



## UvA-DARE (Digital Academic Repository)

### The spectra of supersymmetric states in string theory

Cheng, M.C.N.

**Publication date**  
2008

[Link to publication](#)

#### **Citation for published version (APA):**

Cheng, M. C. N. (2008). *The spectra of supersymmetric states in string theory*. [Thesis, fully internal, Universiteit van Amsterdam].

#### **General rights**

It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

#### **Disclaimer/Complaints regulations**

If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: <https://uba.uva.nl/en/contact>, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.

---

# 3

## K3 Compactification

---

After discussing the Calabi-Yau compactification of string theories in details, we will be brief in the K3 compactification since many of the basic ideas are fairly similar to the Calabi-Yau case. By discussing the M-theory/type II string theory compactification on K3 manifolds, we will also introduce the toroidally compactified heterotic string theories, which are related to K3 compactified M-theory/type II string theory by dualities.

This chapter is organized as follows. First of all, we assume some basic knowledge about the generic topological properties of K3 manifolds. The readers who are not familiar with them can resort to Appendix A. In the first section we again begin with a world-sheet perspective, introducing the (4,4) superconformal field theory which is relevant for describing the internal CFT with K3 as the target space. With the knowledge that the marginal deformation of the CFT is given by the moduli space of the target space, in section 3.2 we derive the form of the moduli space using a spacetime viewpoint. In section 3.3 we dimensionally reduce type II string theory on  $K3 \times T^2$  and study the low-energy effective theory in four dimensions. From the form of the charge lattice and the moduli space we motivate the existence of a toroidally compactified heterotic string theory which is dual to type II superstring on  $K3 \times T^2$ , and spell out the correspondence of conserved charges in different frames on the heterotic-IIA-M-IIB chain connected by various dualities.

### 3.1 (4,4) Superconformal Field Theory

As we mentioned earlier, a Calabi-Yau manifold with  $n$  complex dimensions can be defined as a Kähler manifold with  $SU(n)$  holonomy. In particular, a K3 manifold has  $SU(2)$  holonomy and is therefore also hyper-Kähler. By decomposing the four dimensional spinor in representations of  $SO(4) = SU(2) \times SU(2)$ , we see that the holonomy preserves 1/2 of the total thirty-

two supersymmetries, as opposed to 1/4 in the case of Calabi-Yau three-folds. From our experience with the relationship between spacetime and world-sheet supersymmetry, it is therefore not surprising that the relevant superconformal field theory now turns out to have (4, 4) instead of (2, 2) world-sheet supersymmetries.

The action of the non-linear sigma model is again given by (2.1.2), supplemented with the coupling to the B-field (2.1.4). Instead of the  $\mathcal{N} = 2$  superconformal algebra (2.1.1), we have now the following (small)  $\mathcal{N} = 4$  superconformal algebra

$$\begin{aligned}
[L_m, L_n] &= (m - n)L_{m+n} + \frac{c}{12}m(m^2 - 1)\delta_{m+n,0} \\
[J_m^i, J_n^j] &= -2i\epsilon^{ijk}J_{m+n}^k + \frac{c}{3}m\delta_{m+n,0}\delta^{ij} \\
[L_n, J_m^i] &= -mJ_{m+n}^i \\
[L_n, G_r^{\pm\pm}] &= \left(\frac{n}{2} - r\right)G_{r+n}^{\pm\pm} \\
[J_n^i, G_r^{\alpha+}] &= \sigma_{\alpha\beta}^i G_{r+n}^{\beta+}, \quad [J_n^i, G_r^{\alpha-}] = -G_{r+n}^{\beta-} \sigma_{\beta\alpha}^i \\
\{G_r^{\alpha+}, G_s^{\beta-}\} &= 2\delta^{\alpha\beta}L_{r+s} + (r - s)\sigma_{\alpha\beta}^i J_{r+s}^i + \frac{c}{3}\left(r^2 - \frac{1}{4}\right)\delta_{r+s,0}\delta^{\alpha\beta},
\end{aligned} \tag{3.1.1}$$

