UNIVERSITY OF AMSTERDAM

UvA-DARE (Digital Academic Repository)

Unemployment and tax design

Hummel, A.J.

Publication date
2019

Document Version
Submitted manuscript

Link to publication

Citation for published version (APA):
Hummel, A. J. (2019). *Unemployment and tax design*. https://www.albertjanhummel.com/wp-content/uploads/2018/09/jmp.pdf

General rights
It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

Disclaimer/Complaints regulations
If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: https://uba.uva.nl/en/contact, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.

UvA-DARE is a service provided by the library of the University of Amsterdam (https://dare.uva.nl)

Download date:29 Sep 2022

Unemployment and Tax Design*

Albert Jan Hummel†

January 9, 2019

Click here for the latest version

Abstract

This paper studies the implications of unemployment for the optimal design of the tax-benefit system. To do so, I develop a directed search model where individuals face heterogeneous and uninsurable unemployment risk. They differ in terms of their skills and participation costs and supply labor on the intensive and extensive margin. Matching frictions give rise to a trade-off for workers between high wages and low unemployment risk. The government affects this trade-off by altering the costs and benefits of searching. The associated changes in unemployment generate fiscal externalities which modify optimal tax formulas. How unemployment affects optimal tax policy depends on the elasticity of unemployment with respect to the marginal and average tax rate and on the hazard rate of the income distribution. I show that optimal employment subsidies (such as the EITC) phase in with income. Moreover, financing unemployment benefits through lump-sum or proportional taxes on labor income – as is commonly assumed in the literature – is sub-optimal even in the absence of a motive for redistribution. I calibrate the model to the US economy and find that unemployment is an important margin to consider when setting tax rates at low levels of income. In my preferred calibration, unemployment generates a negative fiscal externality which lowers the mechanical revenue gain of income taxes by 3%.

JEL classification: H21, J64, J65, J68
Keywords: directed search, optimal taxation, unemployment insurance

*I am grateful to Aart Gerritsen, Bas Jacobs and Dominik Sachs for continuous guidance and support at various stages of the project. Also I would like to thank Björn Brügemann, Peter Diamond, Emmanuel Farhi, Pieter Gautier, Nathan Hendren, Philipp Kircher, Etienne Lehmann, Marcelo Pedroni, Florian Scheuer, Stefanie Stantcheva, Uwe Thümmel and Matthew Weinzierl for many helpful comments and suggestions. Finally, this paper has benefited greatly from discussions at IIPF 2018 Tampere, SaM 2018 Cambridge, ZEW 2018 Mannheim, Erasmus School of Economics, Paris Center for Law and Economics, Tinbergen Institute and VU University Amsterdam.

†Erasmus University Rotterdam and Tinbergen Institute. E-mail: hummel@ese.eur.nl.

1 Introduction

Unemployment is a major policy concern. It leads to significant drops in consumption and life satisfaction and is an important source of inequality (Winkelmann and Winkelmann (1998), Kassenboehmer and Haisken-DeNew (2009)). Governments around the world use complicated systems of taxes and benefits to redistribute income and provide insurance against unemployment risk. When doing so, they face a complex trade-off. On the one hand, unemployment motivates the use of the tax-benefit system because unemployment risk is not insurable and unequally distributed.[1] On the other hand, unemployment limits how much insurance and redistribution the government can provide. This is because the tax-benefit system affects individuals' incentives to search and supply labor and thereby unemployment. This two-way interaction calls for a joint analysis of unemployment and the tax-benefit system. Yet, despite the apparent policy relevance the literature offers little guidance as to how unemployment should be taken into account when designing the tax-benefit system.

This paper characterizes the optimal joint design of unemployment insurance and income redistribution. To do so, I develop a directed search model where individuals differ in terms of their skills and participation costs. They supply labor on the extensive (participation) and intensive (hours, effort) margin and optimally choose whether and where to apply. Matching frictions generate unemployment risk, which is not insurable. The government cannot observe individuals' labor supply and search behavior but only their earnings and employment status. It provides partial insurance and redistributes income by setting an unemployment benefit and a non-linear income tax. I use the model to (i) study how the tax-benefit system affects labor-market outcomes and (ii) derive optimal policy rules. Finally, I do a quantitative analysis by calibrating the model to the US economy. My main findings are the following.

