(Non-)Precautionary Cash Hoarding and the Evolution of Growth Firms

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Abstract

We analyze whether growth firms should delay current investment to hoard cash in order to reduce dilution from external financing. This hoarding motive is the natural counterpart to saving cash as a precaution to help secure funding for future investment opportunities. However, the two motives lead to fundamentally different implications for hoarding and for how cash interacts with key financial and investment decisions. In particular, our paper contributes to understanding why firms choosing private over public financing hoard less, and why product market competition has an ambivalent impact on the public-private choice.

Keywords: cash hoarding; growth firms; public versus private financing; competition; real options

JEL Classification: G31, G32, D92

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1 Introduction

Our knowledge about cash hoarding and investment is mainly framed by a literature that seeks to explain empirical patterns in large and mature firms. Some of the most important rationales for cash hoarding include building up cash reserves for precautionary or tax reasons (Opler et al., 1999; Bates et al., 2009). In this paper, we take a somewhat different perspective: that of a growth firm with investment opportunities already present, but without the necessary cash to undertake these opportunities—arguably one of the most important settings in corporate finance. A typical example is a firm in need of capital to transition from the phase of idea generation and testing to commercialization of this idea at a larger-scale. The relevant question for such a growth firm is whether it should hoard cash first and delay investment to reduce dilution associated with costly external financing (by self-financing more); or not hoard, accept dilution and invest immediately.

A key insight is that there is a stark difference in predictions depending on whether the driver for hoarding are investment opportunities that are already present or anticipated future investment opportunities. In our setting, in which hoarding means delaying current investment, firms with better investment opportunities are less inclined to hoard. Intuitively, the opportunity cost of investment delay is increasing in the attractiveness of the opportunities. By contrast, the main prediction in the literature on precautionary hoarding, which considers hoarding prior to the arrival of future investment opportunities, is that such firms hoard more (Bates et al., 2009). In practice, both motives are likely to be important. However, to the extent that financial decisions in growth firms are predominantly shaped by current rather than future investment considerations, the same is likely to apply to hoarding.[1] In this paper, we refer to hoarding that leads to delay of current investment as non-precautionary to highlight its close relation, but also contrast, to its better-known precautionary counterpart.

The fundamental contrast between the two hoarding motives, coupled with the focus on investment timing, could shed light on several puzzling stylized facts that highlight that there are still gaps in our understanding of how hoarding relates to the evolution of growth firms. For example, it may help explain why firms choosing private over public financing hoard less (Gao et al., 2013; Asker et al., 2015). This finding has been difficult to reconcile with the precautionary view that private firms need to hoard more because of their more-constrained access to external financing (e.g., due to lack of transparency or illiquidity costs). However, this result emerges naturally in our setting. Our analysis also sheds light on the

[1] Indeed, Rampini and Viswanathan (2010) argue that financially constrained growth firms do not hedge, as for such firms the marginal product of current investment is much higher than that of future investments, making the opportunity cost of hedging very high.

contradictory findings that in some studies product market competition increases (Chod and Lyandres, 2011), while in others it reduces the preference for public financing (Chemmanur et al., 2010). We show that stronger competition can push either direction, depending on its effect on hoarding and investment timing.

We derive our insights in a model in which a growth firm, run by an owner-manager (henceforth, manager), wants to make a lumpy investment. External financing is costly, because external financiers have a lower valuation of the firm's growth opportunity.[2] Hence, the manager considers delaying investment in order to hoard cash to reduce her dependence on external financing. To further reduce the firm's cost of finance, the manager could make the firm more transparent. With increasing "transparency" we mean making the firm open to monitoring and interference by financiers, for example, by adjusting its reporting and corporate governance practices. Interference increases firm value from the financier's perspective, but is costly for the manager, as it effectively reduces her autonomy. The trade-off between private and public financing that we consider is that public financing requires a minimum level of transparency, but has a liquidity benefit.

The starting point of our analysis is to flesh out the contrasting insight (vis-à-vis precautionary theories) that firms with better investment opportunities hoard less. Based on this insight, we derive predictions for how hoarding interacts with the preferred level of transparency, the firm's competitive environment, and the choice between public and private financing.

Take, first, the choice of transparency. We show that when the manager accelerates investment by hoarding less and relying more on external financing, she prefers less transparency. The reason is that the manager's stronger reliance on external financing gives the financier extra incentives to monitor because of his larger stake in the firm. This increases the likelihood of interference, which the manager could partially counteract by making the firm less transparent. Though decreasing transparency increases the firm's funding cost, it is still preferable on balance.

Considering now that both hoarding and the choice between public and private financing are endogenous decisions, we show that private financing is associated with less hoarding. The reason is that public financing has minimum requirements for the firm's transparency, while private financing lets the manager optimally choose the desired level of transparency. This makes private financing more attractive for the manager when seeking to limit transparency, which is when she delays and hoards less.

[2] The difference in valuations in our model is due to a difference in vision how to run the firm (Dittmar and Thakor, 2007; van den Steen, 2005, 2010). Other reasons could be a limited redeployability and/or pledgeability of assets or human capital (Boyle and Guthrie, 2003).

Building on this analysis, we show that product market competition has a dual effect on both hoarding and the firm's choice between public and private financing. Specifically, by linking hoarding to investment delay, we highlight that hoarding puts the firm's first-mover advantage at risk. Though this makes hoarding less attractive, there is also a countervailing effect: An increase in competition reduces profits regardless of whether or not the firm is a first-mover. This reduces the opportunity cost of delaying investment and encourages hoarding. Taken together, these countervailing effects imply that competition leads to a reduction of hoarding (in order to accelerate investment) only if having a first-mover advantage is of paramount importance. In this case, the firm aims to invest more quickly, and uses more external financing. Given that this invites more monitoring and interference by financiers, less transparency (and, thus, private financing) becomes preferable to partially counteract the increased scrutiny. Thus, a growth firm rushing to realize a first-mover advantage prefers private financing. By contrast, if having a first-mover advantage is not of paramount importance, stronger competition leads to more delays and hoarding. The lesser dependence on external financing leads to less monitoring and interference, which makes it optimal to lower the cost of funding by choosing more transparency. Public financing is now more likely because the minimum transparency requirement of public financing is less of a burden.

We extend the model along several dimensions. We show that endogenizing the liquidity benefit of public financing (which we take as given in the baseline model) reinforces our results on the choice between public and private financing. Furthermore, we consider information asymmetry about the firm's growth opportunity in addition to the disagreement frictions between the manager and financiers. We show that firms with better investment opportunities will further reduce hoarding to signal quality.

Our results reconcile a number of puzzling empirical findings and give rise to novel empirical predictions. First, our model sheds light on Gao et al.'s (2013) and Asker et al.'s (2015) counter-intuitive findings that private firms hoard less than public firms. We further relate to the evidence that some firms try to achieve the best of both worlds by being public, but making private placements, which typically have lower transparency requirements. In line with our predictions, such firms invest more quickly (Phillips and Sertsios, 2017). Second, our results demonstrate how the public-private choice is affected by product market competition. In particular, the importance of having a first-mover advantage determines not only hoarding, but also whether competition leads to more public or more private financing. This could help reconcile conflicting empirical findings, such as those reported in Chod and Lyandres (2011) and Chemmanur et al. (2010). Indeed, our results on the non-monotonicity between investment delay and competition is consistent with existing empirical evidence (Akdogu and MacKay, 2008). Overall, our analysis highlights the fundamental difference in predictions depending on whether hoarding is driven by current investment (more likely for growth firms) or anticipated future investment considerations (more likely for mature firms).

Our paper mainly relates to the fast growing literature on cash. Firms hoard cash because they may be unable to frictionlessly raise financing for new investments. Agency conflicts are one such important friction (Jensen, 1986).[3] Alternatively, firms may hoard cash as a precautionary measure when anticipating future investment or hedging risk (Tirole, 2006). Bolton et al. (2011) show that firms will keep a positive cash balance even if this necessitates costly external financing, since the marginal benefit of avoiding to seize operations is high. In such cases, firms with stronger cash flow streams need to hoard less (Acharya et al., 2012). Related, Almeida et al. (2004) show that financially constrained firms save more cash out of cash flows.[4] Existing evidence supports the precautionary motive for hoarding cash (Opler et al., 1999; Bates et al., 2009). However, we are not aware of empirical work investigating the delay of investment due to cash hoarding. In this paper, we argue that this channel is important, as such hoarding has contrasting cross-sectional implications compared to precautionary theories.

By highlighting the key differences in predictions for hoarding depending on whether or not an investment opportunity is already present, our analysis provides novel insights about the endogenous relation between cash hoarding, transparency, competition, and the choice between public or private financing. While prior work, such as Boyle and Guthrie (2003), Hugonnier et al. (2015), and Bolton et al. (2013), has analyzed hoarding and investment timing, these broader interactions have been ignored.[5]

Earlier contributions relating hoarding to competition have argued that hoarding insures against negative liquidity shocks in order to secure survival (Hoberg et al., 2014; Morellec et al., 2014). Hoarding is then more important in a competitive environment, leading to an unambiguously positive relationship between hoarding and competition. By contrast, when relating hoarding to investment delay, we show that competition has a dual effect. Specifically, the pressure of competition on future profits makes investment delay and hoarding

[3]Dittmar and Mahrt-Smith (2007) and Pinkowitz et al. (2006) show that cash is worth less when agency problems between inside and outside shareholders are greater, and Nikolov and Whited (2013) identify low managerial ownership as a key factor driving agency costs. In contrast, Opler et al. (1999) and Bates et al. (2009) find no evidence relating agency problems to cash holdings.

[4]In the context of risk management, Acharya et al. (2013) show that firms with high aggregate risk exposure prefer cash to credit lines, while Rampini and Viswanathan (2010) argue that the opportunity cost of risk management is higher for constrained firms. Unlike our focus on financing current growth opportunities, these papers focus on cash and/or credit lines as means of overcoming future liquidity problems. Also note that credit lines are not common for growth firms (Sufi, 2009).

[5]Interestingly, Chemmanur and He (2011) show that, when going public helps firms grab market share, a firm may have incentives to go public to preempt yielding market share to rivals. As in other models analyzing the choice between public and private financing (e.g., Pagano and Röell, 1998; Chemmanur and Fulghieri, 1999; Boot et al., 2008), there is no cash hoarding in Chemmanur and He's (2011) model.

more attractive. However, there is also a force working in the opposite direction, as competition creates incentives to invest more quickly (Grenadier, 2002; Carlson et al., 2006; Novy-Marx, 2007), which leaves less time for hoarding. These effects differ from those in the precautionary literature. Specifically, when competition erodes the profitability of investments, precautionary hoarding becomes less attractive, while in our case with investment opportunities already present, hoarding incentives go up. Strategic considerations differ as well. In our setting, the concern of not being a first-mover would drive the manager to reduce hoarding and delay. Instead, with precautionary hoarding as in Lyandres and Palazzo (2015), the strategic consideration is that a firm might hoard more to increase its likelihood of being able to invest, and in doing so discourage hoarding and investing by its financially constrained competitors.

Our paper is organized as follows. Section 2 presents the model. Section 3 discusses the relation between hoarding and investment delay and relates it to the choice between private and public financing and the effects of competition. In section 4, we analyze various extensions. Section 5 discusses empirical implications. Section 6 concludes. Appendix A contains all proofs, and the supplementary material in Appendix B discusses a number of further extensions and robustness issues.

2 Model

Our baseline model features a growth firm run by a sole owner-manager (henceforth, manager). This firm already generates revenues, but its potentially main profitable expansion is still ahead of it. As mentioned, a good example is a firm that transitions from idea generation and product testing to large-scale production. We model this in the following natural way. Suppose that the firm has an existing asset in place producing stochastic cash flows. If they are not paid out or invested, these cash flows accumulate in the form of cash reserves. The change of the level of these cash reserves over time follows

$$dw_t = \mu w_t dt + \sigma w_t dZ_t,\ w_0 > 0, \tag{1}$$

where $\mu > 0$ and $\sigma \geq 0$ are constant and $(Z_t)_{t\geq 0}$ is a standard Brownian motion. This simple reduced-form formulation for how the level of cash changes within the firm is sufficient for our purposes. A key assumption is that $\mu < r$, where r is the constant discount rate used by all. This assumption, which is standard in the real options literature, implies that the firm has only a weak ability to generate cash, and retaining cash flows within it is costly to insiders. This is precisely the feature we want to capture for a growth firm for which the investment opportunity is the main component of valuation and absent which the firm constitutes an unprofitable business. At the same time, this setting is sufficiently flexible to allow us to discuss payout policies and to capture the likelihood of default (because if w_t hits zero, the firm does not recover).[6] Though for most of the main text, we refer to w_t as cash, an alternative interpretation is that w_t represents the assets the firm builds up over time, which are available as a safe collateral free of any financing frictions.[7]

The reason the manager is willing to keep the firm going is that it has generated a profitable lumpy investment opportunity, requiring an investment of K and generating cash flows with an expected discounted present value of X. Initially, the manager does not have sufficient cash at hand for making the investment, but she has discretion over the timing of the investment. Our approach makes use of the standard real options framework (McDonald and Siegel, 1986; Dixit and Pindyck, 1994), but differs from this framework in one important aspect: the firm is cash-constrained and the manager may not be able or willing to invest in a positive NPV project even if she has access to outside financing.

Specifically, we assume that at the time at which the manager raises capital to make the investment, she is facing a competitive capital market. However, what makes financing expensive for the firm is that the financier and the owner-manager could have different ideas about the best way to run the firm, leading the financier to undervalue the firm from the manager's point of view.

To model this, we assume that at $t = 0$, the manager and the financier observe a signal θ that indicates the project's expected discounted cash flows, with $X(\theta) > K$ at least for some θ and X increasing in θ. What makes financing costly is that, although the financier and the manager observe the same signal, they may interpret it differently. The valuation from the manager's perspective is $X(\theta)$, while the financier believes that there is a probability that the project's value is less than $X(\theta)$, so that his overall valuation is only $E\rho X(\theta)$. In this expression, $0 \leq \rho \leq 1$ is the degree of agreement, and $E \geq 0$ is the financier's monitoring intensity that we define below.[8] While we believe differences in vision to be a key reason for differences in valuation of growth firms, we could interpret $\rho \leq 1$ alternatively as being

[6]The main advantage of (1) is that it allows us to solve most things in closed form. At the cost of losing this tractability, we could specify a cash flow process generating the cash level w_t as in Bolton et al. (2013), but such alternative formulations do not lead to further insights.

[7]Examples could include assets like property, plant and equipment, inventories, and accounts receivables, which the firm accumulates over time.