where  $\alpha, \beta = \pm$ ,  $i = 1, 2, 3$ ,  $\sigma^i$  are the Pauli matrices and the superscripts “ $\pm\pm$ ” of the fermionic currents  $G^{\pm\pm}$  denote the way they transform under the R-symmetry group  $SU(2)$ . Again we have two possible periodic conditions for the fermions

$$\begin{cases} 2r = 0 \pmod{2} & \text{for R sector} \\ 2r = 1 \pmod{2} & \text{for NS sector.} \end{cases} \tag{3.1.2}$$

This  $\mathcal{N} = 4$  superconformal algebra shares some important features with the  $\mathcal{N} = 2$  superconformal algebra (2.1.1). First of all, there is a natural embedding of the  $\mathcal{N} = 2$  algebra into the  $\mathcal{N} = 4$  algebra given by

$$J_m \rightarrow J_m^3, \quad G_r^+ \rightarrow G_r^{++}, \quad G_r^- \rightarrow G_r^{+-}. \tag{3.1.3}$$

As for the representation, a highest weight state is again defined by

$$\begin{aligned}
G_r^{\pm\pm}|h, q\rangle &= J_n^i|h, q\rangle = L_n|h, q\rangle = 0 \quad \text{for all } r, n > 0 \\
L_0|h, q\rangle &= h|h, q\rangle, \quad J_0^3|h, q\rangle = q|h, q\rangle \dots
\end{aligned}$$

As before, a special is played by the “massless representation”, meaning states which are in addition annihilated by

$$J_0^+, G_0^{\pm\pm} \tag{3.1.4}$$

for states in the R-sector and

$$J_0^+, G_{-1/2}^{-+}, G_{-1/2}^{+-} \quad (\text{or } J_0^-, G_{-1/2}^{++}, G_{-1/2}^{--}) \quad (3.1.5)$$

for states in the NS-sector, where we have defined  $J^\pm = \frac{1}{\sqrt{2}}(J^1 \mp iJ^2)$ . These are the counter-part of the R-ground states and the chiral primaries in the  $\mathcal{N} = 2$  case respectively, and it can again be seen from various commutation relations that an unitary massless representation satisfies

$$0 \leq h = \frac{c}{24}, \quad |q| \leq \frac{c}{6} \quad \text{in the R-sector} \quad (3.1.6)$$

and

$$0 \leq h = \frac{|q|}{2} \leq \frac{c}{6} \quad \text{in the R-sector} . \quad (3.1.7)$$

Finally, there is again an automorphism of this  $\mathcal{N} = 4$  algebra which generalises the spectral flow of the  $\mathcal{N} = 2$  algebra (2.1.24) to

$$\begin{aligned} L_n &\rightarrow L_n + \eta J_n + \eta^2 \frac{c}{6} \delta_{n,0} \\ J_n^3 &\rightarrow J_n^3 + \eta \frac{c}{3} \delta_{n,0} \quad , \quad J_n^\pm \rightarrow J_{n \pm 2\eta}^\pm \\ G_r^{\pm+} &\rightarrow G_{r \pm \eta}^{\pm+} \quad , \quad G_r^{\pm-} \rightarrow G_{r \mp \eta}^{\pm-} . \end{aligned} \quad (3.1.8)$$

This in particular implies that the elliptic genus has again the theta-function decomposition as in (2.1.38).

There are of course also differences between the  $\mathcal{N} = 4$  and  $\mathcal{N} = 2$  non-linear sigma models. One important distinction is that, unlike the case for the Calabi-Yau three-folds, the Ricci flat metric is now an exact solution but not just in the leading order of  $\alpha'$ , due to the non-renormalisation theorem brought to us by higher supersymmetries. In the  $\mathcal{N} = 4$  case there is again a notion of mirror symmetry, but since now the complex structure and Kähler moduli are in the same cohomology  $H^{1,1}(X, \mathbb{R})$ , the discussion of the mirror symmetry becomes more involved and we will not include it in the present thesis. See [66] for some discussions of  $\mathcal{N} = 4$  superconformal algebras and [67, 68] for its representations relevant in the present context.