First, the government faces a trade-off between lowering the unemployment rate among low-skilled workers and among individuals with higher skills. This trade-off for the government directly originates from a trade-off individuals face between *wages* and *probabilities*. In particular, individuals are less likely to be matched if they apply for a job which pays a higher wage. The tax-benefit system affects this trade-off and hence unemployment, in two opposing ways. On the one hand, an increase in the *marginal* tax rate makes individuals care less about higher wages. This leads firms to post lower wages and hire more workers. As a result, unemployment declines. I label this the employment-enhancing effect (EEE) of taxation. On the other hand, an increase in the *average* tax rate or unemployment benefit makes individuals care less about finding a job. This puts upward pressure on wages, which leads firms to hire fewer workers and unemployment to increase. I label this the employment-reducing effect (ERE) of taxation. Hence, for a given average tax rate, an increase in the marginal tax rate reduces unemployment and *vice versa*. Both effects are consistent with empirical evidence – as will be made clear below. Moreover, because an increase in the marginal tax rate at some income level mechanically raises average tax rates at higher levels of income, lowering the unemployment rate at some point in the skill distribution comes at the costs of increasing it at higher skill levels.

[1] For example, in the US individuals who completed primary education are approximately three times more likely to be unemployed than those who completed tertiary education. Figures for other OECD countries are comparable (OECD (2018)).

Second, I characterize the optimal tax-benefit system in terms of the income distribution, social welfare weights and behavioral responses. These sufficient-statistics formulas clearly demonstrate how unemployment should be taken into account when designing the tax-benefit system. Changes in unemployment generate fiscal externalities because an unemployed worker receives benefits and does not pay income taxes. These externalities, in turn, call for intuitive adjustments of standard optimal tax formulas. How unemployment affects optimal tax policy depends on two types of statistics: (i) the elasticity of unemployment with respect to the marginal and average tax rate and (ii) the hazard rate of the income distribution. Intuitively, an increase in the marginal tax rate at some income level mechanically raises average tax rates further up in the income distribution. Provided the employment tax (i.e., the sum of the income tax and the unemployment benefit) is non-zero, the associated changes in employment due to the EEE and ERE affect government finances.[2] The revenue effect of the former is proportional to the elasticity of unemployment with respect to the marginal tax rate and the fraction of people whose marginal tax rate is increased (i.e., the density of the income distribution). The latter generates a revenue effect which is proportional to the elasticity of unemployment with respect to the average tax rate and the fraction of individuals whose average tax rate is increased (i.e., one minus the cumulative distribution of income). In the typical case that the employment tax is positive, the EEE (ERE) raises (lowers) the optimal marginal tax rate.

Third, employment subsidies should phase in with income. Put differently, if the *level* of the employment tax for low-income workers is negative, these workers should also face a negative *marginal* tax rate. My model thus provides a rationale for the phase-in region of the EITC. This result complements those from Diamond (1980) and Saez (2002). They show that it is optimal to subsidize employment if the government cares sufficiently about the working poor and labor-supply responses are mostly concentrated on the extensive margin. If unemployment is taken into account, the case for lowering *marginal* tax rates for these workers is then a particularly strong one. Intuitively, doing so induces them to apply for jobs which pay a higher wage, whereas the implied reduction in average tax rates lowers wages for individuals with higher skills. If employment is subsidized at the bottom and taxed at higher levels of income, both the reduction in employment among low-skilled workers and the increase in employment among higher-skilled workers positively affect government finances. As a result, the optimal marginal tax rates for low-income workers are lower if unemployment is taken into account and negative at the very bottom if employment is subsidized.