[8]We could derive ρ from primitives by assuming that the financier believes that the project's value is $X(\theta)$ with probability ρ' and $aX(\theta)$ otherwise (where $0 \leq a, \rho' \leq 1$). Then $\rho = (a + \rho'(1 - a))$. If we had $\rho > 1$, there would be no hoarding (Proposition 1). Disagreement in a corporate finance context is usually introduced by postulating heterogeneous priors in the sense of Kurz (1994a,b), e.g., Boot et al. (2008). However, disagreement can also arise due to overconfidence (Bernardo and Welch, 2001; Daniel et al., 1998), excessive pessimism (Coval and Thakor, 1998), or optimism (Manove and Padilla, 1999).

caused by limited pledgeability or redeployability of the assets outside the firm or by the problem that the manager may not be able to commit her human capital to the project (Boyle and Guthrie, 2003).

All features of our model are common knowledge, and the cash flows and the level of cash are costlessly verifiable. Furthermore, we assume that all parties are risk neutral and protected by limited liability. In our baseline model, we assume that ρ and $X(\theta)$ do not change over time, but we relax these assumptions in Appendix B. In that appendix, we also show that the idea of delaying investment to hoard easily extends also to other financing frictions.

Transparency and Product Market Competition Given the disagreement between the manager and financiers, the manager could make the firm more transparent in an effort to obtain better financing terms. Specifically, in analogy to Burkart et al. (1997), once a financier has provided capital and the firm has invested, the financier monitors the firm and interferes with management with intensity E. Monitoring and interference increases the firm's value from the financier's perspective at a cost to the financier of $\frac{E^2}{2\phi}$. The parameter ϕ reflects the ease of monitoring, determined by the manager's choice of transparency. A higher ϕ implies a higher level of transparency and a lower cost of monitoring. Thus, ϕ captures the extent to which the firm's reporting and corporate governance permit outsiders to influence the way the firm is run.[9] The reason the manager may choose less transparency is that she perceives financier interference as costly, with the cost $\kappa(E)$ increasing and convex in the amount of interference E. Specifically, we assume that $\kappa(E) = \frac{E^2}{2}c$, where $c > 0$. The trade-off between public and private financing that we focus on is that public financing requires a minimum level of transparency $\widehat{\phi}$, while there are no such requirements for private financing. On the positive side, public financing carries a liquidity benefit. In our baseline model, this benefit is exogenous (and infinitesimal), but we endogenize liquidity considerations in Section 4. We let the choice of the amount and type of financing as well as transparency be made together with the investment decision.

The manager's hoarding and financing choices are further affected by the firm's competitive environment. Following Loury (1979) and Weeds (2002), we model competition by assuming that the likelihood that a competitor with a similar idea enters the market before the firm invests follows an exponential distribution with parameter λ. The entry parameter $0 \leq \lambda < \infty$ could be interpreted as a measure of the intensity of competition. The impor-

[9] Arguably, our focus on growth firms (with limited track records) allows us to treat ϕ as a choice variable largely in the hands of the owner-manager. This could be different in a mature firm with a well-understood business model or a truly large firm that would invite substantial information production by investors regardless of its own transparency choice.

tance of competition in our model is that it reduces expected firm profits. This happens in two ways: an overall reduction in profits regardless of whether the firm is a first-mover, and extra losses in case the firm becomes a late-mover. Specifically, if the firm is a first-mover, the expected value from investment is a fraction $\pi_{FM}(\lambda) \leq 1$ of the value without competition, with $\pi'_{FM}(\lambda) \leq 0$. If a competitor enters before the firm, the expected value from investment when being a late-mover is a fraction $\pi_{LM}(\lambda) \leq \pi_{FM}(\lambda)$ of the value without competition, with again $\pi'_{LM}(\lambda) \leq 0$.[10]

3 The Growth Firm's Decision to Hoard Cash

3.1 (Non-)Precautionary Cash Hoarding and Speed of Growth

To make our first point in a simple way, we initially abstract from the choice of transparency and the choice between public and private financing and focus exclusively on hoarding and the timing of investment. To do so, we assume that the choice of monitoring and interference is binary $E \in \{0,1\}$. In this case, the manager can only raise external financing if $E = 1$, which is associated with a financier valuation of ρX. Clearly, in this case, it is optimal for the manager to choose maximal transparency $\phi \to \infty$, as this minimizes the overall cost $\frac{E^2}{2\phi} + K - w$ the financier needs to be compensated for, and in turn the manager's cost of finance. In the next section, we remove the restriction that E is binary, which makes the transparency choice ϕ non-trivial.

The manager raises $K - w$ by selling an equity stake to fund the investment outlay K. Since the market for capital is competitive, the financier only requires to break even and the equity stake that needs to be promised to him is (suppressing the dependence of X on θ),

$$\alpha = \frac{K - w}{\rho X}. \tag{2}$$

The manager's net expected payoff at the point in time that she raises $K - w$ and co-invests w is:

$$V(w, \theta) := \left(1 - \frac{K - w}{\rho X}\right) X - \kappa(1) - w. \tag{3}$$

This payoff is increasing in the co-investment w, the profitability parameter θ, and the agreement parameter ρ.

We now derive how the potential lack of alignment between management and financiers

[10] As in Loury (1979) and Weeds (2002), this reduced-form way of modeling competition captures the main forces behind it without committing us to a specific modeling choice. See Grenadier (2002) and Novy-Marx (2007) for models in which firms compete on quantity and try to time market demand.

affects the timing (delay) of investment, and in turn the hoarding decision. We solve for the value of the real option to invest using standard dynamic programming methods (Dixit and Pindyck, 1994). The problem is that of finding the optimal level of cash holdings w^* at which to stop hoarding that maximizes the value of the option to invest U. This involves trading off the benefit of reducing the funding cost against the time value of money lost from delaying investment, where the manager's expected payoff is

$$U\left(w_t, w^*, \theta\right) = \max E\left[\frac{1}{1+rdt}\left[U\left(w_t + dw_t, w^*, \theta\right)\right]\right]. \tag{4}$$

Applying Ito's lemma, we obtain

$$rU = \mu w \frac{\partial U}{\partial w} + \frac{1}{2}\sigma^2 w^2 \frac{\partial^2 U}{\partial w^2}.$$

This equation is solved subject to the following boundary conditions. First, the manager's expected payoff at the time of investment should be equal to her payoff from investment: $U\left(w_t, w^*, \theta\right)|_{w_t = w^*} = V\left(w_t, \theta\right)|_{w_t = w^*}$. Second, the manager chooses the investment trigger so as to maximize her value at the endogenous investment threshold: $\frac{\partial}{\partial w^*} U\left(w_t, w^*, \theta\right)|_{w_t = w^*} = 0$. Finally, the option to hoard cash becomes worthless as the value of cash tends to zero: $\lim_{w_t \to 0} U\left(w_t, w^*, \theta\right) = \max\left\{0, \left(1 - \frac{K}{\rho X}\right) X\right\}$. Indeed, if the existing business falters ($w_t \to 0$), then it almost surely does not recover (cf. (1)), and the manager can invest only if she raises all financing externally. If that is not possible, the firm has no purpose, and seizes to exist. Using these conditions, we can restate the manager's problem as

$$w^* = \arg\max_{\widehat{w}} \left(\left(1 - \frac{K - \widehat{w}}{\rho X}\right) X - \kappa\left(1\right) - \widehat{w}\right) \left(\frac{w_t}{\widehat{w}}\right)^\beta \tag{5}$$

where β is the positive root of $\frac{1}{2}\sigma^2 y\left(y-1\right) + \mu y - r = 0$, and $\mu < r$ implies that $\beta > 1$. Intuitively, the right-hand side of expression in (5) can be interpreted as the manager's expected payoff from investing at $\widehat{w}$ multiplied by the probability of reaching the cash level $\widehat{w}$ and investing. If, disagreement is not so strong, which we define as $\rho \geq \frac{X - \kappa(1)}{K}$, (5) has no interior solution, and the manager is better off investing immediately. Instead, if disagreement is sufficiently strong ($\rho < \frac{X - \kappa(1)}{K}$), solving the optimization problem (5) involves trading off the marginal cost of delay (due to $\mu < r$) with the potential gains from avoiding dilution by hoarding cash. The value maximizing investment threshold w^* is given by

$$w^* = \frac{\beta}{\beta - 1} \frac{K - \rho\left(X - \kappa\left(1\right)\right)}{\left(1 - \rho\right)}. \tag{6}$$

For completeness, note that since E is binary, raising external financing always entails the monitoring cost $\kappa(1) > 0$ for the manager, implying that she may be better off avoiding external financing altogether and hoarding the entire amount K.[11]

Proposition 1 *If disagreement is sufficiently strong ($\rho < \frac{X-\kappa(1)}{K}$), it is optimal for the manager to hoard cash and delay the investment. If raising external financing is still optimal, the optimal cash level is given by (6), and decreases in the quality of the investment opportunity, i.e., $\frac{\partial w^*}{\partial \theta} < 0$. Furthermore, hoarding increases when there is more disagreement, i.e., $\frac{\partial w^*}{\partial \rho} < 0$.*

Quite naturally, the cost of delay weighs less when there is more disagreement, prompting the manager to hoard more cash. More important, what this proposition points at is that delaying is costlier if the investment opportunity is better. Hence, by building on the simple insight that firms with better investment opportunities seek to invest more quickly, we obtain that they hoard less cash. The robustness and simplicity of this insight is key. As we show next, it is the exact opposite to what can be expected from precautionary hoarding.

Contrast to Precautionary Hoarding and Discussion Suppose (for this discussion only) that the firm did not have yet an investment opportunity at $t = 0$, but expected that such an opportunity may present itself at some future point in time. This is the setting of much of the existing literature where the focus is on precautionary hoarding. To avoid costly delay following the arrival of the investment opportunity, the manager could start hoarding cash prior to its arrival. This would be optimal if the profitability of the investment opportunity is sufficiently high. Thus, in such a precautionary setting, firms with better investment opportunities hoard more. In our setting with investment opportunities already present, we have the opposite result: firms with better opportunities hoard less (Proposition 1).

In practice, both settings are relevant. However, as discussed in the introduction, our model captures the idea that the financial and hoarding decisions in growth firms are mainly determined by current investment needs. By contrast, anticipating future investment needs with precautionary hoarding is more likely to be characteristic for mature firms.

The hoarding decision further depends on other factors, such as the rate at which the firm generates cash from existing operations, which might be correlated with the quality of its investment opportunity. Clearly, a higher rate of cash generation μ would imply more hoarding. Thus, empirical tests need to control not only for a firm's growth options but also for the profitability of its existing assets. It is also possible that scale is adjustable, despite

[11] For the remainder of the analysis we assume that $\rho < \frac{X-\kappa(1)}{K}$.

the lumpy nature of the investment opportunity. Here the effect is less clear cut, as the scale decision would depend on whether the investment features increasing or decreasing returns to scale, as well as whether scale can be added in steps. Nevertheless, it continues to hold that, when the firm's investment opportunity is better, an investment at a given scale is made with a lower proportion of hoarded cash.[12]

3.2 Transparency and the Public-Private Choice

We now remove the restriction that E is binary and let the manager choose the level of transparency vis-à-vis external financiers. Financiers like transparency as it helps them monitor and interfere. In what follows, we derive the endogenous relation between hoarding and transparency, and then relate the analysis to the choice between public and private financing.

Given an equity stake α_m in the firm, the financier's monitoring choice is given by

$$E^* = \arg\max_E \alpha_m E \rho X - \frac{E^2}{2\phi}, \tag{7}$$

resulting in an optimal monitoring level $E^* = \alpha_m \phi \rho X$.[13] Clearly, a larger stake implies more monitoring. Plugging in for E^* on the right-hand side of (7) to obtain the financier's payoff, his break even condition can be stated as

$$\frac{(\alpha_m \phi \rho X)^2}{2\phi} \geq K - w.$$

Since financiers compete, this break even condition is satisfied with equality. Thus, when the financier monitors, his equity stake is given by

$$\alpha_m^* = \frac{\sqrt{2\phi (K - w)}}{\phi \rho X}. \tag{8}$$

We can now determine the optimal level of hoarding and transparency by considering the manager's optimization problem. From (8), we see immediately that more transparency decreases the cost of finance ($\frac{\partial \alpha_m^*}{\partial \phi} < 0$). However, the trade-off is that more transparency invites more interference by the financier, which reduces the manager's payoff. Specifically,

[12] In Appendix B, we show that this insight remains true also when the required investment outlay, the investment opportunity's profitability, or disagreement can change over time.

[13] We assume throughout that the manager finances the firm with equity. In a previous working paper version, we have shown that debt may lead to less hoarding, as it exposes the financier less to the disagreement friction. However, with debt financing, the manager's residual claim becomes very sensitive to disagreement, potentially making debt a suboptimal financing choice.

the manager's optimal transparency and hoarding decisions solve

$$\max_{\phi,\widehat{w}} \left((1-\alpha_m^*) X - \kappa(E^*) - \widehat{w} \right) \left(\frac{w}{\widehat{w}} \right)^{\beta}, \tag{9}$$

where E^* and α_m^* are given by (7) and (8). We now have:

Proposition 2 *The manager chooses a lower level of transparency and hoarding if the firm's investment opportunity is better, i.e.,* $\frac{\partial \phi^*}{\partial \theta} < 0$ *and* $\frac{\partial w_m^*}{\partial \theta} < 0$.

Key for this result is that the financier monitors and interferes more if his stake in the firm is larger. In analogy to Proposition 1, this occurs when the manager has a better investment opportunity. She is then less willing to delay and, thus, needs more external financing, which invites more interference by the financier. Such increased scrutiny induces the manager to choose less transparency, which partially mitigates the extra interference. The exact level of transparency balances the impact on the cost of external financing with the impact on interference.

We can now use Proposition 2 to understand when public or private financing dominates and how it relates to hoarding. Specifically, recall that public financing comes with a minimum transparency requirement $\widehat{\phi}$, while private financing puts no such constraints and lets the manager freely choose the desired level of transparency. The latter discretion is more important if the firm's investment opportunity is better (θ is high), as then the manager prefers a lower transparency level (Proposition 2) to partially counteract the financier's higher scrutiny. By contrast, if θ is low, the minimum transparency level $\widehat{\phi}$ is not constraining, and the manager prefers public financing for its liquidity benefit.

Proposition 3 *Given a minimum level of transparency requirement* $\widehat{\phi}$ *of public financing, there is a threshold* $\widehat{\theta}$*, defined by* $\phi^*(\widehat{\theta}) = \widehat{\phi}$*, such that the manager chooses private financing for* $\theta \geq \widehat{\theta}$*, and public financing otherwise.*

Proposition 3 implies that private financing is associated with better investment opportunities, less hoarding, and less delay.

