Quantifiers in TIME and SPACE: computational complexity of generalized quantifiers in natural language
Szymanik, J.K.

Citation for published version (APA):

General rights
It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

Disclaimer/Complaints regulations
If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: http://uba.uva.nl/en/contact, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.
# Contents

vii

Acknowledgments

xi

Introduction

1

1 Algorithmic Semantics

1.1 Meaning and Comprehension ........................................ 5
1.2 Geography of Perspectives ........................................... 7
   1.2.1 Traditional Semantics and Pragmatics .......................... 7
   1.2.2 The Dynamic Turn in Semantics ................................. 8
   1.2.3 Meaning as Algorithm ........................................... 10
1.3 Our Procedural Perspective on Meaning ............................ 15
   1.3.1 Questions ..................................................... 15
   1.3.2 Methodology .................................................. 15
   1.3.3 Psychological Motivations ..................................... 17
1.4 Algorithms and Computations ........................................ 18
   1.4.1 What is an “Algorithm”? ....................................... 18
   1.4.2 Turing Computability .......................................... 19
1.5 Computability and Cognition ......................................... 24
   1.5.1 Computational Explanation of Cognitive Tasks .................. 25
   1.5.2 Computational Bounds on Cognition ............................ 26
   1.5.3 Common Doubts ............................................... 29
   1.5.4 Explaining the Comprehension Task ............................ 31
1.6 Finite Interpretations ................................................. 32
1.7 Complexity in Linguistics ............................................. 34
   1.7.1 Syntax ....................................................... 34
   1.7.2 Between Syntax and Semantics .................................. 35
1.8 Classifying Meaning by Intractability ............................... 37
## 2 Mathematical Prerequisites

2.1 Basic Notation ......................................... 43

2.2 Generalized Quantifier Theory .......................... 44
   2.2.1 Two Equivalent Concepts of Generalized Quantifiers .... 45
   2.2.2 Branching Quantifiers .............................. 47
   2.2.3 Logic Enriched by Generalized Quantifiers ............ 48
   2.2.4 Definability of Generalized Quantifiers ............... 48
   2.2.5 Linguistic Properties of Generalized Quantifiers ...... 50

2.3 Computability ........................................... 54
   2.3.1 Languages and Automata ............................. 54
   2.3.2 Turing Machines ..................................... 60
   2.3.3 Complexity Classes .................................. 61
   2.3.4 Oracle Machines ..................................... 63
   2.3.5 The Polynomial Hierarchy ............................ 64
   2.3.6 The Counting Hierarchy .............................. 64
   2.3.7 Reductions and Complete Problems .................... 65

2.4 Descriptive Complexity Theory .......................... 66
   2.4.1 Encoding Finite Models .............................. 67
   2.4.2 Logics Capturing Complexity Classes .................. 68

2.5 Quantifiers in Finite Models ............................ 69

## 3 Complexity of Polyadic Quantifiers

3.1 Standard Polyadic Lifts ................................ 72
   3.1.1 Iteration ......................................... 73
   3.1.2 Cumulation ....................................... 74
   3.1.3 Resumption ....................................... 74
   3.1.4 PTIME GQs are Closed under It, Cum, and Res .......... 75

3.2 Branching Quantifiers .................................. 77
   3.2.1 Henkin’s Quantifiers are Mighty ..................... 77
   3.2.2 Proportional Branching Quantifiers are Mighty ........ 78
   3.2.3 Branching Counting Quantifiers are Mighty ........... 79
   3.2.4 Linguistic Remark and Branching of Dissertation ...... 80

3.3 Ramsey Quantifiers ..................................... 81
   3.3.1 Ramsey Theory and Quantifiers ....................... 81
   3.3.2 The Branching Reading of Hintikka’s Sentence ........ 82
   3.3.3 Clique Quantifiers .................................. 83
   3.3.4 Proportional Ramsey Quantifiers ...................... 84
   3.3.5 Tractable Ramsey Quantifiers ......................... 86

3.4 Summary ................................................ 89
4 Complexity of Quantified Reciprocals .............................. 91
  4.1 Reciprocal Expressions .............................................. 92
    4.1.1 Strong Meaning Hypothesis ..................................... 94
  4.2 Reciprocals as Polyadic Quantifiers ................................ 96
    4.2.1 Strong Reciprocal Lift ........................................... 96
    4.2.2 Intermediate Reciprocal Lift .................................... 97
    4.2.3 Weak Reciprocal Lift ............................................. 97
    4.2.4 The Reciprocal Lifts in Action .................................. 98
  4.3 Complexity of Strong Reciprocity .................................. 99
    4.3.1 Counting Quantifiers in the Antecedent ....................... 99
    4.3.2 Proportional Quantifiers in the Antecedent .................. 100
    4.3.3 Tractable Strong Reciprocity .................................. 102
  4.4 Intermediate and Weak Lifts ........................................ 103
  4.5 A Complexity Perspective on the SMH ................................ 105
  4.6 Summary .................................................................. 107

5 Complexity of Collective Quantification ................................ 109
  5.1 Collective Quantifiers ............................................... 110
    5.1.1 Collective Readings in the Natural Language .................. 110
    5.1.2 Modelling Collectivity ............................................ 111
  5.2 Lifting First-order Determiners ..................................... 113
    5.2.1 Existential Modifier .............................................. 113
    5.2.2 The Neutral Modifier ............................................. 115
    5.2.3 The Determiner Fitting Operator ................................ 116
    5.2.4 A Note on Collective Invariance Properties .................. 117
  5.3 Second-order Generalized Quantifiers ................................ 118
    5.3.1 Definability for SOGQs ............................................ 119
  5.4 Defining Collective Determiners by SOGQs .......................... 121
  5.5 Collective Majority .................................................. 123
    5.5.1 An Undefinability Result for SOGQ “MOST” .................... 123
    5.5.2 Consequences of Undefinability ................................ 124
  5.6 Summary .................................................................. 126

6 Hintikka’s Thesis Revisited ............................................ 129
  6.1 Hintikka’s Thesis ..................................................... 130
  6.2 Other Hintikka-like Sentences ...................................... 132
  6.3 Theoretical Discussion ............................................... 133
    6.3.1 A Remark on Possible Readings ................................... 133
    6.3.2 Hintikka-like Sentences are Symmetric ......................... 134
    6.3.3 Inferential Arguments ............................................. 136
    6.3.4 Negation Normality ................................................. 137
    6.3.5 Complexity Arguments .............................................. 138
    6.3.6 Theoretical Conclusions .......................................... 139