Fiscal policy under rules and restrictions

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Chapter 3

The political economy of structural reforms under a deficit restriction*

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"We must recognize that the goal of consolidating public budgets may well conflict in the short term with the goal of enhancing the potential for economic growth. A reformed pact must take this conflict into account, as well as the need to bring improved growth and employment opportunities into line in the long term with sound public budgets."

Gerhard Schröder
"A framework for a stable Europe," *Financial Times*, January 16, 2005.

3.1 Introduction

Many countries are now confronted with the simultaneous need to pursue fiscal discipline and to implement structural reforms, such as making their labor and product markets more flexible and reform of their welfare and pension systems. This is, for example, the case for Germany and France, but also for other countries. The needs for structural reform and fiscal discipline should not be seen in isolation. In fact, the two are tightly related to each other, as structural reforms are conducive to maintaining the long-term sustainability of the public finances.[1] Most directly this is probably the case for pension reform. However, also the supposedly larger growth potential and the greater shock-resilience of an economy with better functioning markets should reduce pressures for fiscal profligacy. Nevertheless, it has been argued that fiscal restrictions, while in principle beneficial for fiscal discipline, may in the short run conflict with the abovementioned structural reform measures.[2] In particular, implementing the necessary structural reforms may require substantial up-front spending, such as compensation for those who stand to lose from the reform.[3] In fact, the conflict between short-run and long-run economic objectives has been an important argument for the recent reform of Europe's Stability and Growth Pact (SGP) (see ECOFIN, 2005; European Commission, 2005b).[4]

This chapter analyses this conflict in the context of a model that combines the presence of a short-run deficit restriction with the possibility of carrying out structural reforms that yield long-run benefits, but impose upfront costs on private agents. Of course, not

[1]This is also recognized by the European Commision (2005a) in its three-pronged strategy for ensuring long-term sustainability and the quality of the public finances. The Commission evaluates what has been done and what is further required in terms of structural reforms. For most countries in the European Union, large parts of the necessary reforms still need to be completed.

[2]An early paper in this vein is Razin and Sadka (2002). They explore the political viability of a pay-as-you-go social security-system in an aging scenario and compare it to a funded system. Their work supports the idea that a deficit ceiling hampers the transition to a funded pension system.

[3]Beetsma and Debrun (2004, 2007) provide examples of such compensation costs (see also Saint-Paul, 2002, and Grüner, 2002).

[4]A large number of reform proposals have been made by academic researchers. See, for instance, Buiter (2003), Buti et al. (2003), Blanchard and Giavazzi (2004), Fatás et al. (2004) and Mathieu and Sterdyniak (2003).

all structural reforms fit this description. However, a number of important reforms in, for instance, labor and product markets do. Examples are the relaxation of firing restrictions or sectoral re-allocations leading to temporary unemployment and a loss in income. To enhance the support for structural reform and therefore its chance to be re-elected, the incumbent government can provide compensation to the voting population. However, such handouts come at the cost of a higher deficit and may thus lead to a violation of the fiscal restriction intended to restrain politically-motivated overspending. We build a political-economy framework for analyzing the feedback effects between fiscal restrictions and the incentives for structural reform in a setting where the policymaker is accountable to the voting population. Eventually, the desirability of a deficit restriction depends on the severity of the political distortion leading to the excessive deficit and the long-run benefit from the structural reform.

We demonstrate that a tighter deficit restriction limits the room for reform-related spending and, hence, reduces the likelihood that a reform will eventually be implemented. We also explore the effects of changes in income uncertainty and income inequality across the voters on the likelihood of structural reforms. In our model, an increase in income uncertainty reduces the likelihood of reforms, while more inequality makes reforms more likely. As we discuss, a more flexible implementation of the deficit restriction that takes explicit account of reform-related spending reduces the conflict between adhering to the restriction and increasing the likelihood of reform.

The remainder of the paper is organized as follows. Section 3.2 presents the basic model. In Section 3.3 we analyze the outcomes under a benevolent policymaker (a "social planner") who is not subject to electoral uncertainty. Then, in Section 3.4, we turn to the solution under a partisan government that is subject to electoral uncertainty. Section 3.5 provides a comparative static analysis of the model, exploring the effects of a tighter fiscal restrictions, changes in income uncertainty and income inequality. Section 3.6 explores how a tighter deficit restriction affects social welfare. Finally, Section 3.7 concludes the paper. Technicalities, proofs and some extensions of our analysis are relegated to the Appendix, which is available upon request or from the authors' websites.

3.2 The model

The model features three periods (0, 1 and 2) and focuses on the interaction between structural reform and the need to adhere to a restriction that limits public deficits. We shall describe an economy that can borrow and lend freely on the international capital market, while taking the real interest rate as given. The country operates as a democracy with two political parties in a set-up that follows Alesina and Tabellini (1990). Partisan politics leads to the emergence of a deficit bias which rationalizes the presence of a fiscal restriction.[5] As a benchmark to this case, we also study a social planner that makes all

[5]There exist a number of potential political mechanisms that all give rise to a deficit bias. An early paper that contains an overview is Roubini and Sachs (1989).

choices. The model in this paper is partially based on Beetsma and Debrun (2004, 2007). However, it differs in some crucial ways from these earlier models. Most importantly, while these earlier models assumed exogenous re-election chances for the incumbent party, in the current set-up the government internalizes a feedback from its structural reform proposal to its re-election chances.

3.2.1 Private agents

Consumption (private and public) takes only place in periods 1 and 2. The economy is populated by a continuum of private individuals with total mass normalized to unity. Individuals differ in terms of their income and in terms of their preferences for public goods. All individuals derive utility from private consumption, while the low-income individuals in addition derive utility from a public good G and high-income individuals in addition obtain utility from a public good F. The other, middle-income, individuals do not benefit from any of these two public goods. Empirical work shows that public spending on various categories can differ strongly across income groups. For example, for the U.S., Ruggles and O'Higgins (1981) found that relatively large shares of public spending on housing and health and hospitals went to the (very) poor, while most of the spending on safety (policy, firearms) went to the high-income groups. O'Higgins and Ruggles (1981) do a similar analysis for the U.K. with roughly similar findings as for the U.S.

Expected utility of agent i from the middle-income group is:

$$\mathrm{E}_0\left[u\left(c_{1i}\right)+u\left(c_{2i}\right)\right], \tag{3.1}$$

where c_{ti} is the amount of the private good consumed in period t (1 and 2), and $\mathrm{E}_0\left[\cdot\right]$ is the expectation conditional on information available at the start of the game, before any of the uncertainties have been resolved. We assume that $u' > 0$ and $u'' \leq 0$. We allow for the possibility that $u'' = 0$ for all consumption levels, in which case the utility from private consumption is linear. For convenience, we abstract from the discounting of the future. Expected utility of an agent i from the low-income, respectively high-income, group is:

$$\mathrm{E}_0\left[u\left(c_{1i}\right)+u\left(c_{2i}\right)+v\left(g_1\right)+v\left(g_2\right)\right],$$
$$\mathrm{E}_0\left[u\left(c_{1i}\right)+u\left(c_{2i}\right)+v\left(f_1\right)+v\left(f_2\right)\right],$$

where g_t and f_t are the amounts of public good G and F, respectively, consumed in period t (1 and 2). We assume that $v' > 0$ and $v'' < 0$. Moreover, we assume that $v\left(0\right) = 0$, $v'\left(0\right) \rightarrow \infty$ and $v'\left(\infty\right) \rightarrow 0$. We assume that the sizes of the low- and high-income groups are λ each. The sizes of these groups are sufficiently smaller than $\frac{1}{2}$, such that the outcome of the election discussed below is solely determined by individuals' income comparisons under the two parties.

Resources available for private consumption depend on whether a structural reform takes place. In the absence of the reform,

$$c_{1i} = (1-\theta)\, y_i(1+\varepsilon) + d, \quad c_{2i} = (1-\theta)\, y_i(1+\varepsilon) - d, \tag{3.2}$$

while with the reform,

$$c_{1i} = (1-\theta)\, y_i(1+\varepsilon) + (\eta - I)\,\gamma + d, \quad c_{2i} = (1-\theta)\, y_i(1+\varepsilon)\,(1+\Gamma) - d, \tag{3.3}$$

where θ is a given (and constant) tax rate, y_i is an exogenous income component specific to individual i, which is, for convenience, assumed to be equal in both periods, ε is a (common) macroeconomic income shock, and d is the amount that the individual borrows in period 1 and needs to pay off in period 2. Differences in income can result from differences in individual labor productivity. Further, $\gamma > 0$ is the size of the reform, $I > 0$ is a short-run (period-1) private cost associated with the reform, $\eta \geq 0$ is a compensation provided by the government to the individual and $\Gamma > 0$ is a boost to second period income resulting from the reform. Hence, while reforms produce costs in the short-run (period 1), they stimulate income in the longer run, for example because markets are made to work more efficiently. We model the total cost of the reform and the total compensation as proportional to the size of the reform. This seems more restrictive than it in fact is. Because the size of the reform is given, we could instead have used a formulation in which we introduce separate parameters for the total cost and the total compensation, without any implications for the results. The current formulation turns out to be slightly more appealing when we introduce functional specifications for a numerical analysis later on. Related to the previous point, one might expect that in reality the size of the future income boost generally depends on the size of the reform, γ. That is, the boost is a function $\Gamma(\gamma)$ of γ. However, because γ is fixed, we shall most of the time suppress the argument of Γ.

While there are many possible types of structural reforms, our analysis captures only those reforms that carry some upfront cost at the individual level before yielding its longer-run benefits. Obviously, this excludes a substantial number of reforms. However, relevant reforms in the context of the current framework, could be certain types of labor market or product market reform, or some types of social security or welfare reform. Key is the assumption that individuals bear a short-run cost of the reform. These costs include among other things the loss of rents, because reforms enhance competition in product and labor markets, thereby eroding wage premia, and salary losses due to temporary unemployment associated with the induced reallocation of resources across sectors or with reductions in employment protection (IMF, 2003). We summarize these costs as foregone private consumption. To focus on the relation between structural reforms and the presence of a deficit restriction, the model assumes that each individual experiences the same cost of the reform and also receives the same compensation in order to support the reforms. However, one should notice that, even if the short-run cost of reform would affect only a fraction of the population, the government might be strongly motivated to provide net transfers in order to prevent social unrest to undermine the broader support for the reform program. A good example was the recent proposal by the French government to introduce employment contracts that would allow employers to fire young workers at will. The proposal was withdrawn after massive popular protest, despite the fact that the group affected by the measure was only a minority of the population. For simplicity, in our model, there are no lobby groups or trade unions, so that people can only express their discontent through the ballot box.

As far as compensation is concerned, in practice it may range from direct monetary transfers to more indirect forms such as active labor market policies designed to enhance the employability of the individual and ease the matching between unemployed individuals and available vacancies. Some evidence of such costs is provided in Beetsma and Debrun (2004, 2007). We assume that the size of the reform is given and that, once it has been implemented, it cannot be reversed.[6] The former assumption contrasts with Beetsma and Debrun (2004, 2007), where the size of the reform follows from optimization on the side of the government. In reality, many (though by no means all) possible reforms are of a size that can only be varied to a limited extent.

Individuals face a very simple decision problem. They optimally choose their borrowing so as to equalize their consumption levels in the two periods. In the absence of reform, this leads to $d = 0$, implying an expected utility from private consumption of [7]

$$U_i^{NR} \equiv 2\mathrm{E}_0 \left[u \left((1-\theta)\, y_i (1+\varepsilon) \right) \right], \tag{3.4}$$

while with reform they set $d = \frac{1}{2}(1-\theta)\, y_i(1+\varepsilon)\Gamma + \frac{1}{2}(I-\eta)\gamma$, implying an expected utility from private consumption of

$$U_i^{R} \equiv 2\mathrm{E}_0 \left[u \left((1-\theta)\, y_i (1+\varepsilon) \left(1+\tfrac{1}{2}\Gamma\right) + \tfrac{1}{2}(\eta - I)\gamma \right) \right]. \tag{3.5}$$

3.2.2 The Parties

There are two political parties, F and G. These parties are exogenous. This assumption seems reasonable as in reality the party landscape usually changes only slowly. Party F (party G) only obtains utility from the provision of public good F (G). Hence, the expected utility of party F (and similar for party G) is:

$$\mathrm{E}_0 \left[v(f_1) + v(f_2) - \Delta_F k \left(b - \bar{b} \right) \right], \tag{3.6}$$

where b is the budget deficit in period 1 and $\bar{b}$ is a reference deficit level. Hence, when in power, party F (party G) will direct all its spending towards public good F (good G)

[6] For an analysis of economic reforms and dynamic political constraints, see Dewatripont and Roland (1992).

[7] In the case where function $u(.)$ is linear, any debt level d is optimal, as long as consumption does not become negative in one of the periods.

and spend nothing on the other public good. We assume that in period 1 the government is subject to some fiscal restriction intended to limit fiscal profligacy. In particular, the government incurs a utility cost from running an *excessive deficit,* that is, a deficit that exceeds its reference level ($b > \bar{b}$). For simplicity, we assume here that the costs associated with an excessive deficit are non-monetary. For example, they can arise from peer pressure by governments of other countries, tight monitoring of (or even control over) the country's budgetary policies by the enforcer of the deficit restriction and public embarrassment for the officials in charge. Appendix 3.G analyzes the case where the cost of the excessive deficit is tangible in the form of a fine, but this will have no implications for the results. In the case of the European Union, both mechanisms play a role. The enforcer of the pact, the Council of Economics and Finance Ministers (ECOFIN) with the help of the European Commission, exerts pressure on individual governments to keep their finances under control, while, in the case of persistently excessive deficits, they have the possibility to impose a fine on the violator of the Excessive Deficit Criterion. Parameter k captures the tightness of the deficit restriction or the severity of sanctions. Finally, Δ_F is an indicator function, taking a value of one when F is in power, and a value of zero, otherwise. In other words, party F only suffers from the implementation of the restriction when it is in office (and runs a too high deficit). It is important to realize that the expectations operator in (3.6) aggregates out all future uncertainties as seen from the start of the game. In particular, this is the case also for the electoral uncertainties discussed below.

Without loss of generality, we assume that at the start of the game, period 0, party F is in power and that this party presents a reform package to be implemented if it is re-elected into government in the first period. This reform package is a combination (γ, η). As mentioned above, the size γ of the reform is given and thus cannot be varied by party F. The amount of compensation is the only variable to be selected by party F at the start of the game. It is assumed to be continuous. As we shall see, it will involve a trade-off for F between enhancing its re-election chances and keeping the budgetary cost (and thus the conflict with the deficit restriction) limited. The other party, G, commits to a platform that does not involve reform (and, thus, also no compensation). We assume that the commitments made in period 0 are binding so that they are executed by the parties when they get to power. While this is not modeled explicitly, one can motivate this by assuming that concerns about their longer-run credibility prevents parties from reneging on their commitments. Since we focus on the consequences of introducing reform, we exclude the possibility that party F chooses a platform with no reform. This possibility would also lead to an indeterminacy because the parties would be preferred by equally-large groups of voters. We also exclude the possibility that party G chooses a reform platform with a certain amount of compensation. This would lead to complications that are not the focus of the current analysis. On the one hand, the parties could be drawn into a bidding contest for voters by promising higher compensation than the other party, while on the other hand, they would be restrained by the fact that higher levels of compensation reduce available resources for their preferred public good. We thus abstract from these complications. We believe that the current set up with only one party proposing a reform is reasonable, since the group of individuals that prefer this party because of the type of public good it provides, is also the group that will benefit most from the reform. Exactly the opposite will be the case for party G. This party provides the public good preferred by the low-income group, which will also be the group that is harmed most by the reform.

In the case where party G is elected to hold office in period 1 and no reform is undertaken, the first- and second-period government budget constraints thus read:

$$f_1 + g_1 = \theta\overline{y}(1+\varepsilon) + b, \quad f_2 + g_2 = \theta\overline{y}(1+\varepsilon) - b, \tag{3.7}$$

where $\overline{y}$ is the average over all y_i. The first term on the right-hand side of each budget constraint is the total tax revenue. In period 1, the government can issue public debt b, which has to be paid off in period 2. Given the absence of initial debt, the debt carried over from period 1 into period 2 is equal to the deficit in period 1. When party F is re-elected and reform is implemented in the first period, the government budget constraints become:

$$f_1 + g_1 = \theta\overline{y}(1+\varepsilon) - \eta\gamma + b, \quad f_2 + g_2 = \theta\overline{y}(1+\varepsilon)(1+\Gamma) - b. \tag{3.8}$$

In order to provide a rationale for the deficit restriction, the model needs to feature a deficit bias. Therefore, we assume that there is another election at the start of the second period. However, since the implementation (or not) of the reform package has already taken place and cannot be reversed, we assume that the incumbent in period 1 is re-elected into office in period 2 with an exogenous probability $0 < p < 1$. As we shall see, this electoral uncertainty, and thus the possibility of not being able to spend the remaining resources in period 2 on its own preferred public good, leads the period-1 government (whatever its identity is) to increase its deficit. The electoral uncertainty at the start of the second period may arise from various imperfections in the political process, such as uncertainty about the voter turnout, which can have a different impact on the two parties. It could also arise from uncertainty about the appeal of the parties' leaders to voters, the occurrence of scandals, and so on. Electoral uncertainty may thus reflect extra-economic events that are not explicitly modelled here, but that the electorate cares about and that may cause part of a party's natural constituency to vote for its opponent.

3.2.3 The timing

Figure 1 summarizes the timing of the model. In Stage 1, period 0, the incumbent government chooses η. In Stage 2, also period 0, the income shock ε materializes. Then, in Stage 3, beginning of period 1, the first election takes place. This is followed by the implementation (or not) of the reform and the compensation if the incumbent is re-elected (or not), as well as the selection of the period-1 deficit (Stage 4). At the beginning of period 2, new elections take place (Stage 5). Finally, the public debt is paid off (Stage 6).

3.3 The social Planner solution

To provide a benchmark for the solution under a partisan government, we consider a utilitarian social planner who is not subject to electoral uncertainty and whose utility is the average of the utilities of all individuals in society. The social planner chooses whether or not to implement the reform. When it decides to implement the reform, it chooses the level of compensation. In addition, the planner selects the public debt (= deficit) level to optimally shift resources between periods 1 and 2.

If the planner chooses not to implement the reform, he maximizes over b, f_1 and f_2:

$$\int U_i^{NR} di + \lambda \mathrm{E}_0 \left[\begin{array}{c} v(f_1) + v(\theta \overline{y}(1+\varepsilon) + b - f_1) + \\ v(f_2) + v(\theta \overline{y}(1+\varepsilon) - b - f_2) \end{array} \right]. \tag{3.9}$$

The first term in this objective function is the aggregate of the utilities from private consumption over periods 1 and 2. The second component is the aggregate of the utilities from public consumption. The first and third terms in this component are the aggregate utilities of the high-income individuals from public consumption in periods 1 and 2 (each F-type experiences equal utility from public consumption). Similarly, the other two terms are the corresponding aggregate utilities of the low-income individuals. The arguments in the public consumption utility functions follow directly from the public budget constraints (3.7) in the two periods, when reform is forsaken.

The planner makes his choices after ε has materialized. Hence, we find b, f_1 and f_2 by maximizing the term in square brackets in (3.9). The outcomes are $f_1 = g_1 = f_2 = g_2 = \frac{1}{2}\theta\overline{y}(1+\varepsilon)$ and $b = 0$. Within each period, the planner spreads the provision of public goods equally over the F- and G-type individuals. Moreover, given the absence of discounting and a zero interest rate on debt, the planner sets debt so as to smooth public goods consumption perfectly over time. Substituting the outcomes into (3.9), we obtain the planner's utility under the optimal solution in the absence of reform:

$$\int U_i^{NR} di + 4\lambda \mathrm{E}_0 \left[v\left(\tfrac{1}{2}\theta\overline{y}(1+\varepsilon)\right) \right]. \tag{3.10}$$

If the planner chooses to conduct reform, his objective is to maximize over η, b, f_1 and f_2:

$$\int U_i^{R} di + \lambda \mathrm{E}_0 \left[\begin{array}{c} v(f_1) + v(\theta \overline{y}(1+\varepsilon) - \eta\gamma + b - f_1) + \\ v(f_2) + v(\theta \overline{y}(1+\varepsilon)(1+\Gamma) - b - f_2) \end{array} \right], \tag{3.11}$$

Again, we solve the planner's problem by working backwards. The outcomes are:

$$f_1 = g_1 = f_2 = g_2 = \tfrac{1}{2}\left[\theta\overline{y}(1+\varepsilon)\left(1+\tfrac{1}{2}\Gamma\right) - \tfrac{1}{2}\eta\gamma\right], \tag{3.12}$$

and

$$b = \tfrac{1}{2}\eta\gamma + \tfrac{1}{2}\theta\overline{y}(1+\varepsilon)\Gamma. \tag{3.13}$$

Through its debt policy, the planner shifts half of the compensation cost to the second period and half of the income gain from reform to the first period. This way it can smooth public consumption perfectly over time. In the following sections, when we explore the partisan outcomes, the deficit will generally differ from the benchmark levels derived here. Then, in the absence of reform, when the deficit is above zero, or, in the presence of reform, when the deficit exceeds (3.13), we can meaningfully speak of a *deficit bias*.

When utility from private consumption is linear, one finds the following simple first-order condition for η:

$$1 = \lambda E_0 \left\{ v' \left[\tfrac{1}{2} \left(\theta \overline{y}(1+\varepsilon) \left(1 + \tfrac{1}{2}\Gamma\right) - \tfrac{1}{2}\eta\gamma \right) \right] \right\},$$

where we have used (3.8) and substituted away b with the help of (3.13). A higher future reform benefit Γ, higher average income $\overline{y}$ and a higher tax rate θ imply an increase in the optimal level of compensation. Using (3.12), the planner's utility then becomes:

$$\int U_i^R di + 4\lambda \mathrm{E}_0 \left\{ v \left[\tfrac{1}{2} \left(\theta \overline{y}(1+\varepsilon) \left(1 + \tfrac{1}{2}\Gamma\right) - \tfrac{1}{2}\eta\gamma \right) \right] \right\}. \tag{3.14}$$

With linear private consumption utility, one has $\int U_i^{NR} di = 2(1-\theta)\overline{y}$ and $\int U_i^R di = (1-\theta)\overline{y}(2+\Gamma) + (\eta - I)\gamma$. Then, if one assumes that $(1-\theta)\overline{y}\Gamma > I\gamma$, it is always optimal for the planner to conduct reform. Setting $\eta = 0$ would already produce higher social welfare under reform, as can be seen by comparing (3.10) with (3.14). The extra flexibility of choosing η allows for a possible further welfare gain.

3.4 Solution with a partisan government

We return now to the case of the partisan solution. According to the timing presented earlier, the only moments at which decisions are taken are Stages 1 and 4 when the level of compensation η and the deficit b, respectively, are chosen. We shall solve the model using backwards induction and thus first derive the optimal deficit level for given compensation and then derive the optimal amount of compensation (assuming that the deficit will be optimally chosen subsequently). Thoughout, we assume that the reference level $\overline{b}$ is set sufficiently low (and k is not too high), so that the equilibrium deficit level exceeds its reference level. This is the relevant case for our purposes.[8]

3.4.1 The deficit decision

The deficit is selected either by party F or party G, depending on which party is in government in period 1. Therefore, we compute the first-order conditions for the deficit selected in period 1 by party F (which implements a reform), respectively party G (which does not implement a reform):

[8] This case applies, for example, if $\overline{b} = 0$, which would be the social planner's debt choice in the absence of reform.

$$v'\left(f_1^R\right) = pv'\left(f_2^R\right) + k, \quad v'\left(g_1^{NR}\right) = pv'\left(g_2^{NR}\right) + k, \tag{3.15}$$

where

$$\begin{aligned} f_1^R &= \theta\overline{y}(1+\varepsilon) - \eta\gamma + b^R, \quad f_2^R = \theta\overline{y}(1+\varepsilon)(1+\Gamma) - b^R, \\ g_1^{NR} &= \theta\overline{y}(1+\varepsilon) + b^{NR}, \quad g_2^{NR} = \theta\overline{y}(1+\varepsilon) - b^{NR}, \end{aligned}$$

and where the superscript R (NR) indicates that reform has (has not) been implemented. The first-order conditions equate the marginal benefit of a higher deficit (in terms of more public good consumption in period 1) with the expected marginal cost in terms of foregone future public good consumption plus the marginal utility cost from the implementation of the deficit restriction. In deriving these first-order conditions, we have made use of the fact that the party in government optimally chooses to spend only on its own public good. Therefore, the period-1 government will rationally calculate that it only foregoes future consumption if it is re-elected in period 2. Hence, the marginal cost in terms of foregone future public spending should be weighted with the re-election probability p in (3.15). Effectively, the period-1 government discounts future utility more heavily than if it were certain to be re-elected.

