{2L}^{NR}\right)\right] = 1,$$

since $\frac{\partial\varepsilon_L}{\partial k} = 0$ and where:

$$g_{1L}^{NR} = \theta\overline{y}(1+\varepsilon_L) + b^{NR}(\varepsilon_L), \quad (3.50)$$
$$g_{2L}^{NR} = \theta\overline{y}(1+\varepsilon_L) - b^{NR}(\varepsilon_L). \quad (3.51)$$

Using (3.39), we can rewrite the last expression as:

$$\frac{\partial\eta}{\partial k}\left[-\frac{\theta\overline{y}\gamma}{(1-\theta)\,y^m\Gamma[\gamma]}\left(v"\left(g_{1L}^{NR}\right) - pv"\left(g_{2L}^{NR}\right)\right)\right] + \frac{\partial b_L^{NR}}{\partial k}\left[v"\left(g_{1L}^{NR}\right) + pv"\left(g_{2L}^{NR}\right)\right] = 1,$$

which can be represented as:

$$C_{1L}\frac{\partial\eta}{\partial k} + C_{2L}\frac{\partial b_L^{NR}}{\partial k} = D_3, \quad (3.52)$$

where

$$C_{1L} = -\frac{\theta\overline{y}\gamma}{(1-\theta)\,y^m\Gamma[\gamma]}\left(v"\left(g_{1L}^{NR}\right) - pv"\left(g_{2L}^{NR}\right)\right) \gtrless 0, \quad (3.53)$$
$$C_{2L} = v"\left(g_{1L}^{NR}\right) + pv"\left(g_{2L}^{NR}\right) < 0, \text{ and} \quad (3.54)$$
$$D_3 = 1 > 0. \quad (3.55)$$

The sign of (3.53) is ambiguous, but if we assume a quadratic utility function for the public good, for example, it is positive.

Solution of the system

We can write (3.40), (3.45) and (3.52) as the system:

$$\begin{cases} A_{1L}\frac{\partial\eta}{\partial k} & +0 & -A_2\int_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma\frac{\partial\eta}{\partial k}+\frac{\partial b^R}{\partial k}\right)v"\left(f_1^R\right)d\varepsilon & +A_{3L}\frac{\partial b_L^{NR}}{\partial k} & = & D_{1L} \\ B_1\frac{\partial\eta}{\partial k} & +B_2\frac{\partial b^R}{\partial k} & +0 & +0 & = & D_2 \\ C_{1L}\frac{\partial\eta}{\partial k} & +0 & +0 & +C_{2L}\frac{\partial b_L^{NR}}{\partial k} & = & D_3 \end{cases}.$$

Further, we can rewrite (3.45) and (3.52) as, respectively

$$\frac{\partial b^R}{\partial k} = \frac{D_2}{B_2}-\frac{B_1}{B_2}\frac{\partial\eta}{\partial k}, \tag{3.56}$$

$$\frac{\partial b_L^{NR}}{\partial k} = \frac{D_3}{C_{2L}}-\frac{C_{1L}}{C_{2L}}\frac{\partial\eta}{\partial k}. \tag{3.57}$$

Substituting these two equations in (3.40), we have:

$$A_{1L}\frac{\partial\eta}{\partial k}-A_2\int_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma\frac{\partial\eta}{\partial k}+\frac{D_2}{B_2}-\frac{B_1}{B_2}\frac{\partial\eta}{\partial k}\right)v"\left(f_1^R\right)d\varepsilon-\frac{A_{3L}C_{1L}}{C_{2L}}\frac{\partial\eta}{\partial k}=D_{1L}-\frac{A_{3L}D_3}{C_{2L}}\Leftrightarrow$$

$$A_{1L}\frac{\partial\eta}{\partial k}-A_2\int_{\varepsilon_L}^{\overline{\varepsilon}}\frac{\partial\eta}{\partial k}\left(-\gamma-\frac{B_1}{B_2}\right)v"\left(f_1^R\right)d\varepsilon-\frac{A_{3L}C_{1L}}{C_{2L}}\frac{\partial\eta}{\partial k}=D_{1L}+A_2\int_{\varepsilon_L}^{\overline{\varepsilon}}\frac{D_2}{B_2}v"\left(f_1^R\right)d\varepsilon-\frac{A_{3L}D_3}{C_{2L}}. \tag{3.58}$$

Sign of $\frac{\partial\eta}{\partial k}$: Substituting the values of (3.41) to (3.44), (3.46)-(3.48) and (3.53)-(3.55), we can rewrite (3.58) as:

$$\frac{\partial\eta}{\partial k}\left\{\begin{array}{c} A_{1L}-\frac{A_{3L}C_{1L}}{C_{2L}}- \\ A_2\int_{\varepsilon_L}^{\overline{\varepsilon}}\left[-\gamma v"\left(f_1^R\right)+\frac{\gamma\left(v"\left(f_1^R\right)\right)^2}{v"\left(f_1^R\right)+pv"\left(f_2^R\right)}\right]d\varepsilon \end{array}\right\}=\left\{\begin{array}{c} D_{1L}+A_2\int_{\varepsilon_L}^{\overline{\varepsilon}}\frac{D_2}{B_2}v"\left(f_1^R\right)d\varepsilon \\ -\frac{A_{3L}D_3}{C_{2L}} \end{array}\right\}\Leftrightarrow$$

$$\frac{\partial\eta}{\partial k}\left\{\begin{array}{c} A_{1L}-\frac{A_{3L}C_{1L}}{C_{2L}}- \\ A_2\int_{\varepsilon_L}^{\overline{\varepsilon}}\left[\frac{-\gamma\left(v"\left(f_1^R\right)\right)^2-\gamma v"\left(f_1^R\right)pv"\left(f_2^R\right)+\gamma\left(v"\left(f_1^R\right)\right)^2}{v"\left(f_1^R\right)+pv"\left(f_2^R\right)}\right]d\varepsilon \end{array}\right\}=\left\{\begin{array}{c} D_{1L}+A_2\int_{\varepsilon_L}^{\overline{\varepsilon}}\frac{D_2}{B_2}v"\left(f_1^R\right)d\varepsilon \\ -\frac{A_{3L}D_3}{C_{2L}} \end{array}\right\}\Leftrightarrow$$

$$\frac{\partial\eta}{\partial k}\left\{\begin{array}{c} -\frac{\theta\overline{y}\gamma}{(1-\theta)y^m\Gamma}\left[\begin{array}{c} v'\left(f_{1L}^R\right)+(1+\Gamma)pv'\left(f_{2L}^R\right) \\ -(1-p)v'\left(f_{2L}^{NR}\right)\end{array}\right] \\ -2\gamma v'\left(f_{1L}^R\right)+\frac{(1-p)\theta\overline{y}\gamma v'\left(f_{2L}^{NR}\right)\left[v"\left(g_{1L}^{NR}\right)-pv"\left(g_{2L}^{NR}\right)\right]}{(1-\theta)y^m\Gamma\left[v"\left(g_{1L}^{NR}\right)+pv"\left(g_{2L}^{NR}\right)\right]} \\ +(1-\theta)y^m\Gamma\gamma\int_{\varepsilon_L}^{\overline{\varepsilon}}\frac{v"\left(f_1^R\right)pv"\left(f_2^R\right)}{v"\left(f_1^R\right)+pv"\left(f_2^R\right)}d\varepsilon \end{array}\right\}=\left\{\begin{array}{c} \left(b_L^R-\overline{b}\right)- \\ \frac{v'\left(f_{2L}^{NR}\right)(1-p)}{v"\left(g_{1L}^{NR}\right)+pv"\left(g_{2L}^{NR}\right)}+ \\ (1-\theta)y^m\Gamma\int_{\varepsilon_L}^{\overline{\varepsilon}}\frac{v"\left(f_1^R\right)}{v"\left(f_1^R\right)+pv"\left(f_2^R\right)}d\varepsilon \end{array}\right\}. \tag{3.59}$$

Note that the overall sign of the term in brackets on the left-hand side of (3.59) is negative if p is sufficiently different from 0. Then the terms $\frac{\theta\overline{y}\gamma}{(1-\theta)y^m\Gamma}(1-p)v'\left(f_{2L}^R\right)$ and $\frac{(1-p)\theta\overline{y}\gamma v'\left(f_{2L}^{NR}\right)\left[v"\left(g_{1L}^{NR}\right)-pv"\left(g_{2L}^{NR}\right)\right]}{(1-\theta)y^m\Gamma\left[v"\left(g_{1L}^{NR}\right)+pv"\left(g_{2L}^{NR}\right)\right]}$ are close enough to zero, so that they do not dominate the other terms. Further, the right-hand side of (3.59) is positive. Hence, for p sufficiently different from 0, $\frac{\partial\eta}{\partial k}<0$.

Sign of $\frac{\partial b^R}{\partial k}$: Given (3.56), the signs of (3.46)-(3.48) and of $\frac{\partial \eta}{\partial k}$, we find:

$$\frac{\partial b^R}{\partial k} < 0. \tag{3.60}$$

Sign of $\frac{\partial b^{NR}}{\partial k}$: Finally, given (3.57), the signs of (3.53)-(3.55) and the sign of $\frac{\partial \eta}{\partial k}$, then:

$$\frac{\partial b_L^{NR}}{\partial k} \lesseqgtr 0. \tag{3.61}$$

However if one assumes that $v"\left(g_{1L}^{NR}\right) - pv"\left(g_{2L}^{NR}\right) < 0$ [25] then the sign of (3.61) is negative.

3.D.2 Effect of k on the probability of re-election

From (3.16) and (3.17), we know that:

$$\frac{d\Pr(R)}{d\eta} = -\left(\frac{1}{2\overline{\varepsilon}}\right)\frac{\partial \varepsilon_L}{\partial \eta}. \tag{3.62}$$

Using this last expression and (3.39), we can write $\frac{d\Pr(R)}{dk}$ as:

$$\begin{aligned}
\frac{d\Pr(R)}{dk} &= -\frac{1}{2\overline{\varepsilon}}\left(\frac{\partial \varepsilon_L}{\partial \eta}\frac{\partial \eta}{\partial k} + \frac{\partial \varepsilon_L}{\partial k}\right) = -\frac{1}{2\overline{\varepsilon}}\frac{\partial \varepsilon_L}{\partial \eta}\frac{\partial \eta}{\partial k} \Leftrightarrow \\
\frac{d\Pr(R)}{dk} &= -\frac{1}{2\overline{\varepsilon}}\left(-\frac{\gamma}{(1-\theta)\,y^m\Gamma}\right)\frac{\partial \eta}{\partial k} < 0.
\end{aligned}$$

3.D.3 Effect of $\overline{\varepsilon}$ on η, b^R and b^{NR}

Given our assumption that ε has a uniform distribution on the interval $[-\overline{\varepsilon}, \overline{\varepsilon}]$, we have:

$$\begin{aligned}
Var(\varepsilon) &= E_0\left\{\varepsilon - E_0\{\varepsilon\}\right\}^2 \Leftrightarrow \\
Var(\varepsilon) &= E_0\left\{\varepsilon^2\right\} = \int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} \varepsilon^2 f(\varepsilon)\, d\varepsilon = \int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} \varepsilon^2 \frac{1}{2\overline{\varepsilon}} d\varepsilon \Leftrightarrow \\
Var(\varepsilon) &= \frac{1}{2\overline{\varepsilon}}\left[\frac{\varepsilon^3}{3}\right]_{-\overline{\varepsilon}}^{\overline{\varepsilon}} = \frac{1}{3}\overline{\varepsilon}^2.
\end{aligned}$$

This last expression demonstrates that the variance of ε is monotonically increasing in $\overline{\varepsilon}$. Hence, by investigating the effect of changes in $\overline{\varepsilon}$, we explore the effects of changes in the variance of ε.

[25] That is the case for a quadratic specification (3.25) of $v(\cdot)$.

Effect on η

Differentiating (3.23) with respect to $\overline{\varepsilon}$, we have:

$$\begin{aligned}
& v'\left(f_{1L}^{R}\right)\left[\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\overline{\varepsilon}}+\frac{\partial\varepsilon_L}{\partial\overline{\varepsilon}}\right)-\gamma\frac{\partial\eta}{\partial\overline{\varepsilon}}+\frac{\partial b_L^R}{\partial\overline{\varepsilon}}\right]+ \\
& pv'\left(f_{2L}^{R}\right)\left[\theta\overline{y}\left(1+\Gamma\right)\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\overline{\varepsilon}}+\frac{\partial\varepsilon_L}{\partial\overline{\varepsilon}}\right)-\frac{\partial b_L^R}{\partial\overline{\varepsilon}}\right] \\
& -k\left(\frac{\partial b_L^R}{\partial\overline{\varepsilon}}\right)-\left(1-p\right)v'\left(f_{2L}^{R}\right)\left[\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\overline{\varepsilon}}+\frac{\partial\varepsilon_L}{\partial\overline{\varepsilon}}\right)-\frac{\partial b_L^{NR}}{\partial\overline{\varepsilon}}\right] \\
& -\left(1-\theta\right)y^m\Gamma\left[\begin{array}{c} v'\left(f_{1\overline{\varepsilon}}^{R}\right)\frac{\partial\overline{\varepsilon}}{\partial\overline{\varepsilon}}-v'\left(f_{1L}^{R}\right)\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\overline{\varepsilon}}+\frac{\partial\varepsilon_L}{\partial\overline{\varepsilon}}\right)+ \\ \int_{\varepsilon_L}^{\overline{\varepsilon}} v"\left(f_{1L}^{R}\right)\left(-\gamma\frac{\partial\eta}{\partial\overline{\varepsilon}}+\frac{\partial b^R}{\partial\overline{\varepsilon}}\right)d\varepsilon \end{array}\right] \\
= \;& 0 \Leftrightarrow
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial\eta}{\partial\overline{\varepsilon}}\left\{\theta\overline{y}\frac{\partial\varepsilon_L}{\partial\eta}\left[v'\left(f_{1L}^{R}\right)+\left(1+\Gamma\right)pv'\left(f_{2L}^{R}\right)-\left(1-p\right)v'\left(f_{2L}^{NR}\right)\right]-\gamma v'\left(f_{1L}^{R}\right)\right\}+ \\
& \frac{\partial b_L^{NR}}{\partial\overline{\varepsilon}}\left[v'\left(f_{2L}^{NR}\right)\left(1-p\right)\right]+ \\
& \theta\overline{y}\frac{\partial\varepsilon_L}{\partial\overline{\varepsilon}}\left[v'\left(f_{1L}^{R}\right)+\left(1+\Gamma\right)pv'\left(f_{2L}^{R}\right)-\left(1-p\right)v'\left(f_{2L}^{NR}\right)\right]+ \\
& \frac{\partial b_L^{R}}{\partial\overline{\varepsilon}}\left[v'\left(f_{1L}^{R}\right)-pv'\left(f_{2L}^{R}\right)-k\right]- \\
& \left(1-\theta\right)y^m\Gamma\left\{-v'\left(f_{1L}^{R}\right)\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\overline{\varepsilon}}+\frac{\partial\varepsilon_L}{\partial\overline{\varepsilon}}\right)+\int_{\varepsilon_L}^{\overline{\varepsilon}} v"\left(f_{1}^{R}\right)\left(-\gamma\frac{\partial\eta}{\partial\overline{\varepsilon}}+\frac{\partial b^R}{\partial\overline{\varepsilon}}\right)d\varepsilon\right\} \\
= \;& \left(1-\theta\right)y^m\Gamma v'\left(f_{1\overline{\varepsilon}}^{R}\right),
\end{aligned}$$

where we have defined:

$$f_{1\overline{\varepsilon}}^{R}=\theta\overline{y}(1+\overline{\varepsilon})-\eta\gamma+b^R\left(\overline{\varepsilon}\right). \tag{3.63}$$

From (3.16) we know that $\frac{\partial\varepsilon_L}{\partial\overline{\varepsilon}}=0$. So, using again (3.39) and the fact that (3.27) is valid for any realization of ε in the interval $[\varepsilon_L,\overline{\varepsilon}]$, we can simplify the latter expression to:

$$\left(\begin{array}{c} \frac{\partial\eta}{\partial\overline{\varepsilon}}\gamma\left\{-\frac{\theta\overline{y}}{(1-\theta)y^m\Gamma}\left[\begin{array}{c} v'\left(f_{1L}^{R}\right)+\left(1+\Gamma\right)pv'\left(f_{2L}^{R}\right) \\ -\left(1-p\right)v'\left(f_{2L}^{NR}\right)\end{array}\right]-2v'\left(f_{1L}^{R}\right)\right\} \\ +\frac{\partial b_L^{NR}}{\partial\overline{\varepsilon}}\left[v'\left(f_{2L}^{NR}\right)\left(1-p\right)\right]-\left(1-\theta\right)y^m\Gamma\int_{\varepsilon_L}^{\overline{\varepsilon}} v"\left(f_{1}^{R}\right)\left(-\gamma\frac{\partial\eta}{\partial\overline{\varepsilon}}+\frac{\partial b^R}{\partial\overline{\varepsilon}}\right)d\varepsilon \end{array}\right)=\left(1-\theta\right)y^m\Gamma v'\left(f_{1\overline{\varepsilon}}^{R}\right),$$

which can be represented by:

$$A_{1L}\frac{\partial\eta}{\partial\overline{\varepsilon}}-A_2\int_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma\frac{\partial\eta}{\partial\overline{\varepsilon}}+\frac{\partial b^R}{\partial\overline{\varepsilon}}\right)v"\left(f_1^R\right)d\varepsilon+A_{3L}\frac{\partial b_L^{NR}}{\partial\overline{\varepsilon}}=I_1. \tag{3.64}$$

A_{1L} is given by (3.41), A_2 by (3.42), A_{3L} by (3.43) and

$$I_1=\left(1-\theta\right)y^m\Gamma v'\left(f_{1\overline{\varepsilon}}^{R}\right)>0. \tag{3.65}$$

Effect on b^R

Differentiating (3.27) with respect to $\overline{\varepsilon}$, we have:

$$\begin{aligned} v"\left(f_1^R\right)\left(-\gamma\frac{\partial\eta}{\partial\overline{\varepsilon}}+\frac{\partial b^R}{\partial\overline{\varepsilon}}\right)-pv"\left(f_2^R\right)\left(-\frac{\partial b^R}{\partial\overline{\varepsilon}}\right) &= 0 \Leftrightarrow \\ \frac{\partial\eta}{\partial\overline{\varepsilon}}\left[-\gamma v"\left(f_1^R\right)\right]+\frac{\partial b^R}{\partial\overline{\varepsilon}}\left[v"\left(f_1^R\right)+pv"\left(f_2^R\right)\right] &= 0, \end{aligned}$$

which can be represented by:

$$B_1\frac{\partial\eta}{\partial\overline{\varepsilon}}+B_2\frac{\partial b^R}{\partial\overline{\varepsilon}}=I_2. \tag{3.66}$$

B_1 is given by (3.46), B_2 by (3.47), and

$$I_2=0. \tag{3.67}$$

Effect on b^{NR}

When the government chooses the optimal η, it takes into account b^{NR} evaluated at $\varepsilon=\varepsilon_L$. Therefore, differentiating (3.49) with respect to $\overline{\varepsilon}$, we have:

$$\begin{aligned} v"\left(g_{1L}^{NR}\right)\left[\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\overline{\varepsilon}}+\frac{\partial\varepsilon_L}{\partial\overline{\varepsilon}}\right)+\frac{\partial b_L^{NR}}{\partial\overline{\varepsilon}}\right]-pv"\left(g_{2L}^{NR}\right)\left\{\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\overline{\varepsilon}}+\frac{\partial\varepsilon_L}{\partial\overline{\varepsilon}}\right)-\frac{\partial b_L^{NR}}{\partial\overline{\varepsilon}}\right\} &= 0 \Leftrightarrow \\ \frac{\partial\eta}{\partial\overline{\varepsilon}}\left[\theta\overline{y}\frac{\partial\varepsilon_L}{\partial\eta}\left(v"\left(g_{1L}^{NR}\right)-pv"\left(g_{2L}^{NR}\right)\right)\right]+\frac{\partial b_L^{NR}}{\partial\overline{\varepsilon}}\left[v"\left(g_{1L}^{NR}\right)+pv"\left(g_{2L}^{NR}\right)\right] &= 0, \end{aligned}$$

where g_{1L}^{NR} is given by (3.50), g_{2L}^{NR} by (3.51) and $\frac{\partial\varepsilon_L}{\partial\overline{\varepsilon}}=0$. Furthermore, using (3.39), the last expression can be represented by:

$$C_{1L}\frac{\partial\eta}{\partial\overline{\varepsilon}}+C_{2L}\frac{\partial b_L^{NR}}{\partial\overline{\varepsilon}}=I_{3L}, \tag{3.68}$$

where C_{1L} is given by (3.53), C_{2L} by (3.54) and

$$I_{3L}=0. \tag{3.69}$$

Solution of the system

We can write the system formed by (3.64), (3.66) and (3.68) as:

$$\left\{\begin{array}{lllllll} A_{1L}\frac{\partial\eta}{\partial\overline{\varepsilon}} & +0 & -A_2\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma\frac{\partial\eta}{\partial\overline{\varepsilon}}+\frac{\partial b^R}{\partial\overline{\varepsilon}}\right)v"\left(f_1^R\right)d\varepsilon & +A_{3L}\frac{\partial b_L^{NR}}{\partial\overline{\varepsilon}} & = & I_1 \\ B_1\frac{\partial\eta}{\partial\overline{\varepsilon}} & +B_2\frac{\partial b^R}{\partial\overline{\varepsilon}} & +0 & +0 & = & I_2 \\ C_{1L}\frac{\partial\eta}{\partial\overline{\varepsilon}} & +0 & +0 & +C_{2L}\frac{\partial b_L^{NR}}{\partial\overline{\varepsilon}} & = & I_{3L} \end{array}\right. .$$

Further, we can rewrite (3.66) and (3.68) as:

$$\frac{\partial b^R}{\partial\overline{\varepsilon}} = -\frac{B_1}{B_2}\frac{\partial\eta}{\partial\overline{\varepsilon}}, \text{ and} \tag{3.70}$$

$$\frac{\partial b_L^{NR}}{\partial\overline{\varepsilon}} = -\frac{C_{1L}}{C_{2L}}\frac{\partial\eta}{\partial\overline{\varepsilon}}. \tag{3.71}$$

Substituting these equations into (3.64), we have:

$$
\begin{aligned}
A_{1L}\frac{\partial \eta}{\partial \overline{\varepsilon}} - A_2 \int_{\varepsilon_L}^{\overline{\varepsilon}} \left(-\gamma \frac{\partial \eta}{\partial \overline{\varepsilon}} - \frac{B_1}{B_2}\frac{\partial \eta}{\partial \overline{\varepsilon}}\right) v\text{"}\left(f_1^R\right) d\varepsilon - A_{3L}\frac{C_{1L}}{C_{2L}}\frac{\partial \eta}{\partial \overline{\varepsilon}} &= I_1 \Leftrightarrow \\
A_1\frac{\partial \eta}{\partial \overline{\varepsilon}} - A_2 \int_{\varepsilon_L}^{\overline{\varepsilon}} \frac{\partial \eta}{\partial \overline{\varepsilon}} \left(-\gamma - \frac{B_1}{B_2}\right) v\text{"}\left(f_1^R\right) d\varepsilon - A_{3L}\frac{C_{1L}}{C_{2L}}\frac{\partial \eta}{\partial \overline{\varepsilon}} &= I_1. \qquad (3.72)
\end{aligned}
$$

Sign of $\frac{\partial \eta}{\partial \overline{\varepsilon}}$: Using the values of (3.41) to (3.43), (3.46), (3.47), (3.53), (3.54) and (3.65), we are able to rewrite (3.72) as:

$$
\begin{aligned}
\frac{\partial \eta}{\partial \overline{\varepsilon}}\left\{\begin{array}{c} A_{1L} - A_{3L}\frac{C_{1L}}{C_{2L}} - \\ A_2 \int_{\varepsilon_L}^{\overline{\varepsilon}} \left[-\gamma v\text{"}\left(f_1^R\right) + \frac{\gamma\left(v\text{"}\left(f_1^R\right)\right)^2}{v\text{"}\left(f_1^R\right)+pv\text{"}\left(f_2^R\right)}\right] d\varepsilon \end{array}\right\} &= I_1 \Leftrightarrow \\
\frac{\partial \eta}{\partial \overline{\varepsilon}}\left[\begin{array}{c} A_{1L} - A_{3L}\frac{C_{1L}}{C_{2L}} - \\ A_2 \int_{\varepsilon_L}^{\overline{\varepsilon}} \left[\frac{-\gamma\left(v\text{"}\left(f_1^R\right)\right)^2 - \gamma v\text{"}\left(f_1^R\right)pv\text{"}\left(f_2^R\right) + \gamma\left(v\text{"}\left(f_1^R\right)\right)^2}{v\text{"}\left(f_1^R\right)+pv\text{"}\left(f_2^R\right)}\right] d\varepsilon \end{array}\right] &= I_1 \Leftrightarrow \\
\frac{\partial \eta}{\partial \overline{\varepsilon}}\left\{\begin{array}{c} -\frac{\theta\overline{y}\gamma}{(1-\theta)y^m\Gamma}\left[\begin{array}{c} v'\left(f_{1L}^R\right) + (1+\Gamma)\,pv'\left(f_{2L}^R\right) \\ -(1-p)\,v'\left(f_{2L}^{NR}\right) \end{array}\right] \\ -2\gamma v'\left(f_{1L}^R\right) + \frac{(1-p)\theta\overline{y}\gamma v'\left(f_{2L}^{NR}\right)\left[v\text{"}\left(g_{1L}^{NR}\right) - pv\text{"}\left(g_{2L}^{NR}\right)\right]}{(1-\theta)y^m\Gamma\left[v\text{"}\left(g_{1L}^{NR}\right)+pv\text{"}\left(g_{2L}^{NR}\right)\right]} \\ +(1-\theta)\,y^m\Gamma\gamma\int_{\varepsilon_L}^{\overline{\varepsilon}} \frac{v\text{"}\left(f_1^R\right)pv\text{"}\left(f_2^R\right)}{v\text{"}\left(f_1^R\right)+pv\text{"}\left(f_2^R\right)} d\varepsilon \end{array}\right\} &= (1-\theta)\,y^m\Gamma v'[f_{1\overline{\varepsilon}}^R] \qquad (3.73)
\end{aligned}
$$

If p is sufficiently different from 0 the overall sign of the term in brackets on the left-hand side of (3.73) is negative. The right-hand side is positive. Hence, for p close enough to 1, $\frac{\partial \eta}{\partial \overline{\varepsilon}} < 0$.

Sign of $\frac{\partial b^R}{\partial \overline{\varepsilon}}$: Given (3.70), the signs of (3.46), (3.47), and the sign of $\frac{\partial \eta}{\partial \overline{\varepsilon}}$, we have

$$\frac{\partial b^R}{\partial \overline{\varepsilon}} < 0. \qquad (3.74)$$

Sign of $\frac{\partial b_L^{NR}}{\partial \overline{\varepsilon}}$: Given (3.71), the signs of (3.53), (3.54) and of $\frac{\partial \eta}{\partial \overline{\varepsilon}}$, the sign of $\frac{\partial b_L^{NR}}{\partial \overline{\varepsilon}}$ is ambiguous. However, if we assume that $v\text{"}\left(g_{1L}^{NR}\right) - pv\text{"}\left(g_{2L}^{NR}\right) < 0$ this ambiguity disappears and a higher variance of the shock of output diminishes the optimal deficit in case of no re-election when $\varepsilon = \varepsilon_L$:

$$\frac{\partial b_L^{NR}}{\partial \overline{\varepsilon}} < 0. \qquad (3.75)$$

3.D.4 Effect of $\overline{\varepsilon}$ on the probability of re-election

With $(1-\theta)\,y^m\Gamma + (\eta - I)\,\gamma > 0$, and using (3.39), we can prove Proposition 3.2 by writing $\frac{d\Pr(R)}{d\overline{\varepsilon}}$ as:

$$
\begin{aligned}
\frac{d\Pr(R)}{d\overline{\varepsilon}} &= \frac{\left(1-\frac{\partial\varepsilon_L}{\partial\overline{\varepsilon}}\right)2\overline{\varepsilon} - (\overline{\varepsilon}-\varepsilon_L)\,2}{(2\overline{\varepsilon})^2} = \frac{-2\left[\overline{\varepsilon}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\overline{\varepsilon}}\right)-\varepsilon_L\right]}{4\overline{\varepsilon}^2} \Leftrightarrow \\
\frac{d\Pr(R)}{d\overline{\varepsilon}} &= \left(\frac{1}{2\overline{\varepsilon}}\right)\left(\frac{\gamma}{(1-\theta)\,y^m\Gamma}\frac{\partial\eta}{\partial\overline{\varepsilon}}\right) + \left(\frac{-\frac{(\eta-I)\gamma}{(1-\theta)y^m\Gamma}-1}{2\overline{\varepsilon}^2}\right) \Leftrightarrow \\
\frac{d\Pr(R)}{d\overline{\varepsilon}} &= \left(\frac{1}{2\overline{\varepsilon}}\right)\left(\frac{\gamma}{(1-\theta)\,y^m\Gamma}\frac{\partial\eta}{\partial\overline{\varepsilon}}\right) + \frac{-\frac{(1-\theta)y^m\Gamma+(\eta-I)\gamma}{(1-\theta)y^m\Gamma}}{2\overline{\varepsilon}^2} \Leftrightarrow \\
\frac{d\Pr(R)}{d\overline{\varepsilon}} &= \left(\frac{1}{2\overline{\varepsilon}}\right)\left(\frac{1}{(1-\theta)\,y^m\Gamma}\right)\left(\gamma\frac{\partial\eta}{\partial\overline{\varepsilon}} - \frac{(1-\theta)\,y^m\Gamma+(\eta-I)\,\gamma}{\overline{\varepsilon}}\right) < 0. \quad (3.76)
\end{aligned}
$$

3.D.5 Effect of y^m on η, b^R and b

Effect on η

Differentiate (3.23) with respect to y^m (holding $\overline{y}$ constant)

$$
\begin{aligned}
& v'\left(f_{1L}^R\right)\left[\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial y^m}+\frac{\partial\varepsilon_L}{\partial y^m}\right)-\gamma\frac{\partial\eta}{\partial y^m}+\frac{\partial b_L^R}{\partial y^m}\right]+ \\
& pv'\left(f_{2L}^R\right)\left[\theta\overline{y}\,(1+\Gamma)\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial y^m}+\frac{\partial\varepsilon_L}{\partial y^m}\right)-\frac{\partial b_L^R}{\partial y^m}\right] \\
& -k\left(\frac{\partial b_L^R}{\partial y^m}\right)-(1-p)\,v'\left(f_{2L}^{NR}\right)\left(\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial y^m}+\frac{\partial\varepsilon_L}{\partial y^m}\right)-\frac{\partial b_L^{NR}}{\partial y^m}\right)- \\
& (1-\theta)\,\Gamma\int_{\varepsilon_L}^{\overline{\varepsilon}} v'\left(f_1^R\right)-(1-\theta)\,y^m\Gamma\left\{\begin{array}{c} v'\left(f_{1\overline{\varepsilon}}^R\right)\frac{\partial\overline{\varepsilon}}{\partial y^m}-v'\left(f_{1L}^R\right)\frac{\partial\varepsilon_L}{\partial y^m}+ \\ \int_{\varepsilon_L}^{\overline{\varepsilon}} v"\left(f_1^R\right)\left(-\gamma\frac{\partial\eta}{\partial y^m}+\frac{\partial b^R}{\partial y^m}\right)d\varepsilon \end{array}\right\}=0\Leftrightarrow
\end{aligned}
$$

$$
\begin{aligned}
& \frac{\partial\eta}{\partial y^m}\left\{\theta\overline{y}\frac{\partial\varepsilon_L}{\partial\eta}\left[v'\left(f_{1L}^R\right)+(1+\Gamma)\,pv'\left(f_{2L}^R\right)-(1-p)\,v'\left(f_{2L}^{NR}\right)\right]-\gamma v'\left(f_{1L}^R\right)\right\}+ \\
& \frac{\partial b_L^{NR}}{\partial y^m}\left[v'\left(f_{2L}^{NR}\right)(1-p)\right]+\theta\overline{y}\frac{\partial\varepsilon_L}{\partial y^m}\left[v'\left(f_{1L}^R\right)+(1+\Gamma)\,pv'\left(f_{2L}^R\right)-(1-p)\,v'\left(f_{2L}^{NR}\right)\right]+ \\
& \frac{\partial b_L^R}{\partial y^m}\left[v'\left(f_{1L}^R\right)-pv'\left(f_{2L}^R\right)-k\right]-(1-\theta)\,y^m\Gamma\left[\begin{array}{c} -v'\left(f_{1L}^R\right)\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial y^m}+\frac{\partial\varepsilon_L}{\partial y^m}\right)+ \\ \int_{\varepsilon_L}^{\overline{\varepsilon}} v"\left(f_1^R\right)\left(-\gamma\frac{\partial\eta}{\partial y^m}+\frac{\partial b^R}{\partial y^m}\right)d\varepsilon \end{array}\right] \\
= {}& (1-\theta)\,\Gamma\int_{\varepsilon_L}^{\overline{\varepsilon}} v'\left(f_1^R\right).
\end{aligned}
$$

Using (3.39) and the fact that (3.27) is valid for any realization of ε in the interval

$(\varepsilon_L, \overline{\varepsilon})$, we can simplify the latter expression to:

$$\begin{aligned}
&\frac{\partial\eta}{\partial y^m}\gamma\left\{-\frac{\theta\overline{y}}{(1-\theta)\,y^m\Gamma}\left[v'\left(f_{1L}^R\right)+(1+\Gamma)\,pv'\left(f_{2L}^R\right)-(1-p)\,v'\left(f_{2L}^{NR}\right)\right]-v'\left(f_{1L}^R\right)\right\}\\
&+\frac{\partial b_L^{NR}}{\partial y^m}\left[v'\left(f_{2L}^{NR}\right)(1-p)\right]+\theta\overline{y}\frac{\partial\varepsilon_L}{\partial y^m}\left[v'\left(f_{1L}^R\right)+(1+\Gamma)\,pv'\left(f_{2L}^R\right)-(1-p)\,v'\left(f_{2L}^{NR}\right)\right]\\
&-(1-\theta)\,y^m\Gamma\left\{\begin{array}{c}-v'\left(f_{1L}^R\right)\left[\left(-\frac{\gamma}{(1-\theta)y^m\Gamma}\right)\frac{\partial\eta}{\partial y^m}+\frac{\partial\varepsilon_L}{\partial y^m}\right]\\+\int_{\varepsilon_L}^{\overline{\varepsilon}}v"\left(f_1^R\right)\left(-\gamma\frac{\partial\eta}{\partial y^m}+\frac{\partial b^R}{\partial y^m}\right)d\varepsilon\end{array}\right\}\\
=\;&(1-\theta)\,\Gamma\int_{\varepsilon_L}^{\overline{\varepsilon}}v'\left(f_1^R\right).
\end{aligned}\tag{3.77}$$

Additionally, taking the derivative of (3.16) with respect to y^m:

$$\frac{\partial\varepsilon_L}{\partial y^m}=\frac{-\left\{-(\eta-I)\,\gamma\,(1-\theta)\,\Gamma\right\}}{\left((1-\theta)\,y^m\Gamma\right)^2}=\frac{(\eta-I)\,\gamma}{(1-\theta)\,(y^m)^2\,\Gamma}<0.\tag{3.78}$$

We can rewrite (3.77) as:

$$\left\{\begin{array}{c}\frac{\partial\eta}{\partial y^m}\left\{\begin{array}{c}-\frac{\gamma\theta\overline{y}}{(1-\theta)y^m\Gamma}\left[\begin{array}{c}v'\left(f_{1L}^R\right)+\\(1+\Gamma)\,pv'\left(f_{2L}^R\right)\\-(1-p)\,v'\left(f_{2L}^{NR}\right)\end{array}\right]\\-2\gamma v'\left(f_{1L}^R\right)\end{array}\right\}\\+\frac{\partial b_L^{NR}}{\partial y^m}\left[v'\left(f_{2L}^{NR}\right)(1-p)\right]-\\(1-\theta)\,y^m\Gamma\int_{\varepsilon_L}^{\overline{\varepsilon}}v"\left(f_1^R\right)\left(-\gamma\frac{\partial\eta}{\partial y^m}+\frac{\partial b^R}{\partial y^m}\right)d\varepsilon\end{array}\right\}=\left\{\begin{array}{c}(1-\theta)\,\Gamma\int_{\varepsilon_L}^{\overline{\varepsilon}}v'\left(f_1^R\right)-\frac{(\eta-I)\gamma v'\left(f_{1L}^R\right)}{y^m}\\-\frac{\theta\overline{y}(\eta-I)\gamma}{(1-\theta)(y^m)^2\Gamma}\left[\begin{array}{c}v'\left(f_{1L}^R\right)+\\(1+\Gamma\,[\gamma])\,pv'\left(f_{2L}^R\right)\\-(1-p)\,v'\left(f_{2L}^{NR}\right)\end{array}\right]\end{array}\right\},$$

which can be represented by:

$$A_{1L}\frac{\partial\eta}{\partial y^m}-A_2\int_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma\frac{\partial\eta}{\partial y^m}+\frac{\partial b^R}{\partial y^m}\right)v"\left(f_1^R\right)d\varepsilon+A_{3L}\frac{\partial b_L^{NR}}{\partial y^m}=H_{1L}.\tag{3.79}$$

A_{1L} is given by (3.41), A_2 by (3.42), A_{3L} by (3.43) and, if p is sufficiently different from 0:

$$H_{1L}=\left\{\begin{array}{c}(1-\theta)\,\Gamma\int_{\varepsilon_L}^{\overline{\varepsilon}}v'\left(f_1^R\right)d\varepsilon-\\\frac{(\eta-I)\gamma}{y^m}\left\{v'\left(f_{1L}^R\right)+\frac{\theta\overline{y}}{(1-\theta)y^m\Gamma}\left[\begin{array}{c}v'\left(f_{1L}^R\right)+(1+\Gamma)\,pv'\left(f_{2L}^R\right)\\-(1-p)\,v'\left(f_{2L}^{NR}\right)\end{array}\right]\right\}\end{array}\right\}>0.\tag{3.80}$$

Effect on b^R

Differentiating (3.27) with respect to y^m, we have:

$$\begin{aligned}
v"\left(f_1^R\right)\left(-\gamma\frac{\partial\eta}{\partial y^m}+\frac{\partial b^R}{\partial y^m}\right)-pv"\left(f_2^R\right)\left(-\frac{\partial b^R}{\partial y^m}\right)&=0\Leftrightarrow\\
\frac{\partial\eta}{\partial y^m}\left[-\gamma v"\left(f_1^R\right)\right]+\frac{\partial b^R}{\partial y^m}\left[v"\left(f_1^R\right)+pv"\left(f_2^R\right)\right]&=0,
\end{aligned}$$

which can be represented by:

$$B_1 \frac{\partial \eta}{\partial y^m} + B_2 \frac{\partial b^R}{\partial y^m} = H_2. \tag{3.81}$$

B_1 is given by (3.46), B_2 by (3.47), and

$$H_2 = 0. \tag{3.82}$$

Effect on b^{NR}

When the government chooses the optimal η, it takes into account b^{NR} evaluated at $\varepsilon = \varepsilon_L$. Therefore, differentiating (3.49) with respect to y^m, we have:

$$v"\left(g_{1L}^{NR}\right)\left[\theta\overline{y}\left(\frac{\partial \varepsilon_L}{\partial \eta}\frac{\partial \eta}{\partial y^m} + \frac{\partial \varepsilon_L}{\partial y^m}\right) + \frac{\partial b_L^{NR}}{\partial k}\right] - pv"\left(g_{2L}^{NR}\right)\left[\theta\overline{y}\left(\frac{\partial \varepsilon_L}{\partial \eta}\frac{\partial \eta}{\partial y^m} + \frac{\partial \varepsilon_L}{\partial y^m}\right) - \frac{\partial b_L^{NR}}{\partial y^m}\right] = 0 \Leftrightarrow$$

$$\begin{aligned} & \frac{\partial \eta}{\partial y^m}\left[\theta\overline{y}\frac{\partial \varepsilon_L}{\partial \eta}\left(v"\left(g_{1L}^{NR}\right) - pv"\left(g_{2L}^{NR}\right)\right)\right] + \frac{\partial b_L^{NR}}{\partial y^m}\left[v"\left(g_{1L}^{NR}\right) + pv"\left(g_{2L}^{NR}\right)\right] \\ = & -\theta\overline{y}\frac{\partial \varepsilon_L}{\partial y^m}\left[v"\left(g_{1L}^{NR}\right) - pv"\left(g_{2L}^{NR}\right)\right], \end{aligned}$$

where g_{1L}^{NR} is given by (3.50), and g_{2L}^{NR} by (3.51). Furthermore, using (3.39) and (3.78), the last expression can be represented as:

$$C_{1L} \frac{\partial \eta}{\partial y^m} + C_{2L} \frac{\partial b_L^{NR}}{\partial y^m} = H_{3L}, \tag{3.83}$$

where C_{1L} is given by (3.53), C_{2L} by (3.54) and

$$H_{3L} = -\frac{\theta\overline{y}\left(\eta - I\right)\gamma}{\left(1-\theta\right)\left(y^m\right)^2\Gamma}\left[v"\left(g_{1L}^{NR}\right) - pv"\left(g_{2L}^{NR}\right)\right]. \tag{3.84}$$

H_{3L} is negative for $v"\left(g_{1L}^{NR}\right) - pv"\left(g_{2L}^{NR}\right) < 0$.[26]

Solution of the system

We can write the system formed by (3.79), (3.81) and (3.83) as:

$$\left\{\begin{array}{llllll} A_{1L}\frac{\partial \eta}{\partial y^m} & +0 & -A_2 \int_{\varepsilon_L}^{\overline{\varepsilon}} \left(-\gamma\frac{\partial \eta}{\partial y^m} + \frac{\partial b^R}{\partial y^m}\right) v"\left(f_1^R\right) d\varepsilon & +A_{3L}\frac{\partial b_L^{NR}}{\partial y^m} & = & H_{1L} \\ B_1\frac{\partial \eta}{\partial y^m} & +B_2\frac{\partial b^R}{\partial y^m} & +0 & +0 & = & H_2 \\ C_{1L}\frac{\partial \eta}{\partial y^m} & +0 & +0 & +C_{2L}\frac{\partial b_L^{NR}}{\partial y^m} & = & H_{3L} \end{array}\right. .$$

Further, we can rewrite (3.81) and (3.83) as:

$$\frac{\partial b^R}{\partial y^m} = -\frac{B_1}{B_2}\frac{\partial \eta}{\partial y^m}, \text{ and} \tag{3.85}$$

$$\frac{\partial b_L^{NR}}{\partial y^m} = \frac{H_{3L}}{C_{2L}} - \frac{C_{1L}}{C_{2L}}\frac{\partial \eta}{\partial y^m}. \tag{3.86}$$

[26] That is the case for a quadratic utility function specification for instance.

Substituting these two previous equations into (3.79), we have:

$$\begin{aligned} A_{1L}\frac{\partial\eta}{\partial y^m} - A_2\int_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma\frac{\partial\eta}{\partial y^m} - \frac{B_1}{B_2}\frac{\partial\eta}{\partial y^m}\right)v"\left(f_1^R\right)d\varepsilon - A_{3L}\frac{C_{1L}}{C_{2L}}\frac{\partial\eta}{\partial y^m} &= H_{1L} - \tfrac{A_{3L}H_{3L}}{C_{2L}} \Leftrightarrow \\ \frac{\partial\eta}{\partial y^m}\left\{A_{1L} - A_{3L}\frac{C_{1L}}{C_{2L}} - A_2\int_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma - \frac{B_1}{B_2}\right)v"\left(f_1^R\right)d\varepsilon\right\} &= H_{1L} - \tfrac{A_{3L}H_{3L}}{C_{2L}} \end{aligned} \tag{3.87}$$

Sign of $\frac{\partial\eta}{\partial y^m}$: Using the values of (3.41) to (3.43), (3.46), (3.47), (3.53), (3.54) and (3.80) we can (3.87) as:

$$\frac{\partial\eta}{\partial y^m}\left\{A_{1L} - A_{3L}\frac{C_{1L}}{C_{2L}} - A_2\int_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma v"\left(f_1^R\right) + \frac{\gamma\left(v"\left(f_1^R\right)\right)^2}{v"\left(f_1^R\right) + pv"\left(f_2^R\right)}\right)d\varepsilon\right\} = H_{1L} - \frac{A_{3L}H_{3L}}{C_{2L}} \Leftrightarrow$$

$$\begin{aligned} &\frac{\partial\eta}{\partial y^m}\left\{\begin{array}{c} -\frac{\gamma\theta\overline{y}}{(1-\theta)y^m\Gamma}\left[\begin{array}{c} v'\left(f_{1L}^R\right) + \\ (1+\Gamma)\,pv'\left(f_{2L}^R\right) \\ -(1-p)\,v'\left(f_{2L}^{NR}\right) \end{array}\right] \\ -2\gamma v'\left(f_{1L}^R\right) + \frac{(1-p)\gamma\theta\overline{y}v'\left(f_{2L}^{NR}\right)\left[v"\left(g_{1L}^{NR}\right) - pv"\left(g_{2L}^{NR}\right)\right]}{(1-\theta)y^m\Gamma\left[v"\left(g_{1L}^{NR}\right) + pv"\left(g_{2L}^{NR}\right)\right]} \\ +(1-\theta)\,y^m\Gamma\gamma\int_{\varepsilon_L}^{\overline{\varepsilon}}\frac{v"\left(f_1^R\right)pv"\left(f_2^R\right)}{v"\left(f_1^R\right) + pv"\left(f_2^R\right)}d\varepsilon \end{array}\right\} \\ &= (1-\theta)\,\Gamma\int_{\varepsilon_L}^{\overline{\varepsilon}}v'\left(f_1^R\right) - \tfrac{(\eta-I)\gamma}{y^m}v'\left(f_{1L}^R\right) - \tfrac{(\eta-I)\gamma}{y^m}\left\{\begin{array}{c} \frac{\theta\overline{y}}{(1-\theta)y^m\Gamma}\left[\begin{array}{c} v'\left(f_{1L}^R\right) + \\ (1+\Gamma)\,pv'\left(f_{2L}^R\right) \\ -(1-p)\,v'\left(f_{2L}^{NR}\right) \end{array}\right] \\ -\frac{\theta\overline{y}(1-p)v'\left(f_{2L}^{NR}\right)}{(1-\theta)y^m\Gamma}\frac{\left[v"\left(g_{1L}^{NR}\right) - pv"\left(g_{2L}^{NR}\right)\right]}{\left[v"\left(g_{1L}^{NR}\right) + pv"\left(g_{2L}^{NR}\right)\right]} \end{array}\right\} \end{aligned} \tag{3.88}$$

If p is sufficiently different from 0, the overall sign of the term in brackets on the left-hand side of (3.88) is negative. The right-hand side is positive. Hence, for p sufficiently different from 0, $\frac{\partial\eta}{\partial y^m} < 0$.

