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THE CIRCULATION OF IDEAS:
FIRMS VERSUS MARKETS

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THE CIRCULATION OF IDEAS: FIRMS VERSUS MARKETS

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ABSTRACT

The Circulation of Ideas: Firms Versus Markets*

We describe new ideas as incomplete concepts for which the innovator needs feedback from agents with complementary skills. Once shared, ideas may be stolen. We compare how different contractual environments support invention and implementation. Markets, as open exchange systems, are good for circulation and thus elaboration, but may fail to reward idea generation. Firms, as controlled idea exchange systems, can reward idea generation but can do so only by restricting their circulation. This identifies a basic trade-off between protecting the rights of invention and the best implementation of ideas. An environment that allows ideas to cross firm boundaries enhances the rate of innovation and creates a symbiotic relationship between markets and firms.

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1 Introduction

The role of innovation in economic growth is well recognized (Romer, 1990), yet the process of generating innovative ideas is still a novel field of inquiry. The literature has focused on the role of intellectual property rights as incentive for invention (Nordhaus, 1969, Gallini and Scotchmer, 2001). We focus here on an earlier stage in the innovation process, where ideas are still half-baked (so they cannot yet be patented), and need to be shared in order to be further screened and elaborated. We study how different contracting environments promote the development of novel concepts, when communicating them exposes the inventor to the risk that they will be stolen (Arrow, 1962; Anton and Yao, 1994). In the process we explore the notion that a free circulation of ideas may be as critical for generating innovation as their protection. Our main result is that the most innovative environment produces a symbiotic interaction between large firms and markets. While firms specialize in incubating ideas, markets (in the sense of open idea exchanges without a governance structure) may be best at elaborating those ideas which do not fit firms.

While the technological success of new ventures in Silicon Valley is often taken as evidence that innovation thrives in a free market environment, it is remarkable how any history of Silicon Valley describes the long list of talented people leaving large firms with novel ideas. Most R&D is still performed in established firms.\(^1\) Clearly, large established firms play some role as incubators for innovation. Specifically, in our approach, they spawn new ventures.

Schumpeter (1926, 1942) argued that new ideas are original combinations of existing factors (Weitzman, 1998). Most random combinations are useless. Valuable ideas are combinations which ”fit” together, aggregating the component resources in a novel, functional way (Biais and Perotti, 2004). Accordingly, we define an early stage idea as an incompletely specified combination, which requires further elaboration before generating value.

Because the idea is both incomplete and novel, it is not obvious who may be the agent with the right skills to complement it. Thus elaborating an idea requires sharing it with various agents, seeking the right match. A broad circulation of ideas is thus critical for the process of innovation, as it allows maximum scope for any concept to find the right complementary match.\(^2\) Yet there is a fundamental problem with the open circulation of ideas, namely that information can be stolen. We assume that an incomplete idea is too vague for an independent patent office to grant exclusive property rights.\(^3\)

\(^1\) For 2003, the National Science Foundation estimated private industrial R&D spending at $180 billion. In comparison, the National Venture Capital Association reported investments in venture capital backed companies amounting to $18 billion, i.e., one tenth the amount.

\(^2\) Saxenian (1994) emphasizes “cross-pollination” and open networking as a central aspect of Silicon Valley’s innovative potential.

\(^3\) The well-known Yale survey found that patent protection is only one among many ways that firms protect their intellectual property, and that patents only matter for a limited set of industries.
that agents cannot contract to cooperate on an idea without knowing its content in advance. We then study the relative merit of hierarchies (firms) versus markets in the trade-off between the need to circulate ideas and to reward their generation, when ideas must be shared before contracting.

In our model, an inventor who has developed an incomplete idea screens other agents on competence, seeking a partner with matching skills (a “complementor”) to assess and elaborate the concept. If the inventor shares his idea with a complementor, this agent will have some information (formally a signal) about the viability of the idea. The complementor can either report this signal truthfully, or not. Either the complementor considers the idea worthless, in which case he provides truthful negative feedback. Or the idea has potential, and elaborating the idea will require the interaction of two agents with complementary skills. The complementor can either steal the idea (by giving false negative feedback), or he can give honest constructive feedback, which signals his type. Since idea elaboration requires two parties with complementary skills, the inventor and the complementor agree to cooperate, and bargain over the surplus.

However, often the inventor will present the idea to an agent whose skills essentially replicate his own. We call such an agent a “substitute.” A substitute is unable to elaborate the idea, but understands it and can match the skills of the inventor. Our first result is that since a substitute has nothing to gain from truthful reporting, he will steal the idea while giving negative feedback. This discourages the inventor, who cannot distinguish a false from a truthful negative response. As a result, an idea may circulate through a sequence of agents until matched to a complementor, who will “resolve” it by either recognizing it as hopeless, or by elaborating it if valuable. Thus from a social perspective, a free circulation of ideas in an open exchange environment is efficient in ensuring a complete screening and elaboration of all ideas. The problem is that idea stealing may deny the inventor a sufficient reward for invention. In such circumstances, no ideas will be shared in an open exchange system, and firms emerge as a solution to this market failure.\(^4\)

In our setting, agents accepting employment in a firm join a closed, “structured” exchange system. Firms incur monitoring costs to control idea stealing among a limited number of employees. As part of their employment contract, employees sign a broad non-compete agreement on ideas generated inside the firm. In exchange, the firm owner commits to a performance-based reward system. When markets fail, firms may still support idea generation, provided they can maintain a reputation for honoring their employment contracts, paying out the promised rewards and refraining from stealing their employees’ ideas. However, firms have two drawbacks. Controlling the flow of ideas requires costly monitoring. And containing ideas within the firm’s

\(^4\)This results echoes Grossman and Stiglitz (1980), who derive the impossibility of perfectly informative financial prices in a frictionless environment, as there are no incentives to generate information.
boundary limits the set of possible matching opportunities.

A firm does not own an idea until the employee chooses to disclose it. Employees prefer to disclose their ideas, instead of leaving with them, if the firm can better protect the idea and promises an adequate reward. The firm allocates the task of “internal champion” or “intrapreneur” to the idea generator. We can think of the firm “registering” the idea, and monitoring its circulation by an internal recording system (read: “bureaucracy”). This creates a verifiable paper trail of internal projects, which supports the firm’s guarantee. Once an idea is recorded, the owner cannot expropriate it without loss of reputation, nor can any employee take it outside the firm. Our theory thus provides a justification for the frequently bemoaned bureaucracy associated with large hierarchical firms (Novaes and Zingales, 2004).

Why would an agent trust a firm, if he refuses to sign a market-based contract with another individual? How can firms commit to reward rather than expropriate ideas? In our setting, any individual can set up a firm, by investing enough resources into building a reputation (see Kreps, 1986). With an infinite time horizon, a reputation for fair dealing allows firms to capture the residual value of multiple ideas. A large sunk investment in reputation, which would be lost in the case of idea expropriation, convinces employees about the firm’s incentive to reward inventors, as well as to fight idea stealing by other employees.

Comparing idea generation by individual entrepreneurs and within firms, we offer a rationale for why individuals often pursue opportunities that require modest budgets, and may account for a larger fraction of radical innovations, whereas firms focus on larger scale, complex ideas which require substantial investments. Interestingly, this result is not a consequence of greater financial constraints faced by individuals (as is often assumed), but of the need to offer adequate rewards to inventors.

After studying idea circulation in markets and firms in isolation, we consider the possibility that ideas may move across firm boundaries. We study two possible forms of mobility: when firms condone the departure of an employee, after internal matching has failed; and when an employee with an idea may leave the firm without disclosing it internally. In the first case, allowing mobility may be efficient for firms, since it allows them to capture part of the value of ideas they cannot develop (Lewis and Yao, 2003). However, increased mobility also produces a general equilibrium effect, increasing the reservation utility for joining a firm. This is because the flow of ideas out of firms makes it more attractive for agents to be active in the market. In the second case, inventors prefer to develop certain ideas externally. For very profitable idea, employees may not trust the firm to honor its commitment not to steal the idea. And for ideas that are a poor fit with the firm’s competencies, employees may resent the delay of first looking for an internal match.

In general, the mobility of ideas from firms to markets enhances efficiency: firms promote idea generation, while markets increase their chances of elaboration, thus

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5Of course, an internal recording does not protect the idea from non employees. However, any employee who shares an internal idea with an outsider would not be able to legally profit from it.
raising the rate of innovation. We conclude that markets and firms are in a natural symbiosis, complementing each other’s weakness.

Relationship to the literature

The literature on the theory of the firm has made significant progress by looking at the role of corporate ownership of assets (Hart, 1995). Ideas are in principle intangible real assets, yet their distinctive feature in our approach is that they are neither verifiable ex ante (i.e. prior to disclosure) nor excludable ex post (prior to patenting). Holmström and Roberts (1998) suggest that ideas and people should belong at the core of any theory of the firm. Our approach partially integrate this view with the asset ownership approach, as it views individual choices as essential to the specification of ownership rights on intangible assets.

In our model, firms are defined as boundaries around people and their ideas. As in the general literature, this boundary emerges because of a market failure, specifically to protect against idea stealing. However, the firm boundary generates its own inefficiency by limiting idea elaboration. Interestingly, markets can then contribute to resolve the firm’s inefficiency. Even when idea generation in markets is not viable, markets can complement the incubating role of firms by refining those ideas that could not be elaborated within firms. Our theory of the firm therefore points to an important symbiotic relationship between firms and markets. This provides a novel perspective on the long-standing debate of firms versus markets (Williamson, 1975).

This paper also contributes to the recent theory literature on innovation. The optimal allocation of control over observable innovative ideas is analyzed in Aghion and Tirole (1994). Anton and Yao (1994) show that inventors can secure some value from their idea by threatening to transmit it further, creating more competitors. Anton and Yao (2002, 2004) considers partial disclosure of ideas. See also Bhattacharya and Guriev (2004) and Cestone and White (2003). Baccara and Razin (2002) examine a complex bargaining game where a team of inventors consider whether to buy out all idea holders, or whether to allow some leakage of ideas. Baccara and Razin (2004) extend that model by examining how a firm can provide additional incentive to prevent idea leakages. Rajan and Zingales (2001) examine how a hierarchy may prevent idea-stealing by granting access to its technology only to dedicated employees. Gromb and Scharfstein (2001) argue that firms have an advantage at knowing an employee’s track record, which reduces the cost of innovation failures. Ueda (2004) and Chemmanur and Chen (2003) examines the trade-off of talking to uninformed investors who cannot steal an idea, versus venture capitalists who can. A large literature has examined R&D spillovers across firms (d’Aspremont and Jacquemin, 1988, Kamien, Müller and Zhang, 1992). Our paper provides an explanation for why spillovers may be asymmetric, and how they are transmitted - namely departing employees. Finally, a recent literature on the open source movement addresses the question of voluntary exchange of ideas. Lerner and Tirole (2002) argue that career concerns induce people to contribute to open source code, while Johnson (2002) emphasizes the private provision of
a public good. Biais and Perotti (2004) show that an early stage, unpatentable idea may be safely shared in a partnership with experts when their screening expertise are highly complementary. The current paper pursues this notion of complementarity one step further - or rather earlier - by allowing the complementary agent also to elaborate the idea.