## 3.2 Moduli Space of K3

Two major differences between the moduli space of Calabi-Yau two-and three-folds are that for the K3 case, first of all there is no clear separation between the complex and Kähler moduli space; now both of them are in the same

## 82 3. K3 Compactification

cohomology class  $H^{1,1}(S, \mathbb{R})$ . Secondly, as we have mentioned before, a K3 manifold is not only Kähler but also hyper-Kähler, which means it has not only one complex structure but a whole  $S^2$  of possible complex structures, rotated to each other by elements of  $SU(2)$ .

Keeping these facts in mind, a simple counting gives the dimension of the moduli space of the non-linear sigma model:

$$\dim \mathcal{M}_\sigma = \dim H^{1,1}(S, \mathbb{Z}) + 2\dim H^{1,1}(S, \mathbb{Z}) + \dim H^2(S, \mathbb{Z}) - 2 = 80 ,$$

where the first three terms account for the moduli space for the Kähler moduli, the complex structure moduli, and the B-field respectively, and the 2 is subtracted to account for the fact that each metric comes with a sphere of complex structures.

To see the structure of this 80-dimensional moduli space, let's first concentrate on the complex structure and Kähler moduli. From

$$\begin{aligned} \int_S J \wedge J, \int_S \Omega \wedge \bar{\Omega} &> 0 \\ \int_S \Omega \wedge \Omega = \int_S J \wedge \Omega &= 0 , \end{aligned}$$

where  $J$  is the Kähler form and  $\Omega = \Omega_1 + i\Omega_2$  is the complex structure, we see that  $J$ ,  $\Omega_1$  and  $\Omega_2$  are three vectors that are all mutually perpendicular, with respect to the bilinear (A.0.18) on the space  $H^{2*}(S, \mathbb{R}) \cong \mathbb{R}^{4,20}$  (A.0.21)

$$(\alpha, \beta) = \int_S \alpha \wedge \beta \tag{3.2.1}$$

and that are all spacelike. In other words,  $J$ ,  $\Omega_1$  and  $\Omega_2$  defines a three-dimensional plane inside  $H^2(S, \mathbb{R}) \cong \mathbb{R}^{3,19}$ . Furthermore, a rotation of the three vectors corresponds to a rotation of the  $S^2$  possibilities of complex structures and therefore does not correspond to a change in the geometry. In other words, the complex structure and Kähler moduli space of K3 is locally a Grassmannian times the positive half of a real line representing the volume  $V$  of the K3. Globally, the moduli space is

$$O(\Gamma^{3,19}) \backslash O(3, 19, \mathbb{R}) / (O(3, \mathbb{R}) \times O(9, \mathbb{R})) \times \mathbb{R}_+ ,$$

where  $O(\Gamma^{3,19})$  is the automorphism group of the lattice  $\Gamma^{3,19}$ .

Now we want to incorporate the moduli space for the B-field, which is not considered in the above discussion. Given a choice of B-field two-form and the volume  $V$ , define a map  $\xi : H^2(S, \mathbb{R}) \cong \mathbb{R}^{3,19} \rightarrow H^{2*}(S, \mathbb{R}) \cong \mathbb{R}^{4,20}$  and an

additional vector  $\xi_4$  by

$$\begin{aligned}\xi(\alpha) &= \alpha - (B, \alpha)\alpha^0 \\ \xi_4 &= \alpha_0 + B + \left(V - \frac{1}{2}(B, B)\right)\alpha^0,\end{aligned}$$