Sign of $\frac{\partial b^R}{\partial y^m}$: Given (3.85), the signs of (3.46), (3.47), and of $\frac{\partial\eta}{\partial y^m}$, we have:

$$\frac{\partial b^R}{\partial y^m} < 0. \tag{3.89}$$

Sign of $\frac{\partial b_L^{NR}}{\partial y^m}$: Finally, given (3.86), the signs of (3.53), (3.54) and of $\frac{\partial\eta}{\partial y^m}$, we have:

$$\frac{\partial b_L^{NR}}{\partial y^m} \lesseqgtr 0. \tag{3.90}$$

If we assume that $v"\left(g_{1L}^{NR}\right) - pv"\left(g_{2L}^{NR}\right) < 0$, the ambiguity in (3.90) can be rewritten as:

$$\begin{aligned} \frac{\partial b_L^{NR}}{\partial y^m} &\lesseqgtr 0 \Leftrightarrow H_{3L} \gtrless -C_{1L}\frac{\partial \eta}{\partial y^m} \Leftrightarrow \\ -\frac{\theta\overline{y}\gamma\left(\eta - I\right)\left(v"\left[g_{1L}\right] - pv"\left[g_{2L}\right]\right)}{\left(1-\theta\right)\left(y^m\right)^2\Gamma\left[\gamma\right]} &\gtrless -\frac{\theta\overline{y}\gamma\left(v"\left[g_{1L}\right] - pv"\left[g_{2L}\right]\right)}{\left(1-\theta\right)y^m\Gamma\left[\gamma\right]}\frac{\partial \eta}{\partial y^m} \Leftrightarrow \\ \frac{\left(\eta - I\right)}{y^m} &\gtrless \frac{\partial \eta}{\partial y^m}. \end{aligned} \tag{3.91}$$

3.D.6 Effect of y^m on the probability of re-election

Using (3.39), (3.62) and (3.78), we have:

$$\begin{aligned} \frac{d\Pr\left(R\right)}{dy^m} &= -\frac{1}{2\overline{\varepsilon}}\left(\frac{\partial \varepsilon_L}{\partial \eta}\frac{\partial \eta}{\partial y^m} + \frac{\partial \varepsilon_L}{\partial y^m}\right) \Leftrightarrow \\ \frac{d\Pr\left(R\right)}{dy^m} &= \frac{1}{2\overline{\varepsilon}}\left\{\left[\frac{\gamma}{\left(1-\theta\right)y^m\Gamma}\frac{\partial \eta}{\partial y^m}\right] + \left[\frac{-\left(\eta - I\right)\gamma\left(1-\theta\right)\Gamma}{\left(\left(1-\theta\right)y^m\Gamma\right)^2}\right]\right\} \Leftrightarrow \\ \frac{d\Pr\left(R\right)}{dy^m} &= \frac{1}{2\overline{\varepsilon}}\left\{\left[\frac{\gamma}{\left(1-\theta\right)y^m\Gamma}\frac{\partial \eta}{\partial y^m}\right] + \left[\frac{-\left(\eta - I\right)\gamma}{\left(1-\theta\right)\left(y^m\right)^2\Gamma}\right]\right\} \Leftrightarrow \\ \frac{d\Pr\left(R\right)}{dy^m} &= \frac{1}{2\overline{\varepsilon}}\left[\frac{\gamma}{\left(1-\theta\right)y^m\Gamma}\right]\left[\frac{\partial \eta}{\partial y^m} - \frac{\left(\eta - I\right)}{y^m}\right] \gtreqless 0. \end{aligned} \tag{3.92}$$

3.E Welfare analysis

Taking a utilitarian perspective in which each individual receives an equal weight and assuming linear utility from private consumption, expected social welfare can be written as:

$$\mathrm{E}_0\left\{U\right\} = \frac{1}{2\overline{\varepsilon}}\left\{\begin{array}{c} \int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\left[\left(1-\theta\right)\overline{y}(1+\varepsilon)\left(2+\Gamma\right) + \left(\eta - I\right)\gamma\right]d\varepsilon + \\ \int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\lambda\left[v\left(f_1^R\right) + pv\left(f_2^R\right) + \left(1-p\right)v\left(g_2^R\right)\right]d\varepsilon + \\ \int\limits_{-\overline{\varepsilon}}^{\varepsilon_L}\left[2\left(1-\theta\right)\overline{y}(1+\varepsilon)\right]d\varepsilon + \int\limits_{-\overline{\varepsilon}}^{\varepsilon_L}\lambda\left[v\left(g_1^{NR}\right) + pv\left(g_2^{NR}\right) + \left(1-p\right)v\left(f_2^{NR}\right)\right]d\varepsilon \end{array}\right\},$$

which leads to equation (3.24).

3.E.1 Effect of k on the expected social welfare

Differentiating (3.24) with respect to k, evaluated at $k = 0$, and applying the Leibnitz's rule:

$$\frac{\partial \mathrm{E}_0\{U\}}{\partial k}=\frac{1}{2\overline{\varepsilon}}\left\{\begin{array}{c}-\left[(\eta-I)\gamma+(1-\theta)\Gamma\overline{y}\right]\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial k}+\frac{\partial\varepsilon_L}{\partial k}\right)+(\overline{\varepsilon}-\varepsilon_L)\gamma\frac{\partial\eta}{\partial k}\\ -\frac{2\varepsilon_L}{2}(1-\theta)\Gamma\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial k}+\frac{\partial\varepsilon_L}{\partial k}\right)+\\ \lambda\left[\begin{array}{c}v\left(f_{1\overline{\varepsilon}}^R\right)\frac{\partial\overline{\varepsilon}}{\partial k}-v\left(f_{1L}^R\right)\frac{\partial\varepsilon_L}{\partial k}+\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\frac{\partial\left[v\left(f_1^R\right)\right]}{\partial k}d\varepsilon+pv\left(f_{2\overline{\varepsilon}}^R\right)\frac{\partial\overline{\varepsilon}}{\partial k}\\ -pv\left(f_{2L}^R\right)\frac{\partial\varepsilon_L}{\partial k}+p\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\frac{\partial\left[v\left(f_2^R\right)\right]}{\partial k}d\varepsilon+(1-p)v\left(g_{2\overline{\varepsilon}}^R\right)\frac{\partial\overline{\varepsilon}}{\partial k}\\ -(1-p)v\left(g_{2L}^R\right)\frac{\partial\varepsilon_L}{\partial k}+(1-p)\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\frac{\partial\left[v\left(g_2^R\right)\right]}{\partial k}d\varepsilon\end{array}\right]+\\ \lambda\left[\begin{array}{c}v\left(g_{1L}^{NR}\right)\frac{\partial\varepsilon_L}{\partial k}-v\left(g_{1-\overline{\varepsilon}}^{NR}\right)\frac{\partial(-\overline{\varepsilon})}{\partial k}+\int\limits_{-\overline{\varepsilon}}^{\varepsilon_L}\frac{\partial\left[v\left(g_1^{NR}\right)\right]}{\partial k}d\varepsilon+pv\left(g_{2L}^{NR}\right)\frac{\partial\varepsilon_L}{\partial k}\\ -pv\left(g_{2-\overline{\varepsilon}}^{NR}\right)\frac{\partial(-\overline{\varepsilon})}{\partial k}+p\int\limits_{-\overline{\varepsilon}}^{\varepsilon_L}\frac{\partial\left[v\left(g_2^{NR}\right)\right]}{\partial k}d\varepsilon+(1-p)v\left(f_{2L}^{NR}\right)\frac{\partial\varepsilon_L}{\partial k}\\ -(1-p)v\left(f_{2-\overline{\varepsilon}}^{NR}\right)\frac{\partial(-\overline{\varepsilon})}{\partial k}+(1-p)\int\limits_{-\overline{\varepsilon}}^{\varepsilon_L}\frac{\partial\left[v\left(f_2^{NR}\right)\right]}{\partial k}d\varepsilon\end{array}\right]\end{array}\right\},$$

where f_{1L}^R is given by (3.20), $f_{2L}^R=g_{2L}^R$ by (3.21), g_{1L}^{NR} by (3.50), $g_{2L}^{NR}=f_{2L}^{NR}$ by (3.51), $f_{1\overline{\varepsilon}}^R$ by (3.63), and

$$f_{2\overline{\varepsilon}}^R = g_{2\overline{\varepsilon}}^R=\theta.\overline{y}.(1+\overline{\varepsilon})(1+\Gamma)-b^R[\overline{\varepsilon}] \tag{3.93}$$

$$g_{1-\overline{\varepsilon}}^{NR} = \theta.\overline{y}.(1-\overline{\varepsilon})+b^{NR}[-\overline{\varepsilon}], \tag{3.94}$$

$$g_{2-\overline{\varepsilon}}^{NR} = f_{2-\overline{\varepsilon}}^{NR}=\theta.\overline{y}.(1-\overline{\varepsilon})-b^{NR}[-\overline{\varepsilon}]. \tag{3.95}$$

Given that in budgetary terms $f_{2\overline{\varepsilon}}^R=g_{2\overline{\varepsilon}}^R$, $f_{2L}^R=g_{2L}^R$, $g_{2L}^{NR}=f_{2L}^{NR}$, $g_{2-\overline{\varepsilon}}^{NR}=f_{2-\overline{\varepsilon}}^{NR}$; and in terms of utility of public good we assume that the same number of individuals λ enjoy public good F and G, we can simplify the last equation to:

$$\frac{\partial \mathrm{E}_0\{U\}}{\partial k}=\frac{1}{2\overline{\varepsilon}}\left\{\begin{array}{c}-\left[(\eta-I)\gamma+(1-\theta)\Gamma\overline{y}\right]\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial k}+\frac{\partial\varepsilon_L}{\partial k}\right)+(\overline{\varepsilon}-\varepsilon_L)\gamma\frac{\partial\eta}{\partial k}\\ -\frac{2\varepsilon_L}{2}(1-\theta)\Gamma\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial k}+\frac{\partial\varepsilon_L}{\partial k}\right)+\\ \lambda\left[\begin{array}{c}v\left(f_{1\overline{\varepsilon}}^R\right)\frac{\partial\overline{\varepsilon}}{\partial k}-v\left(f_{1L}^R\right)\frac{\partial\varepsilon_L}{\partial k}+\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\frac{\partial\left[v\left(f_1^R\right)\right]}{\partial k}d\varepsilon+v\left(f_{2\overline{\varepsilon}}^R\right)\frac{\partial\overline{\varepsilon}}{\partial k}-\\ v\left(f_{2L}^R\right)\frac{\partial\varepsilon_L}{\partial k}+\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\frac{\partial\left[v\left(f_2^R\right)\right]}{\partial k}d\varepsilon+v\left(g_{1L}^{NR}\right)\frac{\partial\varepsilon_L}{\partial k}-v\left(g_{1-\overline{\varepsilon}}^{NR}\right)\frac{\partial(-\overline{\varepsilon})}{\partial k}+\\ \int\limits_{-\overline{\varepsilon}}^{\varepsilon_L}\frac{\partial\left[v\left(g_1^{NR}\right)\right]}{\partial k}d\varepsilon+v\left(g_{2L}^{NR}\right)\frac{\partial\varepsilon_L}{\partial k}-v\left(g_{2-\overline{\varepsilon}}^{NR}\right)\frac{\partial(-\overline{\varepsilon})}{\partial k}+\int\limits_{-\overline{\varepsilon}}^{\varepsilon_L}\frac{\partial\left[v\left(g_2^{NR}\right)\right]}{\partial k}d\varepsilon\end{array}\right]\end{array}\right\},$$

Since $\frac{\partial \overline{\varepsilon}}{\partial k}=0, \frac{\partial \varepsilon_L}{\partial k}=0$, and knowing (3.39), we can rewrite the last expression as:

$$\frac{\partial \mathrm{E}_0\{U\}}{\partial k} = \frac{1}{2\overline{\varepsilon}}\left\{\begin{array}{c} -\left[(\eta-I)\gamma+(1-\theta)\Gamma\overline{y}\right]\left(-\frac{\gamma}{(1-\theta)y^m\Gamma}\right)\frac{\partial\eta}{\partial k}+(\overline{\varepsilon}-\varepsilon_L)\gamma\frac{\partial\eta}{\partial k} \\ -\varepsilon_L(1-\theta)\Gamma\overline{y}\left(-\frac{\gamma}{(1-\theta)y^m\Gamma}\right)\frac{\partial\eta}{\partial k} \\ \lambda\left[\begin{array}{c} -v\left(f_{1L}^R\right)\left(-\frac{\gamma}{(1-\theta)y^m\Gamma}\right)\frac{\partial\eta}{\partial k}+\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}v'\left(f_1^R\right)\left(-\gamma\frac{\partial\eta}{\partial k}+\frac{\partial b^R}{\partial k}\right)d\varepsilon \\ -v\left(f_{2L}^R\right)\left(-\frac{\gamma}{(1-\theta)y^m\Gamma}\right)\frac{\partial\eta}{\partial k}+\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}v'\left(f_2^R\right)\left(-\frac{\partial b^R}{\partial k}\right)d\varepsilon+ \\ v\left(g_{1L}^{NR}\right)\left(-\frac{\gamma}{(1-\theta)y^m\Gamma}\right)\frac{\partial\eta}{\partial k}+\int\limits_{-\overline{\varepsilon}}^{\varepsilon_L}v'\left(g_1^{NR}\right)\left(+\frac{\partial b^{NR}}{\partial k}\right)d\varepsilon+ \\ v\left(g_{2L}^{NR}\right)\left(-\frac{\gamma}{(1-\theta)y^m\Gamma}\right)\frac{\partial\eta}{\partial k}+\int\limits_{-\overline{\varepsilon}}^{\varepsilon_L}v'\left(g_2^{NR}\right)\left(-\frac{\partial b^{NR}}{\partial k}\right)d\varepsilon \end{array}\right] \end{array}\right\} \Leftrightarrow \tag{3.96}$$

$$\frac{\partial \mathrm{E}_0\{U\}}{\partial k} = \frac{1}{2\overline{\varepsilon}}\left\{\begin{array}{c} \gamma\left\{\begin{array}{c} \frac{\partial\eta}{\partial k}\left[\frac{1}{(1-\theta)y^m\Gamma}\right]\left\{\begin{array}{c}(1-\theta)\Gamma\overline{y}(1+\varepsilon_L)+(\eta-I)\gamma \\ +\lambda\left[\begin{array}{c} v\left(f_{1L}^R\right)+v\left(f_{2L}^R\right) \\ -v\left(g_{1L}^{NR}\right)-v\left(g_{2L}^{NR}\right)\end{array}\right]\end{array}\right\} \\ +\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\frac{\partial\eta}{\partial k}\left[1-\lambda v'\left(f_1^R\right)\right]d\varepsilon \end{array}\right\} \\ +\lambda\left[\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\frac{\partial b^R}{\partial k}\left[v'\left(f_1^R\right)-v'\left(f_2^R\right)\right]d\varepsilon+\int\limits_{-\overline{\varepsilon}}^{\varepsilon_L}\frac{\partial b^{NR}}{\partial k}\left[v'\left(g_1^{NR}\right)-v'\left(g_2^{NR}\right)\right]d\varepsilon\right] \end{array}\right\}.$$

3.F Numerical check of Proposition 3.1 for $p < 1$

Given that we are only able to formally prove Proposition 3.1 for p sufficiently close to 1, we perform a numerical check of the results of the proposition for values of $p<1$. We base this check on the specifications (3.25) for $v(.)$ and (3.26) for $\Gamma(.)$.

The baseline parameter combination for the calibration is the following: $\xi = 1.03$, $\overline{y}=10$, $p=\theta=\gamma=0.5$, $I=2$, $q=0.25$, $\overline{\varepsilon}=1.2$, $y^m=0.8y$, $\overline{b}=0.3$. This baseline is to some extent motivated by a rough assessment of the European situation, being one where deficit restrictions are actually implemented. Average income (GDP), $\overline{y}$, captures the size of the economy and can be freely chosen; ξ should be sufficiently close to unity to ensure positive marginal utilities for public consumption; with $q=0.25$, the resulting equilibrium value for γ would imply 8.4% of additional GDP in the long run compared to the case in which no reform takes place. This is in line with the existing estimates for the benefits from structural reform in Europe – see IMF (2003). In addition, the value of $\overline{\varepsilon}$ implies a standard deviation of $(1/3)^{0.5}\,\overline{\varepsilon}=0.69$.[27] Median income, y^m, is 80% of average income across the population, which is in line with data for the EU-25 – see Boix (2004). Finally, $\overline{b}$ equals 3% of average GDP, which is roughly in line with Europe's SGP (where the deficit limit, however, is expressed as a share of actual, rather than natural, GDP).

We confirm the validity of Proposition 3.1 for the baseline parameter combination for

[27]For an assessment of the magnitude of business cycle fluctuations in Europe, see Artis et al. (2003) or Mitchell and Mouratidis (2004).

$k = 0$ and $k = 0.2$. (The value of k should not be too high to ensure that there is still a deficit bias, which is the relevant situation for our purpose.) Next, we assess the robustness of the results by allowing parameters to deviate from their baseline values. In particular, we investigate the validity of Proposition 3.1 for all parameter combinations formed by the Carthesian product of the parameter sets $p \in \{0.35, 0.8\}$, $\theta \in \{0.25, 0.7\}$, $\gamma \in \{0.1, 2\}$, $I \in \{0.5, 4\}$, $\overline{\varepsilon} \in \{0.8, 1.6\}$, $q \in \{0.1, 0.5\}$, $y^m \in \{0.5\overline{y}, 0.95\overline{y}\}$, and $\overline{b} \in \{0.1, 1\}$, while the remaining parameters are kept at their baseline values.[28] The set of possible parameter combinations covers a wide range of the relevant parameter space. We find that for all possible parameter combinations, both at $k = 0$ and $k = 0.2$, a small increase in k reproduces all results stated in Proposition 3.1.

3.G Extensions and a variation

In this appendix, we vary our basic model in a number of ways and explore how these changes affect the results.

3.G.1 A contingent deficit restriction

As we have seen, while our deficit restriction improves the intertemporal allocation of public spending, its disadvantage is that it discourages structural reform, by making it more expensive for the government to provide up-front compensation. The question then is whether it is possible to device a restriction that limits its possible damage to the likelihood of structural reform. Therefore, we consider a slightly more complicated arrangement with

$$\overline{b} = \overline{b^c} + \delta_1 \eta - \delta_2 \varepsilon, \quad \delta_1, \delta_2 \geq 0, \tag{3.97}$$

so that the *actual* reference deficit level $\overline{b}$ is now a combination of a constant component $\overline{b^c}$ and two components that take account of the compensation expenditures and of the macroeconomic shock (the "business cycle"). Both larger compensation outlays η and a worsening of the business cycle (a fall in ε) lead to an increase in the reference deficit level. This arrangement reflects some features of the recent reform of the SGP, which in its implementation now takes more explicitly into account both the short-run costs of structural reform as well as the business cycle situation (see European Commission, 2005b, and ECOFIN, 2005).

[28] We want to confine ourselves to parameter combinations under which the initial incumbent would benefit from the implementation of the reform (and would have an incentive in the first place to introduce reform). This requires p to be larger than 30%. If p is too low, the chance that the initial incumbent would be able to spend the additional second-period resources associated with reform would become too low.

With our contingent deficit restriction (3.97), the first-order condition for η becomes:

$$\left\{ \begin{array}{c} v\left(f_{1L}^{R}\right)+pv\left(f_{2L}^{R}\right)- \\ k\left(b_{L}^{R}-\overline{b^{c}}-\delta_{1}\eta+\delta_{2}\varepsilon_{L}\right)-(1-p)\,v\left(f_{2L}^{NR}\right) \end{array} \right\} = (1-\theta)\,y^{m}\Gamma(\gamma)\left[\int_{\varepsilon_{L}}^{\overline{\varepsilon}} v'[f_{1}^{R}]d\varepsilon - \frac{k\delta_{1}}{\gamma}\left(\overline{\varepsilon}-\varepsilon_{L}\right)\right]. \tag{3.98}$$

This last expression yields at most one solution when p is sufficiently large and both δ_1 and δ_2 are small enough. The implications of making the deficit restriction more flexible for given k are summarized in the following proposition:

Proposition 3.4 *Let p be sufficiently large. If $\delta_1 > 0$ ($\delta_2 > 0$) is small enough, then a stronger contingency of the reference deficit level on reform compensation or the business cycle (i.e., a higher δ_1, respectively δ_2) leads to an increase in the amount of compensation and improves the likelihood of re-election of the initial incumbent and thus of reform. Moreover, for any given shock ε, an increase in δ_1 (δ_2) implies a higher deficit in case the initial incumbent is re-elected and reform materializes.*

Proof. See Appendices 3.H.1 and 3.H.1. ■

By raising the reference deficit level in response to reform-related compensation, the initial government is induced to provide more compensation, because the marginal cost associated with the additional debt needed to spread the compensation over time is smaller. Of course, the enhanced likelihood of reform is bought at the cost of a higher deficit when reform actually takes place. Similarly, if the reference deficit level is raised in response to a bad shock ($\varepsilon < 0$), the initial government is induced to offer more compensation since any compensation that has to be paid under relatively bad economic circumstances will hurt less if the marginal cost of issuing debt is reduced.

Of course, proper implementation of a contingent deficit restriction in reality requires more detailed budgetary and macroeconomic information than a non-contingent restriction. For example, the enforcer of the restriction needs to identify what share of spending was specifically needed as compensation for the reform and what was the size of the original output shock (rather than the output movement itself, which is subject to the policies followed by the government). Further, and in relation to these requirements, the arrangement should be designed in such a way that governments have no incentive to abuse the enhanced flexibility of the restriction, for example by disguising government consumption as reform expenditure.

Given that we can only formally prove Proposition 3.4 when p is sufficiently close to 1, we resort again to a numerical evaluation. The numerical results confirm the effects described in Proposition 3.4 of an increase in the contingency parameter δ_1 or δ_2 (starting at $\delta_1 = 0$, respectively $\delta_2 = 0$) for all parameter combinations considered earlier.

3.G.2 Targeting compensation

In the main text we assumed that every individual receives the same amount of compensation. From the perspective of the initial incumbent it is suboptimal to disregard the income distribution when deciding about compensation. Some people (those with sufficiently high incomes) would in any case vote for the incumbent, whether they receive compensation or not. Others would in any case *not* vote for the government, unless they receive extremely high compensation. Loosely speaking, the best strategy would be to target compensation only at those who are likely to change their voting behavior as a result of the compensation. That is, the government should target compensation at those who are not too far towards the extremes of the income distribution. We show that, under certain conditions, if the government targets compensation, its probability of re-election is still given by (3.17), implying that the preceding analysis carries through (although the equilibrium outcome for compensation is now lower).

To this end, let us rank individuals by income and index them by $i \in [0,1]$, where the lowest value of i corresponds to the individuals with the lowest income. Compare the argument in the utility functions in (3.4) and (3.5), and define $\underline{i}$ such that, if $i = \underline{i}$, then $(1-\theta)\, y_i(1+\varepsilon)\Gamma + (\eta - I)\,\gamma = 0$ and $\overline{i}$ such that, if $i = \overline{i}$, then $(1-\theta)\, y_i(1+\varepsilon)\Gamma - I\gamma = 0$. Hence, for given values of compensation η and income shock $\varepsilon \in [-\overline{\varepsilon}, \overline{\varepsilon}]$, we can divide individuals into the following three intervals:[29]

- Individuals in the interval $[0, \underline{i}]$ are net losers from the reform.
- Individuals in the interval $\left[\underline{i}, \overline{i}\right]$ have a net benefit from the reform *as a result of the compensation.*
- Individuals in the interval $\left[\overline{i}, 1\right]$ are net beneficiaries from the reform also without compensation ($\eta = 0$).

For the specific case in which individual income increases linearly in the index:

$$y_i \equiv y(i) = y.i,$$

and combining this with the assumption that the individuals who do receive compensation from the government, all receive the same amount, we find that:

$$\underline{i} = \frac{(I-\eta)\,\gamma}{(1-\theta)\,y(1+\varepsilon)\Gamma} \text{ and } \overline{i} = \frac{I\gamma}{(1-\theta)\,y(1+\varepsilon)\Gamma}.$$

Hence, the larger is the reform benefit Γ or the smaller is the marginal reform cost I, the smaller is the interval $\left[\underline{i}, \overline{i}\right]$ and thus the number of individuals to be compensated in order to switch to supporting the initial incumbent and thus the reform.

[29] This requires that for all possible realizations of ε, $\lambda < \underline{i}$ and $1 - \lambda > 1 - \overline{i}$, while, of course, the condition in Footnote 10 should continue to hold.

Finally, denoting the density of individuals on the interval $[0,1]$ by $h(i)$, we can compute the share α of the population to which compensation should be targeted as:

$$\alpha = \int_{\underline{i}}^{\overline{i}} h(i)\, di.$$

Total compensation provided by the initial incumbent is then $\eta\gamma\alpha$. Writing $\alpha = \alpha(\eta, \varepsilon)$, the probability that Party F is re-elected in period 1 becomes
$\Pr(R) = \Pr\left[\alpha(\eta,\varepsilon) + \int_{\overline{i}}^{1} h(i)\, di \geq 0.5\right]$. Finally, if we assume that individuals are uniformly distributed on the interval $[0,1]$, then $\alpha = \overline{i} - \underline{i}$ and

$$\Pr(R) = \Pr\left[1 - \underline{i} \geq 0.5\right] = \Pr\left[\varepsilon \geq -\frac{2(\eta - I)\gamma}{(1-\theta)\, y\Gamma} - 1\right],$$

which, for given η, is equal to (3.17).[30] Therefore, when $\varepsilon = \varepsilon_L$, $\underline{i}$ represents the median voter. Of course, one should realize that, while for given η, ε_L is the same as before, total compensation will be lower when compared to the baseline case in which compensation was not targeted. Hence, for a given total outlay on compensation, a larger number of individuals can now be persuaded to switch towards favoring reforms.

3.G.3 Pecuniary sanctions

In the final extension we model the deficit restriction as an explicit fine in the government budget constraint. We allow for monetary sanctions in period 2, that is, sanctions are ex-post to the realization of the deficit in the first period. For example, in the case of Europe's SGP, fines can only materialize some time (at least two years) after the excessive deficit violation has taken place. We now drop the cost of the deficit restriction from the government's utility (3.6) – that is, we set $\Delta_F = 0$. Hence, the second-period government budget constraints in the case of reform and no-reform become:

$$f_2^R + g_2^R = \theta\overline{y}(1+\varepsilon)(1+\Gamma) - b^R - k\left(b^R - \overline{b}\right), \quad f_2^{NR} + g_2^{NR} = \theta\overline{y}(1+\varepsilon) - b^{NR} - k\left(b^{NR} - \overline{b}\right). \tag{3.99}$$

when $b^R \geq \overline{b}$ and $b^{NR} \geq \overline{b}$, while the final term in both constraints drops out when the deficit is below $\overline{b}$. As before, we assume that this reference level $\overline{b}$ is set sufficiently low, so that the equilibrium deficit level exceeds its reference level. The new formulation corresponds partly to the operation of the SGP in Europe, where sanctions for excessive deficits can culminate into fines imposed on the governments. Obviously, the comparison can only be limited. For example, the fines under the SGP are only linear in the degree of excessiveness of the deficit over a certain range, while here they are linear over the entire range of deficits above $\overline{b}$. The linearity assumption obviously simplifies matters a lot and allows us to obtain analytical results. The optimal sanction scheme in this framework could well be non-linear. However, in reality, there is a strong case for simple schemes,

[30] Note that, in this case, $\overline{y} = y^m$.

both for the understanding of the general public and for the politicians' understanding of the consequences of their fiscal behavior.

With a new utility function and government budget constraints, the first-order conditions for the optimal deficit, when parties F and G, respectively, assume power in period 1 are:

$$v'\left(f_1^R\right) = (1+k)\,pv'\left(f_2^R\right), \quad v'\left(g_1^{NR}\right) = (1+k)\,pv'\left(g_2^{NR}\right), \tag{3.100}$$

with f_1^R and g_1^{NR} as described before and f_2^R and g_2^{NR} as given in (3.99) above. We see that an increase in the tightness of sanctions, k, causes a shift in public spending away from the first period towards the second period. This is intuitive, because an increase in k raises the cost of running a deficit and thus, effectively, makes first-period public spending more expensive relative to second-period public spending. Appendix 3.I shows that also the other results of Lemma 3.1 continue to hold.

To find the overall effect of an increase in k on the deficit (taking account of the effect via η), we first need to find the optimal level of compensation. The new first-order condition for η differs only slightly from (3.23) – see Appendix 3.I. By differentiating the first-order conditions further, one can again check that the effects of an increase in k, as stated in Proposition 3.1, continue to hold.

3.H A contingent deficit restriction – derivations

With (3.97) we have:

$$\max_{\eta} \frac{1}{2\overline{\varepsilon}} \left\{ \begin{array}{c} \int_{\varepsilon_L}^{\overline{\varepsilon}} v\left(\theta\overline{y}(1+\varepsilon) - \eta\gamma + b^R\right) d\varepsilon + p. \int_{\varepsilon_L}^{\overline{\varepsilon}} v\left(\theta\overline{y}(1+\varepsilon)(1+\Gamma) - b^R\right) d\varepsilon + \\ \int_{\varepsilon_L}^{\overline{\varepsilon}} -k\left(b^R - \overline{b^c} - \delta_1\eta + \delta_2\varepsilon\right) d\varepsilon + \int_{-\overline{\varepsilon}}^{\varepsilon_L} 0 d\varepsilon + (1-p)\,. \int_{-\overline{\varepsilon}}^{\varepsilon_L} v\left(\theta\overline{y}(1+\varepsilon) - b^{NR}\right) d\varepsilon \end{array} \right\}.$$

We use the same procedure as before, maximizing this last expression on η and applying Leibnitz's rule. Using again the facts that $\frac{\partial\overline{\varepsilon}}{\partial\eta} = 0$, $v'\left(f_1^R\right) - pv'\left(f_2^R\right) - k = 0$, $\frac{\partial b^{NR}}{\partial\eta} = 0$ and using (3.20) to (3.22), we derive the new first-order condition:

$$-\frac{\partial\varepsilon_L}{\partial\eta}\left[v\left(f_{1L}^R\right) + pv\left(f_{2L}^R\right) - k\left(b_L^R - \overline{b^c} - \delta_1\eta + \delta_2\varepsilon_L\right) - (1-p)\,v\left(f_{2L}^{NR}\right)\right] + k\delta_1\left(\overline{\varepsilon} - \varepsilon_L\right)$$
$$= \gamma \int_{\varepsilon_L}^{\overline{\varepsilon}} v'\left(f_1^R\right) d\varepsilon.$$

Using (3.39) and simplifying the last expression, we arrive to (3.98).

3.H.1 Comparative statics

Effect of δ_1 on η, b^R , b^{NR}

We differentiate (3.98) with respect to δ_1:

$$
\begin{aligned}
& v'\left(f_{1L}^R\right)\left[\theta\overline{y}\left(\tfrac{\partial\varepsilon_L}{\partial\eta}\tfrac{\partial\eta}{\partial\delta_1}+\tfrac{\partial\varepsilon_L}{\partial\delta_1}\right)-\gamma\tfrac{\partial\eta}{\partial\delta_1}+\tfrac{\partial b_L^R}{\partial\delta_1}\right]+pv'\left(f_{2L}^R\right)\left[\theta\overline{y}\left(\tfrac{\partial\varepsilon_L}{\partial\eta}\tfrac{\partial\eta}{\partial\delta_1}+\tfrac{\partial\varepsilon_L}{\partial\delta_1}\right)(1+\Gamma)-\tfrac{\partial b_L^R}{\partial\delta_1}\right] \\
& -k\left[\tfrac{\partial b_L^R}{\partial\delta_1}-\eta-\delta_1\tfrac{\partial\eta}{\partial\delta_1}+\delta_2\left(\tfrac{\partial\varepsilon_L}{\partial\eta}\tfrac{\partial\eta}{\partial\delta_1}+\tfrac{\partial\varepsilon_L}{\partial\delta_1}\right)\right]-(1-p)\,v'\left(f_{2L}^{NR}\right)\left\{\theta\overline{y}\left(\tfrac{\partial\varepsilon_L}{\partial\eta}\tfrac{\partial\eta}{\partial\delta_1}+\tfrac{\partial\varepsilon_L}{\partial\delta_1}\right)-\tfrac{\partial b_L^{NR}}{\partial\delta_1}\right\} \\
& -(1-\theta)\,y^m\Gamma\left\{\begin{array}{c} v'\left(f_{1\overline{\varepsilon}}^R\right)\tfrac{\partial\overline{\varepsilon}}{\partial\delta_1}-v'\left(f_{1L}^R\right)\left(\tfrac{\partial\varepsilon_L}{\partial\eta}\tfrac{\partial\eta}{\partial\delta_1}+\tfrac{\partial\varepsilon_L}{\partial\delta_1}\right)+\int_{\varepsilon_L}^{\overline{\varepsilon}}v"\left(f_1^R\right)\left(-\gamma\tfrac{\partial\eta}{\partial\delta_1}+\tfrac{\partial b^R}{\partial\delta_1}\right)d\varepsilon \\ -\tfrac{k}{\gamma}\left(\overline{\varepsilon}-\varepsilon_L\right)+\tfrac{k\delta_1}{\gamma}\left(\tfrac{\partial\varepsilon_L}{\partial\eta}\tfrac{\partial\eta}{\partial\delta_1}+\tfrac{\partial\varepsilon_L}{\partial\delta_1}\right)\end{array}\right\} \\
= \; & 0 \Leftrightarrow
\end{aligned}
$$

$$
\begin{aligned}
& \frac{\partial\eta}{\partial\delta_1}\left\{-\frac{\theta\overline{y}\gamma}{(1-\theta)\,y^m\Gamma}\left[\begin{array}{c} v'\left(f_{1L}^R\right)+(1+\Gamma)\,pv'\left(f_{2L}^R\right) \\ -k\delta_2-(1-p)\,v'\left(f_{2L}^{NR}\right)\end{array}\right]+2k\delta_1-2\gamma v'\left(f_{1L}^R\right)\right\} \\
& +\frac{\partial b_L^R}{\partial\delta_1}\left[v'\left(f_{1L}^R\right)-pv'\left(f_{2L}^R\right)-k\right]+\frac{\partial b_L^{NR}}{\partial\delta_1}\left[v'\left(f_{2L}^{NR}\right)(1-p)\right] \\
& -(1-\theta)\,y^m\Gamma\int_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma\frac{\partial\eta}{\partial\delta_1}+\frac{\partial b^R}{\partial\delta_1}\right)v"\left(f_1^R\right)d\varepsilon \\
= \; & -k\eta-\frac{(1-\theta)\,y^m\Gamma k}{\gamma}\left(\overline{\varepsilon}-\varepsilon_L\right).
\end{aligned}
$$

This latter expression can be represented in the same way as in (3.40) by:

$$
A_{1L}'\frac{\partial\eta}{\partial\delta_1}-A_2\int_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma\frac{\partial\eta}{\partial\delta_1}+\frac{\partial b^R}{\partial\delta_1}\right)v"\left(f_1^R\right)d\varepsilon+A_{3L}\frac{\partial b_L^{NR}}{\partial\delta_1}=M_{1L}, \tag{3.101}
$$

with the new parameters:[31]

$$
\begin{aligned}
A_{1L}' &= -\tfrac{\theta\overline{y}\gamma}{(1-\theta)y^m\Gamma[\gamma]}\left[\begin{array}{c} v'\left(f_{1L}^R\right)+(1+\Gamma)\,pv'\left(f_{2L}^R\right) \\ -k\delta_2-(1-p)\,v'\left(f_{2L}^{NR}\right)\end{array}\right]+2k\delta_1-2\gamma v'\left(f_{1L}^R\right)<0, & (3.102) \\
M_{1L} &= -k\eta-\frac{(1-\theta)\,y^m\Gamma k}{\gamma}\left(\overline{\varepsilon}-\varepsilon_L\right)<0. & (3.103)
\end{aligned}
$$

Effect on b^R Differentiating (3.27) with respect to δ_1, we have:

$$
v"\left(f_1^R\right)\left(-\gamma\frac{\partial\eta}{\partial\delta_1}+\frac{\partial b^R}{\partial\delta_1}\right)-pv"\left(f_2^R\right)\left(-\frac{\partial b^R}{\partial\delta_1}\right)=0.
$$

This last expression can be represented by:

$$
B_1\frac{\partial\eta}{\partial\delta_1}+B_2\frac{\partial b^R}{\partial\delta_1}=0. \tag{3.104}
$$

[31] The sign of A_{1L}' holds if p is sufficiently different from 0, and δ_2 and δ_1 are small enough.

Effect on b^{NR} Differentiating (3.49) with respect to δ_1:

$$v"\left(g_{1L}^{NR}\right)\left[\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\delta_1}+\frac{\partial\varepsilon_L}{\partial\delta_1}\right)+\frac{\partial b_L^{NR}}{\partial\delta_1}\right]-pv"\left(g_{2L}^{NR}\right)\left[\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\delta_1}+\frac{\partial\varepsilon_L}{\partial\delta_1}\right)-\frac{\partial b_L^{NR}}{\partial\delta_1}\right]=0,$$

and, since $\frac{\partial\varepsilon_L}{\partial\delta_1}=0$ and using (3.39), we can represent the last as:

$$C_{1L}\frac{\partial\eta}{\partial\delta_1}+C_{2L}\frac{\partial b_L^{NR}}{\partial\delta_1}=0. \tag{3.105}$$

Solution of the system We can write the system (3.101), (3.104), and (3.105) as:

$$\left\{\begin{array}{ccccccc} A'_{1L}\frac{\partial\eta}{\partial\delta_1} & +0 & -A_2\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma\frac{\partial\eta}{\partial\delta_1}+\frac{\partial b^R}{\partial\delta_1}\right)v"\left(f_1^R\right)d\varepsilon & +A_{3L}\frac{\partial b_L^{NR}}{\partial\delta_1} & = & M_{1L} \\ B_1\frac{\partial\eta}{\partial\delta_1} & +B_2\frac{\partial b^R}{\partial\delta_1} & +0 & +0 & = & 0 \\ C_{1L}\frac{\partial\eta}{\partial\delta_1} & +0 & +0 & +C_{2L}\frac{\partial b_L^{NR}}{\partial\delta_1} & = & 0 \end{array}\right. .$$

Further, we can rewrite (3.104) and (3.105) as

$$\frac{\partial b^R}{\partial\delta_1}=-\frac{B_1}{B_2}\frac{\partial\eta}{\partial\delta_1},\quad \text{and}\quad \frac{\partial b_L^{NR}}{\partial\delta_1}=-\frac{C_{1L}}{C_{2L}}\frac{\partial\eta}{\partial\delta_1}. \tag{3.106}$$

Substituting these two equations in (3.101), we have:

$$A'_{1L}\frac{\partial\eta}{\partial\delta_1}-A_2\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\frac{\partial\eta}{\partial\delta_1}\left(-\gamma-\frac{B_1}{B_2}\right)v"\left(f_1^R\right)d\varepsilon-\frac{A_{3L}C_{1L}}{C_{2L}}\frac{\partial\eta}{\partial\delta_1}=M_{1L}. \tag{3.107}$$

Sign of $\frac{\partial\eta}{\partial\delta_1}$: Substituting the values of (3.42), (3.43), (3.46), (3.47), (3.53), (3.54), (3.102) and (3.103), we are able to rewrite (3.107) as:

$$\frac{\partial\eta}{\partial\delta_1}\left\{\begin{array}{c} A'_{1L}-\frac{A_{3L}C_{1L}}{C_{2L}}- \\ A_2\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\left[\frac{-\gamma\left(v"\left(f_1^R\right)\right)^2-\gamma v"\left(f_1^R\right)pv"\left(f_2^R\right)+\gamma\left(v"\left(f_1^R\right)\right)^2}{v"\left(f_1^R\right)+pv"\left(f_2^R\right)}\right]d\varepsilon \end{array}\right\}=M_{1L}\Leftrightarrow$$

$$\frac{\partial\eta}{\partial\delta_1}\left\{\begin{array}{c} -\frac{\theta\overline{y}\gamma}{(1-\theta)y^m\Gamma[\gamma]}\left[\begin{array}{c} v'\left(f_{1L}^R\right)+(1+\Gamma)\,pv'\left(f_{2L}^R\right) \\ -k\delta_2-(1-p)\,v'\left(f_{2L}^{NR}\right)\end{array}\right]+ \\ 2k\delta_1-2\gamma v'\left(f_{1L}^R\right)+\frac{(1-p)\theta\overline{y}\gamma v'\left(f_{2L}^{NR}\right)\left[v"\left(g_{1L}^{NR}\right)-pv"\left(g_{2L}^{NR}\right)\right]}{(1-\theta)y^m\Gamma\left[v"\left(g_{1L}^{NR}\right)+pv"\left(g_{2L}^{NR}\right)\right]} \\ +(1-\theta)\,y^m\Gamma\gamma\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\frac{v"\left(f_1^R\right)pv"\left(f_2^R\right)}{v"\left(f_1^R\right)+pv"\left(f_2^R\right)}d\varepsilon \end{array}\right\}=\left\{\begin{array}{c} -k\eta- \\ \frac{(1-\theta)y^m\Gamma k}{\gamma}\left(\overline{\varepsilon}-\varepsilon_L\right)\end{array}\right\}. \tag{3.108}$$

The overall sign of the term in brackets on the left-hand side of (3.108) is negative if p is sufficiently different from 0, and δ_2 and δ_1 are small enough. The right-hand side of that expression is also negative. Hence, for p sufficiently different from 0, $\frac{\partial\eta}{\partial\delta_1}>0$. Further, given (3.106), we know that $\frac{\partial b^R}{\partial\delta_1}>0$ and, in addition, if we suppose that $v"\left(g_{1L}^{NR}\right)-pv"\left(g_{2L}^{NR}\right)<0$, then $\frac{\partial b_L^{NR}}{\partial\delta_1}>0$.

Welfare Effect: The welfare effect of δ_1 can be found by differentiating (3.24) with respect to δ_1:

$$\frac{\partial \mathrm{E}_0\{U\}}{\partial \delta_1} = \frac{1}{2\overline{\varepsilon}} \left\{ \begin{array}{c} \gamma \left\{ \begin{array}{c} \frac{\partial \eta}{\partial \delta_1}\left[\frac{1}{(1-\theta)y^m\Gamma}\right]\left[\begin{array}{c}(1-\theta)\Gamma\overline{y}(1+\varepsilon_L)+(\eta-I)\gamma \\ +v\left[f_{1L}^R\right]+v\left[f_{2L}^R\right]-v\left[g_{1L}^{NR}\right]-v\left[g_{2L}^{NR}\right]\end{array}\right] \\ +\int_{\varepsilon_L}^{\overline{\varepsilon}} \frac{\partial \eta}{\partial \delta_1}\left[1-v'\left(f_1^R\right)\right]d\varepsilon \end{array}\right\} \\ +\int_{\varepsilon_L}^{\overline{\varepsilon}} \frac{\partial b^R}{\partial \delta_1}\left[v'\left(f_1^R\right)-v'\left(f_2^R\right)\right]d\varepsilon + \int_{-\overline{\varepsilon}}^{\varepsilon_L} \frac{\partial b^{NR}}{\partial \delta_1}\left[v'\left(g_1^{NR}\right)-v'\left(g_2^{NR}\right)\right]d\varepsilon \end{array}\right\}.$$

The sign of this derivative is ambiguous and will be solved numerically.

Effect of δ_2 on η, b^R and b^{NR} under a contingent deficit restriction

Differentiating (3.98) with respect to δ_2:

$$\begin{array}{l} v'\left(f_{1L}^R\right)\left[\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\delta_2}+\frac{\partial\varepsilon_L}{\partial\delta_2}\right)-\gamma\frac{\partial\eta}{\partial\delta_2}+\frac{\partial b_L^R}{\partial\delta_2}\right]+pv'\left(f_{2L}^R\right)\left[\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\delta_2}+\frac{\partial\varepsilon_L}{\partial\delta_2}\right)(1+\Gamma)-\frac{\partial b_L^R}{\partial\delta_2}\right] \\ -k\left[\frac{\partial b_L^R}{\partial\delta_2}-\delta_1\frac{\partial\eta}{\partial\delta_2}+\varepsilon_L+\delta_2\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\delta_2}+\frac{\partial\varepsilon_L}{\partial\delta_2}\right)\right]-(1-p)\,v'\left(f_{2L}^{NR}\right)\left\{\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\delta_2}+\frac{\partial\varepsilon_L}{\partial\delta_2}\right)-\frac{\partial b_L^{NR}}{\partial\delta_2}\right\} \\ -(1-\theta)\,y^m\Gamma\left\{\begin{array}{c} v'\left(f_{1\overline{\varepsilon}}^R\right)\frac{\partial\overline{\varepsilon}}{\partial\delta_2}-v'\left(f_{1L}^R\right)\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\delta_2}+\frac{\partial\varepsilon_L}{\partial\delta_2}\right)+ \\ \int_{\varepsilon_L}^{\overline{\varepsilon}} v"\left(f_1^R\right)\left(-\gamma\frac{\partial\eta}{\partial\delta_2}+\frac{\partial b^R}{\partial\delta_2}\right)d\varepsilon+\frac{k\delta_1}{\gamma}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\delta_2}+\frac{\partial\varepsilon_L}{\partial\delta_2}\right)\end{array}\right\} \\ = \; 0 \Leftrightarrow \end{array}$$

$$\begin{array}{l} \frac{\partial\eta}{\partial\delta_2}\left\{-\frac{\theta\overline{y}\gamma}{(1-\theta)\,y^m\Gamma}\left[\begin{array}{c} v'\left(f_{1L}^R\right)+(1+\Gamma)\,pv'\left(f_{2L}^R\right) \\ -k\delta_2-(1-p)\,v'\left(f_{2L}^{NR}\right)\end{array}\right]+2k\delta_1-2\gamma v'\left(f_{1L}^R\right)\right\} \\ +\frac{\partial b_L^R}{\partial\delta_2}\left[v'\left(f_{1L}^R\right)-pv'\left(f_{2L}^R\right)-k\right]+\frac{\partial b_L^{NR}}{\partial\delta_2}\left[v'\left(f_{2L}^{NR}\right)(1-p)\right] \\ -(1-\theta)\,y^m\Gamma\int_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma\frac{\partial\eta}{\partial\delta_2}+\frac{\partial b^R}{\partial\delta_2}\right)v"\left(f_1^R\right)d\varepsilon \\ = \; k\varepsilon_L. \end{array}$$

This latter expression can be represented in the same way as in (3.101) by:

$$A_{1L}'\frac{\partial\eta}{\partial\delta_2}-A_2\int_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma\frac{\partial\eta}{\partial\delta_2}+\frac{\partial b^R}{\partial\delta_2}\right)v"\left(f_1^R\right)d\varepsilon+A_{3L}\frac{\partial b_L^{NR}}{\partial\delta_2}=N_{1L}, \tag{3.109}$$

with A_{1L}' is given by (3.102) and

$$N_{1L}=k\varepsilon_L<0. \tag{3.110}$$

Effect on b^R Differentiating (3.27) with respect to δ_2, we have:

$$v"\left(f_1^R\right)\left(-\gamma\frac{\partial\eta}{\partial\delta_2}+\frac{\partial b^R}{\partial\delta_2}\right)-pv"\left(f_2^R\right)\left(-\frac{\partial b^R}{\partial\delta_2}\right)=0.$$

This last expression can be represented by:

$$B_1 \frac{\partial \eta}{\partial \delta_2} + B_2 \frac{\partial b^R}{\partial \delta_2} = 0. \tag{3.111}$$

Effect on b^{NR} Differentiating (3.49) with respect to δ_2:

$$v"\left(g_{1L}^{NR}\right)\left[\theta \overline{y}\left(\frac{\partial \varepsilon_L}{\partial \eta}\frac{\partial \eta}{\partial \delta_2} + \frac{\partial \varepsilon_L}{\partial \delta_2}\right) + \frac{\partial b_L^{NR}}{\partial \delta_2}\right] - pv"\left(g_{2L}^{NR}\right)\left[\theta \overline{y}\left(\frac{\partial \varepsilon_L}{\partial \eta}\frac{\partial \eta}{\partial \delta_2} + \frac{\partial \varepsilon_L}{\partial \delta_2}\right) - \frac{\partial b_L^{NR}}{\partial \delta_2}\right] = 0,$$

since $\frac{\partial \varepsilon_L}{\partial \delta_2} = 0$ and using (3.39), we can represent the last as:

$$C_{1L} \frac{\partial \eta}{\partial \delta_2} + C_{2L} \frac{\partial b_L^{NR}}{\partial \delta_2} = 0. \tag{3.112}$$

Solution of the system We can write the system (3.109), (3.111), and (3.112) as:

$$\left\{\begin{array}{cccccc} A'_{1L}\frac{\partial \eta}{\partial \delta_2} & +0 & -A_2 \int\limits_{\varepsilon_L}^{\overline{\varepsilon}} \left(-\gamma \frac{\partial \eta}{\partial \delta_2} + \frac{\partial b^R}{\partial \delta_2}\right) v"\left(f_1^R\right) d\varepsilon & +A_{3L}\frac{\partial b_L^{NR}}{\partial \delta_2} & = & N_{1L} \\ B_1 \frac{\partial \eta}{\partial \delta_2} & +B_2\frac{\partial b^R}{\partial \delta_2} & +0 & +0 & = & 0 \\ C_{1L}\frac{\partial \eta}{\partial \delta_2} & +0 & +0 & +C_{2L}\frac{\partial b_L^{NR}}{\partial \delta_2} & = & 0 \end{array}\right. .$$

Further, we can rewrite (3.111) and (3.112) as

$$\frac{\partial b^R}{\partial \delta_2} = -\frac{B_1}{B_2}\frac{\partial \eta}{\partial \delta_2}, \quad \text{and} \quad \frac{\partial b_L^{NR}}{\partial \delta_2} = -\frac{C_{1L}}{C_{2L}}\frac{\partial \eta}{\partial \delta_2}. \tag{3.113}$$

Substituting these two equations in (3.109), we have:

$$A'_{1L}\frac{\partial \eta}{\partial \delta_2} - A_2 \int\limits_{\varepsilon_L}^{\overline{\varepsilon}} \frac{\partial \eta}{\partial \delta_2}\left(-\gamma - \frac{B_1}{B_2}\right) v"\left(f_1^R\right) d\varepsilon - \frac{A_{3L}C_{1L}}{C_{2L}}\frac{\partial \eta}{\partial \delta_2} = N_{1L}. \tag{3.114}$$

Sign of $\frac{\partial \eta}{\partial \delta_2}$: Substituting the values of (3.42), (3.43), (3.46), (3.47), (3.53), (3.54), (3.102) and (3.110), we are able to rewrite (3.114) as:

$$\frac{\partial \eta}{\partial \delta_2}\left\{\begin{array}{c} -\frac{\theta \overline{y}\gamma}{(1-\theta)y^m\Gamma[\gamma]}\left[\begin{array}{c} v'\left(f_{1L}^R\right) + (1+\Gamma)\,pv'\left(f_{2L}^R\right) \\ -k\delta_2 - (1-p)\,v'\left(f_{2L}^{NR}\right)\end{array}\right] + \\ 2k\delta_1 - 2\gamma v'\left(f_{1L}^R\right) + \frac{(1-p)\theta\overline{y}\gamma v'\left(f_{2L}^{NR}\right)\left[v"\left(g_{1L}^{NR}\right) - pv"\left(g_{2L}^{NR}\right)\right]}{(1-\theta)y^m\Gamma\left[v"\left(g_{1L}^{NR}\right)+pv"\left(g_{2L}^{NR}\right)\right]} \\ +(1-\theta)\,y^m\Gamma\gamma\int\limits_{\varepsilon_L}^{\overline{\varepsilon}} \frac{v"\left(f_1^R\right)pv"\left(f_2^R\right)}{v"\left(f_1^R\right)+pv"\left(f_2^R\right)}d\varepsilon \end{array}\right\} = k\varepsilon_L. \tag{3.115}$$

The overall sign of the term in brackets on the left-hand side of (3.115) is negative if p is sufficiently different from 0, and δ_2 and δ_1 are small enough. The right-hand side of that expression is also negative. Hence, for p sufficiently different from 0, $\frac{\partial \eta}{\partial \delta_2} > 0$. Further, given (3.113), we know that $\frac{\partial b^R}{\partial \delta_2} > 0$ and, in addition, if we suppose that $v"\left(g_{1L}^{NR}\right) - pv"\left(g_{2L}^{NR}\right) < 0$, then $\frac{\partial b_L^{NR}}{\partial \delta_2} > 0$.