Fourth, the optimal provision of unemployment insurance (UI) is closely linked to the shape of the tax schedule. My results are close in spirit to those derived in Baily (1978) and Chetty (2006). Importantly, however, I do not restrict UI payments to be financed through lump-sum or proportional taxes on labor income. In fact I show that doing so is sub-optimal. To see why, suppose there is no *ex ante* heterogeneity. The tax-benefit system is then merely used for insurance (and not redistributive) purposes. If individuals are risk-averse, the government optimally provides an unemployment benefit, which is financed through a positive *average* tax

[2] The literature typically uses the term participation tax to refer to the sum of the income tax and the unemployment benefit. However, as pointed out by Kroft et al. (2017), the term employment tax is more appropriate in case there are individuals who participate (i.e., who look for a job), but nevertheless remain unemployed.

rate on earnings. As in the Baily-Chetty framework, this form of insurance leads to an upward distortion in unemployment (through the ERE) and optimal policy balances the insurance benefits against the distortionary costs. I complement this result by showing that the optimal *marginal* tax rate is positive as well. Intuitively, raising the marginal tax rate reduces wage pressure. The associated increase in employment (through the EEE) partially off-sets the upward distortion in unemployment generated by UI. The optimal marginal tax rate satisfies a simple inverse-elasticity rule and increases in the size of the employment tax and the elasticity of unemployment with respect to the marginal tax rate.

Finally, I calibrate the model to the US economy and find that unemployment is an important margin to consider when setting tax rates at low levels of income. In my preferred calibration, the government loses close to 3 cents on the dollar due to unemployment responses if – starting from the *current* tax-benefit system – marginal tax rates for low-income workers are increased. This is because (i) unemployment is more responsive to changes in the average than to changes in the marginal tax rate and (ii) the hazard rate of the income distribution is low at low levels of income. Consequently, raising the marginal tax rates for low-skilled workers improves the employment prospects of only a few individuals, whereas the implied increase in average tax rate reduces employment at virtually all other skill levels. Despite this, the quantitative implications of unemployment for the *optimal* tax-benefit system appear to be modest.

1.1 Related literature

Taxation in imperfect labor markets

Theory There is an extensive literature which analyzes the impact of the tax-benefit system on labor-market outcomes in imperfectly competitive labor markets (see, e.g., Bovenberg and van der Ploeg (1994) and Picard and Toulemonde (2003) for overviews). A robust finding is that – for a given average tax rate – an increase in the marginal tax rate reduces unemployment. Conversely, for a given marginal tax rate, an increase in the average tax rate or unemployment benefit raises unemployment. I label the first of these the employment-enhancing effect (EEE) of taxation and the second the employment-reducing effect (ERE). These results are obtained in union bargaining models (Hersoug (1984), Lockwood and Manning (1993), Koskela and Vilmunen (1996)), in matching models with individual bargaining (Pissarides (1985, 1998)) and in models where firms pay efficiency wages (Pisauro (1991)). I contribute to this literature by showing the results also hold if firms post vacancies to attract workers and matching frictions generate unemployment. Moreover, because in the framework I analyze workers differ in their skills, the EEE and ERE imply that lowering the unemployment rate at some skill level by raising the marginal tax rate comes at the costs of raising unemployment among individuals with higher skills.

Evidence There exists ample evidence that benefit generosity positively affects unemployment and unemployment duration, in line with the ERE (see, e.g., Meyer (1990), Chetty (2008) and Card et al. (2015)). Moreover, many macro-empirical studies document a positive effect of income taxes on unemployment (see, e.g., Nickell and Layard (1999), Blanchard and Wolfers (2000), Daveri and Tabellini (2000), Griffith et al. (2007), Bassanini and Duval (2009)). However, because these studies do not distinguish between marginal and average tax rates, the results are not very informative about the EEE and ERE. The only study I know which empirically separates these channels is a recent paper by Lehmann et al. (2016). Using data on a panel of OECD countries, they find that average tax rates positively affect unemployment, whereas marginal tax rates have the opposite effect. Similarly, Manning (1993) shows that a single index of tax progressivity (which increases in the marginal tax rate and decreases in the average tax rate) is negatively associated with unemployment in the UK.

More indirect evidence comes from studies which analyze the impact of income taxes on wages. A typical finding in the macro-empirical literature is that an increase in the average tax rate is associated with an increase in the hourly wage, whereas an increase in the marginal tax rate has the opposite effect (see, e.g., Malcomson and Sartor (1987), Lockwood and Manning (1993), Holmlund and Kolm (1995)). Using micro-level data, Blomquist and Selin (2010) exploit a series of Swedish tax reforms and find a strong negative effect of marginal tax rates on wages, in line with the EEE. Schneider (2005) and Rattenhuber (2017) obtain similar results using data on German workers. The results from these studies are consistent with the predictions from my model.

