Man muss immer umkehren!

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Man Muss Immer Umkehren!

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1 Introduction

The 19th century geometrist Jacobi famously said that one should always try to invert every geometrical theorem. But his advice applies much more widely! Choose any class of relational frames, and you can study its valid modal axioms. But now turn the perspective around, and fix some modal axiom beforehand. You can then find the class of frames where the axiom is guaranteed to hold by 'modal correspondence' analysis – and we all know the famous examples of that. It may look as if this style of analysis is tied to one particular semantics, say relational frames: but it is not. Correspondence analysis also works on neighbourhood models, telling us, e.g., just which modal axioms collapse these to binary relational frames. We will show how this same style of inverse thinking also applies to modern dynamic logics of information change. Basic axioms for knowledge after information update *!A* tell us what sort of operation *must* be used for updating a given model ***M*** to a new one incorporating *A*. Likewise, we will show how modal axioms for (conditional) beliefs that hold after revision actions **A* actually fix one particular operation of changing the relative plausibility orderings which agents have on the universe of possible worlds. And finally, going back to the traditional heartland of logic, we show how we can read standard predicate-logical axioms as constraints on the sort of abstract 'process models' that lie at the heart of first-order semantics, properly understood. In all these cases, in order for the inversion to work and illuminate a given subject, we need to step back and reconsider our standard modeling. But *that*, I think, is what Shahid Rahman is all about.

2 Standard modal frame correspondences

One of the most attractive features of the semantics of modal logic is the match between modal axioms and corresponding patterns in the accessibility relation between worlds. This can be seen by giving a class of models, say temporal or epistemic, and then axiomatizing its set of modal validities. On top of the minimal modal logic which holds under all circumstances, one gets additional axioms reflecting more specific structure. For general background to modal completeness theory, as well as the rest of this paper, we refer to the *Handbook of Modal Logic* (P. Blackburn, J. van Benthem & F. Wolter, eds.) which has just come out with Elsevier Science Publishers, Amsterdam, 1997.

Now, as a counterpoint to the completeness, the point of *modal correspondence theory* (van Benthem 1983) is that one can also invert this line of thought. One takes some appealing modal axiom whose validity is to be guaranteed, and then finds out which accessibility patterns must then be assumed. Just to get into the spirit, consider the perennial modal *K4-axiom* $[]p \to [][]p$. Let us call a modal formula ϕ true at a point s in a semantic frame $\boldsymbol{F} = (W, R)$ if it is true at s under all atomic valuations V on $\boldsymbol{F}$. Here is perhaps the mother of all correspondences:

Fact 1 $\boldsymbol{F}, s \models []p \to [][]p$ iff $\boldsymbol{F}$'s accessibility relation R is *transitive* at s: i.e., $\boldsymbol{F}, s \models \forall y(Rxy \to \forall z(Ryz \to Rxz))$.

Proof If the relation is transitive, $[]p \to [][]p$ clearly holds under every valuation. Conversely, let $\boldsymbol{F}, s \models []p \to [][]p$. In particular, the *K4*-axiom will hold if we take $V(p)$ to be $\{y \mid Rsy\}$. But then, the antecedent $[]p$ holds at s, and hence so does the consequent $[][]p$. And the latter states the transitivity, by the definition of $V(p)$. ♣

The theory behind this example involves the Sahlqvist Theorem: all axioms of the right syntactic shape allow for systematic first-order translation. As a beneficial side-effect, the inversion in perspective also makes us look differently at familiar modal axioms, and see patterns unnoticed before! One famous very non-Sahlqvist principle is *Löb's axiom* $[]([]p \to p) \to []p$ expressing a basic principle of arithmetical provability logic. This expresses an interesting higher-order feature of accessibility patterns:

Fact 2 Löb's Axiom is true at the point s in a frame $\boldsymbol{F} = (W, R)$ iff (a) $\boldsymbol{F}$ is upward R-well-founded at s, and (b) $\boldsymbol{F}$ is transitive at s.