An immediate extension to this analysis is to consider private financing by firms that are already public. Specifically, new equity issues come with additional disclosure requirements, but these requirements are typically not as stringent for private placements. This can be modeled by assuming that there is an extra transparency obligation dictated by the new issue that can be limited if a private placement is chosen. Since it remains true that firms with better investment opportunities prefer less interference and hoard (and delay) less, we obtain:

Corollary 1 *Growth firms with better investment opportunities that are already public prefer financing sources with lower transparency requirements, such as private placements.*

Discussion: Relation to Precautionary Hoarding We have shown that growth firms with better current investment opportunities are more likely to stay private and hoard less. Extending this result to a setting in which the firm transitions to precautionary hoarding as it matures would suggest that these firms effectively delay going public until they mature. Indeed, once growth firms have matured and hoarding is dictated by anticipated future investment needs rather than current investments, firms with better future investment opportunities are likely to hoard *more* and rely less on external financing, which makes being public more attractive. The higher transparency requirements of public financing are then less cumbersome, as issuing smaller stakes leads to less interference. In general, by making the firm less dependent on external financing, precautionary hoarding is likely to (weakly) increase the preference for public financing.

3.3 Product Market Competition and the Public-Private Choice

We now relate the choice between public and private financing to the firm's competitive environment. By relating hoarding to investment delay, we show that competition exerts countervailing effects on the incentive to hoard. On the one hand, competition leads to an urgency to accelerate investment and reduce hoarding as a strategic move to preempt the potential loss of the first-mover advantage ($\pi_{FM} \geq \pi_{LM}$). This effect is reinforced by the fact that stronger competition not only increases the likelihood of being a late-mover, but also erodes the payoffs late-movers can make (i.e., $\pi'_{LM}(\lambda) \leq 0$). Since these two forces work in the same direction, we call them jointly the first-mover benefit effect.

On the other hand, stronger product market competition erodes the firm's profitability even it is a first-mover (i.e., $\pi'_{FM}(\lambda) \leq 0$)—call this the first-mover erosion effect. Following the intuition of Proposition 1, this second effect implies that stronger competition reduces the opportunity cost of hoarding, which makes hoarding more attractive.

Facing these two countervailing effects, the manager increases investment delay and hoarding in the face of stronger competition if the first-mover benefit effect is either not very important or is simply irrelevant, such as when the firm is a late-mover.

Proposition 4 *Stronger product market competition has two countervailing effects: (i) a first-mover benefit effect, which calls for reducing hoarding and speeding up investment; and (ii) a first-mover erosion effect, which calls for increasing hoarding and delaying investment. The first-mover benefit effect dominates (and, hence,* $\frac{\partial w^*_{FM}}{\partial \lambda} < 0$*) if* $\pi_{LM}(\lambda)$ *and* $\pi'_{LM}(\lambda)$ *are* *sufficiently low compared to $\pi_{FM}(\lambda)$ and $\pi'_{FM}(\lambda)$ (as defined in the Appendix). Otherwise, stronger competition increases hoarding ($\frac{\partial w^*_{FM}}{\partial \lambda} > 0$).*

We can now combine the insights of Section 3.2 with those from Proposition 4. Depending on whether the overall effect of product market competition leads to more or less hoarding and, thus, to a lesser or stronger dependence on external financing, we have different predictions for the choice between public and private financing.[14]

Corollary 2 *The effect of product market competition on transparency and the choice between public and private financing is as follows: (a) If the first-mover benefit effect dominates, competition lowers the manager's preferred level of transparency ($\frac{\partial \phi^*_{FM}}{\partial \lambda} < 0$), which increases the attractiveness of private financing (i.e., $\widehat{\theta}$ decreases in λ). (b) If the first-mover erosion effect dominates, competition increases the manager's preference for transparency ($\frac{\partial \phi^*_{FM}}{\partial \lambda} > 0$), which increases the attractiveness of public financing (i.e., $\widehat{\theta}$ increases in λ).*

Corollary 2 shows that competition can affect the choice between public and private financing by affecting the firm's choice of hoarding investment delay. Before continuing with the model's extensions, we briefly relate again to the precautionary hoarding motive. Studying the effect of competition on this motive, Hoberg et al. (2014) and Morellec et al. (2014) argue that, by compressing margins, competition reduces the capacity of firms to deal with liquidity shocks and would, thus, increase the need for precautionary hoarding. Incorporating this prediction into our setting would mean that the preference for public financing (weakly) increases.[15]

4 Extensions and Robustness

In this section we discuss extensions and robustness issues. In Section 4.1, we expand on the liquidity benefit that we have associated with public financing and how it is related to the

[14] Relating to the discussion in Section 3.1, competition may also have other effects that are not specific to our setting. First, it could erode the firm's existing business and, thus, the rate at which cash accumulates. This would strengthen the negative effect of competition on hoarding. Second, to the extent that the lumpy investment opportunity is scalable, adjusting the scale would mechanically affect the need for cash, but it would not change that (for any given scale) better investment opportunities will drive firms to invest with a higher proportion of external financing.

[15] Note that the higher level of hoarding reduces the need for outside financing, which makes public financing more desirable. Other precautionary theories have discussed how the firm's hoarding strategy deters rivals' willingness to invest (Lyandres and Palazzo, 2015). In this work, the effect of competition on precautionary hoarding is ambiguous. None of these papers has discussed the link between hoarding and the choice between public and private financing.

choice of transparency and the intensity of monitoring interference. Subsequently, in Section 4.2, we introduce asymmetric information and study its effect on hoarding.[16]

4.1 Liquidity and the Public-Private Choice

In our analysis of the choice between public and private financing, we assumed that public financing has a (infinitesimal) liquidity benefit. This benefit served as a tie breaker, inducing the manager to choose public financing when not burdened by its minimum transparency requirements. We will now expand on the liquidity benefit and analyze its relation to transparency and monitoring.

What we see as liquidity benefit of public financing is that it is easier to find financiers in public markets. Specifically, suppose that, after funding the project and interfering, the financier may encounter a liquidity shock with probability q, in which case he needs to sell his stake in the firm to a new financier with a potentially lower valuation. We let the new financier's expected valuation be a fraction ζ of that of the initial financier. Thus, when buying a stake α, the initial financier values this stake at $(1-q+q\zeta)\,\alpha E\rho X$. We stipulate that under public financing, $\zeta = \zeta_{pub} = 1$ (i.e., a liquidity shock is not costly for the financier), while under private financing $\zeta = \zeta_{priv} < 1$.

In analogy to Section 3.2, the financier's monitoring level is $E^* = \alpha_m\,(1-q+q\zeta)\,\phi\rho X$, and his break even condition

$$\frac{\left((1-q+q\zeta)\,\alpha_m\phi\rho X\right)^2}{2\phi} \geq K - w$$

will be satisfied with equality, resulting in an equity stake of

$$\alpha_m^* = \frac{\sqrt{2\phi\,(K-w)}}{(1-q+q\zeta)\,\phi\rho X}.$$

Observe that greater liquidity (higher ζ) has some similarity to a more aligned valuation (higher ρ), which lowers the cost of external financing. Thus, if under public financing, the manager would optimally choose $\phi^* > \widehat{\phi}$, it is a clear-cut decision to choose public financing. However, if the manager's optimal transparency choice under public financing is below $\widehat{\phi}$, she faces a trade-off between receiving a higher valuation from financiers (in case of public financing) and being able to optimally set the firm's transparency level (with private

[16] Appendix B offers further extensions, such as allowing for investment delay to reduce uncertainty, allowing for the profitability of the investment opportunity to vary over time, and extending our results to a setting in which the financing friction is an incentive problem à la Holmstrom and Tirole (1997) rather than disagreement.

financing).

The main insight here is that, while the marginal benefit of liquidity is the same for every dollar of external financing, the marginal cost of interference to the manager increases in the amount of external financing she wants to raise. When the manager seeks to raise more external financing, she faces more monitoring and interference, which she would like to partially offset by choosing less transparency. This may not be possible with public financing given its minimum transparency requirement. As a result, the higher the quality of the firm's investment opportunity θ, the bigger the gap between the transparency requirement of public financing and the transparency level preferred by the manager. Thus, as in Proposition 3, we obtain that there is a threshold $\widehat{\theta}_l$, such that the manager prefers public financing if and only if θ is below $\widehat{\theta}_l$.

Proposition 5 *Consider a model extension in which public financing makes reselling equity stakes easier. The manager chooses public financing if and only if $\theta < \widehat{\theta}_l$, and private financing otherwise.*

Discussion: Free Rider Problems A related issue is that financiers in public markets might be passive due to free rider problems. Specifically, if the manager raises equity from multiple financiers, no single investor might have incentives to monitor, or with a large financier being present, that large financier might monitor while others are freeriding. Allowing for such behavior does not change the main insights. Since monitoring and interference increase firm value from the financiers' perspective, the liquidity in public markets would facilitate that passive (small) financiers sell their shares to a (large) financier who does all the monitoring. Thus, the manager would still choose public financing only if interference is less likely to be a big burden.[17]

4.2 Can Cash Hoarding Reveal Growth Prospects?

Sofar, the friction between the outside financier and the manager was limited to disagreement about whether the value of the investment opportunity is $X(\theta)$ or $\rho X(\theta)$. We now also consider the possibility that the manager is better informed about the signal θ. We will show that the manager's hoarding choice mitigates this problem, as it helps convey valuable

[17] These value-increasing trades may fail if small shareholders hold out hoping that the value of their stake appreciates when others sell. There are various ways to deal with this problem (Grossman and Hart, 1980; Holmstrom and Nalebuff, 1992). However, if the problem persists, being able to fine tune how much monitoring she would face (by choosing a particular distribution of shares over investors) would make public financing more attractive for the manager.

information to financiers regarding the firm's growth prospects.[18]

To convey the main idea, we abstract from the transparency choice by assuming again that monitoring is binary, i.e., $E \in \{0, 1\}$, in which case the manager can raise outside financing only if $E = 1$ and her transparency choice is trivial ($\phi \to \infty$). We introduce asymmetric information by making the parameter θ privately known to the manager, but not to financiers. It is common knowledge that θ is drawn from a CDF F on $[\underline{\theta}, \overline{\theta}]$. This gives rise to a game of signaling, in which the manager signals her type through her choice of hoarding.

An equilibrium candidate in pure strategies for the signaling game can be characterized with a triple of functions $(w_\theta^{**}, \mu^*, \alpha)$, where w_θ^{**} is the cash level that a manager of type θ chooses as target for hoarding; μ^* is the financier's posterior belief that maps w_θ^{**} into the set of probability distributions over the type set $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$; $\alpha \in [0, 1]$ is the equity stake offered by the financier in return for funding $K - w_\theta^{**}$. In a competitive market for capital, this stake is such that the financier breaks even for his posterior believes. Our equilibrium concept is that of a Perfect Bayesian Equilibrium.

Summarizing, the manager maximizes (4) subject to the condition that the proposed contract is individually rational for a financier who makes zero profit and who uses Bayes rule on the equilibrium path to form his posterior beliefs μ^* when drawing an inference $\widehat{\theta}$ about the firm's type. We assume pessimistic out-of-equilibrium beliefs that assign probability one to the lowest type if the financiers observe an off-equilibrium hoarding level.

In a separating equilibrium of the resulting game, the proposed contract must be incentive compatible. More formally, suppose that there is a monotonic differentiable function w^{**}, which outside financiers use to infer the manager's type given her choice of investment threshold. Then, if the manager decides to exercise at $\widehat{w} \in w^{**}\left(\left[\underline{\theta}, \overline{\theta}\right]\right)$, outside financiers infer that the type is $\widehat{\theta} = w^{**-1}(\widehat{w})$ and the manager's expected payoff is

$$U(w_t, \widehat{w}, w^{**-1}(\widehat{w}), \theta) = \left(\left(1 - \frac{K - \widehat{w}}{\rho X\left(w^{**-1}(\widehat{w})\right)}\right) X - \kappa\,(1) - \widehat{w}\right) \left(\frac{w_t}{\widehat{w}}\right)^\beta,$$

which generalizes (5). Since the investment decision must be on the optimal path, w_θ^{**} solves:

$$w_\theta^{**} = \arg \max_{\widehat{w} \in w^{**}\left(\left[\underline{\theta}, \overline{\theta}\right]\right)} U(w_t, \widehat{w}, w^{**-1}(\widehat{w}), \theta) \tag{10}$$

where, assuming that a separating equilibrium exists, we evaluate the respective first-order

[18] This extension relates to the work of Grenadier and Malenko (2011) who analyze real options signaling games. Morellec and Schürhoff (2011) and Bouvard (2014) analyze financial contracting and real options financing under asymmetric information.

condition at $w^{**-1}(\widehat{w}) = \theta$. This problem is well-behaved. Lemma A.1 in the Appendix shows that single crossing with respect to cash hoarding holds. Intuitively, while hoarding helps to reduce the dependence on external financing, it is costly (as $\mu < r$) and firms with better investment opportunities face higher costs of delay than firms with worse investment opportunities. At any level of hoarding and for all beliefs $\widehat{\theta}$, a manager with a better investment opportunity would gain more (or lose less) from reducing hoarding. Hence, delaying is most costly for good types.

Consider now the following equilibrium candidate. The lowest type chooses the same hoarding level as under symmetric information, i.e., $w^{**}(\underline{\theta}) = w^{*}(\underline{\theta})$. Intuitively, there is no reason for the lowest type to distort its hoarding policy, given that no type has an incentive to pretend being the lowest type. Higher types choose a hoarding level $w^{**}(\theta) < w^{*}(\theta)$, defined by the first-order condition (10), evaluated at $w^{**-1}(\widehat{w}) = \theta$. The reason the manager needs to distort her hoarding policy downward relative to the case with symmetric information is to avoid being mimicked by lower types. Indeed, the single crossing condition guarantees that such downward deviations can make mimicking prohibitively costly for lower types. We verify in the Appendix that such a separating equilibrium exists and is unique if $\rho \geq \frac{K}{X(\underline{\theta})}$. If the latter condition does not hold, there is still a continuum of semi-separating equilibria in which higher types hoard (weakly) less than lower types.

Proposition 6 *(i) A sufficient condition for a unique fully separating equilibrium is that $\rho \geq \frac{K}{X(\underline{\theta})}$. In this equilibrium, better types separate from lower types by hoarding and delaying investment less. There is less hoarding than under symmetric information: $w^{**}(\theta) \leq w^{*}(\theta)$ with the inequality being strict for all $\theta > \underline{\theta}$. (ii) Regardless of whether $\rho \geq \frac{K}{X(\underline{\theta})}$ holds, there is a continuum of semi-separating equilibria in which higher types hoard (weakly) less than lower types.*

5 Empirical Implications

We conclude with a discussion of the main empirical implications stemming from our model. Our innovation is to ask: If growth firms have investment opportunities present, but not the funds to finance them, will they delay investment and hoard cash to reduce dependence on external financing? And how does this hoarding motive interact with other financial decisions such as the choice between public and private financing? Surprisingly, the literature has overlooked that analyzing hoarding with investment opportunities present (focus on current investment) leads to very different predictions when compared to the much-analyzed case of precautionary hoarding, i.e., hoarding driven by future investment opportunities. As emphasized earlier, we believe that our setting better describes growth firms. For mature firms, precautionary hoarding motivated by anticipated future investment needs may be the more relevant description.