Optimal taxation with intensive and extensive labor-supply responses

This paper builds on a literature which studies optimal income taxation when individuals supply labor both on the extensive (participation) and intensive (hours, effort) margin. In an influential paper, Saez (2002) shows that if labor supply is most responsive on the intensive margin the optimal policy features a Negative Income Tax (NIT), i.e., a substantial guaranteed income which is quickly phased out. If labor-supply responses are mostly concentrated along the extensive margin, the optimal tax schedule more closely resembles an Earned Income Tax Credit (EITC) with a low guaranteed income and substantial in-work benefits. Jacquet et al. (2013) generalize this framework and derive conditions under which marginal tax rates and employment taxes are positive. Hansen (2017) analyzes under which conditions marginal tax rates and employment taxes are negative at low levels of income. Jacobs et al. (2017) use a similar framework to derive optimal tax formulas in terms of sufficient statistics. I contribute to this literature by analyzing the implications of unemployment for optimal tax design. My optimal tax formulas generalize those of Jacobs et al. (2017).[3] Moreover, I show that if unemployment is taken into account it is optimal to let employment subsidies (such as the EITC) phase in with income.

Optimal taxation and search

This paper is closely related to a literature which studies optimal taxation with search frictions. Boone and Bovenberg (2004, 2006) analyze a model where workers engage in costly search effort. Recently, Sleet and Yazici (2017) study optimal income taxation in a framework with on and

[3] Kroft et al. (2017) and Hummel and Jacobs (2018) also derive optimal tax formulas in a model where wages and unemployment respond to taxation. In their models there is a discrete set of occupations and the government sets the tax liability in each occupation separately. By contrast, in my model there is a single labor market and the government levies a non-linear income tax. This implies marginal and average tax rates cannot be set independently, which gives rise to the trade-off between the EEE and ERE.

off the job search. In these papers unemployment is not affected by policy choices. By contrast, the interaction between unemployment and the design of the tax-benefit system lies at the heart of the current paper.

Hungerbühler et al. (2006) study optimal income taxation in a framework where wages are determined through Nash bargaining, but labor supply is exogenous. As in my model, an increase (decrease) in the marginal (average) tax rate leads to wage moderation, which reduces unemployment. Hungerbühler and Lehmann (2009), Lehmann et al. (2011) and Jacquet et al. (2014) extend the framework to study minimum wages, endogenous participation and alternative bargaining structures. Golosov et al. (2013) use a similar matching model to analyze the optimal redistribution of residual wage inequality (i.e., wage inequality among equally skilled workers). I contribute to this literature in three ways. First, I study a model where individuals supply labor both on the extensive and intensive margin. Second, I characterize optimal policy in terms of the income distribution and behavioral responses ("sufficient statistics" in the terminology of Chetty (2009)). This clearly illustrates how unemployment affects the optimal design of the tax-benefit system. Third, I study the optimal provision of unemployment insurance and show that it is closely linked to the shape of the tax schedule even in the absence of wage heterogeneity.

Optimal unemployment insurance

Finally, this paper relates to a literature on the optimal provision of unemployment insurance (UI). In a seminal paper, Baily (1978) shows that optimal UI policy balances insurance gains against the adverse effects of UI on job search. Chetty (2006) generalizes this framework and derives a sufficient-statistic formula for the optimal UI benefit which holds in a wide class of models. The framework I present differs in a number of ways from these and most other models used in the literature. I abstract from dynamic considerations but allow UI to not only affect unemployment but also wages (as in Acemoglu and Shimer (1999) and Fredriksson and Holmlund (2001)) and labor supply. Moreover, I do not restrict UI payments to be financed through lump-sum or proportional taxes on labor income. This has two important implications. First, UI does not only affect government finances through unemployment but also through earnings responses. This calls for an intuitive adjustment of the Baily-Chetty formula for the optimal benefit level. Second, the optimal provision of UI is closely linked to the shape of the tax schedule. In particular, I show that financing UI payments through lump-sum or proportional taxes on labor income is sub-optimal.

