Quantifiers in TIME and SPACE : computational complexity of generalized quantifiers in natural language

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Introduction

Everyone who has ever tried to understand, explain, and describe natural language knows how complicated it is. This dissertation is concerned with some aspects of complexity in natural language. In particular, we try to shed some light on the interplay between complexity and expressibility.

Complexity is a very broad, although quite intuitive, notion. There are at least two levels of language complexity — syntactic level and semantic level. The latter is what we will consider in the thesis. Moreover, we will focus on **complexity of meaning (semantics) of natural language quantifiers**. Especially, we are interested in complexity of finding the truth-values of natural language quantified sentences in finite situations. The general question we aim to answer is why the meanings of some sentences are more difficult than the meanings of others. For instance, why we will probably all agree that it is easier to evaluate sentence (1) than sentence (2) and why sentence (3) seems hard while sentence (4) sounds odd.

(1) Every book on the shelf is yellow.

(2) Most of the books on the shelf are yellow.

(3) Less than half of the members of parliament refer to each other.

(4) Some book by every author is referred to in some essay by every critic.

To discuss such differences in a precise manner, in the dissertation we use tools of computability theory to measure complexity. By doing it we commit ourselves to the idea that part of a "meaning" can be identified with an algorithm that checks the truth-value of an expression. Having this in mind we formalize linguistic descriptions in logic. Then we use **descriptive computational complexity** to study properties of formalisms we have defined. Among other things we try to draw a systematic line between easy (tractable) and difficult (intractable) quantifiers with respect to model-checking complexity. Moreover, as we are interested in the empirical value of complexity claims we provide some **linguistic case studies** as well as **psycholinguistic empirical experiments**. We apply complexity analysis in the domain of reciprocal expressions to account for some pragmasemantic phenomena, study various combinations of quantifiers in natural language, and empirically investigate the comprehension of simple quantifier sentences. Our results show that computational complexity can be very useful in investigating natural language semantics.

As far as our research topics and methods are concerned this work is **highly interdisciplinary**. The study can be placed within logic, natural language semantics, philosophy of mind and language, theoretical computer science, and cognitive science. It is an example of applying computational complexity theory to linguistics and cognitive science. Therefore, the intended audience consists mostly of linguists, philosophers and cognitive scientists interested in formal approaches to complexity in natural language. However, we hope that also logicians and computer scientists can find this study a source of inspiration, not only for possible applications, but also for developments in their research fields. The dissertation is intended to be self-contained, but some general background in logic and natural language semantics is assumed. Below we briefly overview every chapter.

Chapter 1, "Algorithmic Semantics", is a proper introduction to the thesis, where we place our study on the map of research themes. We discuss our methodological assumptions there; in particular, we outline the idea of treating **referential meaning as an algorithm**. We also argue that computational complexity can be taken as a reasonable measure for the difficulty of natural language expressions. Moreover, we propose some hypotheses summing up our perspective on the role of computational complexity in linguistics and cognitive science.

Chapter 2, "Mathematical Prerequisites", shows the background of the technical work of the thesis. We introduce here our main logical tools: the notions of generalized quantifier theory, computability theory, and descriptive complexity theory. The chapter may be skipped by readers with background knowledge on generalized quantifiers and computability. For the rest of the readers it can serve as a basic reference to the mathematics used elsewhere in the dissertation.

Chapter 3, "Complexity of Polyadic Quantifiers", is devoted to a logical study of the computational complexity of **polyadic generalized quantifiers**. We focus here on constructions that are interesting from the semantic point of view. We prove that quantifier iteration, cumulation and resumption do not carry us outside polynomial time computability. Other polyadic quantifiers often used in linguistics, like branching and Ramsey quantifiers, lead to NP-complete natural language sentences. This chapter prepares the ground for the more linguistic discussion in the next parts of the thesis, particularly in Chapters 4 and 6.

Chapter 4, "Complexity of Quantified Reciprocals", is concerned with the linguistic case study. We investigate the computational complexity of different interpretations of **reciprocal expressions**, like "each other", in English. Especially, we show that Ramsey quantifiers express the semantics of reciprocal sentences with a quantifier in the antecedent. As a result we find a computational dichotomy between different interpretations of reciprocals: some of them stay in PTIME when others are NP-complete. This dichotomy is consistent with well-known semantic distinctions among different interpretations of "each other". We discuss the impact of this dichotomy on the so-called Strong Meaning Hypothesis proposed as a pragmatic explanation for shifts occurring among different reciprocal readings.

Chapter 5 "Complexity of Collective Quantification" discusses yet another form of quantification in natural language. We investigate logic of **collective quantifiers** and propose to analyze them in terms of second-order generalized quantifiers. In particular, our research shows that the widely accepted type-shifting approach to modelling collectivity in natural language is probably not expressive enough to cover all instances. Additionally, it is also extremely complex and, as such, implausible. As a result we suggest to study algebraic (many-sorted) formalisms. Another interpretation of our results is that computational complexity restricts expressibility of everyday language and as a result collective readings of some quantifiers, e.g., proportional determiners, is not realized in that fragment of natural language.

Chapter 6, "Hintikka's Thesis Revisited", is the first empirical fragment of our work. We study Hintikka's well-known claim concerning the necessity of a **branching interpretation** for some natural language sentences, e.g., "Some relatives of each villager and some relatives of each townsman hate each other" or "Most boys and most girls from my class dated each other". We argue against Hintikka's Thesis and propose our own reading of these sentences, which we call the conjunctional reading. Next, we show how to investigate such a problem empirically using insights from linguistics, logic and computability. The results of our tests confirm that the conjunctional reading is a widely accepted interpretation for these sentences.

Chapter 7, "Processing Simple Quantifiers", is about a computational semantics for simple (monadic) quantifiers in natural language. In this case we can model the meaning of quantifiers using finite and push-down automata. Our aim is to show the empirical relevance of computational descriptions. We start by presenting the neurological research studying an automata-theoretic model of **simple quantifier comprehension**. Next we criticize the methodology of this research and propose a better set of hypotheses. We use these to conduct our own empirical study comparing the reaction times needed for recognizing the truth-value of sentences with simple quantifiers. The results strongly confirm the complexity predictions of the computational model for simple quantifiers in natural language. Hence, this chapter directly links computational predictions to linguistic processing.

Chapter 8, "Conclusions and Perspectives", closes the dissertation with a short summary of results achieved from the perspective of the grand issues recognized in the first chapter. This is followed by a discussion of related open questions and directions for further research. This final discussion focuses on general loose ends, as particular research questions directly implied by our technical work are discussed at the end of the corresponding chapters.