Correspondence theory is still alive and expanding today. Van Benthem 2005, 2006A show how Löb's Axiom leads to a systematic analysis of structural properties of accessibility definable in *LFP(FO)*, first-order logic with added *fixed-point operators*. As a consequence, one can also analyze well-known modal fixed-point languages like the μ–calculus in new ways. But further modal axioms define accessibility patterns still beyond this level, with the *McKinsey Axiom* $[]\langle\rangle p \to \langle\rangle[]p$ as a prime example.

3 Modal distribution and neighborhood models

Some people think correspondence analysis is tied up exclusively with one particular view of what modal models must be like, viz. directed graphs. But it will work on any sort of structure, even ones that look 'higher-order'. E.g., Rodenburg 1986 showed how one can do correspondence analysis of intuitionistic axioms on Beth models, taking points and branches as primitive objects. Here is an example closer to modal logic itself (van Benthem 1992, 1996B, Chapter 11). *Neighbourhood models* generalize directed graphs by having accessibility relations RxY relating points to sets of points. These structures have concrete motivations in scenarios of 'deductive support' in logic programs, topological semantics and modal logics of space, or modal logics of players' powers of reaching outcomes in games. One can then interpret the key modality via the following generalization of the usual truth condition:

$$\boldsymbol{M}, s \models \langle\rangle\phi \text{ iff there is a set of points } Y \text{ with } RsY \text{ and for all } y\in Y, \boldsymbol{M}, y \models \phi$$

The resulting minimal logic loses distributivity of the modality over either conjunction or disjunction, though it retains upward monotonicity. Moreover, its *SAT* complexity drops from *Pspace-complete* for modal *K* to *NP*-complete: i.e., 'from worse to bad'. But this move to a more general semantics now means that formerly minimally valid principles now acquire substantial content. In particular, we have this simple

Fact 3 The distribution axiom $\langle\rangle(p\vee q) \leftrightarrow (\langle\rangle p \vee \langle\rangle q)$ is valid on a frame iff that frame is generated by a binary relation Rxy with RxY iff $\{y:Rxy\}\subseteq Y$.

Van Benthem 1992 investigates correspondences over neighbourhood frames in more details, and finds appropriate generalizations for the correspondence-theoretic content of major modal principles, such as the above *K4*-axiom. Consider its existential version:

Fact 4 $\langle\rangle\langle\rangle p \to \langle\rangle p$ corresponds to a rule of *Cut* ('Generalized Transitivity'):

$$\forall x \forall Y \forall \{Z_y \mid y\in Y\}: ((R\, xY \wedge R\, yZ_y \text{ (for all } y\in Y)) \to R\, x \bigcup\{Z_y \mid y\in Y\})$$

Just as over directed graphs, such correspondences can be computed automatically by a substitution algorithm (cf. Blackburn, de Rijke & Venema 2000), this time, producing relational conditions in a weak sub-language of second-order logic.

4 Geometry: two-sorted modal logic

Another source of correspondence thinking which goes 'out of the box' is in geometry. Van Benthem 1996A, 1999 makes a plea for a many-sorted view of space, with points and lines, or points and arrows, on a par. Matching modal languages will now be two-sorted, with one kind of formulas referring to points, and another to lines or arrows. Of course, relations can be of many kinds here, beyond the original idea of accessibility for worlds. The resulting modal geometry has appealing concrete correspondences between modal axioms and spatial patterns. We give two examples here.

Consider modal *Arrow Logic*, a two-sorted language describing both points and transitions between them as first-class semantic citizens. We focus on the latter here. Basic arrow models are of the form $\boldsymbol{M} = (A, C^3, R^2, I^1, V)$ with A a set of 'arrows' with three predicates: C^3x,yz (x is a composition of y and z), R^2x,y (y is a reversal of x), I^1x (x is an identity arrow). The modal language is interpreted with these key clauses:

$\boldsymbol{M}, x \mid= \phi\bullet\psi$ iff there are y, z with $C\,x, yz$ and $\boldsymbol{M}, y \mid= \phi$, $\boldsymbol{M}, z \mid= \psi$

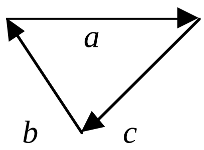
$\boldsymbol{M}, x \mid= \phi^{\smile}$ iff there exists y with $R\,x, y$ and $\boldsymbol{M}, y \mid= \phi$