The first-order conditions imply the following results:

Lemma 3.1 *(a) The deficit is higher under reform than under no reform, (b) holding everything else equal in the reform case, more compensation, η, and a larger reform package, γ, both imply a higher deficit, (c) a higher re-election probability p for period 2 reduces the deficit both under reform and under no reform, and (d) tighter sanctions (an increase in k) reduce the deficit both under reform and under no reform.*

Proof. See Appendices 3.A and 3.B. ■

The intuition for part (a) is that implementing structural reforms requires compensating transfers financed from the period-1 public budget, while, moreover, future tax revenues rise due to the income gain brought about by the reform. For both reasons, it is optimal to shift resources to period 1, so that $b^R > b^{NR}$. We can explain part (b) by noting that more compensation, given the reform level, and a larger reform package, given the amount of compensation, both reduce first-period resources for public consumption and thus lead the government to rebalance resources over time by issuing more debt.[9] Better re-election chances effectively make the government in period 1 less myopic and, thus, induce it to issue less public debt – see part (c). Finally, tighter sanctions raise the government's marginal utility cost of issuing public debt and induce the government to cut the deficit.

[9] If second-period resources also increase because Γ is positively related to the size of the reform package, then the effect of an increase in γ on debt will be strengthened.

3.4.2 The choice of compensation

We now move further back into the game and explore the choice of the optimal compensation level η. Before so doing, let us first look at the electoral preferences of the private agents (given the reform package and the shock ε), who have to cast their votes at the start of the first period. Each individual votes for the candidate that maximizes his utility. As the benefit from the structural reform is proportional to individual income, while the private cost is fixed over the individuals, the higher the income, the more likely it is that the individual will prefer reform to no reform, ceteris paribus. Therefore, low-income individuals would always vote for party G, while high-income individuals would always vote for party F. Assuming that the election outcome is based on a majority vote, the choice of the voter with the median income, which we denote by y^m, then becomes decisive.[10] Based on (3.4) and (3.5), it is easy to see that party F is re-elected when $(1-\theta)\, y^m(1+\varepsilon)\left(1+\frac{1}{2}\Gamma\right)+\frac{1}{2}\,(\eta-I)\,\gamma > (1-\theta)\, y^m\,(1+\varepsilon)$ or $(1-\theta)\, y^m(1+\varepsilon)\Gamma > (I-\eta)\,\gamma$. For a given level of compensation η, structural reform is more likely, the higher is the median income level, the more favorable is the income shock and the more effective (as measured Γ) is the structural reform.

To keep the algebra as simple as possible, we assume that the income shock has a uniform distribution on the interval $[-\bar{\varepsilon}, \bar{\varepsilon}]$. Thus, party F will be in government in period 1 if $\varepsilon \geq \varepsilon_L$, where

$$\varepsilon_L \equiv \frac{(I-\eta)\,\gamma}{(1-\theta)\, y^m\Gamma} - 1. \tag{3.16}$$

Therefore, the probability that party F is re-elected in period 1, and reform takes place, is:

$$\Pr(R) = \frac{1}{2\bar{\varepsilon}}\left[1+\bar{\varepsilon}+\frac{(\eta-I)\,\gamma}{(1-\theta)\, y^m\Gamma}\right]. \tag{3.17}$$

Not surprisingly, an increase in compensation raises the probability of re-election.

In Stage 1 of the game, party F faces the problem of maximizing over η:

[10] For the median voter theorem to hold, the richest among the individuals that still benefit from the public good provided by party G, must prefer party G to party F. That is, the following condition must hold:

$$\begin{aligned} & 2u\left((1-\theta)\, y_\lambda(1+\varepsilon)\right)+v\left(g_1^{NR}\right)+pv\left(g_2^{NR}\right) \\ > \; & 2u\left((1-\theta)\, y_\lambda(1+\varepsilon)\left(1+\tfrac{1}{2}\Gamma\right)+\tfrac{1}{2}\,(\eta-I)\,\gamma\right)+(1-p)\,v\left(g_2^{R}\right). \end{aligned}$$

where y_λ is the income level of individuals in the λ-th percentile of the income distribution. The first term on both sides of the inequality is the utility from private consumption over the two periods of such an individual under no reform and reform, respectively. If party G takes office in period 1, there is no reform and the individual benefits from the corresponding level of provision of the G-type public good in the first period and with probability p in the second period. If party F takes office in period 1, there is reform, but the individual does not benefit from public good provision in period 1 and benefits with probability $1-p$ from the provision of the G-type public good in period 2.

$$\max_{\eta} \left\{ \begin{array}{c} \Pr(R)\,\mathrm{E}_{\varepsilon}\left[v\left(f_1^R\right) + pv\left(f_2^R\right) - k\left(b^R - \overline{b}\right) \mid \varepsilon \geqslant \varepsilon_L\right] + \\ \left[1 - \Pr(R)\right]\mathrm{E}_{\varepsilon}\left[(1-p)\,v\left(f_2^{NR}\right) \mid \varepsilon < \varepsilon_L\right] \end{array} \right\}, \tag{3.18}$$

where the operator $\mathrm{E}_{\varepsilon}[.]$ indicates that expectations are only taken over ε (the electoral uncertainty at the start of the second period has been integrated out). In optimizing, the government takes account of the effect of a change in η on the probability of re-election. Applying Leibnitz' rule and using the first-order conditions for b^R and b^{NR}, (3.15), the first-order condition for η is:

$$-\frac{\partial \varepsilon_L}{\partial \eta}\left[v\left(f_{1L}^R\right) + pv\left(f_{2L}^R\right) - k\left(b_L^R - \overline{b}\right) - (1-p)\,v\left(f_{2L}^{NR}\right)\right] = \gamma \int_{\varepsilon_L}^{\overline{\varepsilon}} v'\left(f_1^R\right) d\varepsilon, \tag{3.19}$$

where

$$f_{1L}^R \equiv \theta\overline{y}(1+\varepsilon_L) - \eta\gamma + b_L^R, \tag{3.20}$$

$$f_{2L}^R \equiv \theta\overline{y}(1+\varepsilon_L)(1+\Gamma) - b_L^R, \tag{3.21}$$

$$f_{2L}^{NR} \equiv \theta\overline{y}(1+\varepsilon_L) - b_L^{NR}. \tag{3.22}$$

Here, b_L^R and b_L^{NR} are the deficit levels under reform, respectively no reform, when $\varepsilon = \varepsilon_L$. Condition (3.19) equates the marginal benefit of an increase in compensation – the left-hand side – with its marginal cost. The marginal benefit is (minus) the marginal reduction of the lower bound on the shock that admits F's re-election, multiplied by the utility difference to F of re-election versus no re-election. The marginal cost is the expected marginal utility loss in the first period from having to pay a higher compensation for any ε in the interval $[\varepsilon_L, \overline{\varepsilon}]$, for which re-election takes place. Substituting for $\partial\varepsilon_L/\partial\eta$, one can rewrite (3.19) as

$$v\left(f_{1L}^R\right) + pv\left(f_{2L}^R\right) - k\left(b_L^R - \overline{b}\right) - (1-p)\,v\left(f_{2L}^{NR}\right) = (1-\theta)\,y^m\Gamma(\gamma)\int_{\varepsilon_L}^{\overline{\varepsilon}} v'\left(f_1^R\right) d\varepsilon. \tag{3.23}$$

One can show that equation (3.23) has only one solution in η (when taking account of the subsequent effect of η on b_L^R).[11]

3.5 Comparative statics

Having derived the first-order conditions for the model, we can now turn to the comparative static analysis. In particular, we shall explore the implications of a tighter deficit restriction. We shall also study the consequences of more income uncertainty and less income inequality.

[11] See Appendix 3.C.1.

3.5.1 Tighter enforcement of the deficit restriction

We are particularly interested in how the enforcement of the deficit restriction influences the equilibrium deficit, compensation, and re-election (reform) chances. These effects are summarized in the following proposition:

Proposition 3.1 *If the re-election probability pat the start of the second period is sufficiently high (and, in particular, if $p = 1$), then tighter enforcement of the deficit restriction (i.e., a higher k) leads to (a) a lower level of reform compensation for private agents, (b) a lower deficit both under reform and (at least for quadratic utility of public consumption) under no reform, and (c) a lower probability of reform.*

Proof. (Summary) The proof of parts (a) and (b) is obtained by differentiating, with respect to k, the first-order conditions for the optimal deficits both under reform and no reform (evaluated at $\varepsilon = \varepsilon_L$), and the first-order condition for optimal compensation (3.23). Then, by solving the resulting system of equations, we obtain the reduced form effect of k on each one of the three endogenous variables. The quadratic specification mentioned under no reform in part (b) is discussed below. Finally, differentiating (3.17) with respect to k and using the result of part (a), one can prove part (c). See Appendices 3.D.1 and 3.D.2 for a formal proof of this proposition. ■

The intuition for part (a) is as follows. In the case of reform, the period-1 government issues additional debt in order to rebalance the compensation cost over the two periods. However, as tighter enforcement of the deficit restriction raises the cost of doing so, the government is forced to alter the time profile of public goods provision in a way it does not like. This exerts a negative effect on the utility it expects from providing the public good. To mitigate this effect, the period-1 government reduces the amount of compensation it offers. In so doing, it trades off an improvement in the time profile of its public spending against a reduction in its electoral chances. The case of no reform in part (b) was already covered by Lemma 3.1. Under reform, the effect of a tighter deficit restriction is now twofold, because the increase in enforcement also affects compensation, which exerts an indirect effect on the deficit. The overall effect of tighter enforcement is a reduction in the deficit. Finally, given that tightness k only affects the initial government's re-election chances through the choice of η, it follows immediately that tighter enforcement reduces the likelihood that the initial government is re-elected and thus that reforms will be implemented.[12]

In the recent past some countries have found it difficult to adhere to the 3% deficit ceiling of Europe's SGP. This may in part be explained by these countries' attempts to introduce reforms that require some compensation to render them politically acceptable. As discussed above, governments have an incentive to increase the deficit to spread the

[12] Given that we are only able to formally prove Proposition 3.1 for p sufficiently close to 1, we have performed a numerical check for a large set of parameter combinations of the results of the proposition for values of $p < 1$. The details are in Appendix 3.F.

compensation costs over time. This undermines the compliance with the deficit restriction. This may be particularly relevant for Germany, which is trying to implement several reforms (such as reforms in pensions and in the tax system).[13]

3.5.2 Increased income uncertainty

We can also explore the implications of an increase in income uncertainty. Income uncertainty can be summarized by the variance of the shock ε, which, in turn, is monotonically increasing in $\overline{\varepsilon}$. We formalize the implications of an increase in $\overline{\varepsilon}$ in the following proposition:

Proposition 3.2 *If the re-election probability pat the start of the second period is sufficiently high (and, in particular, if $p = 1$), an increase in income uncertainty (a rise in $\overline{\varepsilon}$), leads to less compensation η, a lower deficit after reform, and, if $(1 - \theta)\, y^m \Gamma > (I - \eta)\, \gamma$, a lower probability that the initial incumbent is re-elected at the start of the first period.*

Proof. Differentiate the first-order conditions with respect to $\overline{\varepsilon}$. See Appendices 3.D.3 and 3.D.4 for the details. ■

The intuition for these, perhaps rather surprising, results is the following. An increase in $\overline{\varepsilon}$ does not directly affect ε_L, but increases the range of income shocks over which compensation must be paid. As a result, the government cuts back on compensation. This improves the public budget in period 1 for any shock that leads to the re-election of the incumbent and thus to reform. Then, less debt is issued in period 1, and the deficit will be lower. Of course, the reduction in compensation affects the re-election chances of the initial government negatively, and, hence, also reduces the chances that reforms will be implemented. Given that the results in the proposition are only formally confirmed for p sufficiently close to 1, for $p < 1$ we confirm them numerically for the setting and parameter combinations described in Appendix 3.F (for both $k = 0$ and $k = 0.2$).

3.5.3 Reduced income inequality

The degree of income inequality is often thought to affect the political acceptability of structural reforms. This may be relevant for the European Union, especially after the entry of the new members, which are characterized by relatively high income inequality, as measured by the ratio of median and average income (see Boix, 2004).[14] These countries are expected to become part of the EMU area in the foreseeable future and will then be subject to criteria of the SGP.[15]

The effects of a change in income inequality in the model are summarized in the following proposition:

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[13]For two anecdotal examples for Germany that also allude to the political obstacles in the implementation of reforms, see Eichel (2004) and Schröder (2005).

[14]For a discussion of the increase in inequality in Eastern Europe, see Forster et al. (2002).

[15]In fact, the SGP requires the "pre-ins" already to submit Convergence Programs. These programs project the path for the government budget until the medium run.

Proposition 3.3 *Suppose that average income $\bar{y}$is held constant. Then, for p sufficiently large (and, in particular, if $p = 1$), a reduction in income inequality, as measured by an increase in median income, (a) reduces compensation for reform η, (b) reduces the deficit after reform and (c) has an ambiguous effect on the likelihood of re-election of the initial incumbent.*

Proof. Appendices 3.D.5 and 3.D.6 provide the proofs. ∎

The intuition for parts (a) and (b) is as follows. For a richer median voter, reform becomes acceptable at a lower level of compensation. Hence, if reform takes place, the government offers less compensation, which improves the public budget in period 1 and, thus enables it to run a lower deficit. As far as part (c) is concerned, we notice that, on the one hand, an increase in the median voter's income renders reform more attractive to this voter, ceteris paribus. On the other hand, the reduced compensation given to the voters makes reform and thus re-election of the incumbent less attractive. The results in parts (a) and (b) of Proposition 3.3 are confirmed by a numerical evaluation based on the parameter combinations and setting described in Appendix 3.F. Moreover, this numerical evaluation shows that a reduction in income inequality reduces the likelihood that the initial incumbent is re-elected and that reform takes place.

3.6 Welfare effects

In this section we confine ourselves to linear private consumption utility and assume that $(1-\theta)\,\overline{y}\Gamma > I\gamma$, implying that reform is beneficial from a utilitarian social welfare perspective. By focussing on linear private consumption utility, we abstract from distributional issues and the concern is only with the "size of the cake" for private consumption. Distributional issues are beyond the scope of this paper in any case, so that with this specification not too much is lost in terms of the intuitions that we want to highlight. The concavity of public goods consumption *is* important here, though, because restrictions on deficits only make sense when deficits are harmful. That is, when the implied shift in the intertemporal public spending profile is harmful. For this to be the case, public consumption utility should be non-linear.

Taking a utilitarian perspective, hence each individual receives an equal weight, expected social welfare can be written as (see Appendix 3.E):

$$2\,(1-\theta)\,\overline{y} + \left[\frac{\overline{\varepsilon}-\varepsilon_L}{2\overline{\varepsilon}}\right]\left[(\eta - I)\,\gamma + (1-\theta)\,\Gamma\overline{y}\right] + \frac{1}{4\overline{\varepsilon}}\left(\overline{\varepsilon}^2 - \varepsilon_L^2\right)(1-\theta)\,\Gamma\overline{y} +$$
$$\frac{\lambda}{2\overline{\varepsilon}}\int_{-\overline{\varepsilon}}^{\varepsilon_L}\left[v\left(g_1^{NR}\right) + pv\left(g_2^{NR}\right) + (1-p)\,v\left(f_2^{NR}\right)\right] d\varepsilon +$$
$$\frac{\lambda}{2\overline{\varepsilon}}\int_{\varepsilon_L}^{\overline{\varepsilon}}\left[v\left(f_1^{R}\right) + pv\left(f_2^{R}\right) + (1-p)\,v\left(g_2^{R}\right)\right] d\varepsilon. \tag{3.24}$$

The first line of this expression is expected private consumption over the two periods. The second line is society's expected utility from public goods consumption when party G comes to office and thus reforms do not take place. Party G stays in office in the second period with probability p and loses it again to F with probability $1-p$. The corresponding second period utilities from public consumption are given by the second and third term, respectively, in this line. Similarly, the third line of (3.24) is society's expected utility from public goods consumption when party F retains office after the first election.

We explore the consequences of tighter enforcement of the deficit restriction (a higher k) on social welfare. The social welfare effect can be separated into an effect on private consumption utility and an effect on public consumption utility. Crucial for the welfare effect is the effect of the aforementioned changes on the probability of re-election of the initial incumbent and thus the probability that reform takes place. This probability of re-election (reform) is increasing in the amount of compensation. Any exogenous change leading to an increase in compensation has a positive effect on the utility from private consumption for two reasons. First, an increase in compensation raises consumption when reform does actually take place. Second, average consumption is higher under reform than under no reform, while the increase in compensation raises the chance that reform indeed takes place (notice that the increase in η reduces ε_L, implying that both the second and the third term in the first line increase).[16]

A priori, the implications of an increase in compensation (and thus of the likelihood of reform) on the expected utility from public spending are ambiguous. On the one hand, more compensation for private agents means that there are less resources for public consumption when reform actually takes. On the other hand, if a reform leads to an increase in resources for public consumption,[17] then the higher likelihood of reform raises the expected utility from public consumption.

As we saw earlier from Proposition 3.1, a tighter deficit restriction implies a reduction in the likelihood of reform when p is sufficiently large, while this is confirmed numerically for a wide range of parameter combinations when $p < 1$. Hence, through the effect on the reform likelihood the increase in k impacts negatively on private consumption utility and in an ambiguous manner on the utility from public consumption. However, as is clear from (3.24), the change in k affects these welfare components not only through the probability of reform. Most importantly, a tighter deficit restriction can have a separate, beneficial effect on public consumption utility by reducing the deficit bias and ensuring a public consumption profile that is better balanced over time.

[16] Indeed, more formally, taking the derivative of the first line of (3.24) with respect to η yields:

$$-\frac{1}{2\overline{\varepsilon}}\left[(\eta - I)\,\gamma + (1-\theta)\,\Gamma(\gamma)\,\overline{y}\right]\frac{d\varepsilon_L}{d\eta} + \frac{\overline{\varepsilon}-\varepsilon_L}{2\overline{\varepsilon}}\gamma - \frac{1}{2\overline{\varepsilon}}(1-\theta)\,\Gamma(\gamma)\,\overline{y}\frac{d\varepsilon_L}{d\eta}.$$

Given that $d\varepsilon_L/d\eta < 0$ and that $(\eta - I)\,\gamma + (1-\theta)\,\Gamma(\gamma)\,\overline{y} > 0$, by assumption, for linear private consumption utility, this expression is positive.

[17] The present discounted values of public resources under no reform and reform are $2\theta\overline{y}\,(1+\varepsilon)$ and $2\theta\overline{y}\,(1+\varepsilon) + \theta\overline{y}\,(1+\varepsilon)\,\Gamma - \eta\gamma$, respectively. Hence, public resources benefit from reform if $\theta\overline{y}\,(1+\varepsilon)\,\Gamma > \eta\gamma$.

To assess the overall effect of changes in k on the social welfare components, we resort to a numerical analysis based on the following functional specifications for public consumption in both periods, respectively the additional income effect Γ in the second period:

$$v(x) = \omega\left[-(\xi-1)x^2/2+\xi x\right], \quad \xi>1 \text{ and } x<\xi/(\xi-1), \tag{3.25}$$
$$\Gamma(\gamma) = \gamma^q, \quad 0\leqslant q\leqslant 1. \tag{3.26}$$

The restriction $x<\xi/(\xi-1)$ in (3.25) ensures that the marginal utilities of public consumption are always positive. Further, the parameter ω regulates the desirability of public spending relative to private consumption.[18] The baseline parameter combination is $\xi=1.03$, $\overline{y}=10$, $p=\theta=\gamma=0.5$, $I=2$, $q=0.25$, $\overline{\varepsilon}=1.2$, $y^m=0.8y$ and $\overline{b}=0.3$. It is selected to be roughly in line with the estimated benefits of structural reform in Europe (see IMF, 2003) and evidence on the magnitude of business cycle fluctuations for Europe (Artis et al., 2003, and Mitchell and Mouratidis, 2004). More detail is provided in Appendix 3.F.

Table 3.1 shows the effects of an increase in the tightness of the deficit restriction, both for the baseline parameter combination and for settings in which we vary each time one parameter away from its baseline value. We consider large perturbations of parameter values in order to cover a wide range of the parameter space.[19] As argued above, an increase in k in all instances leads to a reduction in the probability of re-election of the initial incumbent (and thus of reform) and a reduction in the utility from private consumption. We also see that, with one exception, in all instances, an increase in k has a positive effect on the utility from public consumption, indicating that the better intertemporal smoothing of public consumption (and the reduction in compensation) outweighs the potential reduction in available resources for public consumption associated with less frequent reform. We do not sum the effects on private and public consumption utility into an overall effect of a change in k on expected social welfare, because the relative weights of the two welfare components depend on λ and ω. Unless one could make specific assumptions about the values of these parameters, the resulting assessment would not be very meaningful.

Table 3.1 also shows whether the deficit in each case exceeds the corresponding deficit (3.13) under the social planner, given the outcome for compensation. We see that this is always the case, except when $p=0.8$ and $k=0.2$. With the re-election probability so high, the incentive for shifting spending from period 2 towards period 1 is already relatively weak. Adding to this the pressure of a sufficiently high value of k, the bias towards a too high deficit may actually turn into an underbias for the deficit.[20] An increase in k worsens

[18] For given values of the other parameters, ω can always be chosen such that the condition in Footnote 10 holds.

[19] As a robustness check, we repeated this numerical assessment for all possible *combinations* of parameter deviations and found that the effects of an increase in k generally correspond to those we report in Table 3.1.

[20] Although not reported in Table 3.1, the transition from $k=0$ to $k=0.2$ in all instances leads to a

the underbias and further distorts the time profile of public spending, thereby leading to a reduction in the utility from public consumption, as reported in Table 3.1.

3.7 Conclusion

This paper has explored the incentives for a government to implement structural reform in the presence of electoral uncertainty and a deficit restriction that reduces the scope for providing short-run compensation to the losers from the reform. Such compensation impacts on the government's budget and makes it more difficult to obey the deficit restriction. In solving for the optimal level of compensation and thus the likelihood of reform actually taking place, a trade-off arises between the marginal benefits of compensation in terms of better re-election chances for a government willing to execute reforms and the marginal costs associated with an increase in the deficit. While the deficit restriction is effective in restraining the actual public deficit, it reduces the likelihood of structural reform by forcing a reduction in compensation spending for the losers from the reform. As a result, social welfare may be negatively affected because the future reform benefits are more likely to be foregone. We also find that more individual income uncertainty reduces the likelihood of reform, because the range of shocks for which compensation payments need to be made becomes wider, forcing the initial incumbent to promise lower compensation for when it is actually re-elected.

This result suggests that the political feasibility of reforms is enhanced by making the deficit restriction contingent on the business cycle. In an extension which, for the sake of space, we did not analyze in the main text, we also explored an arrangement in which the reference deficit level was made contingent on both compensation spending and the business cycle shock.[21] We found that both types of contingency make reform more likely. Of course, such an arrangement would be more demanding in terms of the budgetary and macroeconomic information needed to take informed decisions for such a more flexible implementation of the deficit restriction. In addition, and in relation to this, mechanisms would need to be devised to ensure that governments do not abuse the flexibility of the fiscal restriction as an escape route for their lack of discipline (for example, by classifying government consumption as reform expenditure).

Appendix 3.G analyzes two additional extensions. In one of them, we make reform compensation income dependent and assume that it is targeted at those voters that as a result of the compensation are likely to switch from being opponents to being supporters of the reform. For given overall compensation costs, this would enable the incumbent to pursuade more voters to support its reforms. Our findings remain qualitatively unchanged. The other extension assumes that the deficit restriction had a direct budgetary effect rather than an effect on the government's utility. Also in this case, our main findings remain

reduction in the difference between the partisan deficit and the social planner's deficit, given the level of compensation.

[21] This, and other, extensions are analyzed in Appendix 3.G.

unchanged.

Our analysis is relevant for the current debate about the relation between structural reforms and Europe's SGP. While there is a consensus about the need to meet the objectives set by the Lisbon Agenda, most European countries fail in implementing the necessary structural reforms. Part of the blame is often put on the unpopularity of these structural reforms and the difficulty of carrying them out when at the same time countries are bound by the fiscal restrictions embedded in the SGP. After its reform in 2005, the preventive arm of the SGP now allows for deviations from the normal adjustment path to the medium term deficit objective (MTO). In the corrective arm (which specifies possible sanctions), structural reforms are explicitly mentioned as a relevant factor permitting a freeze or at least a slowdown of the Excessive Deficit Procedure for countries violating the deficit criterion.

Of course, one needs to be careful in translating the preceding analysis into the context of the SGP. The SGP is implemented in a setting in which countries have given up their monetary autonomy, but have retained their fiscal autonomy. Many experts view the SGP as an alternative for the missing formal fiscal coordination mechanism. The benefits and costs of relaxing Europe's SGP are more far-reaching than for the deficit restriction in our model.[22] While the latter was intended to reduce the costs of an intertemporal misallocation of public spending in a national context, the SGP also aims at limiting the consequences of negative fiscal spill-overs between the members of Europe's monetary union. The benefit of a such fiscal restriction is likely to be larger than in a national context. However, enforcement is also more complicated and typically involves peer pressure from other countries. Making such an international arrangement more flexible in the way suggested above is more complicated than in a purely national context, because the information requirements on the countries' economic situations become more stringent and countries have to trust each other not to abuse the more flexible arrangement. The need for structural reforms adds another dimension of complexity, because of the international consequences of those reforms. Europe's supranational institutions, such as the ECB and the European Commission repeatedly emphasize the need for reform. By reducing the incentives for fiscal profligacy, structural reform limits the potential negative fiscal policy spill-overs between countries and benefits the conduct of a credible monetary policy. In addition, the enhanced flexibility of the labor and product markets also reduces the pressure for protectionist stances of European governments, thereby contributing to further economic integration. Obviously, a fully-fledged analysis of the interaction between fiscal restrictions and structural reforms in a monetary union should take these external spill-over effects into account.