Welfare Effect: The welfare effect of δ_2 can be found by differentiating (3.24) with respect to δ_2:

$$\frac{\partial \mathrm{E}_0\left\{U\right\}}{\partial \delta_2} = \frac{1}{2\overline{\varepsilon}}\left\{\begin{array}{c} \gamma\left\{\begin{array}{c} \frac{\partial \eta}{\partial \delta_2}\left[\frac{1}{(1-\theta)y^m\Gamma}\right]\left[\begin{array}{c}(1-\theta)\Gamma\overline{y}\left(1+\varepsilon_L\right)+\left(\eta-I\right)\gamma \\ +v\left[f_{1L}^R\right]+v\left[f_{2L}^R\right]-v\left[g_{1L}^{NR}\right]-v\left[g_{2L}^{NR}\right]\end{array}\right] \\ +\int_{\varepsilon_L}^{\overline{\varepsilon}}\frac{\partial \eta}{\partial \delta_2}\left[1-v'\left(f_1^R\right)\right]d\varepsilon \end{array}\right\} \\ +\int_{\varepsilon_L}^{\overline{\varepsilon}}\frac{\partial b^R}{\partial \delta_2}\left[v'\left(f_1^R\right)-v'\left(f_2^R\right)\right]d\varepsilon+\int_{-\overline{\varepsilon}}^{\varepsilon_L}\frac{\partial b^{NR}}{\partial \delta_2}\left[v'\left(g_1^{NR}\right)-v'\left(g_2^{NR}\right)\right]d\varepsilon \end{array}\right\}.$$

The sign of this derivative is ambiguous and we solve it numerically.

3.I Fines in the government budget constraint – derivations

With the set up of Section 3.G.3, the new expected utility of party F (similar to party G) becomes:

$$\mathrm{E}_0\left[v\left(f_1\right)+v\left(f_2\right)\right].$$

Since the sanctions are imposed in the second period, the first-period government budget constraints do not change, while those for the second period are given by (3.99). Therefore, as in Appendices 3.A and 3.B, we maximize the above expected utility with respect to the deficit in case of no re-election of the initial incumbent in period 1 $\left(b^{NR}\right)$ and in case of its re-election in period 1 $\left(b^R\right)$ leading to, respectively:

$$\begin{aligned} v'\left(\theta\overline{y}(1+\varepsilon)-\eta\gamma+b^R\right) &= (1+k)*p*v'\left(\theta\overline{y}(1+\varepsilon)\left(1+\Gamma\right)-b^R-k\left(b^R-\overline{b}\right)\right), \text{ and} \\ v'\left(\theta\overline{y}(1+\varepsilon)+b^{NR}\right) &= (1+k)*p*v'\left(\theta\overline{y}(1+\varepsilon)-b^{NR}-k\left(b^{NR}-\overline{b}\right)\right). \end{aligned}$$

These two first-order conditions are also described in (3.100). We differentiate them with respect to k and η:

$$\begin{aligned} \frac{\partial b^R}{\partial k} &= \frac{pv'\left(f_2^R\right)}{v"\left(f_1^R\right)+(1+k)^2pv"\left(f_2^R\right)}-\frac{(1+k)\,pv"\left(f_2^R\right)\left(b^R-\overline{b}\right)}{v"\left(f_1^R\right)+(1+k)^2pv"\left(f_2^R\right)} \lesseqgtr 0, \\ \frac{\partial b^{NR}}{\partial k} &= \frac{pv'\left(g_2^{NR}\right)}{v"\left(g_1^{NR}\right)+(1+k)^2pv"\left(g_2^{NR}\right)}-\frac{(1+k)\,pv"\left(g_2^{NR}\right)\left(b^{NR}-\overline{b}\right)}{v"\left(g_1^{NR}\right)+(1+k)^2pv"\left(g_2^{NR}\right)} \lesseqgtr 0, \\ \frac{\partial b^R}{\partial \eta} &= \frac{\gamma v"\left(f_1^R\right)}{v"\left(f_1^R\right)+(1+k)^2pv"\left(f_2^R\right)} > 0,\ \frac{\partial b^{NR}}{\partial \eta}=0, \\ \frac{\partial b^R}{\partial p} &= \frac{(1+k)\,v'\left(f_2^R\right)}{v"\left(f_1^R\right)+(1+k)^2pv"\left(f_2^R\right)},\ \frac{\partial b^{NR}}{\partial p}=\frac{(1+k)\,v'\left(g_2^{NR}\right)}{v"\left(g_1^{NR}\right)+(1+k)^2pv"\left(g_2^{NR}\right)}. \end{aligned}$$

The signs of $\frac{\partial b^R}{\partial \eta}$ and $\frac{\partial b^{NR}}{\partial \eta}$ remain the same as those of (3.29) and (3.33). The effect of an increase in k on b^R and b^{NR} depends on whether $\left(b^R-\overline{b}\right)$ and $\left(b^{NR}-\overline{b}\right)$, respectively are greater than zero. We assumed that $\overline{b}$ is sufficiently tight that $b^R>\overline{b}$ and $b^{NR}>\overline{b}$, so that we obtain the same qualitative effects of k as in (3.28) and (3.32).

To find the optimal η, we now solve:

$$\max_{\eta} \frac{1}{2\overline{\varepsilon}} \left\{ \begin{array}{c} \int_{\varepsilon_L}^{\overline{\varepsilon}} v\left(\theta\overline{y}(1+\varepsilon)-\eta\gamma+b^R\right)d\varepsilon + p.\int_{\varepsilon_L}^{\overline{\varepsilon}} v\left(\theta\overline{y}(1+\varepsilon)\left(1+\Gamma\right)-b^R-k\left(b^R-\overline{b}\right)\right)d\varepsilon \\ +\int_{-\overline{\varepsilon}}^{\varepsilon_L} 0d\varepsilon + (1-p).\int_{-\overline{\varepsilon}}^{\varepsilon_L} v\left(\theta\overline{y}(1+\varepsilon)-b^{NR}-k\left(b^{NR}-\overline{b}\right)\right)d\varepsilon \end{array} \right\}.$$

We take the first-order condition and apply Leibnitz' rule. Thus, using the facts that $\frac{\partial\overline{\varepsilon}}{\partial\eta}=0$, $v'\left(f_1^R\right)-(1+k)\,pv'\left(f_2^R\right)=0$ as given by (3.100) and $\frac{\partial b^{NR}}{\partial\eta}=0$, the new first-order condition for η becomes:

$$-\frac{\partial\varepsilon_L}{\partial\eta}\left[v\left(f_{1L}^R\right)+pv\left(f_{2L}^R\right)-(1-p)\,v\left(f_{2L}^{NR}\right)\right]=\gamma\int_{\varepsilon_L}^{\overline{\varepsilon}} v'\left(f_1^R\right)d\varepsilon,$$

where f_{1L}^R is the same as (3.20) and $f_{2L}^R=\theta\overline{y}(1+\varepsilon_L)\left(1+\Gamma\right)-b_L^R-k\left(b_L^R-\overline{b}\right)$ and $f_{2L}^{NR}=\theta\overline{y}(1+\varepsilon_L)-b_L^{NR}-k\left(b_L^{NR}-\overline{b}\right)$.

Using (3.39) and simplifying the last expression, we arrive at:

$$v\left(f_{1L}^R\right)+pv\left(f_{2L}^R\right)-(1-p)\,v\left(f_{2L}^{NR}\right)=(1-\theta)\,y^m\Gamma\left(\gamma\right)\int_{\varepsilon_L}^{\overline{\varepsilon}} v'\left(f_1^R\right)d\varepsilon, \tag{3.116}$$

3.I.1 Effect of k on η, b^R, b^{NR} and $\Pr(R)$

Differentiate (3.116) with respect to k:

$$\begin{aligned} & v'\left(f_{1L}^R\right)\left[\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial k}+\frac{\partial\varepsilon_L}{\partial k}\right)-\gamma\frac{\partial\eta}{\partial k}+\frac{\partial b_L^R}{\partial k}\right]+ \\ & pv'\left(f_{2L}^R\right)\left[\theta\overline{y}\left(1+\Gamma\right)\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial k}+\frac{\partial\varepsilon_L}{\partial k}\right)-\frac{\partial b_L^R}{\partial k}(1+k)-\left(b_L^R-\overline{b}\right)\right]- \\ & (1-p)\,v'\left(f_{2L}^{NR}\right)\left[\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial k}+\frac{\partial\varepsilon_L}{\partial k}\right)-\frac{\partial b_L^{NR}}{\partial k}(1+k)-\left(b_L^{NR}-\overline{b}\right)\right]- \\ & (1-\theta)\,y^m\Gamma\left[-v'\left(f_{1L}^R\right)\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial k}+\frac{\partial\varepsilon_L}{\partial k}\right)+\int_{\varepsilon_L}^{\overline{\varepsilon}} v"\left(f_1^R\right)\left(-\gamma\frac{\partial\eta}{\partial k}+\frac{\partial b^R}{\partial k}\right)d\varepsilon\right] \\ = & \;0. \end{aligned}$$

This expression can be written as:

$$\begin{aligned}
&\frac{\partial\eta}{\partial k}\left\{\theta\overline{y}\frac{\partial\varepsilon_L}{\partial\eta}\left[v'\left(f_{1L}^R\right)+(1+\Gamma)\,pv'\left(f_{2L}^R\right)-(1-p)\,v'\left(f_{2L}^{NR}\right)\right]-\gamma v'\left(f_{1L}^R\right)\right\}\\
&+\frac{\partial b_L^{NR}}{\partial k}v'\left(f_{2L}^{NR}\right)(1-p)\,(1+k)\\
&+\theta\overline{y}\frac{\partial\varepsilon_L}{\partial k}\left[v'\left(f_{1L}^R\right)+(1+\Gamma)\,pv'\left(f_{2L}^R\right)-(1-p)\,v'\left(f_{2L}^{NR}\right)\right]+\frac{\partial b_L^R}{\partial k}\left[v'\left(f_{1L}^R\right)-pv'\left(f_{2L}^R\right)(1+k)\right]\\
&-(1-\theta)\,y^m\Gamma\left[-v'\left(f_{1L}^R\right)\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial k}+\frac{\partial\varepsilon_L}{\partial k}\right)+\int_{\varepsilon_L}^{\overline{\varepsilon}}v"[f_1^R]\left(-\gamma\frac{\partial\eta}{\partial k}+\frac{\partial b^R}{\partial k}\right)d\varepsilon\right]\\
=\;&pv'\left(f_{2L}^R\right)\left(b_L^R-\overline{b}\right)-(1-p)\,v'\left(f_{2L}^{NR}\right)\left(b_L^{NR}-\overline{b}\right).
\end{aligned}$$

Following the same steps as before, we obtain:

$$A_{1L}\frac{\partial\eta}{\partial k}-A_2\int_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma\frac{\partial\eta}{\partial k}+\frac{\partial b^R}{\partial k}\right)v"\left(f_1^R\right)d\varepsilon+A_{3L}\frac{\partial b_L^{NR}}{\partial k}=D"_{1L}, \tag{3.117}$$

where:

$$D"_{1L}=pv'\left(f_{2L}^R\right)\left(b_L^R-\overline{b}\right)-(1-p)\,v'\left(f_{2L}^{NR}\right)\left(b_L^{NR}-\overline{b}\right)\gtrless 0.$$

The coefficients (3.47) and (3.48) of equation (3.45) are also altered to:

$$B"_2 = v"\left(f_1^R\right)+(1+k)^2\,pv"\left(f_2^R\right)<0, \tag{3.118}$$

$$D"_2 = pv'\left(f_2^R\right)-(1+k)\,pv"\left(f_2^R\right)\left(b_L^R-\overline{b}\right)>0,\text{ if } b_L^R-\overline{b}>0. \tag{3.119}$$

Finally, the three coefficients of (3.52) are also modified:

$$C"_{1L} = -\frac{\theta\overline{y}\gamma}{(1-\theta)\,y^m\Gamma\,[\gamma]}\left(v"\left(g_{1L}^{NR}\right)-(1+k)\,pv"\left(g_{2L}^{NR}\right)\right)\gtrless 0, \tag{3.120}$$

$$C"_{2L} = v"\left(g_{1L}^{NR}\right)+(1+k)^2\,pv"\left(g_{2L}^{NR}\right)<0, \tag{3.121}$$

$$D"_3 = pv'\left(g_{2L}^{NR}\right)-(1+k)\,pv"\left(g_{2L}^{NR}\right)\left(b_L^{NR}-\overline{b}\right)>0,\text{ if } b_L^{NR}-\overline{b}>0. \tag{3.122}$$

Thus, solving the system with the three equations (3.40), (3.45) and (3.52) using these new coefficients, we find the effect of a tighter sanction on compensation as:

$$\frac{\partial\eta}{\partial k}\left\{\begin{array}{c}-\frac{\theta\overline{y}\gamma}{(1-\theta)y^m\Gamma}\left[v'\left(f_{1L}^R\right)+(1+\Gamma)\,pv'\left(f_{2L}^R\right)-(1-p)\,v'\left(f_{2L}^{NR}\right)\right]-2\gamma v'\left(f_{1L}^R\right)+\\ \frac{(1-p)\theta\overline{y}\gamma v'\left(f_{2L}^{NR}\right)\left[v"\left(g_{1L}^{NR}\right)-(1+k)pv"\left(g_{2L}^{NR}\right)\right]}{(1-\theta)y^m\Gamma\left[v"\left(g_{1L}^{NR}\right)+(1+k)^2pv"\left(g_{2L}^{NR}\right)\right]}+(1-\theta)\,y^m\Gamma\gamma\int_{\varepsilon_L}^{\overline{\varepsilon}}\frac{v"\left(f_1^R\right)(1+k)^2pv"\left(f_2^R\right)}{v"\left(f_1^R\right)+(1+k)^2pv"\left(f_2^R\right)}d\varepsilon\end{array}\right\}$$

$$=\left\{\begin{array}{c}pv'\left(f_2^R\right)\left(b_L^R-\overline{b}\right)-\frac{v'\left(f_{2L}^{NR}\right)(1-p)\left[pv'\left(g_{2L}^{NR}\right)+\left(b_L^{NR}-\overline{b}\right)\left(v"\left(g_{1L}^{NR}\right)+k(1+k)pv"\left(g_{2L}^{NR}\right)\right)\right]}{v"\left(g_{1L}^{NR}\right)+(1+k)^2pv"\left(g_{2L}^{NR}\right)}\\ +(1-\theta)\,y^m\Gamma\int_{\varepsilon_L}^{\overline{\varepsilon}}\frac{\left[pv'\left(f_2^R\right)-(1+k)pv"\left(f_2^R\right)\left(b_L^R-\overline{b}\right)\right]v"\left(f_1^R\right)}{v"\left(f_1^R\right)+(1+k)^2pv"\left(f_2^R\right)}d\varepsilon\end{array}\right\}.$$

For $b_L^R-\overline{b}>0$ and p close to 1, we observe that the left-hand side of the last equation is negative and the right-hand side positive, so that $\frac{\partial\eta}{\partial k}<0$. Further, by (3.56) and using the new coefficients (3.118) and (3.119), we obtain that $\frac{\partial b^R}{\partial k} < 0$. Finally, from (3.57), (3.120), (3.121) and (3.122), we have as before that $\frac{\partial b^{NR}}{\partial k} \gtrless 0$. However, with the quadratic utility specification (3.25) and $k \geq 0$ not too large, $\frac{\partial b^{NR}}{\partial k} < 0$. With respect to the probability of re-election, given (3.62) and the fact that $\frac{\partial \eta}{\partial k} < 0$, again we obtain: $\frac{d \Pr(R)}{dk} < 0$.

Chapter 4

How effective have Europe's fiscal restrictions been?

4.1 Introduction

In the last fifteen years, fiscal policy in Europe has been subject to restrictions. Far from unique in the "developed" world,[1] these rules have been very controversial. The fiscal framework is enshrined in the Maastricht Treaty (MT), which was signed by the Finance and Foreign ministers of the European Union in February of 1992. Treaty Articles 101 to 104 were designed to keep public deficits low and to ensure budgetary discipline on the part of member states. Further, to guarantee that sound fiscal policies would be continued during Stage 3 of the Economic and Monetary Union and to make the Treaty provisions more precise and operational, in June of 1997, the European Council accepted a draft resolution of the Stability and Growth Pact (SGP). In its first draft, the pact comprised two Council Regulations and a Resolution of the European Council, which together formed a package with two main branches, one aimed at the surveillance of fiscal policy and one aimed at the dissuasion of fiscal profligacy. The surveillance part entered in force on 1 July 1998, whereas the dissuasive arm effectively came into force on 1 January 1999.

So far, most of the literature assessing empirically the performance of these restrictions finds significant differences in the level of fiscal deficits between the MT-period (1992-1997) and the SGP-period (1998-2004).[2] After a strong increase in fiscal discipline in most of the nineties due to entry criteria for admission to the Euro-area, the SGP-period has witnessed a fatigue in fiscal consolidation as suggested by the rising deficits (Fatás and Mihov, 2003, and Hughes-Hallet and Lewis, 2005). The empirical conclusions regarding the fiscal responses to business cycle fluctuations, however, are still mixed. Some authors, such as Galí and Perotti (2003) do not find any change in cyclical fiscal behavior after the Maastricht Treaty was signed, whereas others like Marinheiro (2004) find that the EMU policy rules have reinforced the countercyclical behaviour of fiscal policy.

Therefore, this chapter investigates how effective the fiscal framework has been in disciplining fiscal policy in the Euro zone. In accordance with Fatás and Mihov (2003) and Fatás (2005), we concentrate on two types of biases that are the result of poor fiscal policy management and we assess how the EU fiscal framework has affected them. The first type of bias is the possibility of *excessive deficits* that arise either when governments do not internalize the cost of additional debt or when they postpone fiscal adjustment after a cyclical downturn. The second bias is the possibility of fiscal policy being *procyclical*. The argument is that in good times spending goes up in excess of the rise in tax revenues due to the misinterpretation by politicians of cyclical increases in revenues as being structural.

The analysis separates the MT-period and the SGP-period, disentangling the effects of each set of restrictions and isolating the fiscal impacts stemming from the efforts of

[1] For a survey of some of the fiscal restrictions and rules implemented recently in other developed countries, such as Australia and Canada, see Kennedy and Robbins (2001). For the particular case of Japan, see Von Hagen (2005).

[2] For a theoretical analysis of those restrictions see, for instance, Beetsma and Debrun (2007), Buiter (2005), Fatás et al. (2004), Buti et al.(2003), Beetsma and Uhlig (1999), and Chari and Kehoe (1997).

European countries to enter the Euro zone.[3,4] Specifically to the SGP, the failure of some countries to comply with the deficit target imposed by the pact have added concerns about whether the Pact is indeed an effective instrument in reducing fiscal profligacy. Clearly, any assessment can only be preliminary, as the Pact and the Euro have only been in existence for a few years now, which is less than a complete business cycle for some countries.

So, after controlling for relevant economic and political variables, we examine for the cyclically adjusted deficit (as a measure of the fiscal stance) whether (i) its average level and (ii) its response to the output gap have changed during the MT- and SGP periods, and (iii) how it reacted when the reference deficit level of the Treaty (or Pact) were exceeded. These reactions are estimated using pooling and instrumental variables techniques. They are also compared with responses of other "industrialized" OECD countries, putting the European experience with the MT and the SGP into a broader perspective.

Our main findings are that both the MT and the SGP have been effective in reducing fiscal profligacy whenever the deficit limit was exceeded, i.e. they were effective in inducing a contraction of the fiscal stance in response to *excessive deficits*. This study also indicates that only during the MT-period the average fiscal stance in the Euro-11 contracted. Nevertheless, this contraction coincided with a fiscal tightening in other "industrialized" OECD countries during the same period, suggesting a common trend in the fiscal stance of developed countries rather than an isolated effect of the MT. In addition, we find that neither the MT nor the SGP have altered the cyclical behaviour of the Euro-11 fiscal authorities. Therefore, if the enforcement of countercyclical fiscal policy in the Euro zone is seen as an objective of the Pact, the analysis implies the need for improvements in the current fiscal framework. These results survive extensive robustness testing.

The remainder of the paper is organized as follows. Section 4.2 details the empirical strategy and the methodology used in this paper. In this section we also describe the dataset and present some descriptive statistics of the main variables. Section 4.3 reports and discusses our empirical findings. In Section 4.4 these findings are subjected to further robustness testing. At last, Section 4.5 concludes the main body of the paper.

4.2 Empirical strategy and methodology

4.2.1 Research questions

In order to assess how effective the European's fiscal restrictions have been, we start by detailing our main research questions. We focus the analysis on the effects of the MT

[3]More precisely, the Treaty applies during the entire period (1992-2004) under consideration, while the Pact has been introduced to give an operational content to the Treaty provisions.

[4]Several authors argue that throughout the MT-period the fiscal targets were more binding and resulted in more fiscal discipline than during the SGP-period. The reason is that during the MT period the EU countries had to restrain their fiscal behavior in order to qualify for entry into the Eurozone. Once in, the incentive to adhere to the fiscal limits weakened.

and of the SGP on eleven member countries of the Euro zone (the "Euro-11"): Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, The Netherlands, Portugal and Spain. Luxembourg is left out of the analysis due to missing data.

To investigate the behaviour of fiscal policy in those countries, we use the cyclically adjusted primary deficit (CAPD). This variable shows how the fiscal authorities have reacted to the restrictions of the MT and SGP, as it purges the actual fiscal stance of automatic fiscal responses to business cycles developments. Hence, the first research question we address is:

Question 4.1 *Has the average cyclically adjusted primary deficit in the Euro-11 fallen during the MT-period and/or the SGP-period when compared with: (i) the level during the subperiod 1980-1991? (ii) the level of the corresponding variable for our two control groups of countries that have not joined the Euro zone over the same period?*

The motivation for addressing this issue is that the MT and the SGP should have had an impact on reducing the average level of the cyclically-adjusted fiscal deficit in the Euro zone.[5] Thus, to verify this conjecture, we compare the average level of this variable during the MT–period (1992-1997) and SGP-period (1998-2004) with its average over the period 1980-1991 for the same set of Euro-area countries. In connection with Question 4.1-ii, we compare the average level of the CAPD during the MT- and SGP-periods with that of the (a) EU-3 (Denmark, Sweden and UK) – the set of countries that have been in the EU since at least 1995 but do not participate in the Euro zone; (b) The OECD-6 (Australia, Canada, Iceland, Japan, Norway and the US) – a sample of "industrialized" OECD countries that do not participate in the European Union. This comparison is useful to check the potential presence (or not) of some common external factor driving the fiscal stances of all countries in the sample. For example, if during the SGP period the Euro-area fiscal stances and those of the other countries all contract alike, then it is likely that the observed fiscal contraction for the Euro countries can be ascribed also to this common factor, rather than only due to the SGP.

Our second research question is:

Question 4.2 *Has the cyclically adjusted primary deficit response to the business cycle in the Euro-11 become more or less countercyclical during the MT-period and/or the SGP-period when compared with: (i) its level in the previous subperiod (1980-1991)? (ii) the level of the corresponding variable for the two control groups of countries (EU-3 and OECD-6) over the same period?*

[5] In our analysis, we assume that the fiscal restrictions are exogenous. Poterba (1994) and Braun and Tommasi (2004) account for the possibility that states or countries in which voters have a preference for fiscal prudence not only tend to have low deficits but also pass balanced budget rules. However, given the heterogeneity of the EU countries in terms of fiscal discipline at the moment of initial adoption of the EU fiscal restrictions, we believe that this possibility is not relevant for our sample.

The idea behind Question 4.2 is that the loss of monetary independence should have strengthened the need for more countercyclical fiscal stabilization. Furthermore, the MT and the SGP should have given the Euro-11 fiscal authorities an incentive to become more countercyclical when compared to the average discretionary fiscal policy in EU-3 and OECD-6. At least the SGP requires countries to adopt minimal benchmarks (i.e. median-term fiscal balances) that are sufficiently prudent to allow for some leeway to the 3% deficit limit when the business cycle worsens.

Finally, it might be the case that, albeit the overall average level of discretionary primary deficit has not decreased, the sanctions of the MT and/or the SGP have been effective in leading to fiscal adjustments when the deficit ceilings imposed by these restrictions were exceeded (i.e. became binding). This is what motivates our final research question:

Question 4.3 *(i) How did the cyclically adjusted primary deficit react when the constraints of the Maastricht Treaty and/or the SGP were violated? (ii) How has this response differed between the countries belonging to the Euro-11 and the two other control groups, EU-3 and OECD-6?*

In connection with Question 4.3-ii, we again contrast the results for the Euro-11 with those for our control groups to see whether those groups acted differently when their deficit exceeded the 3% level. When the Euro-11 behaves in the same way as the control groups, then the argument that the deficit ceiling has disciplined the Euro-11 fiscal authorities is undermined.

4.2.2 The fiscal reaction function

We address empirical Questions 4.1, 4.2 and 4.3 via the estimation of a fiscal reaction function of the format:

$$pedfay_{i,t} = \alpha_i + \lambda_t + \beta * X_{i,t} + \gamma * Z_{i,t} + \varepsilon_{i,t}, \tag{4.1}$$

where subscripts $i = 1, ..., N$ and $t = 1, ..., T$ denote the country and year of the observation, respectively. α_i represents the country-fixed effects and λ_t the time-fixed effects. $X_{i,t}$ is a vector of economic and political variables that, in accordance with the economic literature, explains deficit behavior in industrialized countries. $Z_{i,t}$ is a vector of testing variables related to our empirical questions. Finally $\varepsilon_{i,t}$ is a error term.

The dependent variable, $pdefay_{i,t}$, is the "cyclically adjusted primary deficit as a percentage of potential GDP" (CAPD). It is equal to minus the "cyclically adjusted government primary balance as percentage of potential GDP", which is provided by the OECD (2005).[6] Hence, $pdefay_{i,t}$, filters out the automatic stabilizers built into the OECD tax

[6] We use revised data for this variable as well as for all other variables employed in our estimations. An interesting extension would be to use real-time data, so that we could better evaluate the intentional stance

systems and unemployment compensation schemes and yields an approximation of discretionary fiscal policy in OECD countries.[7]

Control variables

The vector $X_{i,t}$ contains five economic and political control variables in our main estimations. Those, as well as the testing variables (vector $Z_{i,t}$), are presented in Table 4.1 and discussed in detail in Appendix 4.A. The first control variable is the lagged cyclically adjusted primary deficit, pdefay(-1). It accounts for the autocorrelation of the dependent variable and is generally included in this type of empirical analysis[8]. Its coefficient provides an estimate of the amount of inertia of fiscal policy. The second control variable, the lagged government debt (as % of actual output), ggflq(-1), deals with potential debt stabilization policies guiding the determination of CAPD. As suggested by Bohn (1998), a negative estimate for its coefficient indicates a fiscal policy aiming at debt sustainability.

To control for the effects of the inflation of private consumption in the conduct of fiscal policy, we have also included the variable *inf*. The literature identifies several reasons to control fiscal deficits for inflation. In sum, they can be grouped into two conflicting effects on discretionary fiscal policy. On the one hand, the cyclically adjusted primary deficit falls with inflation due to bracket creep in taxes (tax brackets are not fully adjusted or only adjusted with a lag to inflation), seigniorage revenues and, to the extent that it is unexpected, the effect on the real debt servicing costs when debt is nominal. On the other hand, higher inflation can also increase the CAPD because of its effect on the nominal interest rate, as Fatás and Mihov (2003), Woo (2003) and Claeys (2005) claim. In fact, the increase in the nominal interest rate can be larger than the inflation increase if the central bank applies the Taylor principle.[9] An additional argument – motivated by the Optimum Currency Area (OCA) literature – for controlling for inflation is that in a monetary union with asymmetric shocks or diverging inflation preferences, national fiscal policy makers should take over the role of monetary policy in stabilizing the country-specific component of inflation (see Beetsma and Jensen, 2005, and Claeys, 2005).

of fiscal policy based upon all the information available to policymakers at the time of fiscal planning (see, for example, Cimadomo, 2007; Giuliodori and Beetsma, 2007; and Golinelli and Momigliano, 2006). Further information about the CAPD can be found in Appendix 4.A, OECD (2004) and Giorno et al. (1995). For more information about the dataset and the construction of the variables employed in (4.1) see the next two subsections, Table 4.1, and again Appendix 4.A. All appendices are available upon request.

[7]For a criticism of that variable as an approximation of discretionary fiscal policy, see Alberola et al. (2003), Larch and Salto (2005), and Mélitz (2005). Roughly, those authors (in particular, Mélitz, 2005) claim that CAPD does not take into account several other fiscal variables (such as payments for pensions, health, subsistence and subsidies of all sorts) that respond automatically to the cycle.

[8]See Fatás and Mihov (2003), Galí and Perotti (2003), Afonso (2005) and Claeys (2005), for instance.

[9]In addition, the empirical literature on fiscal policy often makes a distinction between anticipated inflation (which leads to a lower deficit due to seigniorage, but does not affect real debt servicing costs due to the reaction of the nominal interest rate) and unanticipated inflation (which leads to a reduction in the real value of the debt-servicing costs of nominal debt). However, since the effects of inflation on our deficit variable are not the focus of this paper, we disregard this difference and only control for the effect of the ex-post inflation rate.

Political factors can also affect fiscal deficits. Thus, we have also considered potential political budget cycles (PBC) by including the dummy *ele*. This dummy equals one in years of parliamentary elections in a country and zero otherwise. The intuition is that political circumstances can also explain fiscal policies changes during the MT- and/or SGP-period. The output gap, *gap*, is also included in the set of explanatory variables for the CAPD since fiscal authorities may react in a systematic way to changes in the output gap, in addition to besides the presence of automatic stabilizers. Other control variables that can explain CAPD, such as the real interest rate and trade openness, are relegated to Section 4.4, which tests the robustness of the results.

At last, when analyzing part (ii) of Questions 4.1, 4.2 and 4.3, some of the control variables are interacted with the country dummies *deu3* and *doecd6*, representing our control groups of countries, EU-3 and OECD-6 (see Table 4.1). This interaction is introduced because different groups of countries can present divergent responses to some of the control variables.

Testing variables

Regarding the testing variables, we use time dummies referring to the period of the Maastricht Treaty and SGP (d9297 and d9804 respectively) to address item (i), and their interaction with the country dummies *deu3* and *doecd6* to answer item (ii) of Question 4.1. Question 4.2 is addressed interacting those time and country dummies with the output gap (*gap*).

Our final test (Question 4.3) concerns the impacts of the Treaty and SGP when their deficit ceiling is binding. So, in order to capture the effects of the Treaty, we construct the following variable based on Forni and Momigliano (2004) and Giuliodori and Beetsma (2007):

$$\begin{cases} for\ t < 1992 \text{ and } t > 1997: & mas_{i,t} = 0 & \\ for\ 1992 \leq t \leq 1997: & mas_{i,t} = \frac{tdefy_{i,t-1} - 3\%}{1998 - t} & if\ \ tdefy_{i,t-1} \geq 3\% \\ for\ 1992 \leq t \leq 1997: & mas_{i,t} = 0 & if\ \ tdefy_{i,t-1} < 3\% \end{cases} . \quad (4.2)$$

This variable accounts only for cases when the total deficit (as a percentage of GDP) in the previous year, $tdefy_{i,t-1}$, exceeded the reference level of 3%. In addition, starting in 1992 (the first year in which the Maastricht Treaty applied), the bigger is the time gap between 1998 (the starting date of the Euro zone) and the year that a particular country surpassed the fiscal target, the longer was the amount of time available for the country to adjust its deficit to the ceiling imposed by the Treaty and, hence, the smaller is $mas_{i,t}$.[10]

Likewise, we create another variable capturing the cases when the deficit exceeds the

[10]Here, as a simplification, we assume that the disciplinary effect of the Maastricht Treaty is linear in the difference of the total deficit to the reference level if $pdefy_{it-1} \geq 3\%$, and linear in the time gap between the year of the violation of the rule and the deadline to enter the Eurozone - 1998.

reference level during the SGP-period. It is given by:[11]

$$\begin{cases} for\ t < 1998: & sgp_{i,t} = 0 & \\ for\ 1998 \leq t \leq 2004: & sgp_{i,t} = \frac{tdefy_{i,t-1} - 3\%}{2} & if\ \ tdefy_{i,t-1} \geq 3\% \\ for\ 1998 \leq t \leq 2004: & sgp_{i,t} = 0 & if\ \ tdefy_{i,t-1} < 3\% \end{cases} . \qquad (4.3)$$

Here, when $tdefy_{i,t-1} \geq 3\%$, the variable $sgp_{i,t}$ is divided by two since the Excessive Deficit Procedure allows for a two-year period to eliminate the excess in the deficit before financial sanctions take place.[12]

4.2.3 Estimation procedure

In order to address the endogeneity of some explanatory variables, we estimate the fiscal reaction function (4.1) for the period 1980-2004 via Two-Stage Least Square (TSLS)[13] with country- and period-fixed effects. In our main estimations, we use instrumental variables for *pdefay(-1)*, *inf* and *gap*. Although predetermined, *pdefay(-1)* is instrumented given that its inclusion in equation (4.1) leads to autocorrelation, common in dynamic panel data estimations.[14,15]

The instrument variables are found by running OLS regressions of those three variables on potential proxies for all samples under consideration. The significant proxies are then included as instruments in the estimation of (4.1) by TSLS. Table 4.1 of Appendix 4.B shows the results. We see that *pdefay(-2)* is a highly significant explanatory variable for *pdefay(-1)* in all cases. Hence, we use it as instrument for *pdefay(-1)*. Further, given the results of Table 4.1, we instrument *inf* and *gap* by the first two lags of inflation, *inf(-1)* and *inf(-2)*, the first two lags of the long-term interest rate, *irlrc(-1)* and *irlrc(-2)*, and the first two lags of output gap, *gap(-1)* and *gap(-2)*.[16]

To test for the validity of the instruments, we report a Hausman-test of endogeneity for all regressions.[17] The standard errors of the coefficients of equation (4.1) are based on White's (1980) correction. This procedure corrects for autocorrelation, which typically arises in panels with a large time span. Further, since we are interested in comparing differences in fiscal behavior between different groups of countries, we perform Wald tests

[11]For simplicity, we compute this variable for Greece also from the year 1998 onwards, although that country joined the Eurozone only at the beginning of 2001.

[12]Again, by constructing this variable in this way, we assume for simplicity that the sanctions of the SGP hit the country linearly, and that the amount of adjustment in the deficit is equal in each of the two years.

[13]Similar results are obtained using Aurellano-Bond and Blundell-Bond estimators.

[14]For a discussion, see Baltagi (2005, pp. 135), or Judson and Owen (1999).

[15]For more information on econometric issues related to the explanatory variables, see Appendices 4.A and 4.G.

[16]In the EU-3 case, inf(-2) is insignificant and, therefore, not included as an instrument. In that sample, we also exclude irlrc(-2), since the explanatory power of the Hausman test for the validity of the instruments (see below) fell abruptly when that variable was included.

[17]For more information on the Hausman test see Johnston and Dinardo (1997), pp. 338-342, or Wooldridge (2002), pp. 118-122. Loosely speaking, the null hypothesis of this test is that the regressors of the estimation are exogenous and, therefore, that OLS estimation is more adequate than 2SLS where the variables are instrumented.

to see whether the corresponding regression coefficients differ across those groups. Moreover, to check the robustness of our results, we shall also include additional economic and political variables that could potentially explain our dependent variable.

Further, we assume that the coefficients in the fiscal reaction function (4.1) are equal across countries. Because our sample covers a large time span and because standard pooled estimators (such as fixed effect models) for our dynamic panel model are subject to potential bias when the parameters are heterogenous across countries and the regressors are serially correlated, it may be preferable to estimate the coefficients of some of the control variables separately for each country. Therefore, Appendix 4.F.3 reports the individual-country estimates of the coefficients of the output gap and inflation for the Euro-11 sample, while in Section 4.4 the results of this robustness test are briefly discussed. For the EU-14 and OECD-17 samples, Section 4.3 already controls for the heterogeneity of particular variables at the group level (the Euro-11 versus the EU-3 and the Euro-11 versus the OECD-6).

4.2.4 Data and descriptive statistics

Most of the data is from OECD (2005) and AMECO (2005). To limit further the potential cross-country heterogeneity discussed above, we exclude the Czech Republic, Hungary, Poland, Slovakia, Mexico, South Korea and Turkey. These countries are or were relatively less developed and joined the OECD much later than the countries in our sample. Moreover, some of them have been during a long period part of a different economic system. Luxembourg, Switzerland and New Zealand are also excluded due to missing observations. We end up with a set of 20 countries: the Euro-11 (described before), the EU-3 (U.K., Denmark and Sweden), and the OECD-6 (Australia, Canada, Iceland, Japan, Norway, and the USA). The parliamentary election dummy (*ele*), is obtained from the site of the International Institute for Democracy and Electoral Assistance (IDEA – http://www.idea.int/vt/parl.cfm) combined with the information from the site http://electionresources.org. As mentioned earlier, we restrict our sample to the period 1980-2004. The number of missing observations would become too large if we extend the sample to earlier years.

Tables 4.2 displays the unweighted averages of our dependent variable for all OECD countries during four different periods: 1980-1991, 1992-1997, 1998-2004, and the entire time span 1980-2004. While the CAPD differs across countries, there seems to be a general decline over time, with the cyclically-adjusted balance turning to surpluses (indicated by a negative sign in the table) during the MT and the SGP periods. This is the case, for instance, for Italy and Spain where the average CAPDs as percentage of potential GDP are respectively 2.86% and 1.69% during the period 1980-1991, -4.12% and -0.79% during 1992-1997, and -2.89% and -2.07% during 1998-2004. By contrast, the countries in the OECD-6 show rather mixed developments during those periods.

Further, the last four lines of Table 4.2 convey averages for each group of countries.

For all groups, the average cyclically adjusted primary deficit decreases over time going from 0.73% for the Euro-11, -0.9% for the EU-3 and 1.15% for the OECD-6 during the period 1980-1991 to, respectively, -1.86%, -2.19% and 0.32% during the SGP period.[18]

The same effect can be observed in Figures 4.1 and 4.2. Figure 4.1 shows the dynamics of the average CAPD for the three groups of countries under consideration, whereas Figure 4.2 displays the dynamics of that variable for each country individually. Both figures portray a rather synchronized behavior in "industrialized" OECD countries with respect to their cyclically adjusted fiscal policy. In particular, Figure 4.1 evinces that just after 1992 there was an abrupt fall in the averages values of the CAPD for all groups of countries. During the SGP period, however, the CAPDs have gone up, albeit they stayed at a lower level than during the period 1980-1991 (except for the OECD-6, for which the CAPDs have on average returned to a level similar to that during the first subperiod). That figure also shows that the difference between the average CAPD levels of Euro-11 and EU-3 was larger throughout 1992-1997, and has become significantly smaller in the recent period. Conversely, after 1997, the OECD-6 has run higher cyclically-adjusted levels of deficit than the other two groups.

Next, Table 4.3 and Figure 4.3 provide a simple analysis of Question 4.2 and the discretionary fiscal policy response to the business cycle in the OECD "industrialized" countries. Table 4.3 is organized in the same way as Table 4.2. It displays the unweighted averages of the output gap among OECD developed countries during the period 1980-2004 and its subperiods. There, we observe that for all three groups of countries, the MT-period was characterized by a recession with large negative average values of the output gap (-2.28% in the Euro-11, -2.4% in the EU-3 and -2.01% in the OECD-6). By contrast, the period 1998-2004 constitutes an upturn phase with a boost in GDP growth rates in the end of the nineties in those economies. As a consequence, the average output gaps for all three groups of countries were positive (0.45% in the Euro-11, 0.12% in the EU-3 and 0.07% in the OECD-6).[19]

Figure 4.3, in turn, displays scatter plots of the CAPD against the output gap. The charts are separated by group of countries and period of analysis. Each one of them presents a regression line estimated by simple OLS. The figure and the non-significance of the regression lines in the figure reveal the heterogeneous behavior of fiscal authorities with respect to the business cycle. This is specially the case for Euro-11 and OECD-6. For the first period of analysis (1980-1991), the scatter plot suggests that Euro-11 fiscal authorities provided on average a discretionary countercyclical response to the output gap. The CAPD generally went up when the output gap fell. This outcome, which

[18]Although the numbers for the CAPD look rather small during the first subperiod and the MT period, in both periods the stock of debt grew fast, especially for the Euro-11 and the EU-3. Table 4.9 of Appendix 4.A illustrates this fact by displaying large total deficit averages during those two subperiods for these groups of countries. The difference between the CAPD and total deficit is accounted for by the effect of the automatic stabilizers and interest outlays on the stock of debt.

[19]The positive average output gap during 1998-2004 for the Euro-11 in Table 4.3 and Figure 4.3 seems to be driven in large part by Ireland.

contradicts the findings of Gali and Perotti (2003), is reinforced by the downward (albeit non significant) slope of the regression line. For the OECD-6, however, fiscal policy is procyclical during the period 1980-1991.

Throughout the time span 1992-1997, the relationship between discretionary fiscal policy and the output gap among Euro-11 countries has become even more heterogeneous. This initial result for the Euro-11 suggests that the Maastricht Treaty did not lead the fiscal authorities in the Euro zone to respond cohesively to the business cycle. For the EU-3 and the OECD-6 fiscal policy evolves into more countercyclical and procyclical responses, respectively, conveying a clear distinction between those two groups during that period. During the SGP period, however the Euro-11 discretionary fiscal response is more consistent, presenting a clearer procyclical trend. This outcome goes against what one would expect if the provisions of the SGP are abided, in particular its aim for countries to strive for medium term balance or surplus. This procyclical tendency is also shared by the EU-3 group of countries, whereas OECD-6 shows a more disperse response after 1998. Those results suggest that procyclicality has become a European trend after 1998 and that the SGP fiscal rules have not been able to correct this undesired behavior for Euro-11.

Finally, Figure 4.4 displays the OECD measure of the total deficit (in percent of actual GDP) for our sample of countries during the period 1992 to 2004. This time span covers the MT and SGP periods, which are separated for each country in the figure by a vertical dashed line in the year 1998. Further, a horizontal dashed line also marks the 3% deficit ceiling. From the figure we see that several countries started in 1992 with total deficit higher than 3% of GDP. After 1998, the number of countries above that level dropped to just a few. Among EU-3, only the United Kingdom exceeds the 3% level and only on just two recent occasions (2003 and 2004). Among OECD-6, after 1998, the 3% deficit level was only exceeded by Japan and the U.S. in several years. During the period of implementation of the SGP, six countries have at some point in time exceeded the 3% deficit level : Germany, France, Greece, Italy, Netherlands and Portugal. These figures already point out that, while during the MT period there was a trend towards tighter fiscal discipline in "industrialized" countries, during the SGP period, the trend reversed in the direction of a relaxation of fiscal policy in those countries. This seems to be the case in particular for the Euro zone countries, which again casts doubts about the efficacy of the SGP in disciplining the fiscal authorities of those countries.

In sum, Figures 4.1, 4.2, 4.3 and 4.4 do not convey a positive message about the effectiveness of the SGP. This judgment is motivated by the increase in the average level of the cyclically adjusted primary deficit as well as the total deficit of the Euro-11 during the SGP period. Moreover, the negligible difference of those average levels for the Euro-11 with the EU-3 average and some countries of the OECD-6 indicates that other "industrialized" countries obtained the same fiscal records in the absence of supranational fiscal restrictions.

4.3 Estimation results

4.3.1 Effects of the MT and the SGP on the Euro-11

We start by estimating equation (4.1) using only the Euro-11 sample of countries. Table 4.4 displays the results. There, each column reports the results of a different specification of (4.1), using various combinations of testing variables. In all of them, we report the average fixed-effect for the regression, α, as well as the vector $X_{i,t}$ of control variables. Thus, column (1) reports the results of the regression in which only the control variables are included. Column (2), in addition, includes the time dummies for the MT-period and the SGP-period. These time dummies account for differences in the average value of the cyclically adjusted primary deficit for the Euro-11 during the two periods, 1992-1997 and 1998-2004, when compared to the previous period of 1980-1991. Column (3) accounts for differences in the responses of our discretionary deficit variable to the output gap during the periods of the fiscal rules. This is done by interacting our time dummies with the output gap variable. Column (4) incorporates the two aforementioned sets of variables together. Column (5) estimates equation (4.1) including as additional variables the (adjusted) excessiveness of the deficits when the deficit ceiling is binding during the MT- and SGP-periods. Columns (6) and (7) combine this set of testing variables with each of the previous testing sets, namely the dummies for the MT- and SGP-periods and their interactions with the output gap, respectively. Finally, in Column (8) the three sets of testing variables are all jointly included.

Regarding the estimation results for the control variables, we observe that in all columns the average fixed effect α for Euro-11 is positive and between 1 and 2 percent of potential GDP. The lagged cyclically adjusted primary deficit, *pdefay(-1)*, is highly significant with a positive sign and a value of at least 0.75 in all cases. This outcome demonstrates the strong persistence in the primary deficit in the Euro-11 countries. The coefficient of *ggflq(-1)* is negative in all cases and also highly significant. Its value is roughly 0.02 in all columns. Therefore, an increase of one percentage point in the lagged government debt/GDP ratio causes a decrease in CAPD by 0.02%.[20]

Private consumption inflation is insignificant in all of the cases, even though its coefficient is always estimated to be negative. This result might be related to the expected conflicting effects of inflation in discretionary fiscal policy explained previously, and therefore, the heterogeneity of the fiscal responses to inflation among the Euro-11 countries. Further, the highly significant coefficient for *ele* in all columns indicates the existence of political budget cycles in the Euro-11. In electoral years the average CAPD rises by around 0.63 percentage points in those countries. Finally, the response of our deficit variable to the output gap is not statistically significant for the Euro-11 in any of the columns of Table 4.4. As Figure 4.3 suggests, this outcome might be attributable to the large heterogeneity in the discretionary fiscal responses to the output gap in those countries.

[20] Annett (2006) obtains similar estimates.

Did the MT and the SGP affect the average level of the cyclically-adjusted primary deficit in the Euro-11?

This subsection addresses the effects of the MT and the SGP on the level of the cyclically adjusted primary deficit in the Euro-11 (item (i) of Question 4.1). As mentioned before, this is done by including the time dummies *d9297* and *d9804* in our regression equation.

Thus, in Table 4.4, we observe two different results. When those time dummies are estimated without the presence of the variables related to the binding cases of the MT and SGP (columns (2) and (4)), *d9297* is statistically significant and negative. However, if we insert *mas*, the dummy loses significance (columns (6) and (8)). This finding, confirmed by the results of robustness tests later on in Section 4.4, leads us to the conclusion that the decrease in the CAPD during the MT-period in Euro-11 was caused in large part by fiscal contraction when the deficit ceiling was binding.

The dummy relating to the SGP period, *d9804*, fails to be significant in any of the cases. Hence, during the SGP-period, the CAPD was on average not significantly lower than during the period 1980-1991.

We can summarize the above outcomes as follows:

Result 4.1 *Compared to the period 1980-1991, the average level of the CAPD in the Euro-11: (i) was lower during the Maastricht Treaty period, 1992-1997 (in particular, in cases when the deficit ceiling was binding); but (ii) was not statistically different during the SGP period.*

Therefore, Result 4.1 implies that the Maastricht Treaty during the run-up to EMU had a disciplinary impact on the Euro-11, while the SGP has generally not been effective in reducing the CAPD.

Did the MT and the SGP affect the fiscal responses to the business cycle in the Euro-11?

This issue is explored by estimating the coefficients on the variables *gap*, *d9297*gap*, *d9804*gap* in our regression framework – see columns (3), (4), (7) and (8) of Table 4.4. The coefficient on *gap* represents the response of the CAPD during the period 1980-1991. The two interaction terms correspond to differences in the responses during the MT- and SGP-periods. We obtain the following result:

Result 4.2 *For the Euro-11 sample, there is on average no statistical difference in the response of the CAPD to the output gap during the two periods under consideration, 1992-1997 and 1998-2004, when compared with the period 1980-1991.*

Therefore, neither the MT, nor the SGP, seem to have significantly affected the cyclicality of the fiscal authorities' responses to the business cycle. As for the benchmark period 1980-1991, the responses are mixed, albeit after 1992 a subtle tendency towards procyclicality can be observed in the data (see Figure 4.3). This contradicts the findings of Marinheiro (2004). Thus, the fiscal restrictions have not forced countries to consistently "save in good times and spend in bad times".