Here is the content of two famous principles for converse and composition from Tarski's Relational Algebra, re-stated as modal axioms of Arrow Logic:

Fact 5 $(\phi\bullet\psi)^{\smile} \to \psi^{\smile}\bullet\phi^{\smile}$ corresponds to $\forall xyz\colon\ C\,x, yz \to C\,r(x), r(z)r(y)$

$\phi \bullet \neg(\phi^{\smile}\bullet\psi) \to \neg\psi$ corresponds to $\forall xyz\colon\ C\,x, yz \to C\,z, r(y)x$

Given these properties of our relations, we can view composition triangles like the one depicted here from any arrow we please taking reversals:



Basic arrow logics are decidable. But an ominous threshold is the existential principle of *associativity*, which makes these logics undecidable, just like relational set algebra:

Fact 6 The associativity axiom $(\phi\bullet\psi)\bullet\chi \to \phi\bullet(\psi\bullet\chi)$ corresponds to

$\forall xyzuv\colon (C\,x, yz\ \&\ C\,y, uv) \to \exists w\colon (C\,x, uw\ \&\ C\,w, vz))$

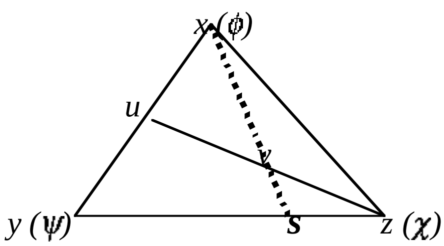
This says that structures have to be rich enough to admit of 'recombination'.

Complex or not, this same axiom is highly appealing from a geometrical standpoint. Our second example comes from modal logics of space (Aiello & van Benthem 2003). Here is what Associativity says when we shift our correspondence analysis to modal logics of geometry where the ternary relation now rather stands for affine *betweenness* $B\,x, yz$: *x lies on the segment y–z*. We use the following modality:

$\boldsymbol{M}, s \mid= C\phi\psi$ iff $\exists t, u\colon\ Bs, tu\ \&\ \boldsymbol{M}, t\mid=\phi\ \ \&\ \boldsymbol{M}, u\mid=\psi$

Fact 7 The associativity law $C(C\phi\psi)\chi \to C\phi(C\psi\chi))$ corresponds to *Pasch' Axiom*

$\forall xyzuv\colon (Bu, xy\ \&\ Bv, uz) \to \exists s\,(Bv, xs\ \&\ Bs, yz))$.

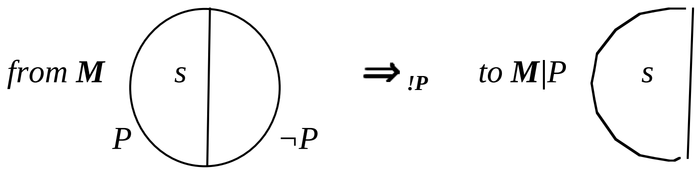


Thus, depending on the semantic environment, correspondence analysis may reveal very different content for well-known modal principles, linking them up in surprising ways with mathematical structure known from other sources. Let's now move elsewhere.

5 Knowledge and Information Update

Another paradigm for modal logic is the analysis of *knowledge* and other information-related attitudes. Here, models stand for information patterns describing current states of one or more agents in interaction. The modal language is as usual: $p \mid \neg\phi \mid \phi \vee \psi \mid K_i\phi$ and perhaps common knowledge $C_G\phi$, while models $M = (W, \{\sim_i \mid i \in G\}, V)$ have worlds W, accessibility relations $\sim_i$, and a valuation V. The standard epistemic truth condition reads: $M, s \models K_i\phi$ iff for all t with $s \sim_i t$: $M, t \models \phi$. The usual modal frame correspondences apply here, both for knowledge modalities and for the common knowledge, treated as a fixed-point operator in the sense of Section 2. So far, so good.