The current analysis offers various other possibilities for further research. First, we have assumed that the tax rate was exogenous and constant. While there is some merit

[22]See Chari and Kehoe (1997) and Beetsma and Uhlig (1999) for an analysis of fiscal restrictions in a monetary union.

in this assumption as the level of taxes captures the population's preference about the size of the public sector (which is beyond the scope of this analysis), obviously in reality governments possess some freedom to change the level taxes. Introducing this possibility into our current model would reduce the degree to which the deficit restriction is binding, because the first-period government could finance its overspending on public goods by raising taxes rather than issuing debt. However, a fully-fledged analysis along this line should also take account of the fact that varying taxes over time raises the income losses from distortions (Barro, 1979) and that, ceteris paribus, major tax hikes have adverse implications for the government's re-election chances. Indeed, in reality, we often see that governments finance additional expenditure with higher deficits. We conjecture that an analysis that takes proper account of these limitations to changing taxes, would in qualitative terms reproduce our main results. Second, in this paper, we have neglected issues pertaining to the credibility of the enforcement of the deficit restriction. The recent events with the SGP make clear that this would be an important matter to address. Also, it would be interesting to extend the model to a longer horizon, so that one can address the issue of the optimal timing of the implementation of the structural reform and explore how the dynamics of the debt are then affected by a deficit restriction. Such an extension could further take account of the fact that a very strict implementation of the restriction now might undermine its future credibility, if such a strict implementation discourages reforms and thus lowers future tax revenues in this way.

3.8 Table

Table 3.1: Marginal effects of an increase in the tightness of the deficit restriction

	$k = 0$				$k = 0.2$			
case[1]	$\frac{d\Pr(R)}{dk}$	$\frac{dVc}{dk}$	$\frac{dVp}{dk}$	Δb^R	$\frac{d\Pr(R)}{dk}$	$\frac{dVc}{dk}$	$\frac{dVp}{dk}$	Δb^R
Baseline	-	-	+	>0	-	-	+	>0
$p = 0.35$	-	-	+	>0	-	-	+	>0
$p = 0.80$	-	-	+	>0	-	-	-	<0
$\theta = 0.25$	-	-	+	>0	-	-	+	>0
$\theta = 0.7$	-	-	+	>0	-	-	+	>0
$\gamma = 0.1$	-	-	+	>0	-	-	+	>0
γ=2	-	-	+	>0	-	-	+	>0
$I = 0.5$	-	-	+	>0	-	-	+	>0
$I = 4$	-	-	+	>0	-	-	+	>0
$q = 0.1$	-	-	+	>0	-	-	+	>0
$q = 0.5$	-	-	+	>0	-	-	+	>0
$y^m = 0.5y$	-	-	+	>0	-	-	+	>0
$y^m = 0.95y$	-	-	+	>0	-	-	+	>0
$\overline{\varepsilon} = 0.8$	-	-	+	>0	-	-	+	>0
$\overline{\varepsilon} = 1.6$	-	-	+	>0	-	-	+	>0
$b = 0.1$	-	-	+	>0	-	-	+	>0
$b = 1$	-	-	+	>0	-	-	+	>0

Notes: [1] We always vary one parameter (indicated in the first column), while keeping the others at their baseline values. $\frac{d\Pr(R)}{dk}$ = marginal effect of k on the probability of reform. $\frac{dVc}{dk}$ = marginal effect of k on private consumption utility. $\frac{dVp}{dk}$ = marginal effect of k on public consumption utility. Δb^R = difference under reform between deficit under partisan government and under social planner.

Appendices to Chapter 3

3.A Optimal deficit under re-election of initial incumbent in period 1

After the first election, if the initial incumbent remains in power, it chooses the public debt b^R in order to maximize:

$$\max_{b^R} v\left(f_1^R\right) + pv\left(f_2^R\right) + (1-p)\, v(0) - k\left(b^R - \overline{b}\right).$$

The first-order condition is written as follows:

$$v'\left(\theta\overline{y}(1+\varepsilon) - \eta\gamma + b^R\right) - k = pv'\left(\theta\overline{y}(1+\varepsilon)\left(1+\Gamma(\gamma)\right) - b^R\right). \tag{3.27}$$

By differentiating (3.27), we obtain:

$$\begin{aligned}
\frac{\partial b^R}{\partial k} &= \frac{1}{v"\left(f_1^R\right) + pv"\left(f_2^R\right)} < 0, && (3.28)\\
\frac{\partial b^R}{\partial \eta} &= \frac{\gamma v"\left(f_1^R\right)}{v"\left(f_1^R\right) + pv"\left(f_2^R\right)} > 0, && (3.29)\\
\frac{\partial b^R}{\partial y^m}|_{y=\overline{y}} &= 0, && (3.30)\\
\frac{\partial b^R}{\partial \gamma} &= \frac{pv"\left(f_2^R\right)\left(\theta\overline{y}(1+\varepsilon)\Gamma'(\gamma)\right) + \eta v"\left(f_1^R\right)}{v"\left(f_1^R\right) + pv"\left(f_2^R\right)} > 0,\\
\frac{\partial b^R}{\partial p} &= \frac{v'\left(f_2^R\right)}{v"\left(f_1^R\right) + pv"\left(f_2^R\right)} < 0.
\end{aligned}$$

3.B Optimal deficit under no re-election in period 1

In case of no re-election, the new party on power G chooses public debt b^{NR} in order to maximize:

$$\max_{b^{NR}} v\left(g_1^{NR}\right) + pv\left(g_2^{NR}\right) + (1-p)\, v(0) - k\left(b^{NR} - \overline{b}\right).$$

The first-order condition is written as follows:

$$v'\left(\theta\overline{y}(1+\varepsilon) + b^{NR}\right) - k = pv'\left(\theta\overline{y}(1+\varepsilon) - b^{NR}\right). \tag{3.31}$$

By differentiating (3.31), we obtain:

$$\begin{aligned}
\frac{\partial b^{NR}}{\partial k} &= \frac{1}{v"\left(g_1^{NR}\right)+pv"\left(g_2^{NR}\right)}<0, &(3.32)\\
\frac{\partial b^{NR}}{\partial \eta} &= 0, &(3.33)\\
\frac{\partial b^{NR}}{\partial y^m}|_{y=\overline{y}} &= 0, &(3.34)\\
\frac{\partial b^{NR}}{\partial \gamma} &= 0,\\
\frac{\partial b^{NR}}{\partial p} &= \frac{v'\left(g_2^{NR}\right)}{v"\left(g_1^{NR}\right)+pv"\left(g_2^{NR}\right)}<0.
\end{aligned}$$

3.C Choice of η

Before one starts to solve the maximization problem given in (3.18), it is useful to compute the conditional probability density functions for ε when $\varepsilon \geqslant \varepsilon_L$ or $\varepsilon < \varepsilon_L$. Since the shock ε is uniformly distributed, these functions are given by:

$$h_R(\varepsilon) = \begin{cases} \frac{1}{\overline{\varepsilon}-\varepsilon_L} & if \quad \varepsilon_L \leqslant \varepsilon \leqslant \overline{\varepsilon} \\ 0 & if \quad -\overline{\varepsilon} < \varepsilon \leqslant \varepsilon_L \end{cases},$$

and

$$h_{NR}(\varepsilon) = \begin{cases} 0 & if \quad \varepsilon_L < \varepsilon \leqslant \overline{\varepsilon} \\ \frac{1}{\varepsilon_L-(-\overline{\varepsilon})} & if \quad -\overline{\varepsilon} \leqslant \varepsilon \leqslant \varepsilon_L \end{cases}.$$

Using (3.16), the two expressions above can be rewritten respectively as:

$$h_R(\varepsilon) = \frac{1}{\overline{\varepsilon}-\varepsilon_L} = \frac{1}{\overline{\varepsilon}-\left(-\frac{(\eta-I)\gamma}{(1-\theta)y^m\Gamma}-1\right)} = \frac{1}{1+\overline{\varepsilon}+\frac{(\eta-I)\gamma}{(1-\theta)y^m\Gamma}}, \tag{3.35}$$

and

$$h_{NR}(\varepsilon) = \frac{1}{\varepsilon_L-(-\overline{\varepsilon})} = \frac{1}{\left(-\frac{(\eta-I)\gamma}{(1-\theta)y^m\Gamma}-1\right)+\overline{\varepsilon}} = \frac{1}{\overline{\varepsilon}-1-\frac{(\eta-I)\gamma}{(1-\theta)y^m\Gamma}}. \tag{3.36}$$

Using this, (3.18) can be rewritten as:

$$\max_{\eta}\left\{\begin{array}{c} \Pr(R)\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\left\{v\left(f_1^R\right)+pv\left(f_2^R\right)-k\left(b^R-\overline{b}\right)\right\}h_R(\varepsilon)\,d\varepsilon+ \\ (1-\Pr(R))\int\limits_{-\overline{\varepsilon}}^{\varepsilon_L}\left\{v(0)+(1-p)\,v\left(f_2^{NR}\right)\right\}h_{NR}(\varepsilon)\,d\varepsilon \end{array}\right\}, \tag{3.37}$$

where ε_L, $h_R(\varepsilon)$, and $h_{NR}(\varepsilon)$ are given by (3.16), (3.35) and (3.36) respectively. Further, in (3.35) and (3.36) ε does not appear. So, we can take these two terms out of the integrand and compute $\Pr(Re) * h_R(\varepsilon)$ and $(1-\Pr(Re)) * h_{NR}(\varepsilon)$. Using (3.17), we have:

$$\Pr(R) h_R(\varepsilon) = \frac{1+\overline{\varepsilon}+\frac{(\eta-I)\gamma}{(1-\theta)y^m\Gamma}}{2\overline{\varepsilon}} * \frac{1}{1+\overline{\varepsilon}+\frac{(\eta-I)\gamma}{(1-\theta)y^m\Gamma}} = \frac{1}{2\overline{\varepsilon}}$$

and

$$(1-\Pr(R))\, h_{NR}(\varepsilon) = \left[1+\frac{1}{2\overline{\varepsilon}}\left(-\overline{\varepsilon}-1-\frac{(\eta-I)\gamma}{(1-\theta)y^m\Gamma}\right)\right]\left[\frac{1}{\overline{\varepsilon}-1-\frac{(\eta-I)\gamma}{(1-\theta)y^m\Gamma}}\right] \Leftrightarrow$$

$$(1-\Pr(R))\, h_{NR}(\varepsilon) = \left[1+\frac{1}{2\overline{\varepsilon}}\left(\frac{(1-\theta)y^m\Gamma(-\overline{\varepsilon}-1)-(\eta-I)\gamma}{(1-\theta)y^m\Gamma}\right)\right]\left[\frac{1}{\overline{\varepsilon}-1-\frac{(\eta-I)\gamma}{(1-\theta)y^m\Gamma}}\right] \Leftrightarrow$$

$$(1-\Pr(R))\, h_{NR}(\varepsilon) = \left[\frac{2\overline{\varepsilon}(1-\theta)y^m\Gamma + (1-\theta)y^m\Gamma(-\overline{\varepsilon}-1)-(\eta-I)\gamma}{2\overline{\varepsilon}(1-\theta)y^m\Gamma}\right]\left[\frac{1}{\frac{(1-\theta)y^m\Gamma(\overline{\varepsilon}-1)-(\eta-I)\gamma}{(1-\theta)y^m\Gamma}}\right] \Leftrightarrow$$

$$(1-\Pr(R))\, h_{NR}(\varepsilon) = \left[\frac{(1-\theta)y^m\Gamma(2\overline{\varepsilon}-\overline{\varepsilon}-1)-(\eta-I)\gamma}{2\overline{\varepsilon}(1-\theta)y^m\Gamma}\right]\left[\frac{1}{\frac{(1-\theta)y^m\Gamma(\overline{\varepsilon}-1)-(\eta-I)\gamma}{(1-\theta)y^m\Gamma}}\right] \Leftrightarrow$$

$$(1-\Pr(R))\, h_{NR}(\varepsilon) = \left[\frac{(1-\theta)y^m\Gamma(\overline{\varepsilon}-1)-(\eta-I)\gamma}{2\overline{\varepsilon}(1-\theta)y^m\Gamma}\right]\left[\frac{1}{\frac{(1-\theta)y^m\Gamma(\overline{\varepsilon}-1)-(\eta-I)\gamma}{(1-\theta)y^m\Gamma}}\right] = \frac{1}{2\overline{\varepsilon}}$$

$$\Longrightarrow$$

$$\Pr(R) h_R(\varepsilon) = (1-\Pr(R))\, h_{NR}(\varepsilon) = \frac{1}{2\overline{\varepsilon}}. \tag{3.38}$$

With (3.38), we can rewrite (3.37):

$$\max_{\eta}\left\{\begin{array}{c} \frac{1}{2\overline{\varepsilon}}\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\left\{v\left(\theta\overline{y}(1+\varepsilon)-\eta\gamma+b^R\right)+pv\left(\theta\overline{y}(1+\varepsilon)(1+\Gamma)-b^R\right)-k\left(b^R-\overline{b}\right)\right\}d\varepsilon \\ +\frac{1}{2\overline{\varepsilon}}\int\limits_{-\overline{\varepsilon}}^{\varepsilon_L}\left\{v(0)+(1-p)\,v\left(\theta\overline{y}(1+\varepsilon)-b^{NR}\right)\right\}d\varepsilon \end{array}\right\}$$

$$\Leftrightarrow$$

$$\max_{\eta}\frac{1}{2\overline{\varepsilon}}\left\{\begin{array}{c} \int\limits_{\varepsilon_L}^{\overline{\varepsilon}} v\left(\theta\overline{y}(1+\varepsilon)-\eta\gamma+b^R\right)d\varepsilon + p.\int\limits_{\varepsilon_L}^{\overline{\varepsilon}} v\left(\theta\overline{y}(1+\varepsilon)(1+\Gamma)-b^R\right)d\varepsilon \\ +\int\limits_{\varepsilon_L}^{\overline{\varepsilon}} -k\left(b^R-\overline{b}\right)d\varepsilon + \int\limits_{-\overline{\varepsilon}}^{\varepsilon_L} 0 d\varepsilon + (1-p)\,.\int\limits_{-\overline{\varepsilon}}^{\varepsilon_L} v\left(\theta\overline{y}(1+\varepsilon)-b^{NR}\right)d\varepsilon \end{array}\right\}.$$

Applying Leibnitz's rule[23] and maximizing this last expression on η:

$$\frac{1}{2\overline{\varepsilon}}\left\{\begin{array}{c} v\left(\theta\overline{y}(1+\overline{\varepsilon})-\eta\gamma+b^{R}\left(\overline{\varepsilon}\right)\right)\frac{\partial\overline{\varepsilon}}{\partial\eta}-v\left(\theta\overline{y}(1+\varepsilon_{L})-\eta\gamma+b^{R}\left(\varepsilon_{L}\right)\right)\frac{\partial\varepsilon_{L}}{\partial\eta}\\ +\int\limits_{\varepsilon_{L}}^{\overline{\varepsilon}}\frac{\partial v\left(\theta\overline{y}(1+\varepsilon)-\eta\gamma+b^{R}\right)}{\partial\eta}d\varepsilon+\\ p\left\{\begin{array}{c} v\left(\theta\overline{y}(1+\overline{\varepsilon})\left(1+\Gamma\right)-b^{R}\left(\overline{\varepsilon}\right)\right)\frac{\partial\overline{\varepsilon}}{\partial\eta}-v\left(\theta\overline{y}(1+\varepsilon_{L})\left(1+\Gamma\right)-b^{R}\left(\varepsilon_{L}\right)\right)\frac{\partial\varepsilon_{L}}{\partial\eta}\\ +\int\limits_{\varepsilon_{L}}^{\overline{\varepsilon}}\frac{\partial v\left(\theta\overline{y}(1+\varepsilon)(1+\Gamma)-b^{R}\right)}{\partial\eta}d\varepsilon\end{array}\right\}\\ -k\left(b^{R}\left(\overline{\varepsilon}\right)-\overline{b}\right)\frac{\partial\overline{\varepsilon}}{\partial\eta}+k\left(b^{R}\left(\varepsilon_{L}\right)-\overline{b}\right)\frac{\partial\varepsilon_{L}}{\partial\eta}+\int\limits_{\varepsilon_{L}}^{\overline{\varepsilon}}\frac{-\partial k\left(b^{R}-\overline{b}\right)}{\partial\eta}d\varepsilon+\\ (1-p)\left\{\begin{array}{c} v\left(\theta\overline{y}(1+\varepsilon_{L})-b^{NR}\left(\varepsilon_{L}\right)\right)\frac{\partial\varepsilon_{L}}{\partial\eta}-v\left(\theta\overline{y}(1-\overline{\varepsilon})-b^{NR}\left(-\overline{\varepsilon}\right)\right)\frac{\partial(-\overline{\varepsilon})}{\partial\eta}\\ +\int\limits_{-\overline{\varepsilon}}^{\varepsilon_{L}}\frac{\partial v\left(\theta\overline{y}(1+\varepsilon)-b^{NR}\right)}{\partial\eta}d\varepsilon\end{array}\right\}\end{array}\right\}=0.$$

Since $\frac{\partial\overline{\varepsilon}}{\partial\eta}=0$ and using the notation in (3.20) - (3.22), we have:

$$\left\{\begin{array}{c} -v\left(f_{1L}^{R}\right)\frac{\partial\varepsilon_{L}}{\partial\eta}+\int\limits_{\varepsilon_{L}}^{\overline{\varepsilon}}v'\left(f_{1}^{R}\right)\left(-\gamma+\frac{\partial b^{R}}{\partial\eta}\right)d\varepsilon+\\ p\left\{0-v\left(f_{2L}^{R}\right)\frac{\partial\varepsilon_{L}}{\partial\eta}+\int\limits_{\varepsilon_{L}}^{\overline{\varepsilon}}v'\left(f_{2}^{R}\right)\left(-\frac{\partial b^{R}}{\partial\eta}\right)d\varepsilon\right\}\\ -k\left(b^{R}\left(\overline{\varepsilon}\right)-\overline{b}\right).0-k\left(b_{L}^{R}-\overline{b}\right)\left(-\frac{\partial\varepsilon_{L}}{\partial\eta}\right)+\int\limits_{\varepsilon_{L}}^{\overline{\varepsilon}}-k\left(\frac{\partial b^{R}}{\partial\eta}\right)d\varepsilon\\ +(1-p)\left\{v\left(f_{2L}^{NR}\right)\frac{\partial\varepsilon_{L}}{\partial\eta}-0+\int\limits_{-\overline{\varepsilon}}^{\varepsilon_{L}}v'\left(f_{2}^{NR}\right)\left(-\frac{\partial b^{NR}}{\partial\eta}\right)d\varepsilon\right\}\end{array}\right\} = 0 \Longrightarrow$$

$$\left\{\begin{array}{c} \frac{\partial\varepsilon_{L}}{\partial\eta}\left[-v\left(f_{1L}^{R}\right)-pv\left(f_{2L}^{R}\right)+k\left(b_{L}^{R}-\overline{b}\right)+(1-p)\,v\left(f_{2L}^{NR}\right)\right]\\ -\gamma\int\limits_{\varepsilon_{L}}^{\overline{\varepsilon}}v'\left(f_{1}^{R}\right)d\varepsilon+\int\limits_{\varepsilon_{L}}^{\overline{\varepsilon}}\frac{\partial b^{R}}{\partial\eta}\left[v'\left(f_{1}^{R}\right)-pv'\left(f_{2}^{R}\right)-k\right]d\varepsilon\\ +\int\limits_{-\overline{\varepsilon}}^{\varepsilon_{L}}\frac{\partial b^{NR}}{\partial\eta}\left[-(1-p)\,v'\left(f_{2}^{NR}\right)\right]d\varepsilon\end{array}\right\} = 0.$$

[23]Leibnitz's rule: *Let* $f(x,r)$ *be continuous with respect to* x *for every value of* r*, with a continuous derivative* $\frac{\partial f(x,r)}{\partial r}$ *with respect to* x *and* r *in the rectangle* $a\leq x\leq b$*,* $\underline{r}\leq r\leq\overline{r}$ *of the* $x-r$ *plane. Let the functions* $A(r)$ *and* $B(r)$ *have continuous derivatives. If* $V(r)=\int\limits_{A(r)}^{B(r)}f(x,r)\,dx$*, then*

$$V'(r)=f(B(r),r)\,B'(r)-f(A(r),r)\,A'(r)+\int\limits_{A(r)}^{B(r)}\frac{\partial f(x,r)}{\partial r}dx.$$

For example,

$$V(r) = \int\limits_{r^{2}}^{r}e^{-rs}P(s)\,ds\Longrightarrow$$

$$\frac{dV(r)}{dr} = P(r)\,e^{-r^{2}}-2P\left(r^{2}\right)re^{-r^{3}}-\int\limits_{r^{2}}^{r}se^{-rs}P(s)\,ds.$$

From (3.27) we know that $v'\left(f_1^R\right)-pv'\left(f_2^R\right)-k=0$. Further, from (3.33), $\frac{\partial b^{NR}}{\partial\eta}=0$. So, the last expression is rewritten as (3.19). Next, we can substitute

$$\frac{\partial\varepsilon_L}{\partial\eta}=\frac{\partial\left(-\frac{(\eta-I)\gamma}{(1-\theta)y^m\Gamma}-1\right)}{\partial\eta}=-\frac{\gamma}{(1-\theta)\,y^m\Gamma}<0, \tag{3.39}$$

into (3.19):

$$\frac{\gamma}{(1-\theta)\,y^m\Gamma}\left[v\left(f_{1L}^R\right)+pv\left(f_{2L}^R\right)-k\left(b_L^R-\overline{b}\right)-(1-p)\,v\left(f_{2L}^{NR}\right)\right]=\gamma\int_{\varepsilon_L}^{\overline{\varepsilon}}v'\left(f_1^R\right)d\varepsilon,$$

which results in (3.23).

In (3.23) both sides must have the same sign for an internal solution to exist. Clearly, the right-hand side of that expression is positive. The left-hand side is positive, since we have the deficit bias in the first period $b^R>\overline{b}$. Further, the probability of being in power in the second period p is the same for both parties and assumed to be sufficiently different from 0.

3.C.1 Proof that (3.23) has at most one solution

The derivative of the left-hand side of (3.23) with respect to η is:

$$\left\{\begin{array}{c} v'\left(f_{1L}^R\right)\left(\theta\overline{y}\frac{\partial\varepsilon_L}{\partial\eta}-\gamma+\frac{\partial b_L^R}{\partial\eta}\right)+pv'\left(f_{2L}^R\right)\left(\theta\overline{y}\left(1+\Gamma\right)\frac{\partial\varepsilon_L}{\partial\eta}-\frac{\partial b_L^R}{\partial\eta}\right)\\ -k\left(\frac{\partial b_L^R}{\partial\eta}\right)-(1-p)\,v'\left(f_{2L}^{NR}\right)\left(\theta\overline{y}\frac{\partial\varepsilon_L}{\partial\eta}-\frac{\partial b_L^{NR}}{\partial\eta}\right)\end{array}\right\}$$

$$=\left\{\begin{array}{c} \frac{\partial b_L^R}{\partial\eta}\left[v'\left(f_{1L}^R\right)-pv'\left(f_{2L}^R\right)-k\right]+\frac{\partial b_L^{NR}}{\partial\eta}\left[(1-p)\,v'\left(f_2^{NR}\right)\right]-\gamma v'\left(f_{1L}^R\right)\\ +\theta\overline{y}\frac{\partial\varepsilon_L}{\partial\eta}\left[v'\left(f_{1L}^R\right)+(1+\Gamma)\,pv'\left(f_{2L}^R\right)-(1-p)\,v'\left(f_{2L}^{NR}\right)\right]\end{array}\right\}.$$

We know that $v'\left(f_{1L}^R\right)-pv'\left(f_{2L}^R\right)-k=0$ since (3.27) is valid for any realization of ε in the interval $(\varepsilon_L,\overline{\varepsilon})$. Further, by (3.39) and if we assume that p is sufficiently different from 0, we can rewrite the last expression as:

$$\left\{\begin{array}{c} +(1-p)\,v'\left(f_2^{NR}\right)\frac{\partial b_L^{NR}}{\partial\eta}-\gamma v'\left(f_{1L}^R\right)\\ -\frac{\theta\overline{y}\gamma}{(1-\theta)y^m\Gamma}\left[v'\left(f_{1L}^R\right)+(1+\Gamma)\,pv'\left(f_{2L}^R\right)-(1-p)\,v'\left(f_{2L}^{NR}\right)\right]\end{array}\right\}<0.$$

Hence, an increase in η reduces the left-hand side of (3.23). The derivative of the right side of (3.23) with respect to η is:

$$(1-\theta)\,y^m\Gamma\frac{\partial}{\partial\eta}\left[\int_{\varepsilon_L}^{\overline{\varepsilon}}v'\left(\theta\overline{y}(1+\varepsilon)-\eta\gamma+b^R\right)d\varepsilon\right].$$

Using again Leibnitz's rule, we can write this as:

$$(1-\theta)\,y^m\Gamma\left[\begin{array}{c} v'\left(\theta\overline{y}(1+\overline{\varepsilon})-\eta\gamma+b^R\left(\overline{\varepsilon}\right)\right)\frac{\partial\overline{\varepsilon}}{\partial\eta}-v'\left(\theta\overline{y}(1+\varepsilon_L)-\eta\gamma+b^R\left(\varepsilon_L\right)\right)\frac{\partial\varepsilon_L}{\partial\eta}\\ +\int_{\varepsilon_L}^{\overline{\varepsilon}}v"\left(\theta\overline{y}(1+\varepsilon)-\eta\gamma+b^R\right)\left(-\gamma+\frac{\partial b^R}{\partial\eta}\right)d\varepsilon\end{array}\right]$$

$$\Leftrightarrow\ (1-\theta)\,y^m\Gamma\left[-v'\left(f_{1L}^R\right)\frac{\partial\varepsilon_L}{\partial\eta}+\int_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma+\frac{\partial b^R}{\partial\eta}\right)v"\left(f_1^R\right)d\varepsilon\right].$$

Substituting (3.29) and (3.39) to this last expression, we get:

$$(1-\theta)\, y^m\Gamma \left\{ -v'\left(f_{1L}^R\right)\left[-\frac{\gamma}{(1-\theta)\, y^m\Gamma}\right] + \int_{\varepsilon_L}^{\overline{\varepsilon}} \left[-\gamma + \frac{\gamma . v"\left(f_1^R\right)}{v"\left(f_1^R\right) + pv"\left(f_2^R\right)}\right] v"\left(f_1^R\right) d\varepsilon \right\} \Leftrightarrow$$

$$\gamma\left\{ v\prime\left(f_{1L}^R\right) + (1-\theta)\, y^m\Gamma \int_{\varepsilon_L}^{\overline{\varepsilon}} \left[\frac{-pv"\left(f_2^R\right)}{v"\left(f_1^R\right) + pv"\left(f_2^R\right)}\right] v"\left(f_1^R\right) d\varepsilon \right\} > 0.$$

Hence, the right-hand side of (3.23) is positive. This shows that η has an maximum value.