where  $\alpha_0, \alpha^0$  are the dual basis for  $H^0(S, \mathbb{Z}), H^4(S, \mathbb{Z})$  introduced in (A.0.21). It can be checked easily that  $\xi_4$  is perpendicular to all  $\xi(\alpha)$  with  $\alpha \in H^2(S, \mathbb{R})$  and that  $\xi(J), \xi(\Omega_1), \xi(\Omega_2)$  are again three mutually perpendicular space-like vectors, but now in the larger space  $\mathbb{R}^{4,20}$ . Furthermore, the spacelike four-dimensional plane spanned by  $\xi(J), \xi(\Omega_1), \xi(\Omega_2)$  and  $\xi_4$  contains the same information as the three-dimensional plane spanned by  $J, \Omega_1, \Omega_2$ , when a choice of  $B, V$  and  $\alpha_0$  is given. On the other hand, the B-field moduli can be thought of as the moduli of embedding  $\mathbb{R}^{3,19} \cong H^2(S, \mathbb{R})$  into the larger space  $\mathbb{R}^{4,20} \cong H^2 * (S, \mathbb{R})$ .

Let's now consider an integral shift of the B-field

$$B \rightarrow B + \beta \quad , \quad \beta \in H^2(S, \mathbb{Z}),$$

which must be a symmetry of the theory. Equivalently, it can be seen as a change in the choice of  $\alpha_0$

$$\alpha_0 \mapsto \alpha_0 + \beta - \frac{(\beta, \beta)}{2}\alpha^0$$

together with the following shift of the two-form

$$\alpha \mapsto \alpha - (\alpha, \beta)\alpha^0.$$

In other words, when the B-fields are incorporated, the symmetry group involves the whole automorphism group of the larger lattice  $H^{2*}(S, \mathbb{Z}) \cong \Gamma^{4,20}$ . Putting the above together, we then conclude that the moduli space of the K3 non-linear sigma model given by a Grassmannian as

$$\mathcal{M}_\sigma = O(\Gamma^{4,20}) \backslash O(4, 20, \mathbb{R}) / (O(4, \mathbb{R}) \times O(20, \mathbb{R})). \quad (3.2.2)$$

See [69, 70, 71] and references therein for discussions about the above moduli space.

### 3.3 Four-Dimensional Theories and Heterotic String Dualities

As mentioned before, the  $SU(2)$  holonomy of K3 leads to the breaking of half of the supersymmetries. At the low energy limit, type II string theories

compactified on a K3 manifold therefore yield six-dimensional supergravity theories with sixteen supercharges. However, we will be interested in four-dimensional theories instead of six-dimensional ones.

For concreteness, we will begin with considering type IIA string theory compactified on the internal manifold  $K3 \times T^2$  down to four dimensions. The torus has trivial holonomy and thus does not break supersymmetry any further. The four-dimensional theory has now  $\mathcal{N} = 4$  supersymmetry and we anticipate to obtain some  $\mathcal{N} = 4, d=4$  supergravity theory at low energy. We will therefore begin this section by discussing the generalities of these  $\mathcal{N} = 4, d = 4$  supergravity theories.

### 3.3.1 $\mathcal{N} = 4, d = 4$ Supergravity

There are two kinds of supermultiplets relevant in  $\mathcal{N} = 4, d = 4$  supergravity theories, namely the supergravity and the matter multiplets. From their bosonic field contents we then expect the bosonic field content of our low energy effective action to be

$$(g_{\mu\nu}, A_{\mu}^{m=1,\dots,6}, \lambda) \text{ and } n \times (A_{\mu}, \phi^{m=1,\dots,6}),$$

where  $\lambda$  is a complex scalar and  $m$  is an  $SU(4) = SO(6)$  R-symmetry index. Furthermore, the 2 and  $6n$  scalars parametrise the scalar manifold [72]

$$\frac{SL(2)}{U(1)} \times \frac{SO(6, n)}{SO(6) \times SO(n)}.$$

To study the supergravity theory obtained by the IIA/ $K3 \times T^2$  compactification, first we would like to determine the number of matter multiplets in the theory. We will do this by counting the number of scalars by dimensionally reducing the massless fields of type IIA string theory (Table 1.1) using the harmonic forms of the internal manifold. The result is