Effects of the MT and the SGP when the deficit ceiling was binding

In view of the highly significant and negative coefficients of *mas* and *sgp* in columns (5) to (8) of Table 4.4, we conclude that:

Result 4.3 *Fiscal policy as measured by the primary cyclically adjusted deficit has been contractionary in instances when the deficit ceiling was binding, both during the run-up towards the Euro and after the formation of the monetary union in Europe.*

Therefore, Result 4.3 suggests that during both periods, 1992-1997 and 1998-2004, fiscal discipline increased whenever the ceilings of the MT and SGP were binding. Hence, even though the SGP has not led to a significant change in the average level of CAPD (as evinced by Result 4.1), it seems to have been effective at least to some extent in disciplining countries that violated the fiscal restrictions of the SGP.

4.3.2 Comparison of the Euro-11 with other OECD countries

We also compare fiscal policy behaviour of the Euro-11 throughout the MT and SGP periods with that of the EU-3 and OECD-6 groups of countries. The aim is to investigate whether the fiscal behaviour of the Euro-11 during these periods has been different from that of other countries with roughly similar economic and political characteristics, but that were not constrained by Europe's fiscal restrictions.

The Euro-11 versus the EU-3

First, we merge the Euro-11 and EU-3 samples into an EU-14 sample. Given that some of the coefficients of the control variables can differ between the Euro-11 and the EU-3 samples, we estimate this merged sample allowing for differences in the coefficients of each control variable between those two groups of countries[21]. For that, we interact the coefficients of those variables with the dummy *deu3*, which assumes a value of 1 for Denmark, Sweden and the UK, and 0, otherwise. The results, relegated to Table 4.3 in Appendix 4.C, show that the coefficients of *ggflq(-1)*, *inf*, and *gap* are indeed statistically different for the Euro-11 and the EU-3. Hence, when comparing the effects of the MT and SGP between Euro-11 and EU-3, we include those controls while allowing for their coefficients to differ between the two groups.

[21] We also estimate equation (4.1) for the aggregate sample EU-14 without any distinction between the coefficients of the control variables for Euro-11 and EU-3. The estimates are displayed in Table 4.2 in Appendix 4.C. Roughly, the coefficients are closer to those in Table 4.4 since Euro-11 sample contains almost four times more countries than EU-3.

To distinguish between the Euro-11 and EU-3 in terms of the effects of our testing variables, we interact these variables with dummy *deu3*. This procedure leads to Table 4.5, where, in addition, only the control variables for which the coefficients were significantly different between the Euro-11 and EU-3 (as was reported in Table 4.3) are interacted with *deu-3*. Thus the regressions in Table 4.5 involve two types of testing variables. Those that are not interacted with *deu3* measure the differences in fiscal behaviour between the Euro-11 over the relevant period (1992-1997 and/or 1998-2004) and the average for the EU-14 over the period 1980-1991. Those that are interacted with *deu3* estimate departures of the EU-3 sample from the outcomes of the Euro-11 during the period under examination. Furthermore, since the interaction terms only check for statistical differences between the EU-3 and the Euro-11, at the end of Table 4.5, we also sum the coefficients of a particular variable with and without the interaction term and then test via a Wald coefficient test if this sum is statistically different from zero. This analysis checks whether the coefficient of the particular variable is statistically different from zero for the EU-3. This is done, for instance, for the variable *inf* by summing the coefficients of *inf* and *inf*deu3* and testing via a Wald coefficient test whether this sum is equal to zero. The results in Table 4.5 show indeed that the CAPD reacts negatively to inflation in the EU-3 countries for most of the estimations. The same analysis is also performed for the output gap. The Wald test shows that if we consider the entire period sample (1980-2004), the EU-3 has followed countercyclical fiscal policy (*gap* + *gap*deu3* is statistically different from zero).[22] Other than this, we arrange Table 4.5 in the same way as Table 4.4.

In discussing the outcomes of Table 4.5 for our testing variables, we start by addressing item (ii) of Question 4.1 concerning the differences between Euro-11 and EU-3 of the effects of the MT and the SGP on the average fiscal stance. Thus, the focus here is on the estimates of the coefficients of the time dummies *d9297* and *d9804*, and the interaction terms *d9297*deu3* and *d9804*deu3*. We have the following result:

Result 4.4 *(i) During the MT-period, the average fiscal stance of the Euro-11 was tighter than that for the EU-14 over the period 1980-1991 (especially in cases when the deficit ceiling was binding), but in none of the specifications it was statistically different from that for the EU-3 over the same period. (ii) During the SGP period, only when the deficit ceiling was binding the average fiscal stance of the Euro-11 was significantly tighter than that for the EU-14 over the period 1980-1991, otherwise no statistical difference is observed between those two average fiscal stances. In turn, the EU-3 average fiscal stance seems slightly tighter than that for Euro-11 over the SGP-period (albeit this result does not hold when the deficit ceiling was binding).*

From the significance of *d9297* in Columns (1) and (3), we infer the difference in the fiscal stance between the Euro-11 during the MT-period and the EU-14 over the period

[22]We do not explicitly test whether the coefficient of *ggflq(-1)* is different from zero in Table 4.5, since the coefficient for the EU-3 is statistically different and more negative than the one for the Euro-11.

1980-1991. We observe that, when the variable *mas* is included from Columns (4) to (7), *d9297* loses significance, indicating that the extra tightening of the fiscal stance is concentrated among cases in which the Maastricht deficit ceiling was exceeded. Moreover, the marginal significance and negative sign of *d9804*deu3* in Columns (3) and (7) of Table 4.5 suggests that, during the SGP-period, the EU-3 might have had a slightly tighter average fiscal policy than the Euro-11 over the same period.[23] Wald tests are also used to check whether during the SGP period, the EU-3 average fiscal stance (the sum of the variables *d9804* and *d9804*deu3*) was different from zero. The results show that only in two regressions (columns 3 and 8) this test is significant, which suggests that the average fiscal stance of the EU-3 has not contracted significantly after the start of the implementation of the SGP.

Therefore, Result 4.4 suggests that in neither of the two periods, 1992-1997 and 1998-2004, the Euro-11 tightened its fiscal stance on average more than the EU-3.

Comparison of the fiscal responses to the business cycle in the full EU sample
Next, we investigate how the Euro-11 and EU-3 differ in the response of their fiscal stances to the output gap during the MT and SGP periods. To this end, we incorporate into the regressions the interaction terms *d9297*gap*, *d9804*gap*, *d9297*deu3*gap*, and *d9804*deu3*gap*. With these terms, we examine the respective differences in the fiscal responses to the output gap for the Euro-11 and EU-3 during the MT- and SGP-periods.

When the coefficients of *gap* and *gap*deu3* are estimated in the absence of the other interaction terms, they provide us with the cyclically-adjusted fiscal responses to the output gap for Euro-11 and EU-3 over the entire sample period (1980-2004). The coefficient estimates of *gap*deu3* in Columns (1), (4) and (5) of Table 4.5 are statistically significant, indicating a significant difference between the Euro-11 and EU-3 in their responses. In particular, Denmark, Sweden and the UK seem to have followed a more countercyclical fiscal response on average than the Euro zone countries. It seems that this difference can be attributed to the more countercyclical response of the EU-3 countries during the MT period, as the highly significant coefficient of *d9297*gap*deu3* in Columns (2) and (6) of Table 4.5 suggests.[24] The coefficient of *d9297*gap* indicates that on average the fiscal response of the Euro-11 during the MT period was not different from that in the period before 1991. Further, for both groups of countries, the fiscal responses to the output gap during the SGP period do not seem to have differed from that during the period 1980-1991, nor from each other.[25]. This similar cyclical fiscal behaviour during the SGP-period is confirmed by a Wald test of the equality of the coefficients of *d9804*gap* and *d9804*deu3*gap*. So, we can conclude that:

[23]Nevertheless this result is not robust over all columns, and therefore, no strong conclusions can be drawn about this difference in the fiscal behaviour between the EU-3 and the Euro-11.

[24]However, when we control for the difference in the average CAPD levels during the MT- and the SGP-periods (columns (3) and (7) of Table 4.5) this effect vanishes.

[25]Only in one regression (Column (3)) the coefficient on *d9804*gap*deu3* is significant.

Result 4.5 *(i) During the MT-period, the fiscal response to the output gap has on average been more countercyclical for the EU-3 than for the Euro-11. (ii) During the SGP-period, responses to the output gaps were similar for the two groups and for both groups they were similar to those during the period 1980-1991.*

Hence, the SGP has not led to significant differences in fiscal responses between the two groups of countries. Beyond that, during this recent period, countries seem to have shifted to a slightly more procyclical fiscal behavior. Wald coefficient tests also confirm that the EU-3 fiscal policy was significantly countercyclical during the MT period and pro-cyclical during the SGP-period.

Comparison of fiscal behaviour when the deficit ceiling was binding By including the interaction terms *mas*deu3* and *sgp*deu3* into the regression specification of Table 4.5, we aim to address item (ii) of Question 4.3, and capture the difference of the effect of the MT or the SGP for the EU-3 when compared to the Euro-11. Of course, one can argue that the computation of those variables for the EU-3 group is meaningless. However, since our objective here is to compare the response of our "treated" group, Euro-11, with a control group, EU-3, differences in the coefficient estimates of those variables become a relevant testable hypothesis. However, this comparison is not very meaningful for the SGP period, given that on only one occasion during this period an EU-3 country exceeded the 3% deficit level. The negative and highly significant coefficient of mas and, in addition, the non-significant coefficient of mas*deu3 imply that:

Result 4.6 *During the MT-period, the fiscal stance of both the EU-3 and the Euro-11, was similarly tightened whenever the deficit ceiling was violated.*

A possible explanation for the similarity in behaviour could be that countries did not know in advance if they would try to join the Euro area at some point. In particular, Denmark and Sweden held referenda regarding potential participation in the Euro zone.[26] Hence, fiscal policy in those countries might have been influenced at least to some extent by the provisions of the Maastricht Treaty.

Regarding the variable *sgp* in Table 4.5, it is again significant and negative for the Euro-11 sample, corroborating Result 4.3 of the previous subsection. In turn, for the EU-3 sample, the coefficient of *sgp*deu3* is insignificant and displays very large positive values most likely caused by a small sample bias. In fact, as Figure 4.4 reveals, only on one occasion during the period 1998-2004 an EU-3 country exceeded the 3% total deficit level (this was the UK in 2003). Hence, no clear interpretations and conclusions can be drawn from that variable.

[26]In a referendum on September 28, 2000, the Danish rejected with a narrow margin the proposal to join the Euro. The same happened in Sweden on September 14, 2003.

The Euro-11 versus the OECD-6

At last, we perform the same analysis as we did in the comparison between the EU-3 and the Euro-11, but now substituting the OECD-6 for the EU-3. The idea is that while the EU-3 countries are members of the European Union and, therefore, bound by the Treaty, the OECD-6 countries are not subject to the Treaty, but exhibit roughly similar economic and political structures as the Euro-11 countries and are at a roughly similar stage of development as the Euro-11.

Thus, we estimate the same model used in Table 4.4 for OECD-6. This investigation is also done by merging the samples Euro-11 and OECD-6 (sample OECD-17), and estimating once more the model of Table 4.4 to test for statistically significant differences in each one of the control variables between the two groups of countries. The distinction between them is carried out via the interaction of each one of the control variables with the dummy *doecd6*. This dummy assumes the value 1 throughout the entire sample period for the countries Australia, Canada, Iceland, Japan, Norway and the US, and 0 otherwise (see Table 4.1). The results of both procedures are presented in Appendices 4.D and 4.E – see Tables 4.5 and 4.6, respectively.[27] The coefficients of the control variables that present relevant statistical differences between Euro-11 and OECD-6 in those tables are then estimated separately when we analyze item (ii) of our empirical Questions 4.1, 4.2 and 4.3.

The results are presented in Table 4.6. There, we observe that only the dummy for political budget cycles, *ele*, and the output gap, *gap*, exert a significantly different effect for the two groups of countries. The political budget cycle is significantly different and weaker for the OECD-6 than for the Euro-11. This is also confirmed by a Wald test that shows that the coefficient of the election variable for the OECD-6 is not significantly different from zero, and therefore, does not help to explain the CAPD in that sample of countries.

As regards to the comparison of the average fiscal stances, Table 4.6 shows that (similar to Result 4.4 for the comparison with the EU-3), the average CAPD levels of the Euro-11 and the OECD-6 are not statistically different during the MT-period. However, when we examine CAPD average levels discrepancies during the SGP-period, we obtain the following result:

Result 4.7 *During the SGP-period, the Euro-11 displayed on average a tighter fiscal stance than the OECD-6 (especially in those cases when the total deficit ratio to GDP exceeded the level of 3%).*

This result, which shows up as the positive and significant coefficient of *d9804*doecd6* in Table 4.6 (when *sgp* and *sgp*doecd6* are not simultaneously included) and which is corroborated by Figure 4.1, indicates that the fiscal restrictions of the SGP have slowed

[27] Appendix 4.E presents estimations similar to those underlying Table 4.4 for the OECD-17 with homogenous coefficients on the control variables, i.e. without allowing for differences between the Euro-11 and the OECD-6 as regards to their control variables. Table 4.7 shows the results.

increases in the average CAPD in Euro-11 after 1998, whereas this did not occur in Japan and US. As Figure 4.4 shows, these countries pushed OECD-6 average CAPD up.

Measured over the entire sample period 1980-2004, the response of the CAPD to the output gap is on average more countercyclical for the OECD-6 than for the Euro-11. This is corroborated by the Wald test at the end of Table 4.6. The difference is particularly pronounced when we control for level differences in the CAPD during the MT- and SGP-periods as Columns (1), (3), (5) and (7) of Table 4.6 shows. For the MT- and SGP-periods, however, the responses of the two groups of countries do not differ between each other. That conclusion (analogous to the one contrasting the Euro-11 with the EU-3 - Result 4.5) evinces that the fiscal constraints did not induce the Euro-11 to react differently to the output gap than the other "industrialized" countries.

Moreover, comparing the fiscal behaviour of the two country groups in cases when the total deficit ratio was higher than 3%, we obtain the following result:

Result 4.8 *During both the MT- and the SGP-period, the average Euro-11 fiscal stance became significantly tighter than that of the OECD-6, whenever the total deficit exceeded the level of 3% of actual GDP.*

Result 4.8 reinforces our findings for the Euro-11 and EU-14 of strong fiscal contractions in the case of excessive deficits (recall Results 4.3 and 4.6, respectively). The results indicate that the Euro-11 was more disciplined than other "industrialized" countries (in particular, the US and Japan, as Figure 4.4 shows) in instances when the deficit ceiling was binding. In addition, Wald tests show that, as expected, the variable *mas* is not significantly different from zero for the OECD-6 group of countries, and the variable *sgp* is only marginally significant, although with the wrong (positive) sign. This last result is again related to the profligate fiscal behaviour of Japan and US.

Therefore, comparing the fiscal behaviour of the Euro-11 with that of other OECD countries, we obtain mixed results. The EU-3, even though not subject to the sanctions of the SGP, has shown more fiscal discipline than the Euro-11 countries during the SGP period. Conversely, compared to other "industrialized" OECD countries outside the European Union (in particular, the US and Japan), the Euro-11 seemed to be slightly more disciplined. Nevertheless, discretionary fiscal responses to the business cycle in the Euro-11 have not changed after the Treaty was signed and, if anything, they have been more procyclical than in other "industrialized" countries.

4.4 Robustness tests

To explore the robustness of Results 4.1 to 4.8, we include in the estimation of (4.1) five economic and six political additional variables potentially relevant in explaining fiscal policy. Table 4.7 defines these variables, while Appendix 4.F details their construction and intuition.[28]

The economic variables are the ex-post real long-term interest rate based on the private consumption deflator (*irlrc*), the share of non-working population (*nwp*), trade openness (*open*), the relative economic country size defined as the ratio of real GDP to the sum of the real GDPs of the countries in the relevant sample (*size*), and economic volatility defined as the standard deviation of real economic growth over the preceding 10 years (*vol*). The literature identifies two opposite reasons to control for real long-term interest rate. On the one hand, as Roubini and Sachs (1989) explain, the budget deficit may be a positive function of *irlrc*, because an increase in this variable reflects higher debt servicing costs, which, if transitory, should be accommodated by a temporary increase in the budget deficit. On the other hand, Fatás and Mihov (2003) point out that, besides its direct effect on interest payments, interest rates may also affect the budget negatively via their effects on public infrastructure investments. The higher is the real interest rate, the smaller is the net present value of the investment and thus the weaker is the incentive to invest.

The share of the non-working population (i.e. the sum of those younger than 14 and older than 64 divided by the total population) captures potential implications of "baby booms" and ageing for the budget, because these demographic variables affect spending on education, health care and pension benefits.[29] Trade openness is included in the analysis, because there are reasons to believe that this variable interacts with fiscal policy. For instance, Rodrik (1998) argues that open economies are particularly vulnerable to risk. Hence, it may be important for the government to facilitate consumption smoothing by operating a counter-cyclical fiscal policy.

Two arguments support the use of the relative economic country size as a control variable. As Annett (2006) explains, from the political side, small countries are simply more accustomed to external influences over policy. They also tend to have less bargaining power so that the loss of reputation from violating the fiscal rule is greater. Second, smaller countries could also fear tangible pecuniary losses such as reductions in structural funds. Large countries may view the cost of profligate fiscal policy to be low, given that they suffer little diminution in reputation. Further, the economic costs of fiscal consolidation tend to be higher in large countries, given their larger fiscal multipliers.

Finally, we also control for macroeconomic volatility. Talvi and Vegh (2000) predict that fiscal procyclicality is positively correlated with the degree of output volatility. Their argument is based on the political infeasibility of running large surpluses during booms. For a high-volatility country, the appropriate (from the viewpoint of smoothing government consumption and other expenditures) surplus during a boom may be quite large as a share of GDP. However a large surplus may also unleash intense political pressure

[28] Analogous to Tables 4.4, 4.5 and 4.6, this appendix also provides the estimation results when the additional variables are included (see Tables 4.9 to 4.41).

[29] Because large peaks in the number of children are absent in our sample, it would be more interesting to control only for the effects of the ageing process. However, this variable is not available on a regular basis for all the countries of our sample. Therefore, we have used *nwp* instead. See Appendix 4.F for more details.

to increase public spending. In contrast, the required surplus in a low-volatility country may be quite small and may not attract the same degree of political opposition. The net result is that fiscal procyclicality is more likely for countries or periods in which the amplitude of the business cycle is large. Another argument to control for macroeconomic volatility comes from Anett (2006), who claims that the SGP may act as an external anchor for countries prone to macroeconomic volatility. In this sense, the pact can garner credibility for more volatile countries and take over the role once played by exchange rate coordination mechanisms such as the Bretton Woods system.

The six political variables, extracted from Armingeon et al. (2005) and available until 2003, are the cabinet composition, *gpart*; the new party composition of the cabinet, *gnew*; the ideological gap between the old and new cabinet, *ggap*; the annual number of changes in government, *gchan*, the type of government, *gtype*;[30] and the index of fractionalization of the party system, *rae*. The first three variables capture the political ideology (left or right) of the cabinet in power, and changes in this ideology due to new cabinet formations. These variables might affect fiscal policy because different ideological views about the government's role affect the amount of public spending. So, given the demands of their electorate, we would expect countries with a predominance of left governments (or recent changes into this ideological direction) to be associated with higher public spending. The frequency of changes in the government is often used to explain budget deficits.[31] As Roubini and Sachs (1989) argue, the shorter is the expected tenure of the government, the more difficult it may be to achieve cooperation among the coalition partners. Thus, a higher frequency changes in government exposes the fragility of the political governance in a country, increasing the effective rate at which politicians discount the future.[32]. In the same way, the type of government and the index of fractionalization of the party system capture the fragility of the government and the political structure of the country, thereby affecting the determination of fiscal deficits.[33] This argument is in line once more with Roubini and Sachs (1989) who find that the size and persistence of the budget deficits in the industrial countries in the seventies were greater when the government was divided (for example, in the case of multiparty fragmented coalitions rather than one-party governments or governments with fewer and stronger parties).

To isolate the impact of the various additional variables, each time we have included only one of these additional variables in the estimations. Further, the comparison of the results for the Euro-11 with the EU-3 and the OECD-6 followed the same methodology

[30] The classification of this variable takes into account whether the government has a majority in the parliament as well as the number of parties that forms the coalition. The intuition is that if a government has a majority and the lower is the number of parties forming it, the greater is its governability. For details, see Appendix 4.F.

[31] Woo (2003), for example, expects that public deficits should be larger in countries with more frequent changes in the governing party.

[32] This variable is also strongly correlated with the frequency of elections in countries with a parliamentary system, which forms the majority of the countries in our sample. So, whenever *gchan* was included, the dummy for parliamentary elections *ele* was removed from the regressions.

[33] The index of fractionalization of the party system is computed using the formula in Rae (1968).

as before. Thus, when adding a new variable in the estimations, we tested whether the coefficients were different for the different groups of countries. Whenever this was the case, we interacted the extra variable with the dummies *deu3* and *doecd6* and estimated the specific coefficients for each group of countries.

The main conclusions of these robustness tests are summarized by Table 4.8. It reports for each of the samples under consideration (the Euro-11, EU-14 and OECD-17), the additional variables that are included, their significance and, whenever they are significant, the sign of their coefficients. Further, it also reports whether the original empirical results are robust or not. So, the term "Robust" in the line "robustness" of the table indicates that the inclusion of the additional variable did not qualitatively affect the results for the case under consideration. However, whenever any of the empirical results is altered by the inclusion of a variable, the change (for instance, a coefficient becoming (in)significant when it was not before or a change in its sign) is indicated in Table 4.8.

Table 4.8 shows that *nwp*, *open*, *size*, and *gchan* enter with significant coefficients in the Euro-11 estimations. The share of the non-working population has a positive and marginally significant coefficient. However, when we compare the results for the Euro-11 with those for the EU-3 and the OECD-6, the coefficients of *nwp* for the latter two samples differ significantly from the corresponding coefficient for the Euro-11. This indicates that only in the Euro area, the additional expenditures on children and in particular on the elderly people impact positively on the CAPD. Obviously, this poses additional concerns about the fiscal effects of the ageing process that this group of countries will face in the near future. Further, the number of changes in the government per year is also positive and significant for the Euro-11 as well as for the EU-14 sample. This corroborates the hypothesis that political volatility reduces the effective discount factor of politicians and leads to a deficit bias. For the OECD-6, however, the response of the CAPD to this variable is significantly different and more negative than the response of the Euro-11. This suggests that for the other OECD countries the frequency of changes in the government do not have a strong effect on the CAPD.

In turn, the coefficient of trade openness is negative and highly significant not only for the Euro-11, but for also for the EU-3 and the OECD-6. This outcome suggests a negative relationship between trade openness and the CAPD among "industrialized" countries. One possible explanation, shared by Annett (2006), is that the economic vulnerability associated with more openness forces the fiscal authorities to be more disciplined, to obtain more flexibility in dealing with negative trade shocks. The relative economic size of the country is also negative and highly significant for the Euro-11 and EU-14. However for both samples, this variable becomes significantly less negative after 1998 as the coefficient of the variable *size*d9804* indicates. Those results bear out the fiscal discipline of the large countries specially during the eighties and their looser behaviour after the implementation of the SGP. Moreover, the coefficient of *size* for the other OECD countries is significantly different and positive when compared to Euro-11. This indicates the lack of fiscal discipline of the non-European larger countries (US and Japan), especially when compared with the European countries.

The variables *ggap*, *gtype* and *rae* are not significant for the Euro-11 and the OECD-17 samples, but show differences for the Euro-11 and the EU-3 in the estimations for the EU-14. The response of the CAPD to *ggap* and *gtype* for the EU-3 sample is significantly more negative, while the response to *rae* is significantly more positive. As a consequence, the ideological gap between the old and the new government affects the fiscal policy of the EU-3 countries more than the fiscal policy of the Euro-11. The same is the case for the fragmentation of the party-system that induces a significant increase in the CAPD for EU-3 and is not relevant for Euro-11. Finally, the fact that the type of government induces a significantly more negative response of the CAPD for the EU-3 than for Euro-11 indicates that minority coalitions in the EU-3 are less prone to a fiscal deficit on average than in the Euro-11.

Table 4.8 shows that item (i) of Result 4.1 is weakened when the political variables *gtype* and *gchan* are included in the regressions. This suggests that the reduction in the average level of the CAPD during the MT might have been caused by more powerful coalitions in (more governable) governments or less party fragmentation in parliaments. For the Euro-11, when *size* is included, the level effect of the SGP becomes statistically more negative. This contradicts Result 4.1-ii and shows that after controlling for the size, the fiscal stance of Euro-11 tightens after the introduction of the SGP. In turn, Result 4.4-i is weakened by the inclusion of *nwp*, *open*, *size*, *vol*, *ggap*, *gtype*, and *rae*, which indicates that, in fact, during the MT-period the average fiscal stance of the Euro-11 was not firmly tighter than that of the EU-14 over the period 1980-1991. Again, for the EU-14, the fiscal stance tightens post-SGP when we control for the economic size for the country. Result 4.7 is also invalidated by inserting *irlrc*, *nwp*, *open*, *gchan* and *rae*. Thus, the robustness tests undermine the conclusion that, during the SGP, the Euro-11 had on average a tighter fiscal stance than the OECD-6.

Finally, we also estimate (4.1) for the Euro-11 sample allowing for individual-country coefficients for the output gap and inflation. However, the outcomes, reported in Tables 4.42 and 4.43 of Appendix 4.F.3, do not display relevant differences from our previous results.

Summarizing, we can conclude that our earlier findings on the effects of the MT and SGP on the average fiscal stance of the Euro-11 are not so strong under the various robustness tests. Nevertheless, the finding that excessive deficits are met by significant fiscal contractions remains present and unaffected.

4.5 Conclusion

The Maastricht Treaty has now been in existence for fifteen years. Its creation marked a new era for European fiscal policy with the introduction of strict fiscal rules and restrictions that were later reinforced with the introduction of the Stability and Growth Pact. This chapter investigates the economic effects of those fiscal rules and restrictions on discretionary fiscal policy in Europe.

Based on our empirical results, we reach the following conclusions:

a. On average, the level of the CAPD in the Euro-11 slightly decreased during the MT-period. This small reduction, however, also occurred in other groups of developed countries (EU-3 and OECD-6), suggesting a common trend of deficit reduction among "industrialized" countries rather than an unique impact of the MT in the Euro-11. Moreover, during the SGP-period, no reduction in the average CAPD is observed for the Euro-11 group of countries, even though it has decreased for the EU-3. This indicates that the sanctions of the SGP have not been effective in keeping Euro-11's fiscal stance in tandem with those of other OECD countries (with the exception of Japan and US that have shown an increase in fiscal profligacy throughout the SGP-period).

b. The MT and the SGP have also not succeeded in making Euro zone fiscal policy countercyclical. In addition, during the MT-period fiscal policy in the EU-3 became more countercyclical than that of the Euro-11 and the average fiscal response to the business cycle of the OECD-6 did not differ from that of the Euro-11. During the SGP-period the fiscal responses of both groups (EU-3 and OECD-6) were also not statistically different from that of the Euro-11.

c. Favorably, our results demonstrate that the MT and the SGP were on average effective in reducing the CAPD when the actual deficit ceiling was exceeded in the Euro-11. This fiscal behaviour contrasts with that of the other "industrialized" OECD countries when their deficit ratio exceeded the 3% level (especially, Japan and the US) and so, evinces the singular reaction of Euro-11 fiscal authorities in confronting excessive deficits. During the MT period, EU-3 also shared this disciplined behaviour in cases that the 3% deficit limit was exceeded.

This chapter demonstrates, therefore, that both the MT and the SGP were effective in inducing a fiscal tightening in response to *excessive deficits*. However, if the reduction of total average deficits and/or the change of fiscal policy to countercyclical are also considered as measures of effectiveness of the EU fiscal framework, then our verdict is less positive. In addition, the MT during the period preceding EMU seems to have been more stringent than the SGP, although this fiscal stance contraction was also observed in other "industrialized" countries (even those not subject to fiscal constraints). This common trend among OECD countries in reducing deficits during the MT-period is also highlighted by Kennedy and Robbins (2001).

Furthermore, our conclusions imply the need for improvements of the SGP, especially if the enforcement of countercyclical fiscal policies in the Euro zone is seen as an objective of the SGP. An amended pact should include incentives to produce lower deficits (or higher surpluses) during boom phases of the business cycle and more flexibility in the application of sanctions during recessions. This is in line with the recent revision of the Pact in 2005 (see European Commission, 2005b, and Buti and Sapir, 2006). However, as these latter authors assess, the successful implementation of the new Pact will depend on the political will and the trade off between the perceived negative externalities of fiscal misbehavior against the political costs of attempting to limit the partner countries' room for manoeuvre.

The analysis also leaves some empirical questions open to further examination. For example, a comparison of the Euro-11 only with other countries that have also adopted strict fiscal rules (Australia, Canada and the UK, for instance) would help us to understand differences in their outcomes. Second, it would be important to investigate which tools (increase in taxation or cut in expenditures) fiscal authorities have used to reduce the CAPD in cases of excessive deficits.

4.6 Tables and figures

Table 4.1: List of variables

pdefay	Cyclically adjusted primary deficit.
α	Average fixed-effect or constant in the estimation.
pdefay(-1)	Lagged cyclically adjusted primary deficit.
ggflq(-1)	Lagged government debt (as % of actual output).
inf	Inflation of private consumption.
ele	Dummy for years of parliamentary elections: equals one in years of parliamentary elections in a country and zero otherwise.
gap	Output gap.
d9297	Time dummy for the period 1992 to 1997 (First Phase of EMU or Maastricht Treaty): equals one during the years 1992 to 1997 for all countries and zero otherwise.
d9804	Time dummy for the period 1998 to 2004 (Stability and Growth Pact): equals one during the years 1998 to 2004 for all countries and zero otherwise.
*d9297*gap*	Interaction term between Maastricht Treaty and *gap*: equals the output gap for any sample of countries during the period 1992-1997.
*d9804*gap*	Interaction term between SGP and *gap*: equals the output gap for any sample of countries during the period 1998-2004.
mas	Constructed variable that accounts for the effects of the Maastricht Treaty in cases when the deficit exceeded the 3% limit. See equation (4.2).
sgp	Constructed variable that accounts for the effects of the Stability and Growth Pact in cases when the deficit exceeded the 3% limit. See equation (4.3).
deu3	Country dummy for three EU member countries that do not belong to the Eurozone: equals one for Denmark, Sweden and UK in all years, and zero otherwise.
doecd6	Country dummy for six OECD countries that does not belong to the Eurozone: equals one for Australia, Canada, Iceland, Japan, Norway and the US in all years and zero otherwise.
*d9297*deu3*	Interaction term: equals 1 for EU-3 during the years 1992 to 1997 and zero otherwise.
*d9297*doecd6*	Interaction term: equals 1 for OECD-6 countries during the years 1992 to 1997 and zero otherwise.
*d9804*deu3*	Interaction term: equals 1 for EU-3 during the years 1998 to 2004 and zero otherwise.
*d9804*doecd6*	Interaction term: equals 1 for OECD-6 countries during the years 1998 to 2004 and zero otherwise.
*d9297*gap*deu3*	Interaction term between Maastricht Treaty, *gap* and *deu3*: equals the output gap for EU-3 sample of countries during the period 1992-1997 and zero otherwise.
*d9804*gap*deu3*	Interaction term between SGP, *gap* and *deu3*: equals the output gap for EU-3 sample of countries during the period 1998-2004, and zero otherwise.
*d9297*gap*doecd6*	Interaction term between Maastricht Treaty, *gap* and *doecd6*: equals the output gap for OECD-6 sample of countries during the period 1992-1997 and and zero otherwise.
*d9804*gap*doed6*	Interaction term between SGP, *gap* and *doecd6*: equals the output gap for OECD-6 sample of countries during the period 1998-2004 and zero otherwise.
*mas*deu3*	Interaction term between the variable *mas* and *deu3*: equals the variable *mas* for EU-3 sample of countries, and zero otherwise.
*mas*doecd6*	Interaction term between the variable *mas* and *doecd6*: equals the variable *mas* for OECD-6 sample of countries, and zero otherwise.
*sgp*deu3*	Interaction term between the variable *sgp* and *deu3*: equals the variable *sgp* for EU-3 sample of countries, and zero otherwise.
*sgp*doecd6*	Interaction term between the variable *sgp* and *doecd6*: equals the variable *sgp* for OECD-6 sample of countries, and zero otherwise.

Table 4.2: Unweighted average - cyclically-adjusted primary deficit (as % of pot. GDP) in OECD countries

Country	1980-1991	1992-1997	1998-2004	1980-2004
AUT	0.64	0.51	-0.96	0.16
BEL	-0.30	-5.06	-6.16	-3.08
DEU	0.06	-0.50	-0.07	-0.11
ESP	1.69	-0.79	-2.07	0.04
FIN	-2.08	-0.32	-4.24	-2.26
FRA	-0.26	0.94	-0.26	0.03
GRC	4.22	-2.81	-1.38	0.96
IRE	1.43	-4.20	-1.39	-0.71
ITA	2.86	-4.12	-2.89	-0.43
NLD	0.27	-1.56	-1.34	-0.62
PRT	-0.51	-1.32	0.28	-0.48
DNK	-1.61	-1.27	-2.60	-1.81
GBR	-1.62	1.93	-1.38	-0.70
SWE	0.54	2.93	-2.60	0.23
AUS	-0.74	-0.81	-2.35	-1.21
CAN	2.60	-1.04	-3.79	-0.06
ISL	1.59	-0.30	-1.47	0.28
JPN	-1.18	2.19	4.78	1.30
NOR	3.69	6.22	5.67	4.85
USA	0.95	-0.50	-0.92	0.08
Euro-11	0.73	-1.75	-1.86	-0.59
EU-3	-0.90	1.20	-2.19	-0.76
OECD-6	1.15	0.96	0.32	0.87
OECD-20	0.61	-0.49	-1.26	-0.18

Source: OECD (2005) and own calculations

Table 4.3: Unweighted average - output gap (in %) in OECD countries

Country	1980-1991	1992-1997	1998-2004	1980-2004
AUT	-0.70	-0.23	0.80	-0.17
BEL	-0.92	-1.64	-0.20	-0.89
DEU	-0.29	-0.72	0.04	-0.30
ESP	-1.72	-3.16	-0.26	-1.66
FIN	1.13	-7.83	0.13	-1.30
FRA	-1.05	-1.47	-0.43	-0.98
GRC	-0.48	-3.02	0.51	-0.81
IRE	-2.18	-3.91	2.40	-1.31
ITA	-1.06	-1.97	0.32	-0.89
NLD	-0.52	0.04	1.03	0.05
PRT	-2.63	-1.21	0.67	-1.36
DNK	-0.65	-0.83	0.12	-0.48
GBR	-1.25	-1.73	0.23	-0.95
SWE	-0.97	-4.65	0.01	-1.58
AUS	-1.69	-2.54	1.03	-1.13
CAN	-1.08	-2.51	0.43	-1.00
ISL	1.52	-4.61	-0.55	-0.53
JPN	-0.07	0.83	-1.81	-0.34
NOR	-0.59	-1.85	1.76	-0.24
USA	-1.28	-1.38	-0.46	-1.07
Euro-11	-0.95	-2.28	0.45	-0.87
EU-3	-0.96	-2.40	0.12	-1.00
OECD-6	-0.53	-2.01	0.07	-0.72
OECD-20	-0.82	-2.22	0.29	-0.85

Source: OECD (2005) and own calculations

Table 4.4: Effects of the MT and the SGP on the CAPD - Euro-11 (1980 - 2004)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
α	1.53**	2.00*	1.20*	1.74	1.44**	1.83*	1.05	1.64
	(0.68)	(1.05)	(0.68)	(1.09)	(0.68)	(1.02)	(0.68)	(1.04)
pdefay(-1)	0.77***	0.77***	0.78***	0.78***	0.75***	0.75***	0.76***	0.76***
	(0.04)	(0.04)	(0.04)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)
ggflq(-1)	-0.03***	-0.02***	-0.02***	-0.02**	-0.02***	-0.02**	-0.02**	-0.02**
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
inf	-0.05	-0.03	-0.03	-0.02	-0.07	-0.06	-0.05	-0.05
	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.06)
ele	0.65***	0.61***	0.64***	0.61***	0.66***	0.63***	0.65***	0.63***
	(0.17)	(0.15)	(0.17)	(0.16)	(0.15)	(0.14)	(0.16)	(0.15)
gap	-0.03	-0.01	-0.03	-0.02	-0.03	-0.01	-0.02	-0.02
	(0.06)	(0.07)	(0.11)	(0.11)	(0.06)	(0.07)	(0.11)	(0.12)
d9297		-0.90**		-1.01**		-0.57		-0.72
		(0.44)		(0.44)		(0.49)		(0.49)
d9804		-0.44		-0.48		-0.54		-0.62
		(0.43)		(0.47)		(0.46)		(0.49)
*d9297*gap*			-0.01	-0.04			-0.03	-0.06
			(0.12)	(0.13)			(0.13)	(0.12)
*d9804*gap*			0.10	0.18			0.20	0.20
			(0.18)	(0.14)			(0.20)	(0.15)
mas					-0.58***	-0.57***	-0.63***	-0.61***
					(0.21)	(0.17)	(0.20)	(0.15)
sgp					-0.78**	-0.88***	-0.94**	-0.88**
					(0.31)	(0.31)	(0.47)	(0.37)
Adjusted R^2	0.80	0.80	0.80	0.80	0.81	0.81	0.81	0.81
Hausman Statistic[a]	41.19***	44.00***	19.50***	20.76***	43.15***	45.67***	21.21***	22.49***
Cross-Section	11	11	11	11	11	11	11	11
Observations	263	263	263	263	263	263	263	263

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Table 4.5: Comparison - effects of the MT and the SGP on the CAPD - EU-14 (1980 - 2004)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
α	2.82***	2.16***	2.84***	2.34***	2.69***	2.14***	2.80***
	(0.96)	(0.55)	(0.71)	(0.62)	(0.90)	(0.51)	(0.63)
pdefay(-1)	0.71***	0.74***	0.74***	0.73***	0.73***	0.74***	0.76***
	(0.05)	(0.04)	(0.05)	(0.05)	(0.05)	(0.04)	(0.04)
ggflq(-1)	-0.02***	-0.02***	-0.02**	-0.02***	-0.02**	-0.01**	-0.01*
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
*ggflq(-1)*deu3*	-0.09***	-0.09***	-0.11***	-0.08***	-0.09***	-0.09***	-0.11***
	(0.02)	(0.03)	(0.03)	(0.03)	(0.02)	(0.03)	(0.02)
inf (A)	-0.02	-0.02	-0.01	-0.05	-0.05	-0.06	-0.05
	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.05)	(0.05)
*inf*deu3 (B)*	-0.03	-0.09*	-0.18**	-0.08	-0.07	-0.10**	-0.21**
	(0.21)	(0.05)	(0.07)	(0.07)	(0.17)	(0.05)	(0.08)
ele	0.60***	0.59***	0.60***	0.62***	0.64***	0.61***	0.65***
	(0.12)	(0.14)	(0.14)	(0.13)	(0.12)	(0.13)	(0.14)
gap (C)	0.01	0.02	0.04	0.01	0.01	0.01	0.02
	(0.08)	(0.10)	(0.10)	(0.07)	(0.07)	(0.09)	(0.09)
*gap*deu3 (D)*	-0.20**	-0.19	-0.25	-0.28*	-0.21**	-0.16	-0.22
	(0.09)	(0.14)	(0.16)	(0.16)	(0.11)	(0.16)	(0.18)
d9297	-0.69*		-0.81**		-0.35		-0.52
	(0.41)		(0.40)		(0.43)		(0.42)
*d9297*deu3*	1.47		0.89		1.54		0.82
	(1.24)		(0.89)		(1.03)		(0.50)
d9804 (E)	-0.33		-0.27		-0.39		-0.41
	(0.38)		(0.38)		(0.40)		(0.39)
*d9804*deu3 (F)*	-0.16		-0.88*		-0.38		-1.06**
	(1.18)		(0.45)		(0.99)		(0.47)
*d9297*gap (G)*		-0.02	-0.11			-0.03	-0.10
		(0.13)	(0.11)			(0.12)	(0.10)
*d9297*gap*deu3 (H)*		-0.33***	0.04			-0.55***	-0.10
		(0.07)	(0.29)			(0.10)	(0.24)
*d9804*gap (I)*		0.11	0.13			0.21	0.18
		(0.16)	(0.12)			(0.19)	(0.13)
*d9804*gap*deu3 (J)*		0.13	0.41**			0.22	0.36
		(0.29)	(0.19)			(0.27)	(0.22)
mas				-0.68***	-0.59***	-0.75***	-0.63***
				(0.25)	(0.18)	(0.22)	(0.16)
*mas*deu3*				0.27	-0.37	-0.47	-0.53*
				(0.37)	(0.27)	(0.54)	(0.29)
sgp				-0.65*	-0.82**	-1.00**	-0.92***
				(0.38)	(0.35)	(0.48)	(0.35)
*sgp*deu3*				4.78	4.59	1.05	1.73
				(3.36)	(2.92)	(1.93)	(2.09)
inf EU-3 (A+B)[a]	-0.05	-0.12**	-0.19**	-0.13**	-0.11	-0.16***	-0.26***
gap EU-3 (C+D)[a]	-0.19***	-0.17	-0.22*	-0.27*	-0.21***	-0.15	-0.21
d9804 EU-3 (E+F)[a]	-0.49		-1.16*		-0.77		-1.47***
*d9297*gap EU-3 (G+H)*[a]		-0.35**	-0.07			-0.58***	-0.20
*d9804*gap EU-3 (I+J)*[a]		0.25	0.54***			0.43	0.53***
Adjusted R^2	0.79	0.79	0.79	0.79	0.80	0.80	0.80
Hausman Statistic[b]	34.98***	13.89***	14.98***	34.25***	36.28***	14.84***	15.31***
Cross-Section	14	14	14	14	14	14	14
Observations	337	337	337	337	337	337	337

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Wald test of coefficient restrictions whose null hypothesis is that the estimated coefficient is equal to zero. [b] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Table 4.6: Comparison - effects of the MT and the SGP on the CAPD - OECD-17 (1980 - 2004)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
α	1.58**	1.01**	1.49**	1.46**	1.83**	1.19**	1.60**
	(0.67)	(0.42)	(0.63)	(0.63)	(0.84)	(0.51)	(0.76)
pdefay(-1)	0.79***	0.82***	0.80***	0.77***	0.76***	0.78***	0.78***
	(0.04)	(0.04)	(0.04)	(0.05)	(0.04)	(0.05)	(0.05)
ggflq(-1)	-0.02***	-0.02***	-0.02**	-0.02***	-0.02***	-0.02***	-0.02***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
inf	0.00	0.00	0.01	-0.05	-0.03	-0.04	-0.02
	(0.04)	(0.04)	(0.04)	(0.05)	(0.06)	(0.05)	(0.05)
ele (A)	0.63***	0.66***	0.63***	0.65***	0.64***	0.66***	0.65***
	(0.15)	(0.17)	(0.16)	(0.16)	(0.15)	(0.16)	(0.16)
*ele*doecd6 (B)*	-0.55**	-0.57*	-0.53*	-0.58**	-0.56**	-0.55*	-0.54**
	(0.25)	(0.30)	(0.28)	(0.27)	(0.25)	(0.28)	(0.27)
gap (C)	0.00	0.00	0.01	-0.04	-0.01	-0.01	0.01
	(0.05)	(0.06)	(0.07)	(0.04)	(0.05)	(0.07)	(0.07)
*gap*doecd6 (D)*	-0.26***	-0.17*	-0.25***	-0.12	-0.23***	-0.18*	-0.24**
	(0.07)	(0.09)	(0.09)	(0.08)	(0.08)	(0.10)	(0.09)
d9297	-0.87***		-1.02***		-0.59		-0.75*
	(0.33)		(0.35)		(0.37)		(0.39)
*d9297*doecd6*	0.28		0.50		-0.13		0.10
	(0.64)		(0.82)		(0.68)		(0.92)
d9804	-0.39		-0.40		-0.49		-0.52
	(0.33)		(0.36)		(0.34)		(0.37)
*d9804*doecd6*	0.88*		0.89**		0.67		0.78
	(0.47)		(0.44)		(0.48)		(0.50)
*d9297*gap*		-0.03	-0.09			-0.07	-0.10
		(0.10)	(0.09)			(0.10)	(0.08)
*d9297*gap*doecd6*		0.09	0.13			0.17*	0.13
		(0.10)	(0.20)			(0.09)	(0.22)
*d9804*gap*		0.01	0.12			0.07	0.15
		(0.13)	(0.11)			(0.16)	(0.12)
*d9804*gap*doecd6*		-0.21	-0.27			-0.07	-0.25
		(0.25)	(0.22)			(0.22)	(0.25)
mas (E)				-0.40**	-0.50***	-0.44***	-0.52***
				(0.18)	(0.18)	(0.17)	(0.16)
*mas*doecd6 (F)*				0.47	0.67*	0.57*	0.64*
				(0.46)	(0.38)	(0.34)	(0.37)
sgp (G)				-0.79**	-0.64*	-0.83**	-0.62*
				(0.34)	(0.35)	(0.40)	(0.35)
*sgp*doecd6 (H)*				1.62***	1.17**	1.54***	0.92*
				(0.55)	(0.52)	(0.59)	(0.52)
ele OECD-6 (A+B)[a]	0.08	0.09	0.10	0.07	0.09	0.11	0.11
gap OECD-6 (C+D)[a]	-0.26***	-0.18***	-0.24***	-0.16***	-0.23***	-0.18***	-0.23***
mas OECD-6 (E+F)[a]				0.06	0.17	0.12	0.12
sgp OECD-6 (G+H)[a]				0.83**	0.53*	0.71**	0.30
Adjusted R^2	0.84	0.85	0.84	0.85	0.85	0.85	0.85
Hausman Statistic[b]	64.63***	22.81***	25.51***	53.24***	58.54***	21.90***	23.69***
Cross-Section	17	17	17	17	17	17	17
Observations	390	390	390	390	390	390	390

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Wald test of coefficient restrictions whose null hypothesis is that the estimated coefficient is equal to zero. [b] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Table 4.7: List of variables of the robustness tests

Economic	*irlrc*	Ex-post real long-term interest rate, based on Private Consumption Deflator, in %
	nwp	Share of the non-working population.
	open	Trade Openness.
	size	Relative country size in terms of GDP
	vol	GDP volatility in the previous ten years.
Political	*gpart*	Cabinet Composition: index representing the political color (right or left) of the Cabinet in power.
	gnew	New party composition of cabinet.
	ggap	"Ideological gap" between new cabinet and old one.
	gchan	Number of changes in government per year.
	gtype	Type of Government Coalition.
	rae	Index of fractionalization of the party-system according to Rae (1968).

Table 4.8: Results summary of the robustness tests

Variables	Tests	Euro-11 results	Euro-14 results	OECD-17 results
irlrc	coefficient	Non-significant.	Non-significant.	Non-significant.
	robustness	Robust.	Robust.	*d9804*doecd6* less significant, Result 4.7 weakened.
nwp	coefficient	Positive and marginally significant (10%)	Euro-11 differ from EU-3. More negative for EU-3.	Euro-11 differ from OECD-6. More negative for OECD-6.
	robustness	Robust.	*d9297* less significant. *d9297*deu3*: signif. positive. Result 4.4-i weakened.	*d9804*doecd6* less significant. Result 4.7 weakened.
open	coefficient	Negative and highly significant.	Negative and highly significant for EU-14.	Negative and highly significant for OECD-17.
	robustness	Robust.	*d9297* less significant. Result 4.4-i weakened.	*d9804*doecd6* less significant. Result 4.7 weakened. *d9804*doecd6*gap* significant. and negative. OECD more countercyclical during SGP.
size	coefficient	Negative and highly significant. After 1998, *size*d9804*, the coefficient becomes significantly less negative.	Negative and highly significant. After 1998, *size*d9804*, the coefficient becomes significantly less negative.	Euro-11 differ from OECD-6. Positive and highly significant for OECD-6.
	robustness	*d9804* negative and highly significant. Result 4.1-ii weakened.	*d9297* less significant. *d9804* negative and highly significant. Result 4.4 weakened.	Robust.
vol	coefficient	Non-significant.	Non-significant.	Non-significant.
	robustness	Robust.	*d9297* less significant. *d9804*deu3* less significant. Result 4.4 weakened.	Robust.
gpart	coefficient	Non-significant.	Non-significant.	Non-significant.
	robustness	Robust.	Robust.	Robust.
gnew	coefficient	Non-significant.	Non-significant.	Non-significant.
	robustness	Robust.	Robust.	Robust.
ggap	coefficient	Non-significant.	Euro-11 differ from EU-3. More negative for EU-3.	Non-significant.
	robustness	Robust.	*d9297* less significant. Result 4.4-i weakened.	Robust.
gchan	coefficient	Positive and highly significant.	Positive and highly significant for EU-14.	Euro-11 differ from OECD-6. More negative for OECD-6.
	robustness	Robust.	Robust.	*d9804*doecd6* less significant. Result 4.7 weakened.
gtype	coefficient	Non-significant.	Euro-11 differ from EU-3. More negative for EU-3.	Non-significant.
	robustness	*d9297* less significant. Result 4.1-i weakened.	*d9297* less significant. Result 4.4-i weakened.	Robust.
rae	coefficient	Non-significant.	Euro-11 differ from EU-3. More positive for EU-3.	Non-significant.
	robustness	*d9297* less significant. Result 4.1-i weakened.	*d9297* less significant. Result 4.4-i weakened.	*d9804*doecd6* less significant. Result 4.7 weakened.