3.D Comparative statics

3.D.1 Effect of k on η, b^R and b^{NR}

FOC of η

We differentiate (3.23) with respect to k using Leibnitz's rule and the notations (3.20) to (3.22):

$$\begin{aligned}
& v'\left(f_{1L}^R\right)\left[\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial k} + \frac{\partial\varepsilon_L}{\partial k}\right) - \gamma\frac{\partial\eta}{\partial k} + \frac{\partial b_L^R}{\partial k}\right] + \\
& pv'\left(f_{2L}^R\right)\left[\theta\overline{y}\left(1+\Gamma\right)\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial k} + \frac{\partial\varepsilon_L}{\partial k}\right) - \frac{\partial b_L^R}{\partial k}\right] - \left(b_L^R - \overline{b}\right) \\
& -k\left(\frac{\partial b_L^R}{\partial k}\right) - (1-p)\, v'\left(f_{2L}^{NR}\right)\left[\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial k} + \frac{\partial\varepsilon_L}{\partial k}\right) - \frac{\partial b_L^{NR}}{\partial k}\right] - \\
& (1-\theta)\, y^m\Gamma\left[-v'\left(f_{1L}^R\right)\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial k} + \frac{\partial\varepsilon_L}{\partial k}\right) + \int_{\varepsilon_L}^{\overline{\varepsilon}} v"\left(f_1^R\right)\left(-\gamma\frac{\partial\eta}{\partial k} + \frac{\partial b^R}{\partial k}\right) d\varepsilon\right] = 0.
\end{aligned}$$

This expression can be written as:

$$\begin{aligned}
& \frac{\partial\eta}{\partial k}\left\{\theta\overline{y}\frac{\partial\varepsilon_L}{\partial\eta}\left[\begin{array}{c} v'\left(f_{1L}^R\right) + (1+\Gamma)\, pv'\left(f_{2L}^R\right) \\ -(1-p)\, v'\left(f_{2L}^{NR}\right)\end{array}\right] - \gamma v'\left(f_{1L}^R\right)\right\} + \frac{\partial b_L^{NR}}{\partial k} v'\left(f_{2L}^{NR}\right)(1-p) \\
& +\theta\overline{y}\frac{\partial\varepsilon_L}{\partial k}\left[v'\left(f_{1L}^R\right) + (1+\Gamma)\, pv'\left(f_{2L}^R\right) - (1-p)\, v'\left(f_{2L}^{NR}\right)\right] + \frac{\partial b_L^R}{\partial k}\left[v'\left(f_{1L}^R\right) - pv'\left(f_{2L}^R\right) - k\right] \\
& -(1-\theta)\, y^m\Gamma\left[-v'\left(f_{1L}^R\right)\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial k} + \frac{\partial\varepsilon_L}{\partial k}\right) + \int_{\varepsilon_L}^{\overline{\varepsilon}} v"[f_1^R]\left(-\gamma\frac{\partial\eta}{\partial k} + \frac{\partial b^R}{\partial k}\right) d\varepsilon\right] = \left(b_L^R - \overline{b}\right).
\end{aligned}$$

The first and second terms on the previous expression show the effect of k on the variables η and b^{NR} respectively, evaluated at $\varepsilon = \varepsilon_L$. The second line shows the direct effect of k on ε_L and b^R. This line disappears since $\frac{\partial\varepsilon_L}{\partial k} = 0$ and (3.27) is valid for any realization of ε in the interval $(\varepsilon_L, \overline{\varepsilon})$. Therefore, using (3.39), and if we assume that p is sufficiently different from 0, we can simplify this last expression to:

$$\left\{\begin{array}{c} \frac{\partial\eta}{\partial k}\left\{\theta\overline{y}\left(-\frac{\gamma}{(1-\theta)y^m\Gamma}\right)\left[\begin{array}{c} v'\left(f_{1L}^R\right)+(1+\Gamma)\,pv'\left(f_{2L}^R\right) \\ -(1-p)\,v'\left(f_{2L}^{NR}\right)\end{array}\right]-\gamma v'\left(f_{1L}^R\right)\right\} \\ +\frac{\partial b_L^{NR}}{\partial k}\left[v'\left(f_{2L}^{NR}\right)(1-p)\right]-(1-\theta)\,y^m\Gamma\left[\begin{array}{c} -v'\left(f_{1L}^R\right)\left(-\frac{\gamma}{(1-\theta)y^m\Gamma}\right)\frac{\partial\eta}{\partial k} \\ +\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma\frac{\partial\eta}{\partial k}+\frac{\partial b^R}{\partial k}\right)v"\left(f_1^R\right)d\varepsilon\end{array}\right]\end{array}\right\} = \left(b_L^R-\overline{b}\right)$$

$$\Leftrightarrow$$

$$\left\{\begin{array}{c} \frac{\partial\eta}{\partial k}\left\{-\frac{\theta\overline{y}\gamma}{(1-\theta)y^m\Gamma}\left[\begin{array}{c} v'\left(f_{1L}^R\right)+(1+\Gamma)\,pv'\left(f_{2L}^R\right) \\ -(1-p)\,v'\left(f_{2L}^{NR}\right)\end{array}\right]-2\gamma v'\left(f_{1L}^R\right)\right\} \\ -(1-\theta)\,y^m\Gamma\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma\frac{\partial\eta}{\partial k}+\frac{\partial b^R}{\partial k}\right)v"\left(f_1^R\right)d\varepsilon+\frac{\partial b_L^{NR}}{\partial k}\left[v'\left(f_{2L}^{NR}\right)(1-p)\right]\end{array}\right\} = \left(b_L^R-\overline{b}\right),$$

which can be represented by:

$$A_{1L}\frac{\partial\eta}{\partial k}-A_2\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma\frac{\partial\eta}{\partial k}+\frac{\partial b^R}{\partial k}\right)v"\left(f_1^R\right)d\varepsilon+A_{3L}\frac{\partial b_L^{NR}}{\partial k}=D_{1L}, \tag{3.40}$$

where:[24]

$$A_{1L} = -\frac{\theta\overline{y}\gamma}{(1-\theta)\,y^m\Gamma}\left[\begin{array}{c} v'\left(f_{1L}^R\right)+(1+\Gamma)\,pv'\left(f_{2L}^R\right) \\ -(1-p)\,v'\left(f_{2L}^{NR}\right)\end{array}\right]-2\gamma v'\left(f_{1L}^R\right), \tag{3.41}$$

$$A_2 = (1-\theta)\,y^m\Gamma>0, \tag{3.42}$$

$$A_{3L} = v'\left(f_{2L}^{NR}\right)(1-p)>0, \tag{3.43}$$

$$D_{1L} = b_L^R-\overline{b}>0. \tag{3.44}$$

We use a subscript L to indicate that the evaluation takes place at $\varepsilon=\varepsilon_L$. The reason is that the effect of k on the endogenous variable η will affect the probability of re-election via shifts in the inferior support of the distribution, ε_L. The sign of A_{1L} is generally ambiguous. However, for p sufficiently large, this ambiguity vanishes and $A_{1L}<0$.

FOC of b^R

Differentiating (3.27) with respect to k, we have:

$$\begin{aligned} v"\left(f_1^R\right)\left(-\gamma\frac{\partial\eta}{\partial k}+\frac{\partial b^R}{\partial k}\right)-1-pv"\left(f_2^R\right)\left(-\frac{\partial b^R}{\partial k}\right) &= 0\Leftrightarrow \\ \frac{\partial\eta}{\partial k}\left[-\gamma v"\left(f_1^R\right)\right]+\frac{\partial b^R}{\partial k}\left[v"\left(f_1^R\right)+pv"\left(f_2^R\right)\right] &= 1. \end{aligned}$$

This last expression can be represented by:

$$B_1\frac{\partial\eta}{\partial k}+B_2\frac{\partial b^R}{\partial k}=D_2, \tag{3.45}$$

[24] If p is sufficiently different from 0.

where:

$$B_1 = -\gamma v"\left(f_1^R\right) > 0, \quad (3.46)$$
$$B_2 = v"\left(f_1^R\right) + pv"\left(f_2^R\right) < 0, \quad (3.47)$$
$$D_2 = 1 > 0. \quad (3.48)$$

FOC of b^{NR}

When the government chooses the optimal η, it takes into account b^{NR} evaluated at $\varepsilon = \varepsilon_L$. Therefore, (3.31) evaluated at $\varepsilon = \varepsilon_L$ yields:

$$v'\left(\theta\overline{y}(1+\varepsilon_L) + b_L^{NR}\right) - k = pv'\left(\theta\overline{y}(1+\varepsilon_L) - b_L^{NR}\right). \quad (3.49)$$

Differentiating (3.49) with respect to k:

$$\begin{aligned} v"\left(g_{1L}^{NR}\right)\left[\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial k} + \frac{\partial\varepsilon_L}{\partial k}\right) + \frac{\partial b_L^{NR}}{\partial k}\right] - 1 - pv"\left(g_{2L}^{NR}\right)\left[\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial k} + \frac{\partial\varepsilon_L}{\partial k}\right) - \frac{\partial b_L^{NR}}{\partial k}\right] &= 0 \\ &\Leftrightarrow \\ \frac{\partial\eta}{\partial k}\left\{\theta\overline{y}\frac{\partial\varepsilon_L}{\partial\eta}\left[v"\left(g_{1L}^{NR}\right) - pv"\left(g_{2L}^{NR}\right)\right]\right\} + \frac{\partial b_L^{NR}}{\partial k}\left[v"\left(g_{1L}^{NR}\right) + pv"\left(g_{2L}^{NR}\right)\right] &= 1, \end{aligned}$$

since $\frac{\partial\varepsilon_L}{\partial k} = 0$ and where:

$$g_{1L}^{NR} = \theta\overline{y}(1+\varepsilon_L) + b^{NR}(\varepsilon_L), \quad (3.50)$$
$$g_{2L}^{NR} = \theta\overline{y}(1+\varepsilon_L) - b^{NR}(\varepsilon_L). \quad (3.51)$$

Using (3.39), we can rewrite the last expression as:

$$\frac{\partial\eta}{\partial k}\left[-\frac{\theta\overline{y}\gamma}{(1-\theta)\,y^m\Gamma[\gamma]}\left(v"\left(g_{1L}^{NR}\right) - pv"\left(g_{2L}^{NR}\right)\right)\right] + \frac{\partial b_L^{NR}}{\partial k}\left[v"\left(g_{1L}^{NR}\right) + pv"\left(g_{2L}^{NR}\right)\right] = 1,$$

which can be represented as:

$$C_{1L}\frac{\partial\eta}{\partial k} + C_{2L}\frac{\partial b_L^{NR}}{\partial k} = D_3, \quad (3.52)$$

where

$$C_{1L} = -\frac{\theta\overline{y}\gamma}{(1-\theta)\,y^m\Gamma[\gamma]}\left(v"\left(g_{1L}^{NR}\right) - pv"\left(g_{2L}^{NR}\right)\right) \gtrless 0, \quad (3.53)$$
$$C_{2L} = v"\left(g_{1L}^{NR}\right) + pv"\left(g_{2L}^{NR}\right) < 0, \text{ and} \quad (3.54)$$
$$D_3 = 1 > 0. \quad (3.55)$$

The sign of (3.53) is ambiguous, but if we assume a quadratic utility function for the public good, for example, it is positive.

Solution of the system

We can write (3.40), (3.45) and (3.52) as the system:

$$\left\{\begin{array}{lllll} A_{1L}\frac{\partial \eta}{\partial k} & +0 & -A_2\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma\frac{\partial \eta}{\partial k}+\frac{\partial b^R}{\partial k}\right)v"\left(f_1^R\right)d\varepsilon & +A_{3L}\frac{\partial b_L^{NR}}{\partial k} & = D_{1L} \\ B_1\frac{\partial \eta}{\partial k} & +B_2\frac{\partial b^R}{\partial k} & +0 & +0 & = D_2 \\ C_{1L}\frac{\partial \eta}{\partial k} & +0 & +0 & +C_{2L}\frac{\partial b_L^{NR}}{\partial k} & = D_3 \end{array}\right. .$$

Further, we can rewrite (3.45) and (3.52) as, respectively

$$\frac{\partial b^R}{\partial k} = \frac{D_2}{B_2}-\frac{B_1}{B_2}\frac{\partial \eta}{\partial k}, \tag{3.56}$$

$$\frac{\partial b_L^{NR}}{\partial k} = \frac{D_3}{C_{2L}}-\frac{C_{1L}}{C_{2L}}\frac{\partial \eta}{\partial k}. \tag{3.57}$$

Substituting these two equations in (3.40), we have:

$$A_{1L}\frac{\partial \eta}{\partial k}-A_2\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma\frac{\partial \eta}{\partial k}+\frac{D_2}{B_2}-\frac{B_1}{B_2}\frac{\partial \eta}{\partial k}\right)v"\left(f_1^R\right)d\varepsilon-\frac{A_{3L}C_{1L}}{C_{2L}}\frac{\partial \eta}{\partial k}=D_{1L}-\frac{A_{3L}D_3}{C_{2L}}\Leftrightarrow$$

$$A_{1L}\frac{\partial \eta}{\partial k}-A_2\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\frac{\partial \eta}{\partial k}\left(-\gamma-\frac{B_1}{B_2}\right)v"\left(f_1^R\right)d\varepsilon-\frac{A_{3L}C_{1L}}{C_{2L}}\frac{\partial \eta}{\partial k}=D_{1L}+A_2\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\frac{D_2}{B_2}v"\left(f_1^R\right)d\varepsilon-\frac{A_{3L}D_3}{C_{2L}}. \tag{3.58}$$

Sign of $\frac{\partial \eta}{\partial k}$: Substituting the values of (3.41) to (3.44), (3.46)-(3.48) and (3.53)-(3.55), we can rewrite (3.58) as:

$$\frac{\partial \eta}{\partial k}\left\{\begin{array}{c} A_{1L}-\frac{A_{3L}C_{1L}}{C_{2L}}- \\ A_2\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\left[-\gamma v"\left(f_1^R\right)+\frac{\gamma\left(v"\left(f_1^R\right)\right)^2}{v"\left(f_1^R\right)+pv"\left(f_2^R\right)}\right]d\varepsilon \end{array}\right\} = \left\{\begin{array}{c} D_{1L}+A_2\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\frac{D_2}{B_2}v"\left(f_1^R\right)d\varepsilon \\ -\frac{A_{3L}D_3}{C_{2L}} \end{array}\right\}\Leftrightarrow$$

$$\frac{\partial \eta}{\partial k}\left\{\begin{array}{c} A_{1L}-\frac{A_{3L}C_{1L}}{C_{2L}}- \\ A_2\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\left[\frac{-\gamma\left(v"\left(f_1^R\right)\right)^2-\gamma v"\left(f_1^R\right)pv"\left(f_2^R\right)+\gamma\left(v"\left(f_1^R\right)\right)^2}{v"\left(f_1^R\right)+pv"\left(f_2^R\right)}\right]d\varepsilon \end{array}\right\} = \left\{\begin{array}{c} D_{1L}+A_2\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\frac{D_2}{B_2}v"\left(f_1^R\right)d\varepsilon \\ -\frac{A_{3L}D_3}{C_{2L}} \end{array}\right\}\Leftrightarrow$$

$$\frac{\partial \eta}{\partial k}\left\{\begin{array}{c} -\frac{\theta\overline{y}\gamma}{(1-\theta)y^m\Gamma}\left[\begin{array}{c} v'\left(f_{1L}^R\right)+(1+\Gamma)pv'\left(f_{2L}^R\right) \\ -(1-p)v'\left(f_{2L}^{NR}\right) \end{array}\right] \\ -2\gamma v'\left(f_{1L}^R\right)+\frac{(1-p)\theta\overline{y}\gamma v'\left(f_{2L}^{NR}\right)\left[v"\left(g_{1L}^{NR}\right)-pv"\left(g_{2L}^{NR}\right)\right]}{(1-\theta)y^m\Gamma\left[v"\left(g_{1L}^{NR}\right)+pv"\left(g_{2L}^{NR}\right)\right]} \\ +(1-\theta)y^m\Gamma\gamma\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\frac{v"\left(f_1^R\right)pv"\left(f_2^R\right)}{v"\left(f_1^R\right)+pv"\left(f_2^R\right)}d\varepsilon \end{array}\right\} = \left\{\begin{array}{c} \left(b_L^R-\overline{b}\right)- \\ \frac{v'\left(f_{2L}^{NR}\right)(1-p)}{v"\left(g_{1L}^{NR}\right)+pv"\left(g_{2L}^{NR}\right)}+ \\ (1-\theta)y^m\Gamma\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\frac{v"\left(f_1^R\right)}{v"\left(f_1^R\right)+pv"\left(f_2^R\right)}d\varepsilon \end{array}\right\}. \tag{3.59}$$

Note that the overall sign of the term in brackets on the left-hand side of (3.59) is negative if p is sufficiently different from 0. Then the terms $\frac{\theta\overline{y}\gamma}{(1-\theta)y^m\Gamma}(1-p)v'\left(f_{2L}^R\right)$ and $\frac{(1-p)\theta\overline{y}\gamma v'\left(f_{2L}^{NR}\right)\left[v"\left(g_{1L}^{NR}\right)-pv"\left(g_{2L}^{NR}\right)\right]}{(1-\theta)y^m\Gamma\left[v"\left(g_{1L}^{NR}\right)+pv"\left(g_{2L}^{NR}\right)\right]}$ are close enough to zero, so that they do not dominate the other terms. Further, the right-hand side of (3.59) is positive. Hence, for p sufficiently different from 0, $\frac{\partial \eta}{\partial k}<0$.

Sign of $\frac{\partial b^R}{\partial k}$: Given (3.56), the signs of (3.46)-(3.48) and of $\frac{\partial \eta}{\partial k}$, we find:

$$\frac{\partial b^R}{\partial k} < 0. \tag{3.60}$$

Sign of $\frac{\partial b^{NR}}{\partial k}$: Finally, given (3.57), the signs of (3.53)-(3.55) and the sign of $\frac{\partial \eta}{\partial k}$, then:

$$\frac{\partial b_L^{NR}}{\partial k} \lesseqgtr 0. \tag{3.61}$$

However if one assumes that $v"\left(g_{1L}^{NR}\right) - pv"\left(g_{2L}^{NR}\right) < 0$ [25] then the sign of (3.61) is negative.

3.D.2 Effect of k on the probability of re-election

From (3.16) and (3.17), we know that:

$$\frac{d\Pr(R)}{d\eta} = -\left(\frac{1}{2\overline{\varepsilon}}\right)\frac{\partial \varepsilon_L}{\partial \eta}. \tag{3.62}$$

Using this last expression and (3.39), we can write $\frac{d\Pr(R)}{dk}$ as:

$$\begin{aligned}
\frac{d\Pr(R)}{dk} &= -\frac{1}{2\overline{\varepsilon}}\left(\frac{\partial \varepsilon_L}{\partial \eta}\frac{\partial \eta}{\partial k} + \frac{\partial \varepsilon_L}{\partial k}\right) = -\frac{1}{2\overline{\varepsilon}}\frac{\partial \varepsilon_L}{\partial \eta}\frac{\partial \eta}{\partial k} \Leftrightarrow \\
\frac{d\Pr(R)}{dk} &= -\frac{1}{2\overline{\varepsilon}}\left(-\frac{\gamma}{(1-\theta)\, y^m \Gamma}\right)\frac{\partial \eta}{\partial k} < 0.
\end{aligned}$$

3.D.3 Effect of $\overline{\varepsilon}$ on η, b^R and b^{NR}

Given our assumption that ε has a uniform distribution on the interval $[-\overline{\varepsilon}, \overline{\varepsilon}]$, we have:

$$\begin{aligned}
Var(\varepsilon) &= E_0\left\{\varepsilon - E_0\{\varepsilon\}\right\}^2 \Leftrightarrow \\
Var(\varepsilon) &= E_0\left\{\varepsilon^2\right\} = \int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} \varepsilon^2 f(\varepsilon)\, d\varepsilon = \int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} \varepsilon^2 \frac{1}{2\overline{\varepsilon}} d\varepsilon \Leftrightarrow \\
Var(\varepsilon) &= \frac{1}{2\overline{\varepsilon}}\left[\frac{\varepsilon^3}{3}\right]_{-\overline{\varepsilon}}^{\overline{\varepsilon}} = \frac{1}{3}\overline{\varepsilon}^2.
\end{aligned}$$

This last expression demonstrates that the variance of ε is monotonically increasing in $\overline{\varepsilon}$. Hence, by investigating the effect of changes in $\overline{\varepsilon}$, we explore the effects of changes in the variance of ε.

[25] That is the case for a quadratic specification (3.25) of $v(\cdot)$.

