The spectra of supersymmetric states in string theory

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The goals of the thesis, apart from for the author to become a doctor, are the following: 1. To summarise the main results of my research of the past three years. 2. To provide a compact and self-contained survey of the relevant materials for beginning graduate students or researchers in other sub-fields as a shortcut to the frontline of the current research in this area of string theory.

Motivation for the First Goal

My personal motivation to pursue this line of research has two sides. First of all, in order to understand the structure of a theory, it is important to know the spectrum of the theory. Just like the spectrum of a hydrogen atom holds the key to understanding quantum mechanics, we hope that the same might be true for string theory. For a very complex theory as string theory is, the supersymmetric part of the spectrum is usually the part which is most accessible to us due to the great simplification supersymmetry offers. Nevertheless, as I hope I will convince the readers in this thesis, it is still a far from trivial task to study this part of the spectrum. In other words, we hope that the study of the spectrum of supersymmetric states of string theory will be a feasible step towards furthering our understanding of string theory.

In the other direction, it has been a great challenge since the invention of Einstein gravity and quantum mechanics to understand the quantum aspects of gravity. A fundamental question since the work of Bekenstein and Hawking in the 70’s, is why black holes have entropy. Only when we can answer this question can we ever claim that we understand the nature of quantum gravity. Conversely, because of the challenging nature of the question, once we can answer this question we have a reason to believe that we are on the right track to the goal of quantising, in one way or the other, Einstein gravity. String theory, at the time of writing, still scores highest in the challenge of explaining the thermodynamical entropy of the black holes, while it is also true that most of the work done along this trajectory still focuses on black holes with supersymmetry, which are unlikely to be directly observable in
nature. From this point of view, to study the supersymmetric spectrum of string theory and to use it as information about the black hole entropy, is a part of the effort towards a deeper understanding of the nature of quantum gravity.

Motivation for the Second Goal

Now I will move on to explain the motivation to achieve the second goal of the thesis: providing a self-contained material serving as a shortcut to the current research on the topic.

In the course of development of string theory since its birth in the 70’s, it has expanded into an extremely broad and sometimes very complicated field. According Mr. Peter Woit, there might be around 30,000 papers written on the subject so far. While exactly this property makes it, in my opinion, a sufficiently fun field to be working in, it is no good news for beginners. In order to work on a topic in a specific sub-field, she or he is likely to find herself having to go through the labyrinth of a large amount of papers on various totally different but yet somehow inter-connected topics in physics and mathematics, with conflicting notations and conventions.

As my own experience seems to suggest that it could be fairly time-consuming and frustrating a process, I would like to take the chance of writing my PhD thesis to provide a service for great string theorists yet to materialise, or for researchers specialising on other topics, by making an attempt at a relatively compact and self-contained exposition of some of the should-know’s for performing research related to the subjects I have worked on in the past three years.

Reader’s Manual

But the attempt to be self-contained also brings some drawbacks to this thesis, namely that a fair portion of it is probably unnecessary for expert readers only interested in the results of my research. To cure this problem we now give a rough reader’s manual so the advanced readers will be less likely finding themselves wasting their time on the introductions.

This thesis is organised as follows. In the first part we introduce some basic concepts of superstring theories in 30 pages, focusing on the perturbative aspects in the first section and the non-perturbative aspects in the second. The readers who are sufficiently familiar with superstring theory can safely skip this part.

The second part of the thesis is about string theory compactified on Calabi-Yau two- and three-folds. The readers who are not yet familiar with basic con-
cepts of differential and algebraic geometry might want to first read Appendix A before going into this part. Basically all the mathematical background for understanding this thesis, in particular part II and IV, is summarised in Appendix A with some explanations but without proofs.

The readers who are familiar with basic concepts of differential and algebraic geometry can go directly to chapter two where Calabi-Yau (three-fold) compactification is discussed. This chapter is relevant for understanding the discussion of the BPS spectrum of the $d = 4, \mathcal{N} = 2$ theories. But this chapter can again be skipped if the reader is sufficiently familiar with generalities of Calabi-Yau compactifications. The same goes for chapter three, where K3 compactification is discussed and which is relevant for the understanding the discussion of the BPS spectrum of the $d = 4, \mathcal{N} = 4$ theories.

After chapter four, the reader can go directly to the part III, IV or part V of the thesis, depending on whether she or he is interested in $\mathcal{N} = 2$ or $\mathcal{N} = 4$ theories, and which aspects of them. The material presented here are mostly original. I wish the readers who have gone through the first half can now enjoy the fruit of reading the hundred pages of “preliminary knowledge”, and the expert readers can find this half of the thesis somewhat interesting after having skipped the first half.