Figure 4.1: Cyclically-adjusted primary deficit (as % of potential GDP) - OECD regions

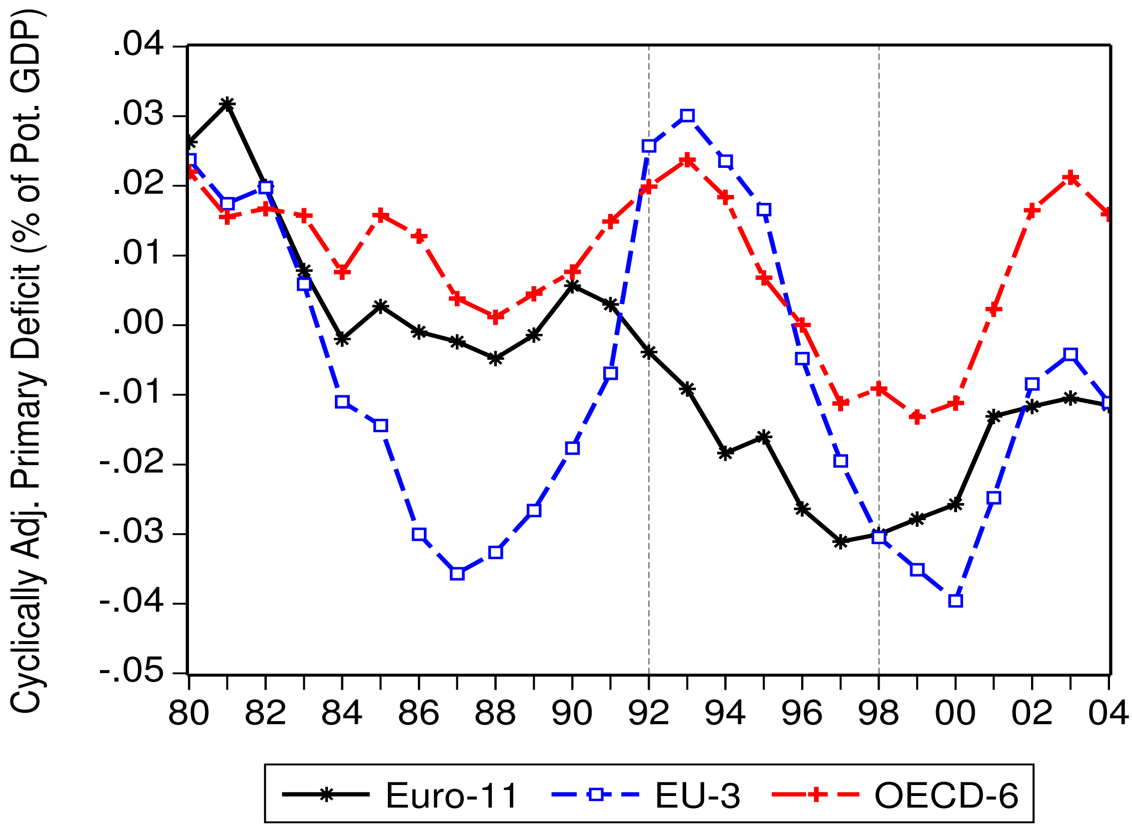


Figure 4.2: Cyclically-adjusted primary deficit (as % of potential GDP) - OECD countries

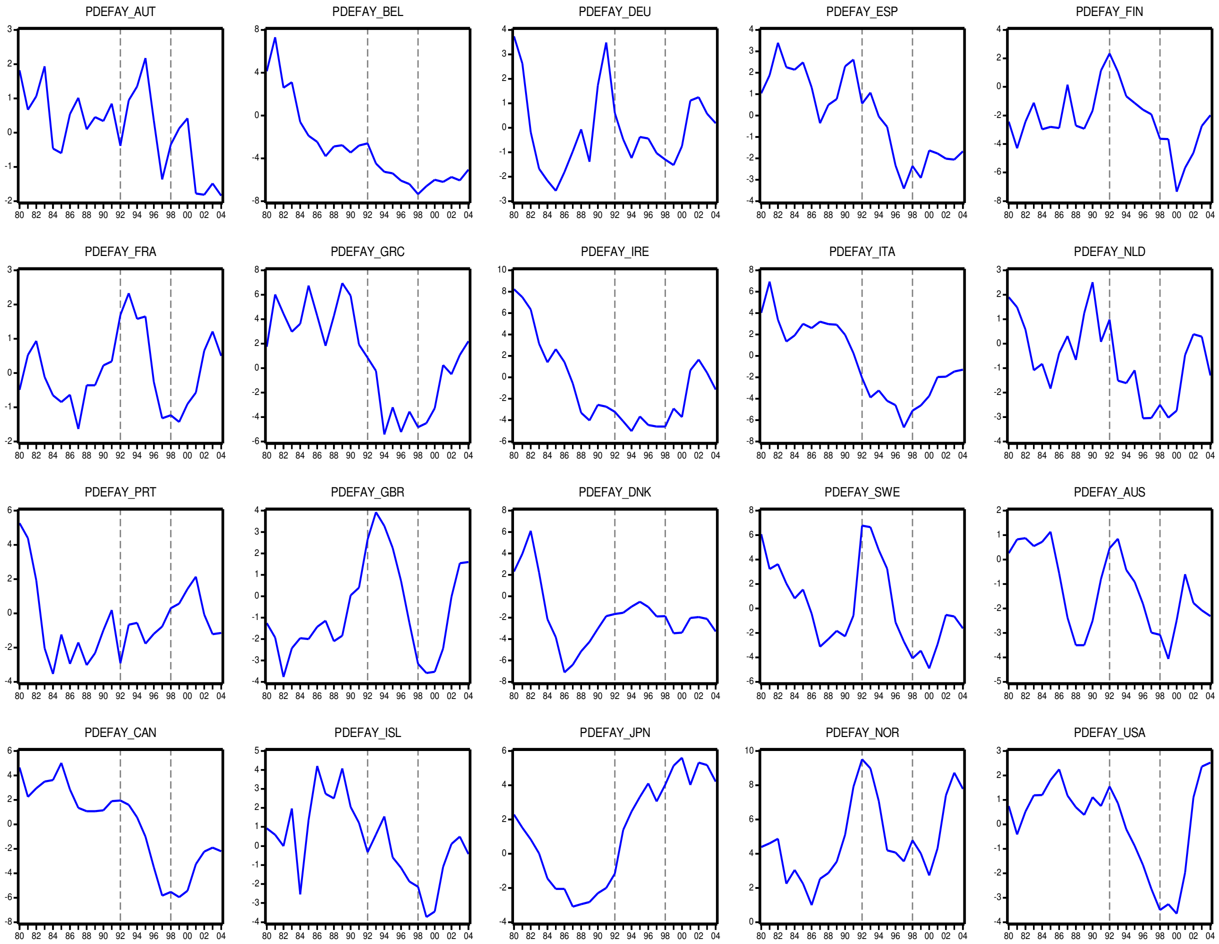


Figure 4.3: Scatter plots - cyclically-adjusted primary deficit x output gap

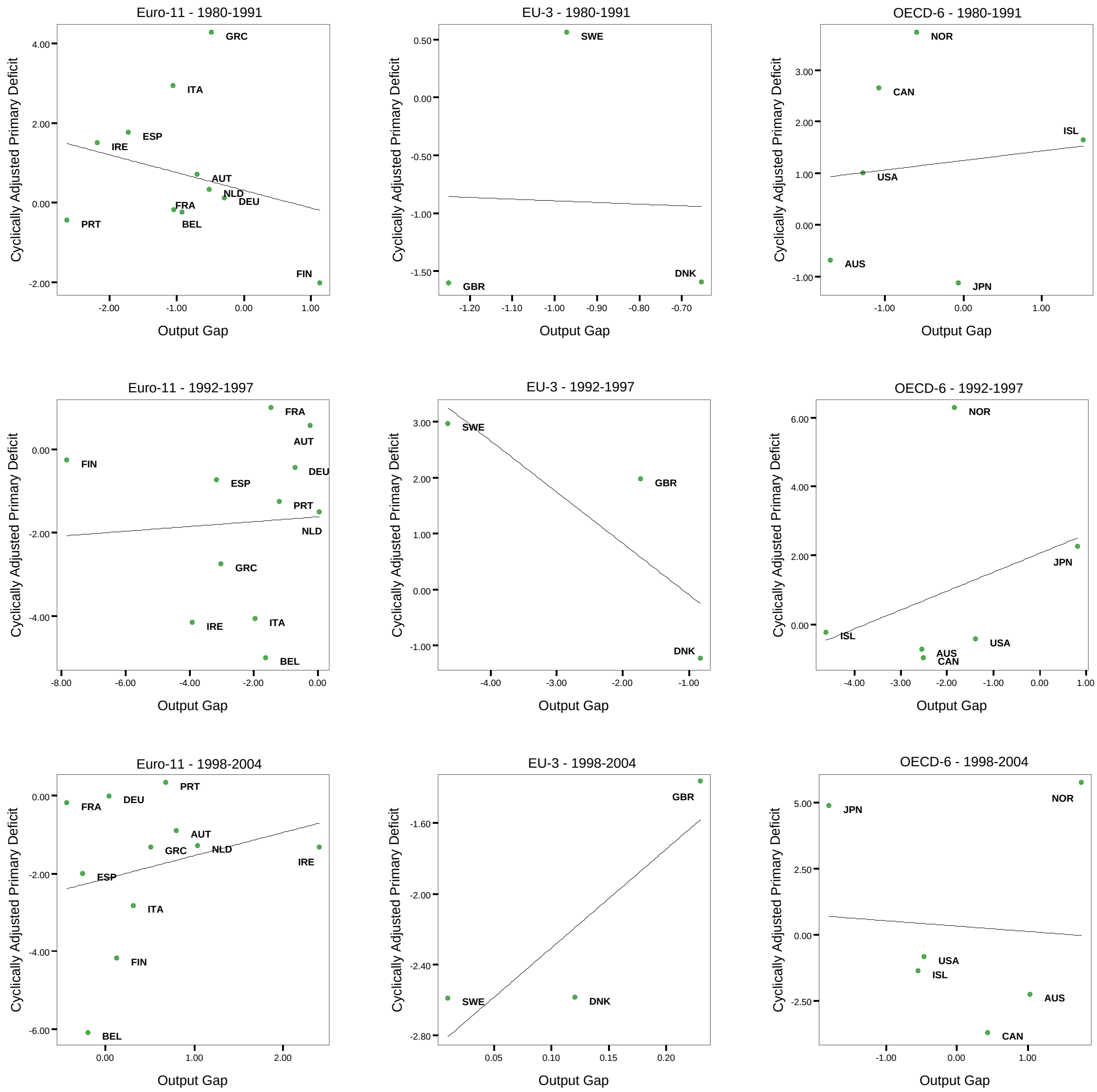
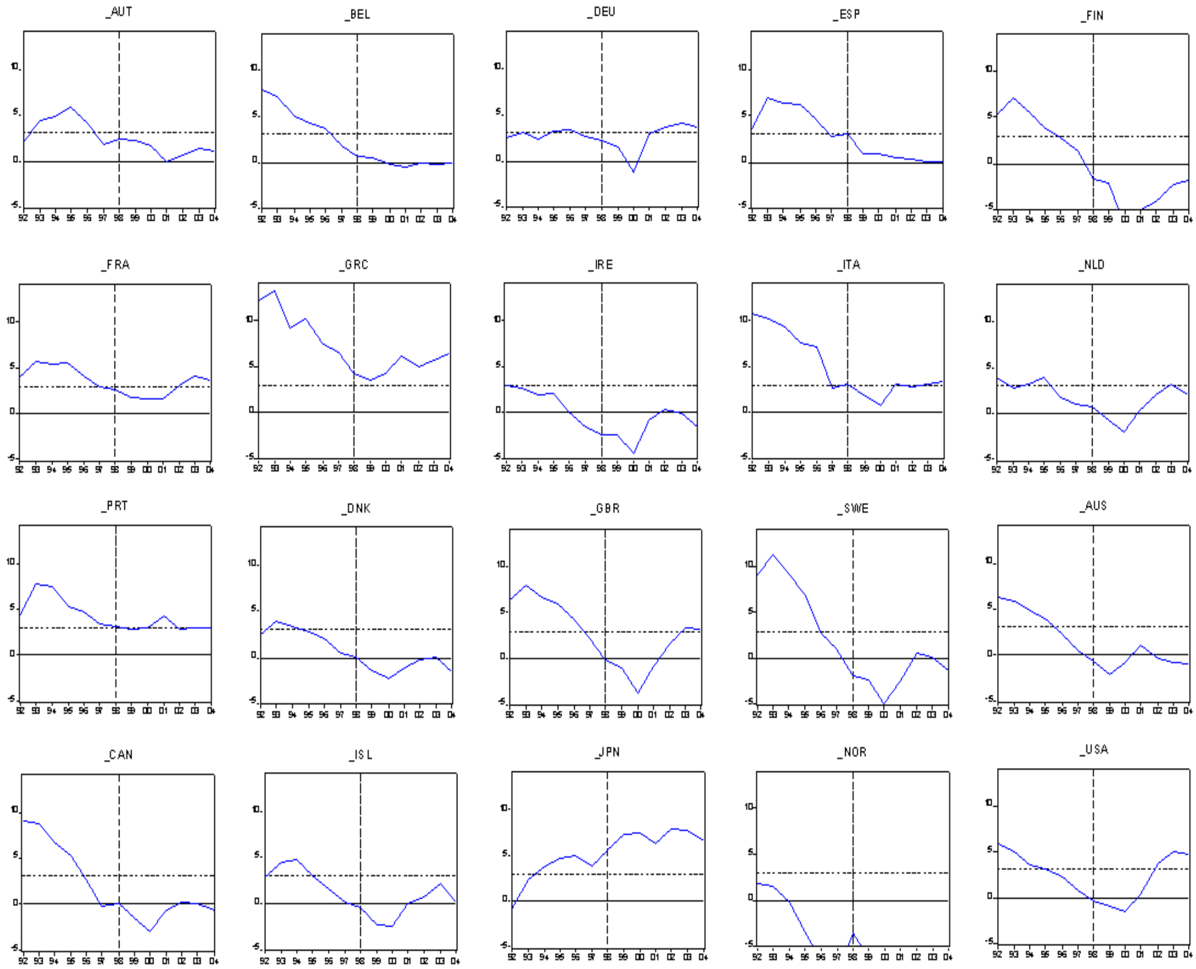


Figure 4.4: Total deficit (as % of actual GDP) - OECD countries



Appendices to Chapter 4

4.A Dataset

The variables explained below are mainly extracted from the OECD Economic Outlook (2005) and AMECO (2005). The parliamentary election variable is constructed with the dataset of the website of the International Institute for Democracy and Electoral Assistance (IDEA – http://www.idea.int/vt/parl.cfm) combined with the information from the website (http://electionresources.org) for.

4.A.1 List of variables

In this subsection, we describe the dependent, control and other support variables used in the estimation of (4.1), their source and construction, the intuition for using them and, finally, the sign with which we expect them to show up in the regressions.

Dependent variable

Variable: *Cyclically Adjusted Primary Deficit (as % of potential output)*: $PDEFAY_{it}$.
Source: OECD Economic Outlook (2005)

Construction: Minus the Primary Government Balance, Cyclically Adjusted, as % of potential GDP: $-NLGXQA_{i,t}$.

It uses a reference value of GDP (Y^*) whose method is explained in Giorgio *et al.* (1995) or OECD 2004, and estimated values for the output elasticity of tax revenues and spending α and β as:

$$\frac{T^*_{j,t}}{T_{j,t}} = \left(\frac{Y^*_t}{Y_t}\right)^{\alpha_j}; \quad \frac{G^*_t}{G_t} = \left(\frac{Y^*_t}{Y_t}\right)^{\beta},$$

where t is the year of observation, j corresponds to four different tax categories[34], T_j is the actual tax revenues for the jth tax category; G is actual government expenditures (excluding capital spending), Y is the level of actual output, Y^* is the level of potential output, α_j is the elasticity of the jth tax category with respect to output and β is the elasticity of current government expenditures with respect to output. The OECD measure of the cyclically adjusted primary deficit is then given by (see also Giorgio *et al.* (1995))

$$PDEFA = G_t \left(\frac{Y^*_t}{Y_t}\right)^{\beta} + capital\ spending - \sum_{j=1}^{4} T_{j,t} \left(\frac{Y^*_t}{Y_t}\right)^{\alpha_j}, \qquad \alpha_j > 0 \text{ and } \beta < 0.$$

Finally, the equation above is divided by the potential output Y^*_t and multiplied by 100 to obtain the Cyclically Adjusted Primary Deficit as % of potential output.

[34]For a detailed explanation of each tax category see Giorgio *et al.* (1995).

Control variables

Variable: *Lagged Cyclically Adjusted Primary Deficit (as % of potential output)*: $PDEFAY_{i,t-1} = -NLGXQA_{i,t-1}$. **Source:** OECD Economic Outlook (2005).

Variable: *Lagged Debt (as % of potential output).* **Source:** OECD Economic Outlook (2005).

Construction: OECD Lagged Value of General Government gross financial liabilities (Gross Government Debt) divided by the lagged value of Potential Output of Total Economy at current prices multiplied by 100: $GGFL_{i,t-1}/GDPTR_{i,t-1}$.

Potential output is used as a deflator of all variables, instead of actual output, to reduce endogeneity problems and to minimize the influence of current GDP on the evolution of the fiscal ratios.

Variable *Inflation of private consumption:* $INF_{i,t}$. **Source:** OECD Economic Outlook (2005).

Construction: Uses OECD deflator for private consumption as follows: $INF_{i,t} = (PCP_{i,t}/PCP_{i,t-1} - 1) * 100$.

Variable: *Elections.* **Source:** Website of the International Institute for Democracy and Electoral Assistance- IDEA (http://www.idea.int/vt/parl.cfm) combined with the most recent information of the website (http://electionresources.org).

Construction: A dummy variable ($ELE_{i,t}$) that takes on value 1 if in country i there were elections for the parliament in t and zero otherwise. Since the United States has a clear presidential regime, this dummy assumes value 1 only in the years of presidential elections. For France, following Afonso (2005), we use the dates of the parliamentary elections instead of the presidential ones, since the latter followed in the past a longer political cycle resulting in a smaller number of observations.

Variable: *Output Gap:* $GAP_{i,t}$. **Source**: OECD Economic Outlook (2005).

Construction: The OECD computes its measure of the output gap using as potential output a production function-based method. In its simplest form, a two-factor Cobb Douglas production function for the business sector is estimated for each country, for given average labour shares over the sample (see Giorno et al., 1995). The estimated residuals from these equations are then smoothed to give measures of trend total factor productivity. This measure of trend factor productivity is then combined with the actual capital stock and estimates of "'potential"' employment, using the same estimated production function to calculate the potential output for the business sector. The chosen measure of "'potential"' employment is defined as the level of labour resources that might be used without resulting in additional inflation. This amounts to adjusting the actual labour input used in estimated production function for the gap between actual unemployment and the estimated NAWRU level.

Additional support variables for the estimation of (4.1)

Variable: *Indicator of participation in the European Union, but not in the Eurozone:* $DEU3_i$. **Source:** Constructed by the authors.

Construction: This variable takes on the value of 1 over the entire sample period if country i currently participates in the European Union but *not* in its monetary union and zero, otherwise. Therefore, this variable assumes a value of 1 from 1980 to 2004 for Denmark, Sweden and United Kingdom.

Variable: *Indicator of "industrialized" OECD countries that are not members of the Euro zone:* $DOECD6_i$. **Source:** Constructed by the authors.

Construction: This variable assumes the value of 1 from 1980 to 2004 for Australia, Canada, Iceland, Japan, Norway and the USA.

Variable: *First Phase of European Monetary Union or Dummy Maastricht Treaty:* $D9297_t$. **Source:** Constructed by the authors.

Construction: This is a time dummy variable that takes on a value of 1 for all countries during the years 1992 until 1997 and zero otherwise. As the name indicates, it refers to the period after the Maastricht Treaty until the implementation of the SGP.

Variable: *Time dummy for the period of existence of the Stability and Growth Pact*: $D9804_t$. **Source:** Constructed by the authors.

Construction: This variable takes on a value of 1 for all countries during the years 1998 until 2004 (the final year of our dataset) and zero otherwise.

Variable: *Total Deficit (as % of actual output)*: $TDEFY_{it}$. **Source**: OECD Economic Outlook (2005)

Construction: Minus the OECD variable *government net lending, as % of actual GDP*, or $-NLGQ_{i,t}$. It is used to construct the testing variables $mas_{i,t}$ and $sgp_{i,t}$ via equations (4.2) and (4.3). Table 4.9 below displays respectively the unweighted averages of this variable for all OECD countries during four different periods: 1980-1991, 1992-1997, 1998-2004, and the entire time span 1980-2004. In addition, its last four lines also convey averages for each of the three group of countries analyzed.

Table 4.9: Unweighted average - total deficit (as % of actual GDP) in OECD countries

Country	1980-1991	1992-1997	1998-2004	1980-2004
AUT	3.20	3.80	1.34	2.83
BEL	9.81	4.99	0.00	5.91
DEU	2.19	2.82	2.36	2.39
ESP	4.37	5.15	0.83	3.56
FIN	-3.52	4.38	-3.50	-1.62
FRA	2.03	4.60	2.63	2.82
GRC	9.78	9.84	5.04	8.47
IRE	8.40	1.38	-1.59	3.92
ITA	11.14	7.94	2.60	7.98
NLD	4.51	2.81	0.78	3.06
PRT	6.48	5.52	3.12	5.31
DNK	2.72	2.48	-0.97	1.63
GBR	2.34	5.57	0.37	2.56
SWE	1.36	6.74	-1.81	1.77
AUS	3.43	3.92	-0.68	2.40
CAN	6.21	5.41	-0.84	4.04
ISL	1.23	2.77	-0.37	1.15
JPN	0.58	3.15	6.92	2.97
NOR	-4.52	-2.43	-9.60	-5.44
USA	4.21	3.40	1.56	3.27
Euro-11	5.31	4.84	1.24	4.06
EU-3	2.14	4.93	-0.80	1.99
OECD-6	1.86	2.70	-0.50	1.40
OECD-20	3.80	4.21	0.41	2.95

Source: OECD Economic Outlook n^{o}78 and own calculations

4.B Instrumental variables

Table 4.1: Instrumental variables estimations

	Euro-11			*EU-3*			*EU-14*			*OECD-17*		
	pdefay(-1)	*inf*	*gap*	*pdefay(-1)*	*inf*	*gap*	*pdefay(-1)*	*inf*	*gap*	*pdefay(-1)*	*inf*	*gap*
a	-0.22**	0.38	0.54**	-0.23	0.22	-0.15	-0.21**	0.36	0.39**	-0.09	0.31	0.60***
	(0.09)	(0.28)	(0.21)	(0.22)	(0.58)	(0.39)	(0.09)	(0.25)	(0.19)	(0.07)	(0.22)	(0.19)
pdefay(-2)	0.88***			0.96***			0.91***			0.97**		
	(0.06)			(0.11)			(0.05)			(0.05)		
pdefay(-3)	-0.05			-0.25**			-0.11**			-0.10*		
	(0.06)			(0.11)			(0.05)			(0.05)		
gap(-1)		0.29***	1.26***		0.40***	1.11***		0.33***	1.25***		0.32***	1.17***
		(0.06)	(0.05)		(0.14)	(0.09)		(0.06)	(0.04)		(0.05)	(0.04)
gap(-2)		-0.17***	-0.58***		-0.09	-0.48***		-0.17***	-0.57***		-0.18***	-0.52***
		(0.06)	(0.05)		(0.14)	(0.09)		(0.06)	(0.04)		(0.05)	(0.04)
inf(-1)		1.04***	-0.23***		0.90***	-0.34***		0.99***	-0.24***		1.05***	-0.23***
		(0.08)	(0.06)		(0.17)	(0.12)		(0.07)	(0.05)		(0.06)	(0.06)
inf(-2)		-0.13	0.15**		0.01	0.23*		-0.09	0.17***		-0.14**	0.15***
		(0.08)	(0.06)		(0.18)	(0.12)		(0.08)	(0.06)		(0.07)	(0.06)
irlrc(-1)		0.09	-0.27***		0.37**	-0.34***		0.16**	-0.29***		0.14**	-0.26***
		(0.09)	(0.06)		(0.18)	(0.12)		(0.08)	(0.06)		(0.07)	(0.06)
irlrc(-2)		-0.12	0.14**		-0.33**	0.38***		-0.18**	0.20***		-0.15**	0.13**
		(0.08)	(0.06)		(0.16)	(0.11)		(0.07)	(0.05)		(0.06)	(0.05)
Adj. R-squared	0.74	0.91	0.81	0.61	0.78	0.79	0.71	0.90	0.80	0.78	0.91	0.77
Cross-Section	11	11	11	3	3	3	14	14	14	17	17	17
Observations	264	268	268	75	75	75	339	343	343	412	404	404

Notes: Regressions estimated by Ordinary Least Squares (OLS). ***,**,* Level of significance at 1%, 5% and 10% respectively. Estimated standard errors in parenthesis.

4.C Estimations for EU-14

Table 4.2: Effects of the Maastricht Treaty and the SGP on the CAPD for the EU-14 (1980 - 2004) - homogeneous control variable coefficients

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
α	2.05***	2.69***	1.90***	2.74***	2.01***	2.55***	1.83***	2.65***
	(0.70)	(0.95)	(0.62)	(0.89)	(0.71)	(0.92)	(0.66)	(0.87)
pdefay(-1)	0.73***	0.71***	0.76***	0.75***	0.73***	0.71***	0.75***	0.75***
	(0.04)	(0.05)	(0.04)	(0.04)	(0.05)	(0.05)	(0.04)	(0.04)
ggflq(-1)	-0.03***	-0.04***	-0.03***	-0.03**	-0.03***	-0.03***	-0.03***	-0.03***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
inf	-0.07	-0.04	-0.07	-0.06	-0.08	-0.06	-0.10*	-0.09*
	(0.05)	(0.05)	(0.04)	(0.05)	(0.05)	(0.06)	(0.05)	(0.05)
ele	0.58***	0.57***	0.59***	0.59***	0.60***	0.60***	0.60***	0.63***
	(0.14)	(0.12)	(0.14)	(0.13)	(0.13)	(0.12)	(0.13)	(0.13)
gap	-0.10**	-0.08	-0.11	-0.12	-0.10**	-0.08	-0.11	-0.11
	(0.04)	(0.05)	(0.08)	(0.08)	(0.05)	(0.05)	(0.09)	(0.08)
d9297		-0.81**		-0.93***		-0.42		-0.61
		(0.32)		(0.31)		(0.39)		(0.37)
d9804		-0.57*		-0.70**		-0.67**		-0.84**
		(0.32)		(0.34)		(0.34)		(0.34)
*d9297*gap*			0.02	0.02			-0.02	-0.03
			(0.12)	(0.11)			(0.12)	(0.11)
*d9804*gap*			0.17	0.27**			0.24	0.29**
			(0.18)	(0.13)			(0.19)	(0.13)
mas					-0.56***	-0.64***	-0.63***	-0.71***
					(0.19)	(0.16)	(0.16)	(0.14)
sgp					-0.46	-0.45	-0.79*	-0.70*
					(0.40)	(0.45)	(0.43)	(0.37)
Adjusted R^2	0.77	0.76	0.77	0.76	0.77	0.77	0.78	0.78
Hausman Statistic[a]	59.50***	68.23***	27.90***	32.73***	61.38***	69.45***	30.25***	34.01***
Cross-Section	14	14	14	14	14	14	14	14
Observations	337	337	337	337	337	337	337	337

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Table 4.3: Effects of the MT and the SGP on the CAPD for the EU-14 (1980 - 2004) - heterogeneous control variable coefficients

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
α	2.33***	2.52***	2.07***	2.40***	2.35***	2.51***	2.09***	2.49***
	(0.63)	(0.90)	(0.52)	(0.73)	(0.64)	(0.88)	(0.55)	(0.72)
pdefay(-1)	0.76***	0.76***	0.77***	0.77***	0.74***	0.74***	0.74***	0.74***
	(0.04)	(0.04)	(0.04)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)
*pdefay(-1)*deu3*	-0.05	-0.09	-0.06	-0.07	0.00	0.00	0.03	0.05
	(0.11)	(0.12)	(0.11)	(0.11)	(0.12)	(0.11)	(0.11)	(0.09)
ggflq(-1)	-0.02***	-0.02**	-0.02**	-0.02**	-0.02**	-0.02**	-0.01*	-0.01
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
*ggflq(-1)*deu3*	-0.08**	-0.08**	-0.09**	-0.09**	-0.08***	-0.08***	-0.09***	-0.10***
	(0.03)	(0.03)	(0.04)	(0.04)	(0.03)	(0.03)	(0.03)	(0.03)
inf	-0.04	-0.02	-0.02	-0.01	-0.05	-0.04	-0.04	-0.04
	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.06)	(0.06)
*inf*deu3*	-0.08	-0.07	-0.11	-0.12*	-0.10	-0.10	-0.16***	-0.18***
	(0.09)	(0.09)	(0.07)	(0.07)	(0.08)	(0.08)	(0.05)	(0.05)
ele	0.65***	0.64***	0.66***	0.65***	0.67***	0.66***	0.67***	0.67***
	(0.18)	(0.16)	(0.18)	(0.17)	(0.17)	(0.15)	(0.17)	(0.17)
*ele*deu3*	-0.28	-0.20	-0.29	-0.19	-0.22	-0.12	-0.20	-0.09
	(0.23)	(0.23)	(0.22)	(0.22)	(0.21)	(0.21)	(0.21)	(0.21)
gap	0.02	0.04	0.07	0.08	0.02	0.03	0.07	0.07
	(0.07)	(0.07)	(0.12)	(0.12)	(0.07)	(0.07)	(0.12)	(0.11)
*gap*deu3*	-0.32*	-0.35*	-0.35*	-0.36*	-0.30	-0.31	-0.33	-0.31*
	(0.19)	(0.19)	(0.20)	(0.19)	(0.20)	(0.19)	(0.21)	(0.18)
d9297		-0.31		-0.56		-0.01		-0.32
		(0.55)		(0.46)		(0.53)		(0.45)
d9804		-0.26		-0.35		-0.43		-0.62
		(0.45)		(0.46)		(0.46)		(0.46)
*d9297*gap*			-0.10	-0.13			-0.15	-0.16
			(0.15)	(0.14)			(0.14)	(0.12)
*d9804*gap*			0.06	0.15			0.16	0.20
			(0.18)	(0.13)			(0.20)	(0.13)
mas					-0.63***	-0.66***	-0.75***	-0.77***
					(0.23)	(0.19)	(0.22)	(0.18)
sgp					-0.60	-0.68*	-0.80*	-0.77**
					(0.37)	(0.36)	(0.48)	(0.37)
Adjusted R^2	0.78	0.78	0.78	0.78	0.79	0.79	0.79	0.79
Hausman Statistic[a]	26.90***	29.42***	15.02***	17.31***	28.44***	30.78***	16.21***	17.71***
Cross-Section	14	14	14	14	14	14	14	14
Observations	337	337	337	337	337	337	337	337

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Table 4.4: Comparison - Effects of the MT and the SGP on the CAPD for the EU-14 (1980 - 2004) - homogeneous control variable coefficients

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
α	2.76**	1.80***	2.78***	1.99***	2.60**	1.76***	2.61***
	(1.07)	(0.67)	(1.01)	(0.72)	(1.00)	(0.67)	(0.92)
pdefay(-1)	0.69***	0.72***	0.72***	0.71***	0.70***	0.74***	0.75***
	(0.06)	(0.05)	(0.05)	(0.06)	(0.05)	(0.05)	(0.05)
ggflq(-1)	-0.04***	-0.03***	-0.03***	-0.03***	-0.03***	-0.03***	-0.03***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
inf	-0.04	-0.06	-0.06	-0.08	-0.06	-0.09*	-0.09*
	(0.06)	(0.05)	(0.05)	(0.05)	(0.06)	(0.05)	(0.05)
ele	0.57***	0.58***	0.58***	0.60***	0.61***	0.61***	0.63***
	(0.12)	(0.14)	(0.13)	(0.13)	(0.12)	(0.13)	(0.13)
gap	-0.08	-0.10	-0.11	-0.10**	-0.09	-0.11	-0.11
	(0.06)	(0.09)	(0.09)	(0.05)	(0.06)	(0.09)	(0.09)
d9297	-1.05***		-1.09***		-0.75**		-0.81**
	(0.31)		(0.35)		(0.38)		(0.40)
*d9297*deu3*	1.07***		0.30		1.49**		0.62
	(0.35)		(0.67)		(0.65)		(0.44)
d9804	-0.52		-0.65*		-0.59*		-0.75**
	(0.32)		(0.33)		(0.32)		(0.31)
*d9804*deu3*	-0.43		-0.37		-0.45		-0.37
	(0.42)		(0.41)		(0.36)		(0.36)
*d9297*gap*		0.06	0.03			0.07	0.02
		(0.12)	(0.10)			(0.11)	(0.10)
*d9297*gap*deu3*		-0.37***	-0.27			-0.68***	-0.41**
		(0.10)	(0.16)			(0.12)	(0.16)
*d9804*gap*		0.18	0.24*			0.24	0.27**
		(0.18)	(0.13)			(0.19)	(0.13)
*d9804*gap*deu3*		-0.29	-0.01			-0.21	-0.02
		(0.27)	(0.11)			(0.23)	(0.09)
mas				-0.61***	-0.52***	-0.59***	-0.56***
				(0.23)	(0.19)	(0.21)	(0.15)
*mas*deu3*				0.27	-0.60	-1.04*	-0.99*
				(0.31)	(0.43)	(0.62)	(0.51)
sgp				-0.49	-0.53	-0.77*	-0.77**
				(0.41)	(0.41)	(0.45)	(0.33)
*sgp*deu3*				6.41***	6.51***	4.74**	4.60***
				(2.01)	(1.81)	(2.29)	(1.24)
Adjusted R^2	0.77	0.77	0.77	0.77	0.77	0.79	0.78
Hausman Statistic[a]	38.00***	19.20***	19.31***	58.81***	65.06***	28.36***	22.01***
Cross-Section	14	14	14	14	14	14	14
Observations	337	337	337	337	337	337	337

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

4.D Estimations for OECD-6

Table 4.5: Effects of the Maastricht Treaty and the SGP on the CAPD - OECD-6 (1980 - 2004)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
α	0.26	-0.03	0.20	0.52	0.68	0.19	0.36	0.64
	(0.64)	(0.78)	(0.85)	(0.40)	(1.23)	(1.55)	(1.29)	(1.22)
pdefay(-1)	0.88***	0.86***	0.91***	0.89***	0.86***	0.86***	0.90***	0.88***
	(0.07)	(0.07)	(0.06)	(0.07)	(0.09)	(0.09)	(0.10)	(0.10)
ggflq(-1)	-0.02**	-0.02***	-0.01	-0.03***	-0.02***	-0.03*	-0.01	-0.03**
	(0.01)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
inf	0.22	0.39**	0.16	0.32***	0.20	0.38**	0.15	0.32**
	(0.24)	(0.16)	(0.18)	(0.11)	(0.25)	(0.17)	(0.21)	(0.13)
ele	0.12	-0.01	0.13	0.00	0.11	-0.02	0.13	0.00
	(0.25)	(0.26)	(0.25)	(0.26)	(0.25)	(0.27)	(0.25)	(0.27)
gap	-0.21***	-0.30***	-0.22***	-0.24***	-0.20**	-0.29***	-0.22***	-0.24***
	(0.07)	(0.06)	(0.02)	(0.03)	(0.08)	(0.07)	(0.01)	(0.03)
d9297		0.21		-0.02		0.25		0.01
		(0.71)		(0.68)		(0.81)		(0.82)
d9804		1.44**		1.18**		1.39**		1.14*
		(0.59)		(0.49)		(0.68)		(0.64)
*d9297*gap*			0.23*	-0.01			0.23**	-0.01
			(0.12)	(0.11)			(0.11)	(0.11)
*d9804*gap*			0.09	-0.10			0.12	-0.08**
			(0.25)	(0.16)			(0.26)	(0.15)
mas					0.04	-0.03	-0.10	-0.04
					(0.45)	(0.45)	(0.38)	(0.43)
sgp					0.35	0.19	0.19	0.14
					(0.57)	(0.66)	(0.69)	(0.66)
Adjusted R^2	0.90	0.89	0.90	0.89	0.90	0.88	0.90	0.89
Hausman Statistic[a]	29.98***	35.79***	14.01***	15.98***	25.73***	27.67***	12.33***	13.26***
Cross-Section	6	6	6	6	6	6	6	6
Observations	127	127	127	127	127	127	127	127

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

4.E Estimation for the OECD-17

Table 4.6: Effects of the MT and the SGP on the CAPD for the OECD-17 (1980 - 2004) - heterogeneous control variable coefficients

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
α	1.14**	1.57**	1.04**	1.33*	1.13*	1.58*	1.02*	1.33*
	(0.54)	(0.79)	(0.46)	(0.70)	(0.62)	(0.85)	(0.59)	(0.77)
pdefay(-1)	0.78***	0.79***	0.79***	0.81***	0.77***	0.77***	0.78***	0.79***
	(0.04)	(0.04)	(0.04)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)
*pdefay(-1)*doecd6*	0.06	0.03	0.05	0.03	0.07	0.03	0.06	0.03
	(0.07)	(0.07)	(0.06)	(0.07)	(0.07)	(0.08)	(0.07)	(0.08)
ggflq(-1)	-0.03***	-0.02**	-0.02***	-0.02**	-0.02***	-0.02**	-0.02***	-0.02**
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
*ggflq(-1)*doecd6*	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
inf	-0.03	0.00	-0.01	0.02	-0.03	0.00	-0.01	0.01
	(0.05)	(0.05)	(0.04)	(0.05)	(0.05)	(0.06)	(0.05)	(0.05)
*inf*doecd6*	0.02	0.05	0.00	0.05	0.02	0.04	0.01	0.05
	(0.07)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.09)	(0.09)
ele	0.65***	0.62***	0.65***	0.63***	0.66***	0.63***	0.66***	0.64***
	(0.17)	(0.15)	(0.17)	(0.16)	(0.17)	(0.15)	(0.17)	(0.16)
*ele*doecd6*	-0.56*	-0.56**	-0.56*	-0.57**	-0.58**	-0.59**	-0.58**	-0.60**
	(0.30)	(0.27)	(0.30)	(0.27)	(0.29)	(0.26)	(0.29)	(0.27)
gap	-0.04	-0.02	-0.03	-0.01	-0.05	-0.02	-0.03	-0.01
	(0.04)	(0.05)	(0.07)	(0.07)	(0.04)	(0.05)	(0.07)	(0.07)
*gap*doecd6*	-0.13*	-0.17**	-0.15*	-0.18**	-0.12	-0.16*	-0.14*	-0.17**
	(0.08)	(0.08)	(0.07)	(0.07)	(0.08)	(0.08)	(0.07)	(0.08)
d9297		-0.82**		-0.89**		-0.63*		-0.72*
		(0.32)		(0.37)		(0.36)		(0.40)
d9804		-0.15		-0.11		-0.22		-0.19
		(0.34)		(0.35)		(0.33)		(0.35)
*d9297*gap*			-0.01	-0.05			-0.03	-0.07
			(0.10)	(0.09)			(0.10)	(0.10)
*d9804*gap*			-0.02	0.08			-0.01	0.09
			(0.14)	(0.11)			(0.15)	(0.11)
mas					-0.29*	-0.32*	-0.29	-0.34**
					(0.18)	(0.17)	(0.18)	(0.16)
sgp					0.01	0.15	0.03	0.12
					(0.48)	(0.51)	(0.50)	(0.53)
Adjusted R^2	0.85	0.84	0.85	0.84	0.85	0.84	0.85	0.84
Hausman Statistic[a]	37.21***	43.29***	24.54***	26.61***	35.72***	40.28***	23.75***	25.07***
Cross-Section	17	17	17	17	17	17	17	17
Observations	390	390	390	390	390	390	390	390

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Table 4.7: Effects of the MT and the SGP on the CAPD for the OECD-17 (1980 - 2004) - homogeneous control variable coefficients

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
α	1.16*	1.84**	1.12**	1.64*	1.38**	2.06**	1.28**	1.85**
	(0.63)	(0.88)	(0.55)	(0.84)	(0.58)	(0.85)	(0.57)	(0.83)
pdefay(-1)	0.81***	0.81***	0.83***	0.83***	0.79***	0.78***	0.80***	0.79***
	(0.05)	(0.05)	(0.05)	(0.06)	(0.05)	(0.05)	(0.05)	(0.05)
ggflq(-1)	-0.02***	-0.02***	-0.02***	-0.02**	-0.02***	-0.02***	-0.02***	-0.02***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
inf	-0.03	0.00	-0.02	0.01	-0.03	0.00	-0.02	0.00
	(0.05)	(0.05)	(0.04)	(0.04)	(0.05)	(0.05)	(0.04)	(0.04)
ele	0.44***	0.40***	0.44***	0.40***	0.44***	0.41***	0.44***	0.41***
	(0.15)	(0.14)	(0.15)	(0.14)	(0.15)	(0.14)	(0.15)	(0.15)
gap	-0.09***	-0.07**	-0.09**	-0.08*	-0.09***	-0.07*	-0.08*	-0.08
	(0.03)	(0.04)	(0.04)	(0.05)	(0.03)	(0.04)	(0.04)	(0.05)
d9297		-1.05***		-1.09***		-0.82**		-0.90**
		(0.34)		(0.40)		(0.40)		(0.45)
d9804		-0.31		-0.31		-0.44		-0.45
		(0.38)		(0.39)		(0.34)		(0.36)
*d9297*gap*			0.02	-0.01			0.00	-0.03
			(0.09)	(0.09)			(0.09)	(0.10)
*d9804*gap*			-0.03	0.11			0.00	0.13
			(0.12)	(0.10)			(0.10)	(0.09)
mas					-0.26	-0.31*	-0.25	-0.32**
					(0.17)	(0.16)	(0.18)	(0.15)
sgp					0.45	0.54	0.42	0.54
					(0.45)	(0.44)	(0.42)	(0.44)
Adjusted R^2	0.84	0.83	0.84	0.83	0.84	0.84	0.84	0.83
Hausman Statistic[a]	79.62***	94.70***	40.09***	46.10***	71.58***	82.77***	38.16***	41.86***
Cross-Section	17	17	17	17	17	17	17	17
Observations	390	390	390	390	390	390	390	390

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Table 4.8: Comparison - effects of the MT and the SGP on the CAPD for the OECD-17 (1980 - 2004) - homogeneous control variable coefficients

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
α	1.93**	1.19**	1.96***	1.66***	2.41***	1.41**	2.16***
	(0.80)	(0.49)	(0.65)	(0.59)	(0.85)	(0.54)	(0.76)
pdefay(-1)	0.79***	0.82***	0.80***	0.75***	0.74***	0.77***	0.77***
	(0.04)	(0.05)	(0.04)	(0.05)	(0.04)	(0.05)	(0.04)
ggflq(-1)	-0.02***	-0.02***	-0.02***	-0.03***	-0.03***	-0.02***	-0.02***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
inf	-0.01	-0.02	-0.01	-0.06	-0.04	-0.06	-0.04
	(0.05)	(0.04)	(0.04)	(0.05)	(0.06)	(0.05)	(0.05)
ele	0.40***	0.45***	0.42***	0.44***	0.42***	0.45***	0.44***
	(0.14)	(0.16)	(0.15)	(0.15)	(0.14)	(0.15)	(0.14)
gap	-0.08**	-0.08*	-0.09*	-0.08***	-0.07*	-0.09*	-0.09*
	(0.04)	(0.04)	(0.05)	(0.03)	(0.04)	(0.05)	(0.05)
d9297	-1.23***		-1.19***		-0.89**		-0.93**
	(0.33)		(0.34)		(0.38)		(0.39)
*d9297*doecd6*	0.48		0.28		0.01		-0.11
	(0.56)		(0.83)		(0.66)		(0.94)
d9804	-0.54		-0.62*		-0.66*		-0.74**
	(0.36)		(0.35)		(0.35)		(0.35)
*d9804*doecd6*	0.58		0.71		0.26		0.51
	(0.53)		(0.47)		(0.50)		(0.50)
*d9297*gap*		0.04	0.02			0.01	0.00
		(0.08)	(0.08)			(0.09)	(0.07)
*d9297*gap*doecd6*		-0.05	-0.10			0.03	-0.07
		(0.09)	(0.20)			(0.09)	(0.21)
*d9804*gap*		0.10	0.24**			0.16	0.25**
		(0.11)	(0.10)			(0.14)	(0.10)
*d9804*gap*doecd6*		-0.39*	-0.50***			-0.18	-0.40**
		(0.24)	(0.18)			(0.19)	(0.20)
mas				-0.43**	-0.50***	-0.44***	-0.52***
				(0.17)	(0.17)	(0.16)	(0.14)
*mas*doecd6*				0.64*	0.74**	0.59*	0.71**
				(0.37)	(0.37)	(0.32)	(0.36)
sgp				-0.80**	-0.63*	-0.89**	-0.69*
				(0.35)	(0.35)	(0.43)	(0.36)
*sgp*doecd6*				1.85***	1.69***	1.80***	1.28***
				(0.51)	(0.50)	(0.60)	(0.48)
Adjusted R^2	0.84	0.84	0.84	0.85	0.84	0.85	0.85
Hausman Statistic[a]	41.00***	27.43***	29.00***	72.78***	82.64***	37.13***	28.64***
Cross-Section	17	17	17	17	17	17	17
Observations	390	390	390	390	390	390	390

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

4.F Robustness tests

In this subsection, we describe the variables used in the robustness tests, their source and the way they are constructed. The economic variables are extracted from OECD Economic Outlook (2005). The political variables, available until the year 2003, are extracted from the dataset "Comparative Political Data Set 1960-2003" (Armingeon et al., 2005).

4.F.1 Additional economic variables

Real interest rate

Variable: *Ex-post real long-term interest rate, based on private consumption deflator, in %:* $IRLRC_{i,t}$. **Source:** OECD Economic Outlook (2005).

Construction: We use $IRL_{i,t}$ (the long-term interest rate) and $PCP_{i,t}$ (the private final consumption expenditure deflator) to compute in the same way as OECD (2004):[35]

$$IRLRC_{i,t} = IRL_{i,t} - 100 * (PCP_{i,t}/PCP_{i,t-1} - 1)\,.$$

[35]Although the internet database OECDSource does not contain $IRLRC_{i,t}$, it contains the variables necessary for its construction, $IRL_{i,t}$ and $PCP_{i,t}$. So, we used those two variables to construct $IRLRC_{i,t}$.