But now consider a modern trend, the analysis of informational actions which *change* a current epistemic model (van Benthem 2006B). For instance, a *public announcement* $!P$ works as follows: learning P eliminates the worlds where P is false. In a picture:



To describe this, we need a *dynamic-epistemic* logic, with a key operator

$$\boldsymbol{M}, s \models [!P]\,\phi \quad \text{iff} \quad \text{if } \boldsymbol{M}, s \models P, \text{ then } \boldsymbol{M}|P, s \models \phi$$

The logic of this can be axiomatized completely. In particular, one compositional law explains when agents acquire knowledge after an announcement of some 'hard fact' P:

Fact 8 The following modal axiom is sound for public announcement:

$$[!P]K_i\phi \leftrightarrow (P \rightarrow K_i(P \rightarrow [!P]\phi))$$

Well-understood, this axiom expresses non-trivial assumptions about epistemic agents. In particular, the interchange of knowledge and observed events expresses their capacity of *perfect memory*. This is indeed presupposed in the following argument.

Proof Compare two models: $(\boldsymbol{M}, s)$ and $(\boldsymbol{M}|P, s)$ before and after the update: it helps to draw pictures. The formula $[P!]K_i\phi$ says that, in $\boldsymbol{M}|P$, all worlds $\sim_i$-accessible from s satisfy ϕ. The matching worlds in $\boldsymbol{M}$ are those $\sim_i$-accessible from *s and which satisfy P*. Moreover, truth values of formulas may change in an update step, say from ignorance to knowledge. Hence the correct description of these worlds in $\boldsymbol{M}$ is not that that they satisfy ϕ (which they do in $\boldsymbol{M}|P$), but rather $[P!]\phi$: they become ϕ. after the update. Finally, a small detail. $!P$ is a partial operation, as P has to be true for its truthful public announcement. Thus, we make our assertion on the right only if $!P$ is executable, i.e., P is true. Altogether then, $[P!]K_i\phi$ says the same as $P \rightarrow K_i(P \rightarrow [P!]\phi$. ♣

The use of pictures here is not just a convenience. These pictures also reflect genuine intuitions concerning what might be called *the geometry of knowledge* and update.

But now, Jacobi's advice once more. We have seen that treating public announcement of hard facts as world elimination validates the above axiom. What about a converse. Suppose that this axiom looks independently plausible for information update, *which operations on models* would validate it? The answer is again a correspondence argument (van Benthem 2007). We consider abstract model-changing operations ♥p taking epistemic models $\boldsymbol{M}$ with sets of worlds p inside to new models $\boldsymbol{M}$♥p – with some mild conditions on available worlds for their domains. A simple proof then shows

Fact 9 Eliminative update is the only model-changing operation which satisfies the equivalence $[$♥$p]\ K\ q \leftrightarrow (p \rightarrow K(p \rightarrow [$♥$p]\ q))$.

Proof From left to right, the formula implies the following. Take q equal to the set of worlds which are ~-accessible from the current one s inside the set p. Assume also that the world s is in p. Then the right-hand side says that all worlds still ~-accessible from s after the operation ♥p are in q: i.e., they were accessible before, and they were members of p. Thus, the relation change leaves only already existing links from p-worlds to p-worlds. By a similar argument in the converse direction, we see that indeed, *all* such links are preserved into the new model after the operation ♥p. This is precisely the link-cutting version of epistemic update described before. ♣

This argument can be sharpened up, defining the universe of relevant epistemic frames and transition relations explicitly, and stipulating how individual worlds can be related across frames. In such a setting, *three axioms* capture eliminative update for public announcement. First, the equivalence (a) $<!p>T \leftrightarrow p$ makes sure that inside a given model $\boldsymbol{M}$, the only worlds surviving into $\boldsymbol{M}$ p are those in the set denoted by p. Next, a reduction axiom (b) $<!p>Eq \leftrightarrow p \wedge E<!p>q$ for the *existential modality* Eq ("q is true in some world") says that the domain of $\boldsymbol{M}$ p contain no objects beyond the set p in $\boldsymbol{M}$. Finally, the above axiom (c) for knowledge ensures that the epistemic relations are the same in $\boldsymbol{M}$ and $\boldsymbol{M}$ p, so that our update operation really takes a *submodel*.