Our model of coexistence is consistent with the empirical evidence on firms spawning innovative ventures. According to Bhidé (2000), over 70% of the founders of firms in the Inc. 500 list of fast growing young firms replicated or modified ideas encountered in their previous employment. Gompers, Lerner and Scharfstein (2004) show how a large number of new entrepreneurs in high tech firms started their firms after leaving large firms. Klepper and Sleeper (2002) provide similar evidence from the laser industry. Azoulay (2004) finds that pharmaceutical firms, while actively outsourcing in many other areas, maintain strong firm boundaries around knowledge intensive projects. Kremp and Mairesse (2004) find a positive relationship between firms' internal knowledge management systems and their innovative performance.

In section 2 we explain the basic setup. In section 3 we contrast markets (open systems) and firms (closed systems). In section 4 we present the model where firms and markets interact, and we study the mobility of ideas across firm boundaries. Section 5 discusses the implication of the extensive model, and is followed by some conclusions and thoughts for further research.

2 The Base Model

2.1 Ideas and Agents

In our model, incomplete ideas are generated by individuals at some cost. Ideas are private information, and are too incomplete to be patentable. To be elaborated, they need feedback by someone with a set of skills complementary to those of the inventor. Since these ideas are novel and incomplete, it is not obvious what the complementary skills are, or who might have them. So inventors need to seek agents with the right match of skills. By incurring a search cost, inventors can identify 'competent' agents, who a priori have a chance to be able to elaborate the idea.

Competent agents can be of two types, which cannot be distinguished ex ante. The first type are "complementors," who understand whether the idea has promise, and if so are able to work on elaboration. The second type are "substitutes," who do not know whether an idea has promise, or how it can be elaborated. However, once they hear the idea, they have the same skills as the inventor. Whether an agent is a complementor or substitute is idea-specific, and cannot be ascertained by his history. Our basic result will be that because substitutes have nothing to contribute, they have no incentive to report truthfully their ignorance, and they are likely to steal the idea.
2.2 Contracting in Firms and Markets

We contrast sharing ideas in open idea exchanges (markets) and within closed idea exchanges (such as firms). Market interaction offers inventors unrestricted matching with an infinite set of free market agents, not subject to any governance structure. Within a closed system such as a firm, ideas circulate only inside a boundary, subject to some rules.

Idea stealing is sometimes explained by assuming that ideas are not contractible. This is not fully convincing, since any idea which can be communicated can presumably be written down, and thus be put into a contract. However, contractual commitments not to use information which will be disclosed only ex post are difficult to specify. In practice, so called “non-disclosure agreements” or NDAs are rarely used. Anton and Yao (1994) report that they are difficult to enforce, and that many agents involved in the circulation of ideas, such as venture capitalists, routinely refuse to sign them. The work by Arrow (1962) and Anton and Yao (1994, 2002) suggests a reason. Individuals will not agree to sign a contract that forfeits their right to use some idea, before they have seen its content. Signing such a blind contract may unduly restrict the agent’s future opportunities and/or expose him to extortion risks. For instance, a contract may contain ideas and knowledge that the agent already possesses, thus preventing him from pursuing his regular activities. We thus assume that agents are unwilling to sign such blind contracts. Incomplete ideas must be shared before any contractual commitments are made. Once agents agree to cooperate on developing a specific idea, they can create a reliable partnership with properly specified NDA clauses. Ex ante, however, there is no contractual protection for ideas before they are shared.

Agents can also interact within firms (intended here as established multi-project firms, rather than single-project start-ups). The boundaries of the firm is given by the set of individuals who contract with the firm owner to become employees. The employment contract specifies that the owner has an ownership claim on all the ideas of all employees. Thus employees sign away the right to use any of the ideas circulating within the firm or to transfer them across its boundary.

The firm owner has two important tasks. First, she must monitor the generation and use of ideas. Yet claiming ownership requires that inventors report their ideas to the firm. Thus the second task is that the firm must commit to attribute ideas to their true inventors, and reward successful ideas. We argue that firms keep records of internal initiatives to produce verifiable evidence (a “paper trail”) on what ideas are generated within the firm, and who generated them. Thus “bureaucracy” helps the firm to make claims on internally generated ideas, and to define a compensation system for inventors.

We argue that the ability of a firm to commit to a reward for inventors stems from a desire to uphold its reputation. A sufficiently large sunk investment in reputation guarantees that the firm does not want to steal an employee’s idea, or withhold his reward. The fixed cost of setting up a firm, denoted by $K$, includes the sunk cost
of establishing a reputation (in addition to setting up a bureaucracy that tracks the
generation and use of ideas). Moreover, we assume that every period the firms incurs
monitoring costs $m(E)$, where $E$ is the number of employees in the firm, with $m' > 0$
and $m'' > 0$ for sufficiently large $E$. Thus monitoring costs are (eventually) convex,
reflecting an increasing complexity of managing a large organization. The number of
firms is endogenous. It is determined by a free entry condition, which identifies the
number of agents willing to incur the cost of building a reputation.

2.3 Stage game

Table 1 summarizes the notation used throughout the paper. Our model has an
infinite sequence of dates in which agents interact. We assume there is a unit mass
of agents whose discount factor is $\delta$.

The assumptions for our stage game are chosen to keep it sufficiently simple to
retain stationarity. We assume that agents can only carry one idea from one period
to the next. They can thus talk at most about one idea, and listen at most to one
idea. This structure allows us to treat talking and listening as independent activities.

Figure 1 illustrates the structure of the stage game. At the beginning of each
period, an agent may try to generate an idea at a private cost $\psi$. He finds an
(incomplete) idea with probability $\gamma$. The idea may be promising, with prior chance
$p$, or bad. A promising idea requires elaboration by a team of two agents with
complementary skills. The expected value of a promising idea (net of any elaboration
cost) is denoted by $z$; a bad one is worthless. Ex ante, it is not known what the
complementary piece is, so the idea must circulate among competent agents to find
one with the complementary skills. Since competence is idea-specific (rather than
an individual quality) the carrier of an idea incurs some search cost $c$ to identify
competent agents. We assume that it is always optimal to incur this cost to avoid
being matched with agents who certainly lack the necessary skills.\footnote{Since the talker discounts future payoffs, he is willing to incur some search costs to screen ideas faster. The screening also avoids that incompetent agents may circulate an idea, which they do not understand, to potential competitors.} Thus $c$ is a
measure of the (local) frequency of competent agents. After hearing the idea, the
listener receives a private signal on its quality and gives a report (feedback). The
idea is then either elaborated, or dropped, or it is taken to other listeners next period
by one or both agents. If two agents agree to cooperate on elaborating and idea, we
assume that they negotiate on sharing the expected returns, using Nash bargaining.\footnote{In the appendix we also show that our result do not depend on Nash bargaining, and continue to hold for outside option bargaining à la Rubinstein (1982).}

Competent listeners understand the idea, but only some of them are able to com-
plement them. With probability $\phi$ the listener has the right complementary skills
to work with the talker on elaborating the idea. We term such an agent a comple-
mentor. If the idea is promising, the complementor can signal his skills by providing
some positive constructive feedback. With probability $\bar{\phi}$ the listener understands the idea, but receives an uninformative signal about its quality. At this point, this agent has the same skills and the same information as the inventor. We term such agents substitutes. In order to elaborate an idea, both types of skills (those of any complementor, and of the generator or any of his substitutes) are required. We say that an idea is “resolved” if it is either abandoned, or if two parties agree to cooperate on elaboration.

Because signals are private information, both complementors and substitutes may lie. A substitute who reports truthfully admits that he has nothing to contribute to the idea. We will show that he has an incentive to steal the idea and search for a complementor himself (so in the next period he will become a talker). A complementor who reports truthfully a good signal reveals the idea to be promising. In other words, positive constructive feedback makes it self-evident that the idea is promising and that the listener is a complementor. Since both types of skills are needed to elaborate an idea, the complementor can either offer the talker to collaborate, or steal the idea and look for a substitute. If a complementor recognizes the idea as bad, he has no reason to pursue it further, and gives a honest negative feedback. We assume that a negative report by a complementor cannot be distinguished from that of a substitute.⁹

For simplicity, we assume that when two teams compete, both teams get zero returns because of Bertrand competition. Moreover, once a team decides to elaborate on an idea, it can release enough information to convince any other potential team not to duplicate its efforts.

Throughout the paper we denote the per period utilities of agents with lower cases $u$ and $v$, and lifetime utilities (i.e., the net present value of the current and all future per-period utilities) with upper cases $U$ and $V$. $u$ and $U$ pertain to agents in the market, while $v$ and $V$ pertain to agents inside firms.

3 Analysis of the model

3.1 Analysis of markets

The stage game

We now introduce an intuitive assumption to maintain a stationary process of idea circulation. We restrict attention to circumstances when in equilibrium, an

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⁸Throughout the paper, a bar over any variable signifies $1−$ that variable, so $\bar{\phi} = 1 − \phi$.

⁹The intuition why it may be difficult to distinguish honest and false negative feedback, is that negative feedback would typically consist not of a conclusive proof that the idea is infeasible, but of a subjective assessment that the odds of success are too low to warrant further investments in idea elaboration.
idea is circulated in each period by only one agent. Specifically, we impose a so-called discouragement condition, so that the talker stops pursuing the idea after a negative report. Negative feedback could either come from an honest complementor or a dishonest substitute. Negative feedback is always informative, and leads to a downward revision in the talker’s beliefs on the quality of the idea.

A talker receiving negative feedback may be discouraged and abandon the idea when the utility of generating a new idea is greater than the utility of further pursuing the current idea. This discouragement condition, which is formally derived in the appendix, is always satisfied for $\phi$ sufficiently low, and also for $\phi$ sufficiently large. Since discouragement happens after the first negative feedback, it prevents parallel dissemination and allows us to maintain a stationary stage game. The intuition for discouragement with a large $\phi$ is that it negative feedback is very credible. In contrast, a low $\phi$ discourages further searching because it is too difficult to find a match.

With the discouragement condition in place, we can examine the listener’s choice of providing honest or false feedback.

**Lemma 1:**
(i) A substitute always provides false negative feedback and steals the idea.
(ii) A complementor always provides honest feedback and cooperates with the talker.

The proof of Lemma 1 is instructive, so we derive it in the main text. Consider first a substitute listener. He understands the concept, but cannot elaborate it by himself. He can either provide false negative feedback, or else disclose his ignorance. If he provides false feedback, the discouragement condition guarantees that he can take the idea forward alone. The substitute then becomes a talker next period, and gets a utility $\delta u_T$ (derived below). If the substitute discloses his ignorance to the talker, then both are equally capable of taking it forward. It is then efficient to flip a fair coin, to determine who is to take the idea forward. The utility of disclosing ignorance is thus given by $\frac{1}{2}\delta u_T$. This is always less than the utility of stealing the idea.