Our starting point is to show that firms with better investment opportunities will hoard less. The intuition is as simple as it is robust: Once investment opportunities have arrived, delaying investment to avoid dilution is costlier if the opportunities are better (Proposition 1).

Implication 1 *Growth firms with better current investment opportunities at hand hoard less, i.e., such firms invest with a higher proportion of external financing, as they want to minimize investment delay.*

Implication 1 provides a sharp contrast with the insights from a precautionary hoarding perspective that firms with better future opportunities hoard more. Thus, our analysis implies that cash hoarding is very much dependent on the firm's life-cycle phase. Growth firms with better investment opportunities follow a low cash strategy in their growth phase (in the sense that they invest with a higher proportion of external financing), even though they might end up cash rich as they mature. This life-cycle pattern finds support in Drobetz et al. (2015).

One of the innovations of our paper is to study the interaction of hoarding with the choice between public and private financing. The key driver for this choice is that public financing dictates a minimum level of transparency, which encourages monitoring and interference. Since firms with substantial external financing (low hoarding) would be particularly exposed to interference, they may choose to partially mitigate this by opting for private financing in combination with less transparency (Proposition 3). With such self-selection, one may observe that private firms are more closely monitored than public firms. However, this is misleading, because a firm choosing private financing would have faced even higher scrutiny with public financing. Hence, taking into account that both hoarding and the public-private choice are endogenous decisions, we show that firms choosing private financing delay current investments less and hoard less (Proposition 3).[19]

Implication 2 *A growth firm in a position to choose between public and private financing (i.e., for which the financing choice is an endogenous decision) delays current investment less and hoards less when choosing private financing.*

[19] Focusing on the endogeneity of the public-private decision means that we are *not* comparing the average public with the average private firm. In particular, note that introducing fixed costs of public financing will imply that public financing will be preferable only for larger issues (Pagano and Röell, 1998).

Implication 2 finds strong support in a recent empirical study by Gao et al. (2013) that explicitly takes into account the endogeneity of the choice between public and private financing. It shows that public firms hoard up to twice as much cash as comparable private firms. Further in line with our theory, Asker et al. (2015) find that private firms not only have less cash, but also react more quickly to new growth opportunities.[20]

In a similar vein, some firms may wish to gain the best of both worlds (Corollary 1) by being public, but choosing private placements when their investment opportunities are better (Gomes and Phillips, 2012; Phillips and Sertsios, 2017). Implication 3 captures this

Implication 3 *For a firm that is already public, choosing private financing (e.g., a private placement) would go hand-in-hand with less hoarding and delay.*

It is important to stress that Implication 2 may look different for mature firms, for which hoarding seeks to address future investment needs, i.e., precautionary hoarding. For such firms, anticipated better future investment opportunities would stimulate hoarding and reduce the subsequent reliance on external financing. This would increase the attractiveness of public financing. Applying our insights to a life-cycle prediction tracking a growth firm to maturity, we expect (following Implication 2) better firms to rely more on external financing in the growth phase which dictates private financing and delaying public financing. Yet in the subsequent more mature phase, precautionary hoarding kicks in, and better firms would be willing to go for public financing.

Implication 4 *Firms with better current investment opportunities wait to go public until they mature, at which stage hoarding is mainly driven by future investment opportunities.*[21]

Another novel insight of our model concerns the interaction of competition, hoarding, and the choice between public and private financing. Consider, first, the effect of competition on hoarding. The prior literature on cash and competition has not focused on investment delay. By allowing for delay of current investment opportunities, the first effect of competition that we consider is that hoarding makes it more likely that the firm loses a first-mover advantage related to such opportunities. We show that this effect would call for speeding up investment and less hoarding, which would favor private financing. However, we also show that competition could have the opposite effect: If securing a first-mover advantage becomes less important, competition will lead to less hoarding and will favor public financing. This

[20] In addition, firms choosing private over public financing are less likely to wait for uncertainty to unravel before investing (see Appendix B.2).

[21] Note that firms with better investment opportunities undertake these opportunities more quickly and, thus, mature faster. Thus, they do not necessarily go public more slowly.

is because, by reducing profitability, competition makes hoarding more attractive (Corollary 2).

Implication 5 *The effects of stronger product market competition are as follows: (i) If having a first-mover advantage is of paramount importance for a growth firm, it prefers private financing, as such financing is more attractive if the firm reacts to competition by lowering hoarding and delaying current investment less. (ii) By contrast, public financing is preferable if product market competition significantly erodes the profitability even for first-movers.*

Implication 5 is consistent with findings documenting a U-shaped relationship between competition and investment delay (Akdogu and MacKay, 2008). In particular, an explanation consistent with our model is that an increase in competition in highly concentrated industries makes securing a first-mover advantage more important, leading to an acceleration of investment. By contrast, first-mover considerations are less relevant in already competitive industries. Thus, an increase in competition mainly leads to a further erosion of profitability and a slowdown in investment. Based on such evidence, *we expect a corresponding U-shaped relation between competition and hoarding*. The differential effect predicted by Implication 5 could further help explain why some empirical studies find that stronger product market competition leads to more public financing (Chod and Lyandres, 2011) while others find the opposite (Chemmanur et al., 2010).[22]

We conclude this section by noting that testing these predictions would require carefully controlling for a number of factors. As discussed in Sections 3.1 and 3.3, empirical tests would have to control for the profitability of the firm's assets in place and the firm's investment scale. Furthermore, recall that our predictions are about the fraction of hoarded cash (relative to external financing) used by the firm to finance any given investment outlay and how this depends on the firm's growth prospects. In particular, these predictions do not easily translate into predictions for cash-to-assets ratios, as, trivially, firms with more investment opportunities would mechanically hoard more cash.[23] Thus, empirical tests of our predictions may want to focus on exogenous shocks affecting the value of already existing investment opportunities. Controlling for the firm's stage of development is also important, as we expect that our predictions will apply less well for mature firms, for which precautionary hoarding might be a better description. A further empirical challenge would be to take into

[22] As discussed at the end of Section 3.3, to the extent that product market competition increases precautionary hoarding, such competition is likely to increase the preference for public financing.

[23] Thus, the evidence that firms with a high Tobin's Q hoard more cash (Opler et al., 1999; Bates et al., 2009) could be consistent with hoarding driven by current or anticipated future investment opportunities (see Implication 1).

account that infrequent balance sheet data might not capture the dynamics of hoarding and building up cash first, and then using the hoarded cash to invest. Furthermore, better growth firms will more quickly make the transition to becoming mature and cash rich as a sign of their success. For such firms the precautionary motive might more quickly become a better description. As emphasized, the hoarding predictions would then be the opposite. Finally, it would be important to consider non-linear effects, in particular for the predicted U-shaped relationship between competition and cash.

6 Conclusion

We develop a theory that analyzes whether a growth firm will choose to delay investments in order to hoard cash and depend less on outside financing. Our perspective is one where investment opportunities are already present, but funding is not. This perspective is of primary importance for growth firms, but surprisingly ignored in the literature, which has largely focused on hoarding in anticipation of a future investment opportunity. The distinction is far from trivial, as the two types of hoarding have very different implications.

In our model, entrepreneurs try to avoid external financing because they are reluctant to see their stake diluted. Our starting point is to show that firms with better investment opportunities hoard less and finance a higher fraction of new investments with outside financing. The key reason is that they find it more costly to delay a more profitable opportunity. By comparison, in a precautionary setting, firms hoarding cash in anticipation of the arrival of future investment opportunities, hoard more when these prospects are better. Thus, the cross-sectional predictions are the opposite. Expanding on this simple insight, we show a number of novel results that question the extent to which standard arguments developed for mature firms (focusing on precautionary hoarding) apply to growth firms that seek to satisfy immediate funding needs for investment opportunities at hand.

One of our main insights is that firms with better opportunities are more likely to go for private financing. The reason is that these firms depend less on internally generated cash and more on external financing, which encourages monitoring and interference by financiers. With private financing, this interference can be managed by lowering transparency, contrary to public financing, which has minimum levels of mandatory transparency.

Another prediction is that product market competition can have opposing effects on hoarding and the choice between public and private financing. One effect is that competition gives firms an incentive to accelerate investment to hold on to their first-mover advantage. This leaves less time for hoarding. However, there is a countervailing effect, which could easily dominate: competition is likely to reduce profitability regardless of whether or not the firm is a first-mover. Investment is then less lucrative, making delay less costly and hoarding more attractive. Combining these insights with our results on public versus private financing, we predict that, when product market competition strongly erodes profits and drives firms to delay and hoard more cash, firms are more likely to raise public financing. Alternatively, if product market competition leads firms to accelerate investment, it will reinforce the benefit of raising private financing

Several extensions of our model yield further insights into how cash hoarding affects the evolution of growth firms. In particular, we show that introducing asymmetric information leads to less hoarding. This is because hoarding conveys a signal about the firm's prospects, which induces firms to choose less hoarding in order to signal better prospects.

Our results further provide insights on the dynamics of firm evolution and cash holdings. Our analysis focuses on growth firms that are short on cash and operate in an uncertain environment. The ones with better investment opportunities will choose to grow rapidly using outside funding, and, relative to their lesser peers, will be cash-poor. However, on average, they will be more profitable and successful. This implies that in the follow-up stage after these firms have established themselves, they may start earning cash at a higher rate than needed for investment and growth. High cash holdings are then a sign of past success. This would imply that growth firms striving to become the next Google, Microsoft, or Apple should not try copying the large cash holdings of these already mature firms; as *growth* firms they should hoard little to realize more quickly current investment opportunities.[24] Another implication is that since firms with better opportunities also invest more rapidly, reinforcing effects are present. The result resembles an accelerated Darwinian survival process with "winners taking it all."

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Appendix A Omitted Proofs

Proof of Proposition 1. Differentiating (5) with respect to $\widehat{w}$ we obtain

$$\left(-\frac{\beta}{\widehat{w}}\left(\left(1-\frac{K-\widehat{w}}{\rho X}\right)X-\kappa\left(1\right)-\widehat{w}\right)+\left(\frac{1}{\rho}-1\right)\right)\left(\frac{w_t}{\widehat{w}}\right)^{\beta}. \tag{A.1}$$

The first term shows the time-value-loss of waiting for w to increase, while the second term shows the benefit from obtaining cheaper financing when increasing the co-investment. Note that since $\beta > 1$, expression (A.1) is negative if $\rho > \frac{X-\kappa(1)}{K}$. In this case, hoarding cash is never optimal. Hence, the manager only hoards cash if the disagreement with the financier is sufficiently strong and $\rho \leq \frac{X-\kappa(1)}{K}$. Then, the first-order condition (FOC) yields:

$$w^{*}=\frac{\beta}{(\beta-1)}\left(\frac{\rho\kappa\left(1\right)+K-\rho X}{1-\rho}\right).$$

For completeness, note that the second order condition is

$$\begin{aligned}&-\frac{\beta}{\widehat{w}^{2}}\frac{\left(\rho\kappa\left(1\right)+K-\rho X\right)}{\rho}\\&-\frac{\beta}{\widehat{w}}\left(-\frac{\beta}{\widehat{w}}\left(\left(1-\frac{K-\widehat{w}}{\rho X}\right)X-\kappa\left(1\right)-\widehat{w}\right)+\left(\frac{1}{\rho}-1\right)\right)\left(\frac{w_t}{\widehat{w}}\right)^{\beta}.\end{aligned} \tag{A.2}$$

At the interior optimum (when the FOC holds), which is the case when $\rho X < K$, the second line is zero, and the expression is negative. Clearly, if the initial cash at hand is $w_0 > w^*$, the manager invests immediately. Observe now that w^* decreases in ρ. Furthermore, w^* decreases in θ, as w^* decreases in X and X increases in θ.

Finally, observe that, since $\kappa(1) > 0$, the manager's expected payoff (5) when hoarding (6) needs to be compared to her payoff when she hoards all of K. That is, the manager hoards w^* if

$$\left(\left(1 - \frac{K - w^*}{\rho X}\right) X - \kappa(1) - w^*\right)\left(\frac{w}{w^*}\right)^\beta \geq (X - K)\left(\frac{w}{K}\right)^\beta,$$

and otherwise hoards K. **Q.E.D.**

Proof of Proposition 2. Using that $\alpha_m^* = \frac{\sqrt{2\phi(K-\widehat{w})}}{\phi\rho X}$ from (8), the manager's payoff in (9) can be stated as

$$U = \left(\left(1 - \frac{\sqrt{2\phi(K - \widehat{w})}}{\phi\rho X}\right) X - \kappa(E) - \widehat{w}\right)\left(\frac{w}{\widehat{w}}\right)^\beta. \tag{A.3}$$

We derive, first, the conditions characterizing the optimal level of hoarding w_m^* and transparency ϕ^* (Step 1). Then, we show that for the so optimally chosen w_m^* and ϕ^*, firms with better growth prospects choose less transparency and less hoarding (Step 2). For use below, observe that $E^*(w, \phi) = \alpha_m \phi\rho X = \sqrt{2\phi(K - w)}$.

Step 1. The optimal level of transparency is given by the first-order condition of (A.3) with respect to ϕ

$$U_\phi = \left(\frac{(K - \widehat{w})}{\rho\phi^{\frac{3}{2}}\sqrt{2(K - \widehat{w})}} - \frac{\partial\kappa(E)}{\partial E}\frac{\sqrt{2(K - \widehat{w})}}{2\sqrt{\phi}}\right)\left(\frac{w}{\widehat{w}}\right)^\beta = 0, \tag{A.4}$$

where the subscripts of U denote partial derivatives. The second-order condition is

$$\begin{aligned} U_{\phi\phi} &= \left(-\frac{3}{2}\frac{(K-w)}{\rho\phi^{\frac{5}{2}}\sqrt{2(K-w)}} - \left(\frac{\partial^2\kappa(E)}{\partial E^2}\left(\frac{\partial E}{\partial\phi}\right)^2 - \frac{\partial\kappa(E)}{\partial E}\frac{1}{4\phi^{\frac{3}{2}}}\sqrt{2(K-w)}\right)\right)\left(\frac{w}{\widehat{w}}\right)^\beta \\ &= \left(-\frac{3}{2}\frac{(K-w)}{\rho\phi^{\frac{5}{2}}\sqrt{2(K-w)}} - \frac{\partial^2\kappa(E)}{\partial E^2}\left(\frac{\partial E}{\partial\phi}\right)^2 + \frac{(K-w)}{2\rho\phi^{\frac{5}{2}}\sqrt{2(K-w)}}\right)\left(\frac{w}{\widehat{w}}\right)^\beta < 0, \end{aligned} \tag{A.5}$$

where the second equality follows after plugging in from the first-order condition (A.4) that at the optimal choice of transparency, we must have $\frac{\partial\kappa(E)}{\partial E} = \frac{1}{\rho\phi}$, and using that $\frac{\partial^2\kappa(E)}{\partial E^2} \geq 0$.