80	$g, B$ on $K3$
4	$g, B$ on $T^2$
2	$C^{(1)}$
44	$C^{(3)}$
2	$C^{(3)}$ to spatial one-forms and dualize to scalars
2	$\Phi, B_{\mu\nu}$ (axion-dilaton)

which implies in this case  $n = 22$ , namely that the massless field content of the four-dimensional theory is one  $\mathcal{N} = 4$  supergravity multiplet together with 22 matter multiplets.

One can easily check that there are 28 vector fields upon compactification, which again decompose into a supergravity multiplet together with 22 matter multiplets. With respect to these vector fields, there are 28 electric and 28 magnetic conserved charges, forming a charge lattice

$$\begin{pmatrix} P \\ Q \end{pmatrix} \in \Gamma^{6,22} \oplus \Gamma^{6,22} . \quad (3.3.1)$$

The Grassmannian part of the scalar manifold

$$\frac{SO(6, 22)}{SO(6) \times SO(22)}$$

can be thought of as the moduli space of different ways to separate the above charges into the “left-moving” and “right-moving” parts, such that

$$P_L^2 - P_R^2 = P^2 \quad (3.3.2)$$

and similarly for the electric charges. Explicitly, we can therefore parametrise this part of the moduli space by a  $28 \times 6$  matrix  $\mu_{a=1, \dots, 28}^{m=1, \dots, 6}$ , such that

$$P_L^m = \mu_a^m P^a \quad (3.3.3)$$

and similarly for the  $Q$ 's. Notice that  $\mu$  is only defined up to rotations which leave all  $P_L^2$  invariant.

For a very simple example of a moduli space which is a Grassmannian, let's consider string theory compactified on a circle with radius  $R$  and consider the states with winding number  $w$  and momentum  $k$  along the circle. Then the left- and right-charges are

$$\begin{aligned} P_L &= k/R + wR \\ P_R &= k/R - wR \quad , \quad P_L^2 - P_R^2 = P^2 = 4kw . \end{aligned}$$

Notice that  $P_L^2 - P_R^2$  does not depend on the radius  $R$  while both  $P_L$  and  $P_R$  do.

The Grassmannian  $SO(1, 1, \mathbb{R})$  is a one-dimensional space parametrised by a real number  $\eta$  as

$$\begin{pmatrix} P_L \\ P_R \end{pmatrix} = \begin{pmatrix} \cosh \eta & \sinh \eta \\ \sinh \eta & \cosh \eta \end{pmatrix} \begin{pmatrix} k + w \\ k - w \end{pmatrix} . \quad (3.3.4)$$

Then we see that the modulus of the compactification circle, in this case the radius  $R$ , is related to  $\eta$  by

$$\cosh \eta = \frac{1}{2} \left( R + \frac{1}{R} \right) \quad , \quad \sinh \eta = -\frac{1}{2} \left( R - \frac{1}{R} \right) . \quad (3.3.5)$$

Finally let's turn to the first factor of the scalar manifold

$$\frac{SL(2)}{U(1)} \cong \mathcal{H}_1 . \quad (3.3.6)$$

As discussed in section (1.3.5), this is nothing but the upper half-plane and we will parametrise it by  $\tau \in \mathbb{C}$ ,  $\text{Im}\tau > 0$  as in (1.3.41).

In our present setting of IIA/ $K3 \times T^2$  compactification, this complex scalar  $\lambda$  is the complexified Kähler moduli of the torus. As we will see in the following subsection, it becomes the complex structure moduli in the IIB/ $K3 \times T^2$  compactification and the axion-dilaton moduli in the heterotic/ $T^6$  compactification, when we apply a chain of dualities.