Consider next the choice facing a complementor, who knows whether the idea is promising or bad. If the idea is bad, he has no interest in pursuing it, and he is willing to provide honest negative feedback to the talker. The interesting case is when the idea is promising. Suppose first that the complementor reveals its expertise by providing constructive feedback. This signals that the idea is promising, and that he and the talker could elaborate the idea together. Below we derive the equilibrium bargaining shares, denoted by $a$ for the talker and $\pi$ for the complementor. If instead the complementor provides false negative feedback, the talker is discouraged. We denote the complementor’s utility of taking the idea forward into the next period by
\( \zeta_C \). In the next period, the complementor talks to a substitute (who has the missing skills for the complementor) with probability \( \overline{\phi} \), or to another complementor (who has duplicate skills). Finding a substitute leads to bargaining over the division of the return which yields a utility \( \overline{az} \). Finding another complementor creates a dilemma, since both agents are equally capable, but they need the skills of a substitute to elaborate the idea. Once again it is efficient to flip an even coin, to determine who is to take the idea further. The utility of providing negative feedback and taking the idea further alone is thus given by

\[
\zeta_C = \delta(\phi \frac{\zeta_C}{2} + \overline{\phi}az - c) \Leftrightarrow \zeta_C = \frac{\delta(\overline{\phi}az - c)}{1 - \delta \frac{\phi}{2}}
\]

To see that the complementor always prefers to give honest feedback, it is easy to verify that \( \overline{az} > \zeta_C \). In fact, we can rewrite this inequality as \( \overline{az} > \delta(\frac{\phi}{2}az + \overline{\phi}az - c) \). This helps us to see why stealing has several disadvantages for a complementor: it causes delay (as captured by \( \delta \)), it requires additional costly search (as captured by \( c \)), it is risky, since he may talk to a rival complementor (as captured by \( \frac{\phi}{2}az \)), and at best it only leads to meeting another substitute (as captured by \( \overline{\phi}az \)). This completes the proof of Lemma 1.

Lemma 1 explains the fundamental problem of sharing an idea in the market. Interestingly, substitutes are the talker’s biggest problem: they have nothing to add, and so are eager to steal the idea. Complementors actually need a partner with the talker’s skills, so cooperation is efficient.

We are now in a position to examine the bargaining game between a talker and a complementor who signals his type by providing constructive feedback. To derive the outside options, we consider the continuation game where the talker and complementor fail to agree. Depending on parameters, this continuation game may have three types of equilibria. In one equilibrium, the complementor takes the idea further. Knowing this, the talker prefers to stop. This is similar to the discouragement, except that here the talker actually knows that the idea is promising. What is stopping him here is the threat of competition from the complementor. In a second equilibrium, it is the talker that takes the idea further, and the complementor prefers to stop. Finally, in a third equilibrium, both talker and complementor take the idea further, even though there may be competition. Unlike the first two equilibria, the third equilibrium is no longer stationary, and is analytically not easily tractable.\(^{10}\) Since our goal is to describe circulation rather than dissemination of ideas, we identify parameter ranges which produce a stationary equilibrium. We obtain sufficient conditions by placing an upper bound on the utilities associated to the non-stationary continuation game.

\(^{10}\)For an analysis of the effect of competition, see the work by Anton and Yao (1994), Baccara and Razin (2002, 2004) and Biais and Perotti (2004).
Lemma 2:
(i) If $\phi$ is sufficiently small, the only equilibrium of the continuation game has the complementor continuing, and the talker stopping.
(ii) If $\phi$ is sufficiently large, the only equilibrium of the continuation game has the talker continuing, and the complementor stopping.

The proof is in the appendix. The key intuition is that the agent with the scarcer skill is in the stronger position, since he is more likely to win the race of finding a suitable partner. The weaker partner prefers to stop, rather than incurring search costs.

We are now in a position to solve the continuation game, which determines the outside options for the bargaining game between the talker and the complementor. For $\phi$ sufficiently small, the talker has an outside option of 0, and the complementor has an outside option of $\zeta_C$. Using Nash bargaining, the talker gets $az = \frac{z - \zeta_C}{2}$ and complementor gets $\bar{a}z = \frac{z + \zeta_C}{2}$. For $\phi$ sufficiently large, the talker’s outside option is denoted by $\zeta_T$. The talker gets $az = \frac{z + \zeta_T}{2}$ and complementor gets $\bar{a}z = \frac{z - \zeta_T}{2}$.

Lemma 3:
(i) For $\phi$ sufficiently small, the Nash bargaining share of talker is given by
\[
a = \frac{1}{2} - \frac{\zeta_C}{2z} \quad \text{where} \quad \zeta_C = \frac{\delta(\phi z - 2c)}{2 - \delta}.
\]

Note that $a$ is larger when the chance of meeting another complementor is higher (higher $\phi$), when search costs are higher (higher $c$), and when the delay costs are larger (lower $\delta$).

(ii) For $\phi$ sufficiently large, the Nash bargaining share of talker is given by
\[
a = \frac{1}{2} + \frac{\zeta_T}{2z} \quad \text{where} \quad \zeta_T = \frac{\delta(\phi z - 2c)}{2 - \delta}.
\]

Here, $a$ is larger when the chance of encountering another complementor is higher (higher $\phi$), when search costs are lower (lower $c$), and when the delay costs are smaller (higher $\delta$).

$^{11}$To exclude trivial cases, we assume that search costs $c$ are not too high to prevent the party in question from wanting to continue. Formally, this requires $\zeta_C > 0 \Leftrightarrow c < \frac{\phi z}{2}$ (for the case where $\phi$ sufficiently low) and $\zeta_T > 0 \Leftrightarrow c < \frac{\phi z}{2}$ (for the case where $\phi$ sufficiently high).
The proof is in the appendix.

Lemma 3 shows that bargaining shares depend on the relative scarcity of skills. If complementors are sufficiently rare, the market rewards them more than the generators of ideas (i.e. \( a < \frac{1}{2} \)). Moreover, the sharing of rents does not depend on the cost of generating ideas. Thus the reward for the inventor does not depend on its ex ante investment, but only on its ex post bargaining strength.

Equilibrium behavior

We now compute the return to generating and stealing ideas. Conditional on having an idea, the utility of the talker is equal to her share of the profits conditional on meeting a complementor and the idea being promising.

\[
u_T = \phi p a z - c
\]

The expected utility of an idea inventor is given by

\[
u_G = \gamma u_T - \psi
\]

Part of the expected benefit of being a listener comes from the chance of becoming a complementor, which equals

\[
u_L = \phi p \overline{a} z
\]

Below we derive the total benefit of being a listener, which also includes the benefit of stealing the idea.

To derive the steady state properties of the market, denote by \( s \) the fraction of agents who just stole an idea. The number of people who have an idea are all those who stole an idea (\( s \)), and those who generated a new one (\( \overline{s} \)). Thus the number of people talking is given by \( t = s + \overline{s} \gamma \). Naturally, this is also the number of people listening.

In any period, \( tp \) ideas get elaborated, and \( tp \) ideas get dropped because complementors realize they are not feasible. Finally, in any period, \( t \overline{\phi} \) ideas get stolen. Thus \( s = t \overline{\phi} \). Using \( t = s + \overline{s} \gamma \) we get the following result.

**Lemma 4:** The steady state number of agents talking (\( t \)) and stealing (\( s \)) are given by

\[
t = \frac{\gamma}{\phi + \gamma - \gamma \overline{\phi}} \quad \text{and} \quad s = \frac{\gamma - \gamma \phi}{\phi + \gamma - \gamma \phi}.
\]

Both \( t \) and \( s \) are increasing in the probability of generating an idea (\( \gamma \)), and decreasing in the probability of finding a complementor (\( \overline{\phi} \)).
Consider now the ex-ante utility of an agent in such a system, assuming that everyone invests in generating ideas. With probability $s$ the agent steals an idea, and gets $u_T$. With probability $\overline{s}\gamma$, the agent has a new idea and also gets $u_T$. With probability $\overline{s}\overline{\gamma}$, the agent has no idea and gets nothing. In addition, with probability $t$, an agent becomes a listener and gets $u_L$. We therefore get\(^\text{12}\)

$$U = \frac{\overline{s}u_G + s u_T + tu_L}{1 - \delta}$$

$U$ is increasing in $p, z, \delta, \phi$ and $\gamma$, which is very intuitive.

The market is an efficient environment to resolve ideas, as they circulate until they find a complementary match; they are then either elaborated if promising, or dropped if bad. However, in such a system an inventor may receive only a small fraction of the payoff from his own idea. Thus in the market, listening is very attractive: it may lead to becoming a complementor, or to stealing an idea. However, agents may not have an incentive to generate ideas. The critical cost threshold such that inventors break even is given by

$$\psi_M = \gamma u_T = \gamma(\phi p a z - c)$$

**Proposition 1** Consider an open exchange system (i.e., the market).

- For $\psi < \psi_M$, the market achieves a first-best outcome, where all ideas are resolved (i.e. they circulate until they are either elaborated if promising, or dropped if bad).
- For $\psi > \psi_M$, the expected reward of generating an idea is too low because of idea stealing. No ideas are generated and no ideas circulate.
- The market is more likely to fail to support idea generation:
  1. if finding an idea is rare (low $\gamma$),
  2. if the idea is likely to be bad (low $p$),
  3. if ideas have little value (low $z$),
  4. if finding a right match is rare (low $\phi$),
  5. if it is costly to search for a match (high $c$)

\(^{12}\)To see this, denote the lifetime utility with a stolen idea by $U_T$ and without a stolen idea by $U_G$, then $U_T = u_T + tu_L + \overline{s}\delta u_G + s\delta u_T$ and $U_G = u_G + tu_L + \overline{s}\delta u_G + s\delta u_T$. Using $U = \overline{s}u_G + s u_T$, we rewrite this as $U_T = u_T + tu_L + \delta U$ and $U_G = u_G + tu_L + \delta U$, so that $U = \pi(u_G + tu_L + \delta U) + s(u_T + tu_L + \delta U) \Leftrightarrow U(1 - \delta) = \overline{s}u_G + su_T + tu_L$. 

14
The proof is in the appendix.

In markets the generation of ideas is harder to achieve when ideas are less valuable, when inventors have less bargaining power (in particular, when their skills are more common), and when there are high search costs (perhaps because there are few competent agents).

3.2 Analysis of firms

We now introduce the firm as a possible solution to the weakness of the market, when $\psi > \psi_M$. As defined, a firm is a closed exchange system in which employees sign away their rights to all internally registered ideas. We assume that the firm boundaries are perfectly monitored, so that once disclosed, ideas never escape.