Evaluating the cross-partial $\frac{\partial^2 V_A}{\partial\widehat{w}\partial\phi}$ at the optimal transparency level, $\phi = \phi^*$, we further have

$$U_{\phi\widehat{w}} = -\frac{\beta}{\widehat{w}}\left(\frac{(K-\widehat{w})}{\rho\phi^{\frac{3}{2}}\sqrt{2(K-\widehat{w})}}-\frac{\partial\kappa(E)}{\partial E}\frac{\sqrt{2(K-\widehat{w})}}{2\sqrt{\phi}}\right)\left(\frac{w}{\widehat{w}}\right)^{\beta} \tag{A.6}$$

$$+\left(-\frac{1}{2\phi^{\frac{3}{2}}\rho\sqrt{2(K-w)}}+\frac{\partial^2\kappa(E)}{\partial E^2}\frac{1}{2}+\frac{\partial\kappa(E)}{\partial E}\frac{1}{2\sqrt{2\phi(K-w)}}\right)\left(\frac{w}{\widehat{w}}\right)^{\beta}$$

$$= \left(-\frac{\partial\kappa(E)}{\partial E}\frac{1}{2\sqrt{2\phi(K-w)}}+\frac{\partial^2\kappa(E)}{\partial E^2}\frac{1}{2}+\frac{\partial\kappa(E)}{\partial E}\frac{1}{2\sqrt{2\phi(K-w)}}\right)\left(\frac{w}{\widehat{w}}\right)^{\beta} > 0,$$

where the last equality follows after using that at the optimal choice of transparency ϕ^*, the first line of (A.6) is zero and we must have again that $\frac{\partial\kappa(E)}{\partial E}=\frac{1}{\rho\phi}$. Hence, the manager chooses more transparency when hoarding more.

Similar to Proposition 1, the optimal hoarding level, w^*, is defined by the first-order condition of (9) with respect to $\widehat{w}$

$$\left(\begin{array}{c} -\frac{\beta}{\widehat{w}}\left(\left(1-\frac{\sqrt{2\phi(K-\widehat{w})}}{\phi\rho X}\right)X-\kappa(E)-\widehat{w}\right) \\ +\frac{1}{\rho\sqrt{2\phi(K-\widehat{w})}}+\frac{\partial\kappa(E)}{\partial E}\frac{\phi}{\sqrt{2\phi(K-\widehat{w})}}-1 \end{array}\right)\left(\frac{w}{\widehat{w}}\right)^{\beta}=0. \tag{A.7}$$

The first-order conditions (A.4) and (A.7) define the necessary conditions for local maxima. A sufficient condition is that at these points $U_{\phi\phi}<0$ (which holds by (A.5)), $U_{\widehat{w}\widehat{w}}<0$ and $H\equiv\begin{vmatrix} U_{\widehat{w}\widehat{w}} & U_{\phi\widehat{w}} \\ U_{\widehat{w}\phi} & U_{\phi\phi}\end{vmatrix}=U_{\widehat{w}\widehat{w}}U_{\phi\phi}-U_{\phi\widehat{w}}U_{\widehat{w}\phi}>0$. A limit to our closed-form analysis is that H and $U_{\widehat{w}\widehat{w}}$ cannot be signed in general. However, we have verified numerically that there are wide parameter ranges for which these conditions are satisfied and, thus, for which an interior solution for w^* and ϕ^* exists. In what follows, we limit attention to these cases. The comparative statics when there is no interior solution (i.e., w_m^* is zero or K) are trivial.

Step 2. From the first-order conditions (A.4) and (A.7), we can apply standard comparative statics arguments to obtain that, when interior solutions for w_m^* and ϕ^* exist, then at the optimal hoarding and transparency levels it must hold

$$\frac{dw_m^*}{dX} = \frac{U_{\phi\widehat{w}}U_{\phi X}-U_{\phi\phi}U_{\widehat{w}X}}{H} \tag{A.8}$$

$$\frac{d\phi^*}{dX} = \frac{U_{\phi\widehat{w}}U_{\widehat{w}X}-U_{\widehat{w}\widehat{w}}U_{\phi X}}{H}. \tag{A.9}$$

As noted above, we limit attention to the cases in which $U_{\phi\phi}$, $U_{\widehat{w}\widehat{w}}<0$, and $H>0$. We have shown in (A.6) that $U_{\phi\widehat{w}} > 0$ at w_m^* and ϕ^*. Furthermore

$$U_{\phi X} = \left(-\frac{\partial\kappa^2(E)}{\partial E\partial X}\frac{\sqrt{2(K-w_m^*)}}{2\sqrt{\phi^*}}\right)\left(\frac{w}{w_m^*}\right)^{\beta} \tag{A.10}$$

$$U_{\widehat{w}X} = \left(-\frac{\beta}{\widehat{w}}\left(1-\frac{\partial\kappa(E)}{\partial X}\right)+\frac{\partial^2\kappa(E)}{\partial E\partial X}\frac{\phi}{\sqrt{2\phi(K-\widehat{w})}}\right)\left(\frac{w}{w_m^*}\right)^{\beta}. \tag{A.11}$$

Using that $\frac{\partial\kappa(E)}{\partial X} = \frac{\partial\kappa^2(E)}{\partial E\partial X} = 0$ to sign (A.10) and (A.11), we obtain that both (A.8) and (A.9) are negative, implying that ϕ^* and w_m^* decrease in X. In turn, since X increases in θ, the optimal transparency choice and hoarding level decrease in θ.[25]

Finally, for completeness, observe that from the first-order condition (A.4), we have

$$\phi^* = \frac{1}{(\rho c)^{\frac{2}{3}}(2(K-\widehat{w}))^{\frac{1}{3}}}. \tag{A.12}$$

Hence, the financier's interference is given by $E = \sqrt{2\phi^*(K-w_m^*)} = \left(\frac{2(K-w_m^*)}{\rho c}\right)^{\frac{1}{3}}$, which is decreasing in w_m^*. Since w_m^* decreases in θ, the financier's interference increases in θ (i.e., though a higher θ leads to choosing less transparency, this does not fully offset the increased willingness to monitor and interfere). **Q.E.D.**

Proof of Proposition 3. From Proposition 2, we have that the optimal transparency level $\phi^*(\theta)$ decreases in θ. Hence, there is a value $\widehat{\theta}$, defined by $\phi^*(\theta) = \widehat{\phi}$, such that for values $\theta \geq \widehat{\theta}$ the manager would optimally choose $\phi^* < \widehat{\phi}$, making the minimum transparency level $\widehat{\phi}$ suboptimally high for these types. For values $\theta < \widehat{\theta}$, this level is not binding and the manager prefers public financing for its liquidity benefit. As noted, we endogenize this benefit in Section 4. **Q.E.D.**

Proof of Proposition 4. The proof proceeds in two steps. It starts by showing that if the firm could still enjoy a first-mover advantage, the manager hoards less cash than she would

[25] Our analysis could be extended to the case in which the manager's cost of interference increases in X by assuming that $\kappa(E) = \frac{E^2}{2}cX$. To see this, note that $\frac{\partial^2\kappa(E)}{\partial\phi\partial X} > 0$, so that (A.10) is negative, and that the cross-partial corresponding to (A.11) can be stated as

$$\begin{aligned} U_{\widehat{w}X} &= \frac{1}{X}\left(-\frac{\beta}{\widehat{w}}\left(X-\frac{E^2}{2}cX\right)+\frac{\partial\kappa(E)}{\partial E}\frac{\phi}{\sqrt{2\phi(K-\widehat{w})}}\right) \\ &< \frac{1}{X}\left(-\frac{\beta}{\widehat{w}}(X-\kappa(E))+\frac{\partial\kappa(E)}{\partial E}\frac{\phi}{\sqrt{2\phi(K-\widehat{w})}}+\frac{\beta}{\widehat{w}}\frac{\sqrt{2\phi(K-\widehat{w})}}{\phi\rho}+\frac{1}{\rho\sqrt{2\phi(K-\widehat{w})}}+\beta-1\right)=0 \end{aligned}$$

with the last equality following from the first-order condition (A.7).

hoard if the firm had lost the first-mover advantage (Step 1). Then, it shows that stronger competition exerts a dual effect on hoarding incentives if the firm has not lost yet the first-mover advantage (Step 2). We proceed as before by solving first for the binary monitoring case in which $E = 1$, and extend the analysis to continuous monitoring and interference for the discussion of public and private financing (Corollary 2).

Step 1. Let w_{LM}^* denote the firm's optimal hoarding level if it is a late-mover. In this case, the manager's expected payoff can be derived analogously to Proposition 1 as

$$U_{LM} = \left(\left(1 - \frac{K - w_{LM}^*}{\pi_{LM}\rho X}\right)\pi_{LM}X - \kappa(1) - w_{LM}^*\right)\left(\frac{w}{w_{LM}^*}\right)^{\beta}, \tag{A.13}$$

where for notational simplicity we suppress in what follows the dependence of π_{FM} and π_{LM} on the competition variable λ. By the same arguments as Proposition 1, we have that $w_{LM}^* = \frac{\beta}{\beta-1}\frac{K-\rho(\pi_{LM}X-\kappa(1))}{1-\rho}$.

Let w_{FM}^* be the manager's optimal cash hoarding level while still being a potential first-mover. In what follows, we argue to a contradiction that $w_{FM}^* \leq w_{LM}^*$. Suppose not. If $w_{LM}^* < w_{FM}^*$, we have two cases, depending on whether the manager would optimally invest immediately if the firm loses the first-mover advantage (given the cash she has hoarded by then). Given that the likelihood of this occurring follows an exponential distribution with parameter λ, applying Ito's lemma modified for jump processes we have

$$rU(w) = \mu w U_w(w) + \frac{1}{2}\sigma^2 w^2 U_{ww}(w) + \lambda\left(\overline{U}(w) - U(w)\right). \tag{A.14}$$

If $w_{LM}^* \leq w \leq w_{FM}^*$, the manager's expected payoff takes the form $A_1 w^{\beta_1} + B_1 w^{\beta_2} + C_1 w + D_1$, where β_1 and β_2 are the positive and, respectively, negative root of $(r+\lambda) - \mu y - \frac{1}{2}\sigma^2 y(y-1) = 0$, and where observe that $\frac{\partial}{\partial\lambda}\beta_1 > 0$ implies that $\beta_1 > \beta$. Using the value matching condition $U_{FM_1}(w_{FM}^*) = \left(1 - \frac{K-w_{FM}^*}{\pi_{FM}\rho X}\right)\pi_{FM}X - \kappa(1) - w_{FM}^*$ to derive A_1 and (A.14) to derive C_1 and D_1, the manager's expected payoff when $w_{LM}^* \leq w \leq w_{FM}^*$ is

$$\begin{aligned} U_{FM_1}(w) &= A_1 w^{\beta_1} + B_1 w^{\beta_2} + C_1 w + D_1 \\ &= \begin{pmatrix} \left(1 - \frac{K-w_{FM}^*}{\pi_{FM}\rho X}\right)\pi_{FM}X - \kappa(1) - w_{FM}^* \\ -B_1\left(w_{FM}^*\right)^{\beta_2} - \frac{\lambda}{r+\lambda-\mu}\left(\frac{1}{\rho} - 1\right)w_{FM}^* - \frac{\lambda}{r+\lambda}\frac{\rho(\pi_{LM}X-\kappa(1))-K}{\rho} \end{pmatrix}\left(\frac{w}{w_{FM}^*}\right)^{\beta_1} \\ &\quad + B_1 w^{\beta_2} + \frac{\lambda}{r+\lambda-\mu}\left(\frac{1}{\rho} - 1\right)w + \frac{\lambda}{r+\lambda}\left(\frac{\rho(\pi_{LM}X - \kappa(1)) - K}{\rho}\right). \end{aligned}$$

If, instead, $w \leq w_{LM}^* \leq w_{FM}^*$, then the manager's expected payoff takes the form $A_2 w^{\beta_1} + B_2 w^{\beta_2} + C_2 w^{\beta}$. Since the option value of investing is zero for $w \to 0$, we must have $B_2 = 0$.

Furthermore, using (A.14) to derive C_2, we have

$$U_{FM_2}(w) = A_2 w^{\beta_1} + \left(\left(1 - \frac{K - w_{LM}^*}{\pi_{LM}\rho X}\right)\pi_{LM} X - \kappa(1) - w_{LM}^*\right)\left(\frac{w}{w_{LM}^*}\right)^{\beta}.$$

Finally, we can obtain A_2 and B_1 from the value matching condition $U_{FM_1}(w_{LM}^*) = U_{FM_2}(w_{LM}^*)$, and the smooth pasting condition $\frac{\partial}{\partial w}U_{FM_1}(w)|_{w=w_{LM}^*} = \frac{\partial}{\partial w}U_{FM_2}(w)|_{w=w_{LM}^*}$. In particular, expressing A_2 from the value matching condition and plugging it into the smooth pasting condition, we obtain B_1 after some reformulations as

$$B_1 = \frac{1}{\beta_1 - \beta_2}\left(-\beta_1 \frac{\beta - 1}{\beta}\frac{r}{r+\lambda} + (\beta_1 - 1)\left(\frac{r-\mu}{r+\lambda-\mu}\right)\right)\left(\frac{1}{\rho} - 1\right)\frac{1}{(w_{LM}^*)^{\beta_2 - 1}}.$$

Suppose now that $w_{LM}^* \leq w \leq w_{FM}^*$. Taking the FOC of $U_{FM_1}(w)$ with respect to w_{FM}^*, we obtain

$$\begin{aligned}
0 &= -\frac{\beta_1}{w_{FM}^*}\left(-(K - \rho(\pi_{FM}X - \kappa(1))) + \frac{\lambda}{r+\lambda}(K - \rho(\pi_{LM}X - \kappa(1)))\right)\frac{1}{\rho} \\
&\quad + \frac{\beta_1 - \beta_2}{w_{FM}^*}B_1 (w_{FM}^*)^{\beta_2} - (\beta_1 - 1)\frac{r-\mu}{r+\lambda-\mu}\left(\frac{1}{\rho} - 1\right) \\
&\leq \frac{\beta_1}{w_{FM}^*}\frac{r}{r+\lambda}(K - \rho(\pi_{FM}X - \kappa(1)))\frac{1}{\rho} + \frac{\beta_1 - \beta_2}{w_{FM}^*}B_1 (w_{FM}^*)^{\beta_2} - (\beta_1 - 1)\frac{r-\mu}{r+\lambda-\mu}\left(\frac{1}{\rho} - 1\right) \\
&= \frac{1-\rho}{\rho}\left(\begin{array}{c} \frac{w_{LM}^*}{w_{FM}^*}\left(\beta_1 \frac{r}{r+\lambda}\frac{\beta-1}{\beta} + \left(-\beta_1 \frac{\beta-1}{\beta}\frac{r}{r+\lambda} + (\beta_1 - 1)\left(\frac{r-\mu}{r+\lambda-\mu}\right)\right)\left(\frac{w_{FM}^*}{w_{LM}^*}\right)^{\beta_2}\right) \\ -(\beta_1 - 1)\frac{r-\mu}{r+\lambda-\mu} \end{array}\right) \\
&< \frac{1-\rho}{\rho}\frac{w_{LM}^*}{w_{FM}^*}\left(\beta_1 \frac{r}{r+\lambda}\frac{\beta-1}{\beta} - (\beta_1 - 1)\frac{r-\mu}{r+\lambda-\mu}\right)\left(1 - \left(\frac{w_{FM}^*}{w_{LM}^*}\right)^{\beta_2}\right) < 0
\end{aligned}$$

where the first inequality (in the second line) follows from $\pi_{LM} \leq \pi_{FM}$; the second equality (in the third line) follows after plugging in for B_1 and using that from $w_{LM}^* = \frac{\beta}{\beta-1}\frac{K - \rho(\pi_{FM}X - \kappa(1))}{1-\rho}$, we can replace $K - \rho(\pi_{LM}X - \kappa(1))$ with $w_{LM}^*(1-\rho)\frac{\beta-1}{\beta}$; the second inequality (in the fourth line) follows from the contradiction assumption that $w_{FM}^* > w_{LM}^*$; the last inequality follows from the fact that $\beta_1 \frac{\beta-1}{\beta}\frac{r}{r+\lambda} - (\beta_1 - 1)\frac{r-\mu}{r+\lambda-\mu}$ is negative (as it is zero for $\sigma = 0$ and decreasing in σ) and $1 > \left(\frac{w_{FM}^*}{w_{LM}^*}\right)^{\beta_2}$ (as $\beta_2 < 0$). Thus, we obtain a contradiction, so we must have that $w_{FM}^* \leq w_{LM}^*$.