Effect on η

Differentiating (3.23) with respect to $\overline{\varepsilon}$, we have:

$$\begin{aligned} & v'\left(f_{1L}^R\right)\left[\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\overline{\varepsilon}}+\frac{\partial\varepsilon_L}{\partial\overline{\varepsilon}}\right)-\gamma\frac{\partial\eta}{\partial\overline{\varepsilon}}+\frac{\partial b_L^R}{\partial\overline{\varepsilon}}\right]+ \\ & pv'\left(f_{2L}^R\right)\left[\theta\overline{y}\left(1+\Gamma\right)\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\overline{\varepsilon}}+\frac{\partial\varepsilon_L}{\partial\overline{\varepsilon}}\right)-\frac{\partial b_L^R}{\partial\overline{\varepsilon}}\right] \\ & -k\left(\frac{\partial b_L^R}{\partial\overline{\varepsilon}}\right)-\left(1-p\right)v'\left(f_{2L}^R\right)\left[\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\overline{\varepsilon}}+\frac{\partial\varepsilon_L}{\partial\overline{\varepsilon}}\right)-\frac{\partial b_L^{NR}}{\partial\overline{\varepsilon}}\right] \\ & -\left(1-\theta\right)y^m\Gamma\left[\begin{array}{c} v'\left(f_{1\overline{\varepsilon}}^R\right)\frac{\partial\overline{\varepsilon}}{\partial\overline{\varepsilon}}-v'\left(f_{1L}^R\right)\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\overline{\varepsilon}}+\frac{\partial\varepsilon_L}{\partial\overline{\varepsilon}}\right)+ \\ \int_{\varepsilon_L}^{\overline{\varepsilon}} v"\left(f_{1L}^R\right)\left(-\gamma\frac{\partial\eta}{\partial\overline{\varepsilon}}+\frac{\partial b^R}{\partial\overline{\varepsilon}}\right)d\varepsilon \end{array}\right] \\ = {} & 0 \Leftrightarrow \end{aligned}$$

$$\begin{aligned} & \frac{\partial\eta}{\partial\overline{\varepsilon}}\left\{\theta\overline{y}\frac{\partial\varepsilon_L}{\partial\eta}\left[v'\left(f_{1L}^R\right)+\left(1+\Gamma\right)pv'\left(f_{2L}^R\right)-\left(1-p\right)v'\left(f_{2L}^{NR}\right)\right]-\gamma v'\left(f_{1L}^R\right)\right\}+ \\ & \frac{\partial b_L^{NR}}{\partial\overline{\varepsilon}}\left[v'\left(f_{2L}^{NR}\right)\left(1-p\right)\right]+ \\ & \theta\overline{y}\frac{\partial\varepsilon_L}{\partial\overline{\varepsilon}}\left[v'\left(f_{1L}^R\right)+\left(1+\Gamma\right)pv'\left(f_{2L}^R\right)-\left(1-p\right)v'\left(f_{2L}^{NR}\right)\right]+ \\ & \frac{\partial b_L^R}{\partial\overline{\varepsilon}}\left[v'\left(f_{1L}^R\right)-pv'\left(f_{2L}^R\right)-k\right]- \\ & \left(1-\theta\right)y^m\Gamma\left\{-v'\left(f_{1L}^R\right)\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\overline{\varepsilon}}+\frac{\partial\varepsilon_L}{\partial\overline{\varepsilon}}\right)+\int_{\varepsilon_L}^{\overline{\varepsilon}} v"\left(f_1^R\right)\left(-\gamma\frac{\partial\eta}{\partial\overline{\varepsilon}}+\frac{\partial b^R}{\partial\overline{\varepsilon}}\right)d\varepsilon\right\} \\ = {} & \left(1-\theta\right)y^m\Gamma v'\left(f_{1\overline{\varepsilon}}^R\right), \end{aligned}$$

where we have defined:

$$f_{1\overline{\varepsilon}}^R=\theta\overline{y}(1+\overline{\varepsilon})-\eta\gamma+b^R\left(\overline{\varepsilon}\right). \tag{3.63}$$

From (3.16) we know that $\frac{\partial\varepsilon_L}{\partial\overline{\varepsilon}}=0$. So, using again (3.39) and the fact that (3.27) is valid for any realization of ε in the interval $[\varepsilon_L,\overline{\varepsilon}]$, we can simplify the latter expression to:

$$\left(\begin{array}{c} \frac{\partial\eta}{\partial\overline{\varepsilon}}\gamma\left\{-\frac{\theta\overline{y}}{(1-\theta)y^m\Gamma}\left[\begin{array}{c} v'\left(f_{1L}^R\right)+\left(1+\Gamma\right)pv'\left(f_{2L}^R\right) \\ -\left(1-p\right)v'\left(f_{2L}^{NR}\right)\end{array}\right]-2v'\left(f_{1L}^R\right)\right\} \\ +\frac{\partial b_L^{NR}}{\partial\overline{\varepsilon}}\left[v'\left(f_{2L}^{NR}\right)\left(1-p\right)\right]-\left(1-\theta\right)y^m\Gamma\int_{\varepsilon_L}^{\overline{\varepsilon}} v"\left(f_1^R\right)\left(-\gamma\frac{\partial\eta}{\partial\overline{\varepsilon}}+\frac{\partial b^R}{\partial\overline{\varepsilon}}\right)d\varepsilon \end{array}\right)=\left(1-\theta\right)y^m\Gamma v'\left(f_{1\overline{\varepsilon}}^R\right),$$

which can be represented by:

$$A_{1L}\frac{\partial\eta}{\partial\overline{\varepsilon}}-A_2\int_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma\frac{\partial\eta}{\partial\overline{\varepsilon}}+\frac{\partial b^R}{\partial\overline{\varepsilon}}\right)v"\left(f_1^R\right)d\varepsilon+A_{3L}\frac{\partial b_L^{NR}}{\partial\overline{\varepsilon}}=I_1. \tag{3.64}$$

A_{1L} is given by (3.41), A_2 by (3.42), A_{3L} by (3.43) and

$$I_1=\left(1-\theta\right)y^m\Gamma v'\left(f_{1\overline{\varepsilon}}^R\right)>0. \tag{3.65}$$

Effect on b^R

Differentiating (3.27) with respect to $\overline{\varepsilon}$, we have:

$$
\begin{aligned}
v"\left(f_1^R\right)\left(-\gamma\frac{\partial\eta}{\partial\overline{\varepsilon}}+\frac{\partial b^R}{\partial\overline{\varepsilon}}\right)-pv"\left(f_2^R\right)\left(-\frac{\partial b^R}{\partial\overline{\varepsilon}}\right) &= 0 \Leftrightarrow \\
\frac{\partial\eta}{\partial\overline{\varepsilon}}\left[-\gamma v"\left(f_1^R\right)\right]+\frac{\partial b^R}{\partial\overline{\varepsilon}}\left[v"\left(f_1^R\right)+pv"\left(f_2^R\right)\right] &= 0,
\end{aligned}
$$

which can be represented by:

$$B_1\frac{\partial\eta}{\partial\overline{\varepsilon}}+B_2\frac{\partial b^R}{\partial\overline{\varepsilon}}=I_2. \tag{3.66}$$

B_1 is given by (3.46), B_2 by (3.47), and

$$I_2=0. \tag{3.67}$$

Effect on b^{NR}

When the government chooses the optimal η, it takes into account b^{NR} evaluated at $\varepsilon=\varepsilon_L$. Therefore, differentiating (3.49) with respect to $\overline{\varepsilon}$, we have:

$$
\begin{aligned}
v"\left(g_{1L}^{NR}\right)\left[\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\overline{\varepsilon}}+\frac{\partial\varepsilon_L}{\partial\overline{\varepsilon}}\right)+\frac{\partial b_L^{NR}}{\partial\overline{\varepsilon}}\right]-pv"\left(g_{2L}^{NR}\right)\left\{\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\overline{\varepsilon}}+\frac{\partial\varepsilon_L}{\partial\overline{\varepsilon}}\right)-\frac{\partial b_L^{NR}}{\partial\overline{\varepsilon}}\right\} &= 0 \Leftrightarrow \\
\frac{\partial\eta}{\partial\overline{\varepsilon}}\left[\theta\overline{y}\frac{\partial\varepsilon_L}{\partial\eta}\left(v"\left(g_{1L}^{NR}\right)-pv"\left(g_{2L}^{NR}\right)\right)\right]+\frac{\partial b_L^{NR}}{\partial\overline{\varepsilon}}\left[v"\left(g_{1L}^{NR}\right)+pv"\left(g_{2L}^{NR}\right)\right] &= 0,
\end{aligned}
$$

where g_{1L}^{NR} is given by (3.50), g_{2L}^{NR} by (3.51) and $\frac{\partial\varepsilon_L}{\partial\overline{\varepsilon}}=0$. Furthermore, using (3.39), the last expression can be represented by:

$$C_{1L}\frac{\partial\eta}{\partial\overline{\varepsilon}}+C_{2L}\frac{\partial b_L^{NR}}{\partial\overline{\varepsilon}}=I_{3L}, \tag{3.68}$$

where C_{1L} is given by (3.53), C_{2L} by (3.54) and

$$I_{3L}=0. \tag{3.69}$$

Solution of the system

We can write the system formed by (3.64), (3.66) and (3.68) as:

$$
\left\{
\begin{array}{lllllll}
A_{1L}\frac{\partial\eta}{\partial\overline{\varepsilon}} & +0 & -A_2\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma\frac{\partial\eta}{\partial\overline{\varepsilon}}+\frac{\partial b^R}{\partial\overline{\varepsilon}}\right)v"\left(f_1^R\right)d\varepsilon & +A_{3L}\frac{\partial b_L^{NR}}{\partial\overline{\varepsilon}} & = & I_1 \\
B_1\frac{\partial\eta}{\partial\overline{\varepsilon}} & +B_2\frac{\partial b^R}{\partial\overline{\varepsilon}} & +0 & +0 & = & I_2 \\
C_{1L}\frac{\partial\eta}{\partial\overline{\varepsilon}} & +0 & +0 & +C_{2L}\frac{\partial b_L^{NR}}{\partial\overline{\varepsilon}} & = & I_{3L}
\end{array}
\right. .
$$

Further, we can rewrite (3.66) and (3.68) as:

$$\frac{\partial b^R}{\partial\overline{\varepsilon}} = -\frac{B_1}{B_2}\frac{\partial\eta}{\partial\overline{\varepsilon}}, \text{ and} \tag{3.70}$$

$$\frac{\partial b_L^{NR}}{\partial\overline{\varepsilon}} = -\frac{C_{1L}}{C_{2L}}\frac{\partial\eta}{\partial\overline{\varepsilon}}. \tag{3.71}$$

Substituting these equations into (3.64), we have:

$$\begin{aligned} A_{1L}\frac{\partial \eta}{\partial \overline{\varepsilon}} - A_2 \int_{\varepsilon_L}^{\overline{\varepsilon}} \left(-\gamma \frac{\partial \eta}{\partial \overline{\varepsilon}} - \frac{B_1}{B_2}\frac{\partial \eta}{\partial \overline{\varepsilon}} \right) v"\left(f_1^R\right) d\varepsilon - A_{3L}\frac{C_{1L}}{C_{2L}}\frac{\partial \eta}{\partial \overline{\varepsilon}} &= I_1 \Leftrightarrow \\ A_1\frac{\partial \eta}{\partial \overline{\varepsilon}} - A_2 \int_{\varepsilon_L}^{\overline{\varepsilon}} \frac{\partial \eta}{\partial \overline{\varepsilon}} \left(-\gamma - \frac{B_1}{B_2} \right) v"\left(f_1^R\right) d\varepsilon - A_{3L}\frac{C_{1L}}{C_{2L}}\frac{\partial \eta}{\partial \overline{\varepsilon}} &= I_1. \end{aligned} \tag{3.72}$$

Sign of $\frac{\partial \eta}{\partial \overline{\varepsilon}}$: Using the values of (3.41) to (3.43), (3.46), (3.47), (3.53), (3.54) and (3.65), we are able to rewrite (3.72) as:

$$\begin{aligned} \frac{\partial \eta}{\partial \overline{\varepsilon}} \left\{ \begin{array}{c} A_{1L} - A_{3L}\frac{C_{1L}}{C_{2L}} - \\ A_2 \int_{\varepsilon_L}^{\overline{\varepsilon}} \left[-\gamma v"\left(f_1^R\right) + \frac{\gamma\left(v"\left(f_1^R\right)\right)^2}{v"\left(f_1^R\right)+pv"\left(f_2^R\right)} \right] d\varepsilon \end{array} \right\} &= I_1 \Leftrightarrow \\ \frac{\partial \eta}{\partial \overline{\varepsilon}} \left[\begin{array}{c} A_{1L} - A_{3L}\frac{C_{1L}}{C_{2L}} - \\ A_2 \int_{\varepsilon_L}^{\overline{\varepsilon}} \left[\frac{-\gamma\left(v"\left(f_1^R\right)\right)^2 - \gamma v"\left(f_1^R\right)pv"\left(f_2^R\right) + \gamma\left(v"\left(f_1^R\right)\right)^2}{v"\left(f_1^R\right)+pv"\left(f_2^R\right)} \right] d\varepsilon \end{array} \right] &= I_1 \Leftrightarrow \\ \frac{\partial \eta}{\partial \overline{\varepsilon}} \left\{ \begin{array}{c} -\frac{\theta\overline{y}\gamma}{(1-\theta)y^m\Gamma} \left[\begin{array}{c} v'\left(f_{1L}^R\right) + (1+\Gamma)\, pv'\left(f_{2L}^R\right) \\ -(1-p)\, v'\left(f_{2L}^{NR}\right) \end{array} \right] \\ -2\gamma v'\left(f_{1L}^R\right) + \frac{(1-p)\theta\overline{y}\gamma v'\left(f_{2L}^{NR}\right)\left[v"\left(g_{1L}^{NR}\right) - pv"\left(g_{2L}^{NR}\right)\right]}{(1-\theta)y^m\Gamma\left[v"\left(g_{1L}^{NR}\right)+pv"\left(g_{2L}^{NR}\right)\right]} \\ +(1-\theta)\, y^m\Gamma\gamma \int_{\varepsilon_L}^{\overline{\varepsilon}} \frac{v"\left(f_1^R\right)pv"\left(f_2^R\right)}{v"\left(f_1^R\right)+pv"\left(f_2^R\right)} d\varepsilon \end{array} \right\} &= (1-\theta)\, y^m\Gamma v'[f_{1\overline{\varepsilon}}^R] \end{aligned} \tag{3.73}$$

If p is sufficiently different from 0 the overall sign of the term in brackets on the left-hand side of (3.73) is negative. The right-hand side is positive. Hence, for p close enough to 1, $\frac{\partial \eta}{\partial \overline{\varepsilon}} < 0$.

Sign of $\frac{\partial b^R}{\partial \overline{\varepsilon}}$: Given (3.70), the signs of (3.46), (3.47), and the sign of $\frac{\partial \eta}{\partial \overline{\varepsilon}}$, we have

$$\frac{\partial b^R}{\partial \overline{\varepsilon}} < 0. \tag{3.74}$$

Sign of $\frac{\partial b_L^{NR}}{\partial \overline{\varepsilon}}$: Given (3.71), the signs of (3.53), (3.54) and of $\frac{\partial \eta}{\partial \overline{\varepsilon}}$, the sign of $\frac{\partial b_L^{NR}}{\partial \overline{\varepsilon}}$ is ambiguous. However, if we assume that $v"\left(g_{1L}^{NR}\right) - pv"\left(g_{2L}^{NR}\right) < 0$ this ambiguity disappears and a higher variance of the shock of output diminishes the optimal deficit in case of no re-election when $\varepsilon = \varepsilon_L$:

$$\frac{\partial b_L^{NR}}{\partial \overline{\varepsilon}} < 0. \tag{3.75}$$

3.D.4 Effect of $\overline{\varepsilon}$ on the probability of re-election

With $(1-\theta)\, y^m\Gamma + (\eta - I)\,\gamma > 0$, and using (3.39), we can prove Proposition 3.2 by writing $\frac{d\Pr(R)}{d\overline{\varepsilon}}$ as:

$$\frac{d\Pr(R)}{d\overline{\varepsilon}} = \frac{\left(1-\frac{\partial \varepsilon_L}{\partial \overline{\varepsilon}}\right)2\overline{\varepsilon} - (\overline{\varepsilon}-\varepsilon_L)\,2}{(2\overline{\varepsilon})^2} = \frac{-2\left[\overline{\varepsilon}\left(\frac{\partial \varepsilon_L}{\partial \eta}\frac{\partial \eta}{\partial \overline{\varepsilon}}\right)-\varepsilon_L\right]}{4\overline{\varepsilon}^2} \Leftrightarrow$$

$$\frac{d\Pr(R)}{d\overline{\varepsilon}} = \left(\frac{1}{2\overline{\varepsilon}}\right)\left(\frac{\gamma}{(1-\theta)\,y^m\Gamma}\frac{\partial \eta}{\partial \overline{\varepsilon}}\right) + \left(\frac{-\frac{(\eta-I)\gamma}{(1-\theta)y^m\Gamma}-1}{2\overline{\varepsilon}^2}\right) \Leftrightarrow$$

$$\frac{d\Pr(R)}{d\overline{\varepsilon}} = \left(\frac{1}{2\overline{\varepsilon}}\right)\left(\frac{\gamma}{(1-\theta)\,y^m\Gamma}\frac{\partial \eta}{\partial \overline{\varepsilon}}\right) + \frac{-\frac{(1-\theta)y^m\Gamma+(\eta-I)\gamma}{(1-\theta)y^m\Gamma}}{2\overline{\varepsilon}^2} \Leftrightarrow$$

$$\frac{d\Pr(R)}{d\overline{\varepsilon}} = \left(\frac{1}{2\overline{\varepsilon}}\right)\left(\frac{1}{(1-\theta)\,y^m\Gamma}\right)\left(\gamma\frac{\partial \eta}{\partial \overline{\varepsilon}} - \frac{(1-\theta)\,y^m\Gamma+(\eta-I)\,\gamma}{\overline{\varepsilon}}\right) < 0. \quad (3.76)$$

3.D.5 Effect of y^m on η, b^R and b

Effect on η

Differentiate (3.23) with respect to y^m (holding $\overline{y}$ constant)

$$v'\left(f_{1L}^R\right)\left[\theta\overline{y}\left(\frac{\partial \varepsilon_L}{\partial \eta}\frac{\partial \eta}{\partial y^m}+\frac{\partial \varepsilon_L}{\partial y^m}\right)-\gamma\frac{\partial \eta}{\partial y^m}+\frac{\partial b_L^R}{\partial y^m}\right]+$$
$$pv'\left(f_{2L}^R\right)\left[\theta\overline{y}\,(1+\Gamma)\left(\frac{\partial \varepsilon_L}{\partial \eta}\frac{\partial \eta}{\partial y^m}+\frac{\partial \varepsilon_L}{\partial y^m}\right)-\frac{\partial b_L^R}{\partial y^m}\right]$$
$$-k\left(\frac{\partial b_L^R}{\partial y^m}\right)-(1-p)\,v'\left(f_{2L}^{NR}\right)\left(\theta\overline{y}\left(\frac{\partial \varepsilon_L}{\partial \eta}\frac{\partial \eta}{\partial y^m}+\frac{\partial \varepsilon_L}{\partial y^m}\right)-\frac{\partial b_L^{NR}}{\partial y^m}\right)-$$
$$(1-\theta)\,\Gamma\int_{\varepsilon_L}^{\overline{\varepsilon}} v'\left(f_1^R\right)-(1-\theta)\,y^m\Gamma\left\{\begin{array}{c} v'\left(f_{1\overline{\varepsilon}}^R\right)\frac{\partial \overline{\varepsilon}}{\partial y^m}-v'\left(f_{1L}^R\right)\frac{\partial \varepsilon_L}{\partial y^m}+ \\ \int_{\varepsilon_L}^{\overline{\varepsilon}} v"\left(f_1^R\right)\left(-\gamma\frac{\partial \eta}{\partial y^m}+\frac{\partial b^R}{\partial y^m}\right)d\varepsilon \end{array}\right\}=0 \Leftrightarrow$$

$$\frac{\partial \eta}{\partial y^m}\left\{\theta\overline{y}\frac{\partial \varepsilon_L}{\partial \eta}\left[v'\left(f_{1L}^R\right)+(1+\Gamma)\,pv'\left(f_{2L}^R\right)-(1-p)\,v'\left(f_{2L}^{NR}\right)\right]-\gamma v'\left(f_{1L}^R\right)\right\}+$$
$$\frac{\partial b_L^{NR}}{\partial y^m}\left[v'\left(f_{2L}^{NR}\right)(1-p)\right]+\theta\overline{y}\frac{\partial \varepsilon_L}{\partial y^m}\left[v'\left(f_{1L}^R\right)+(1+\Gamma)\,pv'\left(f_{2L}^R\right)-(1-p)\,v'\left(f_{2L}^{NR}\right)\right]+$$
$$\frac{\partial b_L^R}{\partial y^m}\left[v'\left(f_{1L}^R\right)-pv'\left(f_{2L}^R\right)-k\right]-(1-\theta)\,y^m\Gamma\left[\begin{array}{c} -v'\left(f_{1L}^R\right)\left(\frac{\partial \varepsilon_L}{\partial \eta}\frac{\partial \eta}{\partial y^m}+\frac{\partial \varepsilon_L}{\partial y^m}\right)+ \\ \int_{\varepsilon_L}^{\overline{\varepsilon}} v"\left(f_1^R\right)\left(-\gamma\frac{\partial \eta}{\partial y^m}+\frac{\partial b^R}{\partial y^m}\right)d\varepsilon \end{array}\right]$$
$$= (1-\theta)\,\Gamma\int_{\varepsilon_L}^{\overline{\varepsilon}} v'\left(f_1^R\right).$$

Using (3.39) and the fact that (3.27) is valid for any realization of ε in the interval

$(\varepsilon_L, \overline{\varepsilon})$, we can simplify the latter expression to:

$$\begin{aligned}
&\frac{\partial \eta}{\partial y^m}\gamma\left\{-\frac{\theta\overline{y}}{(1-\theta)\,y^m\Gamma}\left[v'\left(f_{1L}^R\right)+(1+\Gamma)\,pv'\left(f_{2L}^R\right)-(1-p)\,v'\left(f_{2L}^{NR}\right)\right]-v'\left(f_{1L}^R\right)\right\}\\
&+\frac{\partial b_L^{NR}}{\partial y^m}\left[v'\left(f_{2L}^{NR}\right)(1-p)\right]+\theta\overline{y}\frac{\partial \varepsilon_L}{\partial y^m}\left[v'\left(f_{1L}^R\right)+(1+\Gamma)\,pv'\left(f_{2L}^R\right)-(1-p)\,v'\left(f_{2L}^{NR}\right)\right]\\
&-(1-\theta)\,y^m\Gamma\left\{\begin{array}{c}-v'\left(f_{1L}^R\right)\left[\left(-\frac{\gamma}{(1-\theta)y^m\Gamma}\right)\frac{\partial\eta}{\partial y^m}+\frac{\partial\varepsilon_L}{\partial y^m}\right]\\+\int_{\varepsilon_L}^{\overline{\varepsilon}}v"\left(f_1^R\right)\left(-\gamma\frac{\partial\eta}{\partial y^m}+\frac{\partial b^R}{\partial y^m}\right)d\varepsilon\end{array}\right\}\\
=\;&(1-\theta)\,\Gamma\int_{\varepsilon_L}^{\overline{\varepsilon}}v'\left(f_1^R\right). \qquad (3.77)
\end{aligned}$$

Additionally, taking the derivative of (3.16) with respect to y^m:

$$\frac{\partial\varepsilon_L}{\partial y^m}=\frac{-\{-(\eta-I)\,\gamma\,(1-\theta)\,\Gamma\}}{((1-\theta)\,y^m\Gamma)^2}=\frac{(\eta-I)\,\gamma}{(1-\theta)\,(y^m)^2\,\Gamma}<0. \qquad (3.78)$$

We can rewrite (3.77) as:

$$\left\{\begin{array}{c}\frac{\partial\eta}{\partial y^m}\left\{\begin{array}{c}-\frac{\gamma\theta\overline{y}}{(1-\theta)y^m\Gamma}\left[\begin{array}{c}v'\left(f_{1L}^R\right)+\\(1+\Gamma)\,pv'\left(f_{2L}^R\right)\\-(1-p)\,v'\left(f_{2L}^{NR}\right)\end{array}\right]\\-2\gamma v'\left(f_{1L}^R\right)\end{array}\right\}\\+\frac{\partial b_L^{NR}}{\partial y^m}\left[v'\left(f_{2L}^{NR}\right)(1-p)\right]-\\(1-\theta)\,y^m\Gamma\int_{\varepsilon_L}^{\overline{\varepsilon}}v"\left(f_1^R\right)\left(-\gamma\frac{\partial\eta}{\partial y^m}+\frac{\partial b^R}{\partial y^m}\right)d\varepsilon\end{array}\right\}=\left\{\begin{array}{c}(1-\theta)\,\Gamma\int_{\varepsilon_L}^{\overline{\varepsilon}}v'\left(f_1^R\right)-\frac{(\eta-I)\gamma v'\left(f_{1L}^R\right)}{y^m}\\-\frac{\theta\overline{y}(\eta-I)\gamma}{(1-\theta)(y^m)^2\Gamma}\left[\begin{array}{c}v'\left(f_{1L}^R\right)+\\(1+\Gamma\,[\gamma])\,pv'\left(f_{2L}^R\right)\\-(1-p)\,v'\left(f_{2L}^{NR}\right)\end{array}\right]\end{array}\right\},$$

which can be represented by:

$$A_{1L}\frac{\partial\eta}{\partial y^m}-A_2\int_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma\frac{\partial\eta}{\partial y^m}+\frac{\partial b^R}{\partial y^m}\right)v"\left(f_1^R\right)d\varepsilon+A_{3L}\frac{\partial b_L^{NR}}{\partial y^m}=H_{1L}. \qquad (3.79)$$

A_{1L} is given by (3.41), A_2 by (3.42), A_{3L} by (3.43) and, if p is sufficiently different from 0:

$$H_{1L}=\left\{\begin{array}{c}(1-\theta)\,\Gamma\int_{\varepsilon_L}^{\overline{\varepsilon}}v'\left(f_1^R\right)d\varepsilon-\\\frac{(\eta-I)\gamma}{y^m}\left\{v'\left(f_{1L}^R\right)+\frac{\theta\overline{y}}{(1-\theta)y^m\Gamma}\left[\begin{array}{c}v'\left(f_{1L}^R\right)+(1+\Gamma)\,pv'\left(f_{2L}^R\right)\\-(1-p)\,v'\left(f_{2L}^{NR}\right)\end{array}\right]\right\}\end{array}\right\}>0. \qquad (3.80)$$

Effect on b^R

Differentiating (3.27) with respect to y^m, we have:

$$\begin{aligned}
v"\left(f_1^R\right)\left(-\gamma\frac{\partial\eta}{\partial y^m}+\frac{\partial b^R}{\partial y^m}\right)-pv"\left(f_2^R\right)\left(-\frac{\partial b^R}{\partial y^m}\right)&=0\Leftrightarrow\\
\frac{\partial\eta}{\partial y^m}\left[-\gamma v"\left(f_1^R\right)\right]+\frac{\partial b^R}{\partial y^m}\left[v"\left(f_1^R\right)+pv"\left(f_2^R\right)\right]&=0,
\end{aligned}$$

which can be represented by:

$$B_1 \frac{\partial \eta}{\partial y^m} + B_2 \frac{\partial b^R}{\partial y^m} = H_2. \tag{3.81}$$

B_1 is given by (3.46), B_2 by (3.47), and

$$H_2 = 0. \tag{3.82}$$

Effect on b^{NR}

When the government chooses the optimal η, it takes into account b^{NR} evaluated at $\varepsilon = \varepsilon_L$. Therefore, differentiating (3.49) with respect to y^m, we have:

$$v"\left(g_{1L}^{NR}\right)\left[\theta\overline{y}\left(\frac{\partial \varepsilon_L}{\partial \eta}\frac{\partial \eta}{\partial y^m} + \frac{\partial \varepsilon_L}{\partial y^m}\right) + \frac{\partial b_L^{NR}}{\partial k}\right] - pv"\left(g_{2L}^{NR}\right)\left[\theta\overline{y}\left(\frac{\partial \varepsilon_L}{\partial \eta}\frac{\partial \eta}{\partial y^m} + \frac{\partial \varepsilon_L}{\partial y^m}\right) - \frac{\partial b_L^{NR}}{\partial y^m}\right] = 0 \Leftrightarrow$$

$$\begin{aligned} & \frac{\partial \eta}{\partial y^m}\left[\theta\overline{y}\frac{\partial \varepsilon_L}{\partial \eta}\left(v"\left(g_{1L}^{NR}\right) - pv"\left(g_{2L}^{NR}\right)\right)\right] + \frac{\partial b_L^{NR}}{\partial y^m}\left[v"\left(g_{1L}^{NR}\right) + pv"\left(g_{2L}^{NR}\right)\right] \\ = \ & -\theta\overline{y}\frac{\partial \varepsilon_L}{\partial y^m}\left[v"\left(g_{1L}^{NR}\right) - pv"\left(g_{2L}^{NR}\right)\right], \end{aligned}$$

where g_{1L}^{NR} is given by (3.50), and g_{2L}^{NR} by (3.51). Furthermore, using (3.39) and (3.78), the last expression can be represented as:

$$C_{1L}\frac{\partial \eta}{\partial y^m} + C_{2L}\frac{\partial b_L^{NR}}{\partial y^m} = H_{3L}, \tag{3.83}$$

where C_{1L} is given by (3.53), C_{2L} by (3.54) and

$$H_{3L} = -\frac{\theta\overline{y}\left(\eta - I\right)\gamma}{\left(1-\theta\right)\left(y^m\right)^2\Gamma}\left[v"\left(g_{1L}^{NR}\right) - pv"\left(g_{2L}^{NR}\right)\right]. \tag{3.84}$$

H_{3L} is negative for $v"\left(g_{1L}^{NR}\right) - pv"\left(g_{2L}^{NR}\right) < 0$.[26]

Solution of the system

We can write the system formed by (3.79), (3.81) and (3.83) as:

$$\begin{cases} A_{1L}\frac{\partial \eta}{\partial y^m} & +0 & -A_2\int_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma\frac{\partial \eta}{\partial y^m} + \frac{\partial b^R}{\partial y^m}\right)v"\left(f_1^R\right)d\varepsilon & +A_{3L}\frac{\partial b_L^{NR}}{\partial y^m} & = & H_{1L} \\ B_1\frac{\partial \eta}{\partial y^m} & +B_2\frac{\partial b^R}{\partial y^m} & +0 & +0 & = & H_2 \\ C_{1L}\frac{\partial \eta}{\partial y^m} & +0 & +0 & +C_{2L}\frac{\partial b_L^{NR}}{\partial y^m} & = & H_{3L} \end{cases}.$$

Further, we can rewrite (3.81) and (3.83) as:

$$\frac{\partial b^R}{\partial y^m} = -\frac{B_1}{B_2}\frac{\partial \eta}{\partial y^m}, \text{ and} \tag{3.85}$$

$$\frac{\partial b_L^{NR}}{\partial y^m} = \frac{H_{3L}}{C_{2L}} - \frac{C_{1L}}{C_{2L}}\frac{\partial \eta}{\partial y^m}. \tag{3.86}$$

[26] That is the case for a quadratic utility function specification for instance.