Table 4.9: Inclusion of IRLRC - Euro-11 (1980 - 2004)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
α	1.43**	1.83*	1.34*	1.97*	1.08	1.47	0.82	1.48
	(0.67)	(1.02)	(0.66)	(1.07)	(0.73)	(1.04)	(0.75)	(1.11)
pdefay(-1)	0.77***	0.77***	0.77***	0.77***	0.75***	0.75***	0.75***	0.76***
	(0.04)	(0.04)	(0.04)	(0.04)	(0.05)	(0.04)	(0.05)	(0.05)
ggflq(-1)	-0.03***	-0.03***	-0.02***	-0.02**	-0.02***	-0.02**	-0.02**	-0.02**
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
inf	-0.05	-0.03	-0.04	-0.03	-0.06	-0.05	-0.04	-0.05
	(0.04)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.06)
ele	0.62***	0.57***	0.62***	0.58***	0.63***	0.60***	0.63***	0.61***
	(0.15)	(0.14)	(0.15)	(0.15)	(0.15)	(0.13)	(0.15)	(0.15)
gap	-0.02	-0.01	-0.04	-0.04	0.00	0.00	0.00	-0.01
	(0.06)	(0.06)	(0.09)	(0.10)	(0.06)	(0.06)	(0.09)	(0.10)
irlrc	0.04	0.04	-0.04	-0.03	0.12	0.07	0.06	0.03
	(0.08)	(0.08)	(0.09)	(0.08)	(0.09)	(0.09)	(0.09)	(0.09)
d9297		-0.84*		-1.10**		-0.43		-0.67
		(0.46)		(0.47)		(0.49)		(0.50)
d9804		-0.30		-0.65		-0.25		-0.50
		(0.42)		(0.47)		(0.50)		(0.55)
*d9297*gap*			0.00	-0.03			-0.05	-0.06
			(0.11)	(0.11)			(0.11)	(0.10)
*d9804*gap*			0.12	0.19			0.18	0.19
			(0.18)	(0.13)			(0.19)	(0.13)
mas					-0.67***	-0.59***	-0.66***	-0.60***
					(0.18)	(0.16)	(0.16)	(0.14)
sgp					-0.94***	-0.92***	-0.95*	-0.83**
					(0.40)	(0.35)	(0.52)	(0.41)
Adjusted R^2	0.80	0.79	0.79	0.79	0.81	0.80	0.81	0.81
Hausman Statistic[a]	29.3***	31.5***	15.1***	16.2***	30.8***	32.8***	17.1***	18.0***
Cross-Section	11	11	11	11	11	11	11	11
Observations	262	262	262	262	262	262	262	262

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Table 4.10: Inclusion of IRLRC - EU-14 (1980 - 2004)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
α	2.76***	2.27***	2.88***	2.21***	2.49***	1.91***	2.51***
	(0.82)	(0.58)	(0.70)	(0.67)	(0.77)	(0.61)	(0.67)
pdefay(-1)	0.71***	0.73***	0.74***	0.73***	0.72***	0.73***	0.75***
	(0.05)	(0.04)	(0.05)	(0.05)	(0.04)	(0.04)	(0.04)
ggflq(-1)	-0.03***	-0.02**	-0.02**	-0.02***	-0.02**	-0.02**	-0.02**
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
*ggflq(-1)*deu3*	-0.09***	-0.09***	-0.11***	-0.08***	-0.10***	-0.10***	-0.11***
	(0.02)	(0.03)	(0.02)	(0.03)	(0.02)	(0.03)	(0.02)
inf	-0.02	-0.04	-0.01	-0.05	-0.04	-0.05	-0.04
	(0.05)	(0.04)	(0.05)	(0.05)	(0.06)	(0.05)	(0.05)
*inf*deu3*	-0.04	-0.09*	-0.18**	-0.08	-0.09	-0.09*	-0.20**
	(0.20)	(0.06)	(0.07)	(0.06)	(0.15)	(0.05)	(0.09)
ele	0.57***	0.56***	0.58***	0.59***	0.61***	0.59***	0.62***
	(0.11)	(0.13)	(0.12)	(0.11)	(0.11)	(0.12)	(0.13)
gap	0.01	0.01	0.03	0.02	0.01	0.02	0.03
	(0.07)	(0.10)	(0.09)	(0.06)	(0.06)	(0.09)	(0.08)
*gap*deu3*	-0.19**	-0.18	-0.24	-0.28*	-0.20	-0.14	-0.20
	(0.09)	(0.15)	(0.17)	(0.17)	(0.12)	(0.17)	(0.18)
irlrc	0.04	-0.03	0.00	0.06	0.07	0.08	0.07
	(0.08)	(0.07)	(0.06)	(0.10)	(0.08)	(0.07)	(0.07)
d9297	-0.67*		-0.83**		-0.28		-0.45
	(0.39)		(0.40)		(0.39)		(0.41)
*d9297*deu3*	1.43		0.89		1.49		0.88*
	(1.18)		(0.87)		(0.92)		(0.47)
d9804	-0.23		-0.29		-0.15		-0.19
	(0.32)		(0.32)		(0.36)		(0.36)
*d9804*deu3*	-0.28		-0.90*		-0.55		-1.06**
	(1.18)		(0.47)		(0.94)		(0.50)
*d9297*gap*		-0.02	-0.11			-0.03	-0.10
		(0.13)	(0.11)			(0.13)	(0.10)
*d9297*gap*deu3*		-0.34***	0.02			-0.59***	-0.12
		(0.08)	(0.30)			(0.10)	(0.25)
*d9804*gap*		0.13	0.13			0.21	0.17
		(0.16)	(0.12)			(0.19)	(0.12)
*d9804*gap*deu3*		0.12	0.39*			0.20	0.32
		(0.30)	(0.20)			(0.27)	(0.23)
mas				-0.71***	-0.61***	-0.80***	-0.64***
				(0.24)	(0.17)	(0.22)	(0.15)
*mas*deu3*				0.30	-0.36	-0.48	-0.53*
				(0.37)	(0.26)	(0.53)	(0.28)
sgp				-0.65	-0.85**	-1.02*	-0.91**
				(0.46)	(0.39)	(0.53)	(0.38)
*sgp*deu3*				4.14	4.04	0.38	1.52
				(3.80)	(2.87)	(2.19)	(1.91)
Adjusted R^2	0.79	0.78	0.78	0.79	0.80	0.80	0.80
Hausman Statistic[a]	28.69***	11.83***	13.12***	28.04***	30.00***	13.17***	13.65***
Cross-Section	14	14	14	14	14	14	14
Observations	336	336	336	336	336	336	336

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Table 4.11: Inclusion of IRLRC - OECD-17 (1980 - 2004)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
α	1.39**	1.03**	1.48**	1.26*	1.53*	1.03	1.37
	(0.68)	(0.46)	(0.70)	(0.68)	(0.87)	(0.64)	(0.87)
pdefay(-1)	0.79***	0.82***	0.80***	0.76***	0.76***	0.78***	0.77***
	(0.04)	(0.04)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)
ggflq(-1)	-0.02***	-0.02***	-0.02***	-0.02***	-0.02***	-0.02***	-0.02***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
inf	0.00	-0.01	0.01	-0.04	-0.02	-0.03	-0.01
	(0.05)	(0.03)	(0.04)	(0.05)	(0.06)	(0.05)	(0.05)
ele	0.60***	0.64***	0.60***	0.62***	0.61***	0.63***	0.62***
	(0.14)	(0.15)	(0.15)	(0.15)	(0.14)	(0.15)	(0.15)
*ele*doecd6*	-0.52**	-0.54*	-0.50*	-0.55**	-0.52**	-0.53*	-0.51*
	(0.23)	(0.29)	(0.27)	(0.26)	(0.23)	(0.27)	(0.26)
gap	0.01	-0.01	0.01	-0.03	0.00	0.00	0.01
	(0.05)	(0.06)	(0.07)	(0.04)	(0.05)	(0.06)	(0.07)
*gap*doecd6*	-0.25***	-0.16*	-0.24***	-0.12	-0.22**	-0.17*	-0.22**
	(0.07)	(0.09)	(0.09)	(0.08)	(0.08)	(0.10)	(0.10)
irlrc	0.04	0.00	0.01	0.06	0.06	0.05	0.05
	(0.07)	(0.08)	(0.08)	(0.09)	(0.08)	(0.09)	(0.09)
d9297	-0.84**		-1.02***		-0.52		-0.70*
	(0.34)		(0.37)		(0.36)		(0.39)
*d9297*doecd6*	0.29		0.49		-0.12		0.11
	(0.63)		(0.82)		(0.67)		(0.91)
d9804	-0.26		-0.38		-0.27		-0.35
	(0.35)		(0.39)		(0.39)		(0.43)
*d9804*doecd6*	0.85*		0.88*		0.60		0.73
	(0.49)		(0.45)		(0.50)		(0.52)
*d9297*gap*		-0.03	-0.09			-0.07	-0.10
		(0.10)	(0.09)			(0.10)	(0.08)
*d9297*gap*doecd6*		0.08	0.12			0.15*	0.13
		(0.10)	(0.21)			(0.09)	(0.22)
*d9804*gap*		0.02	0.13			0.07	0.14
		(0.13)	(0.11)			(0.16)	(0.11)
*d9804*gap*doecd6*		-0.22	-0.29			-0.09	-0.27
		(0.25)	(0.23)			(0.23)	(0.25)
mas				-0.43***	-0.51***	-0.46***	-0.52***
				(0.16)	(0.16)	(0.15)	(0.14)
*mas*doecd6*				0.51	0.70*	0.59*	0.66*
				(0.47)	(0.36)	(0.35)	(0.36)
sgp				-0.79**	-0.65*	-0.80*	-0.60*
				(0.36)	(0.35)	(0.42)	(0.35)
*sgp*doecd6*				1.67***	1.27**	1.53**	0.95*
				(0.57)	(0.55)	(0.60)	(0.53)
Adjusted R^2	0.84	0.84	0.84	0.85	0.85	0.85	0.85
Hausman Statistic[a]	50.36***	19.81***	22.27***	41.04***	45.23***	18.81***	20.45***
Cross-Section	17	17	17	17	17	17	17
Observations	389	389	389	389	389	389	389

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Share of non-working population

Variable: *Share of the non-working population:* $NWP_{i,t}$. **Source:** OECD Economic Outlook (2005).

Construction: $NWP_{i,t} = \left(1 - \frac{POPT_{i,t}}{POP_{i,t}}\right) * 100$, where $POPT_{i,t}$ is the working age population between 14 and 65 years and $POP_{i,t}$ is the total population. Variable also used in Woo (2003).

Table 4.12: Inclusion of NWP - Euro-11 (1980 - 2004)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
α	-2.54	-1.95	-2.48	-2.14	-2.14	-1.49	-2.58	-1.83
	(2.33)	(2.36)	(2.52)	(2.78)	(2.16)	(2.06)	(2.19)	(2.32)
pdefay(-1)	0.75***	0.75***	0.75***	0.76***	0.73***	0.74***	0.73***	0.74***
	(0.05)	(0.05)	(0.05)	(0.06)	(0.06)	(0.05)	(0.06)	(0.06)
ggflq(-1)	-0.03***	-0.03***	-0.03***	-0.03***	-0.03***	-0.03***	-0.02***	-0.02***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
inf	-0.10***	-0.09**	-0.08**	-0.07**	-0.12***	-0.11**	-0.11***	-0.11***
	(0.03)	(0.04)	(0.03)	(0.03)	(0.04)	(0.04)	(0.04)	(0.04)
ele	0.58***	0.54***	0.58***	0.55***	0.60***	0.57***	0.59***	0.58***
	(0.17)	(0.15)	(0.17)	(0.17)	(0.16)	(0.15)	(0.17)	(0.17)
gap	-0.03	-0.02	-0.02	-0.02	-0.03	-0.02	-0.02	-0.02
	(0.06)	(0.06)	(0.10)	(0.11)	(0.06)	(0.06)	(0.10)	(0.11)
nwp	0.15*	0.15*	0.13*	0.14*	0.13*	0.12*	0.13*	0.13*
	(0.08)	(0.08)	(0.08)	(0.08)	(0.07)	(0.07)	(0.07)	(0.07)
d9297		-0.93**		-1.06**		-0.66		-0.80
		(0.47)		(0.50)		(0.53)		(0.55)
d9804		-0.52		-0.57		-0.60		-0.72
		(0.48)		(0.52)		(0.51)		(0.53)
*d9297*gap*			-0.02	-0.06			-0.04	-0.06
			(0.11)	(0.12)			(0.12)	(0.12)
*d9804*gap*			0.11	0.17			0.22	0.21
			(0.17)	(0.14)			(0.20)	(0.15)
mas					-0.52***	-0.52***	-0.59***	-0.57***
					(0.20)	(0.17)	(0.19)	(0.15)
sgp					-0.95***	-1.00***	-1.14***	-1.02***
					(0.24)	(0.23)	(0.36)	(0.26)
Adjusted R^2	0.81	0.81	0.81	0.81	0.82	0.82	0.82	0.82
Hausman Statistic[a]	38.3***	41.0***	19.5***	20.9***	41.3***	43.9***	21.7***	22.8***
Cross-Section	11	11	11	11	11	11	11	11
Observations	251	251	251	251	251	251	251	251

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Table 4.13: Inclusion of NWP - EU-14 (1980 - 2004)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
α	6.87***	7.57***	6.79**	5.66	5.01	4.94*	4.74
	(2.97)	(2.53)	(3.28)	(3.74)	(3.46)	(2.57)	(3.43)
pdefay(-1)	0.73***	0.75***	0.74***	0.73***	0.73***	0.74***	0.74***
	(0.05)	(0.05)	(0.05)	(0.05)	(0.04)	(0.05)	(0.05)
ggflq(-1)	-0.03***	-0.03***	-0.03***	-0.02***	-0.02***	-0.02***	-0.02***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
*ggflq(-1)*deu3*	-0.09***	-0.09***	-0.10***	-0.08***	-0.09***	-0.09***	-0.10***
	(0.01)	(0.02)	(0.01)	(0.02)	(0.01)	(0.01)	(0.01)
inf	-0.07	-0.07***	-0.06**	-0.09**	-0.09*	-0.11***	-0.10***
	(0.04)	(0.03)	(0.03)	(0.04)	(0.05)	(0.04)	(0.03)
ele	0.58***	0.57***	0.59***	0.59***	0.61***	0.59***	0.63***
	(0.13)	(0.14)	(0.14)	(0.13)	(0.13)	(0.14)	(0.14)
gap	0.01	0.02	0.05	0.01	0.00	0.00	0.02
	(0.07)	(0.08)	(0.09)	(0.06)	(0.06)	(0.08)	(0.08)
*gap*deu3*	-0.24**	-0.18	-0.21*	-0.28*	-0.21*	-0.12	-0.15
	(0.10)	(0.12)	(0.11)	(0.15)	(0.12)	(0.13)	(0.11)
nwp	0.11	0.10	0.11	0.11*	0.09	0.10**	0.10
	(0.08)	(0.07)	(0.08)	(0.06)	(0.06)	(0.05)	(0.06)
*nwp*deu3*	-0.91***	-1.03***	-0.94***	-0.81**	-0.62*	-0.71***	-0.63*
	(0.23)	(0.18)	(0.23)	(0.35)	(0.35)	(0.24)	(0.32)
d9297	-0.64		-0.85*		-0.40		-0.63
	(0.42)		(0.45)		(0.47)		(0.49)
*d9297*deu3*	1.38***		1.31*		1.59**		1.33**
	(0.50)		(0.68)		(0.65)		(0.52)
d9804	-0.28		-0.34		-0.39		-0.55
	(0.41)		(0.42)		(0.44)		(0.42)
*d9804*deu3*	-0.25		-0.20		-0.22		-0.20**
	(0.49)		(0.46)		(0.44)		(0.40)
*d9297*gap*		-0.03	-0.13			-0.03	-0.12
		(0.11)	(0.10)			(0.11)	(0.10)
*d9297*gap*deu3*		-0.46***	-0.07			-0.63***	-0.20
		(0.11)	(0.24)			(0.15)	(0.23)
*d9804*gap*		0.10	0.12			0.22	0.17
		(0.15)	(0.12)			(0.19)	(0.13)
*d9804*gap*deu3*		0.08	0.33**			0.15	0.22
		(0.27)	(0.16)			(0.24)	(0.15)
mas				-0.62***	-0.54***	-0.71***	-0.61***
				(0.23)	(0.17)	(0.21)	(0.15)
*mas*deu3*				0.55	-0.18	-0.27	-0.29
				(0.38)	(0.33)	(0.60)	(0.40)
sgp				-0.79**	-0.96***	-1.21***	-1.07***
				(0.32)	(0.28)	(0.37)	(0.23)
*sgp*deu3*				1.95	2.16	-0.61	0.65
				(3.84)	(3.55)	(2.31)	(3.49)
Adjusted R^2	0.80	0.80	0.80	0.80	0.80	0.81	0.81
Hausman Statistic[a]	39.11***	14.36***	15.74***	40.93***	40.60***	15.78***	17.05***
Cross-Section	14	14	14	14	14	14	14
Observations	325	325	325	325	325	325	325

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Table 4.14: Inclusion of NWP - OECD-17 (1980 - 2004)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
α	4.42	4.25**	4.36**	6.77***	6.91***	6.29**	6.84***
	(2.78)	(2.12)	(2.16)	(2.09)	(2.16)	(2.05)	(1.71)
pdefay(-1)	0.80***	0.82***	0.80***	0.75***	0.75***	0.76***	0.76***
	(0.05)	(0.06)	(0.05)	(0.05)	(0.05)	(0.06)	(0.05)
ggflq(-1)	-0.02**	-0.02**	-0.02***	-0.02***	-0.02**	-0.02***	-0.02***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
inf	-0.02	-0.02	-0.01	-0.09*	-0.06	-0.08**	-0.06
	(0.04)	(0.03)	(0.03)	(0.05)	(0.05)	(0.04)	(0.04)
ele	0.59***	0.63***	0.60***	0.62***	0.60***	0.63***	0.62***
	(0.16)	(0.18)	(0.17)	(0.17)	(0.16)	(0.18)	(0.17)
*ele*doecd6*	-0.53**	-0.56*	-0.52*	-0.57**	-0.55**	-0.55*	-0.56**
	(0.25)	(0.31)	(0.29)	(0.28)	(0.25)	(0.29)	(0.28)
gap	0.00	0.01	0.02	-0.04	-0.01	0.00	0.01
	(0.05)	(0.06)	(0.07)	(0.04)	(0.05)	(0.06)	(0.07)
*gap*doecd6*	-0.25***	-0.17**	-0.25***	-0.13	-0.21**	-0.18**	-0.21**
	(0.07)	(0.08)	(0.09)	(0.09)	(0.08)	(0.08)	(0.10)
nwp	0.04	0.05	0.05	0.07	0.06	0.08	0.07
	(0.08)	(0.08)	(0.07)	(0.07)	(0.08)	(0.07)	(0.07)
*nwp*doecd6*	-0.38**	-0.46***	-0.43***	-0.68***	-0.65***	-0.70***	-0.69***
	(0.18)	(0.09)	(0.15)	(0.13)	(0.12)	(0.13)	(0.11)
d9297	-0.89**		-1.04***		-0.61		-0.79*
	(0.36)		(0.39)		(0.40)		(0.44)
*d9297*doecd6*	0.29		0.42		-0.22		-0.12
	(0.57)		(0.76)		(0.58)		(0.83)
d9804	-0.38		-0.41		-0.51		-0.57
	(0.37)		(0.40)		(0.39)		(0.40)
*d9804*doecd6*	0.80		0.82*		0.33		0.46
	(0.49)		(0.46)		(0.48)		(0.53)
*d9297*gap*		-0.05	-0.10			-0.09	-0.11
		(0.09)	(0.08)			(0.09)	(0.08)
*d9297*gap*doecd6*		0.04	0.07			0.11*	0.05
		(0.07)	(0.18)			(0.06)	(0.19)
*d9804*gap*		-0.01	0.12			0.09	0.15
		(0.11)	(0.11)			(0.15)	(0.12)
*d9804*gap*doecd6*		-0.25	-0.31			-0.07	-0.22
		(0.22)	(0.20)			(0.15)	(0.21)
mas				-0.41**	-0.50***	-0.46***	-0.54***
				(0.16)	(0.17)	(0.16)	(0.15)
*mas*doecd6*				0.69*	0.88**	0.78*	0.89**
				(0.37)	(0.34)	(0.33)	(0.34)
sgp				-0.96***	-0.84***	-1.01***	-0.84***
				(0.32)	(0.31)	(0.37)	(0.28)
*sgp*doecd6*				2.16***	1.84***	2.15***	1.67***
				(0.53)	(0.52)	(0.53)	(0.48)
Adjusted R^2	0.85	0.85	0.85	0.86	0.85	0.86	0.86
Hausman Statistic[a]	62.27***	22.17***	24.53***	49.80***	54.21***	21.35***	22.42***
Cross-Section	17	17	17	17	17	17	17
Observations	378	378	378	378	378	378	378

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Trade openness

Variable: *Trade openness: $OPEN_{i,t}$*. **Source:** OECD Economic Outlook (2005).

Construction: Constructed with OECD variables via the formula $OPEN_{i,t} = \frac{XGSV_{i,t}+MGSV_{i,t}}{GDPVTR_{i,t}}$, where $XGSV_{i,t}$ is the volume of exports of goods and services, $MGSV_{i,t}$ is the volume of imports of goods and services, and $GDPVTR_{i,t}$ is the volume of potential output of total economy. This variable was also used in Lane (2003).

Table 4.15: Inclusion of trade openness - Euro-11 (1980 - 2004)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
α	4.53***	4.12***	4.48***	4.52***	4.30***	3.56**	4.43***	4.06***
	(1.42)	(1.50)	(1.47)	(1.40)	(1.49)	(1.52)	(1.51)	(1.44)
pdefay(-1)	0.72***	0.73***	0.72***	0.72***	0.71***	0.72***	0.70***	0.72***
	(0.04)	(0.04)	(0.04)	(0.04)	(0.05)	(0.05)	(0.06)	(0.05)
ggflq(-1)	-0.04***	-0.03***	-0.03***	-0.03***	-0.03***	-0.03***	-0.03***	-0.03***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
inf	-0.05	-0.03	-0.02	-0.02	-0.06	-0.06	-0.04	-0.05
	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.05)	(0.05)	(0.05)
ele	0.60***	0.57***	0.59***	0.56***	0.62***	0.60***	0.60***	0.59***
	(0.17)	(0.15)	(0.18)	(0.16)	(0.16)	(0.14)	(0.16)	(0.16)
gap	-0.05	-0.02	-0.03	-0.03	-0.04	-0.02	-0.02	-0.03
	(0.07)	(0.07)	(0.10)	(0.11)	(0.08)	(0.07)	(0.11)	(0.11)
open	-3.06**	-2.29**	-3.50***	-3.11***	-2.92**	-1.85*	-3.60***	-2.68***
	(1.26)	(1.05)	(1.33)	(0.95)	(1.30)	(0.97)	(1.37)	(0.87)
d9297		-0.82*		-0.97**		-0.54		-0.71
		(0.46)		(0.46)		(0.50)		(0.51)
d9804		-0.06		0.01		-0.22		-0.18
		(0.46)		(0.49)		(0.44)		(0.44)
*d9297*gap*			-0.06	-0.08			-0.09	-0.08
			(0.10)	(0.11)			(0.12)	(0.11)
*d9804*gap*			0.14	0.24*			0.23	0.26*
			(0.16)	(0.13)			(0.20)	(0.15)
mas					-0.56**	-0.53***	-0.63***	-0.55***
					(0.22)	(0.17)	(0.22)	(0.15)
sgp					-0.71**	-0.88***	-0.88*	-0.87**
					(0.34)	(0.30)	(0.50)	(0.38)
Adjusted R^2	0.80	0.80	0.81	0.80	0.81	0.81	0.82	0.81
Hausman Statistic[a]	36.5***	40.6***	17.5***	19.0***	38.4***	42.7***	19.3***	20.2***
Cross-Section	11	11	11	11	11	11	11	11
Observations	263	263	263	263	263	263	263	263

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Table 4.16: Inclusion of trade openness - EU-14 (1980 - 2004)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
α	5.25***	4.81***	5.56***	4.55***	4.47***	4.37***	4.71***
	(1.06)	(0.99)	(0.88)	(1.13)	(1.07)	(1.09)	(0.97)
pdefay(-1)	0.68***	0.70***	0.70***	0.71***	0.70***	0.70***	0.73***
	(0.05)	(0.04)	(0.05)	(0.05)	(0.04)	(0.05)	(0.04)
ggflq(-1)	-0.03***	-0.03***	-0.03***	-0.03***	-0.03***	-0.02***	-0.02***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
*ggflq(-1)*deu3*	-0.08***	-0.07**	-0.10***	-0.07**	-0.09***	-0.08***	-0.11***
	(0.02)	(0.03)	(0.02)	(0.03)	(0.02)	(0.03)	(0.02)
inf	-0.03	-0.02	0.00	-0.05	-0.05	-0.05	-0.04
	(0.04)	(0.04)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)
*inf*deu3*	-0.03	-0.06	-0.17***	-0.05	-0.07	-0.07	-0.20***
	(0.21)	(0.05)	(0.06)	(0.07)	(0.17)	(0.05)	(0.07)
ele	0.55***	0.54***	0.55***	0.58***	0.60***	0.57***	0.60***
	(0.12)	(0.15)	(0.14)	(0.13)	(0.12)	(0.13)	(0.13)
gap	0.01	0.03	0.05	0.01	0.00	0.03	0.03
	(0.09)	(0.11)	(0.10)	(0.08)	(0.07)	(0.09)	(0.09)
*gap*deu3*	-0.22**	-0.23	-0.30	-0.31*	-0.23*	-0.20	-0.25
	(0.10)	(0.17)	(0.19)	(0.17)	(0.12)	(0.17)	(0.19)
open	-2.97***	-3.18***	-3.52***	-2.55**	-2.13**	-2.68**	-2.43***
	(1.01)	(0.98)	(0.81)	(1.15)	(0.86)	(1.11)	(0.70)
d9297	-0.51		-0.68		-0.27		-0.46
	(0.46)		(0.44)		(0.47)		(0.45)
*d9297*deu3*	1.34		1.08		1.42		0.91*
	(1.14)		(0.78)		(0.98)		(0.48)
d9804	0.23		0.38		0.02		0.05
	(0.48)		(0.45)		(0.44)		(0.36)
*d9804*deu3*	-0.43		-1.05**		-0.57		-1.17***
	(1.22)		(0.42)		(1.01)		(0.40)
*d9297*gap*		-0.08	-0.17			-0.08	-0.14
		(0.14)	(0.11)			(0.13)	(0.10)
*d9297*gap*deu3*		-0.33***	0.15			-0.52***	-0.01
		(0.12)	(0.29)			(0.09)	(0.24)
*d9804*gap*		0.13	0.18			0.23	0.21
		(0.16)	(0.12)			(0.19)	(0.13)
*d9804*gap*deu3*		0.05	0.35			0.15	0.32
		(0.33)	(0.22)			(0.31)	(0.24)
mas				-0.65***	-0.54***	-0.75***	-0.57***
				(0.24)	(0.18)	(0.23)	(0.15)
*mas*deu3*				0.32	-0.30	-0.34	-0.40
				(0.40)	(0.28)	(0.59)	(0.30)
sgp				-0.63	-0.85**	-0.97*	-0.93***
				(0.41)	(0.34)	(0.49)	(0.35)
*sgp*deu3*				4.02	4.24	0.37	1.46
				(3.26)	(2.59)	(2.23)	(1.92)
Adjusted R^2	0.79	0.79	0.79	0.79	0.80	0.80	0.80
Hausman Statistic[a]	31.69***	12.23***	13.22***	30.49***	33.25***	13.34***	13.47***
Cross-Section	14	14	14	14	14	14	14
Observations	337	337	337	337	337	337	337

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Table 4.17: Inclusion of trade openness - OECD-17 (1980 - 2004)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
α	3.60***	4.02***	4.12***	4.44***	4.37***	4.70***	4.63***
	(1.04)	(0.94)	(0.85)	(1.14)	(1.22)	(1.07)	(1.06)
pdefay(-1)	0.75***	0.76***	0.74***	0.71***	0.70***	0.70***	0.70***
	(0.04)	(0.05)	(0.04)	(0.05)	(0.04)	(0.05)	(0.04)
ggflq(-1)	-0.03***	-0.03***	-0.03***	-0.03***	-0.03***	-0.03***	-0.03***
	(0.01)	(0.01)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)
inf	0.00	0.01	0.02	-0.05	-0.03	-0.02	-0.01
	(0.04)	(0.03)	(0.03)	(0.05)	(0.05)	(0.04)	(0.04)
ele	0.58***	0.60***	0.57***	0.60***	0.58***	0.59***	0.58***
	(0.15)	(0.17)	(0.17)	(0.16)	(0.15)	(0.17)	(0.16)
*ele*doecd6*	-0.50**	-0.50	-0.46	-0.51*	-0.50**	-0.46	-0.47*
	(0.24)	(0.30)	(0.29)	(0.28)	(0.25)	(0.29)	(0.28)
gap	0.00	0.01	0.02	-0.04	-0.01	0.01	0.01
	(0.06)	(0.06)	(0.07)	(0.06)	(0.07)	(0.07)	(0.07)
*gap*doecd6*	-0.26***	-0.17**	-0.24***	-0.16*	-0.21**	-0.18**	-0.21**
	(0.07)	(0.08)	(0.09)	(0.09)	(0.08)	(0.08)	(0.09)
open	-2.87**	-3.87***	-3.73***	-3.70***	-3.29***	-4.47***	-4.03***
	(1.18)	(1.17)	(1.00)	(1.33)	(1.25)	(1.31)	(1.10)
d9297	-0.75**		-0.97***		-0.51		-0.74*
	(0.35)		(0.37)		(0.39)		(0.43)
*d9297*doecd6*	0.38		0.69		-0.04		0.27
	(0.55)		(0.73)		(0.60)		(0.81)
d9804	0.11		0.20		0.06		0.12
	(0.40)		(0.40)		(0.39)		(0.39)
*d9804*doecd6*	0.70		0.69		0.32		0.42
	(0.48)		(0.44)		(0.44)		(0.45)
*d9297*gap*		-0.08	-0.14*			-0.14	-0.16*
		(0.09)	(0.08)			(0.10)	(0.09)
*d9297*gap*doecd6*		0.00	0.14			0.12	0.15
		(0.09)	(0.19)			(0.08)	(0.20)
*d9804*gap*		0.09	0.20**			0.16	0.23*
		(0.11)	(0.10)			(0.15)	(0.12)
*d9804*gap*doecd6*		-0.42**	-0.47**			-0.29	-0.39*
		(0.20)	(0.19)			(0.19)	(0.22)
mas				-0.42**	-0.44**	-0.50***	-0.47***
				(0.18)	(0.19)	(0.18)	(0.16)
*mas*doecd6*				0.75*	0.81**	0.94***	0.82**
				(0.39)	(0.35)	(0.32)	(0.35)
sgp				-0.82**	-0.72**	-0.85*	-0.69*
				(0.39)	(0.35)	(0.46)	(0.38)
*sgp*doecd6*				1.78***	1.61***	1.67***	1.35***
				(0.53)	(0.50)	(0.59)	(0.52)
Adjusted R^2	0.85	0.85	0.85	0.86	0.85	0.86	0.86
Hausman Statistic[a]	60.85***	20.82***	23.54***	45.74***	52.69***	19.91***	21.17***
Cross-Section	17	17	17	17	17	17	17
Observations	390	390	390	390	390	390	390

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Economic country size

Variable: *Size of the country:* $SIZE_{i,t}$. **Source:** OECD Economic Outlook (2005).

Construction: Constructed as the ratio of the real GDP of country i divided by the sum of the real GDPs of all countries in the specific (sub)sample under consideration:

$$SIZE_{i,t} = \frac{GDP_{i,t}}{\sum_{i}^{N} GDP_{i,t}},$$

where $\{1,..,N\}$ is the set of countries in this (sub)sample. Hence, for the Euro-11 the denominator of this expression is the sum of the real GDPs of the Euro-11 countries. Similarly, for the EU-14 and the OECD-17.

Table 4.18: Inclusion of country size - Euro-11 (1980 - 2004)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
α	3.29***	3.69***	3.02***	3.68***	3.29***	3.68***	3.16***	3.84***
	(0.81)	(1.22)	(0.81)	(1.21)	(0.90)	(1.32)	(0.88)	(1.26)
pdefay(-1)	0.74***	0.75***	0.75***	0.76***	0.72***	0.73***	0.72***	0.73***
	(0.05)	(0.04)	(0.04)	(0.04)	(0.06)	(0.05)	(0.06)	(0.06)
ggflq(-1)	-0.03***	-0.03***	-0.03***	-0.03***	-0.02***	-0.02***	-0.02***	-0.02**
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
inf	-0.10**	-0.07*	-0.09*	-0.08	-0.11**	-0.10*	-0.12**	-0.11**
	(0.04)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.05)
ele	0.67***	0.63***	0.66***	0.63***	0.68***	0.65***	0.67***	0.66***
	(0.16)	(0.14)	(0.17)	(0.16)	(0.15)	(0.14)	(0.15)	(0.15)
gap	-0.02	0.00	-0.06	-0.05	-0.01	0.00	-0.05	-0.05
	(0.07)	(0.07)	(0.11)	(0.11)	(0.07)	(0.07)	(0.11)	(0.11)
size	-14.24***	-12.59***	-14.03***	-12.86***	-15.40***	-14.47***	-17.06***	-15.71***
	(3.26)	(3.29)	(2.77)	(2.94)	(4.16)	(4.59)	(3.85)	(3.71)
*size*d9804*	3.36***	3.36**	3.70*	3.72**	2.87**	2.80**	3.21**	3.26**
	(1.73)	(1.69)	(1.91)	(1.81)	(1.28)	(1.32)	(1.33)	(1.36)
d9297		-0.94**		-1.10***		-0.62		-0.81*
		(0.45)		(0.42)		(0.44)		(0.44)
d9804		-0.90**		-1.06***		-0.96*		-1.19**
		(0.40)		(0.40)		(0.52)		(0.52)
*d9297*gap*			0.04	0.00			0.01	-0.02
			(0.11)	(0.12)			(0.11)	(0.11)
*d9804*gap*			0.21	0.25*			0.31	0.27*
			(0.18)	(0.14)			(0.20)	(0.15)
mas					-0.57***	-0.58***	-0.65***	-0.63***
					(0.22)	(0.18)	(0.20)	(0.15)
sgp					-0.79***	-0.88***	-1.07***	-0.91***
					(0.23)	(0.24)	(0.34)	(0.24)
Adjusted R^2	0.81	0.81	0.81	0.81	0.82	0.82	0.83	0.82
Hausman Statistic[a]	41.4***	43.7***	20.8***	21.2***	43.7***	45.9***	23.7***	24.2***
Cross-Section	11	11	11	11	11	11	11	11
Observations	263	263	263	263	263	263	263	263

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Table 4.19: Inclusion of country size - EU-14 (1980 - 2004)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
α	3.57***	3.17***	3.54***	4.00***	3.84***	3.68***	3.94***
	(1.12)	(0.79)	(0.95)	(0.91)	(1.13)	(0.75)	(0.85)
pdefay(-1)	0.70***	0.73***	0.74***	0.72***	0.71***	0.73***	0.75***
	(0.05)	(0.04)	(0.04)	(0.05)	(0.05)	(0.04)	(0.04)
ggflq(-1)	-0.03***	-0.03***	-0.03***	-0.03***	-0.03***	-0.02***	-0.02***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
*ggflq(-1)*deu3*	-0.09***	-0.08**	-0.11***	-0.08***	-0.09***	-0.09***	-0.11***
	(0.02)	(0.03)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
inf	-0.05	-0.06	-0.04	-0.07	-0.07	-0.09*	-0.07
	(0.05)	(0.05)	(0.04)	(0.05)	(0.06)	(0.05)	(0.05)
*inf*deu3*	0.00	-0.07	-0.16**	-0.06	-0.03	-0.08	-0.18**
	(0.21)	(0.06)	(0.07)	(0.07)	(0.18)	(0.06)	(0.08)
ele	0.61***	0.59***	0.61***	0.63***	0.65***	0.62***	0.65***
	(0.12)	(0.14)	(0.13)	(0.12)	(0.12)	(0.13)	(0.14)
gap	0.02	0.00	0.02	0.02	0.02	-0.01	0.00
	(0.08)	(0.10)	(0.10)	(0.07)	(0.07)	(0.09)	(0.09)
*gap*deu3*	-0.20**	-0.18	-0.24	-0.30**	-0.22**	-0.17	-0.21
	(0.09)	(0.14)	(0.16)	(0.15)	(0.10)	(0.15)	(0.17)
size	-4.63	-8.58	-3.06	-17.14***	-10.36*	-15.28***	-9.29*
	(5.91)	(6.77)	(6.30)	(5.71)	(5.87)	(5.68)	(5.37)
*size*d9804*	6.45***	6.39***	6.12***	5.83***	5.65***	5.85***	5.53***
	(1.55)	(1.77)	(1.46)	(1.52)	(1.37)	(1.44)	(1.10)
d9297	-0.65		-0.79*		-0.26		-0.44
	(0.42)		(0.41)		(0.41)		(0.40)
*d9297*deu3*	1.46		0.84		1.44		0.68
	(1.22)		(0.91)		(1.06)		(0.52)
d9804	-0.85**		-0.80**		-0.86*		-0.91**
	(0.38)		(0.36)		(0.44)		(0.41)
*d9804*deu3*	-0.09		-0.86**		-0.21		-0.93***
	(1.34)		(0.35)		(1.11)		(0.32)
*d9297*gap*		0.02	-0.08			0.02	-0.07
		(0.13)	(0.11)			(0.11)	(0.10)
*d9297*gap*deu3*		-0.32***	0.02			-0.51***	-0.12
		(0.07)	(0.27)			(0.11)	(0.23)
*d9804*gap*		0.18	0.17			0.28	0.22*
		(0.17)	(0.13)			(0.19)	(0.13)
*d9804*gap*deu3*		0.13	0.37*			0.22	0.32
		(0.30)	(0.19)			(0.27)	(0.21)
mas				-0.70***	-0.61***	-0.77***	-0.65***
				(0.25)	(0.19)	(0.22)	(0.16)
*mas*deu3*				0.15	-0.32	-0.48	-0.47
				(0.33)	(0.28)	(0.52)	(0.30)
sgp				-0.45	-0.63*	-0.85**	-0.77**
				(0.35)	(0.33)	(0.43)	(0.30)
Adjusted R^2	0.79	0.80	0.79	0.80	0.80	0.81	0.80
Hausman Statistic[a]	43.22***	15.89***	11.29***	41.72***	43.88***	17.19***	17.08***
Cross-Section	14	14	14	14	14	14	14
Observations	337	337	337	337	337	337	337

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Table 4.20: Inclusion of country size - OECD-17 (1980 - 2004)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
α	1.58*	0.73	1.42*	1.22	1.91*	0.96	1.62*
	(0.83)	(0.68)	(0.77)	(0.84)	(0.97)	(0.77)	(0.90)
pdefay(-1)	0.81***	0.83***	0.81***	0.78***	0.78***	0.79***	0.79***
	(0.04)	(0.05)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)
ggflq(-1)	-0.02***	-0.02***	-0.02***	-0.02***	-0.02***	-0.02***	-0.02**
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
inf	0.00	0.00	0.01	-0.05	-0.02	-0.03	-0.02
	(0.04)	(0.04)	(0.04)	(0.05)	(0.06)	(0.05)	(0.05)
ele	0.65***	0.67***	0.65***	0.67***	0.67***	0.67***	0.67***
	(0.15)	(0.17)	(0.16)	(0.16)	(0.15)	(0.16)	(0.16)
*ele*doecd6*	-0.59**	-0.60**	-0.57**	-0.62**	-0.60***	-0.59**	-0.59**
	(0.23)	(0.28)	(0.26)	(0.25)	(0.23)	(0.25)	(0.25)
gap	0.01	-0.01	0.01	-0.03	0.00	-0.01	0.00
	(0.05)	(0.06)	(0.07)	(0.04)	(0.05)	(0.07)	(0.07)
*gap*doecd6*	-0.26***	-0.16*	-0.23***	-0.12	-0.22***	-0.17*	-0.21**
	(0.06)	(0.08)	(0.09)	(0.08)	(0.07)	(0.09)	(0.09)
size	-25.81*	-16.86	-21.26	-20.10	-30.49**	-20.32	-24.89
	(13.30)	(14.11)	(14.87)	(15.84)	(14.42)	(16.38)	(15.59)
*size*doecd6*	37.37**	28.49*	33.01**	32.99*	42.93***	33.23*	37.45**
	(14.57)	(0.08)	(15.83)	(17.20)	(15.78)	(18.13)	(16.71)
d9297	-0.86***		-1.01***		-0.55*		-0.72**
	(0.30)		(0.32)		(0.32)		(0.35)
*d9297*doecd6*	0.23		0.35		-0.21		-0.08
	(0.48)		(0.62)		(0.51)		(0.67)
d9804	-0.44		-0.45		-0.56*		-0.60*
	(0.29)		(0.32)		(0.31)		(0.34)
*d9804*doecd6*	0.84**		0.85**		0.62		0.71
	(0.41)		(0.40)		(0.41)		(0.44)
*d9297*gap*		-0.03	-0.07			-0.06	-0.08
		(0.08)	(0.08)			(0.08)	(0.08)
*d9297*gap*doecd6*		0.05	0.05			0.13*	0.05
		(0.10)	(0.15)			(0.09)	(0.16)
*d9804*gap*		0.02	0.12			0.09	0.14
		(0.13)	(0.11)			(0.16)	(0.13)
*d9804*gap*doecd6*		-0.19	-0.25			-0.02	-0.22
		(0.25)	(0.22)			(0.22)	(0.25)
mas				-0.42**	-0.53***	-0.47***	-0.55***
				(0.18)	(0.18)	(0.17)	(0.16)
*mas*doecd6*				0.47	0.69*	0.56*	0.67*
				(0.40)	(0.37)	(0.32)	(0.37)
sgp				-0.81**	-0.61*	-0.86**	-0.62*
				(0.33)	(0.34)	(0.41)	(0.35)
*sgp*doecd6*				1.68***	1.19**	1.68***	1.02**
				(0.53)	(0.50)	(0.61)	(0.51)
Adjusted R^2	0.85	0.85	0.85	0.85	0.85	0.85	0.85
Hausman Statistic[a]	63.96***	22.44***	25.30***	51.71***	57.63***	21.59***	23.42***
Cross-Section	17	17	17	17	17	17	17
Observations	390	390	390	390	390	390	390

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Volatility of GDP

Variable: *GDP Volatility:* $VOL_{i,t}$. **Source:** OECD Economic Outlook (2005).

Construction: This variable is the standard deviation of real economic growth over the preceding 10 years.

Table 4.21: Inclusion of GDP volatility - Euro-11 (1980 - 2004)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
α	1.22*	1.77	0.85	1.58	1.09	1.60	0.66	1.48
	(0.71)	(1.10)	(0.79)	(1.20)	(0.73)	(1.10)	(0.81)	(1.18)
pdefay(-1)	0.75***	0.76***	0.76***	0.78***	0.73***	0.74***	0.74***	0.76***
	(0.05)	(0.04)	(0.05)	(0.05)	(0.06)	(0.05)	(0.06)	(0.06)
ggflq(-1)	-0.03***	-0.02***	-0.02***	-0.02**	-0.02**	-0.02**	-0.02**	-0.02**
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
inf	-0.05	-0.03	-0.02	-0.02	-0.06	-0.05	-0.05	-0.05
	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.06)
ele	0.65***	0.61***	0.64***	0.61***	0.66***	0.63***	0.65***	0.64***
	(0.17)	(0.15)	(0.17)	(0.16)	(0.15)	(0.14)	(0.16)	(0.15)
gap	-0.01	0.00	0.00	-0.01	-0.01	0.00	0.01	-0.01
	(0.06)	(0.07)	(0.12)	(0.13)	(0.07)	(0.07)	(0.13)	(0.14)
vol	9.21	8.98	9.38	6.12	10.44**	9.12*	10.50*	5.87
	(6.56)	(5.44)	(6.75)	(6.56)	(5.10)	(5.47)	(6.00)	(7.25)
d9297		-0.91*		-1.02**		-0.59		-0.72
		(0.48)		(0.46)		(0.52)		(0.51)
d9804		-0.44		-0.46		-0.55		-0.61
		(0.46)		(0.50)		(0.49)		(0.51)
*d9297*gap*			-0.03	-0.05			-0.06	-0.06
			(0.13)	(0.14)			(0.14)	(0.14)
*d9804*gap*			0.09	0.16			0.18	0.19
			(0.20)	(0.17)			(0.22)	(0.19)
mas					-0.59***	-0.58***	-0.64***	-0.61***
					(0.20)	(0.17)	(0.19)	(0.15)
sgp					-0.78**	-0.82**	-0.93*	-0.84**
					(0.34)	(0.34)	(0.49)	(0.42)
Adjusted R^2	0.80	0.80	0.80	0.80	0.81	0.81	0.81	0.81
Hausman Statistic[a]	40.3***	43.4***	19.1***	20.1***	42.3***	44.8***	20.8***	21.5***
Cross-Section	11	11	11	11	11	11	11	11
Observations	263	263	263	263	263	263	263	263

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Table 4.22: Inclusion of GDP volatility - EU-14 (1980 - 2004)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
α	2.60**	1.96***	2.69***	2.12***	2.46**	1.94***	2.69***
	(1.02)	(0.57)	(0.79)	(0.62)	(0.99)	(0.55)	(0.74)
pdefay(-1)	0.71***	0.73***	0.74***	0.73***	0.72***	0.73***	0.76***
	(0.05)	(0.04)	(0.05)	(0.05)	(0.05)	(0.04)	(0.05)
ggflq(-1)	-0.02***	-0.02***	-0.02**	-0.02**	-0.02**	-0.01**	-0.01**
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
*ggflq(-1)*deu3*	-0.09***	-0.09**	-0.11***	-0.09***	-0.10***	-0.10***	-0.11***
	(0.03)	(0.04)	(0.03)	(0.03)	(0.02)	(0.03)	(0.02)
inf	-0.02	-0.02	-0.01	-0.05	-0.04	-0.06	-0.05
	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.05)	(0.05)
*inf*deu3*	-0.01	-0.09*	-0.17**	-0.07	-0.04	-0.10**	-0.20**
	(0.22)	(0.05)	(0.07)	(0.07)	(0.18)	(0.05)	(0.08)
ele	0.60***	0.59***	0.61***	0.62***	0.64***	0.61***	0.65***
	(0.12)	(0.14)	(0.13)	(0.12)	(0.12)	(0.13)	(0.13)
gap	0.02	0.04	0.05	0.03	0.02	0.03	0.02
	(0.08)	(0.11)	(0.11)	(0.07)	(0.07)	(0.10)	(0.11)
*gap*deu3*	-0.20**	-0.21	-0.26	-0.28*	-0.21**	-0.18	-0.22
	(0.08)	(0.14)	(0.16)	(0.16)	(0.10)	(0.16)	(0.18)
vol	8.91	7.65	6.01	8.08*	8.93	7.48	4.40
	(5.53)	(6.45)	(6.47)	(4.62)	(5.52)	(5.33)	(6.94)
d9297	-0.70		-0.82*		-0.36		-0.52
	(0.44)		(0.42)		(0.46)		(0.43)
*d9297*deu3*	1.60		1.01		1.70		0.90*
	(1.25)		(0.88)		(1.06)		(0.54)
d9804	-0.33		-0.27		-0.40		-0.41
	(0.41)		(0.40)		(0.42)		(0.41)
*d9804*deu3*	-0.01		-0.78		-0.22		-0.98*
	(1.18)		(0.52)		(1.00)		(0.52)
*d9297*gap*		-0.03	-0.12			-0.04	-0.10
		(0.14)	(0.12)			(0.13)	(0.11)
*d9297*gap*deu3*		-0.33***	0.05			-0.54***	-0.10
		(0.08)	(0.29)			(0.09)	(0.24)
*d9804*gap*		0.10	0.11			0.20	0.16
		(0.18)	(0.15)			(0.20)	(0.16)
*d9804*gap*deu3*		0.19	0.41**			0.28	0.35
		(0.28)	(0.19)			(0.26)	(0.22)
mas				-0.69***	-0.59***	-0.76***	-0.63***
				(0.24)	(0.17)	(0.22)	(0.15)
*mas*deu3*				0.31	-0.37	-0.44	-0.53*
				(0.38)	(0.27)	(0.55)	(0.30)
sgp				-0.64	-0.76*	-0.99**	-0.89**
				(0.40)	(0.38)	(0.49)	(0.39)
Adjusted R^2	0.79	0.79	0.79	0.82	0.80	0.83	0.82
Hausman Statistic[a]	34.24***	13.60***	14.78***	33.71***	35.32***	14.46***	14.93***
Cross-Section	14	14	14	14	14	14	14
Observations	337	337	337	337	337	337	337

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Table 4.23: Inclusion of GDP volatility - OECD-17 (1980 - 2004)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
α	1.42**	0.78*	1.36**	1.24*	1.67*	0.96*	1.48*
	(0.69)	(0.43)	(0.68)	(0.64)	(0.87)	(0.54)	(0.81)
pdefay(-1)	0.78***	0.81***	0.79***	0.75***	0.75***	0.77***	0.77***
	(0.04)	(0.05)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)
ggflq(-1)	-0.02***	-0.02***	-0.02***	-0.02***	-0.02***	-0.02***	-0.02***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
inf	0.00	0.00	0.01	-0.05	-0.03	-0.03	-0.02
	(0.04)	(0.03)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)
ele	0.63***	0.67***	0.64***	0.65***	0.65***	0.66***	0.65***
	(0.15)	(0.17)	(0.16)	(0.16)	(0.15)	(0.16)	(0.16)
*ele*doecd6*	-0.56**	-0.58*	-0.54*	-0.59**	-0.56**	-0.56**	-0.55**
	(0.24)	(0.30)	(0.28)	(0.27)	(0.24)	(0.28)	(0.27)
gap	0.01	0.01	0.02	-0.03	0.00	0.01	0.02
	(0.05)	(0.07)	(0.07)	(0.04)	(0.05)	(0.07)	(0.08)
*gap*doecd6*	-0.26***	-0.17*	-0.25***	-0.11	-0.23***	-0.18*	-0.23**
	(0.07)	(0.09)	(0.09)	(0.08)	(0.08)	(0.10)	(0.09)
vol	6.29	7.77*	6.57	7.45	6.83	8.59*	6.27
	(4.57)	(4.16)	(4.02)	(6.52)	(5.26)	(5.12)	(4.72)
d9297	-0.88**		-1.03***		-0.60		-0.76*
	(0.35)		(0.37)		(0.39)		(0.41)
*d9297*doecd6*	0.26		0.53		-0.15		0.13
	(0.64)		(0.82)		(0.69)		(0.92)
d9804	-0.38		-0.39		-0.50		-0.52
	(0.35)		(0.38)		(0.36)		(0.38)
*d9804*doecd6*	0.92*		0.92**		0.71		0.81
	(0.48)		(0.44)		(0.48)		(0.51)
*d9297*gap*		-0.04	-0.09			-0.07	-0.10
		(0.10)	(0.09)			(0.11)	(0.09)
*d9297*gap*doecd6*		0.12	0.16			0.21*	0.16
		(0.10)	(0.21)			(0.10)	(0.23)
*d9804*gap*		0.00	0.11			0.06	0.13
		(0.14)	(0.12)			(0.17)	(0.14)
*d9804*gap*doecd6*		-0.22	-0.30			-0.07	-0.27
		(0.25)	(0.23)			(0.22)	(0.25)
mas				-0.40**	-0.50***	-0.44***	-0.52***
				(0.17)	(0.17)	(0.16)	(0.15)
*mas*doecd6*				0.45	0.67*	0.58*	0.64*
				(0.46)	(0.35)	(0.31)	(0.35)
sgp				-0.77**	-0.59	-0.80*	-0.58
				(0.35)	(0.36)	(0.41)	(0.38)
*sgp*doecd6*				1.67***	1.16**	1.57***	0.90*
				(0.56)	(0.54)	(0.60)	(0.54)
Adjusted R^2	0.84	0.85	0.85	0.85	0.85	0.85	0.85
Hausman Statistic[a]	63.08***	22.28***	25.29***	51.60***	56.85***	21.31***	23.09***
Cross-Section	17	17	17	17	17	17	17
Observations	390	390	390	390	390	390	390

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

4.F.2 Additional political variables

Cabinet composition

Variable: *Cabinet composition*: $GPART_{i,t}$. **Source:** Armingeon et al. (2005).