An open problem is taking this correspondence analysis to the current world of dynamic epistemic logics for larger families of informative events, including partial observation and hiding. In particular, one would want to show that the basic *product update* mechanism of Baltag, Moss & Solecki 1998 for such more sophisticated scenarios is essentially the only model construction in some suitable abstract space validating the general *DEL* axiom $[\boldsymbol{E}, e]K_i\phi \leftrightarrow (PRE_e \rightarrow \bigwedge \{ K_i[\boldsymbol{E}, f]\phi)) \mid f \sim_i e \text{ in } \boldsymbol{A}\})$.

6 Geometry of Belief Revision

We do not just receive information which smoothly updates our current knowledge. There are also more dramatic episodes of facts which challenge our current beliefs, and lead to dynamic processes of belief revision. Here, too, the preceding considerations can be brought to bear. Beliefs can be interpreted over modal models with a comparison relation of *relative plausibility* between worlds. The key modality then becomes:

$\boldsymbol{M}, s \models B_i\phi$ iff $\boldsymbol{M}, t \models \phi$ for all worlds t *minimal* in the ordering $\lambda xy. \leq_{i,s} xy$.

But soon, this turns out less than what one needs – and a more general notion of *conditional belief* helps 'pre-encode' beliefs we would have if we learnt certain things:

$\boldsymbol{M}, s \models B_i(\phi/ \psi)$ iff $\boldsymbol{M}, t \models \phi$ for all worlds t which
are minimal for $\lambda xy. \leq_{i,s} xy$ in the set $\{u \mid \boldsymbol{M}, u \models \psi\}$.

The resulting logic the standard principles of the minimal conditional logic. Now, as for the 'hard facts' of Section 5, it is easy to see that the following axiom holds:

$$[!P]\, B_i(\phi/ \psi) \quad \leftrightarrow \quad P \rightarrow B_i(\, [!P]\phi \mid P \wedge [!P]\psi)$$

But more interesting is the response of agents to 'soft triggers', events which make a proposition more plausible, though not definitively ruling out that it might be false. Such triggers will not eliminate worlds, but they will *change the plausibility pattern*.

One typical response of this sort is *lexicographic upgrade* $\Uparrow P$, described variously as what a trusting, or a radical agent might do. This changes the current model $\boldsymbol{M}$ to $\boldsymbol{M}\Uparrow P$:

P-worlds become better than all $\neg P$-worlds; within zones, the old order remains.

The complete dynamic logic of this operation of model change can be axiomatized – first bringing it into the language through this matching modality:

$$\boldsymbol{M}, s \models [\Uparrow P]\phi \qquad \text{iff} \qquad \boldsymbol{M}\Uparrow P, s \models \phi.$$

Then the following key principle emerges for the conditional beliefs which agents will have after a lexicographic plausibility change occurred for some soft trigger with P:

$$\begin{aligned}[\Uparrow P]\, B(\phi|\psi) \leftrightarrow\ & (\boldsymbol{E}(P \wedge [\Uparrow P]\psi) \wedge B([\Uparrow P]\phi \mid P \wedge [\Uparrow P]\psi)) \\ & \vee \neg(\boldsymbol{E}(P \wedge [\Uparrow P]\psi) \wedge B([\Uparrow P]\phi \mid [\Uparrow P]\psi)\end{aligned}$$

Here E is again the global existential modality – or a similar epistemic modality. This time, we will not go into the details of the soundness argument. But we do note that, extending the analysis for information update, a nice modal correspondence can be proved, showing that we have captured the essence here. Again, we are working in a universe of frames connected by abstract relation changing operations ♥p.

Fact 10 The formula $[♥p]\, B(q|r) \leftrightarrow (E(p \wedge r) \wedge B(q \mid p \wedge r) \vee (\neg(E(p \wedge r) \wedge B(q \mid r))$ holds in a universe of frames iff the operation ♥p is lexicographic upgrade.