Employees can disclose or “register” their ideas to the firm, who can reward such activity. Let $b$ be the percentage share of profits offered as reward to an inventor. To ensure registration of ideas, the firm only needs to cover the inventor’s generation cost. This is enough to induce the employee to disclose his idea, since the reward in the market is too low to support generation. Once an inventor registers an idea, he receives the task of finding an internal match. In managerial terms, he becomes an “internal project champion” or “intrapreneur.” Since the idea is now registered as an internal project, the firm can ensure that no employee can profit from leaking it outside the firm. This allows the champion to obtain reliable feedback from all internal listeners. Note also that the firm does not need to compensate complementors, since they cannot take ideas elsewhere. The utility of listening to ideas inside the firm is therefore zero, i.e., giving feedback to others’ ideas is part of the job.

We first assume that the inventor must abandon the idea if no internal match is found; later we relax this assumption. For comparability with the market, we assume that the search cost inside firms is the same as in market, even though we believe that in reality it may be lower (such a cost differential would by itself constitute a reason for firms).

We denote the per period utility by $v$ and the lifetime utility by $V$. The expected utility of being a talker in any period is given by

$$v_T = \phi pbz - c.$$ 

To calculate the lifetime utility, we need to take into account how long an idea has already circulated. Suppose that for a given idea, the firm has $\bar{e}$ competent employees. We simplify the closure of the internal search by assuming that it is known when the whole set of competent employees has been consulted, i.e., we assume that $\bar{e}$ is a

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13Making compensation contingent on finding a complementor ensures that employees cannot benefit from registering spurious ideas.
known constant. Naturally, the number of competent employees is non-decreasing in $E$, i.e., $\frac{d\sigma}{dE} \geq 0$.

Let $V_G$ denote the agent’s utility when he does not have an idea, and thus needs to generate a new one. Let $e$ be the $e^{th}$ round of talking about an idea, so $V_T^e$ is the lifetime utility of an employee who is about to talk to the $e^{th}$ listener. For any $e$ with $1 \leq e < \bar{e}$, we have

$$V_T^e = v_T + \phi \delta V_G + \bar{\phi} \delta V_T^{e+1}$$

This says that in the $e^{th}$ round an agent has an expect per-period return of $v_T$. With probability $\phi$ the idea is resolved (the idea is either elaborated or dropped). The agent then gets $V_G$, i.e., he has to start afresh and generate a new idea. With probability $\bar{\phi}$ the idea is still unresolved. The agent continues to circulate his idea, and gets $V_T^{e+1}$. At $e = \bar{e}$, the agent has no one left to talk to, so

$$V_T^{\bar{e}} = V_G$$

where $V_G = -\psi + \gamma V_1^T + \overline{\phi} \delta V_G$

Clearly, these utilities form a recursive system. In the appendix we explain how to solve this system. We show that the life-time utility of an agent joining a firm is given by $V_G = \rho_T v_T - \rho_G \psi$, where $\rho_T$ is the (appropriately discounted) number of times that an employee has an idea to talk about, and $\rho_G$ is the (appropriately discounted) number of times that an employee generates an idea. $V_G$ thus equals the discounted value of talking about ideas, minus the cost of generating them. Is is easy to show that $V_G$ is increasing in $\bar{e}$, the intuition being that a larger firm provides more matching opportunities.

Consider now a firm’s optimization problem. The owner maximizes the value of the firm by choosing the optimal number of employees $E$. The firm’s net present value of profits (excluding the fixed cost $K$) is given by

$$\Pi = E(1 - b)\rho_T \phi pz - \frac{m(E)}{1 - \delta}$$

The firm’s objective is to maximize $\Pi$, subject to the agent’s incentive constraint. For this, she chooses the optimal size $E$, as well as a compensation package $b$.

The incentive constraint requires that employees are willing to generate ideas, and is given by $V_G \geq 0$. $V_G$ is increasing in $b$ (through $v_T$). Since the firm wants to minimize $b$, we get

$$V_G = 0 \iff b^* = \frac{c + \psi \rho_G}{\rho_T \phi pz}$$

Substituting $b^*$ into $\Pi$ yields

\[\text{14} \] This simplifies the exposition. For the results, it makes little difference whether the last competent employee is identified in advance or not.
\[ \Pi = E \rho_T (\phi pz - c) - E \rho_C \psi - \frac{m(E)}{1 - \delta} \]

where \( E \rho_T (\phi pz - c) \) is the number of employees times the fraction with an idea, times the expected value of an idea (net of search costs), and \( E \rho_C \psi \) is the total cost of generating ideas. Note that \( \Pi \) is the same as the joint utility of the firm and all its employees.

**Proposition 2** The firm’s optimal choice of \( E \) is

- decreasing in the marginal cost of monitoring \((m')\).
- decreasing in the cost of generating ideas \((\psi)\)
- decreasing in the internal search cost \((c)\)
- and increasing in the profitability of new ideas \((p \text{ and } z)\).

The optimal compensation \( b^* \) is

- increasing in the cost of generating ideas \((\psi)\)
- increasing in the internal search cost \((c)\)
- decreasing in the value of new ideas \((p \text{ and } z)\).

The proof is in the appendix.

The optimal size of the firm decreases in the cost of monitoring employees and in the cost of generating ideas (high \( \psi \)), as the marginal benefit of having another employee is lower. In contrast, the more profitable are agents’ ideas (higher \( p \) or \( z \)), the larger are firms. Finally, higher (internal) search costs \((c)\) reduce firm size.

The comparative statics of \( b^* \) are also intuitive. If the cost of generating an idea is greater (higher \( \psi \), lower \( \gamma \)), inventors need to be compensated more. Interestingly, in addition to this direct effect, there is also an indirect effect. Higher generation costs reduce the optimal firm size \( E^* \), which in turn reduces the return to generating an idea. The change in the optimal reward \( b^* \) must take this into account.

To close the model, we derive the number of firms endogenously. Suppose there is free entry, and agents have different (but known) costs, drawn from a general distribution of sunk costs \( \Omega(K) \). The number of firms is given by an indifference condition for the marginal firm owner, who is indifferent between being an employee versus setting up a firm. The number of agents \( n_F \) who become firm owners is then given by

\[ n_F = \Omega(\hat{K}) \text{ where } \Pi - \hat{K} = 0 \]
The reputational component of the cost $K$ has to be large enough to dissuade owners from reneging on their commitment to inventors. An owner could try to steal all reported ideas and elaborate them outside the firm. This is actually a dominated strategy, since the owner would still need to find complementors in the market. The best reneging strategy is to steal all returns from successful innovations by refusing to pay the generator his share $b$. A sufficient condition for firms to maintain their reputation is thus given by

$$\Pi > E b z.$$  

This says that the value of firms exceeds the maximum gain from reneging in the extreme case where all $E$ employees elaborate promising ideas at the same time.$^{15}$

### 3.3 Comparative advantages of firms and markets

Under what circumstances do firms emerge? As long as markets can support idea generation ($\psi \leq \psi_M$), there is no role for firms, since markets are more efficient at elaborating ideas. Firms emerge only when markets fail. Naturally, whether or not firms emerge depends on their own viability. Indeed, $\Pi$ is also decreasing in $\psi$. We define $\psi_F$ so that $\Pi(E^*, \psi_F) = K$, where $E^*$ is the optimal choice of $E$. $\psi_F$ is the critical value above which firms are not viable.

In general, we cannot say whether $\psi_F$ is greater or smaller than $\psi_M$, since this depends on the costs of monitoring ($m$), as well as the fixed costs ($K$). We will focus on the non-trivial case where the costs of operating firms are not preventive.

Figure 2 depicts the feasible regions of firm and market governance. The vertical axis measures generation costs ($\psi$), while the horizontal axis measures the value of ideas ($z$). Higher values of $z$ make idea generation more profitable and allow therefore enable a higher value of $\psi_M$. Similar for $\psi_F$. The following proposition distills the main insights from Figure 2.

**Proposition 3**

- **For a given generation cost ($\psi$), markets work well for valuable ideas (high $z$), while firms are necessary for less valuable idea.**

- **For a given value of ideas ($z$), markets work well for low generation costs (low $\psi$), while firms are necessary for higher generation costs (higher $\psi$).**

This result provides some interesting insights into the comparative advantages of firms versus markets. Markets work well for very promising but not too expensive ideas (high $z$, low $\psi$). This correspond to a common perception that radical innovations are often developed outside firms, while incremental innovations fall into the

$^{15}$This condition can be relaxed. For example, the firm could commit to a capacity constraint of never implementing more than $\bar{E} < E$ projects per period, by delaying the implementation of some project.
realm of established corporations. In addition, entrepreneurs may be able to develop only ideas that are simple to generate, whereas companies are better positioned to develop ideas that require substantial investment. Intuitively, individual entrepreneurs pursue opportunities that require modest budgets, whereas firms focus on larger scale, complex ideas which require substantial investments. Yet in our model this result is not, as it is often assumed, a consequence of financial constraints faced by individuals.

Notice also that we do not imply that markets are better than firms. A lower return to ideas inside firms is not a sign of inefficiency, but rather a sign of market failure. Firms emerge precisely when the return to ideas is not high enough for markets to function.

4 Coexistence of firms and markets

4.1 Motivation

The previous section compares firms and markets in isolation. We now discuss how firms and markets may interact, in particular so that some ideas may flow across firm boundaries.

In our basic model, employees commit not to take any internal idea outside the firm. Such a simple rule has the benefit of simplicity, and may keep monitoring costs down. Yet the rule is also inefficient, as it implies that some idea will not be allowed to be resolved. Because the firm has a finite size, a champion may not find an internal match for some promising idea, and thus be forced to give it up.¹⁶

Consider now the case of more flexible rules, where a firm may permit an internal champion to pursue his idea outside the firm, once he has exhausted all potential matches with his colleagues. By allowing the agent to pursue the idea in the market, the firm can improve incentives to generate ideas, and thus capture some of their value by reducing his compensation, or by retaining some stake in the spin off. Since letting employees bring internal ideas to other firms creates potential competition, we assume that firms allow their employees only in a start-up, but not with another firm.

The coexistence of firms and markets creates an interesting symbiotic relationship between firms and market. We maintain our assumption that $\psi > \psi_M$, so that the market is unable by itself to sustain idea generation. Yet employee mobility allow the market to feed off ideas generated inside firms. So, while firms emerge to compensate for market failure, markets can thrive by compensating for the limited efficiency of firms. Thus, in this model firms and markets truly complement one another, each compensating for the inefficiency of the other.

¹⁶Formally, the probability that an internal idea cannot get resolved is given by $1 - \sum_{i=0}^{\psi-2} \phi(\psi)^i > 0$. 
4.2 The coexistence equilibrium

In the model there are three types of agents, namely firm owners, employees and agents in the free market. Their respective numbers are denoted by $n_F$, $n_E$ and $n_M$ (where unit mass implies $n_M = 1 - n_F - n_E$). We focus on stationary equilibria where firm size is constant, so that the flow of agents leaving firms must equal the flow of agent joining firms to maintain the optimal firm size. The new employees are recruited from the market. We characterize the equilibrium flow of agents from firms to markets and vice versa.