Construction: Variable based on the Schmidt-Index that assume growing values the more the Cabinet is composed by left, social-democratic parties. Classification: (1) hegemony of right-wing parties, (2) dominance of right-wing (and centre) parties, (3) partition between left and right, (4) dominance of social-democratic and other left parties, (5) hegemony of social-democratic and other left parties. Available until 2003.

Table 4.24: Inclusion of GPART - Euro-11 (1980 - 2003)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
α	1.67***	2.22**	1.35**	1.90*	1.56**	2.04**	1.17**	1.79*
	(0.63)	(0.98)	(0.56)	(0.97)	(0.62)	(0.94)	(0.58)	(0.94)
pdefay(-1)	0.77***	0.77***	0.78***	0.78***	0.75***	0.75***	0.75***	0.76***
	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.06)	(0.06)	(0.06)
ggflq(-1)	-0.03***	-0.03***	-0.03***	-0.03***	-0.02***	-0.02***	-0.02***	-0.02***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
inf	-0.05	-0.04	-0.03	-0.02	-0.07	-0.07	-0.06	-0.06
	(0.04)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.06)
ele	0.65***	0.62***	0.65***	0.63***	0.66***	0.63***	0.66***	0.65***
	(0.17)	(0.15)	(0.17)	(0.16)	(0.15)	(0.14)	(0.16)	(0.15)
gap	-0.05	-0.04	-0.03	-0.03	-0.04	-0.04	-0.03	-0.04
	(0.07)	(0.06)	(0.10)	(0.11)	(0.07)	(0.06)	(0.11)	(0.11)
gpart	0.06	0.06	0.05	0.05	0.07	0.06	0.06	0.05
	(0.04)	(0.04)	(0.04)	(0.05)	(0.04)	(0.05)	(0.04)	(0.04)
d9297		-0.92**		-0.99*		-0.64		-0.74
		(0.43)		(0.45)		(0.48)		(0.49)
d9804		-0.35		-0.40		-0.47		-0.59
		(0.46)		(0.53)		(0.50)		(0.55)
*d9297*gap*			-0.01	-0.05			-0.03	-0.06
			(0.11)	(0.12)			(0.13)	(0.13)
*d9804*gap*			0.05	0.14			0.16	0.18
			(0.19)	(0.17)			(0.22)	(0.18)
mas					-0.55**	-0.54***	-0.59**	-0.59***
					(0.23)	(0.19)	(0.23)	(0.17)
sgp					-1.07***	-1.12***	-1.12***	-1.08***
					(0.33)	(0.35)	(0.39)	(0.32)
Adjusted R^2	0.80	0.80	0.80	0.80	0.81	0.81	0.81	0.81
Hausman Statistic[a]	36.2***	38.9***	17.3***	18.3***	39.1***	41.7***	19.5***	20.6***
Cross-Section	11	11	11	11	11	11	11	11
Observations	251	251	251	251	251	251	251	251

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Table 4.25: Inclusion of GPART - EU-14 (1980 - 2003)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
α	3.10***	2.31***	3.01***	2.59***	3.00***	2.27***	2.97***
	(0.86)	(0.57)	(0.63)	(0.64)	(0.78)	(0.54)	(0.56)
pdefay(-1)	0.72***	0.73***	0.75***	0.74***	0.74***	0.74***	0.76***
	(0.05)	(0.04)	(0.05)	(0.06)	(0.05)	(0.05)	(0.05)
ggflq(-1)	-0.03***	-0.02***	-0.02***	-0.02***	-0.02***	-0.02**	-0.02**
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
*ggflq(-1)*deu3*	-0.09***	-0.09**	-0.11***	-0.09***	-0.10***	-0.10***	-0.12***
	(0.02)	(0.04)	(0.03)	(0.03)	(0.02)	(0.03)	(0.02)
inf	-0.03	-0.02	-0.01	-0.06	-0.05	-0.06	-0.06
	(0.05)	(0.04)	(0.04)	(0.06)	(0.06)	(0.05)	(0.05)
*inf*deu3*	-0.07	-0.11	-0.19**	-0.11	-0.12	-0.11*	-0.22**
	(0.19)	(0.07)	(0.09)	(0.08)	(0.15)	(0.07)	(0.09)
ele	0.58***	0.58***	0.60***	0.59***	0.62***	0.60***	0.64***
	(0.12)	(0.15)	(0.14)	(0.12)	(0.12)	(0.13)	(0.13)
gap	-0.01	0.02	0.03	0.01	-0.01	0.00	0.01
	(0.08)	(0.10)	(0.09)	(0.08)	(0.07)	(0.09)	(0.09)
*gap*deu3*	-0.23**	-0.21	-0.26	-0.32*	-0.24**	-0.17	-0.22
	(0.09)	(0.15)	(0.17)	(0.18)	(0.12)	(0.16)	(0.18)
gpart	0.04	0.04	0.03	0.02	0.04	0.04	0.04
	(0.08)	(0.07)	(0.06)	(0.08)	(0.07)	(0.06)	(0.06)
d9297	-0.64		-0.77*		-0.36		-0.52
	(0.41)		(0.42)		(0.43)		(0.43)
*d9297*deu3*	1.15		0.78		1.24		0.73*
	(1.15)		(0.85)		(0.94)		(0.47)
d9804	-0.21		-0.20		-0.28		-0.38
	(0.40)		(0.43)		(0.43)		(0.44)
*d9804*deu3*	-0.38		-0.92*		-0.61		-1.05**
	(1.29)		(0.52)		(1.03)		(0.50)
*d9297*gap*		-0.03	-0.12			-0.03	-0.10
		(0.13)	(0.10)			(0.12)	(0.10)
*d9297*gap*deu3*		-0.28***	0.05			-0.50***	-0.10
		(0.08)	(0.27)			(0.10)	(0.22)
*d9804*gap*		0.11	0.10			0.23	0.16
		(0.17)	(0.14)			(0.19)	(0.14)
*d9804*gap*deu3*		-0.19	0.27			-0.14	0.19
		(0.32)	(0.18)			(0.30)	(0.20)
mas				-0.63**	-0.56***	-0.72***	-0.62***
				(0.25)	(0.19)	(0.24)	(0.17)
*mas*deu3*				0.15	-0.40	-0.43	-0.50
				(0.43)	(0.32)	(0.55)	(0.33)
sgp				-0.97**	-1.07***	-1.23***	-1.12***
				(0.38)	(0.38)	(0.38)	(0.30)
Adjusted R^2	0.79	0.79	0.78	0.79	0.80	0.80	0.80
Hausman Statistic[a]	31.69***	12.44***	13.33***	31.90***	33.86***	14.07***	14.37***
Cross-Section	14	14	14	14	14	14	14
Observations	322	322	322	322	322	322	322

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Table 4.26: Inclusion of GPART - OECD-17 (1980 - 2003)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
α	1.68***	1.15***	1.54***	1.59**	1.98**	1.26***	1.69**
	(0.64)	(0.37)	(0.56)	(0.61)	(0.79)	(0.41)	(0.67)
pdefay(-1)	0.80***	0.82***	0.81***	0.76***	0.76***	0.78***	0.78***
	(0.04)	(0.05)	(0.04)	(0.05)	(0.05)	(0.06)	(0.05)
ggflq(-1)	-0.02***	-0.02***	-0.02***	-0.03***	-0.03***	-0.02***	-0.02***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
inf	-0.01	0.00	0.01	-0.06	-0.04	-0.04	-0.02
	(0.04)	(0.03)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)
ele	0.65***	0.68***	0.66***	0.64***	0.65***	0.66***	0.67***
	(0.16)	(0.18)	(0.17)	(0.16)	(0.14)	(0.17)	(0.16)
*ele*doecd6*	-0.52*	-0.59*	-0.50	-0.56**	-0.51*	-0.54*	-0.51*
	(0.27)	(0.32)	(0.31)	(0.28)	(0.26)	(0.29)	(0.29)
gap	-0.02	0.00	0.01	-0.05	-0.03	-0.01	0.01
	(0.05)	(0.06)	(0.07)	(0.05)	(0.05)	(0.07)	(0.07)
*gap*doecd6*	-0.24***	-0.18**	-0.25***	-0.10	-0.20***	-0.18**	-0.23***
	(0.06)	(0.09)	(0.09)	(0.09)	(0.07)	(0.09)	(0.09)
gpart	0.06	0.04	0.06	0.06	0.06*	0.07*	0.06*
	(0.04)	(0.04)	(0.04)	(0.04)	(0.03)	(0.04)	(0.04)
d9297	-0.85***		-0.98***		-0.62*		-0.76*
	(0.32)		(0.35)		(0.36)		(0.39)
*d9297*doecd6*	0.23		0.48		-0.16		0.12
	(0.62)		(0.79)		(0.67)		(0.90)
d9804	-0.28		-0.28		-0.40		-0.42
	(0.36)		(0.42)		(0.37)		(0.43)
*d9804*doecd6*	0.93*		0.95**		0.69		0.81
	(0.50)		(0.47)		(0.48)		(0.53)
*d9297*gap*		-0.05	-0.10			-0.08	-0.11
		(0.09)	(0.08)			(0.10)	(0.08)
*d9297*gap*doecd6*		0.10	0.16			0.20**	0.17
		(0.10)	(0.20)			(0.10)	(0.21)
*d9804*gap*		-0.07	0.07**			0.02	0.09
		(0.14)	(0.14)			(0.17)	(0.14)
*d9804*gap*doecd6*		-0.12	-0.23			0.06	-0.17
		(0.25)	(0.25)			(0.21)	(0.28)
mas				-0.38**	-0.46**	-0.43**	-0.48***
				(0.19)	(0.19)	(0.19)	(0.17)
*mas*doecd6*				0.48*	0.68*	0.61*	0.65*
				(0.47)	(0.40)	(0.35)	(0.39)
sgp				-1.13***	-0.87**	-1.12***	-0.79**
				(0.38)	(0.39)	(0.38)	(0.33)
*sgp*doecd6*				2.08***	1.52**	2.04***	1.24**
				(0.65)	(0.61)	(0.65)	(0.56)
Adjusted R^2	0.84	0.84	0.84	0.85	0.84	0.85	0.84
Hausman Statistic[a]	59.80***	20.74***	23.29***	48.33***	54.11***	20.28***	21.95***
Cross-Section	17	17	17	17	17	17	17
Observations	372	372	372	372	372	372	372

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

New composition of cabinet

Variable: *New party composition of the cabinet*: $GNEW_{i,t}$. **Source:** Armingeon et al. (2005).

Construction: Measures yearly changes in the party composition of the Cabinet: (0) no change (1) change, if cabinet composition ($GPART_{i,t}$) changed from last to present year. Available until 2003.

Table 4.27: Inclusion of GNEW - Euro-11 (1980 - 2003)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
α	1.86***	2.37**	1.53**	2.03*	1.76**	2.19**	1.34**	1.90*
	(0.71)	(1.05)	(0.64)	(1.05)	(0.71)	(1.04)	(0.66)	(1.03)
pdefay(-1)	0.77***	0.77***	0.78***	0.78***	0.75***	0.75***	0.76***	0.76***
	(0.05)	(0.04)	(0.05)	(0.05)	(0.06)	(0.05)	(0.06)	(0.06)
ggflq(-1)	-0.03***	-0.03***	-0.03***	-0.03***	-0.03***	-0.02***	-0.02***	-0.02**
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
inf	-0.05	-0.04	-0.03	-0.02	-0.07	-0.06	-0.06	-0.06
	(0.04)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.06)
ele	0.67***	0.63***	0.68***	0.64***	0.67***	0.63***	0.67***	0.65***
	(0.16)	(0.14)	(0.17)	(0.15)	(0.15)	(0.13)	(0.16)	(0.15)
gap	-0.06	-0.04	-0.04	-0.04	-0.05	-0.03	-0.04	-0.03
	(0.06)	(0.06)	(0.10)	(0.11)	(0.07)	(0.06)	(0.11)	(0.12)
gnew	-0.13	-0.12	-0.12	-0.11	-0.09	-0.07	-0.06	-0.04
	(0.20)	(0.19)	(0.20)	(0.17)	(0.23)	(0.22)	(0.22)	(0.21)
d9297		-0.91**		-0.99**		-0.63		-0.73
		(0.42)		(0.44)		(0.48)		(0.49)
d9804		-0.33		-0.40		-0.46		-0.60
		(0.44)		(0.52)		(0.47)		(0.54)
*d9297*gap*			-0.01	-0.04			-0.03	-0.05
			(0.11)	(0.12)			(0.13)	(0.13)
*d9804*gap*			0.06	0.15			0.17	0.20
			(0.20)	(0.17)			(0.23)	(0.18)
mas					-0.54**	-0.53***	-0.59***	-0.58***
					(0.22)	(0.19)	(0.22)	(0.17)
sgp					-1.02***	-1.08***	-1.08***	-1.04***
					(0.31)	(0.34)	(0.40)	(0.33)
Adjusted R^2	0.80	0.80	0.80	0.80	0.81	0.81	0.81	0.81
Hausman Statistic[a]	36.1***	39.2***	17.1***	18.5***	38.7***	41.6***	19.2***	20.5***
Cross-Section	11	11	11	11	11	11	11	11
Observations	252	252	252	252	252	252	252	252

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Table 4.28: Inclusion of GNEW - EU-14 (1980 - 2003)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
α	3.17***	2.43***	3.06***	2.64***	3.06***	2.37***	3.01***
	(0.90)	(0.50)	(0.66)	(0.59)	(0.82)	(0.48)	(0.60)
pdefay(-1)	0.72***	0.74***	0.75***	0.74***	0.74***	0.74***	0.76***
	(0.05)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)
ggflq(-1)	-0.03***	-0.02***	-0.02***	-0.02***	-0.02***	-0.01**	-0.02**
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
*ggflq(-1)*deu3*	-0.09***	-0.09***	-0.11***	-0.09***	-0.10***	-0.10***	-0.12***
	(0.02)	(0.03)	(0.03)	(0.03)	(0.02)	(0.03)	(0.02)
inf	-0.03	-0.02	-0.01	-0.06	-0.05	-0.06	-0.05
	(0.05)	(0.04)	(0.04)	(0.05)	(0.06)	(0.05)	(0.05)
*inf*deu3*	-0.07	-0.12*	-0.20**	-0.12*	-0.12	-0.13**	-0.23**
	(0.18)	(0.06)	(0.09)	(0.07)	(0.14)	(0.06)	(0.10)
ele	0.59***	0.58***	0.60***	0.58***	0.61***	0.59***	0.63***
	(0.12)	(0.14)	(0.13)	(0.12)	(0.11)	(0.12)	(0.13)
gap	-0.01	0.02	0.04	0.02	-0.01	0.01	0.02
	(0.07)	(0.10)	(0.09)	(0.07)	(0.06)	(0.09)	(0.09)
*gap*deu3*	-0.22**	-0.21	-0.27	-0.32*	-0.24*	-0.17	-0.23
	(0.09)	(0.15)	(0.17)	(0.17)	(0.12)	(0.16)	(0.18)
gnew	-0.05	0.00	0.01	0.06	0.02	0.08	0.10
	(0.17)	(0.18)	(0.17)	(0.20)	(0.20)	(0.20)	(0.20)
d9297	-0.65		-0.77*		-0.36		-0.50
	(0.40)		(0.41)		(0.43)		(0.43)
*d9297*deu3*	1.19		0.80		1.25		0.73*
	(1.09)		(0.88)		(0.84)		(0.52)
d9804	-0.20		-0.20		-0.28		-0.40
	(0.39)		(0.42)		(0.42)		(0.43)
*d9804*deu3*	-0.30		-0.87**		-0.55		-1.00**
	(1.10)		(0.43)		(0.85)		(0.46)
*d9297*gap*		-0.03	-0.12			-0.02	-0.10
		(0.13)	(0.11)			(0.13)	(0.10)
*d9297*gap*deu3*		-0.27***	0.07			-0.49***	-0.08
		(0.08)	(0.29)			(0.10)	(0.24)
*d9804*gap*		0.11	0.11			0.24	0.18
		(0.18)	(0.15)			(0.19)	(0.15)
*d9804*gap*deu3*		-0.21	0.26			-0.14	0.21
		(0.33)	(0.19)			(0.30)	(0.19)
mas				-0.65**	-0.55***	-0.74***	-0.62***
				(0.26)	(0.20)	(0.24)	(0.18)
*mas*deu3*				0.13	-0.43	-0.45	-0.51
				(0.40)	(0.30)	(0.52)	(0.31)
sgp				-0.96***	-1.05***	-1.20***	-1.10***
				(0.36)	(0.37)	(0.38)	(0.31)
Adjusted R^2	0.79	0.79	0.79	0.80	0.80	0.80	0.80
Hausman Statistic[a]	31.50***	12.13***	13.23***	31.38***	33.60***	13.81***	14.22***
Cross-Section	14	14	14	14	14	14	14
Observations	323	323	323	323	323	323	323

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Table 4.29: Inclusion of GNEW - OECD-17 (1980 - 2003)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
α	1.81***	1.23***	1.68***	1.70**	2.11**	1.40***	1.81**
	(0.68)	(0.44)	(0.59)	(0.66)	(0.84)	(0.50)	(0.73)
pdefay(-1)	0.80***	0.82***	0.81***	0.77***	0.77***	0.78***	0.78***
	(0.04)	(0.05)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)
ggflq(-1)	-0.02***	-0.02***	-0.02***	-0.03***	-0.03***	-0.02***	-0.02***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
inf	-0.01	0.00	0.01	-0.05	-0.04	-0.04	-0.02
	(0.04)	(0.04)	(0.04)	(0.05)	(0.06)	(0.05)	(0.05)
ele	0.64***	0.67***	0.65***	0.64***	0.63***	0.65***	0.65***
	(0.15)	(0.17)	(0.16)	(0.15)	(0.14)	(0.16)	(0.15)
*ele*doecd6*	-0.51*	-0.58*	-0.49	-0.54*	-0.49*	-0.53*	-0.49*
	(0.27)	(0.32)	(0.30)	(0.28)	(0.26)	(0.30)	(0.29)
gap	-0.01	0.00	0.01	-0.05	-0.02	-0.01	0.01
	(0.05)	(0.06)	(0.07)	(0.04)	(0.05)	(0.07)	(0.07)
*gap*doecd6*	-0.24***	-0.18**	-0.25***	-0.11	-0.21***	-0.18*	-0.23***
	(0.06)	(0.09)	(0.09)	(0.08)	(0.07)	(0.09)	(0.09)
gnew	0.00	0.01	0.00	-0.01	0.02	0.00	0.03
	(0.17)	(0.19)	(0.17)	(0.20)	(0.19)	(0.19)	(0.19)
d9297	-0.86***		-0.99***		-0.63*		-0.76*
	(0.32)		(0.35)		(0.37)		(0.39)
*d9297*doecd6*	0.26		0.48		-0.12		0.12
	(0.62)		(0.81)		(0.67)		(0.91)
d9804	-0.29		-0.30		-0.41		-0.45
	(0.35)		(0.41)		(0.36)		(0.42)
*d9804*doecd6*	0.88*		0.92**		0.65		0.79
	(0.48)		(0.46)		(0.47)		(0.53)
*d9297*gap*		-0.05	-0.10			-0.07	-0.10
		(0.09)	(0.08)			(0.10)	(0.08)
*d9297*gap*doecd6*		0.09	0.14			0.17*	0.14
		(0.10)	(0.20)			(0.09)	(0.22)
*d9804*gap*		-0.05	0.09			0.04	0.11
		(0.14)	(0.14)			(0.18)	(0.14)
*d9804*gap*doecd6*		-0.15	-0.26			0.00	-0.21
		(0.26)	(0.25)			(0.22)	(0.28)
mas				-0.38**	-0.45**	-0.42**	-0.48***
				(0.18)	(0.19)	(0.18)	(0.17)
*mas*doecd6*				0.49	0.65*	0.58*	0.62
				(0.46)	(0.38)	(0.34)	(0.38)
sgp				-1.07***	-0.82**	-1.04***	-0.74**
				(0.37)	(0.40)	(0.38)	(0.36)
*sgp*doecd6*				2.00***	1.48**	1.91***	1.16**
				(0.63)	(0.62)	(0.63)	(0.57)
Adjusted R^2	0.84	0.84	0.84	0.85	0.84	0.85	0.84
Hausman Statistic^a	59.52***	20.70***	23.32***	48.08***	53.86***	20.19***	21.82***
Cross-Section	17	17	17	17	17	17	17
Observations	373	373	373	373	373	373	373

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. ^{a} Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

'Ideological gap' between new cabinet and old one

Variable: *'Ideological gap' between new cabinet and old one*: $GGAP_{i,t}$. **Source:** Armingeon et al. (2005).

Construction: The gap is calculated as the difference of the index value ($GPART_{i,t}$) of the outgoing and the incoming government. Available until 2003.

Table 4.30: Inclusion of GGAP - Euro-11 (1980 - 2003)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
α	1.80**	2.34**	1.48**	2.00*	1.72**	2.19**	1.33**	1.91*
	(0.70)	(1.04)	(0.63)	(1.03)	(0.71)	(1.02)	(0.66)	(1.01)
pdefay(-1)	0.77***	0.77***	0.78***	0.79***	0.75***	0.76***	0.76***	0.76***
	(0.05)	(0.04)	(0.05)	(0.05)	(0.06)	(0.06)	(0.06)	(0.06)
ggflq(-1)	-0.03***	-0.03***	-0.03***	-0.03***	-0.02***	-0.02***	-0.02***	-0.02**
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
inf	-0.05	-0.04	-0.03	-0.02	-0.07	-0.07	-0.06	-0.06
	(0.04)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.06)
ele	0.65***	0.61***	0.65***	0.63***	0.66***	0.63***	0.66***	0.65***
	(0.16)	(0.14)	(0.17)	(0.16)	(0.15)	(0.13)	(0.16)	(0.15)
gap	-0.05	-0.04	-0.03	-0.03	-0.04	-0.03	-0.03	-0.04
	(0.06)	(0.06)	(0.10)	(0.11)	(0.07)	(0.06)	(0.11)	(0.11)
ggap	0.06	0.04	0.05	0.03	0.07	0.05	0.06	0.04
	(0.12)	(0.12)	(0.12)	(0.12)	(0.12)	(0.12)	(0.12)	(0.12)
d9297		-0.91**		-0.99**		-0.64		-0.74
		(0.42)		(0.44)		(0.48)		(0.49)
d9804		-0.35		-0.41		-0.46		-0.60
		(0.44)		(0.53)		(0.48)		(0.55)
*d9297*gap*			0.00	-0.04			-0.02	-0.05
			(0.11)	(0.12)			(0.13)	(0.12)
*d9804*gap*			0.06	0.15			0.18	0.20
			(0.20)	(0.17)			(0.22)	(0.17)
mas					-0.55**	-0.54***	-0.59***	-0.59***
					(0.23)	(0.19)	(0.22)	(0.16)
sgp					-1.01***	-1.08***	-1.08***	-1.04***
					(0.31)	(0.34)	(0.39)	(0.33)
Adjusted R^2	0.80	0.80	0.80	0.80	0.81	0.81	0.81	0.81
Hausman Statistic[a]	36.7***	39.1***	17.5***	18.5***	39.5***	41.8***	19.6***	20.7***
Cross-Section	11	11	11	11	11	11	11	11
Observations	251	251	251	251	251	251	251	251

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Table 4.31: Inclusion of GGAP - EU-14 (1980 - 2003)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
α	3.39***	2.55***	3.09***	2.80***	3.25***	2.49***	3.07***
	(0.82)	(0.44)	(0.67)	(0.53)	(0.78)	(0.45)	(0.60)
pdefay(-1)	0.74***	0.75***	0.76***	0.74***	0.75***	0.74***	0.76***
	(0.04)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)
ggflq(-1)	-0.03***	-0.02***	-0.02***	-0.02**	-0.02***	-0.02**	-0.02**
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
*ggflq(-1)*deu3*	-0.10***	-0.10***	-0.11***	-0.10***	-0.11***	-0.10***	-0.12***
	(0.02)	(0.03)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
inf	-0.03	-0.03	-0.01	-0.06	-0.06	-0.06	-0.05
	(0.05)	(0.04)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)
*inf*deu3*	-0.18	-0.16***	-0.23***	-0.17***	-0.19	-0.15***	-0.23***
	(0.14)	(0.04)	(0.07)	(0.06)	(0.12)	(0.04)	(0.08)
ele	0.59***	0.58***	0.61***	0.60***	0.62***	0.61***	0.64***
	(0.12)	(0.14)	(0.13)	(0.11)	(0.11)	(0.12)	(0.12)
gap	-0.01	0.01	0.03	0.02	-0.01	0.00	0.01
	(0.07)	(0.10)	(0.09)	(0.08)	(0.06)	(0.09)	(0.09)
*gap*deu3*	-0.26**	-0.24	-0.29*	-0.34*	-0.26**	-0.20	-0.24
	(0.10)	(0.15)	(0.17)	(0.18)	(0.13)	(0.16)	(0.17)
ggap	0.03	0.03	0.02	0.06	0.05	0.05	0.04
	(0.12)	(0.11)	(0.12)	(0.11)	(0.11)	(0.11)	(0.12)
*ggap*deu3*	-0.39***	-0.43***	-0.35**	-0.43**	-0.33*	-0.35	-0.27
	(0.14)	(0.16)	(0.15)	(0.14)	(0.17)	(0.24)	(0.22)
d9297	-0.61		-0.73*		-0.34		-0.49
	(0.41)		(0.42)		(0.43)		(0.43)
*d9297*deu3*	0.79		0.91		0.91		0.79
	(0.75)		(0.86)		(0.64)		(0.59)
d9804	-0.14		-0.15		-0.23		-0.36
	(0.41)		(0.45)		(0.44)		(0.45)
*d9804*deu3*	-0.69		-0.80**		-0.79		-0.91**
	(0.76)		(0.34)		(0.65)		(0.42)
*d9297*gap*		-0.01	-0.11			-0.01	-0.10
		(0.14)	(0.11)			(0.13)	(0.10)
*d9297*gap*deu3*		-0.22***	0.12			-0.40***	-0.03
		(0.08)	(0.25)			(0.12)	(0.24)
*d9804*gap*		0.11	0.11			0.23	0.17
		(0.17)	(0.14)			(0.18)	(0.14)
*d9804*gap*deu3*		-0.33	0.11			-0.24	0.08
		(0.43)	(0.28)			(0.42)	(0.31)
mas				-0.64**	-0.56***	-0.71***	-0.62***
				(0.25)	(0.19)	(0.23)	(0.17)
*mas*deu3*				0.27	-0.25	-0.20	-0.33
				(0.46)	(0.34)	(0.60)	(0.38)
sgp				-1.01***	-1.10***	-1.19***	-1.09***
				(0.34)	(0.34)	(0.37)	(0.30)
Adjusted R^2	0.79	0.79	0.79	0.80	0.80	0.80	0.80
Hausman Statistic[a]	37.63***	13.38***	10.00***	37.70***	39.90***	15.19***	15.48***
Cross-Section	14	14	14	14	14	14	14
Observations	322	322	322	322	322	322	322

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Table 4.32: Inclusion of GGAP - OECD-17 (1980 - 2003)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
α	1.87***	1.30***	1.75***	1.73***	2.18**	1.45***	1.89**
	(0.68)	(0.42)	(0.61)	(0.66)	(0.86)	(0.49)	(0.75)
pdefay(-1)	0.80***	0.82***	0.81***	0.77***	0.77***	0.78***	0.78***
	(0.04)	(0.05)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)
ggflq(-1)	-0.02***	-0.02***	-0.02***	-0.03***	-0.03**	-0.02***	-0.02***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
inf	-0.01	-0.01	0.01	-0.06	-0.04	-0.04	-0.02
	(0.04)	(0.04)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)
ele	0.64***	0.68***	0.65***	0.65***	0.64***	0.66***	0.66***
	(0.15)	(0.17)	(0.16)	(0.15)	(0.14)	(0.16)	(0.15)
*ele*doecd6*	-0.52*	-0.61*	-0.50	-0.58*	-0.52*	-0.56*	-0.52*
	(0.28)	(0.34)	(0.32)	(0.30)	(0.27)	(0.31)	(0.30)
gap	-0.02	-0.01	0.01	-0.05	-0.02	-0.02	0.00
	(0.04)	(0.06)	(0.06)	(0.04)	(0.05)	(0.06)	(0.07)
*gap*doecd6*	-0.24***	-0.17*	-0.24***	-0.11	-0.21***	-0.17*	-0.22**
	(0.06)	(0.09)	(0.09)	(0.08)	(0.07)	(0.09)	(0.09)
ggap	0.04	0.11	0.04	0.12	0.05	0.12	0.05
	(0.12)	(0.12)	(0.12)	(0.12)	(0.12)	(0.11)	(0.12)
*ggap*doecd6*	-0.19	-0.30*	-0.18	-0.28	-0.19	-0.26	-0.19
	(0.17)	(0.09)	(0.18)	(0.17)	(0.17)	(0.18)	(0.19)
d9297	-0.89***		-1.01***		-0.66*		-0.79**
	(0.33)		(0.37)		(0.38)		(0.40)
*d9297*doecd6*	0.23		0.44		-0.13		0.10
	(0.63)		(0.83)		(0.69)		(0.93)
d9804	-0.31		-0.31		-0.42		-0.46
	(0.37)		(0.43)		(0.38)		(0.44)
*d9804*doecd6*	0.87*		0.90**		0.63		0.76
	(0.48)		(0.45)		(0.46)		(0.51)
*d9297*gap*		-0.04	-0.09			-0.06	-0.10
		(0.09)	(0.08)			(0.10)	(0.08)
*d9297*gap*doecd6*		0.08	0.13			0.16*	0.14
		(0.10)	(0.20)			(0.09)	(0.22)
*d9804*gap*		-0.05	0.09			0.05	0.12
		(0.14)	(0.13)			(0.17)	(0.14)
*d9804*gap*doecd6*		-0.15	-0.26			-0.01	-0.21
		(0.24)	(0.24)			(0.21)	(0.27)
mas				-0.38*	-0.46**	-0.41**	-0.48***
				(0.20)	(0.19)	(0.20)	(0.17)
*mas*doecd6*				0.45	0.63	0.54	0.60
				(0.47)	(0.39)	(0.35)	(0.38)
sgp				-1.03***	-0.80**	-1.03***	-0.73**
				(0.35)	(0.40)	(0.36)	(0.35)
*sgp*doecd6*				1.94***	1.47**	1.86***	1.15**
				(0.60)	(0.61)	(0.60)	(0.56)
Adjusted R^2	0.84	0.84	0.84	0.85	0.84	0.85	0.84
Hausman Statistic[a]	59.44***	20.69***	22.68***	48.30***	53.62***	20.08***	21.31***
Cross-Section	17	17	17	17	17	17	17
Observations	372	372	372	372	372	372	372

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Changes in the government per year

Variable: *Number of changes in government per year*: $GCHAN_{i,t}$. **Source:** Armingeon et al. (2005).

Construction: This number counts the number of changes in the government per year. Three possible modifications are summed: (a) change of Prime Minister; (b) change in the party composition of the Cabinet or; (c) re-formation of the government after elections with the same Prime Minister and party composition. Available until 2003.

Table 4.33: Inclusion of GCHAN - Euro-11 (1980 - 2003)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
α	1.69**	2.29**	1.14	2.02*	1.63**	2.14**	1.25**	1.92*
	(0.69)	(1.06)	(0.81)	(1.03)	(0.67)	(1.03)	(0.57)	(0.99)
pdefay(-1)	0.77***	0.77***	0.78***	0.78***	0.75***	0.75***	0.75***	0.75***
	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.06)	(0.06)	(0.06)
ggflq(-1)	-0.03***	-0.03***	-0.02***	-0.02***	-0.02***	-0.02***	-0.02***	-0.02***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
inf	-0.05	-0.04	-0.04	-0.03	-0.08	-0.07	-0.07	-0.06
	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.06)
gap	-0.04	-0.03	-0.01	-0.04	-0.03	-0.02	-0.04	-0.04
	(0.06)	(0.06)	(0.10)	(0.11)	(0.07)	(0.06)	(0.10)	(0.11)
gchan	0.50***	0.48***	0.93	0.48***	0.50***	0.48***	0.51***	0.48***
	(0.12)	(0.11)	(0.84)	(0.13)	(0.12)	(0.12)	(0.13)	(0.13)
d9297		-0.96**		-1.05**		-0.69		-0.80*
		(0.44)		(0.45)		(0.47)		(0.47)
d9804		-0.38		-0.46		-0.50		-0.65
		(0.49)		(0.56)		(0.53)		(0.59)
*d9297*gap*			-0.01	-0.03			-0.01	-0.03
			(0.09)	(0.12)			(0.13)	(0.12)
*d9804*gap*			0.04	0.16			0.19	0.21
			(0.09)	(0.16)			(0.21)	(0.17)
mas					-0.55***	-0.52***	-0.60***	-0.57***
					(0.21)	(0.18)	(0.21)	(0.16)
sgp					-1.02***	-1.06***	-1.09***	-1.03***
					(0.31)	(0.35)	(0.40)	(0.34)
Adjusted R^2	0.80	0.80	0.79	0.80	0.81	0.81	0.81	0.81
Hausman Statistic[a]	37.2***	39.4***	16.7***	18.8***	39.4***	42.0***	19.8***	20.8***
Cross-Section	11	11	11	11	11	11	11	11
Observations	252	252	252	252	252	252	252	252

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Table 4.34: Inclusion of GCHAN - EU-14 (1980 - 2003)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
α	3.11***	2.35***	3.07***	2.56***	3.03***	2.32***	3.05***
	(0.87)	(0.51)	(0.66)	(0.57)	(0.79)	(0.47)	(0.59)
pdefay(-1)	0.72***	0.74***	0.75***	0.74***	0.74***	0.74***	0.76***
	(0.05)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)
ggflq(-1)	-0.03***	-0.02***	-0.02***	-0.02***	-0.02***	-0.01**	-0.02**
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
*ggflq(-1)*deu3*	-0.09***	-0.09***	-0.11***	-0.09***	-0.10***	-0.10***	-0.11***
	(0.02)	(0.03)	(0.03)	(0.03)	(0.02)	(0.03)	(0.02)
inf	-0.03	-0.03	-0.01	-0.06	-0.05	-0.07	-0.06
	(0.05)	(0.04)	(0.04)	(0.05)	(0.06)	(0.05)	(0.05)
*inf*deu3*	-0.07	-0.12*	-0.20**	-0.12*	-0.12	-0.12**	-0.22**
	(0.16)	(0.06)	(0.09)	(0.07)	(0.12)	(0.06)	(0.10)
gap	0.00	0.01	0.03	0.02	0.00	0.00	0.01
	(0.07)	(0.10)	(0.09)	(0.07)	(0.06)	(0.09)	(0.09)
*gap*deu3*	-0.25***	-0.22	-0.27*	-0.33*	-0.26**	-0.18	-0.23
	(0.09)	(0.15)	(0.16)	(0.17)	(0.13)	(0.16)	(0.18)
gchan	0.44***	0.45***	0.43***	0.45***	0.44***	0.45***	0.43***
	(0.12)	(0.14)	(0.14)	(0.14)	(0.12)	(0.13)	(0.14)
d9297	-0.70*		-0.82**		-0.42		-0.58
	(0.41)		(0.40)		(0.42)		(0.41)
*d9297*deu3*	1.07		0.69		1.16		0.64
	(0.97)		(0.79)		(0.75)		(0.44)
d9804	-0.23		-0.24		-0.30		-0.43
	(0.44)		(0.46)		(0.47)		(0.49)
*d9804*deu3*	-0.29		-0.83**		-0.50		-0.95**
	(1.04)		(0.41)		(0.81)		(0.44)
*d9297*gap*		-0.02	-0.10			-0.01	-0.08
		(0.13)	(0.11)			(0.12)	(0.10)
*d9297*gap*deu3*		-0.27***	0.04			-0.50***	-0.11
		(0.09)	(0.28)			(0.11)	(0.23)
*d9804*gap*		0.11	0.12			0.24	0.18
		(0.17)	(0.14)			(0.19)	(0.14)
*d9804*gap*deu3*		-0.22	0.21			-0.16	0.14
		(0.30)	(0.17)			(0.28)	(0.19)
mas				-0.63***	-0.53***	-0.71***	-0.59***
				(0.23)	(0.18)	(0.22)	(0.16)
*mas*deu3*				0.14	-0.39	-0.46	-0.48
				(0.42)	(0.31)	(0.53)	(0.31)
sgp				-0.96***	-1.03***	-1.21***	-1.10***
				(0.35)	(0.37)	(0.38)	(0.30)
Adjusted R^2	0.79	0.79	0.79	0.80	0.80	0.80	0.80
Hausman Statistic[a]	30.55***	11.91***	12.75***	31.01***	32.30***	13.32***	13.47***
Cross-Section	14	14	14	14	14	14	14
Observations	323	323	323	323	323	323	323

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Table 4.35: Inclusion of GCHAN - OECD-17 (1980 - 2003)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
α	1.82***	1.26***	1.76***	1.67***	2.15**	1.42***	1.91**
	(0.68)	(0.40)	(0.61)	(0.63)	(0.85)	(0.46)	(0.74)
pdefay(-1)	0.80***	0.82***	0.81***	0.76***	0.76***	0.77***	0.78***
	(0.04)	(0.05)	(0.04)	(0.06)	(0.05)	(0.06)	(0.05)
ggflq(-1)	-0.02***	-0.02***	-0.02***	-0.03***	-0.03***	-0.02***	-0.02***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
inf	-0.01	-0.01	0.00	-0.06	-0.04	-0.05	-0.03
	(0.04)	(0.03)	(0.04)	(0.05)	(0.06)	(0.05)	(0.05)
gap	-0.01	-0.01	0.00	-0.05	-0.02	-0.02	-0.01
	(0.05)	(0.07)	(0.07)	(0.05)	(0.05)	(0.07)	(0.08)
*gap*doecd6*	-0.24***	-0.16	-0.22**	-0.11	-0.20**	-0.16	-0.20**
	(0.07)	(0.10)	(0.10)	(0.09)	(0.08)	(0.10)	(0.10)
gchan	0.48***	0.51***	0.47***	0.48***	0.47***	0.49***	0.47***
	(0.13)	(0.15)	(0.14)	(0.14)	(0.13)	(0.14)	(0.14)
*gchan*doecd6*	-0.56**	-0.64**	-0.56*	-0.60*	-0.57**	-0.59*	-0.56*
	(0.27)	(0.32)	(0.29)	(0.31)	(0.27)	(0.32)	(0.30)
d9297	-0.89***		-1.00***		-0.67*		-0.79**
	(0.33)		(0.35)		(0.35)		(0.37)
*d9297*doecd6*	0.23		0.41		-0.13		0.06
	(0.63)		(0.83)		(0.69)		(0.94)
d9804	-0.29		-0.32		-0.41		-0.46
	(0.39)		(0.44)		(0.41)		(0.46)
*d9804*doecd6*	0.79		0.82*		0.54		0.69
	(0.49)		(0.45)		(0.47)		(0.52)
*d9297*gap*		-0.02	-0.07			-0.05	-0.08
		(0.10)	(0.09)			(0.11)	(0.09)
*d9297*gap*doecd6*		0.06	0.10			0.14	0.11
		(0.11)	(0.20)			(0.10)	(0.22)
*d9804*gap*		-0.03	0.10			0.05	0.13
		(0.14)	(0.13)			(0.17)	(0.14)
*d9804*gap*doecd6*		-0.19	-0.30			-0.05	-0.25
		(0.25)	(0.25)			(0.22)	(0.29)
mas				-0.37**	-0.44**	-0.40**	-0.46***
				(0.18)	(0.18)	(0.17)	(0.16)
*mas*doecd6*				0.47	0.63	0.54	0.60
				(0.48)	(0.41)	(0.36)	(0.41)
sgp				-1.04***	-0.81**	-1.03***	-0.75**
				(0.37)	(0.40)	(0.37)	(0.35)
*sgp*doecd6*				1.94***	1.50**	1.84***	1.17**
				(0.65)	(0.63)	(0.64)	(0.56)
Adjusted R^2	0.84	0.84	0.84	0.85	0.85	0.85	0.85
Hausman Statistic[a]	59.45***	21.02***	23.32***	48.08***	53.58***	20.37***	21.78***
Cross-Section	17	17	17	17	17	17	17
Observations	373	373	373	373	373	373	373

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Type of government

Variable: *Type of government*: $GTYPE_{i,t}$. **Source:** Armingeon et al. (2005).

Construction: Index representing the type of government of the country in a particular year. Its classification takes into account whether the government has a majority in the parliament as well as the number of parties that form the coalition. The intuition is that if a government has a majority in the parliament and the lower is the number of parties forming it, the greater is its governability.

Classification: (1) single party majority government; (2) minimal-party winning coalition majority government; (3) exceeding-party coalition majority government (4) single party minority government (5) multi party minority government (6) caretaker provisory government. The indicator refers to that type of government that was in office for the longest period each year. Available until 2003.

Table 4.36: Inclusion of GTYPE - Euro-11 (1980 - 2003)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
α	1.83**	2.20*	1.23	1.36	1.52*	1.79	0.76	0.95
	(0.85)	(1.13)	(0.81)	(1.42)	(0.84)	(1.11)	(0.83)	(1.42)
pdefay(-1)	0.79***	0.79***	0.81***	0.81***	0.79***	0.79***	0.80***	0.80***
	(0.06)	(0.06)	(0.07)	(0.08)	(0.07)	(0.07)	(0.09)	(0.09)
ggflq(-1)	-0.03***	-0.03***	-0.02**	-0.02*	-0.02**	-0.02**	-0.02	-0.02
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
inf	-0.03	-0.01	0.01	0.03	-0.03	-0.02	0.01	0.01
	(0.07)	(0.07)	(0.07)	(0.06)	(0.08)	(0.08)	(0.07)	(0.07)
ele	0.47***	0.43**	0.48***	0.44**	0.46***	0.44**	0.47**	0.45**
	(0.17)	(0.17)	(0.18)	(0.20)	(0.17)	(0.19)	(0.19)	(0.21)
gap	-0.03	-0.01	0.12	0.12	-0.01	0.00	0.15	0.15
	(0.08)	(0.07)	(0.21)	(0.21)	(0.09)	(0.08)	(0.21)	(0.22)
gtype	-0.11	-0.09	-0.07	-0.05	-0.03	-0.01	0.01	0.03
	(0.08)	(0.07)	(0.09)	(0.10)	(0.07)	(0.07)	(0.08)	(0.10)
d9297		-0.80*		-0.81		-0.44		-0.45
		(0.44)		(0.56)		(0.49)		(0.60)
d9804		-0.26		-0.09		-0.29		-0.16
		(0.45)		(0.64)		(0.47)		(0.68)
*d9297*gap*			-0.18	-0.21			-0.23	-0.24
			(0.16)	(0.21)			(0.17)	(0.21)
*d9804*gap*			-0.13	-0.02			-0.06	-0.03
			(0.24)	(0.20)			(0.25)	(0.20)
mas					-0.61***	-0.59***	-0.69***	-0.65***
					(0.16)	(0.13)	(0.17)	(0.12)
sgp					-1.55***	-1.65***	-1.67***	-1.62***
					(0.38)	(0.34)	(0.38)	(0.41)
Adjusted R^2	0.78	0.78	0.78	0.78	0.79	0.79	0.79	0.79
Hausman Statistic^a	27.3***	28.7***	11.1***	12.8***	30.2***	32.1***	12.9***	14.7***
Cross-Section	11	11	11	11	11	11	11	11
Observations	223	223	223	223	223	223	223	223

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. ^{a} Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Table 4.37: Inclusion of GTYPE - EU-14 (1980 - 2003)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
α	3.61***	3.27***	3.66***	3.15***	3.28***	2.92***	3.37***
	(1.19)	(0.72)	(0.88)	(0.67)	(1.05)	(0.72)	(0.81)
pdefay(-1)	0.73***	0.75***	0.75***	0.76***	0.76***	0.76***	0.78***
	(0.07)	(0.06)	(0.07)	(0.06)	(0.06)	(0.06)	(0.07)
ggflq(-1)	-0.03***	-0.02***	-0.02**	-0.02**	-0.02**	-0.01*	-0.01*
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
*ggflq(-1)*deu3*	-0.09***	-0.10***	-0.12***	-0.09***	-0.10***	-0.11***	-0.12***
	(0.02)	(0.03)	(0.03)	(0.03)	(0.02)	(0.03)	(0.03)
inf	0.01	0.00	0.03	-0.01	0.00	-0.01	0.00
	(0.09)	(0.08)	(0.07)	(0.09)	(0.09)	(0.08)	(0.07)
*inf*deu3*	-0.06	-0.15**	-0.25**	-0.12	-0.12	-0.16**	-0.27**
	(0.21)	(0.07)	(0.12)	(0.08)	(0.16)	(0.07)	(0.13)
ele	0.46***	0.45***	0.47***	0.45***	0.49***	0.46***	0.50***
	(0.14)	(0.16)	(0.17)	(0.14)	(0.15)	(0.16)	(0.18)
gap	0.01	0.12	0.13	0.03	0.01	0.13	0.13
	(0.09)	(0.16)	(0.14)	(0.10)	(0.08)	(0.15)	(0.13)
*gap*deu3*	-0.24**	-0.31*	-0.36*	-0.34*	-0.28**	-0.30	-0.35*
	(0.11)	(0.19)	(0.20)	(0.18)	(0.13)	(0.18)	(0.20)
gtype	-0.13*	-0.10	-0.08	-0.03	-0.03	0.00	0.01
	(0.07)	(0.09)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)
*gtype*deu3*	-0.44**	-0.67**	-0.62**	-0.59***	-0.48**	-0.68**	-0.65**
	(0.21)	(0.29)	(0.27)	(0.17)	(0.19)	(0.34)	(0.31)
d9297	-0.57		-0.69		-0.19		-0.35
	(0.42)		(0.42)		(0.43)		(0.43)
*d9297*deu3*	1.18		0.62		1.08		0.50
	(1.34)		(0.75)		(1.17)		(0.50)
d9804	-0.12		-0.02		-0.11		-0.12
	(0.41)		(0.45)		(0.42)		(0.45)
*d9804*deu3*	-0.12		-0.93		-0.39		-1.05
	(1.35)		(0.74)		(1.15)		(0.75)
*d9297*gap*		-0.17	-0.23*			-0.19	-0.23*
		(0.16)	(0.13)			(0.14)	(0.13)
*d9297*gap*deu3*		-0.18	0.13			-0.32**	0.03
		(0.13)	(0.35)			(0.14)	(0.32)
*d9804*gap*		-0.04	-0.02			0.06	0.01
		(0.22)	(0.14)			(0.22)	(0.13)
*d9804*gap*deu3*		-0.03	0.38*			0.09	0.36*
		(0.31)	(0.21)			(0.27)	(0.21)
mas				-0.69***	-0.60***	-0.80***	-0.68***
				(0.19)	(0.14)	(0.21)	(0.14)
*mas*deu3*				0.21	-0.26	-0.21	-0.27
				(0.34)	(0.26)	(0.42)	(0.28)
sgp				-1.52***	-1.53***	-1.67***	-1.59***
				(0.39)	(0.36)	(0.37)	(0.35)
Adjusted R^2	0.78	0.78	0.77	0.78	0.79	0.79	0.79
Hausman Statistic[a]	33.27***	11.26***	8.85***	35.58***	35.69***	12.81***	13.44***
Cross-Section	14	14	14	14	14	14	14
Observations	294	294	294	294	294	294	294

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Table 4.38: Inclusion of GTYPE - OECD-17 (1980 - 2003)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
α	1.56**	1.02**	1.22*	1.40*	1.70**	1.01*	1.19
	(0.65)	(0.43)	(0.64)	(0.72)	(0.85)	(0.55)	(0.80)
pdefay(-1)	0.81***	0.85***	0.83***	0.78***	0.79***	0.81***	0.81***
	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.06)	(0.06)
ggflq(-1)	-0.02***	-0.02***	-0.02***	-0.02***	-0.02***	-0.02***	-0.02***
	(0.01)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
inf	0.05	0.03	0.08	0.00	0.03	0.02	0.06
	(0.06)	(0.05)	(0.05)	(0.08)	(0.07)	(0.05)	(0.06)
ele	0.48***	0.50***	0.49**	0.46**	0.48**	0.47**	0.49**
	(0.18)	(0.18)	(0.20)	(0.18)	(0.19)	(0.20)	(0.21)
*ele*doecd6*	-0.34	-0.38	-0.33	-0.35	-0.33	-0.33	-0.33
	(0.27)	(0.29)	(0.30)	(0.27)	(0.27)	(0.28)	(0.29)
gap	-0.01	0.08	0.08	-0.04	-0.01	0.09	0.08
	(0.06)	(0.11)	(0.11)	(0.06)	(0.07)	(0.11)	(0.12)
*gap*doecd6*	-0.25***	-0.25**	-0.31***	-0.12	-0.23***	-0.26**	-0.31***
	(0.07)	(0.12)	(0.11)	(0.08)	(0.08)	(0.12)	(0.11)
gtype	-0.04	0.02	-0.03	0.02	-0.01	0.05	0.02
	(0.05)	(0.05)	(0.05)	(0.06)	(0.05)	(0.05)	(0.05)
d9297	-0.77**		-0.88**		-0.46		-0.58
	(0.32)		(0.35)		(0.35)		(0.38)
*d9297*doecd6*	0.29		0.60		-0.10		0.23
	(0.63)		(0.77)		(0.69)		(0.88)
d9804	-0.15		-0.06		-0.22		-0.14
	(0.33)		(0.39)		(0.34)		(0.40)
*d9804*doecd6*	0.90*		0.91**		0.71		0.80
	(0.46)		(0.44)		(0.47)		(0.50)
*d9297*gap*		-0.14	-0.17			-0.17	-0.19
		(0.13)	(0.12)			(0.13)	(0.12)
*d9297*gap*doecd6*		0.16	0.21			0.25*	0.23
		(0.13)	(0.20)			(0.13)	(0.21)
*d9804*gap*		-0.18	-0.01			-0.10	-0.01
		(0.17)	(0.13)			(0.19)	(0.13)
*d9804*gap*doecd6*		-0.01	-0.11			0.14	-0.06
		(0.27)	(0.25)			(0.24)	(0.26)
mas				-0.47***	-0.55***	-0.54***	-0.58***
				(0.16)	(0.12)	(0.15)	(0.12)
*mas*doecd6*				0.52	0.68**	0.64**	0.64*
				(0.44)	(0.33)	(0.29)	(0.34)
sgp				-1.69***	-1.52***	-1.79***	-1.51***
				(0.39)	(0.31)	(0.39)	(0.31)
*sgp*doecd6*				2.52***	2.04***	2.47***	1.81***
				(0.57)	(0.50)	(0.60)	(0.54)
Adjusted R^2	0.84	0.84	0.84	0.85	0.85	0.85	0.84
Hausman Statistic[a]	51.88***	16.58***	19.25***	43.24***	47.77***	16.35***	18.25***
Cross-Section	17	17	17	17	17	17	17
Observations	344	344	344	344	344	344	344

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Fractionalization of the party-system

Variable: *Index of fractionalization of the party-system according to Rae (1968)*: $RAE_{i,t}$.
Source: Armingeon et al. (2005).