Proof Let $\leq_s xy$ in $\boldsymbol{M}$♥p. We show that $\leq_s$ is the relation produced by lexicographic upgrade. Let r be the set $\{x, y\}$ and $q = \{x\}$. Then the left-hand side of our axiom is true. There are two cases on the right-hand side. *Case 1*: one of x, y is in p, and hence $p \wedge r = \{x, y\}$ (1.1) or $\{y\}$ (1.2) or $\{x\}$ (1.3). Moreover, $B(q \mid p \wedge r)$ holds in $\boldsymbol{M}$ at s. If (1.1), we have $\leq_s xy$ in $\boldsymbol{M}$. If (1.2), we must have $y=x$, and again $\leq_s xy$ in $\boldsymbol{M}$. Case (1.3) can only occur when $x \in p$ and $y \notin p$. Thus, all new relational pairs in $\boldsymbol{M}$♥p satisfy the description of the lexicographic reordering. *Case 2* is when we have $\neg(E(p \wedge r)$ and none of x, y are in p. This can be analyzed analogously, using the truth of the disjunct $B(q \mid r)$.

Conversely, we show that all pairs satisfying the description of lexicographic upgrade do make it into the new order. Here is one example; the other case is similar. Suppose that $x \in p$ while $y \notin p$. Then $p \wedge r = \{x\}$. Next, set $r = \{x, y\}$ and $q = \{x\}$. Then we have $B(q \mid r)$ for trivial reasons. The left-hand side formula $[$$p]\, B(q|r)$ is then also true, since our axiom is supposed to hold for any interpretation of the proposition letters q, r – and it tells us that, in the model $\boldsymbol{M}$$p$, the best worlds in $\{x, y\}$ are in $\{x\}$: i.e., $\leq_s xy$. ♣

Again, van Benthem 2007 actually analyzes the technicalities here a bit more carefully.

In the area of belief revision, this correspondence analysis has further attractions, since no single action of plausibility change works once and for all. E.g., more conservative, or less trusting, agents, might respond to a soft trigger by the operation $\uparrow P$, which only puts *the best P-worlds* on top, and leaves everything else as it was in $\boldsymbol{M}$. With a matching modality, one finds the corresponding reduction axiom for this new policy:

$$[\uparrow P]\, B(\phi|\psi) \leftrightarrow (B(\neg[\uparrow P]\psi)/P) \wedge B([\uparrow P]\phi \mid [\uparrow P]\psi))$$
$$\vee (\neg(B(\neg[\uparrow P]\psi)/P) \wedge B([\Uparrow P]\phi \mid (P \wedge [\Uparrow P]\psi))$$

Again a correspondence argument shows this determines the belief change policy $\uparrow P$.

Correspondence theory tells us the exact correlation between principles describing changes of (conditional) beliefs and suitably definable semantic changes in plausibility patterns. Thus, as with knowledge, we get a geometry of belief and belief changes.

7 Modal Foundations for First-Order Logic

My final example of the power of correspondence analysis and inversion goes back to the heartland where it all started. Modal logic started as an extension of, or maybe a fine-structure fragment of, standard *first-order predicate logic*. But well-understood, that system itself is very 'modal'! Consider Tarski's clause for the existential quantifier $\boldsymbol{M}, \alpha \models \exists x\, \phi$ iff *for some* $d \in |\boldsymbol{M}|$: $\boldsymbol{M}, \alpha^x{}_d \models \phi$. Here, the variable assignments α are essential in decomposing quantified statements. But much less than this is needed to give a compositional semantics for first-order quantification, viz. the abstract pattern

$$\boldsymbol{M}, \alpha \models \exists x\, \phi \text{ iff } \textit{for some } \beta\text{: } R_x\alpha\beta \textit{ and } \boldsymbol{M}, \beta \models \phi$$

Here, the assignments become abstract states, and the concrete relation $\alpha =_x \beta$ which holds between α and $\alpha^x{}_d$ has become just a binary relation R_x. Evidently, this is the semantics of a minimal poly-modal language. This state semantics has an independent appeal. First-order evaluation is an informational process that changes computational states (van Benthem 1996B), and formulas are compound procedures over basic atomic actions of testing for a fact, and shifting the value of some variable.