The remainder of this paper is organized as follows. Section 2 presents a directed search model of the labor market. Section 3 discusses the efficiency properties and analyzes how the tax-benefit system affects labor-market outcomes. Section 4 characterizes the optimal tax-benefit system. Section 5 presents the quantitative analysis. Section 6 concludes and discusses directions for future research. All proofs and additional details on the numerical analysis can be found in the appendices.

2 A directed search model of the labor market

This section presents the model which is used in the remainder of the analysis. The main ingredients are the following. There is a continuum of individuals who are heterogeneous in terms of their ability and costs of participating in the labor market, both of which are private information. They supply labor on the extensive (participation) and on the intensive (hours, effort) margin to homogeneous firms. Firms post vacancies in order to attract applicants, and continue to do so until expected profits are zero. Matching frictions generate unemployment, and the risk of becoming unemployed is not privately insurable. The government cares for redistribution but faces asymmetric information regarding individuals' types and their labor supply and search decisions. It only observes an individual's labor earnings and whether or not he is employed. Consequently it can levy a non-linear income tax and provide a uniform unemployment benefit. The latter is paid both to non-participants and to individuals who decided to search but nevertheless remained unemployed. I first characterize the equilibrium for a given tax-benefit system and analyze the government's optimization problem separately in Section 4.

Individuals

The economy is populated by a unit mass of individuals, who are also referred to as workers. They differ both in terms of their ability (or skill), and in their costs of participating in the labor market. Ability is denoted by $n \in [n_0, n_1]$, and participation costs by $\varphi \in [\varphi_0, \varphi_1]$. $F(\varphi, n)$ describes the joint distribution and $f(\varphi, n)$ the corresponding density.

Individuals derive utility from consumption c and disutility from producing output y. To produce y units of output, an individual with ability n must exert y/n units of effort. Each individual can be in three states: employment, unemployment or non-participation. If employed, an individual consumes his earnings z net of taxes $T(z)$. If not, he consumes an unemployment benefit b. Hence, the government does not distinguish between individuals who chose not to participate and those who are (involuntarily) unemployed. Moreover, unemployment risk is not privately insurable.[4] I denote by e the employment rate conditional on participation, which can also be interpreted as the matching probability or the job-finding rate. The unemployment rate is then given by $1 - e$.

The expected utility of an individual (n, φ) who applies for a job where he earns income z, produces output y and becomes employed with probability e, equals:

$$U(n, \varphi) = e \left[u(z - T(z)) - v\left(\frac{y}{n}\right) \right] + (1 - e)u(b) - \varphi. \tag{1}$$

Here, sub-utility over consumption $u(\cdot)$ is assumed to be strictly increasing and weakly concave, and disutility of labor effort $v(\cdot)$ is strictly increasing and strictly convex. The value of $v(0)$ is normalized to zero. Note that participation costs are incurred irrespective of whether or not an

[4] Both the assumption that unemployment risk is not privately insurable and that the government cannot distinguish between non-participants and unemployed workers can be micro-founded by assuming an individual's application strategies are private information and hence, not contractible. See Boadway and Cuff (1999) and Boadway et al. (2003) for an analysis of optimal income taxation if the government can distinguish between non-participants and the involuntary unemployed (e.g., through costly monitoring).

individual finds a job. They can therefore be equivalently interpreted as the costs of searching.

Firms

Firms are homogeneous and post vacancies at unit cost $k > 0$. A vacancy (y, z) specifies (i) how much output y a potential employee is expected to produce, and (ii) the income z he receives as compensation. Because a vacancy specifies output (and not effort), the only source of uncertainty which matters for the firm is *whether* the vacancy is filled, not *by whom*.[5] The firm is therefore indifferent as to whether the vacancy is filled by a high-skilled worker who exerts little effort or by a low-skilled worker who has to work harder to produce the same output.