Let $\lambda$ be the fraction of employees that on average leave a firm every period, a value endogenous to the model. Let $\mu$ be the expected number of employees without an idea at the beginning of the period. Straightforward calculations show that $\mu = \frac{E}{\gamma + \gamma \sum_{i=0}^{\sigma - 2} i \phi}$. The number of employees that leave because they cannot find a match inside the firm is thus $\mu \gamma \phi^{-\sigma - 1}$. The fraction $\lambda$ is therefore given by $\lambda = \frac{\mu \gamma \phi^{-\sigma - 1}}{E}$, i.e.,

$$\lambda = \frac{\gamma \phi^{-\sigma - 1}}{\gamma + \gamma \sum_{i=0}^{\sigma - 2} i \phi}.$$  

(1)

Since $\lambda n_E$ is the fraction of employees leaving firms to explore their ideas in the market, firms must rehire in each period the same number.

Of the $\lambda n_E$ new ideas moving every period to the market, a fraction $\phi$ gets resolved (either elaborated or stopped), while a fraction $\phi$ continues to circulate. The number of ideas that circulated for exactly one period is thus $\lambda n_E \phi$. Similarly, the number of ideas that have circulated for $i$ periods is $\lambda n_E \phi^i$. Of course, the total number of ideas circulating in the market must is the same as the number of people talking. That number is given by

$$t = \lambda n_E \sum_{i=0}^{\infty} \phi^i = \frac{\lambda n_E}{\phi}.$$  

(2)

Consider now the probability that an agent in the market is approached by another agent with an idea. Since there are $t$ ideas, and $n_M$ people to talk to, the probability of becoming a listener is $\frac{t}{n_M}$. We focus on the case where there are more potential

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17 For simplicity we abstract from the possibility that a new employee could revisit an internal idea that the departing employee was previously unable to resolve.

18 With probability $\gamma \phi$ an agent generates a new idea which is not resolved within the period. In steady state $\mu \gamma \phi$ employees start with an idea that is exactly one period old. Similarly, $\mu \gamma \phi^e$ (for $e < \sigma - 1$) employees have an idea that is exactly $e$ periods old. Adding up the total number of employees, we obtain $\mu(1 + \gamma \phi + \gamma \phi^2 + \ldots + \gamma \phi^{\sigma - 2}) = E \iff \mu = \frac{E}{\gamma + \gamma \sum_{i=0}^{\sigma - 2} \phi}$. 

20
listeners than talkers, i.e., where \( t < n_M \). If an agent gets to listen to an idea, he either complements it or steals it.

The utility of a free market agent, who has no idea of his own, is thus given by

\[
U = \frac{t}{n_M} \phi(p\alpha z + \delta U) + \frac{t}{n_M} \phi(p\alpha z - c + \delta U) + (1 - \frac{t}{n_M}) \delta U
\]

After transformations this yields

\[
U = \frac{t}{n_M} \frac{\phi(p\alpha z + \delta(p\alpha z - c))}{1 - \delta}
\] (3)

Next, consider the utilities of departing employees. We assume employees are allowed to retain any profits from ideas they take out of firms. Thus \( V_T^e \) now includes these expected payoff. Since the idea may now be stolen by a substitute, a departing employee receives the expected returns as any inventor in the market, given by \( u_T = \phi p\alpha z - c \).

The utility of the departing employee is first to get an opportunity to talk about his idea (\( u_T \)), and then to reside in the market. Let \( U \) be the lifetime utility of an agent that currently resides in the market and does not have any idea to pursue (which we derive it below). Formally, we have

\[
V_T^e = u_T + U \text{ where } u_T = \phi p\alpha z - c.
\] (4)

The expressions for \( V_T^e \) and \( V_G \) are the same as before, i.e.,

\[
V_T^e = v_T + \phi \delta V_G + \phi \delta V_{T}^{e+1} \text{ and } V_G = -\psi + \gamma V_T^{1} + \pi \delta V_G \text{ where } v_T = \phi pbz - c.
\] (5)

In order to hire employees, firms have to match their utility of being in the market. The utility of a fresh hire is given by \( V_G \), which is increasing in the compensation parameter \( b \). Firms must set their compensation \( b^* \) to match the utility of being in the market, so that

\[
V_G(b^*) = U.
\] (6)

Thus employee compensation inside firms now depends on the utility of agents in the market. We will exploit this insight below.

Next we determine the equilibrium size and number of firms. The optimal size of the firm is found as before, i.e., \( E \) maximizes

\[
\text{For } t > n_M, \text{ the model is the same, except that } U = \frac{\phi(p\alpha z + \delta(p\alpha z - c))}{1 - \delta} \text{ in equation (3) below,}
\]

and \( u_T = \frac{n_M}{t} \phi p\alpha z + \frac{n_M}{t} \delta u_T - c \Leftrightarrow u_T = \frac{n_M}{t} \frac{\phi p\alpha z - c}{1 - \frac{n_M}{t} \delta} \) in equation (4) below.
\[
\Pi = E(1 - b)\rho_{T}\phi_{p}z - \frac{m(E)}{1 - \delta}. \quad (7)
\]

The number of firms is given by the same free entry condition as before, except that the marginal firm owner is now indifferent between spending the fixed costs of setting up a firm versus operating as an agent in the market. Formally, we have

\[n_{F} = \Omega(\hat{K}) \text{ where } \Pi - \hat{K} = U. \quad (8)\]

Finally, with the assumption of a unit mass of agents, the number of employees and free market agent is given by

\[n_{E} = En_{F} \text{ and } n_{M} = 1 - n_{E} - n_{F} = 1 - (1 + E)n_{F}. \quad (9)\]

### 4.3 The consequences of coexistence

Equations (1) through (9) describe the coexistence equilibrium. We now establish its main properties.

**Proposition 4**

- In the coexistence equilibrium, all ideas are generated within firms.
- Employee mobility eliminates the inefficiency of limited circulation of ideas within firms.
- In equilibrium, all ideas are resolved. All promising ideas get elaborated, either within firms or within the market.

This proposition shows that the coexistence of firms and markets is efficient. Markets complement firms by resolving ideas taken out of firms by departing employees.

This result offers a provocative view on the role of markets, playing a complementary, almost subordinate role to firms. Agents in the market receive, and often steal, ideas generated (but not resolved) inside firms. Their role is symbiotic, in the sense that free market agents elaborate those ideas that the firms could not resolve.

**Proposition 5**

- The flow of ideas to the market \((\lambda n_{E})\) is increasing in the number of firms \((n_{E})\), and the rate of idea generation in firms \((\gamma)\),

- The total number of ideas circulating in the market \((t)\) and the utility of residing in the market \((U)\) are also increasing in \(n_{E}\) and \(\gamma\).
This proposition further highlights the symbiotic relationship between firms and markets. The greater is firm density, and the more ideas are generated in firms, the more active the market becomes, with more ideas circulating \((t)\), generating higher benefits for free market agents \((U)\).

Consider now the benefits of employee mobility for firms. From the perspective of an individual firm, it is attractive to let employees go after they exhausted all internal matching opportunities. (We are ignoring any additional costs due to employee turnover). Employee mobility adds a positive term to \(V_G\) in equation (4), increasing the return to generating ideas. Since the firm sets the compensation \(b\) such that \(V_G = U\), the additional term in \(V_G\) allows the firm to lower its reward \(b\). Thus giving employees the benefit of taking their ideas with them reduces the cost of hiring them ex-ante, and is efficient from the perspective of the individual firm.

However, there is also a general equilibrium effect of employee mobility, since the flow of ideas to the market creates rents for free market agents. This raises the utility in the market, which in raises the costs of attracting new employees.

To see this formally, note that equations (2), (3) and (4) can be combined to yield

\[
V_G = \rho_T v_T - \rho_G \psi + \rho_\lambda w_T
\]

where \(\rho_T\) and \(\rho_G\) as before, and where \(\rho_\lambda\) measures the (appropriately discounted) number of times that an employee takes an idea outside the firm.\(^{20}\) Adding \(V_G - U(= 0)\) to \(\Pi = E(1 - b)\rho_T \phi p z - \frac{m(E)}{1 - \delta}\), and using (3') we obtain after standard transformations

\[
\Pi = E \rho_T (\phi p z - c) - E \rho_G \psi - \frac{m(E)}{1 - \delta} + E \rho_\lambda w_T - EU.
\]

As before, \(\Pi\) represents the joint utility of the firm and all its employees. It is given by the expected value of internally elaborating ideas (first term), minus the cost of generating ideas (second term), minus the costs of monitoring (third term), plus the opportunity to take an unresolved idea into the market (fourth term), minus the opportunity cost of having employees work in the firm rather than the market (fifth term). The main difference to the model of section 3 is the addition of the last two terms. Whether the fourth term is larger than the fifth depends on many parameters. We summarize this observation as follows.\(^{21}\)

**Proposition 6** If, in general equilibrium, \(U < \rho_\lambda w_T\), then employee mobility is beneficial to firms, and increases firms’ optimal size \(E^*\). Otherwise, employee mobility reduces profitability and the optimal size for all firms.

\(^{20}\)Standard calculations reveal that \(\rho_\lambda = \frac{\gamma (\bar{\psi} \delta) \bar{\psi}^{-1}}{1 - [\gamma \delta + \gamma (\bar{\psi} \delta) \bar{\psi}^{-1} + \gamma \phi \delta \sum_{i=0}^{\bar{\psi}} (\bar{\psi} \delta)^i]}\).

\(^{21}\)The result about optimal size of the firm follows from the fact that firm owners choose firm size according to a simple cost benefit trade-off.
Under what circumstances is $U$ larger than $\rho_u u T$? Note that an increase in the number of firms $n_F$, say because of lower fixed costs $K$, increases $n_E$ (holding $E$ constant).\footnote{This follows from Equation (9), provided $E$ does not change too much, which is a weak condition.} This raises the number of ideas leaving firms and thus the number of people $t$ talking about ideas in the market. From equation (6), this increases $U$ as well. Firm entry thus has a positive externality for market agents. On the other hand, a greater firm density does not affect $\rho_u u T$, i.e., it does not affect the utility that an employee gets from taking an internally generated idea to the market. The condition $U > \rho_u u T$ therefore holds for larger values of $n_F$. In other words, the negative wage effect dominates when there are sufficiently many “porous” firms (firm that allow their employees to explore unresolved ideas in the market).

This result offers a new perspective on employee mobility. The literature often takes a partial equilibrium perspective (Pakes and Nitzan, 1983). Our result suggests that there might be negative market externality across firms. A higher density of porous firms creates rents for agents in the market, making it more attractive to remain there, and thereby raising the cost of hiring new employees. This is consistent with the famously high labor costs in Silicon Valley, as well as its remarkable employee turnover rates.