Accordingly, the usual validities of first-order logic, which one can look up in a good textbook like Enderton 1972, split into two groups. One group consists of the minimal modal logic: (a) all classical Boolean propositional laws, (b) Modal Distribution: $\forall x\, (\phi \to \psi) \to (\forall x\, \phi \to \forall x\, \psi)$, (c) Modal Necessitation: *if* $\vdash \phi$, *then* $\vdash \forall x\, \phi$, and (d) a definition of $\exists x \phi$ as $\neg \forall x \neg \phi$. Much first-order inference can be described this way. But now, we can also look at further first-order axioms, and see what these say on top of this. What we expect is that they reflect additional properties of the evaluation process.

Again, this may be brought out using modal frame correspondences. The full story is in van Benthem 1997, but we cite a few high-lights here. First, there are some universal properties of the specific relations $\alpha =_x \beta$ among assignments. The fact that these are equivalence relations is reflected in valid *S5*-style axioms such as $\exists x\, \exists x\, \phi \to \exists x\, \phi$. As usual, the latter corresponds to the transitivity. Indeed, the total system corresponding to making these general assumptions, but without any existential ones on 'fullness' of the set of available assignments leads us to an interesting, and still *decidable*, version of first-order logic called *CRS* (Németi 1985), related to generalized relational algebras.

But now about the sources of the undecidability of our usual first-order logic! Modal frame correspondences help us discover these. 'Deconstructing' the further first-order axioms in Enderton 1972, one is quickly left to focus on the valid *interchange laws* for quantifiers. These turn out to correspond (just by virtue of their Sahlqvist forms) to well-known significant existential geometrical properties of the evaluation process:

Fact 11 (a) $\exists y\ \exists x\ \phi \rightarrow \exists x\ \exists y\ \phi$ expresses 'Path Reversal':

$$\forall\alpha\beta\gamma(\ (R_x\alpha\beta\ \&\ R_y\beta\gamma) \rightarrow \exists\delta\ \ (R_y\alpha\delta\ \&\ R_x\delta\gamma))$$

(b) $\exists y\ \forall x\ \phi \rightarrow \forall x\ \exists y\ \phi$ expresses 'Confluence':

$$\forall\alpha\beta\gamma(\ (R_y\alpha\beta\ \&\ R_x\alpha\gamma) \rightarrow \exists\delta\ \ (R_x\beta\delta\ \&\ R_y\gamma\delta))$$

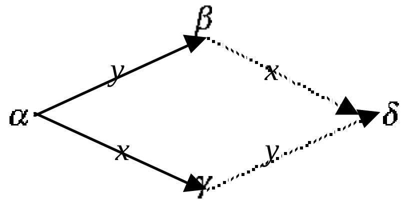


Illustration for Confluence

Thus, by stepping back into a broader class of semantic structures, we give predicate-logical validities different voices. Some remain universally valid – but others express various specific properties of the space of available computational states. And when that space becomes full enough, with grid structures associated with known undecidable Tiling Problems, we get undecidability of its modal theory.

But such a larger universe also brings further rewards. E.g., more distinctions can be made on abstract state models for first-order logic than on standard Tarski models. In particular, there are now separate denotations for *substitution operators [t/x].* Also one can naturally interpret *polyadic quantifiers* $\exists\boldsymbol{x}$ for tuples of variables $\boldsymbol{x}$ to the first-order language, in terms of *simultaneous change* of values in their registers. This genuinely enriches the first-order vocabulary, while still retaining decidability over abstract state models (Andréka, van Benthem & Németi 1998). Of course, modal correspondence analysis applies to principles in these richer languages, too.

8 Conclusion

Modal correspondence analysis arises when we invert an established perspective, look back at the models we have chosen, and ask what sort of semantic content attaches to proposed syntactic axioms. This style of thinking may look like an abstract and somewhat curious interest at times. But we hope to have shown that there are benefits across a wide range of cases. We can check whether proposed axioms really capture what they are supposed to so, we can find surprising new content to familiar principles, and sometimes, we are led to the construction of new semantic domains, and new languages over these. Lot's wife was punished for looking back. There was great unfairness in that – but in logic, looking 'the other way' can only benefit us!

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