Substituting these two previous equations into (3.79), we have:

$$
\begin{aligned}
A_{1L}\frac{\partial\eta}{\partial y^m} - A_2\int_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma\frac{\partial\eta}{\partial y^m} - \frac{B_1}{B_2}\frac{\partial\eta}{\partial y^m}\right)v\text{"}\left(f_1^R\right)d\varepsilon - A_{3L}\frac{C_{1L}}{C_{2L}}\frac{\partial\eta}{\partial y^m} &= H_{1L} - \tfrac{A_{3L}H_{3L}}{C_{2L}} \Leftrightarrow \\
\frac{\partial\eta}{\partial y^m}\left\{A_{1L} - A_{3L}\frac{C_{1L}}{C_{2L}} - A_2\int_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma - \frac{B_1}{B_2}\right)v\text{"}\left(f_1^R\right)d\varepsilon\right\} &= H_{1L} - \tfrac{A_{3L}H_{3L}}{C_{2L}}
\end{aligned}
\tag{3.87}
$$

Sign of $\frac{\partial\eta}{\partial y^m}$: Using the values of (3.41) to (3.43), (3.46), (3.47), (3.53), (3.54) and (3.80) we can (3.87) as:

$$
\frac{\partial\eta}{\partial y^m}\left\{A_{1L} - A_{3L}\frac{C_{1L}}{C_{2L}} - A_2\int_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma v\text{"}\left(f_1^R\right) + \frac{\gamma\left(v\text{"}\left(f_1^R\right)\right)^2}{v\text{"}\left(f_1^R\right) + pv\text{"}\left(f_2^R\right)}\right)d\varepsilon\right\} = H_{1L} - \frac{A_{3L}H_{3L}}{C_{2L}} \Leftrightarrow
$$

$$
\begin{aligned}
&\frac{\partial\eta}{\partial y^m}\left\{\begin{array}{c}
-\frac{\gamma\theta\overline{y}}{(1-\theta)y^m\Gamma}\left[\begin{array}{c} v'\left(f_{1L}^R\right) + \\ (1+\Gamma)\,pv'\left(f_{2L}^R\right) \\ -(1-p)\,v'\left(f_{2L}^{NR}\right)\end{array}\right] \\
-2\gamma v'\left(f_{1L}^R\right) + \frac{(1-p)\gamma\theta\overline{y}v'\left(f_{2L}^{NR}\right)\left[v\text{"}\left(g_{1L}^{NR}\right) - pv\text{"}\left(g_{2L}^{NR}\right)\right]}{(1-\theta)y^m\Gamma\left[v\text{"}\left(g_{1L}^{NR}\right) + pv\text{"}\left(g_{2L}^{NR}\right)\right]} \\
+(1-\theta)\,y^m\Gamma\gamma\int_{\varepsilon_L}^{\overline{\varepsilon}}\frac{v\text{"}\left(f_1^R\right)pv\text{"}\left(f_2^R\right)}{v\text{"}\left(f_1^R\right) + pv\text{"}\left(f_2^R\right)}d\varepsilon
\end{array}\right\} \\
&= (1-\theta)\,\Gamma\int_{\varepsilon_L}^{\overline{\varepsilon}}v'\left(f_1^R\right) - \tfrac{(\eta-I)\gamma}{y^m}v'\left(f_{1L}^R\right) - \tfrac{(\eta-I)\gamma}{y^m}\left\{\begin{array}{c}
\frac{\theta\overline{y}}{(1-\theta)y^m\Gamma}\left[\begin{array}{c} v'\left(f_{1L}^R\right) + \\ (1+\Gamma)\,pv'\left(f_{2L}^R\right) \\ -(1-p)\,v'\left(f_{2L}^{NR}\right)\end{array}\right] \\
-\frac{\theta\overline{y}(1-p)v'\left(f_{2L}^{NR}\right)}{(1-\theta)y^m\Gamma}\frac{\left[v\text{"}\left(g_{1L}^{NR}\right) - pv\text{"}\left(g_{2L}^{NR}\right)\right]}{\left[v\text{"}\left(g_{1L}^{NR}\right) + pv\text{"}\left(g_{2L}^{NR}\right)\right]}
\end{array}\right\}
\end{aligned}
\tag{3.88}
$$

If p is sufficiently different from 0, the overall sign of the term in brackets on the left-hand side of (3.88) is negative. The right-hand side is positive. Hence, for p sufficiently different from 0, $\frac{\partial\eta}{\partial y^m} < 0$.

Sign of $\frac{\partial b^R}{\partial y^m}$: Given (3.85), the signs of (3.46), (3.47), and of $\frac{\partial\eta}{\partial y^m}$, we have:

$$
\frac{\partial b^R}{\partial y^m} < 0. \tag{3.89}
$$

Sign of $\frac{\partial b_L^{NR}}{\partial y^m}$: Finally, given (3.86), the signs of (3.53), (3.54) and of $\frac{\partial\eta}{\partial y^m}$, we have:

$$
\frac{\partial b_L^{NR}}{\partial y^m} \lesseqgtr 0. \tag{3.90}
$$

If we assume that $v"\left(g_{1L}^{NR}\right) - pv"\left(g_{2L}^{NR}\right) < 0$, the ambiguity in (3.90) can be rewritten as:

$$\begin{aligned} \frac{\partial b_L^{NR}}{\partial y^m} &\lesseqgtr 0 \Leftrightarrow H_{3L} \gtrless -C_{1L}\frac{\partial \eta}{\partial y^m} \Leftrightarrow \\ -\frac{\theta\overline{y}\gamma\left(\eta - I\right)\left(v"\left[g_{1L}\right] - pv"\left[g_{2L}\right]\right)}{\left(1-\theta\right)\left(y^m\right)^2\Gamma\left[\gamma\right]} &\gtrless -\frac{\theta\overline{y}\gamma\left(v"\left[g_{1L}\right] - pv"\left[g_{2L}\right]\right)}{\left(1-\theta\right)y^m\Gamma\left[\gamma\right]}\frac{\partial \eta}{\partial y^m} \Leftrightarrow \\ \frac{\left(\eta - I\right)}{y^m} &\gtrless \frac{\partial \eta}{\partial y^m}. \end{aligned} \tag{3.91}$$

3.D.6 Effect of y^m on the probability of re-election

Using (3.39), (3.62) and (3.78), we have:

$$\begin{aligned} \frac{d\Pr\left(R\right)}{dy^m} &= -\frac{1}{2\overline{\varepsilon}}\left(\frac{\partial \varepsilon_L}{\partial \eta}\frac{\partial \eta}{\partial y^m} + \frac{\partial \varepsilon_L}{\partial y^m}\right) \Leftrightarrow \\ \frac{d\Pr\left(R\right)}{dy^m} &= \frac{1}{2\overline{\varepsilon}}\left\{\left[\frac{\gamma}{\left(1-\theta\right)y^m\Gamma}\frac{\partial \eta}{\partial y^m}\right] + \left[\frac{-\left(\eta - I\right)\gamma\left(1-\theta\right)\Gamma}{\left(\left(1-\theta\right)y^m\Gamma\right)^2}\right]\right\} \Leftrightarrow \\ \frac{d\Pr\left(R\right)}{dy^m} &= \frac{1}{2\overline{\varepsilon}}\left\{\left[\frac{\gamma}{\left(1-\theta\right)y^m\Gamma}\frac{\partial \eta}{\partial y^m}\right] + \left[\frac{-\left(\eta - I\right)\gamma}{\left(1-\theta\right)\left(y^m\right)^2\Gamma}\right]\right\} \Leftrightarrow \\ \frac{d\Pr\left(R\right)}{dy^m} &= \frac{1}{2\overline{\varepsilon}}\left[\frac{\gamma}{\left(1-\theta\right)y^m\Gamma}\right]\left[\frac{\partial \eta}{\partial y^m} - \frac{\left(\eta - I\right)}{y^m}\right] \gtreqless 0. \end{aligned} \tag{3.92}$$

3.E Welfare analysis

Taking a utilitarian perspective in which each individual receives an equal weight and assuming linear utility from private consumption, expected social welfare can be written as:

$$\mathrm{E}_0\left\{U\right\} = \frac{1}{2\overline{\varepsilon}}\left\{\begin{array}{c} \int_{\varepsilon_L}^{\overline{\varepsilon}}\left[\left(1-\theta\right)\overline{y}(1+\varepsilon)\left(2+\Gamma\right) + \left(\eta - I\right)\gamma\right]d\varepsilon + \\ \int_{\varepsilon_L}^{\overline{\varepsilon}}\lambda\left[v\left(f_1^R\right) + pv\left(f_2^R\right) + \left(1-p\right)v\left(g_2^R\right)\right]d\varepsilon + \\ \int_{-\overline{\varepsilon}}^{\varepsilon_L}\left[2\left(1-\theta\right)\overline{y}(1+\varepsilon)\right]d\varepsilon + \int_{-\overline{\varepsilon}}^{\varepsilon_L}\lambda\left[v\left(g_1^{NR}\right) + pv\left(g_2^{NR}\right) + \left(1-p\right)v\left(f_2^{NR}\right)\right]d\varepsilon \end{array}\right\},$$

which leads to equation (3.24).

3.E.1 Effect of k on the expected social welfare

Differentiating (3.24) with respect to k, evaluated at $k = 0$, and applying the Leibnitz's rule:

$$\frac{\partial \mathrm{E}_0\{U\}}{\partial k}=\frac{1}{2\overline{\varepsilon}}\left\{\begin{array}{c}-\left[(\eta-I)\gamma+(1-\theta)\Gamma\overline{y}\right]\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial k}+\frac{\partial\varepsilon_L}{\partial k}\right)+(\overline{\varepsilon}-\varepsilon_L)\gamma\frac{\partial\eta}{\partial k}\\ -\frac{2\varepsilon_L}{2}(1-\theta)\Gamma\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial k}+\frac{\partial\varepsilon_L}{\partial k}\right)+\\ \lambda\left[\begin{array}{c}v\left(f_{1\overline{\varepsilon}}^R\right)\frac{\partial\overline{\varepsilon}}{\partial k}-v\left(f_{1L}^R\right)\frac{\partial\varepsilon_L}{\partial k}+\int_{\varepsilon_L}^{\overline{\varepsilon}}\frac{\partial\left[v\left(f_1^R\right)\right]}{\partial k}d\varepsilon+pv\left(f_{2\overline{\varepsilon}}^R\right)\frac{\partial\overline{\varepsilon}}{\partial k}\\ -pv\left(f_{2L}^R\right)\frac{\partial\varepsilon_L}{\partial k}+p\int_{\varepsilon_L}^{\overline{\varepsilon}}\frac{\partial\left[v\left(f_2^R\right)\right]}{\partial k}d\varepsilon+(1-p)v\left(g_{2\overline{\varepsilon}}^R\right)\frac{\partial\overline{\varepsilon}}{\partial k}\\ -(1-p)v\left(g_{2L}^R\right)\frac{\partial\varepsilon_L}{\partial k}+(1-p)\int_{\varepsilon_L}^{\overline{\varepsilon}}\frac{\partial\left[v\left(g_2^R\right)\right]}{\partial k}d\varepsilon\end{array}\right]+\\ \lambda\left[\begin{array}{c}v\left(g_{1L}^{NR}\right)\frac{\partial\varepsilon_L}{\partial k}-v\left(g_{1-\overline{\varepsilon}}^{NR}\right)\frac{\partial(-\overline{\varepsilon})}{\partial k}+\int_{-\overline{\varepsilon}}^{\varepsilon_L}\frac{\partial\left[v\left(g_1^{NR}\right)\right]}{\partial k}d\varepsilon+pv\left(g_{2L}^{NR}\right)\frac{\partial\varepsilon_L}{\partial k}\\ -pv\left(g_{2-\overline{\varepsilon}}^{NR}\right)\frac{\partial(-\overline{\varepsilon})}{\partial k}+p\int_{-\overline{\varepsilon}}^{\varepsilon_L}\frac{\partial\left[v\left(g_2^{NR}\right)\right]}{\partial k}d\varepsilon+(1-p)v\left(f_{2L}^{NR}\right)\frac{\partial\varepsilon_L}{\partial k}\\ -(1-p)v\left(f_{2-\overline{\varepsilon}}^{NR}\right)\frac{\partial(-\overline{\varepsilon})}{\partial k}+(1-p)\int_{-\overline{\varepsilon}}^{\varepsilon_L}\frac{\partial\left[v\left(f_2^{NR}\right)\right]}{\partial k}d\varepsilon\end{array}\right]\end{array}\right\},$$

where f_{1L}^R is given by (3.20), $f_{2L}^R=g_{2L}^R$ by (3.21), g_{1L}^{NR} by (3.50), $g_{2L}^{NR}=f_{2L}^{NR}$ by (3.51), $f_{1\overline{\varepsilon}}^R$ by (3.63), and

$$f_{2\overline{\varepsilon}}^R = g_{2\overline{\varepsilon}}^R=\theta.\overline{y}.(1+\overline{\varepsilon})(1+\Gamma)-b^R[\overline{\varepsilon}] \tag{3.93}$$

$$g_{1-\overline{\varepsilon}}^{NR} = \theta.\overline{y}.(1-\overline{\varepsilon})+b^{NR}[-\overline{\varepsilon}], \tag{3.94}$$

$$g_{2-\overline{\varepsilon}}^{NR} = f_{2-\overline{\varepsilon}}^{NR}=\theta.\overline{y}.(1-\overline{\varepsilon})-b^{NR}[-\overline{\varepsilon}]. \tag{3.95}$$

Given that in budgetary terms $f_{2\overline{\varepsilon}}^R=g_{2\overline{\varepsilon}}^R$, $f_{2L}^R=g_{2L}^R$, $g_{2L}^{NR}=f_{2L}^{NR}$, $g_{2-\overline{\varepsilon}}^{NR}=f_{2-\overline{\varepsilon}}^{NR}$; and in terms of utility of public good we assume that the same number of individuals λ enjoy public good F and G, we can simplify the last equation to:

$$\frac{\partial \mathrm{E}_0\{U\}}{\partial k}=\frac{1}{2\overline{\varepsilon}}\left\{\begin{array}{c}-\left[(\eta-I)\gamma+(1-\theta)\Gamma\overline{y}\right]\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial k}+\frac{\partial\varepsilon_L}{\partial k}\right)+(\overline{\varepsilon}-\varepsilon_L)\gamma\frac{\partial\eta}{\partial k}\\ -\frac{2\varepsilon_L}{2}(1-\theta)\Gamma\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial k}+\frac{\partial\varepsilon_L}{\partial k}\right)+\\ \lambda\left[\begin{array}{c}v\left(f_{1\overline{\varepsilon}}^R\right)\frac{\partial\overline{\varepsilon}}{\partial k}-v\left(f_{1L}^R\right)\frac{\partial\varepsilon_L}{\partial k}+\int_{\varepsilon_L}^{\overline{\varepsilon}}\frac{\partial\left[v\left(f_1^R\right)\right]}{\partial k}d\varepsilon+v\left(f_{2\overline{\varepsilon}}^R\right)\frac{\partial\overline{\varepsilon}}{\partial k}-\\ v\left(f_{2L}^R\right)\frac{\partial\varepsilon_L}{\partial k}+\int_{\varepsilon_L}^{\overline{\varepsilon}}\frac{\partial\left[v\left(f_2^R\right)\right]}{\partial k}d\varepsilon+v\left(g_{1L}^{NR}\right)\frac{\partial\varepsilon_L}{\partial k}-v\left(g_{1-\overline{\varepsilon}}^{NR}\right)\frac{\partial(-\overline{\varepsilon})}{\partial k}+\\ \int_{-\overline{\varepsilon}}^{\varepsilon_L}\frac{\partial\left[v\left(g_1^{NR}\right)\right]}{\partial k}d\varepsilon+v\left(g_{2L}^{NR}\right)\frac{\partial\varepsilon_L}{\partial k}-v\left(g_{2-\overline{\varepsilon}}^{NR}\right)\frac{\partial(-\overline{\varepsilon})}{\partial k}+\int_{-\overline{\varepsilon}}^{\varepsilon_L}\frac{\partial\left[v\left(g_2^{NR}\right)\right]}{\partial k}d\varepsilon\end{array}\right]\end{array}\right\},$$

Since $\frac{\partial \overline{\varepsilon}}{\partial k}=0$, $\frac{\partial \varepsilon_L}{\partial k}=0$, and knowing (3.39), we can rewrite the last expression as:

$$\frac{\partial \mathrm{E}_0\{U\}}{\partial k} = \frac{1}{2\overline{\varepsilon}}\left\{\begin{array}{c} -\left[(\eta-I)\gamma+(1-\theta)\Gamma\overline{y}\right]\left(-\frac{\gamma}{(1-\theta)y^m\Gamma}\right)\frac{\partial\eta}{\partial k}+(\overline{\varepsilon}-\varepsilon_L)\gamma\frac{\partial\eta}{\partial k} \\ -\varepsilon_L(1-\theta)\Gamma\overline{y}\left(-\frac{\gamma}{(1-\theta)y^m\Gamma}\right)\frac{\partial\eta}{\partial k} \\ \lambda\left[\begin{array}{c} -v\left(f_{1L}^R\right)\left(-\frac{\gamma}{(1-\theta)y^m\Gamma}\right)\frac{\partial\eta}{\partial k}+\int_{\varepsilon_L}^{\overline{\varepsilon}} v'\left(f_1^R\right)\left(-\gamma\frac{\partial\eta}{\partial k}+\frac{\partial b^R}{\partial k}\right)d\varepsilon \\ -v\left(f_{2L}^R\right)\left(-\frac{\gamma}{(1-\theta)y^m\Gamma}\right)\frac{\partial\eta}{\partial k}+\int_{\varepsilon_L}^{\overline{\varepsilon}} v'\left(f_2^R\right)\left(-\frac{\partial b^R}{\partial k}\right)d\varepsilon+ \\ v\left(g_{1L}^{NR}\right)\left(-\frac{\gamma}{(1-\theta)y^m\Gamma}\right)\frac{\partial\eta}{\partial k}+\int_{-\overline{\varepsilon}}^{\varepsilon_L} v'\left(g_1^{NR}\right)\left(+\frac{\partial b^{NR}}{\partial k}\right)d\varepsilon+ \\ v\left(g_{2L}^{NR}\right)\left(-\frac{\gamma}{(1-\theta)y^m\Gamma}\right)\frac{\partial\eta}{\partial k}+\int_{-\overline{\varepsilon}}^{\varepsilon_L} v'\left(g_2^{NR}\right)\left(-\frac{\partial b^{NR}}{\partial k}\right)d\varepsilon \end{array}\right] \end{array}\right\} \Leftrightarrow \tag{3.96}$$

$$\frac{\partial \mathrm{E}_0\{U\}}{\partial k} = \frac{1}{2\overline{\varepsilon}}\left\{\begin{array}{c} \gamma\left\{\begin{array}{c} \frac{\partial\eta}{\partial k}\left[\frac{1}{(1-\theta)y^m\Gamma}\right]\left\{\begin{array}{c}(1-\theta)\Gamma\overline{y}(1+\varepsilon_L)+(\eta-I)\gamma \\ +\lambda\left[\begin{array}{c} v\left(f_{1L}^R\right)+v\left(f_{2L}^R\right) \\ -v\left(g_{1L}^{NR}\right)-v\left(g_{2L}^{NR}\right)\end{array}\right]\end{array}\right\} \\ +\int_{\varepsilon_L}^{\overline{\varepsilon}}\frac{\partial\eta}{\partial k}\left[1-\lambda v'\left(f_1^R\right)\right]d\varepsilon \end{array}\right\} \\ +\lambda\left[\int_{\varepsilon_L}^{\overline{\varepsilon}}\frac{\partial b^R}{\partial k}\left[v'\left(f_1^R\right)-v'\left(f_2^R\right)\right]d\varepsilon+\int_{-\overline{\varepsilon}}^{\varepsilon_L}\frac{\partial b^{NR}}{\partial k}\left[v'\left(g_1^{NR}\right)-v'\left(g_2^{NR}\right)\right]d\varepsilon\right] \end{array}\right\}.$$

3.F Numerical check of Proposition 3.1 for $p < 1$

Given that we are only able to formally prove Proposition 3.1 for p sufficiently close to 1, we perform a numerical check of the results of the proposition for values of $p < 1$. We base this check on the specifications (3.25) for $v(.)$ and (3.26) for $\Gamma(.)$.

The baseline parameter combination for the calibration is the following: $\xi = 1.03$, $\overline{y} = 10$, $p = \theta = \gamma = 0.5$, $I = 2$, $q = 0.25$, $\overline{\varepsilon} = 1.2$, $y^m = 0.8y$, $\overline{b} = 0.3$. This baseline is to some extent motivated by a rough assessment of the European situation, being one where deficit restrictions are actually implemented. Average income (GDP), $\overline{y}$, captures the size of the economy and can be freely chosen; ξ should be sufficiently close to unity to ensure positive marginal utilities for public consumption; with $q = 0.25$, the resulting equilibrium value for γ would imply 8.4% of additional GDP in the long run compared to the case in which no reform takes place. This is in line with the existing estimates for the benefits from structural reform in Europe – see IMF (2003). In addition, the value of $\overline{\varepsilon}$ implies a standard deviation of $(1/3)^{0.5}\,\overline{\varepsilon} = 0.69$.[27] Median income, y^m, is 80% of average income across the population, which is in line with data for the EU-25 – see Boix (2004). Finally, $\overline{b}$ equals 3% of average GDP, which is roughly in line with Europe's SGP (where the deficit limit, however, is expressed as a share of actual, rather than natural, GDP).

We confirm the validity of Proposition 3.1 for the baseline parameter combination for

[27] For an assessment of the magnitude of business cycle fluctuations in Europe, see Artis et al. (2003) or Mitchell and Mouratidis (2004).