4.4 Leaving without disclosing the idea

Employees who develop a new idea always have the option to leave the firm before disclosing the idea. In the model so far idea generators always prefer the protected firm environment over the market. In reality, much anecdotal evidence suggests that some employees prefer to leave and pursue their own ideas in the market. We now examine this alternative form of employee mobility, which happens without the employer’s consent.

There are two good reasons why some employees may prefer to leave their employer without disclosing their idea. First, if the idea is very good, they may not trust the firm to keep its commitment to reward idea generation. And second, an idea may not fit well with the competencies of the firm. We briefly sketch how to capture these effects in our model.

Consider first the issue of an employee with an extraordinary idea. Suppose that extraordinary ideas occur with probability $\Gamma$, and that they generate a large expected value, denoted by $Z$. Suppose that the firm can distinguish ordinary and extraordinary ideas, so it could offer a special reward of $BZ$, instead of the usual $bz$ (if firms cannot make this distinction makes it even more likely that employees leave without disclosing). The main issue is that $BZ$ may be so large that the firm wants to renge on its promise to reward idea generation. That is, the firm cannot commit not to take all rents generated by an idea of extraordinary value.

Formally, the firm will always renge if $BZ > \Pi$. Let $B_0 Z$ be the minimum payment that the firm needs to offer to induce disclosure of an extraordinary idea,
which can be derived as follows. The generator’s utility from disclosing an extra-
ordinary idea is given \( V^1_T(BZ) \).23 This is increasing in \( B \), so that \( B_0 \) is given by
\( V^1_T(B_0Z) = u_T(Z) + U \). For sufficiently large values of \( Z \), we have \( B_0Z > \Pi \). This
implies that there exists no payment schedule \( BZ \) that is high enough to induce dis-
closure, but low enough to prevent the firm from stealing the rewards. Under these
circumstances, employees never disclose extraordinary ideas. The structure of the
coexistence equilibrium is the same as before, except that there is one more type of
employee departure, namely employees who leave without disclosing their ideas. Of
course this further increases the return to free agents.

A closely related case when ideas will leave firms without being reported, is when
they do not fit the competence of the firm. Ideas which fit well with the firm’s
competencies have a high probability of finding an internal match (\( \phi_G \)). Other ideas
fall outside of the firm’s focus, so that the probability of finding a match is lower
(\( \phi_B < \phi_G \)). For simplicity, assume that the probability of finding a good match in
the market is always \( \phi_G \) (all that matters is that it sufficiently higher than \( \phi_B \)). We
assume that the fit of the idea is observable to the inventor but not to the firm, so the
reward \( bz \) cannot be made contingent on idea type. An inventor with a good-fitting
idea will seek an internal match, while an employee with a poorly-fitting idea would
have to incur additional search costs and unnecessary delay. Anticipating this, the
employee leaves without disclosing the idea.

To see this more formally, note that the employee’s utility from disclosing a poorly
fitting idea is given by \( V^1_T(\phi_B) \).24 The utility from leaving without disclosing the idea
is given by \( u_T(\phi_G) + U \), where \( u_T \) is evaluated at \( \phi_G \), the chance of a good match in
the market. The employee prefers not to disclose whenever \( V^1_T(\phi_B) < u_T(\phi_G) + U \).
Since \( V_T(\phi_B) \) is an increasing function of \( \phi_B \), this condition is always satisfied for
sufficiently low values of \( \phi_B \). The firm could of course try to retain all employees by
raising its bonus \( b \). However, this would be expensive, since the firm would have to
pay the higher bonus both for good or and poorly fitting ideas. Let \( \gamma_G (\gamma_B) \) be the
probability of finding an idea that fits well (poorly). It is easy to show that as long
as \( \gamma_B \) is sufficiently small relative to \( \gamma_G \), the firm prefers to loose a few employees,
rather than raising its bonus for all ideas. In equilibrium, we have again two types
of employee departures. Some employees leave after having tried to find an internal
match, but others leave without ever disclosing their poorly-fitting ideas.25

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23 Formally, this is derived from \( V^e_T(BZ) = v_T(BZ) + \phi V_G + \bar{\phi} \delta V^{e+1}_T(BZ) \) for \( 1 \leq e < \tau \)
and \( V^\tau_T = u_T(Z) + U \), where \( V_G \) is now given by \( V_G = -\psi + \Gamma \Max[V^1_T(BZ), u_T(Z) + U] + \gamma V^1_T(bz) + 
(1 - \gamma - \Gamma)\delta V_G \).

24 Formally, this is derived from \( V^e_T(\phi_B) = v_T(\phi_B) + \phi_B \delta V_G + \bar{\phi}_B \delta V^{e+1}_T(\phi_B) \) for \( 1 \leq e < \tau \)
and \( V^\tau_T = u_T(\phi_G) + U \), where \( V_G = -\psi + \gamma_B \Max[V^1_T(\phi_B), u_T(\phi_G) + U] + \gamma_G V^1_T(\phi_G) + (1 - \gamma_G - \gamma_B)\delta V_G \).

25 Informally we may add that the problem of employees not disclosing ideas is likely to be higher
for larger firms, and also for firms that are less flexible letting employees depart after they have not
found an internal match.
5 Further Discussion and Open Issues

5.1 Incentives in Firms and Markets

At first the result that markets produce poor incentives relative to firms for idea generation seems counter-intuitive. Typically it is believed that the reward for developing a new venture on one’s own are much greater than for an employee. Reconciling these observations with our model yields some interesting additional insights.

The market offers high potential returns for successful ideas, but the model identifies two important caveats. First, there is a high risk of idea expropriation. Indeed, in the model the inventor gets a single shot at presenting the idea: he either finds a match or gets discouraged. Second, for low $\phi$, the rewards in the market may go disproportionately to the complementor. This is consistent with observing highly successful entrepreneurs in the market (who are not necessarily the idea generator), while at the same time many entrepreneurs fail to achieve an adequate return (Moskowitz and Vissing-Jorgensen, 2002). Contrast this with the rewards inside firms. The compensation of so-called “intrapreneurs” may look less generous, but may be more reliable, because of better protection against idea stealing. Overall, the rewards for intrapreneurs are less risky, but possibly more attractive than for entrepreneurs.

Another observation is that the optimal firm policy may reward inventors, while giving lower rents to complementors. In the model, being a complementor is idea-specific, and all agents have an equal chance of becoming a complementor. Yet if there are agents who a priori are particularly good at idea elaboration (i.e. they are more likely to be complementors), they may prefer to reside in the market. Firms may find it difficult to properly compensate them, because there is only a limited flow of ideas inside firms. Strong complementors would prefer the market where they may draw from a large pool of idea generators. There are several real world analogies to these strong complementors: venture capitalists, seasoned angel investors, experienced entrepreneurs, and professional mentors (including specialized lawyers and consultants). All these agents specialize in working with inventors/entrepreneurs, helping them to turn their ideas into viable businesses (Lee et al., 2000).

5.2 Circulation of Ideas Across Firm Boundaries: Route 128 versus Silicon Valley

Our description of the interaction between firms and markets seem to fit many features of the open circulation of ideas in Silicon Valley, an environment in which many innovative start ups have emerged. Because many such independent ventures have been quite successful, agents in the open market appear to promote more innovation

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26Porter (1978), for example, describes the seemingly endless quest, and eventual success, of an intrapreneur inside the Bendix Corporation, to develop the first electronic fuel injection for automobiles.
than large firms. Our approach suggests a few qualifications. Many innovations (or at least some important preliminary concepts) may have been originally developed in large firms, yet markets were best at developing them. Independent entrepreneurs are often not the original inventors, but rather those who took a promising idea from inventors. Large firms should be seen as a crucial feature of any highly innovative environment, as critical incubators of novel ideas. Indeed, large firms are an important component of the productive structure in Silicon Valley.\footnote{According to Business Week, Silicon Valley accounted for a remarkable 20\% of the largest high tech firms in the world in 1997.} Any account of its success starts with a long list of the remarkable number of novel ventures started by individuals who left employment with a larger firm. Gompers, Lerner and Scharfstein (2004) provide evidence on the role that large corporations play in entrepreneurial spawning. Consistently with our mobility model, they find that more open firms tend to spawn more ventures, which also tend to be on average in less related areas. Enterprises with porous boundaries may create a symbiotic relationship with free market agents operating in an open exchange market. In contrast, a secretive corporate culture which resists interaction with markets may suffocate the circulation and thus subsequent elaboration of internally generated ideas.

The hierarchical approach to R&D in Japan and Europe, as well as in the large high tech companies on Route 128 in Massachusetts, has been unfavorably contrasted with the loosely organized open environment of Silicon Valley in California (Saxenian, 1994; Aoki, 2001), whose success is attributed to a free movement of ideas and individuals creating innovative ventures via informal arrangements. Yet the intense exchange of ideas in Silicon Valley is puzzling, since California actually has a fairly weak tradition of protecting intellectual property, so that is not clear how idea generation may be rewarded.\footnote{See Gilson (1999) and Hyde (1998) for the legal details. Hellmann (2003) provides a theoretical analysis of how intellectual property rights affect the incentives of employees to take ideas out of firms.} Our model offers some insight, by showing that entrepreneurial firm formation and large multi-product firms are actually symbiotic. Large firms are a natural source of innovative ideas, yet many of these ideas can find realization only if they are allowed to move to a free exchange system. In turn, a dynamic market will attract skilled, entrepreneurial individuals only if there are enough firms from which ideas may leave, seeking elaboration. The open environment in Silicon Valley may thus thrive thanks to the historical presence of large firms in the area, which have acted as incubators of new ideas, particularly those which were costly to develop.

The role that venture capitalists play in supporting entrepreneurs who leave firms seems also consistent with the notion suggested (if not fully modelled) in our approach, namely that agents in markets may specialize in implementation rather than origination, while inventors may prefer a more protected corporate environment.\footnote{Academic researchers, who have an independent reward system for novel ideas, seem to play another prominent role in creating innovative ventures.}

The model also suggests that high wages in Silicon Valley may result from a general
equilibrium effect of mobility in the presence of a high density of innovative firms, where operating in the market becomes an attractive outside option for employees.

5.3 Firms as Long Term Bureaucracies

In our model, firms are long term, multi-project institutions. Firms can solve contracting failures better than arm’s length transacting individuals, thanks to their sunk investment in reputation capital, which provides a commitment not to expropriate inventors, and to prosecute idea stealing by employees, i.e. to enforce the firm boundary. To sustain these tasks, the firm needs a formal organizational structure for recording and managing internal initiatives. Its role is to create a formal “paper trail” which facilitates the verifiability by a third party of idea stealing, either by the firm or by employees. Thus the “bureaucratic” nature of firms, while adding costs and possibly slowing down initiatives relative to more informal arrangements, have an essential function in ensuring that ideas are better protected in firms than in markets.