Construction: The index is constructed as follows[36]:

$$RAE_{i,t} = 1 - \sum_{j=1}^{m} t_{i,t}^2,$$

where $t_{i,t}$ is the share of votes for party j in country i and period t, and m the number of parties.

Table 4.39: Inclusion of RAE - Euro-11 (1980 - 2003)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
α	5.71**	6.35**	5.95**	6.59**	5.08*	5.54**	5.14*	5.76**
	(2.61)	(2.69)	(2.63)	(2.71)	(2.70)	(2.77)	(2.64)	(2.74)
pdefay(-1)	0.76***	0.75***	0.76***	0.77***	0.74***	0.74***	0.74***	0.74***
	(0.05)	(0.05)	(0.05)	(0.06)	(0.06)	(0.06)	(0.07)	(0.06)
ggflq(-1)	-0.03***	-0.03***	-0.03***	-0.03***	-0.03***	-0.03***	-0.02***	-0.02***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
inf	-0.03	-0.02	-0.02	-0.01	-0.06	-0.05	-0.05	-0.04
	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.06)	(0.07)	(0.06)
ele	0.68***	0.64***	0.69***	0.66***	0.68***	0.65***	0.69***	0.67***
	(0.16)	(0.15)	(0.17)	(0.16)	(0.15)	(0.15)	(0.16)	(0.16)
gap	-0.05	-0.04	-0.07	-0.07	-0.05	-0.04	-0.06	-0.06
	(0.06)	(0.05)	(0.11)	(0.11)	(0.06)	(0.06)	(0.11)	(0.12)
rae	-0.05	-0.05	-0.06	-0.06*	-0.04	-0.04	-0.05	-0.05
	(0.04)	(0.04)	(0.04)	(0.03)	(0.04)	(0.04)	(0.04)	(0.04)
d9297		-0.78*		-0.83*		-0.54		-0.61
		(0.46)		(0.46)		(0.50)		(0.50)
d9804		-0.21		-0.30		-0.34		-0.50
		(0.49)		(0.55)		(0.53)		(0.56)
*d9297*gap*			0.04	0.01			0.02	-0.01
			(0.13)	(0.13)			(0.15)	(0.13)
*d9804*gap*			0.14	0.22			0.24	0.25
			(0.21)	(0.18)			(0.23)	(0.18)
mas					-0.52**	-0.50**	-0.57***	-0.54***
					(0.22)	(0.20)	(0.21)	(0.16)
sgp					-0.91***	-1.01***	-1.01***	-0.97***
					(0.32)	(0.36)	(0.38)	(0.34)
Adjusted R^2	0.80	0.80	0.80	0.80	0.81	0.81	0.81	0.81
Hausman Statistic[a]	36.8***	38.8***	17.3***	17.8***	39.3***	41.2***	19.3***	20.0***
Cross-Section	11	11	11	11	11	11	11	11
Observations	252	252	252	252	252	252	252	252

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

[36] This formula is also presented in Armingeon et al. (2005), was initially proposed by Rae (1968).

Table 4.40: Inclusion of RAE - EU-14 (1980 - 2003)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
α	4.23*	4.30*	4.09*	3.67*	3.44	3.80	3.48
	(2.28)	(2.22)	(2.25)	(2.17)	(2.22)	(2.30)	(2.29)
pdefay(-1)	0.72***	0.73***	0.75***	0.73***	0.74***	0.74***	0.76***
	(0.05)	(0.05)	(0.05)	(0.06)	(0.05)	(0.05)	(0.05)
ggflq(-1)	-0.03***	-0.02***	-0.02***	-0.02***	-0.02***	-0.02**	-0.02**
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
*ggflq(-1)*deu3*	-0.09***	-0.09**	-0.11***	-0.08***	-0.09***	-0.09***	-0.11***
	(0.02)	(0.03)	(0.03)	(0.03)	(0.02)	(0.03)	(0.02)
inf	-0.01	-0.01	0.00	-0.04	-0.04	-0.05	-0.04
	(0.05)	(0.05)	(0.05)	(0.06)	(0.05)	(0.06)	(0.05)
*inf*deu3*	-0.05	-0.08	-0.17*	-0.07	-0.10	-0.09	-0.19*
	(0.18)	(0.06)	(0.09)	(0.06)	(0.13)	(0.06)	(0.10)
ele	0.60***	0.60***	0.62***	0.60***	0.62***	0.62***	0.65***
	(0.13)	(0.15)	(0.14)	(0.12)	(0.12)	(0.13)	(0.14)
gap	-0.01	0.00	0.02	0.02	-0.01	-0.01	0.00
	(0.07)	(0.10)	(0.10)	(0.08)	(0.06)	(0.10)	(0.09)
*gap*deu3*	-0.26***	-0.24	-0.31*	-0.34*	-0.27**	-0.20	-0.26
	(0.09)	(0.15)	(0.18)	(0.18)	(0.12)	(0.16)	(0.19)
rae	-0.05	-0.05	-0.05	-0.05	-0.04	-0.04	-0.04
	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)
*rae*deu3*	0.18***	0.12	0.18***	0.14***	0.16***	0.10	0.15*
	(0.04)	(0.07)	(0.06)	(0.05)	(0.05)	(0.09)	(0.08)
d9297	-0.43		-0.52		-0.21		-0.34
	(0.51)		(0.47)		(0.49)		(0.45)
*d9297*deu3*	0.92		0.73		1.08		0.75
	(1.03)		(1.06)		(0.69)		(0.69)
d9804	0.02		0.02		-0.08		-0.21
	(0.51)		(0.50)		(0.53)		(0.49)
*d9804*deu3*	-0.61		-1.08***		-0.83		-1.15***
	(0.97)		(0.33)		(0.73)		(0.40)
*d9297*gap*		-0.01	-0.09			0.00	-0.08
		(0.14)	(0.11)			(0.13)	(0.11)
*d9297*gap*deu3*		-0.24***	0.13			-0.47***	-0.03
		(0.08)	(0.33)			(0.09)	(0.27)
*d9804*gap*		0.17	0.14			0.28	0.19
		(0.19)	(0.15)			(0.20)	(0.15)
*d9804*gap*deu3*		-0.20	0.23			-0.15	0.17
		(0.33)	(0.21)			(0.31)	(0.22)
mas				-0.63**	-0.51***	-0.71***	-0.57***
				(0.26)	(0.19)	(0.23)	(0.16)
*mas*deu3*				0.08	-0.51*	-0.49	-0.56*
				(0.44)	(0.30)	(0.52)	(0.28)
sgp				-0.88**	-1.03***	-1.15***	-1.06***
				(0.36)	(0.35)	(0.37)	(0.30)
Adjusted R^2	0.79	0.79	0.79	0.80	0.80	0.80	0.80
Hausman Statistic[a]	37.59***	13.53***	9.99***	38.69***	40.31***	15.37***	15.54***
Cross-Section	14	14	14	14	14	14	14
Observations	323	323	323	323	323	323	323

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Table 4.41: Inclusion of RAE - OECD-17 (1980 - 2003)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
α	5.03**	2.95	4.79*	3.92	5.21**	3.41	4.89*
	(2.42)	(2.41)	(2.65)	(2.56)	(2.59)	(2.68)	(2.90)
pdefay(-1)	0.79***	0.80***	0.80***	0.75***	0.75***	0.77***	0.77***
	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.05)
ggflq(-1)	-0.03***	-0.02***	-0.02***	-0.03***	-0.03***	-0.02***	-0.02***
	(0.01)	(0.00)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)
inf	0.01	0.01	0.02	-0.04	-0.02	-0.02	-0.01
	(0.04)	(0.04)	(0.04)	(0.06)	(0.05)	(0.05)	(0.05)
ele	0.67***	0.71***	0.69***	0.67***	0.67***	0.69***	0.69***
	(0.16)	(0.17)	(0.17)	(0.15)	(0.15)	(0.16)	(0.16)
*ele*doecd6*	-0.53*	-0.63**	-0.53*	-0.57**	-0.51*	-0.57**	-0.52*
	(0.28)	(0.31)	(0.31)	(0.27)	(0.27)	(0.29)	(0.30)
gap	-0.02	0.00	-0.01	-0.05	-0.03	-0.01	-0.02
	(0.04)	(0.06)	(0.07)	(0.04)	(0.05)	(0.07)	(0.08)
*gap*doecd6*	-0.25***	-0.20**	-0.25***	-0.13	-0.21***	-0.19**	-0.23**
	(0.06)	(0.09)	(0.09)	(0.08)	(0.07)	(0.09)	(0.09)
rae	-0.06*	-0.05*	-0.06*	-0.05	-0.05	-0.05	-0.05
	(0.03)	(0.03)	(0.03)	(0.04)	(0.04)	(0.03)	(0.03)
*rae*doecd6*	0.04	0.10	0.05	0.05	0.03	0.07	0.04
	(0.07)	(0.09)	(0.08)	(0.07)	(0.08)	(0.08)	(0.09)
d9297	-0.70*		-0.77**		-0.50		-0.59
	(0.37)		(0.38)		(0.39)		(0.40)
*d9297*doecd6*	0.16		0.25		-0.18		-0.03
	(0.51)		(0.70)		(0.56)		(0.82)
d9804	-0.11		-0.13		-0.24		-0.29
	(0.41)		(0.43)		(0.44)		(0.45)
*d9804*doecd6*	0.80*		0.76		0.56		0.68*
	(0.44)		(0.47)		(0.38)		(0.39)
*d9297*gap*		-0.06	-0.06			-0.07	-0.07
		(0.09)	(0.09)			(0.10)	(0.09)
*d9297*gap*doecd6*		0.15	0.11			0.19**	0.12
		(0.10)	(0.20)			(0.09)	(0.22)
*d9804*gap*		0.01	0.13			0.08	0.15
		(0.16)	(0.15)			(0.18)	(0.15)
*d9804*gap*doecd6*		-0.24	-0.29			-0.08	-0.23
		(0.30)	(0.28)			(0.30)	(0.32)
mas				-0.34*	-0.42**	-0.37**	-0.44***
				(0.18)	(0.19)	(0.17)	(0.16)
*mas*doecd6*				0.40	0.65*	0.47	0.60
				(0.38)	(0.38)	(0.30)	(0.38)
sgp				-0.97***	-0.77**	-0.92**	-0.70**
				(0.37)	(0.39)	(0.36)	(0.35)
*sgp*doecd6*				1.89***	1.50***	1.70**	1.18**
				(0.61)	(0.56)	(0.70)	(0.57)
Adjusted R^2	0.84	0.84	0.84	0.85	0.84	0.85	0.84
Hausman Statistic[a]	59.01***	21.15***	22.59***	47.60***	52.55***	19.95***	20.94***
Cross-Section	17	17	17	17	17	17	17
Observations	373	373	373	373	373	373	373

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

4.F.3 Heterogeneity of output gap and inflation for Euro-11

Table 4.42: Estimation with heterogenous output gap - Euro-11

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
α	1.69**	1.96**	1.85***	1.94*	1.44**	1.69*	1.50**	1.80
	(0.66)	(0.88)	(0.70)	(1.04)	(0.63)	(0.89)	(0.68)	(1.10)
pdefay(-1)	0.77***	0.77***	0.76***	0.77***	0.77***	0.77***	0.76***	0.77***
	(0.06)	(0.05)	(0.05)	(0.05)	(0.06)	(0.05)	(0.06)	(0.06)
ggflq(-1)	-0.03***	-0.03***	-0.03***	-0.03***	-0.02**	-0.02**	-0.02**	-0.02**
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
inf	-0.05	-0.05	-0.05	-0.05	-0.08	-0.08	-0.08	-0.08
	(0.04)	(0.04)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)
ele	0.65***	0.63***	0.64***	0.63***	0.66***	0.64***	0.64***	0.64***
	(0.15)	(0.14)	(0.14)	(0.14)	(0.14)	(0.14)	(0.14)	(0.15)
$gap_{med}{}^{a}$	0.24	0.24	0.19	0.22	0.25	0.24	0.16	0.17
gap_{max}	0.39	0.42	0.36	0.41	0.38	0.41	0.33	0.33
gap_{min}	-0.10	-0.09	-0.21	-0.08	-0.09	-0.08	-0.16	-0.12
d9297		-0.40		-0.45		-0.25		-0.42
		(0.46)		(0.50)		(0.52)		(0.57)
d9804		-0.26		-0.27		-0.37		-0.47
		(0.39)		(0.48)		(0.40)		(0.49)
*d9297*gap*			0.15	-0.02			0.10	0.00
			(0.10)	(0.13)			(0.11)	(0.13)
*d9804*gap*			0.06	0.04			0.14	0.14
			(0.18)	(0.16)			(0.20)	(0.13)
mas					-0.46**	-0.44***	-0.43**	-0.46***
					(0.19)	(0.17)	(0.18)	(0.15)
sgp					-1.01***	-1.00**	-1.09**	-1.01**
					(0.38)	(0.40)	(0.44)	(0.41)
Adjusted R^2	0.81	0.81	0.81	0.81	0.82	0.82	0.82	0.81
Hausman Statistic[b]	1.97**	2.21**	0.71	1.60*	2.5***	2.7***	0.83	1.02
Cross-Section	11	11	11	11	11	11	11	11
Observations	263	263	263	263	263	263	263	263

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Median value of heterogeneous coefficients of *gap* per country. [b] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

Table 4.43: Estimation with heterogenous inflation - Euro-11

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
α	1.92**	2.41**	0.88	2.11**	1.75**	2.35**	0.69	1.47
	(0.80)	(0.97)	(0.67)	(0.99)	(0.86)	(0.99)	(0.78)	(0.98)
pdefay(-1)	0.68***	0.69***	0.69***	0.69***	0.66***	0.67***	0.65***	0.68***
	(0.05)	(0.05)	(0.04)	(0.05)	(0.05)	(0.05)	(0.04)	(0.06)
ggflq(-1)	-0.03***	-0.03***	-0.02***	-0.03***	-0.03***	-0.03***	-0.02**	-0.02***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
ele	0.58***	0.55***	0.58***	0.56***	0.59***	0.58***	0.58***	0.59***
	(0.18)	(0.17)	(0.20)	(0.18)	(0.17)	(0.16)	(0.18)	(0.19)
gap	-0.04	-0.02	-0.02	0.00	-0.03	-0.02	0.00	0.00
	(0.06)	(0.06)	(0.09)	(0.14)	(0.06)	(0.06)	(0.09)	(0.10)
inf^a_{med}	-0.11	-0.05	-0.01	-0.03	-0.11	-0.09	0.00	0.01
inf_{max}	0.20	0.24	0.28	0.25	0.24	0.25	0.34	0.32
inf_{min}	-0.58	-0.45	-0.36	-0.45	-0.53	-0.51	-0.36	-0.31
d9297		-1.08**		-1.23**		-0.67		-0.73
		(0.51)		(0.53)		(0.58)		(0.56)
d9804		-0.72		-0.76		-0.81*		-0.67
		(0.48)		(0.51)		(0.46)		(0.50)
*d9297*gap*			-0.05	-0.09			-0.12	-0.12
			(0.11)	(0.17)			(0.12)	(0.11)
*d9804*gap*			0.18	0.16			0.28	0.18
			(0.17)	(0.21)			(0.20)	(0.16)
mas					-0.69***	-0.69***	-0.81***	-0.72***
					(0.19)	(0.16)	(0.19)	(0.14)
sgp					-1.18***	-1.29***	-1.44***	-1.22***
					(0.28)	(0.22)	(0.42)	(0.32)
Adjusted R^2	0.80	0.80	0.80	0.80	0.81	0.81	0.82	0.81
Hausman Statistic[b]	5.78***	6.23***	3.33***	5.07***	5.87***	6.14***	3.60***	3.57***
Cross-Section	11	11	11	11	11	11	11	11
Observations	263	263	263	263	263	263	263	263

Notes: Regressions estimated by Two-Stage Least Squares (TSLS). ***, **, * Level of significance at 1%, 5% and 10% respectively. White's period robust coefficient standard errors in parenthesis. [a] Median value of heterogeneous coefficients of *gap* per country. [b] Hausman test whose null hypothesis is that the regressors are exogenous, and therefore, OLS is more adequate than 2SLS.

4.G Unit root tests

In this Appendix, we apply panel unit root tests to assess the presence of unit roots in some of our variables. Intuitively, non-stationarity could be a problem for some variables used in Sections 4.3 and 4.4, such as the lagged debt/GDP ratio when the debt path is explosive and unsustainable, the inflation rate, the interest rate and the share of the population that is not in the labour market (due to the ageing process). Following Baltagi (2005), three alternative panel unit root tests are performed. The first is Levin, Lin and Chu (2002) test - LLC. These authors argue that individual unit root tests have limited power against alternative hypotheses with highly persistent deviations from equilibrium. So, they propose a more powerful panel unit root test than performing individual unit root tests for each cross-section. The null hypothesis is that each individual time series contains a unit root against the alternative that each time series is stationary. The augmented Dickey-Fuller (ADF) equation is

$$\Delta y_{it} = \alpha y_{i,t-1} + \Sigma_{j=1}^{k_i} \beta_{ij} \Delta y_{i,t-j} + \gamma_{mi} X_{mt} + \varepsilon_{it}, \qquad m = 1, 2, 3,$$

with X_{mt} indicating the vector of deterministic variables and γ_{mi} the corresponding vector of coefficients for $m = 1, 2, 3$. Assuming $\alpha = \rho - 1$, the null hypothesis of a unit root to be tested is then $H_0 : \alpha = 0$, against the alternative $H_1 : \alpha < 0$.

LLC suggest using their panel unit root test for panels of moderate size with N between 10 and 250 and T between 25 and 250 (our case). However, this test has its limitations. It crucially depends upon the *independence* assumption across cross-sections and is not applicable if cross-sectional correlation is present. Second, the assumption that *all* cross-sections have or do not have a unit root is restrictive.

Instead, Im, Pesaran, and Shin (2003) - IPS - allow for a heterogenous coefficient of $y_{i,t-1}$, i.e. for individual unit root process so that ρ_i in the equation may vary across cross-sections. Hence, they relax the assumption that $\rho_1 = \rho_2 = ... = \rho_N$ and propose an alternative test procedure based on averaging individual unit root statistics. The null hypothesis is that each series in the panel contains a unit root, i.e. $H_0 : \alpha_i = 0$, for all i and the alternative hypothesis allows for some but not all of the individuals series to have unit roots, i.e.,

$$H_1 = \begin{cases} \alpha_i = 0, & for \;\; i = 1, 2, ...N_1 \\ \alpha_i < 0 & for \;\; i = N_1 + 1, N_2 + 2, ..., N \end{cases},$$

implying that some fraction of the individual processes are stationary.

Hadri (2000) derives a residual-based Lagrange multiplier (LM) test where the null hypothesis states that there is no unit root in any one of the series in the panel against the alternative of a common unit root in the panel. In particular, he considers the following two models:

$$\begin{aligned} y_{it} &= r_{it} + \varepsilon_{it} \qquad i = 1, ..., N; \qquad t = 1, ..., T, \text{ and} \\ y_{it} &= r_{it} + \beta_i t + \varepsilon_{it}, \end{aligned} \tag{4.4}$$

where $r_{it} = r_{i,t-1} + u_{it}$ is a random walk. $\varepsilon_{it} \sim IIN\left(0, \sigma_\varepsilon^2\right)$ and $u_{it} \sim \left(0, \sigma_\mu^2\right)$ are mutually independent normal variables that are IID across i and over t. Iterating the expression for r_{it} and substituting the result in (4.4), we obtain:

$$y_{it} = r_{i0} + \beta_i t + \sum_{s=1}^{t} u_{is} + \varepsilon_{it} = r_{i0} + \beta_i t + \nu_{it},$$

where $\nu_{it} = \Sigma_{s=1}^{t} u_{is} + \varepsilon_{it}$. The stationarity hypothesis is then $H_0 : \sigma_u^2 = 0$ for all i, in which case $\nu_{it} = \varepsilon_{it}$.

Therefore, we perform these three tests on the dependent and control variables of Appendix 4.A. Table 4.44 reports the results of the tests. Their null hypotheses for the LLC, the IPS and the Hadri tests are, respectively, a common unit root, an individual unit root and no unit root in any of the series. So, lower P-values reject those null hypotheses. For the two first tests, LLC and IPS, the hypothesis of a unit root can be rejected for all series at least at the 5% significance level. Therefore, although those panel unit root tests have been criticized because they assume cross-sectional independence, Table (4.44) suggests that our dependent and control variables are stationary and can be estimated in level.

Table 4.44: Panel unit root tests - results

Test		pdefay	gap	deb	irlrc	nwp	infoil
	Stat.	−1.72	−6.32	−2.82	−3.47	−5.77	−25.09
LLC	Prob.	0.043**	0.000***	0.002***	0.000***	0.000***	0.000***
	Obs.	615	676	561	794	937	989
	Stat.	−3.31	−8.77	−1.42	−3.13	−2.57	−22.21
IPS	Prob.	0.000***	0.000***	0.077*	0.000***	0.005***	0.000***
	Obs.	615	615	561	794	937	989
	Stat.	7.58	0.31	13.45	5.42	17.22	−1.85
Hadri	Prob.	0.000***	0.377	0.000***	0.000***	0.000***	0.968
	Obs.	615	725	605	834	837	1012

LLC - Levin, Lin & Chu test. Null hypothesis: common unit root. IPS - Im, Pesaran and Shin test. Null hypothesis: individual unit root. Hadri test's null hypothesis: common no-unit root. (***), (**),(*) Significant at 1%, 5% and 10% respectively.

However, for all the variables analyzed, Hadri's test rejects the null hypothesis in favor of a common unit root in all the series.

Chapter 5

Conclusion

With more and more countries world-wide adopting fiscal rules and restrictions, the analysis of the economic effects of such restrictions becomes a key macroeconomic issue. This thesis, therefore, has investigated some of these effects. In particular, the focus of this research has been on restrictions similar to those imposed by Europe's Maastricht Treaty and the Stability and Growth Pact, which are probably the most prominent examples of fiscal rules.

The introduction has presented an overview of the main motivations proposed by the economic literature to explicitly restrict the ability of public authorities (governments) in creating or increasing public deficits. It has shown that fiscal rules and restrictions are used in general to: (i) guarantee fiscal solvency; (ii) reduce the volatility and procyclicality of discretionary fiscal policy; (iii) promote intergenerational fairness and equity; and, especially, (iv) curb excessive deficits. The predominant types of fiscal restrictions, such as numerical rules and procedural restrictions, as well as some of the main issues involving their implementation (flexibility, credibility and enforcement) have also been discussed in Chapter 1. That analysis has shown in particular that fiscal rules should be well defined, transparent, simple, and enforceable. In addition, that chapter has described some of the main rules and restrictions in practice in developed and developing countries around the world, demonstrating the scope and importance of the topic.

Chapter 2 has examined the welfare differences of imposing primary deficit-based sanctions on myopic governments rather than sanctions based on the debt level. As the analysis has shown, both types of sanctions discipline impatient partisan governments and reduce the political deficit bias. The analysis has also shown that economies with debt-based sanctions feature higher social welfare than economies with primary deficit-based sanctions. This finding is reinforced when (i) government myopia is stronger, (ii) the interest rate is higher and (iii) the income shocks have higher variance and are more persistent. Debt-based constraints imply better smoothing of public spending after an income shock. Further, the appropriate debt constraint is more robust against changes in the interest rate than is the appropriate deficit constraint. This suggests that the former type of constraint might be easier to implement in practice, in view of the fact that it would be politically difficult to make frequent and large adjustments to the constraints. These results support the greater emphasis that the SGP puts on debt after its reform.

Chapter 3 has analyzed the incentives of a government facing electoral uncertainty to implement structural reforms in the presence of a deficit that reduces the scope for providing short-run compensation to the losers from the reform. The analysis has shown that in designing a reform package, the government faces a trade-off between enhancing its electoral chances by providing compensation to private individuals and the cost of violating a deficit restriction. While the deficit restriction is effective in restraining the actual public deficit, it reduces the likelihood of structural reform by forcing a reduction in compensation spending for the losers from the reform. As a result, social welfare may be negatively affected because the future reform benefits are more likely to be foregone. That chapter has in addition evinced that more individual income uncertainty reduces the likelihood of reform, indicating that the political feasibility of reforms is enhanced by making the deficit restriction contingent on the business cycle and also on compensation. While one needs to be careful in translating this analysis into the context of the Europe's SGP, the results suggest that the recent reform of the pact went into the right direction by including explicitly in its corrective arm the possibility to exceed the 3% deficit norm in the case of expenses related to the implementation of structural reforms.

The fourth chapter has focused particularly on the European fiscal restrictions and assessed the effectiveness of the Maastricht Treaty and of the Stability and Growth Pact in disciplining fiscal policy in the Euro zone. The results have shown that indeed those restrictions induced a fiscal tightening in response to *excessive deficits*. However, if the reduction of total average deficits and/or the change of fiscal policy to countercyclical are also considered as measures of effectiveness of the EU fiscal framework, then that evaluation becomes less positive. Furthermore, the MT during the period preceding EMU seems to have been more stringent than the SGP, although the fiscal contractions in the EU that preceded the start of EMU were also observed in other "industrialized" countries (also those not subject to fiscal constraints). Therefore, Chapter 4 has called for improvements in the SGP. A reformed Pact should include incentives to produce lower deficits (or higher surpluses) during boom phases of the business cycle and more flexibility in the application of sanctions during recessions, especially if the enforcement of countercyclical fiscal policies in the Euro zone is seen as an objective of the SGP. This is again in line with the recent revision of the Pact in 2005, even though that reform can also undermine the enforceability of the restrictions, making its success strongly dependent of the political will of European fiscal authorities to enforce the Pact.

All in all, this thesis has demonstrated that fiscal rules and restrictions can be an effective instrument in curbing excessive deficits and to guarantee fiscal sustainability. Nevertheless, their design must receive a careful attention in order to avoid providing the wrong incentives to policymakers and to avoid negative spillovers to the economy, which would consequently reduce social welfare. In particular, those restrictions should be flexible enough to accommodate exogenous shocks and to allow the implementation of policies aiming at economic growth (structural reforms and public investment, among others). At the same time, they should be credible and enforceable, and therefore, simple and backed by appropriate legal norms. Chapter 2 has also suggested that debt-based sanctions are preferable to restrictions on the primary deficit.

Regarding Europe's fiscal restrictions, this thesis has evinced that they have been relevant to reduce deficits in the Eurozone during the last fifteen years. Moreover, given all analyses, we conclude that the recent reform of the SGP, even though raising concerns about its enforceability, has increased its flexibility and has given more room for the implementation of the so-needed structural reforms in Europe. Nonetheless, additional measures should be taken to promote the adoption of countercyclical fiscal policies by European fiscal authorities under the new SGP.

This thesis offers various possibilities for further research. While Chapter 2 goes in that direction, the analysis of optimal fiscal rules and their welfare effects could be extended. In particular, the derivation of optimal fiscal rules and restrictions should take into consideration households' optimizing behavior and the interactions between fiscal and monetary policy. Another gap that we identify in the literature is the analysis of the credibility and enforcement of fiscal restrictions. As far as the theoretical side is concerned, more advanced game theoretic frameworks and, as far as the empirical side is concerned, new databases (such as real time data), could be used to tackle such issues and help in designing more credible and easily enforceable restrictions. At last, the analysis of fiscal restrictions and economic growth should also be further researched. A better understanding of how fiscal restrictions affect public investment and productivity, for example, is crucial for their optimal design.

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Samenvatting (summary in Dutch)

De laatste vijftien jaar hebben verschillende landen ter wereld strenge maatregelen met betrekking tot hun begroting genomen, in het bijzonder begrotingsregels en restricties, om begrotingsdiscipline te garanderen (of op zijn minst te signaleren).

Begrotingsregels en restricties zijn geen recent fenomeen. Zoals Basseto en Sargent (2006) vermelden, werden reeds in de achttiende and negentiende eeuw bepaalde typen gouden regels toegepast door verscheidene nationale overheden. Echter, begrotingsrestricties zijn in de laatste jaren waarschijnlijk meer toegepast dan ooit tevoren.

Begrotingsrestricties zijn om verscheidene redenen toegepast, bijvoorbeeld: (i) om macroeconomische stabiliteit te garanderen; (ii) om de geloofwaardigheid van de begrotingsdiscipline te vergroten en als hulp bij eliminatie van het begrotingstekort; (iii) om de duurzaamheid van het begotingsbeleid op lange termijn te ondersteunen, in het bijzonder in het licht van de vergrijzing; of (iv) om negatieve externaliteiten te minimaliseren binnen een federatie of internationaal akkoord.

Ten grondslag aan de meeste begrotingsregels en restricties ligt de idee dat huidige of toekomstige overheden wellicht niet bereid of in staat zijn om optimaal begrotingsbeleid te implementeren zonder externe druk (Kennedy en Robbins, 2001).

Het meest prominente voorbeeld van deze nieuwe generatie van begrotingsrestricties zijn wellicht de begrotingsregels, opgelegd door het verdrag van Maastricht in 1992 en meer in detail gespecificeerd in het Stabiliteits en Groei Pact, dat geratificeerd is in 1998. Maar ook verscheidene landen buiten de EU hebben statuten aangenomen die de EU reglementen nabootsen.

De introductie van dergelijke regels en restricties kan substantiele macroeconomische effecten hebben. Dit proefschrift bestudeert enkele van deze effecten.

Hoofdstuk 1 (Introductie) presenteert een overzicht van de voornaamste motivaties die worden geopperd in de economische literatuur, om expliciet de mogelijkheden te beperken van publieke autoriteiten (overheden) om overheidstekorten te creeren of te vergroten. Dit proefschrift toont dat begrotingsregels en restricties in het algemeen gebruikt worden om: (i) een solvabele begroting te garanderen; (ii) de volatiliteit en procycliciteit van discretionair begrotingsbeleid te reduceren; (iii) eerlijkheid en gelijkheid tussen generaties te bevorderen en in het bijzonder; (iv) excessieve tekorten te voorkomen. De voornaamste typen van begrotingsrestrestricties, zoals numerieke regels en procedurele restricties, evenals een aantal belangrijke kwesties met betrekking tot hun implementatie (flexibiliteit, geloofwaardigheid en handhaving) worden tevens besproken in Hoofdstuk 1. Deze analyse geeft in het bijzonder aan dat begrotingsregels goed gedefinieerd moeten zijn, transparant, simpel en uitvoerbaar. Daarnaast beschrijft dit hoofdstuk enkele hoofdregels en restricties in de praktijk in ontwikkelde en ontwikkelingslanden over de hele wereld, waaruit de omvang en het belang van dit onderwerp blijkt.

Hoofdstuk 2 onderzoekt de welvaartsverschillen tussen enerzijds het opleggen van sancties aan de overheid gebaseerd op het primaire overheidstekort en anderzijds sancties gebaseerd op het schuldniveau. Er worden ook voorstellen gedaan om een meer prominente rol te geven aan schulden in plaats overheidstekorten in het Stabiliteits en Groei Pact. In de literatuur bestaat er geen consensus over het meest wenselijk type restrictie. Voorstanders van restricties ten aanzien van schuldenniveaus claimen dat deze laatste een accurater beeld schetsen van de duurzaamheid van de overheidsfinancieen, dan de tekorten (zie, bijvoorbeeld, Wyplosz, 2005).

Aan de andere kant betogen critici dat zittende overheden niet gestraft zouden moeten worden voor de overaccumulatie van schulden door eerdere bestuurders. Derhalve zouden restricties ten aanzien van het primaire overheidstekort rechtvaardiger zijn, terwijl ze eveneens duurzaamheid van de begroting garanderen. Hoofdstuk 2 onderzoekt daarom deze kwesties door het analyseren van de verschillen in de respons van de economie en in de welvaartsconsequenties in geval van beide typen begrotingsrestricties.

Een cruciaal aspect van deze opzet is dat de overheid de sancties internaliseert, die volgen op de overtreding van een bepaalde limiet ten aanzien van het schuldniveau of het primaire tekort, wat leidt tot gedrag uit voorzorg. Dus, het instellen van de een limiet zal begrotingsbeleid beinvloeden, niet alleen wanneer de limiet bindend is, maar het zal ook effect hebben op de gemiddelde overheidsuitgaven en op de overheidsschuld wanneer de limiet niet bindend is. Ons onderzoek wijst uit dat de economie zich op een zelfde manier gedraagt onder beide type begrenzingen, hoewel het welzijn hoger is onder de begrenzing van de overheidsschuld. Deze bevinding wordt versterkt naarmate: (i) de overheid kortzichtiger is; (ii) de rente hoger is; (iii) inkomensschokken een hogere variantie hebben en langer aanhouden. Beperkingen ten aanzien van het schuldniveau impliceren een betere vereffening (smoothing) van overheidsuitgaven na een inkomensschok. Verder is de gepaste restrictie van het schuldniveau robuuster dan de gepaste restrictie van de overheidsschuld bij veranderingen in de rentevoet. Dit suggereert dat het eerste type restrictie makkelijker te implementeren is in de praktijk, aangezien het op politiek niveau moeilijk zou zijn om frequente en grote veranderingen aan te brengen aan de restricties. Deze resultaten ondersteunen de grotere nadruk die het Stabiliteits en Groei Pact na haar hervorming legt op het schuldniveau.

Variaties in de modelparameters leveren interessante inzichten op. In het bijzonder, een toename van de variantie van inkomensschokken of een verhoging van hun persistentie produceert gemiddeld een lagere schuld onder een schuldbeperking (om een grotere veiligheidsmarge tot het referentieniveau van schuld te handhaven), maar een hogere gemiddelde schuld bij beperking van het primaire tekort. Een hogere gemiddelde schuld correspondeert met een hoger gemiddeld primair surplus, wat een grotere veiligheidsmarge tot het referentieniveau van schuld impliceert, zoals gewenst door de overheid. Om gelijksoortige redenen veroorzaakt een verhoging van risico aversie een lagere gemiddelde schuld onder een schuldbeperking, maar een hogere gemiddelde schuld onder een beperking van het primaire overheidstekort. Tot slot impliceert een kortzichtige overheid een stijging van de gemiddelde schuld onder een beperking van het primaire tekort, maar een daling van de gemiddelde schuld onder een beperking van de schuld. Dit is het resultaat van gedrag uit voorzorg: door zich dicht bij het referentieniveau van schuld te bevinden, zou de overheid het risico lopen om ernstig te snijden in de overheidsuitgaven als er sprake is van een negatieve inkomensschok. Dit is erger voor een overheid die kortzichtiger is, gezien zijn geprefereerde tijdsprofiel ten aanzien van publieke uitgaven. Derhalve, in de aanwezigheid van een schuldbeperking accumuleert een dergelijke overheid gemiddeld minder schuld.

Hoofdstuk 3 bevat een politiek-economische analyse van de consequenties van begrotingsbeperkingen voor structurele hervormingen. Vele landen worden momenteel met de behoefte geconfronteerd om simultaan begrotingsdiscipline na te streven en structurele hervormingen te implementeren, zoals het flexibeler maken van hun arbeids- en produktmarkten en het hervormen van hun welvaarts- en pensioensystemen. Die behoeften zouden niet afzonderlijk van elkaar moeten worden gezien. De twee zijn nauw verwant aan elkaar, aangezien structurele hervormingen bevorderlijk zijn voor het handhaven van de duurzaamheid op lange termijn van de openbare financin. Niettemin heeft men betoogd dat begrotingsbeperkingen, in principe gunstig voor begrotingsdiscipline, op korte termijn kunnen conflicteren met de bereidheid om structurele hervormingen uit te voeren. In het bijzonder kan het uitvoeren van structurele hervormingen substantiele uitgaven vooraf vereisen, zoals het compenseren van diegenen die verliezen zullen lijden als gevolg van de hervormingen. Het conflict tussen korte termijn en lange termijn economische doelstellingen is dan ook een belangrijk argument geweest voor de recente hervorming van Europa's Stabiliteits en Groei Pact.

Hoofdstuk 3 analyseert dit conflict in de context van een model dat de aanwezigheid van een korte termijn beperking van het begrotingstekort combineert met de mogelijkheid om structurele hervormingen uit te voeren die lange termijn voordelen opleveren, maar van tevoren kosten aan private agenten opleggen. Natuurlijk passen niet alle structurele hervormingen binnen deze beschrijving. Echter, voor een aantal belangrijke hervormingen in bijvoorbeeld arbeids- en produktmarkten is dit wel het geval. Voorbeelden zijn het versoepelen van ontslagrestricties of sectorale reallocaties die leiden tot tijdelijke werkeloosheid en inkomensverlies. Om de steun voor structurele hervorming te verbeteren en daarmee de kans om herkozen te worden te vergroten, kan de zittende regering compensatie verschaffen aan de stemmende bevolking. Echter, zulke giften komen op het conto van een hoger begrotingstekort en kunnen zo tot een schending van de begrotingsbeperking leiden, bedoeld om politiek-gemotiveerde te hoge uitgaven in toom te houden. Wij bouwen een politiek-economisch kader om de feedback-effecten tussen begrotingsbeperkingen en de stimulansen voor structurele hervorming te analyseren, in een opzet waar de beleidsvormer verantwoording af moet leggen aan het electoraat. De wenselijkheid van een beperking van het begrotingstekort hangt uiteindelijk af van de mate van de politieke distortie die leidt tot het excessieve tekort en het lange termijn voordeel van de structurele hervorming.

Wij laten zien dat een strictere beperking van het begrotingstekort de ruimte voor uitgaven gerelateerd aan de hervorming beperkt en daarmee de waarschijnlijkheid vermindert dat een hervorming uiteindelijk geimplementeerd zal worden. Wij onderzoeken ook de effecten van veranderingen in inkomensonzekerheid en inkomensongelijkheid bij het electoraat op de kans van structurele hervormingen. In ons model reduceert een toename in inkomensonzekerheid de waarschijnlijkheid van hervormingen, terwijl meer ongelijkheid hervormingen waarschijnlijker maakt. Zoals besproken wordt, verlaagt een flexibeler implementatie van de beperking van het begrotingstekort, waarbij expliciet uitgaven gerelateerd aan de hervormingen in acht worden genomen, het conflict tussen het voldoen aan de beperking en een toename van de waarschijnlijkheid van hervormingen.

Die analyse is nauw verwant aan het debat over de recente hervorming van Europa's Stabiliteits en Groei Pact, die nu meer expliciet rekening houdt met de korte termijn kosten van bepaalde structurele hervormingen. Terwijl men zorgvuldig moet zijn in het vertalen van deze analyse naar de context van Europa's Stabiliteits en Groei Pact, suggereren de resultaten dat de recente hervorming van het pact de juiste richting op geslagen is door expliciet de mogelijkheid op te nemen om de 3% norm voor het tekort te overschrijden in het geval het uitgaven betreft die gerelateerd zijn aan de implementatie van structurele hervormingen.

Vervolgens concentreert Hoofdstuk zich 4 in het bijzonder op de Europese begrotingsrestricties en beoordeelt de effectiviteit van het Verdrag van Maastricht en van het Stabiliteits en Groei Pact in het disciplineren van begrotingsbeleid in de Euro zone. Wij concentreren ons op twee soorten biases die het resultaat zijn van slecht beheer van het begrotingsbeleid en wij onderzoeken hoe het begrotingskader van de EU hen heeft benvloed. Het eerste type bias is de mogelijkheid van bovenmatige tekorten die ontstaan wanneer overheden niet de kosten van additionele schuld internaliseren of wanneer zij aanpassing van de begroting na een periode van laagconjunctuur uitstellen. De tweede bias is de mogelijkheid dat begrotingsbeleid procyclisch is. Het argument is dat in goede tijden bestedingen sterker stijgen dan de belastingopbrengsten als gevolg van de misinterpretatie door politici van cyclische stijgingen van belastingopbrengsten als zijnde structureel. De analyse maakt onderscheid tussen de periode van het Verdrag van Maastricht en de periode van het Stabiliteits en Groei Pact, waarbij de effecten van elke set van beperkingen uit elkaar worden getrokken en de begrotingseffecten geisoleerd worden die afkomstig zijn van de inspanningen van Europese landen om toe te treden tot de Euro zone. Specifiek voor het Stabiliteits en Groei Pact, heeft het falen van sommige landen om aan de doelstelling voor het begrotingstekort te voldoen dat door het pact wordt opgelegd, gezorgd voor bedenkingen omtrent de vraag of het pact wel een effectief instrument is voor het reduceren van losbandigheid ten aanzien van de begroting. Uiteraard kan iedere evaluatie slechts voorlopig zijn, aangezien het Pact en de Euro slechts een paar jaar bestaan, wat voor sommige landen minder is dan een volledige conjunctuurcyclus.

Dus, na het controleren voor relevante economische en politieke variabelen, onderzoeken wij voor het cyclisch aangepaste tekort (als maatstaf voor de houding ten aanzien van de begroting) of (i) het gemiddeld niveau; en (ii) de respons op de output gap zijn veranderd tijdens MT en SGP perioden; en (iii) hoe het reageerde toen het referentieniveau van het tekort van het Verdrag (of Pact) werd overschreden. Deze reacties zijn geschat door gebruik te maken van pooling en instrumental variables technieken. Ze worden ook vergeleken met reacties van andere "gendustrialiseerde" OESO-landen, wat de Europese ervaring met het Verdrag van Maastricht en het Stabiliteits en Groei Pact in een breder perspectief plaatst.

Onze voornaamste bevindingen zijn dat zowel het Verdrag van Maastricht alsook het Stabiliteits en Groei Pact effectief zijn geweest in het verminderen van losbandigheid met betrekking tot de begroting wanneer de grens voor het begrotingstekort werd overschreden, d.w.z. zij waren effectief in het teweeg brengen van een contractie met betrekking tot de begroting als reactie op bovenmatige begrotingstekorten. Deze studie wijst ook uit dat alleen gedurende de periode van het Verdrag van Maastricht er gemiddelde sprake was van een fiscale contractie in het Euro-11 gebied. Niettemin, deze samentrekking viel samen met een beperking van de begroting (fiscal tightening) in andere "gendustrialiseerde" OESO-landen in dezelfde periode, wat eerder een gemeenschappelijke tendens in de houding ten aanzien van de begroting van ontwikkelde landen suggereert dan een gesoleerd effect van het Verdrag van Maastricht. Bovendien vinden wij dat noch het Verdrag van Maastricht, noch het Stabiliteits en Groei Pact het cyclische gedrag van de Euro-11 fiscale autoriteiten heeft veranderd.

Daarom roept Hoofdstuk 4 op voor verbeteringen van het Stabiliteits en Groei Pact. Een hervormd Pact zou stimulansen moeten incorporeren om lagere tekorten (of hogere surplussen) tot stand te brengen tijdens perioden van hoogconjuctuur evenals meer flexibiliteit in de toepassing van sancties tijdens recessies, vooral als de handhaving van anticyclisch begrotingsbeleid in de Euro zone als doelstelling van het Stabiliteits en Groei Pact wordt gezien. Dit is opnieuw in overeenstemming met de recente revisie van het Pact in 2005, zelfs als die hervorming ook de handhaving van de restricties kan ondermijnen, waardoor het succes ervan sterk afhangt van de politieke wil van Europese fiscale autoriteiten om het Pact te handhaven.

Tot slot besluit Hoofdstuk 5 dit proefschrift, met een samenvatting van de belangrijkste resultaten en de beleidsimplicaties die uit de vorige hoofdstukken volgen. Alles bij elkaar toont dit proefschrift aan dat begrotingsregels en restricties een effectief instrument kunnen zijn om excessieve tekorten in bedwang te houden en fiscale duurzaamheid te waarborgen. Niettemin moet hun ontwerp zorgvuldige aandacht krijgen om te voorkomen dat de verkeerde stimulansen worden gegeven aan beleidsmakers en om negatieve spillovers naar de economie te voorkomen, wat vervolgens de sociale welvaart zou doen afnemen. In het bijzonder zouden die restricties flexibel genoeg moeten zijn om exogene schokken te accomoderen en om de implementatie van beleid dat zich richt op economische groei (ondermeer structurele hervormingen en publieke investeringen), toe te staan. Tegelijkertijd zouden zij geloofwaardig en te handhaven moeten zijn en daarom ook eenvoudig en ondersteund door gepaste wettelijke normen. Hoofdstuk 2 heeft ook voorgesteld dat de op schuld gebaseerde sancties te prefereren zijn boven beperkingen van het primaire tekort.

Betreffende de begrotingsrestricties van Europa, heeft dit proefschrift aangetoond dat zij relevant zijn geweest om tekorten in de Eurozone gedurende de laatste vijftien jaar te verminderen. Bovendien, gegeven alle analyses, concluderen we dat de recente hervorming van het Stabiliteits en Groei Pact, alhoewel er zorgen bestaan wat betreft de handhaving ervan, de flexibiliteit van het pact heeft verhoogd en meer ruimte voor de implementatie van de zo nodige structurele hervormingen in Europa heeft gegeven. Desalniettemin zouden extra maatregelen moeten worden getroffen om de adoptie van anticyclisch begrotingsbeleid door Europese fiscale autoriteiten onder het nieuwe Stabiliteits en Groei Pact te bevorderen. Dit proefschrift biedt diverse mogelijkheden voor verder onderzoek. Terwijl Hoofdstuk 2 die richting opgaat, zou de analyse van optimale begrotingsregels en hun welvaartseffecten kunnen worden uitgebreid. In het bijzonder zou de afleiding van optimale begrotingsregels en restricties rekening moeten houden met het optimaliseringsgedrag van huishoudens en de interacties tussen begrotings- en monetair beleid. Een ander hiaat dat wij in de literatuur identificeren is de analyse van de geloofwaardigheid en de handhaving van begrotingsrestricties. Wat betreft de theoretische kant, zouden meer geavanceerdere spel-theoretische kaders en, wat betreft de empirische kant, nieuwe gegevensbestanden (zoals real time gegevens), kunnen worden gebruikt om dergelijke onderwerpen te behandelen en te assisteren bij het ontwerpen van geloofwaardigere en gemakkelijk te handhaven restricties. Tot slot zou de analyse van begrotingsrestricties en economische groei ook verder moeten worden onderzocht. Een beter begrip van hoe begrotingsrestricties publieke investeringen en productiviteit benvloeden, bijvoorbeeld, is essentieel voor hun optimaal ontwerp.

The Tinbergen Institute is the Institute for Economic Research, which was founded in 1987 by the Faculties of Economics and Econometrics of the Erasmus Universiteit Rotterdam, Universiteit van Amsterdam and Vrije Universiteit Amsterdam. The Institute is named after the late Professor Jan Tinbergen, Dutch Nobel Prize laureate in economics in 1969. The Tinbergen Institute is located in Amsterdam and Rotterdam. The following books recently appeared in the Tinbergen Institute Research Series:

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