Step 2. We now analyze the effect of λ on w_{FM}^*. Since $w \leq w_{FM}^* \leq w_{LM}^*$, the manager's expected payoff takes the form $Aw^{\beta_1} + Bw^{\beta_2} + Cw^{\beta}$. We have again $B = 0$, since the option to invest is zero for $w \to 0$, and it can be verified that

$$U_{FM} = U_{LM}(w, w_{LM}^*) + \left(\left(1 - \frac{K - w_{FM}^*}{\pi_{FM}\rho X}\right)\pi_{FM}X - \kappa(1) - w_{FM}^* - U_{LM}(w_{FM}^*, w_{LM}^*)\right)\left(\frac{w}{w_{FM}^*}\right)^{\beta_1}$$

where plugging in from (A.13), w_{FM}^{*} is the solution to the first-order condition

$$\begin{aligned} 0 &= -\frac{\beta_1}{\widehat{w}_{FM}}\left(\left(1-\frac{K-\widehat{w}_{FM}}{\pi_{FM}\rho X}\right)\pi_{FM}X-\kappa(1)-\widehat{w}_{FM}-U_{LM}\left(\widehat{w}_{FM},w_{LM}^{*}\right)\right)\left(\frac{w}{\widehat{w}_{FM}}\right)^{\beta_1} \\ &\quad +\left(\frac{1}{\rho}-1-\frac{\beta}{\widehat{w}_{FM}}U_{LM}\left(\widehat{w}_{FM},w_{LM}^{*}\right)\right)\left(\frac{w}{\widehat{w}_{FM}}\right)^{\beta_1} \end{aligned}$$

By standard monotone comparative statics arguments, the effect of an increase in competition λ on hoarding is given by the sign of the cross partial $\frac{\partial^2 U_{FM_1}}{\partial \widehat{w}_{FM}\partial\lambda}$, which after some transformations becomes

$$\begin{aligned} &-\frac{1}{\widehat{w}_{FM}}\frac{\partial\beta_1}{\partial\lambda}\left(\left(1-\frac{K-\widehat{w}_{FM}}{\pi_{FM}\rho X}\right)\pi_{FM}X-\kappa(1)-\widehat{w}_{FM}-U_{LM}\left(\widehat{w}_{FM},w_{LM}^{*}\right)\right)\left(\frac{\widehat{w}_{FM}}{w_{LM}^{*}}\right)^{\beta_1} \\ &+\frac{\beta_1}{\widehat{w}_{FM}}\left(-\frac{\partial\pi_{FM}}{\partial\lambda}+\frac{\beta_1-\beta}{\beta_1}\frac{\partial\pi_{LM}}{\partial\lambda}\left(\frac{\widehat{w}_{FM}}{w_{LM}^{*}}\right)^{\beta}\right)X\left(\frac{w}{\widehat{w}_{FM}}\right)^{\beta_1}. \end{aligned} \tag{A.15}$$

The first line of (A.15) is negative, as the delay associated with hoarding reduces the likelihood of being a first-mover. For any given $\widehat{w}_{FM}$, the first line of (A.15) is lower if π_{LM} (and so U_{LM}) is lower. Furthermore the term $\frac{\beta_1-\beta}{\beta_1}\frac{\partial\pi_{LM}}{\partial\lambda}\left(\frac{w}{w_{LM}^{*}}\right)^{\beta}$ in the second line of (A.15) is also negative. This term captures that the reduced late-mover profitability increases the loss of not being a first-mover and puts further pressure to reduce hoarding. To see this, note that the cross partial $\frac{\partial^2 U_{FM_1}}{\partial\widehat{w}_{FM}\partial\pi_{LM}}=\frac{\beta_1-\beta}{\beta_1}\left(\frac{w}{w_{LM}^{*}}\right)^{\beta}>0$ captures the effect of losing the first mover advantage on hoarding (lower π_{LM} leads to more hoarding) and that $\frac{\partial\pi}{\partial\lambda}\leq 0$. Together, this two effects comprise the first-mover benefit effect. The lower π_{LM} and $\frac{\partial\pi_{LM}}{\partial\lambda}$, the stronger is the first-mover benefit effect.

The first-mover erosion effect is captured by the term $-\frac{\partial\pi_{FM}}{\partial\lambda}\geq 0$ in the second line. This effect is positive and creates incentives to increase hoarding. This effect works in the opposite direction to $\frac{\partial\pi_{LM}}{\partial\lambda}\leq 0$ in the second line, and is strengthened by the fact that $\frac{\beta_1-\beta}{\beta_1}$ is less than one. Summarizing, the first-mover benefit effect dominates if π_{LM} and $\frac{\partial}{\partial\lambda}(\pi_{LM})$ are sufficiently low to make (A.15) negative. Then, cash hoarding decreases in competition. Otherwise, if the first-mover erosion effect ($-\frac{\partial\pi_{FM}}{\partial\lambda}\geq 0$) dominates, competition increases hoarding. **Q.E.D.**

Proof of Corollary 2. Suppose now that E is a continuous choice. In analogy to the proof of Proposition 2, the manager's expected payoff as a late-mover is

$$U_{LM}=\left(\left(1-\frac{\sqrt{2\phi_{LM}\left(K-\widehat{w}_{LM}\right)}}{\phi_{LM}\pi_{LM}\rho X}\right)\pi_{LM}X-\kappa\left(E_{LM}\right)-w_{LM}^{*}\right)\left(\frac{w}{w_{LM}^{*}}\right)^{\beta}.$$

Furthermore, based on the proof of Proposition 4, one can verify that the manager's expected payoffs as a first-mover is given by[26]

$$U_{FM}=U_{LM}\left(w,w_{LM}^{*}\right)+\left(\begin{array}{c}\left(1-\frac{\sqrt{2\phi_{FM}(K-\widehat{w}_{FM})}}{\phi_{FM}\rho\pi_{FM}X}\right)\pi_{FM}X\\ -\kappa\left(E_{FM}\right)-\widehat{w}_{FM}-U_{LM}\left(\widehat{w}_{FM},w_{LM}^{*}\right)\end{array}\right)\left(\frac{w}{\widehat{w}_{FM}}\right)^{\beta}.$$

Further analogous to Proposition 2, the sign of $\frac{\partial\phi_{LM}^{*}}{\partial\lambda}$ is the same as that of $\frac{\partial^{2}U_{LM}}{\partial\phi_{LM}\partial\widehat{w}_{LM}}\frac{\partial^{2}U_{LM}}{\partial\widehat{w}_{LM}\partial\lambda}$, and the sign of $\frac{\partial\phi_{FM}^{*}}{\partial\lambda}$ is the same as that of $\frac{\partial^{2}U_{FM}}{\partial\phi_{FM}\partial\widehat{w}_{FM}}\frac{\partial^{2}U_{FM}}{\partial\widehat{w}_{FM}\partial\lambda}$ (see (A.9)), with $\frac{\partial^{2}U_{LM}}{\partial\phi_{LM}\partial\widehat{w}_{LM}}>0$ and $\frac{\partial^{2}U_{FM}}{\partial\phi_{FM}\partial\widehat{w}_{FM}}>0$ (see (A.6)). Since $\frac{\partial^{2}U_{LM}}{\partial w_{LM}\partial\lambda}>0$, we have $\frac{\partial\phi_{LM}^{*}}{\partial\lambda}>0$, i.e., competition leads late-movers to choose more transparency. However, if the firm can still be a first-mover, then, just as in (A.15), the sign of $\frac{\partial^{2}U_{FM}}{\partial w_{FM}\partial\lambda}$

$$\begin{aligned}&-\frac{1}{\widehat{w}_{FM}}\frac{\partial\beta_{1}}{\partial\lambda}\left(\left(1-\frac{\sqrt{2\phi_{FM}\left(K-\widehat{w}_{FM}\right)}}{\phi_{FM}\rho\pi_{FM}X}\right)\pi_{FM}X-\kappa\left(E_{FM}\right)-\widehat{w}_{FM}-U_{LM}\left(\widehat{w}_{FM},w_{LM}^{*}\right)\right)\left(\frac{w}{\widehat{w}_{FM}}\right)^{\beta_{1}}\\&+\frac{\beta_{1}}{\widehat{w}_{FM}}\left(-\frac{\partial\pi_{FM}}{\partial\lambda}X+\frac{\beta_{1}-\beta}{\beta_{1}}\frac{\partial\pi_{LM}}{\partial\lambda}X\left(\frac{w_{FM}^{*}}{w_{LM}^{*}}\right)^{\beta}\right)\left(\frac{w}{\widehat{w}_{FM}}\right)^{\beta_{1}}.\end{aligned}$$

depends on the importance of having a first-mover advantage. If the first-mover benefit effect is dominated by the first-mover erosion effect (i.e., $\frac{\partial^{2}U_{FM}}{\partial\widehat{w}_{FM}\partial\lambda}>0$), we have $\frac{\partial\phi_{FM}^{*}}{\partial\lambda}>0$. Then, stronger competition makes higher transparency optimal. Relating to the cutoff $\widehat{\theta}$, implicitly defined by $\phi^{*}(\widehat{\theta})=\widehat{\phi}$, this implies that $\widehat{\theta}$ increases in λ, i.e., public ownership becomes more attractive for a wider range of θ. By contrast, if the first-mover benefit effect dominates (i.e., $\frac{\partial^{2}U_{FM}}{\partial\widehat{w}_{FM}\partial\lambda}<0$), we have $\frac{\partial\phi_{FM}^{*}}{\partial\lambda}<0$. Then, $\widehat{\theta}$ decreases in λ, i.e., stronger competition makes private ownership more attractive for a wider range of θ. **Q.E.D.**

Proof of Proposition 5. Let $p_{pub}=\left(1-q+q\zeta_{pub}\right)$ and $p_{priv}:=\left(1-q+q\zeta_{priv}\right)$. Since $\zeta_{pub}=1$, we have that $p_{pub}=1$ under public financing. The manager's expected payoffs

[26] Note that we have again that $w_{FM}^{*}\leq w_{LM}^{*}$, with w_{LM}^{*} decreasing in π_{LM} ($\frac{\partial^{2}U_{LM}}{\partial w_{LM}^{*}\partial\pi_{LM}}<0$), while w_{FM}^{*} is increasing in π_{LM} ($\frac{\partial^{2}U_{FM}}{\partial w_{FM}^{*}\partial\pi_{LM}}>0$), and $U_{FM}=U_{LM}$ and $w_{FM}^{*}=w_{LM}^{*}$ if $\pi_{LM}=\pi_{FM}$ (i.e., if there is no first-mover advantage).

under public and private financing, respectively, are

$$U^{pub} = \left(\left(1-\frac{\sqrt{2\phi^*_{pub}\left(K-w^*_{pub}\right)}}{\phi^*_{pub}\rho X}\right)X-\phi^*_{pub}\left(K-w^*_{pub}\right)c-w^*_{pub}\right)\left(\frac{w}{w^*_{pub}}\right)^{\beta} \quad \text{(A.16)}$$

$$U^{priv} = \left(\left(1-\frac{\sqrt{2\phi^*_{priv}\left(K-w^*_{priv}\right)}}{\phi^*_{priv}\rho p_{priv}X}\right)X-\phi^*_{priv}\left(K-w^*_{priv}\right)c-w^*_{priv}\right)\left(\frac{w}{w^*_{priv}}\right)^{\beta} \quad \text{(A.17)}$$

where we plug in for $\kappa(E)=\frac{E^2}{2}c=\phi(K-w)c$; and where w^*_{pub}, w^*_{priv}, ϕ^*_{pub}, and ϕ^*_{priv} are the optimal hoarding and transparency levels under public and private financing. In analogy to Proposition 2, these levels decrease in θ, with public ownership involving the additional restriction that $\phi^*_{pub}\geq\widehat{\phi}$. Applying the Envelope theorem, we see immediately that

$$\frac{\partial}{\partial\theta}\left(U^{pub}-U^{priv}\right)=\frac{\partial X}{\partial\theta}\left(\left(\frac{w}{w^*_{pub}}\right)^{\beta}-\left(\frac{w}{w^*_{priv}}\right)^{\beta}\right) \quad \text{(A.18)}$$

implying that the sign of (A.18) is the same as that of $w^*_{priv}-w^*_{pub}$. Furthermore, since the manager's expected payoff increases in p, public financing (with $p=1$) is always preferable if $\phi^*_{pub}\geq\widehat{\phi}$ is not binding. In what follows, we abstract from the extreme cases in which either only public or only private financing is optimal for all values of θ.

Since ϕ^*_{pub} decreases in θ (see Proposition 2), $\phi^*_{pub}\geq\widehat{\phi}$ is not binding and, thus, public ownership is preferable for low values of θ. Let $\widehat{\theta}_l$ be a value of θ for which the preference changes from public to private financing. At this point, it must hold that $\frac{\partial}{\partial\theta}\left(U^{pub}-U^{priv}\right)<0$. Thus, from expression (A.18), we must have that $w^*_{priv}<w^*_{pub}$.