In terms of these scalars  $(\lambda, \mu)$  and the conserved charges  $(P, Q)$ , we can now write down the solutions to this supergravity theory. We will leave the details for the Part V of the thesis.

### 3.3.2 Heterotic String Dualities

In the previous subsection we have seen that the low-energy supergravity theory obtained from compactifying type IIA string theory on  $K3 \times T^2$ , has a scalar manifold which contains the Grassmannian  $SO(6, 22)/SO(6) \times SO(22)$ . This is exactly how the moduli space of a conformal field theory compactified on a  $\Gamma^{6,22}$  lattice looks like locally. Notice that there is one unique (up to isomorphism) lattice of this signature (or any  $\Gamma^{\sigma^+, \sigma^-}$  with  $\sigma^- - \sigma^+ = 0 \pmod{8}$ ) which is even self-dual, or sometimes called unimodular. And an even, self-dual lattice is exactly the kind of lattice required for the one-loop modular invariance of the conformal field theory. Including four free bosons on both sides corresponding to the four non-compact dimensions, this putative conformal field theory should have (10, 26) bosons on the left- and right- moving sector respectively. Notice that they are the critical dimensions, namely the required number of free bosons in order to have total central charge zero with the ghosts included, for  $\mathcal{N} = 1$  and  $\mathcal{N} = 0$  world-sheet supersymmetry respectively. We therefore conclude that only the left-moving sector of this putative conformal field theory has world-sheet supersymmetry. Such conformal field theories are called *heterotic string theories*. One way of interpreting such a conformal field theory geometrically is to say that it has ten spacetime dimensions and the rest of the 16 right-moving bosons are always compactified

on an internal sixteen dimensional even self-dual lattice. In this language, the observation is that the massless fields of type IIA string theory compactified on  $K3 \times T^2$  is the same as that of heterotic string compactified on  $T^6$ . In more details, one can see from matching the low-energy supergravity theory that the complex scalar  $\lambda$  in the supergravity multiplet is now the axion-dilaton field of heterotic string, and the 28 vectors are the 16 gauge bosons present in the massless spectrum of the heterotic string theory and the other 12 coming from compactifying the metric and the B-field on the six-torus.

This motivates the conjecture of the following string duality [73, 74]

$$\text{IIA}/K3 \times T^2 \text{ is dual to heterotic}/T^6 .$$

The U-duality group of the theory is conjectured to be

$$SL(2, \mathbb{Z}) \times SO(6, 22, \mathbb{Z}) ,$$

where the presence of the first factor can be seen from the presence of the modular group of  $T^2$  in the type IIA picture, which then translates into the S-duality (strong-weak-coupling duality) of the heterotic string. This is very reminiscent of the interpretation of the S-duality group of type IIB string theory as the torus modular group in the type M-theory as we saw in section 1.3.5. This group acts on the charges and moduli as

$$\begin{pmatrix} P \\ Q \end{pmatrix} \rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} P \\ Q \end{pmatrix} , \quad \lambda \rightarrow \frac{a\lambda + b}{c\lambda + d} \quad ; \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in PSL(2, \mathbb{Z}) \quad (3.3.7)$$

while leaving the Grassmannian moduli  $\mu$  invariant.

The second group, on the other hand, is nothing but the T-duality group of the  $\Gamma^{6,22}$  compactification of the heterotic string, or equivalently the automorphism group of the charge lattice  $\Gamma^{6,22}$ . In particular, this group rotates the electric and magnetic charges separately and does not create a mix between them. For convenience we will refer to them in the heterotic language as the S- and the T-duality group respectively in the future.