In this paper we do not model explicitly the nature of the convex costs, so that an endogenous determination of the boundaries of firms remains beyond the scope of the current paper. However, it is easy to see that convex costs are a natural aspect of the firm’s monitoring activities. When an employee registers an idea, the firm manager needs to establish that this idea is truly novel. Among other things, the firm manager needs to verify that the idea is not a disguised restatement of an idea that had been previously registered by another employee. Such a check is necessary to eliminate incentives to steal ideas within the firm, by re-registering them in a disguised manner. Suppose that the cost of checking one idea against another employee’s history of ideas is $c$. The total cost of checking one idea is thus $c(E - 1)$. We note that larger firms have higher costs. The expected per period number of employees registering an idea is $\gamma \mu$, where $\mu = \frac{E}{\gamma + \gamma \sum_{i=0}^{e-2} \phi_i}$ as before. Thus the firm’s cost of registering ideas is given by $m(E) = \frac{\gamma c(E - 1)}{\gamma + \gamma \sum_{i=0}^{e-2} \phi_i}$, which is a convex function of $E$.

One open question is exactly what set of rules should govern idea circulation inside firms, especially when ideas are heterogeneous. In the base model, the optimal rule is simple, since ideas are homogenous. Yet many ideas which fit poorly with the firm’s competencies may be taken outside firms without ever been disclosed, precisely to avoid wasting time with the “bureaucratic” process of seeking an unlikely internal match (see section 4.3). This suggests a simple intuitive reason for why firms choose to pursue “narrow” strategies (see Rotemberg and Saloner, 1994) and may have some

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30 Note that $E$ also enters the denominator through $\tau$. This does not matter for large $E$, since for $\tau \rightarrow \infty$, the denominator is bounded above by $\frac{\tau}{\phi}$. 

28
implications for internal flexibility. If firms adopt a rigid approach about forcing inventors to seek all possible internal matches before they may leave the firm, they may discourage reporting when employees are better able to assess specific competence. Flexible rules which allow employees some discretion on how long to search internally would improve incentives to generate and report ideas. With heterogeneous ideas the optimal internal organization will depend on the relative frequency of general and firm specific ideas, and on firms’ specific cluster of competence.

5.4 Other Governance Structures

We have considered only extreme organizational forms, either unstructured or hierarchical ones. We have also assumed that in a market containing an infinite number of agents, no individual reputation process will be enforceable without a large sunk investment. In practice, market exchange include informal arrangements which can protect, to some extent, the individual return to invention.

Suppose that agents earn extra rents from belonging to a closed non-hierarchical system (perhaps because its participants generate on average a larger number of ideas than outsiders). A reputation game may be sustainable if individual behavior is observed by others “within sight” so that opportunistic actions may be punished by ostracism from this group. This would presumably involve some monitoring costs, although less than in a firm. To the extent that such costs increase with size, it will also have a limited scope, producing an outcome similar to a hierarchical system. In fact, precise monitoring may not be feasible except in small, geographically localized circles. In future work we plan to consider different versions of non-hierarchical closed systems based on peer monitoring, such as networks and partnerships. Our conjecture is that such organizational forms, while less precise than firms in controlling attribution of invention, may sometimes support idea generation at a lower cost than hierarchical firms.

6 Conclusions

We have proposed a novel trade-off between the necessity to protect idea generation and the need to share ideas, in order to screen and elaborate them. The free circulation of ideas is efficient for the elaboration of incomplete concepts, but fails to reward personal effort for generating novel concepts. Individuals may voluntarily join close exchange systems, such as firms, to ensure that their initiatives receive support and feedback without being stolen. Firm ownership represent a claim on the use of registered ideas vis-a-vis employees. This creates a legal firm boundary which discourages the escape of ideas. The internal structure of the firm is designed to ensure a controlled circulation of ideas, along with a credible reward system.

Our model suggests that there is a natural symbiosis between the ability of firms to sustain exploration in ambitious ideas and the comparative advantage of market
in screening and elaborating ideas which leave firms.

Much further work is needed to explore this important theme of the circulation of ideas among individuals and institutions. We have sketched some questions and outlined some directions of future research we intend to pursue. Beyond firms and markets, this paper does not examine many other important institutions for the circulation of ideas, such as networks and partnerships. Another compelling question concerns idea generation and circulation among academics. Although academic researchers rarely capture the value created by their discoveries, the academic publication system may ensure some alternative reward mechanism. This suggests that other institutional arrangements may endogenously arise to preserve the free circulation of ideas, which constitutes a valuable public good.
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8 Appendix

8.1 Discouragement condition

To derive the discouragement condition, consider the continuation game after negative feedback. If the talker pursues the idea one more period, he always incurs the search cost $c$. With probability $\frac{\phi \bar{p}}{\phi \bar{p} + \phi}$ the reported negative signal was honest, and he simply wastes his time. With probability $\frac{\phi}{\phi \bar{p} + \phi}$ the negative signal was dishonest. With probability $\bar{\phi}$ the original talker finds no match, and gets no benefits. With probability $\phi^2$ both the talker and the substitute listener find their respective match, each getting the Bertrand payoff, which is zero. And with probability $\phi \bar{\phi}$ the talker finds a match, but the substitute listener does not. In this case the talker gets $pa\bar{z}$. Thus the expected return for the talker from pursuing an idea one more period after negative feedback is $\frac{\phi}{\phi \bar{p} + \phi} \phi \bar{\phi} pa\bar{z} - c$. The discouragement condition ensures that an agent prefers to generate a new idea (obtaining $\gamma(\phi pa\bar{z} - c) - \psi$), rather than pursuing an old idea on which he has received negative feedback. Formally, the condition is therefore given by

$$\frac{\phi^2 \bar{p} a \bar{z}}{\phi \bar{p} + \phi} - c < Max[0, \gamma(\phi pa\bar{z} - c) - \psi]$$

Note that a simpler sufficient condition is $c > \frac{\phi^2 \bar{p} a \bar{z}}{\phi \bar{p} + \phi}$. This condition is always satisfied both for $\phi$ sufficiently small, and sufficiently large.

8.2 Proof of Lemma 2

Let $S$ be the substitute and $C$ be the complementor. The continuation game has three possible equilibria, that we call “$C$ continues alone,” “$S$ continues alone” and “both $C$ and $S$ continue.”

We first show that for $\phi$ sufficiently small, “$C$ continues alone” is the only equilibrium. For this, we derive a set of sufficient conditions that are always satisfied for $\phi$ sufficiently small. Note that we focus on sufficient conditions. Deriving tighter necessary conditions is not tractable, since it would require solving a non-stationary subgame. The sufficient conditions avoid this problem by using appropriate bounds on the utilities of the non-stationary subgame.

To show that “$C$ continues alone” is the unique equilibrium, we provide two conditions. The first condition ensures that “both $C$ and $S$ continue” is not an equilibrium. We show this by providing a sufficient condition for $S$ not to want to continue if $C$ continues. The second condition ensures that “$S$ continues alone” is
also not an equilibrium. We show this by providing a sufficient condition for \( C \) to still want to continue if \( S \) continues.

Consider first the case where both \( C \) and \( S \) continue. With probability \( \phi^2 \), \( C \) finds another complementor and \( S \) also finds a complementor. In this case, \( S \) wins and gets a payoff \( az \). With probability \( \phi^2 \), \( C \) finds a substitute and \( S \) also finds a substitute. In this case \( C \) wins and gets a payoff \( \bar{a}z \). With probability \( \phi \bar{\phi} \), \( C \) finds a substitute and \( S \) finds a complementor. In this case, they compete Bertrand and receive no returns at all. Finally, with \( \phi \bar{\phi} \), \( C \) finds a complementor and \( S \) finds a substitute. In this case, the continuation value is difficult to assess, since it depends on the next continuation game, which is no longer stationary. We therefore find some appropriate bounds on the utilities.

The first condition is harder to satisfy the higher \( S \)'s utility in the equilibrium where both \( C \) and \( S \) continue. We therefore use an upper bound on the \( S \)'s expected utility in that equilibrium. For this, we use \( S \)'s utility under the lucky scenario that \( C \) never finds a match. The upper bound is then given by \( (az - c)[\delta \phi + (\delta \phi)^2 + \ldots] = (az - c)\delta \phi \). The second condition is harder to satisfy the lower \( C \)'s utility in the equilibrium where both \( C \) and \( S \) continue. We therefore use a lower bound on the \( C \)'s utility in that equilibrium. We can use 0 as a lower bound.

With this, the first sufficient condition requires that if \( C \) is continuing, \( S \) does not want to continue. The upper bound on \( S \)'s expected utility from also continuing is given by \( \phi^2 az + \phi \bar{\phi} \frac{(az - c)\delta \phi}{1 - \delta \phi} - c \). The first sufficient condition is thus given by

\[
\phi^2 (az + \frac{(az - c)\delta \phi}{1 - \delta \phi}) < c.
\]

This condition is always satisfied for sufficiently small \( \phi \), i.e., if complementors are sufficiently rare.

The second sufficient condition requires that \( C \) still wants to continue even if \( S \) continues. The lower bound on \( C \)'s expected utility from also continuing is given by \( \phi^2 az - c \). The second sufficient condition is thus given by

\[
\phi < 1 - \sqrt{\frac{c}{az}}.
\]

Again, this condition is always satisfied for sufficiently small \( \phi \). Taken together, these two conditions ensure that for sufficiently small \( \phi \), “\( C \) continues alone” is the unique equilibrium of the continuation game.

The analysis for the second part of the lemma, namely that “\( S \) continues alone” is the unique equilibrium for \( \phi \) sufficiently large is analogous. All we need to do is switch the lower and upper bounds. Straightforward calculations reveal that the two sufficient conditions are now given by

\[
\phi < 1 - \sqrt{\frac{c}{az}}.
\]
Both of these conditions are satisfied for sufficiently large $\phi$.

### 8.3 Proof of Lemma 3

Consider first the case of $\phi$ sufficiently small. Replacing $\overline{\phi}z = \frac{z + \zeta C}{2}$ into $\zeta C = \frac{\delta(\phi \overline{\phi}z - 2c)}{(2 - \delta)}$. From $az = \frac{z - \zeta C}{2}$ we thus obtain $a = \frac{1}{2} - \frac{\delta}{2 - \delta} \frac{\phi}{\overline{\phi} - \frac{c}{z}}$.

If $\phi$ sufficiently large, then the complementor has no outside option. The talker’s outside option is given by $\zeta_T = \delta(\phi az + \phi \frac{\zeta T}{2} - c)$. Note that the talker can no longer be discouraged at this point. Using $az = \frac{z + \zeta_T}{2}$ we get $\zeta_T = \frac{\delta(\phi \overline{\phi}z - 2c)}{2 - \delta}$, and thus $a = \frac{1}{2} + \frac{\delta}{2 - \delta} \frac{\phi}{2 - \frac{c}{z}}$.