Finally, we show that $w^*_{priv}<w^*_{pub}$ whenever the manager chooses private financing. Suppose not. Take the lowest value $\theta'>\widehat{\theta}_l$, for which an increase in θ changes the manager's preference from private to public financing. From expression (A.18), for θ' it must hold $w^*_{priv}>w^*_{pub}$. Hence, for all $\theta\in(\widehat{\theta}_l,\theta')$, we have $U^{priv}\geq U^{pub}$ (by construction), but for some θ in this interval, we have $w^*_{priv}<w^*_{pub}$, while for others $w^*_{priv}\geq w^*_{pub}$. In particular, by (upper hemi-) continuity of w^*_{priv} and w^*_{pub}, there is a value $\theta''\in(\widehat{\theta}_l,\theta')$ for which $w^*_{priv}=w^*_{pub}$. At this value θ'', the inequality $0\leq U^{priv}(w,w^*_{priv})-U^{pub}(w,w^*_{priv})$ can be stated as

$$0\leq\left(K-w^*_{priv}\right)\left(\frac{2}{\rho\sqrt{2\left(K-w^*\right)}}\left(\frac{1}{\sqrt{\widehat{\phi}}}-\frac{1}{p_{priv}\sqrt{\phi^*_{priv}}}\right)+\left(\widehat{\phi}-\phi^*_{priv}\right)c\right)\left(\frac{w}{w^*_{priv}}\right)^{\beta}, \quad \text{(A.19)}$$

where note that, whenever $U^{priv}\geq U^{pub}$, $\widehat{\phi}$ must be binding in case of public financing. In fact, we must also have $\widehat{\phi}\geq\phi^*_{priv}$. Suppose not and $\phi^*_{priv}>\widehat{\phi}$. However, then for the transparency level ϕ^*_{priv}, the manager would not be constrained by the minimum requirements of public financing, but would effectively receive a higher valuation for the firm (cf. (A.16) and (A.17)), implying also a higher payoff than with private financing. This contradicts that $U^{priv} > U^{pub}$ for $\theta \in (\widehat{\theta}_l, \theta'')$.

The derivatives of U^{priv} and U^{pub} with respect to the optimal hoarding level, evaluated at w^*_{priv}, are

$$0 = -\frac{\beta}{w^*_{priv}}\left(\left(1-\frac{\sqrt{2\left(K-w^*_{priv}\right)}}{\sqrt{\widehat{\phi}}\rho X}\right)X-\widehat{\phi}\left(K-w^*_{priv}\right)c-w^*_{priv}\right) + \frac{1}{\rho\sqrt{2\widehat{\phi}\left(K-w^*_{priv}\right)}}+\widehat{\phi}c-1 \tag{A.20}$$

$$0 = -\frac{\beta}{w^*_{priv}}\left(\left(1-\frac{\sqrt{2\left(K-w^*_{priv}\right)}}{\sqrt{\phi^*_{priv}}\rho p_{priv}X}\right)X-\phi^*_{priv}\left(K-w^*_{priv}\right)c-\widehat{w}\right) + \frac{1}{p_{priv}\rho\sqrt{2\phi^*_{priv}\left(K-w^*_{priv}\right)}}+\phi^*_{priv}c-1 \tag{A.21}$$

Subtracting (A.21) from (A.20), we obtain

$$0 = \left(K-w^*_{priv}\right)\frac{\beta}{w^*_{priv}}\left(\frac{2}{\sqrt{2\left(K-w^*_{priv}\right)}\rho}\left(\frac{1}{\sqrt{\widehat{\phi}}}-\frac{1}{p_{priv}\sqrt{\phi^*_{priv}}}\right)+\left(\widehat{\phi}-\phi^*_{priv}\right)c\right) + \frac{1}{\rho\sqrt{2\widehat{\phi}\left(K-w^*_{priv}\right)}}\left(\frac{1}{\sqrt{\widehat{\phi}}}-\frac{1}{p_{priv}\sqrt{\phi^*_{priv}}}\right)+\left(\widehat{\phi}-\phi^*_{priv}\right)c \tag{A.22}$$

Together, conditions (A.19) and (A.22), can only be satisfied if

$$\frac{2}{\rho\sqrt{2\left(K-w^*_{priv}\right)}}\left(\frac{1}{\sqrt{\widehat{\phi}}}-\frac{1}{p_{priv}\sqrt{\phi^*_{priv}}}\right)+\left(\widehat{\phi}-\phi^*_{priv}\right)c \geq 0 \geq \frac{1}{\rho\sqrt{2\left(K-w^*_{priv}\right)}}\left(\frac{1}{\sqrt{\widehat{\phi}}}-\frac{1}{p_{priv}\sqrt{\phi^*_{priv}}}\right)+\left(\widehat{\phi}-\phi^*_{priv}\right)c.$$

These inequalities require that $p_{priv}\sqrt{\phi^*_{priv}} > \sqrt{\widehat{\phi}}$, contradicting $\widehat{\phi} \geq \phi^*_{priv}$, which we established above. **Q.E.D.**

Before proving Proposition 6, we start by showing a useful result.

Lemma A.1 *Single crossing holds because*

$$\frac{\partial}{\partial\theta}\left(-\frac{\frac{\partial}{\partial\widehat{w}}U\left(w_t,\widehat{w},\widehat{\theta},\theta\right)}{\frac{\partial}{\partial\widehat{\theta}}U\left(w_t,\widehat{w},\widehat{\theta},\theta\right)}\right)>0, \tag{A.23}$$

where $\widehat{\theta}$ is the financier's inference about the firm's type θ.

Proof of Lemma A.1. Plugging into the LHS of Expression (A.23), we obtain

$$\begin{aligned}&\frac{\partial}{\partial\theta}\left(-\frac{\left(-\frac{\beta}{\widehat{w}}\left(\left(1-\frac{K-\widehat{w}}{\rho X(\widehat{\theta})}\right)X\left(\theta\right)-\kappa\left(1\right)-\widehat{w}\right)+\frac{X(\theta)}{\rho X(\widehat{\theta})}-1\right)\left(\frac{w_t}{\widehat{w}}\right)^{\beta}}{\frac{K-\widehat{w}}{\left(\rho X(\widehat{\theta})\right)^2}\rho\frac{\partial}{\partial\widehat{\theta}}X(\widehat{\theta})X\left(\theta\right)\left(\frac{w_t}{\widehat{w}}\right)^{\beta}}\right)\\ =\;&\frac{\frac{\partial}{\partial\theta}X\left(\theta\right)\left(w\left(\beta-1\right)+\kappa\left(1\right)\beta\right)}{w\frac{K-\widehat{w}}{\left(\rho X(\widehat{\theta})\right)^2}\rho\frac{\partial}{\partial\widehat{\theta}}X(\widehat{\theta})\left(X\left(\theta\right)\right)^2}>0.\end{aligned} \tag{A.24}$$

Q.E.D.

Proof of Proposition 6. Step 1: *Characterizing fully separating equilibria*

To show existence of a separating equilibrium, we follow standard arguments. Rewriting (10), we obtain

$$w_{\theta}^{**}=\arg\max_{\widehat{w}\in w^{**}\left(\left[\underline{\theta},\overline{\theta}\right]\right)}\left(\left(1-\frac{K-\widehat{w}}{\rho X(w^{**-1}(\widehat{w}))}\right)X\left(\theta\right)-\kappa\left(1\right)-\widehat{w}\right)\left(\frac{w_t}{\widehat{w}}\right)^{\beta}.$$

Taking the FOC and assuming that a separating equilibrium exists, i.e., $w^{**-1}(\widehat{w})=\theta$, we have

$$\frac{dw_{\theta}^{**}}{d\widehat{\theta}}=\frac{-\frac{\partial}{\partial\widehat{\theta}}U(w_t,w_{\widehat{\theta}}^{**},\widehat{\theta},\theta)|_{\widehat{\theta}=\theta}}{-\frac{\beta}{w_{\theta}^{**}}\left(\left(1-\frac{K-w_{\theta}^{**}}{\rho X(\theta)}\right)X\left(\theta\right)-\kappa\left(1\right)-w_{\theta}^{**}\right)\left(\frac{w_t}{w_{\theta}^{**}}\right)^{\beta}+\left(\frac{X(\theta)}{\rho X(\theta)}-1\right)\left(\frac{w_t}{w_{\theta}^{**}}\right)^{\beta}}. \tag{A.25}$$

To solve this equation we need the appropriate boundary condition. Since no type has an incentive to mimic the lowest type, we can set: $w_{\underline{\theta}}^{**}=w_{\underline{\theta}}^{*}$, where $w_{\underline{\theta}}^{*}$ is obtained from expression (6) for $\theta=\underline{\theta}$. For all out-of-equilibrium hoarding levels $w\notin\left[w^{**}\left(\overline{\theta}\right),w^{*}\left(\underline{\theta}\right)\right]$, we stipulate that the financier assigns probability one to the lowest type $\underline{\theta}$. We verify below when the conditions for Theorems 1-3 in Mailath (1987) are satisfied. If these conditions are satisfied, there is a unique (up to the out-of-equilibrium beliefs) separating equilibrium in which w_θ^{**} is continuous and differentiable, satisfies (A.25), and $\frac{dw_\theta^{**}}{d\theta} < 0$ ($\frac{dw_\theta^{**}}{d\theta}$ has the same sign as $\frac{\partial^2}{\partial \widehat{w} \partial \theta} U(w_t, \widehat{w}, \widehat{\theta}, \theta)$). We now show that $w_\theta^{**} < w_\theta^*$ in such a separating equilibrium. To see this, rewrite (A.25) as

$$\begin{aligned} & -\frac{\beta}{w_\theta^{**}} \left(\left(1 - \frac{K - w_\theta^{**}}{\rho X(\theta)} \right) X(\theta) - \kappa(1) - w_\theta^{**} \right) \left(\frac{w_t}{w_\theta^{**}} \right)^\beta + \left(\frac{X(\theta)}{\rho X(\theta)} - 1 \right) \left(\frac{w_t}{w_\theta^{**}} \right)^\beta \\ = & \frac{-\frac{\partial}{\partial \widehat{\theta}} U(w_t, w_\theta^{**}, \widehat{\theta}, \theta)}{\frac{dw_\theta^{**}}{d\widehat{\theta}}}. \end{aligned} \tag{A.26}$$

Compare (A.26) to the optimality condition (A.1) in Proposition 1. The RHS of (A.26) is positive, while it is zero absent information asymmetry. Thus, taking into account that the LHS decreases in w_θ^{**}, we must have $w_\theta^{**} < w_\theta^*$.

Step 2: *Verifying Mailath's (1987) conditions*

Mailath's (1987) regulatory conditions are:

1) Smoothness: $U(w_t, \widehat{w}, \widehat{\theta}, \theta)$ is twice continuously differentiable.

2) Belief monotonicity: $\frac{\partial}{\partial \widehat{\theta}} U(w_t, \widehat{w}, \widehat{\theta}, \theta)$ is either strictly positive or strictly negative.

3) Type monotonicity: $\frac{\partial^2}{\partial \theta \partial \widehat{w}} U(w_t, \widehat{w}, \widehat{\theta}, \theta)$ is either strictly positive or strictly negative.

4) Strict quasiconcavity: $\frac{\partial}{\partial \widehat{w}} U(w_t, \widehat{w}, \theta, \theta) = 0$ has a unique solution in w that maximizes $U(\cdot)\,|_{\widehat{\theta}=\theta}$, and $\frac{\partial^2}{\partial \widehat{w}^2} U(\cdot)\,|_{\widehat{\theta}=\theta} < 0$ at this solution.

5) Boundedness: There is $k > 0$ such that for all $(\theta,\ w) \in \left[\underline{\theta}, \overline{\theta}\right] \times [0, K]$, $\frac{\partial^2}{\partial \widehat{w}^2} U(\cdot)\,|_{\widehat{\theta}=\theta} \geq 0$ implies $\left| \frac{\partial}{\partial \widehat{w}} U(\cdot)_{\widehat{\theta}=\theta} \right| > k$. Note that we restrict attention to $w \in [0, K]$, as the manager has no need of external financing if $w > K$.

Conditions 1)-2) are satisfied. Proposition 1 shows that condition 4) is also satisfied. To check for condition 5), observe that if $\frac{\partial^2}{\partial \widehat{w}^2} U(\cdot)\,|_{\widehat{\theta}=\theta} \geq 0$, then since the first line of the second-order condition (A.2) is negative, we must have $\frac{\partial}{\partial \widehat{w}} U(\cdot)\,|_{\widehat{\theta}=\theta} < 0$. Thus, we can find a k that satisfies condition 5.

Finally, we check when $\frac{\partial^2}{\partial \widehat{w} \partial \theta} U(\cdot) < 0$ holds (i.e., condition 3). We have

$$\frac{\partial^2}{\partial \widehat{w} \partial \theta} U(w_t, \widehat{w}, \widehat{\theta}, \theta) = \left(-\frac{\beta}{\widehat{w}} \left(\rho X(\widehat{\theta}) - K \right) + 1 - \beta \right) \frac{\frac{\partial}{\partial \theta} X(\theta)}{\rho X(\widehat{\theta})} \left(\frac{w_t}{\widehat{w}} \right)^\beta,$$

Since $w^{**}(\theta)$ is lowest for type $\overline{\theta}$, while X is lowest for $\underline{\theta}$, it would be sufficient that

$$w^{**}\left(\overline{\theta}\right) > \underline{w} := \frac{\beta}{\beta - 1} \left(K - \rho X(\underline{\theta}) \right), \tag{A.27}$$

which is always satisfied if $\rho X(\underline{\theta}) \geq K$. Note that since the financier attributes a deviation to a lower cash hoarding strategy to the lowest type, a deviation is unprofitable for all types.