$k = 0$ and $k = 0.2$. (The value of k should not be too high to ensure that there is still a deficit bias, which is the relevant situation for our purpose.) Next, we assess the robustness of the results by allowing parameters to deviate from their baseline values. In particular, we investigate the validity of Proposition 3.1 for all parameter combinations formed by the Carthesian product of the parameter sets $p \in \{0.35, 0.8\}$, $\theta \in \{0.25, 0.7\}$, $\gamma \in \{0.1, 2\}$, $I \in \{0.5, 4\}$, $\overline{\varepsilon} \in \{0.8, 1.6\}$, $q \in \{0.1, 0.5\}$, $y^m \in \{0.5\overline{y}, 0.95\overline{y}\}$, and $\overline{b} \in \{0.1, 1\}$, while the remaining parameters are kept at their baseline values.[28] The set of possible parameter combinations covers a wide range of the relevant parameter space. We find that for all possible parameter combinations, both at $k = 0$ and $k = 0.2$, a small increase in k reproduces all results stated in Proposition 3.1.

3.G Extensions and a variation

In this appendix, we vary our basic model in a number of ways and explore how these changes affect the results.

3.G.1 A contingent deficit restriction

As we have seen, while our deficit restriction improves the intertemporal allocation of public spending, its disadvantage is that it discourages structural reform, by making it more expensive for the government to provide up-front compensation. The question then is whether it is possible to device a restriction that limits its possible damage to the likelihood of structural reform. Therefore, we consider a slightly more complicated arrangement with

$$\overline{b} = \overline{b^c} + \delta_1 \eta - \delta_2 \varepsilon, \quad \delta_1, \delta_2 \geq 0, \tag{3.97}$$

so that the *actual* reference deficit level $\overline{b}$ is now a combination of a constant component $\overline{b^c}$ and two components that take account of the compensation expenditures and of the macroeconomic shock (the "business cycle"). Both larger compensation outlays η and a worsening of the business cycle (a fall in ε) lead to an increase in the reference deficit level. This arrangement reflects some features of the recent reform of the SGP, which in its implementation now takes more explicitly into account both the short-run costs of structural reform as well as the business cycle situation (see European Commission, 2005b, and ECOFIN, 2005).

[28]We want to confine ourselves to parameter combinations under which the initial incumbent would benefit from the implementation of the reform (and would have an incentive in the first place to introduce reform). This requires p to be larger than 30%. If p is too low, the chance that the initial incumbent would be able to spend the additional second-period resources associated with reform would become too low.

With our contingent deficit restriction (3.97), the first-order condition for η becomes:

$$\left\{ \begin{array}{c} v\left(f_{1L}^{R}\right) + pv\left(f_{2L}^{R}\right) - \\ k\left(b_L^R - \overline{b^c} - \delta_1\eta + \delta_2\varepsilon_L\right) - (1-p)\,v\left(f_{2L}^{NR}\right) \end{array} \right\} = (1-\theta)\,y^m\Gamma(\gamma)\left[\int_{\varepsilon_L}^{\overline{\varepsilon}} v'[f_1^R]d\varepsilon - \frac{k\delta_1}{\gamma}\left(\overline{\varepsilon} - \varepsilon_L\right)\right]. \tag{3.98}$$

This last expression yields at most one solution when p is sufficiently large and both δ_1 and δ_2 are small enough. The implications of making the deficit restriction more flexible for given k are summarized in the following proposition:

Proposition 3.4 *Let p be sufficiently large. If $\delta_1 > 0$ ($\delta_2 > 0$) is small enough, then a stronger contingency of the reference deficit level on reform compensation or the business cycle (i.e., a higher δ_1, respectively δ_2) leads to an increase in the amount of compensation and improves the likelihood of re-election of the initial incumbent and thus of reform. Moreover, for any given shock ε, an increase in δ_1 (δ_2) implies a higher deficit in case the initial incumbent is re-elected and reform materializes.*

Proof. See Appendices 3.H.1 and 3.H.1. ■

By raising the reference deficit level in response to reform-related compensation, the initial government is induced to provide more compensation, because the marginal cost associated with the additional debt needed to spread the compensation over time is smaller. Of course, the enhanced likelihood of reform is bought at the cost of a higher deficit when reform actually takes place. Similarly, if the reference deficit level is raised in response to a bad shock ($\varepsilon < 0$), the initial government is induced to offer more compensation since any compensation that has to be paid under relatively bad economic circumstances will hurt less if the marginal cost of issuing debt is reduced.

Of course, proper implementation of a contingent deficit restriction in reality requires more detailed budgetary and macroeconomic information than a non-contingent restriction. For example, the enforcer of the restriction needs to identify what share of spending was specifically needed as compensation for the reform and what was the size of the original output shock (rather than the output movement itself, which is subject to the policies followed by the government). Further, and in relation to these requirements, the arrangement should be designed in such a way that governments have no incentive to abuse the enhanced flexibility of the restriction, for example by disguising government consumption as reform expenditure.

Given that we can only formally prove Proposition 3.4 when p is sufficiently close to 1, we resort again to a numerical evaluation. The numerical results confirm the effects described in Proposition 3.4 of an increase in the contingency parameter δ_1 or δ_2 (starting at $\delta_1 = 0$, respectively $\delta_2 = 0$) for all parameter combinations considered earlier.

3.G.2 Targeting compensation

In the main text we assumed that every individual receives the same amount of compensation. From the perspective of the initial incumbent it is suboptimal to disregard the income distribution when deciding about compensation. Some people (those with sufficiently high incomes) would in any case vote for the incumbent, whether they receive compensation or not. Others would in any case *not* vote for the government, unless they receive extremely high compensation. Loosely speaking, the best strategy would be to target compensation only at those who are likely to change their voting behavior as a result of the compensation. That is, the government should target compensation at those who are not too far towards the extremes of the income distribution. We show that, under certain conditions, if the government targets compensation, its probability of re-election is still given by (3.17), implying that the preceding analysis carries through (although the equilibrium outcome for compensation is now lower).

To this end, let us rank individuals by income and index them by $i \in [0,1]$, where the lowest value of i corresponds to the individuals with the lowest income. Compare the argument in the utility functions in (3.4) and (3.5), and define $\underline{i}$ such that, if $i = \underline{i}$, then $(1-\theta)\, y_i(1+\varepsilon)\Gamma + (\eta - I)\,\gamma = 0$ and $\bar{i}$ such that, if $i = \bar{i}$, then $(1-\theta)\, y_i(1+\varepsilon)\Gamma - I\gamma = 0$. Hence, for given values of compensation η and income shock $\varepsilon \in [-\bar{\varepsilon}, \bar{\varepsilon}]$, we can divide individuals into the following three intervals:[29]

- Individuals in the interval $[0, \underline{i}]$ are net losers from the reform.
- Individuals in the interval $\left[\underline{i}, \bar{i}\right]$ have a net benefit from the reform *as a result of the compensation.*
- Individuals in the interval $\left[\bar{i}, 1\right]$ are net beneficiaries from the reform also without compensation ($\eta = 0$).

For the specific case in which individual income increases linearly in the index:

$$y_i \equiv y(i) = y.i,$$

and combining this with the assumption that the individuals who do receive compensation from the government, all receive the same amount, we find that:

$$\underline{i} = \frac{(I-\eta)\,\gamma}{(1-\theta)\, y(1+\varepsilon)\Gamma} \text{ and } \bar{i} = \frac{I\gamma}{(1-\theta)\, y(1+\varepsilon)\Gamma}.$$

Hence, the larger is the reform benefit Γ or the smaller is the marginal reform cost I, the smaller is the interval $\left[\underline{i}, \bar{i}\right]$ and thus the number of individuals to be compensated in order to switch to supporting the initial incumbent and thus the reform.

[29]This requires that for all possible realizations of ε, $\lambda < \underline{i}$ and $1 - \lambda > 1 - \bar{i}$, while, of course, the condition in Footnote 10 should continue to hold.

Finally, denoting the density of individuals on the interval $[0,1]$ by $h(i)$, we can compute the share α of the population to which compensation should be targeted as:

$$\alpha = \int_{\underline{i}}^{\overline{i}} h(i)\, di.$$

Total compensation provided by the initial incumbent is then $\eta\gamma\alpha$. Writing $\alpha = \alpha(\eta, \varepsilon)$, the probability that Party F is re-elected in period 1 becomes
$\Pr(R) = \Pr\left[\alpha(\eta,\varepsilon) + \int_{\overline{i}}^{1} h(i)\, di \geq 0.5\right]$. Finally, if we assume that individuals are uniformly distributed on the interval $[0,1]$, then $\alpha = \overline{i} - \underline{i}$ and

$$\Pr(R) = \Pr\left[1 - \underline{i} \geq 0.5\right] = \Pr\left[\varepsilon \geq -\frac{2(\eta - I)\gamma}{(1-\theta)\, y\Gamma} - 1\right],$$

which, for given η, is equal to (3.17).[30] Therefore, when $\varepsilon = \varepsilon_L$, $\underline{i}$ represents the median voter. Of course, one should realize that, while for given η, ε_L is the same as before, total compensation will be lower when compared to the baseline case in which compensation was not targeted. Hence, for a given total outlay on compensation, a larger number of individuals can now be persuaded to switch towards favoring reforms.

3.G.3 Pecuniary sanctions

In the final extension we model the deficit restriction as an explicit fine in the government budget constraint. We allow for monetary sanctions in period 2, that is, sanctions are ex-post to the realization of the deficit in the first period. For example, in the case of Europe's SGP, fines can only materialize some time (at least two years) after the excessive deficit violation has taken place. We now drop the cost of the deficit restriction from the government's utility (3.6) – that is, we set $\Delta_F = 0$. Hence, the second-period government budget constraints in the case of reform and no-reform become:

$$f_2^R + g_2^R = \theta\overline{y}(1+\varepsilon)(1+\Gamma) - b^R - k\left(b^R - \overline{b}\right), \quad f_2^{NR} + g_2^{NR} = \theta\overline{y}(1+\varepsilon) - b^{NR} - k\left(b^{NR} - \overline{b}\right). \tag{3.99}$$

when $b^R \geq \overline{b}$ and $b^{NR} \geq \overline{b}$, while the final term in both constraints drops out when the deficit is below $\overline{b}$. As before, we assume that this reference level $\overline{b}$ is set sufficiently low, so that the equilibrium deficit level exceeds its reference level. The new formulation corresponds partly to the operation of the SGP in Europe, where sanctions for excessive deficits can culminate into fines imposed on the governments. Obviously, the comparison can only be limited. For example, the fines under the SGP are only linear in the degree of excessiveness of the deficit over a certain range, while here they are linear over the entire range of deficits above $\overline{b}$. The linearity assumption obviously simplifies matters a lot and allows us to obtain analytical results. The optimal sanction scheme in this framework could well be non-linear. However, in reality, there is a strong case for simple schemes,

[30] Note that, in this case, $\overline{y} = y^m$.

both for the understanding of the general public and for the politicians' understanding of the consequences of their fiscal behavior.

With a new utility function and government budget constraints, the first-order conditions for the optimal deficit, when parties F and G, respectively, assume power in period 1 are:

$$v'\left(f_1^R\right) = (1+k)\, pv'\left(f_2^R\right), \quad v'\left(g_1^{NR}\right) = (1+k)\, pv'\left(g_2^{NR}\right), \tag{3.100}$$

with f_1^R and g_1^{NR} as described before and f_2^R and g_2^{NR} as given in (3.99) above. We see that an increase in the tightness of sanctions, k, causes a shift in public spending away from the first period towards the second period. This is intuitive, because an increase in k raises the cost of running a deficit and thus, effectively, makes first-period public spending more expensive relative to second-period public spending. Appendix 3.I shows that also the other results of Lemma 3.1 continue to hold.

To find the overall effect of an increase in k on the deficit (taking account of the effect via η), we first need to find the optimal level of compensation. The new first-order condition for η differs only slightly from (3.23) – see Appendix 3.I. By differentiating the first-order conditions further, one can again check that the effects of an increase in k, as stated in Proposition 3.1, continue to hold.

3.H A contingent deficit restriction – derivations

With (3.97) we have:

$$\max_{\eta} \frac{1}{2\overline{\varepsilon}} \left\{ \begin{array}{c} \int_{\varepsilon_L}^{\overline{\varepsilon}} v\left(\theta\overline{y}(1+\varepsilon) - \eta\gamma + b^R\right) d\varepsilon + p. \int_{\varepsilon_L}^{\overline{\varepsilon}} v\left(\theta\overline{y}(1+\varepsilon)\left(1+\Gamma\right) - b^R\right) d\varepsilon + \\ \int_{\varepsilon_L}^{\overline{\varepsilon}} -k\left(b^R - \overline{b^c} - \delta_1\eta + \delta_2\varepsilon\right) d\varepsilon + \int_{-\overline{\varepsilon}}^{\varepsilon_L} 0 d\varepsilon + (1-p) . \int_{-\overline{\varepsilon}}^{\varepsilon_L} v\left(\theta\overline{y}(1+\varepsilon) - b^{NR}\right) d\varepsilon \end{array} \right\}.$$

We use the same procedure as before, maximizing this last expression on η and applying Leibnitz's rule. Using again the facts that $\frac{\partial\overline{\varepsilon}}{\partial\eta} = 0$, $v'\left(f_1^R\right) - pv'\left(f_2^R\right) - k = 0$, $\frac{\partial b^{NR}}{\partial\eta} = 0$ and using (3.20) to (3.22), we derive the new first-order condition:

$$\begin{aligned} & -\frac{\partial\varepsilon_L}{\partial\eta}\left[v\left(f_{1L}^R\right) + pv\left(f_{2L}^R\right) - k\left(b_L^R - \overline{b^c} - \delta_1\eta + \delta_2\varepsilon_L\right) - (1-p)\, v\left(f_{2L}^{NR}\right)\right] + k\delta_1\left(\overline{\varepsilon} - \varepsilon_L\right) \\ = \ & \gamma \int_{\varepsilon_L}^{\overline{\varepsilon}} v'\left(f_1^R\right) d\varepsilon. \end{aligned}$$

Using (3.39) and simplifying the last expression, we arrive to (3.98).

3.H.1 Comparative statics

Effect of δ_1 on η, b^R , b^{NR}

We differentiate (3.98) with respect to δ_1:

$$\begin{aligned}
& v'\left(f_{1L}^R\right)\left[\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\delta_1}+\frac{\partial\varepsilon_L}{\partial\delta_1}\right)-\gamma\frac{\partial\eta}{\partial\delta_1}+\frac{\partial b_L^R}{\partial\delta_1}\right]+pv'\left(f_{2L}^R\right)\left[\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\delta_1}+\frac{\partial\varepsilon_L}{\partial\delta_1}\right)(1+\Gamma)-\frac{\partial b_L^R}{\partial\delta_1}\right] \\
& -k\left[\frac{\partial b_L^R}{\partial\delta_1}-\eta-\delta_1\frac{\partial\eta}{\partial\delta_1}+\delta_2\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\delta_1}+\frac{\partial\varepsilon_L}{\partial\delta_1}\right)\right]-(1-p)\,v'\left(f_{2L}^{NR}\right)\left\{\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\delta_1}+\frac{\partial\varepsilon_L}{\partial\delta_1}\right)-\frac{\partial b_L^{NR}}{\partial\delta_1}\right\} \\
& -(1-\theta)\,y^m\Gamma\left\{\begin{array}{c} v'\left(f_{1\overline{\varepsilon}}^R\right)\frac{\partial\overline{\varepsilon}}{\partial\delta_1}-v'\left(f_{1L}^R\right)\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\delta_1}+\frac{\partial\varepsilon_L}{\partial\delta_1}\right)+\int_{\varepsilon_L}^{\overline{\varepsilon}}v"\left(f_1^R\right)\left(-\gamma\frac{\partial\eta}{\partial\delta_1}+\frac{\partial b^R}{\partial\delta_1}\right)d\varepsilon \\ -\frac{k}{\gamma}\left(\overline{\varepsilon}-\varepsilon_L\right)+\frac{k\delta_1}{\gamma}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\delta_1}+\frac{\partial\varepsilon_L}{\partial\delta_1}\right)\end{array}\right\} \\
= & \ 0\Leftrightarrow
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial\eta}{\partial\delta_1}\left\{-\frac{\theta\overline{y}\gamma}{(1-\theta)\,y^m\Gamma}\left[\begin{array}{c} v'\left(f_{1L}^R\right)+(1+\Gamma)\,pv'\left(f_{2L}^R\right) \\ -k\delta_2-(1-p)\,v'\left(f_{2L}^{NR}\right)\end{array}\right]+2k\delta_1-2\gamma v'\left(f_{1L}^R\right)\right\} \\
& +\frac{\partial b_L^R}{\partial\delta_1}\left[v'\left(f_{1L}^R\right)-pv'\left(f_{2L}^R\right)-k\right]+\frac{\partial b_L^{NR}}{\partial\delta_1}\left[v'\left(f_{2L}^{NR}\right)(1-p)\right] \\
& -(1-\theta)\,y^m\Gamma\int_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma\frac{\partial\eta}{\partial\delta_1}+\frac{\partial b^R}{\partial\delta_1}\right)v"\left(f_1^R\right)d\varepsilon \\
= & \ -k\eta-\frac{(1-\theta)\,y^m\Gamma k}{\gamma}\left(\overline{\varepsilon}-\varepsilon_L\right).
\end{aligned}$$

This latter expression can be represented in the same way as in (3.40) by:

$$A_{1L}'\frac{\partial\eta}{\partial\delta_1}-A_2\int_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma\frac{\partial\eta}{\partial\delta_1}+\frac{\partial b^R}{\partial\delta_1}\right)v"\left(f_1^R\right)d\varepsilon+A_{3L}\frac{\partial b_L^{NR}}{\partial\delta_1}=M_{1L}, \tag{3.101}$$

with the new parameters:[31]

$$A_{1L}' = -\frac{\theta\overline{y}\gamma}{(1-\theta)y^m\Gamma[\gamma]}\left[\begin{array}{c} v'\left(f_{1L}^R\right)+(1+\Gamma)\,pv'\left(f_{2L}^R\right) \\ -k\delta_2-(1-p)\,v'\left(f_{2L}^{NR}\right)\end{array}\right]+2k\delta_1-2\gamma v'\left(f_{1L}^R\right)<0, \tag{3.102}$$

$$M_{1L} = -k\eta-\frac{(1-\theta)\,y^m\Gamma k}{\gamma}\left(\overline{\varepsilon}-\varepsilon_L\right)<0. \tag{3.103}$$

Effect on b^R Differentiating (3.27) with respect to δ_1, we have:

$$v"\left(f_1^R\right)\left(-\gamma\frac{\partial\eta}{\partial\delta_1}+\frac{\partial b^R}{\partial\delta_1}\right)-pv"\left(f_2^R\right)\left(-\frac{\partial b^R}{\partial\delta_1}\right)=0.$$

This last expression can be represented by:

$$B_1\frac{\partial\eta}{\partial\delta_1}+B_2\frac{\partial b^R}{\partial\delta_1}=0. \tag{3.104}$$

[31] The sign of A_{1L}' holds if p is sufficiently different from 0, and δ_2 and δ_1 are small enough.

Effect on b^{NR} Differentiating (3.49) with respect to δ_1:

$$v"\left(g_{1L}^{NR}\right)\left[\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\delta_1}+\frac{\partial\varepsilon_L}{\partial\delta_1}\right)+\frac{\partial b_L^{NR}}{\partial\delta_1}\right]-pv"\left(g_{2L}^{NR}\right)\left[\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\delta_1}+\frac{\partial\varepsilon_L}{\partial\delta_1}\right)-\frac{\partial b_L^{NR}}{\partial\delta_1}\right]=0,$$

and, since $\frac{\partial\varepsilon_L}{\partial\delta_1}=0$ and using (3.39), we can represent the last as:

$$C_{1L}\frac{\partial\eta}{\partial\delta_1}+C_{2L}\frac{\partial b_L^{NR}}{\partial\delta_1}=0. \tag{3.105}$$

Solution of the system We can write the system (3.101), (3.104), and (3.105) as:

$$\begin{cases} A'_{1L}\frac{\partial\eta}{\partial\delta_1} & +0 & -A_2\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma\frac{\partial\eta}{\partial\delta_1}+\frac{\partial b^R}{\partial\delta_1}\right)v"\left(f_1^R\right)d\varepsilon & +A_{3L}\frac{\partial b_L^{NR}}{\partial\delta_1} & = & M_{1L} \\ B_1\frac{\partial\eta}{\partial\delta_1} & +B_2\frac{\partial b^R}{\partial\delta_1} & +0 & +0 & = & 0 \\ C_{1L}\frac{\partial\eta}{\partial\delta_1} & +0 & +0 & +C_{2L}\frac{\partial b_L^{NR}}{\partial\delta_1} & = & 0 \end{cases}.$$

Further, we can rewrite (3.104) and (3.105) as

$$\frac{\partial b^R}{\partial\delta_1}=-\frac{B_1}{B_2}\frac{\partial\eta}{\partial\delta_1},\quad\text{and}\quad\frac{\partial b_L^{NR}}{\partial\delta_1}=-\frac{C_{1L}}{C_{2L}}\frac{\partial\eta}{\partial\delta_1}. \tag{3.106}$$

Substituting these two equations in (3.101), we have:

$$A'_{1L}\frac{\partial\eta}{\partial\delta_1}-A_2\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\frac{\partial\eta}{\partial\delta_1}\left(-\gamma-\frac{B_1}{B_2}\right)v"\left(f_1^R\right)d\varepsilon-\frac{A_{3L}C_{1L}}{C_{2L}}\frac{\partial\eta}{\partial\delta_1}=M_{1L}. \tag{3.107}$$

Sign of $\frac{\partial\eta}{\partial\delta_1}$: Substituting the values of (3.42), (3.43), (3.46), (3.47), (3.53), (3.54), (3.102) and (3.103), we are able to rewrite (3.107) as:

$$\frac{\partial\eta}{\partial\delta_1}\left\{\begin{array}{c} A'_{1L}-\frac{A_{3L}C_{1L}}{C_{2L}}- \\ A_2\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\left[\frac{-\gamma\left(v"\left(f_1^R\right)\right)^2-\gamma v"\left(f_1^R\right)pv"\left(f_2^R\right)+\gamma\left(v"\left(f_1^R\right)\right)^2}{v"\left(f_1^R\right)+pv"\left(f_2^R\right)}\right]d\varepsilon\end{array}\right\}=M_{1L}\Leftrightarrow$$

$$\frac{\partial\eta}{\partial\delta_1}\left\{\begin{array}{c} -\frac{\theta\overline{y}\gamma}{(1-\theta)y^m\Gamma[\gamma]}\left[\begin{array}{c} v'\left(f_{1L}^R\right)+(1+\Gamma)\,pv'\left(f_{2L}^R\right) \\ -k\delta_2-(1-p)\,v'\left(f_{2L}^{NR}\right)\end{array}\right]+ \\ 2k\delta_1-2\gamma v'\left(f_{1L}^R\right)+\frac{(1-p)\theta\overline{y}\gamma v'\left(f_{2L}^{NR}\right)\left[v"\left(g_{1L}^{NR}\right)-pv"\left(g_{2L}^{NR}\right)\right]}{(1-\theta)y^m\Gamma\left[v"\left(g_{1L}^{NR}\right)+pv"\left(g_{2L}^{NR}\right)\right]} \\ +(1-\theta)\,y^m\Gamma\gamma\int\limits_{\varepsilon_L}^{\overline{\varepsilon}}\frac{v"\left(f_1^R\right)pv"\left(f_2^R\right)}{v"\left(f_1^R\right)+pv"\left(f_2^R\right)}d\varepsilon\end{array}\right\}=\left\{\begin{array}{c} -k\eta- \\ \frac{(1-\theta)y^m\Gamma k}{\gamma}\left(\overline{\varepsilon}-\varepsilon_L\right)\end{array}\right\}. \tag{3.108}$$

The overall sign of the term in brackets on the left-hand side of (3.108) is negative if p is sufficiently different from 0, and δ_2 and δ_1 are small enough. The right-hand side of that expression is also negative. Hence, for p sufficiently different from 0, $\frac{\partial\eta}{\partial\delta_1}>0$. Further, given (3.106), we know that $\frac{\partial b^R}{\partial\delta_1}>0$ and, in addition, if we suppose that $v"\left(g_{1L}^{NR}\right)-pv"\left(g_{2L}^{NR}\right)<0$, then $\frac{\partial b_L^{NR}}{\partial\delta_1}>0$.

Welfare Effect: The welfare effect of δ_1 can be found by differentiating (3.24) with respect to δ_1:

$$\frac{\partial \mathrm{E}_0\{U\}}{\partial\delta_1}=\frac{1}{2\overline{\varepsilon}}\left\{\begin{array}{c}\gamma\left\{\begin{array}{c}\frac{\partial\eta}{\partial\delta_1}\left[\frac{1}{(1-\theta)y^m\Gamma}\right]\left[\begin{array}{c}(1-\theta)\Gamma\overline{y}(1+\varepsilon_L)+(\eta-I)\gamma\\+v\left[f_{1L}^R\right]+v\left[f_{2L}^R\right]-v\left[g_{1L}^{NR}\right]-v\left[g_{2L}^{NR}\right]\end{array}\right]\\+\int_{\varepsilon_L}^{\overline{\varepsilon}}\frac{\partial\eta}{\partial\delta_1}\left[1-v'\left(f_1^R\right)\right]d\varepsilon\end{array}\right\}\\+\int_{\varepsilon_L}^{\overline{\varepsilon}}\frac{\partial b^R}{\partial\delta_1}\left[v'\left(f_1^R\right)-v'\left(f_2^R\right)\right]d\varepsilon+\int_{-\overline{\varepsilon}}^{\varepsilon_L}\frac{\partial b^{NR}}{\partial\delta_1}\left[v'\left(g_1^{NR}\right)-v'\left(g_2^{NR}\right)\right]d\varepsilon\end{array}\right\}.$$

The sign of this derivative is ambiguous and will be solved numerically.