It should be noted that our results do not rely on the use of the Nash bargaining solution. Binmore, Rubinstein and Wolinsky (1986) provide an extensive form game to justify the use of the Nash bargaining solution. However, there exist other reasonable bargaining solutions, such as “outside option” bargaining. Let us focus on the case of $\phi$ sufficiently small. With “outside option” bargaining, the complementor’s share is given by $\overline{\phi}z = \text{Max}\left[\frac{z}{2}, \zeta C\right]$ where $\zeta C$ as before. For $\zeta C > \frac{z}{2} \Leftrightarrow \frac{c}{z} < \frac{2\delta \overline{\phi} - 2 + \delta}{4\delta}$ we get $\overline{\phi}z = \zeta C = \frac{\delta(\phi \overline{\phi}z - 2c)}{(2 - \delta)}$. For $\zeta C < \frac{z}{2}$ we get $a = \overline{\phi} = \frac{1}{2}$. The complementor gets at least half, and possibly more of the surplus.

### 8.4 Proof of Proposition 1

The only part of Proposition 1 that requires a proof are the comparative statics on $\psi_M = \gamma(\phi p az - c)$. This depends on $a$, which in turn depends on the bargaining equilibrium. Since $a$ does not depend on $\gamma$ and $p$, those comparative statics are straightforward. For a we simply note that even though $a$ may be decreasing in $z$, $az$ is always increasing in $z$, so that $\psi_M$ is increasing in $z$. Next, $a$ is always an increasing function of $\phi$, so that $\psi_M$ is increasing in $\phi$. Finally, for $c$ we note that for $\phi$ sufficiently large, $a$ is decreasing in $c$, so that $\psi_M$ is decreasing in $c$. And for $\phi$ sufficiently small we have $\frac{d\psi_M}{dc} = \gamma(\phi p az \frac{da}{dc} - 1) = \gamma(\phi p \frac{\delta}{2 - \delta} - 1) = \frac{\gamma}{2 - \delta}(\delta(1 + \phi p) - 2) < 0$. 

37
8.5 Proof of Proposition 2

Before proving this proposition, we first establish the following useful Lemma.

**Lemma A1:**

(i) The life-time utility of newly joining a firm is given by $V_G = \rho_T v_T - \rho_G \psi$ where

$$
\rho_T = \frac{\gamma \sum_{i=0}^{i=\tau-2} (\bar{\phi} \delta)^i}{1 - [\bar{\gamma} \delta + \gamma (\bar{\phi} \delta)^{\tau-1} + \gamma \phi \delta \sum_{i=0}^{i=\tau-2} (\bar{\phi} \delta)^i]}
$$

$$
\rho_G = \frac{1}{1 - [\bar{\gamma} \delta + \gamma (\bar{\phi} \delta)^{\tau-1} + \gamma \phi \delta \sum_{i=0}^{i=\tau-2} (\bar{\phi} \delta)^i]}
$$

(ii) $V_G$ is increasing in $\tau$.

(iii) $V_1 > V_2 > \ldots > V_{\tau-1} > V_\tau = V_G$.

For part (i) consider first the case of $\tau = 3$ then $V_T^2 = v_T + \delta V_G$, $V_T^1 = v_T + \phi \delta V_G + \bar{\phi} \delta V_T^2$ and $V_G = -\psi + \gamma V_T^2 + \bar{\gamma} \delta V_G$. This is a system of three variables and three unknown. Solving it, we find that $V_G = -\psi + \bar{\gamma} \delta V_G + \gamma V_T^1 = -\psi + \bar{\gamma} \delta V_G + \gamma v_T + \gamma \phi \delta V_G + \gamma \bar{\phi} \delta V_T + \gamma \bar{\phi} \delta \bar{\phi} \delta V_G = -\psi + \gamma v_T + \gamma \bar{\phi} \delta v_T + \gamma \bar{\phi} \delta V_G + \gamma \phi \delta V_G + \gamma \bar{\phi} \delta V_G + \gamma \bar{\phi} \delta \bar{\phi} \delta V_G$ so that

$$
V_G = \frac{-\psi + \gamma v_T + \gamma \bar{\phi} \delta v_T}{1 - (\bar{\gamma} \delta + \gamma \phi \delta + \gamma \bar{\phi} \delta \bar{\phi} \delta) v_T}
$$

Consider next the case of $\tau = 4$, where $V_T^3 = v_T + \delta V_G$, $V_T^2 = v_T + \phi \delta V_G + \bar{\phi} \delta V_T^2$, $V_T^1 = v_T + \phi \delta V_G + \bar{\phi} \delta V_T^2$ and $V_G = -\psi + \gamma V_T^1 + \bar{\gamma} \delta V_G$. From $V_G = -\psi + \bar{\gamma} \delta V_G + \gamma V_T^1 = -\psi + \bar{\gamma} \delta V_G + \gamma v_T + \gamma \phi \delta V_G + \gamma \bar{\phi} \delta V_T + \gamma \bar{\phi} \delta \phi \delta V_G + \gamma \bar{\phi} \delta \phi \delta V_T = -\psi + \gamma v_T + \gamma \phi \delta V_G + \gamma \bar{\phi} \delta V_T + \gamma \phi \delta \phi \delta V_G + \gamma \bar{\phi} \delta \phi \delta V_T + \gamma \bar{\phi} \delta \phi \delta \delta V_G = -\psi + \gamma v_T + \gamma \bar{\phi} \delta v_T + \gamma \phi \delta \phi \delta v_T + \gamma \phi \delta \delta \delta V_G + \gamma \bar{\phi} \delta \phi \delta \delta V_T + \gamma \bar{\phi} \delta \phi \delta \delta \delta V_G$ we obtain

$$
V_G = \frac{-\psi + \gamma v_T + \gamma \bar{\phi} \delta v_T + \gamma \phi \delta \phi \delta \delta v_T}{1 - (\bar{\gamma} \delta + \gamma \phi \delta + \gamma \bar{\phi} \delta \phi + \gamma \bar{\phi} \delta \phi \delta) v_T}
$$

For the general case, it is now easy to see that

$$
V_G = \frac{-\psi + \gamma v_T \sum_{i=0}^{i=\tau-2} (\bar{\phi} \delta)^i}{1 - [\bar{\gamma} \delta + \gamma (\bar{\phi} \delta)^{\tau-1} + \gamma \phi \delta \sum_{i=0}^{i=\tau-2} (\bar{\phi} \delta)^i]}
$$

where for the denominator we use $[\bar{\gamma} \delta + \gamma (\bar{\phi} \delta)^{\tau-2} + \gamma \phi \delta \sum_{i=0}^{i=\tau-3} (\bar{\phi} \delta)^i] = [\bar{\gamma} \delta + \bar{\phi} \gamma (\bar{\phi} \delta)^{\tau-2} + \gamma \phi \delta \sum_{i=0}^{i=\tau-2} (\bar{\phi} \delta)^i]$. Naturally, this maps directly into $V_G = \rho_T v_T - \rho_G \psi$ where
\[
\begin{align*}
\rho_T &= \frac{\gamma \sum_{i=0}^{\pi-2} (\phi \delta)^i}{1 - [\bar{\gamma} \delta + \gamma (\phi \delta)^{\pi-1} + \gamma \phi \delta \sum_{i=0}^{\pi-2} (\phi \delta)^i]} \\
\rho_G &= \frac{1}{1 - [\bar{\gamma} \delta + \gamma (\phi \delta)^{\pi-1} + \gamma \phi \delta \sum_{i=0}^{\pi-2} (\phi \delta)^i]}
\end{align*}
\]

For part (ii) we simply note that the numerator of \( V_G \) is increasing in \( \bar{\pi} \) (since \( \sum_{i=0}^{\pi-2} (\phi \delta)^i \) increases with \( \bar{\pi} \)) and the numerator is decreasing in \( \phi \) (since \( \gamma (\phi \delta)^{\pi-1} + \gamma \phi \delta \sum_{i=0}^{\pi-2} (\phi \delta)^i \) increases with \( \bar{\pi} \)). Note also that for the same reasons, both \( \rho_T \) and \( \rho_G \) are increasing in \( \bar{\pi} \).

For part (iii) we consider again the case of \( \bar{\pi} = 4 \). To see that \( V^3_T > V^3_G \), assume to the contrary that \( V^3_T < V^3_G \). Then we have \( V^2_T = v_T + \phi \delta v_G + \delta v^3_T < v_T + \delta v_G = V^3_T \), so that \( V^1_T = v_T + \phi \delta v_G + \delta v^2_T < v_T + \phi \delta v_G + \delta v^3_T \) and thus \( V_G = -\psi + \gamma v^1_T + \delta v^3_T < -\psi + \gamma v^3_T + \delta v^3_T = -\psi + v_T + \delta v_G \Leftrightarrow V_G < \frac{\gamma v_T - \psi}{1 - \delta} \).

This is not possible, since \( V_G(\bar{\pi}) > V_G(\bar{\pi} = 2) = \frac{\gamma v_T - \psi}{1 - \delta} \). Thus \( V^3_T > V^3_G \). To see that \( V^2_T > V^3_G \), note that \( V^2_T - V^3_G = v_T + \phi \delta v_G + \delta v^3_T - v_T + \delta v_G = \delta v^2_T - \delta v^3_T > 0 \). For \( V^1_T > V^3_T \) note that \( V^1_T - V^3_T = v_T + \phi \delta v_G + \phi \delta v^3_T - v_T - \phi \delta v_G - \phi \delta v^3_T = \phi \delta (v^3_T - v^3_T) > 0 \). Thus \( V^1_T > V^2_T > V^3_T > V^3_G \). The proof for \( \bar{\pi} > 4 \) is analogous.

We are now in a position to prove Proposition 2. For the comparative statics of the optimal \( E \), we use \( \frac{dE}{d\psi} = \frac{d^2 \Pi}{dEd\psi} \). Since \( \frac{d^2 \Pi}{dE^2} < 0 \) and \( \frac{d^2 \Pi}{dEd\psi} = \frac{dE \rho_T}{dE} < 0 \), we have \( \frac{dE}{d\psi} < 0 \). Similarly, since \( \frac{d^2 \Pi}{dEdc} = \frac{dE \rho_G}{dE} < 0 \), we have \( \frac{dE}{dc} < 0 \); since \( \frac{d^2 \Pi}{dEdpz} = \frac{dE \rho_T}{dE} < 0 \), we have \( \frac{dE}{dP} > 0 \) and \( \frac{dE}{dz} > 0 \). Finally, since \( \frac{d^2 \Pi}{dEdm'} = \frac{-1}{1 - \delta} < 0 \) we have \( \frac{dE}{dm'} < 0 \). The comparative for \( b^* \) are obtained directly from

\[ b^* = \frac{c + \psi \rho_G \rho_T}{\phi z} \]