Step 3: *semi-separating Equilibria*

If, instead, $\rho X(\underline{\theta}) < K$ and if the fully separating equilibrium cannot be supported (note that (A.27) is sufficient, but not necessary), we can still construct a continuum of semi-separating equilibria in which hoarding (weakly) decreases in the manager's type. In what follows, we briefly sketch one such equilibrium: (i) types $(\theta', \overline{\theta}]$ pool at $w_P = \frac{\beta}{\beta-1}\left(K - \rho X(\underline{\theta})\right) + \varepsilon$ (where $\varepsilon > 0$); (ii) types $[\underline{\theta}, \theta']$ separate with a hoarding level $w^{**}(\theta) > w_P$ defined by (10), and where θ' is implicitly defined by

$$\left(\left(1 - \frac{K - w^{**}(\theta')}{\rho X(\theta')}\right) X(\theta') - \kappa(1) - w^{**}(\theta')\right)\left(\frac{w_t}{w^{**}(\theta')}\right)^{\beta} \tag{A.28}$$

$$= \left(\left(1 - \frac{K - w_P}{\rho \int_{\theta'}^{\overline{\theta}} X(\theta) \frac{dF(\theta)}{1-F(\theta')}}\right) X(\theta') - \kappa(1) - w_P\right)\left(\frac{w_t}{w_P}\right)^{\beta}.$$

Note that for all types $\theta > \theta'$, the RHS of expression (A.28) would be larger than its LHS. Furthermore, conditions 1)–5) are satisfied now by construction for all equilibrium hoarding levels. Thus, together with single crossing (Lemma A.1), no type would find it optimal to deviate to the equilibrium hoarding strategy of a different type. (iii) Assuming that the financier places probability one on the deviation coming from the lowest type if the hoarding choice is different from $w_P \cup [w^{**}(\theta'), w^{**}(\underline{\theta})]$ guarantees that there are also no deviations to off-equilibrium hoarding strategies. Thus, the equilibrium can be supported, and it features equilibrium hoarding levels that (weakly) decrease in the manager's type. By varying ε, we obtain a continuum of semi-separating equilibria. **Q.E.D.**

Appendix B Supplementary Material

B.1 Precautionary Hoarding

Assume that the time until the arrival of an investment opportunity follows an exponential distribution with parameter λ_a. It holds:

Proposition B.1 *The attractiveness of the firm's investment opportunity has opposite implications for hoarding depending on whether hoarding occurs in anticipation of an investment opportunity or whether it leads to the delay of an investment opportunity that is already present. In the former case, the manager hoards cash (until the investment opportunity arrives or she has sufficient funds at hand) only if the profitability of the investment opportunity is sufficiently high. In the latter case, the manager follows the hoarding and investment policies set out in Proposition 1.*

Proof of Proposition B.1. We only offer a sketch of the argument. To determine whether the manager should start hoarding, we have to compare the expected payoff from hoarding with paying out w_0. Since the investment opportunity's expected payoff is increasing in θ, there is a threshold $\widetilde{\theta}$, such that starting to hoard is optimal if $\theta > \widetilde{\theta}$.

For completeness, we briefly discuss the optimal hoarding strategy before the investment opportunity's arrival. Let w_0 denote the cash level that the manager has hoarded at the time of the arrival of the firm's investment opportunity. The expected value of the investment opportunity upon its arrival, $U(w_0, w_\theta^*)$, is given by (5), and it is strictly increasing and convex in w_0 for $w_0 < K$. This implies that the optimal hoarding level before arrival is $w_0^* \geq K$. To see this, suppose to a contradiction that the manager stops hoarding at $w_0^* < K$ and pays out $w_t - w_0^*$. Doing so cannot be optimal if hoarding until w_0^* is optimal. First, the probability of arrival is the same at every instant. Second, given that U is convex in w_0, the marginal increase in U is higher for any additionally hoarded unit of cash. In contrast, paying out a unit of cash has the same value to the manager regardless of the previously hoarded amount. Hence, if hoarding dominates paying out for $w_t < w_0^* < K$, it is even more beneficial for $w_t = w_0^*$, giving a contradiction. Hence, we must have that $w_0^* \geq K$.[27] **Q.E.D.**

B.2 Cash Hoarding when Delay Reduces Uncertainty

One of the results from Section 3.2 is that firms that choose private financing delay investment less than firms choosing public financing. We now show that this result is true even if delaying investment helps alleviate the uncertainty and disagreement about the project's fundamentals. As a simple modification to our baseline model, suppose that after receiving her initial signal, the manager believes that the project's value is $X(\theta)$ with probability p_0 (rather than one) and zero otherwise. The financier disagrees, believing that the value is $X(\theta)$ with probability $\rho < p_0$ and zero otherwise. Suppose further that before investing, the firm has a chance of observing a second signal that reveals whether the investment opportunity's value is $X(\theta)$ or zero is correct with certainty and is verifiable to all. The time until such an event follows an exponential distribution with parameter λ_e. If uncertainty disappears, the manager invests immediately if the project's value is $X(\theta)$, and her expected payoff is $X(\theta) - K$. If the project's value is zero, she abandons it and pays out w_t. If there is no signal, the manager can choose between investing and cash hoarding as in the baseline model, but also to continue waiting to observe a signal. For this extension, we assume in analogy to Section 3 that if the manager invests before the second signal arrives,

[27] Since the maximum value of α is one, the term $(1-\alpha)X$ in (5) remains constant for $w_0 \geq K$. Thus, the benefit of hoarding more than K decreases in w_0 for $w_0 \geq K$ and there is a certain hoarding level $w_0^* \geq K$, beyond which the manager pays out all additionally generated cash above w_0^*.

the financier's monitoring and interference choice is $E \in \{0,1\}$ and the manager's cost of interference are $\kappa(1)$ with $\phi \to \infty$. We assume that there is no monitoring and interference in case of the second fully-revealing signal.

The existence of such second reason to delay does not change that delay is less attractive when the firm's investment opportunity is better. If the manager uses delay also for hoarding, we obtain again that the manager delays and hoards less before investment, when choosing private financing. Hence, she is more likely to risk investing under uncertainty.

Proposition B.2 *Suppose that delaying investment could alleviate uncertainty about whether the investment opportunity's value is $X(\theta)$ or zero. When θ is higher, the manager is less willing to delay and is more likely to risk investing under uncertainty.*

Proof of Proposition B.2. We focus on the case in which the manager hoards cash while waiting. The alternative would be to pay out w_0, but this would have no implications for hoarding and the public versus private choice.

Given that the true value of the investment opportunity is revealed with probability λ_e, we can use (A.14) to derive the manager's expected payoff as

$$\frac{\lambda_e p_0 (X-K)}{(r+\lambda_e)} + \left(\left(1-\frac{K-\widehat{w}}{\rho X}\right) p_0 X - \kappa(1) - \widehat{w} - \frac{\lambda_e p_0 (X-K)}{(r+\lambda_e)}\right)\left(\frac{w_t}{\widehat{w}}\right)^{\gamma_e}$$

where γ_e is the positive root to $\frac{1}{2}\sigma^2 y(y-1) + \mu y - r - \lambda_e = 0$. This gives the following first-order condition with respect to $\widehat{w}$

$$0 = \left(-\frac{\gamma_e}{\widehat{w}}\left(\left(1-\frac{K-\widehat{w}}{\rho X}\right) p_0 X - \kappa(1) - \widehat{w} - \frac{\lambda_e p_0 (X-K)}{(r+\lambda_e)}\right) + \left(\frac{p_0}{\rho} - 1\right)\right)\left(\frac{w}{\widehat{w}}\right)^{\gamma_e}$$

implying that the optimal hoarding level is given by

$$w^* = \frac{\gamma_e}{(\gamma_e - 1)}\left(\frac{(\rho\kappa(1) + K - \rho X)\, p_0}{(p_0-\rho)} + \frac{\lambda_e p_0 \rho (X-K)}{(r+\lambda_e)(p_0-\rho)}\right).$$

Differentiating with respect to θ, we have

$$\frac{\partial w^*}{\partial \theta} = \frac{\gamma_e \rho p_0}{(\gamma_e - 1)(p_0-\rho)} \frac{\partial X}{\partial \theta}\left(\frac{\lambda_e}{r+\lambda_e} - 1\right) < 0.$$

Q.E.D.

B.3 Market Timing with Time Varying Profitability

We now let the NPV of the investment opportunity change over time. This is a standard assumption in the related real options literature (Bolton et al., 2013). Define $\widetilde{K}_t := \rho\kappa(1) + K_t - \rho X_t$ and let

$$\frac{d\widetilde{K}}{\widetilde{K}} = \mu_{\widetilde{K}} dt + \sigma_{\widetilde{K}} dZ_{\widetilde{K}}$$

where $Z_{\widetilde{K}}$ is standard Brownian motion and $\sigma_{\widetilde{K}} > 0$ with a correlation ψ to Z (we assume that the change in $\widetilde{K}$ comes from K, but analogous arguments apply if it would come from X). We assume that $\mu_{\widetilde{K}} < 0$ implying that the NPV from the financier's perspective (i.e., $\rho X - K$) increases on average over time. Delay in investment could, then, occur for two reasons: delaying not only to hoard cash, but also to wait for the value of the investment opportunity to increase. Our results remain robust also in such a setting.

Proposition B.3 *Along the optimal investment barrier, the optimal cash level decreases when the firm's investment opportunity is better.*

Proof of Proposition B.3. Following similar steps to Proposition 1, the manager's expected payoff is the solution to the following partial differential equation

$$\begin{aligned} rU &= \mu w U_w + \frac{1}{2}\sigma^2 w^2 U_{ww} + \mu_{\widetilde{K}} \widetilde{K} U_{\widetilde{K}} \\ &\quad + \frac{1}{2}\sigma_{\widetilde{K}}^2 \widetilde{K}^2 U_{\widetilde{K}\widetilde{K}} + \psi\sigma\sigma_{\widetilde{K}} w \widetilde{K} U_{w\widetilde{K}} \end{aligned} \tag{B.1}$$

where the subscripts w and $\widetilde{K}$ denote the partials of U with respect to w and $\widetilde{K}$, respectively. Define $\chi = \frac{w}{\widetilde{K}}$ so that $U(w, \widetilde{K}) = \widetilde{K} U(\chi)$, where we use that U is homogenous of degree one in $(w, \widetilde{K})$ (Doubling $\widetilde{K}$ and doubling w would merely double the manager's expected payoff). We have

$$\begin{aligned} U_w &= U_\chi;\ U_{ww} = \frac{1}{\widetilde{K}} U_{\chi\chi};\ U_{w\widetilde{K}} = -\frac{w}{\widetilde{K}^2} U_{\chi\chi} \\ U_{\widetilde{K}} &= U - \frac{w}{\widetilde{K}} U_\chi;\ U_{\widetilde{K}\widetilde{K}} = \frac{w^2}{\widetilde{K}^3} U_{\chi\chi}. \end{aligned}$$

Plugging into (B.1), we obtain the simple ordinary differential equation

$$\underbrace{\left(r - \mu_{\widetilde{K}}\right)}_{r'} U = \underbrace{\left(\mu - \mu_{\widetilde{K}}\right)}_{\mu'} \chi U_\chi + \underbrace{\left(\frac{1}{2}\sigma^2 + \frac{1}{2}\sigma_{\widetilde{K}}^2 - \psi\sigma\sigma_{\widetilde{K}}\right)}_{\sigma'} \chi^2 U_{\chi\chi} \tag{B.2}$$

with a value matching condition $U(\chi^*) = \left(\frac{\chi^*-1}{\rho}\right) - \chi^*$. Defining φ as the positive root to $\frac{1}{2}\sigma'^2 y(y-1) + \mu' y = r'$ (where r', μ' and σ' are defined in (B.2)), and following the same steps as in Section 3, we obtain

$$\chi^* = \frac{w^*}{\rho\kappa(1) + K^* - \rho X} = \frac{\varphi}{\varphi - 1}\left(\frac{1}{1-\rho}\right).$$

We see, thus, that the optimal co-investment w and the NPV from the financier's point of view are in a constant proportion at the optimal investment barrier. Along this barrier, the optimal cash level w^* increases with the investment cost K^*, and this level is lower when the investment opportunities are better (high θ). **Q.E.D.**

B.4 Dilution of Effort Incentives and Hoarding

We have derived our results assuming that the manager and the financier have different visions. However, hoarding could also be caused by other financing frictions. We have already discussed information asymmetry. Another friction for growth firms is that external financing dilutes ownership and, hence, could reduce the manager's incentives to exert effort, increasing the cost of external financing (Holmstrom and Tirole, 1997).[28]

Specifically, suppose that conditional on being undertaken, the investment succeeds with probability e, in which case it yields X. With probability $1-e$, it fails and yields zero. Let the success probability e reflect the effort exerted by the manager at cost $\frac{e^2}{2\nu}$ after the investment is undertaken. To illustrate the main idea, assume that the firm raises equity from a passive financier. Hence, the financier needs to obtain a stake $\alpha = \frac{K-w}{e^* X}$ to break even, where e^* is the manager's equilibrium effort choice. Conditional on investing, the manager's problem is to choose the optimal level of effort $\widehat{e}$ that maximizes her expected payoff

$$\max_{\widehat{e}} \left(1 - \frac{K-w}{e^* X}\right)\widehat{e}X - w - \frac{\widehat{e}^2}{2\nu}. \tag{B.3}$$

Hence, her effort choice is

$$\widehat{e} = \nu\left(1 - \frac{K-w}{e^* X}\right)X. \tag{B.4}$$

Since in equilibrium, we must have $e^* = \widehat{e}$, we obtain

$$e^* = \frac{1}{2}\nu X + \frac{1}{2}\sqrt{(\nu X)^2 - 4\nu(K-w)}. \tag{B.5}$$

To avoid corner solutions, assume that ν is sufficiently small so that $e^* \leq 1$. We see im-

[28] We thank Andrey Malenko for suggesting this discussion.

mediately that the manager's effort e^* is increasing in her co-investment w. Furthermore, plugging (B.4) into (B.3), we obtain that, prior to investing, the manager chooses her optimal hoarding level w^* to solve

$$U = \max_{w^*} \left(\frac{(e^*)^2}{2\nu} - w^* \right) \left(\frac{w_t}{w^*} \right)^{\beta}. \tag{B.6}$$

It is now straightforward to show that this alternative setting leads to the same qualitative results as our baseline model. In particular, it continues to be true that firms with better investment opportunities hoard less, as delay is more costly for them.

Proposition B.4 *Consider a reformulation of our model as a moral hazard problem, in which the manager's effort determines the probability of the new investment's success. In this setting, hoarding reduces the dilution of the manager's effort incentives, and it continues to be true that firms with better investment opportunities hoard less.*

Proof of Proposition B.4. The optimal level of effort solving (B.5) is

$$e^* = \frac{1}{2}\nu X \pm \frac{1}{2}\sqrt{(\nu X)^2 - 4\nu (K - w)} \tag{B.7}$$

Since, for any given w, the manager's payoff is increasing in e, the global maximum is at $e^* = \frac{1}{2}\nu X + \frac{1}{2}\sqrt{(\nu X)^2 - 4\nu (K - w)}$. This proves (B.6). As usual, we obtain the solution for w^* from the manager's first-order condition

$$\left(-\frac{\beta}{\widehat{w}} \left(\frac{\left(\frac{1}{2}\nu X + \frac{1}{2}\sqrt{(\nu X)^2 - 4\nu (K - \widehat{w})} \right)^2}{2\nu} - \widehat{w} \right) + \left(\frac{\nu X - \sqrt{(\nu X)^2 - 4\nu (K - \widehat{w})}}{2\sqrt{(\nu X)^2 - 4\nu (K - \widehat{w})}} \right) \right) \left(\frac{w}{\widehat{w}} \right)^{\beta} = 0.$$

By standard monotone comparative statics arguments, observe that $\frac{\partial^2 U}{\partial \widehat{w} \partial \theta} < 0$ implies that $\widehat{w}$ is decreasing in θ. Hence, as in our baseline model, firms with better investment opportunities hoard less cash. **Q.E.D.**