Effect of δ_2 on η, b^R and b^{NR} under a contingent deficit restriction

Differentiating (3.98) with respect to δ_2:

$$\begin{array}{l}v'\left(f_{1L}^R\right)\left[\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\delta_2}+\frac{\partial\varepsilon_L}{\partial\delta_2}\right)-\gamma\frac{\partial\eta}{\partial\delta_2}+\frac{\partial b_L^R}{\partial\delta_2}\right]+pv'\left(f_{2L}^R\right)\left[\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\delta_2}+\frac{\partial\varepsilon_L}{\partial\delta_2}\right)(1+\Gamma)-\frac{\partial b_L^R}{\partial\delta_2}\right]\\-k\left[\frac{\partial b_L^R}{\partial\delta_2}-\delta_1\frac{\partial\eta}{\partial\delta_2}+\varepsilon_L+\delta_2\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\delta_2}+\frac{\partial\varepsilon_L}{\partial\delta_2}\right)\right]-(1-p)v'\left(f_{2L}^{NR}\right)\left\{\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\delta_2}+\frac{\partial\varepsilon_L}{\partial\delta_2}\right)-\frac{\partial b_L^{NR}}{\partial\delta_2}\right\}\\-(1-\theta)y^m\Gamma\left\{\begin{array}{c}v'\left(f_{1\overline{\varepsilon}}^R\right)\frac{\partial\overline{\varepsilon}}{\partial\delta_2}-v'\left(f_{1L}^R\right)\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\delta_2}+\frac{\partial\varepsilon_L}{\partial\delta_2}\right)+\\\int_{\varepsilon_L}^{\overline{\varepsilon}}v"\left(f_1^R\right)\left(-\gamma\frac{\partial\eta}{\partial\delta_2}+\frac{\partial b^R}{\partial\delta_2}\right)d\varepsilon+\frac{k\delta_1}{\gamma}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial\delta_2}+\frac{\partial\varepsilon_L}{\partial\delta_2}\right)\end{array}\right\}\\=\ 0\Leftrightarrow\end{array}$$

$$\begin{array}{l}\frac{\partial\eta}{\partial\delta_2}\left\{-\frac{\theta\overline{y}\gamma}{(1-\theta)y^m\Gamma}\left[\begin{array}{c}v'\left(f_{1L}^R\right)+(1+\Gamma)pv'\left(f_{2L}^R\right)\\-k\delta_2-(1-p)v'\left(f_{2L}^{NR}\right)\end{array}\right]+2k\delta_1-2\gamma v'\left(f_{1L}^R\right)\right\}\\+\frac{\partial b_L^R}{\partial\delta_2}\left[v'\left(f_{1L}^R\right)-pv'\left(f_{2L}^R\right)-k\right]+\frac{\partial b_L^{NR}}{\partial\delta_2}\left[v'\left(f_{2L}^{NR}\right)(1-p)\right]\\-(1-\theta)y^m\Gamma\int_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma\frac{\partial\eta}{\partial\delta_2}+\frac{\partial b^R}{\partial\delta_2}\right)v"\left(f_1^R\right)d\varepsilon\\=\ k\varepsilon_L.\end{array}$$

This latter expression can be represented in the same way as in (3.101) by:

$$A'_{1L}\frac{\partial\eta}{\partial\delta_2}-A_2\int_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma\frac{\partial\eta}{\partial\delta_2}+\frac{\partial b^R}{\partial\delta_2}\right)v"\left(f_1^R\right)d\varepsilon+A_{3L}\frac{\partial b_L^{NR}}{\partial\delta_2}=N_{1L},\tag{3.109}$$

with A'_{1L} is given by (3.102) and

$$N_{1L}=k\varepsilon_L<0.\tag{3.110}$$

Effect on b^R Differentiating (3.27) with respect to δ_2, we have:

$$v"\left(f_1^R\right)\left(-\gamma\frac{\partial\eta}{\partial\delta_2}+\frac{\partial b^R}{\partial\delta_2}\right)-pv"\left(f_2^R\right)\left(-\frac{\partial b^R}{\partial\delta_2}\right)=0.$$

This last expression can be represented by:

$$B_1 \frac{\partial \eta}{\partial \delta_2} + B_2 \frac{\partial b^R}{\partial \delta_2} = 0. \tag{3.111}$$

Effect on b^{NR} Differentiating (3.49) with respect to δ_2:

$$v"\left(g_{1L}^{NR}\right)\left[\theta \overline{y}\left(\frac{\partial \varepsilon_L}{\partial \eta}\frac{\partial \eta}{\partial \delta_2} + \frac{\partial \varepsilon_L}{\partial \delta_2}\right) + \frac{\partial b_L^{NR}}{\partial \delta_2}\right] - pv"\left(g_{2L}^{NR}\right)\left[\theta \overline{y}\left(\frac{\partial \varepsilon_L}{\partial \eta}\frac{\partial \eta}{\partial \delta_2} + \frac{\partial \varepsilon_L}{\partial \delta_2}\right) - \frac{\partial b_L^{NR}}{\partial \delta_2}\right] = 0,$$

since $\frac{\partial \varepsilon_L}{\partial \delta_2} = 0$ and using (3.39), we can represent the last as:

$$C_{1L} \frac{\partial \eta}{\partial \delta_2} + C_{2L} \frac{\partial b_L^{NR}}{\partial \delta_2} = 0. \tag{3.112}$$

Solution of the system We can write the system (3.109), (3.111), and (3.112) as:

$$\left\{ \begin{array}{llllll} A'_{1L}\frac{\partial \eta}{\partial \delta_2} & +0 & -A_2 \int\limits_{\varepsilon_L}^{\overline{\varepsilon}} \left(-\gamma \frac{\partial \eta}{\partial \delta_2} + \frac{\partial b^R}{\partial \delta_2}\right) v"\left(f_1^R\right) d\varepsilon & +A_{3L}\frac{\partial b_L^{NR}}{\partial \delta_2} & = & N_{1L} \\ B_1 \frac{\partial \eta}{\partial \delta_2} & +B_2 \frac{\partial b^R}{\partial \delta_2} & +0 & +0 & = & 0 \\ C_{1L}\frac{\partial \eta}{\partial \delta_2} & +0 & +0 & +C_{2L}\frac{\partial b_L^{NR}}{\partial \delta_2} & = & 0 \end{array} \right. .$$

Further, we can rewrite (3.111) and (3.112) as

$$\frac{\partial b^R}{\partial \delta_2} = -\frac{B_1}{B_2}\frac{\partial \eta}{\partial \delta_2}, \quad \text{and} \quad \frac{\partial b_L^{NR}}{\partial \delta_2} = -\frac{C_{1L}}{C_{2L}}\frac{\partial \eta}{\partial \delta_2}. \tag{3.113}$$

Substituting these two equations in (3.109), we have:

$$A'_{1L}\frac{\partial \eta}{\partial \delta_2} - A_2 \int\limits_{\varepsilon_L}^{\overline{\varepsilon}} \frac{\partial \eta}{\partial \delta_2}\left(-\gamma - \frac{B_1}{B_2}\right) v"\left(f_1^R\right) d\varepsilon - \frac{A_{3L}C_{1L}}{C_{2L}}\frac{\partial \eta}{\partial \delta_2} = N_{1L}. \tag{3.114}$$

Sign of $\frac{\partial \eta}{\partial \delta_2}$: Substituting the values of (3.42), (3.43), (3.46), (3.47), (3.53), (3.54), (3.102) and (3.110), we are able to rewrite (3.114) as:

$$\frac{\partial \eta}{\partial \delta_2}\left\{ \begin{array}{c} -\frac{\theta \overline{y}\gamma}{(1-\theta)y^m \Gamma[\gamma]}\left[\begin{array}{c} v'\left(f_{1L}^R\right) + (1+\Gamma)\, pv'\left(f_{2L}^R\right) \\ -k\delta_2 - (1-p)\, v'\left(f_{2L}^{NR}\right) \end{array}\right] + \\ 2k\delta_1 - 2\gamma v'\left(f_{1L}^R\right) + \frac{(1-p)\theta \overline{y}\gamma v'\left(f_{2L}^{NR}\right)\left[v"\left(g_{1L}^{NR}\right) - pv"\left(g_{2L}^{NR}\right)\right]}{(1-\theta)y^m \Gamma\left[v"\left(g_{1L}^{NR}\right) + pv"\left(g_{2L}^{NR}\right)\right]} \\ +(1-\theta)\, y^m \Gamma \gamma \int\limits_{\varepsilon_L}^{\overline{\varepsilon}} \frac{v"\left(f_1^R\right) pv"\left(f_2^R\right)}{v"\left(f_1^R\right) + pv"\left(f_2^R\right)} d\varepsilon \end{array} \right\} = k\varepsilon_L. \tag{3.115}$$

The overall sign of the term in brackets on the left-hand side of (3.115) is negative if p is sufficiently different from 0, and δ_2 and δ_1 are small enough. The right-hand side of that expression is also negative. Hence, for p sufficiently different from 0, $\frac{\partial \eta}{\partial \delta_2} > 0$. Further, given (3.113), we know that $\frac{\partial b^R}{\partial \delta_2} > 0$ and, in addition, if we suppose that $v"\left(g_{1L}^{NR}\right) - pv"\left(g_{2L}^{NR}\right) < 0$, then $\frac{\partial b_L^{NR}}{\partial \delta_2} > 0$.

Welfare Effect: The welfare effect of δ_2 can be found by differentiating (3.24) with respect to δ_2:

$$\frac{\partial \mathrm{E}_0\{U\}}{\partial \delta_2} = \frac{1}{2\overline{\varepsilon}} \left\{ \begin{array}{c} \gamma \left\{ \begin{array}{c} \frac{\partial \eta}{\partial \delta_2}\left[\frac{1}{(1-\theta)y^m\Gamma}\right] \left[\begin{array}{c} (1-\theta)\Gamma\overline{y}(1+\varepsilon_L)+(\eta-I)\gamma \\ +v\left[f_{1L}^R\right]+v\left[f_{2L}^R\right]-v\left[g_{1L}^{NR}\right]-v\left[g_{2L}^{NR}\right] \end{array} \right] \\ +\int_{\varepsilon_L}^{\overline{\varepsilon}} \frac{\partial \eta}{\partial \delta_2}\left[1-v'\left(f_1^R\right)\right] d\varepsilon \end{array} \right\} \\ +\int_{\varepsilon_L}^{\overline{\varepsilon}} \frac{\partial b^R}{\partial \delta_2}\left[v'\left(f_1^R\right)-v'\left(f_2^R\right)\right] d\varepsilon + \int_{-\overline{\varepsilon}}^{\varepsilon_L} \frac{\partial b^{NR}}{\partial \delta_2}\left[v'\left(g_1^{NR}\right)-v'\left(g_2^{NR}\right)\right] d\varepsilon \end{array} \right\}.$$

The sign of this derivative is ambiguous and we solve it numerically.

3.I Fines in the government budget constraint – derivations

With the set up of Section 3.G.3, the new expected utility of party F (similar to party G) becomes:

$$\mathrm{E}_0\left[v\left(f_1\right)+v\left(f_2\right)\right].$$

Since the sanctions are imposed in the second period, the first-period government budget constraints do not change, while those for the second period are given by (3.99). Therefore, as in Appendices 3.A and 3.B, we maximize the above expected utility with respect to the deficit in case of no re-election of the initial incumbent in period 1 $\left(b^{NR}\right)$ and in case of its re-election in period 1 $\left(b^R\right)$ leading to, respectively:

$$\begin{aligned} v'\left(\theta\overline{y}(1+\varepsilon)-\eta\gamma+b^R\right) &= (1+k)*p*v'\left(\theta\overline{y}(1+\varepsilon)(1+\Gamma)-b^R-k\left(b^R-\overline{b}\right)\right), \text{ and} \\ v'\left(\theta\overline{y}(1+\varepsilon)+b^{NR}\right) &= (1+k)*p*v'\left(\theta\overline{y}(1+\varepsilon)-b^{NR}-k\left(b^{NR}-\overline{b}\right)\right). \end{aligned}$$

These two first-order conditions are also described in (3.100). We differentiate them with respect to k and η:

$$\begin{aligned} \frac{\partial b^R}{\partial k} &= \frac{pv'\left(f_2^R\right)}{v"\left(f_1^R\right)+(1+k)^2pv"\left(f_2^R\right)} - \frac{(1+k)pv"\left(f_2^R\right)\left(b^R-\overline{b}\right)}{v"\left(f_1^R\right)+(1+k)^2pv"\left(f_2^R\right)} \lesseqgtr 0, \\ \frac{\partial b^{NR}}{\partial k} &= \frac{pv'\left(g_2^{NR}\right)}{v"\left(g_1^{NR}\right)+(1+k)^2pv"\left(g_2^{NR}\right)} - \frac{(1+k)pv"\left(g_2^{NR}\right)\left(b^{NR}-\overline{b}\right)}{v"\left(g_1^{NR}\right)+(1+k)^2pv"\left(g_2^{NR}\right)} \lesseqgtr 0, \\ \frac{\partial b^R}{\partial \eta} &= \frac{\gamma v"\left(f_1^R\right)}{v"\left(f_1^R\right)+(1+k)^2pv"\left(f_2^R\right)} > 0, \quad \frac{\partial b^{NR}}{\partial \eta} = 0, \\ \frac{\partial b^R}{\partial p} &= \frac{(1+k)v'\left(f_2^R\right)}{v"\left(f_1^R\right)+(1+k)^2pv"\left(f_2^R\right)}, \quad \frac{\partial b^{NR}}{\partial p} = \frac{(1+k)v'\left(g_2^{NR}\right)}{v"\left(g_1^{NR}\right)+(1+k)^2pv"\left(g_2^{NR}\right)}. \end{aligned}$$

The signs of $\frac{\partial b^R}{\partial \eta}$ and $\frac{\partial b^{NR}}{\partial \eta}$ remain the same as those of (3.29) and (3.33). The effect of an increase in k on b^R and b^{NR} depends on whether $\left(b^R-\overline{b}\right)$ and $\left(b^{NR}-\overline{b}\right)$, respectively are greater than zero. We assumed that $\overline{b}$ is sufficiently tight that $b^R > \overline{b}$ and $b^{NR} > \overline{b}$, so that we obtain the same qualitative effects of k as in (3.28) and (3.32).

To find the optimal η, we now solve:

$$\max_{\eta}\frac{1}{2\overline{\varepsilon}}\left\{\begin{array}{c}\int_{\varepsilon_L}^{\overline{\varepsilon}} v\left(\theta\overline{y}(1+\varepsilon)-\eta\gamma+b^R\right)d\varepsilon+p.\int_{\varepsilon_L}^{\overline{\varepsilon}} v\left(\theta\overline{y}(1+\varepsilon)\left(1+\Gamma\right)-b^R-k\left(b^R-\overline{b}\right)\right)d\varepsilon\\ +\int_{-\overline{\varepsilon}}^{\varepsilon_L}0d\varepsilon+(1-p)\,.\int_{-\overline{\varepsilon}}^{\varepsilon_L} v\left(\theta\overline{y}(1+\varepsilon)-b^{NR}-k\left(b^{NR}-\overline{b}\right)\right)d\varepsilon\end{array}\right\}.$$

We take the first-order condition and apply Leibnitz' rule. Thus, using the facts that $\frac{\partial\overline{\varepsilon}}{\partial\eta}=0$, $v'\left(f_1^R\right)-(1+k)\,pv'\left(f_2^R\right)=0$ as given by (3.100) and $\frac{\partial b^{NR}}{\partial\eta}=0$, the new first-order condition for η becomes:

$$-\frac{\partial\varepsilon_L}{\partial\eta}\left[v\left(f_{1L}^R\right)+pv\left(f_{2L}^R\right)-(1-p)\,v\left(f_{2L}^{NR}\right)\right]=\gamma\int_{\varepsilon_L}^{\overline{\varepsilon}} v'\left(f_1^R\right)d\varepsilon,$$

where f_{1L}^R is the same as (3.20) and $f_{2L}^R=\theta\overline{y}(1+\varepsilon_L)\left(1+\Gamma\right)-b_L^R-k\left(b_L^R-\overline{b}\right)$ and $f_{2L}^{NR}=\theta\overline{y}(1+\varepsilon_L)-b_L^{NR}-k\left(b_L^{NR}-\overline{b}\right)$.

Using (3.39) and simplifying the last expression, we arrive at:

$$v\left(f_{1L}^R\right)+pv\left(f_{2L}^R\right)-(1-p)\,v\left(f_{2L}^{NR}\right)=(1-\theta)\,y^m\Gamma\left(\gamma\right)\int_{\varepsilon_L}^{\overline{\varepsilon}} v'\left(f_1^R\right)d\varepsilon, \qquad (3.116)$$

3.I.1 Effect of k on η, b^R, b^{NR} and $\Pr(R)$

Differentiate (3.116) with respect to k:

$$\begin{aligned}
& v'\left(f_{1L}^R\right)\left[\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial k}+\frac{\partial\varepsilon_L}{\partial k}\right)-\gamma\frac{\partial\eta}{\partial k}+\frac{\partial b_L^R}{\partial k}\right]+\\
& pv'\left(f_{2L}^R\right)\left[\theta\overline{y}\left(1+\Gamma\right)\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial k}+\frac{\partial\varepsilon_L}{\partial k}\right)-\frac{\partial b_L^R}{\partial k}\left(1+k\right)-\left(b_L^R-\overline{b}\right)\right]-\\
& (1-p)\,v'\left(f_{2L}^{NR}\right)\left[\theta\overline{y}\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial k}+\frac{\partial\varepsilon_L}{\partial k}\right)-\frac{\partial b_L^{NR}}{\partial k}\left(1+k\right)-\left(b_L^{NR}-\overline{b}\right)\right]-\\
& (1-\theta)\,y^m\Gamma\left[-v'\left(f_{1L}^R\right)\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial k}+\frac{\partial\varepsilon_L}{\partial k}\right)+\int_{\varepsilon_L}^{\overline{\varepsilon}} v"\left(f_1^R\right)\left(-\gamma\frac{\partial\eta}{\partial k}+\frac{\partial b^R}{\partial k}\right)d\varepsilon\right]\\
= & \;0.
\end{aligned}$$

This expression can be written as:

$$\begin{aligned}
&\frac{\partial\eta}{\partial k}\left\{\theta\overline{y}\frac{\partial\varepsilon_L}{\partial\eta}\left[v'\left(f_{1L}^R\right)+(1+\Gamma)\,pv'\left(f_{2L}^R\right)-(1-p)\,v'\left(f_{2L}^{NR}\right)\right]-\gamma v'\left(f_{1L}^R\right)\right\}\\
&+\frac{\partial b_L^{NR}}{\partial k}v'\left(f_{2L}^{NR}\right)(1-p)(1+k)\\
&+\theta\overline{y}\frac{\partial\varepsilon_L}{\partial k}\left[v'\left(f_{1L}^R\right)+(1+\Gamma)\,pv'\left(f_{2L}^R\right)-(1-p)\,v'\left(f_{2L}^{NR}\right)\right]+\frac{\partial b_L^R}{\partial k}\left[v'\left(f_{1L}^R\right)-pv'\left(f_{2L}^R\right)(1+k)\right]\\
&-(1-\theta)\,y^m\Gamma\left[-v'\left(f_{1L}^R\right)\left(\frac{\partial\varepsilon_L}{\partial\eta}\frac{\partial\eta}{\partial k}+\frac{\partial\varepsilon_L}{\partial k}\right)+\int_{\varepsilon_L}^{\overline{\varepsilon}}v"\left[f_1^R\right]\left(-\gamma\frac{\partial\eta}{\partial k}+\frac{\partial b^R}{\partial k}\right)d\varepsilon\right]\\
=\;& pv'\left(f_{2L}^R\right)\left(b_L^R-\overline{b}\right)-(1-p)\,v'\left(f_{2L}^{NR}\right)\left(b_L^{NR}-\overline{b}\right).
\end{aligned}$$

Following the same steps as before, we obtain:

$$A_{1L}\frac{\partial\eta}{\partial k}-A_2\int_{\varepsilon_L}^{\overline{\varepsilon}}\left(-\gamma\frac{\partial\eta}{\partial k}+\frac{\partial b^R}{\partial k}\right)v"\left(f_1^R\right)d\varepsilon+A_{3L}\frac{\partial b_L^{NR}}{\partial k}=D"_{1L}, \tag{3.117}$$

where:

$$D"_{1L}=pv'\left(f_{2L}^R\right)\left(b_L^R-\overline{b}\right)-(1-p)\,v'\left(f_{2L}^{NR}\right)\left(b_L^{NR}-\overline{b}\right)\gtreqless 0.$$

The coefficients (3.47) and (3.48) of equation (3.45) are also altered to:

$$B"_2 = v"\left(f_1^R\right)+(1+k)^2 pv"\left(f_2^R\right)<0, \tag{3.118}$$

$$D"_2 = pv'\left(f_2^R\right)-(1+k)\,pv"\left(f_2^R\right)\left(b_L^R-\overline{b}\right)>0,\text{ if } b_L^R-\overline{b}>0. \tag{3.119}$$

Finally, the three coefficients of (3.52) are also modified:

$$C"_{1L} = -\frac{\theta\overline{y}\gamma}{(1-\theta)\,y^m\Gamma\,[\gamma]}\left(v"\left(g_{1L}^{NR}\right)-(1+k)\,pv"\left(g_{2L}^{NR}\right)\right)\gtreqless 0, \tag{3.120}$$

$$C"_{2L} = v"\left(g_{1L}^{NR}\right)+(1+k)^2 pv"\left(g_{2L}^{NR}\right)<0, \tag{3.121}$$

$$D"_3 = pv'\left(g_{2L}^{NR}\right)-(1+k)\,pv"\left(g_{2L}^{NR}\right)\left(b_L^{NR}-\overline{b}\right)>0,\text{ if } b_L^{NR}-\overline{b}>0. \tag{3.122}$$

Thus, solving the system with the three equations (3.40), (3.45) and (3.52) using these new coefficients, we find the effect of a tighter sanction on compensation as:

$$\begin{aligned}
&\frac{\partial\eta}{\partial k}\left\{\begin{array}{c}-\frac{\theta\overline{y}\gamma}{(1-\theta)y^m\Gamma}\left[v'\left(f_{1L}^R\right)+(1+\Gamma)\,pv'\left(f_{2L}^R\right)-(1-p)\,v'\left(f_{2L}^{NR}\right)\right]-2\gamma v'\left(f_{1L}^R\right)+\\
\frac{(1-p)\theta\overline{y}\gamma v'\left(f_{2L}^{NR}\right)\left[v"\left(g_{1L}^{NR}\right)-(1+k)pv"\left(g_{2L}^{NR}\right)\right]}{(1-\theta)y^m\Gamma\left[v"\left(g_{1L}^{NR}\right)+(1+k)^2pv"\left(g_{2L}^{NR}\right)\right]}+(1-\theta)\,y^m\Gamma\gamma\int_{\varepsilon_L}^{\overline{\varepsilon}}\frac{v"\left(f_1^R\right)(1+k)^2pv"\left(f_2^R\right)}{v"\left(f_1^R\right)+(1+k)^2pv"\left(f_2^R\right)}d\varepsilon\end{array}\right\}\\
&=\left\{\begin{array}{c}pv'\left(f_2^R\right)\left(b_L^R-\overline{b}\right)-\frac{v'\left(f_{2L}^{NR}\right)(1-p)\left[pv'\left(g_{2L}^{NR}\right)+\left(b_L^{NR}-\overline{b}\right)\left(v"\left(g_{1L}^{NR}\right)+k(1+k)pv"\left(g_{2L}^{NR}\right)\right)\right]}{v"\left(g_{1L}^{NR}\right)+(1+k)^2pv"\left(g_{2L}^{NR}\right)}\\
+(1-\theta)\,y^m\Gamma\int_{\varepsilon_L}^{\overline{\varepsilon}}\frac{\left[pv'\left(f_2^R\right)-(1+k)pv"\left(f_2^R\right)\left(b_L^R-\overline{b}\right)\right]v"\left(f_1^R\right)}{v"\left(f_1^R\right)+(1+k)^2pv"\left(f_2^R\right)}d\varepsilon\end{array}\right\}.
\end{aligned}$$

For $b_L^R-\overline{b}>0$ and p close to 1, we observe that the left-hand side of the last equation is negative and the right-hand side positive, so that $\frac{\partial\eta}{\partial k}<0$. Further, by (3.56) and using the new coefficients (3.118) and (3.119), we obtain that $\frac{\partial b^R}{\partial k} < 0$. Finally, from (3.57), (3.120), (3.121) and (3.122), we have as before that $\frac{\partial b^{NR}}{\partial k} \gtrless 0$. However, with the quadratic utility specification (3.25) and $k \geq 0$ not too large, $\frac{\partial b^{NR}}{\partial k} < 0$. With respect to the probability of re-election, given (3.62) and the fact that $\frac{\partial \eta}{\partial k} < 0$, again we obtain: $\frac{d \Pr(R)}{dk} < 0$.