Of course, one can combine the dualities between M- and type IIA, IIB string theories discussed earlier in section 1.3.1 and 1.3.5 with the above new IIA-heterotic dualities and thereby construct a new web of dualities: [73, 74]

$$\text{IIA}/K3 \times T^2 \sim \text{IIB}/K3 \times T^2 \sim \text{M-theory}/K3 \times T^2 \times S^1 \sim \text{heterotic}/T^6 .$$

For later reference we will now write down the charged objects giving the charges  $\begin{pmatrix} P \\ Q \end{pmatrix} \in \Gamma^{6,22} \oplus \Gamma^{6,22}$  in the above different duality frames. Separating the charge lattice into four parts

$$\Gamma^{6,22} \cong \Gamma^{3,19} \oplus \Gamma^{1,1} \oplus \Gamma^{1,1} \oplus \Gamma^{1,1} \cong H^2(K3, \mathbb{Z}) \oplus \Gamma^{1,1} \oplus \Gamma^{1,1} \oplus \Gamma^{1,1} \quad (3.3.8)$$

magnetic and electric charges $(\mathbf{P}, \mathbf{Q}) \in \Gamma^{6,22} \oplus \Gamma^{6,22}$				
	het/ $S^1_{(2)} \times S^1_{(3)} \times S^1_{(4)} \times T^3$	IIA/ $S^1_{(2)} \times S^1_{(3)} \times K3$	M-th/ $S^1_{(1)} \times S^1_{(2)} \times S^1_{(3)} \times K3$	IIB/ $S^1_{(1)} \times S^1_{(3)} \times K3$
$\Gamma^{1,1}$	p(4) F1(4)	D0 D4 (K3)	p(1) M5(1,K3)	F1(1) NS5(1,K3)
$\Gamma^{1,1}$	p(2) F1(2)	p(2) NS5(2,K3)	p(2) M5(2,K3)	D1(1) D5(1,K3)
$\Gamma^{1,1}$	p(3) F1(3)	p(3) NS5 (3,K3)	p(3) M5(3,K3)	p(3) KKM( $\hat{1}$ )
$\Gamma^{3,19}$	$q_A$	D2( $\alpha^A$ )	M2( $\alpha^A$ )	D3(1, $\alpha^A$ )
$\Gamma^{1,1}$	NS5( $\hat{4}$ ) KKM( $\hat{4}$ )	D2(2,3) D6 (2,3,K3)	M2(2,3) TN(2,3,K3)	F1(3) NS5(3,K3)
$\Gamma^{1,1}$	NS5( $\hat{2}$ ) KKM( $\hat{2}$ )	F1(3) KKM( $\hat{2}$ )	M2(1,3) KKM( $\hat{2}$ )	D1(3) D5(3,K3)
$\Gamma^{1,1}$	NS5( $\hat{3}$ ) KKM( $\hat{3}$ )	F1(2) KKM( $\hat{3}$ )	M2(1,2) KKM( $\hat{3}$ )	p(1) KKM( $\hat{3}$ )
$\Gamma^{3,19}$	$p^A$	D4(2, 3, $C_{AB}\alpha^B$ )	M5(1, 2, 3, $C_{AB}\alpha^B$ )	D3(3, $C_{AB}\alpha^B$ )

Table 3.1: A chain of dualities relating the charged objects in the different  $\mathcal{N} = 4, d = 4$  string theories, where  $\alpha_A$ 's are a basis of the twenty-two dimensional lattice  $H^2(K3, \mathbb{Z}) \cong \Gamma^{3,19}$  with the bilinear given by  $C_{AB} = \int_{K3} \alpha_A \wedge \alpha_B$ .

with respective bilinear form given by  $C_{AB} = \int_{K3} \alpha_A \wedge \alpha_B, A, B = 1, \dots, 22$  and  $U = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$  (A.0.20), and using the duality relations between charged objects summarised Table 1.3 and Table 1.4, we obtain the following Table 3.1 of charged objects of the theory (3.3.8) in its different frames.

Since different duality frames gives different perspectives in counting states, we will use this table extensively when we later derive the microscopic degeneracies of BPS states of this theory.