Government

There is a government that cares for redistribution. Its objective is formally defined in Section 4. The government cannot observe individuals' types nor their labor supply or search behavior. Instead, it can only observe an individuals' labor earnings and whether or not he is employed. As a result, the government can levy a non-linear tax $T(z)$ on labor income to finance a (uniform) unemployment benefit b and some exogenous spending G.

Matching

Workers and firms interact through frictional labor markets. Frictions are captured in a reduced-form way through a matching function. The latter relates the number of matches to the number of job-seekers and the number of vacancies. If the matching function features constant returns to scale, the job-finding (or employment) rate depends only on labor market tightness, i.e., the ratio of vacancies to job-seekers. I denote the inverse of this relationship by $\theta = \theta(e)$. Hence, $\theta(e)$ measures how many vacancies relative to job-seekers must be posted for a fraction e of job-seekers to find employment. The probability that a firm is matched is then given by $e/\theta(e)$. The function $\theta(e)$ fully captures the presence and severity of matching frictions. I assume it is strictly increasing, strictly convex, and satisfies $\theta(0) = 0$. For technical convenience, I furthermore assume that the elasticity $\theta'(e)e/\theta(e)$ is non-decreasing.[6]

2.1 Equilibrium

Firms continue to post vacancies until profits are zero in expectation. If a vacancy (y, z) is posted in equilibrium, the following must hold:

$$k = \frac{e}{\theta(e)}(y - z). \tag{2}$$

In words, free entry ensures that the cost of opening a vacancy equals the probability that a vacancy is filled, multiplied by the profit margin. Hence, if a posted vacancy implies a high

[5]This is different from Stantcheva (2014), who analyzes the implications of adverse selection for optimal income taxation. In her model, firms do not know the productivity of workers and screen them through contracts which specify income and working hours (i.e., effort).

[6]This condition holds for virtually all commonly employed matching functions (including the micro-founded ones) and guarantees that $\theta(\cdot)$ is "sufficiently" convex.

profit margin, free entry ensures it is filled with low probability. The conditions on the matching function then imply that workers are matched with a high probability if they apply for such a vacancy. Intuitively, the matching probability of workers is high if there are many vacancies relative to job-seekers, and *vice versa* for firms. However, a vacancy which specifies a high profit margin (and hence a high matching probability for workers) implies a low wage per unit of effort.[7] This trade-off between 'prices and probabilities' is typical in models where search is directed (see, e.g., Wright et al. (2017)). It plays a crucial role in what follows.

Individuals maximize their expected utility (1) by optimally choosing (i) whether to apply and – conditional on participation – (ii) where to apply. Working backward, suppose an individual of type (n, φ) decides to participate. When considering where to apply, the zero-profit condition (2) implies he faces a trade-off not only between income and leisure, but also between a high wage and a low probability of becoming unemployed (i.e., between prices and probabilities). After incurring participation costs φ, he solves:

$$\mathcal{U}(n) \equiv \max_{y,z,e} \left\{ e \left[u(z - T(z)) - v\left(\frac{y}{n}\right) \right] + (1-e)u(b) \;\; \text{s.t.} \;\; k = \frac{e}{\theta(e)}(y-z) \right\}. \tag{3}$$

Because participation costs are sunk, individuals with the same ability n make the same choices conditional on participation. Hence, their expected utility net of participation costs only depends on n. The decision whether or not to participate is then a very simple one: an individual of type (n, φ) participates if and only if his participation costs φ are below the threshold

$$\varphi(n) \equiv \mathcal{U}(n) - u(b). \tag{4}$$

I denote by $\pi(n)$ the participation rate of individuals with ability n. The latter is given by:

$$\pi(n) = \frac{\int_{\varphi_0}^{\varphi(n)} f(\varphi, n) d\varphi}{\int_{\varphi_0}^{\varphi_1} f(\varphi, n) d\varphi}. \tag{5}$$

As mentioned before, the decision where to apply revolves around two trade-offs. The first is between income (i.e., consumption) and leisure. The relevant first-order condition is:[8]

$$u'(z - T(z))n(1 - T'(z)) = v'\left(\frac{y}{n}\right). \tag{6}$$

Equation (6) equates the marginal utility costs of exerting more effort (on the right-hand side) to the marginal benefits in the form of higher income (on the left-hand side). This condition coincides with the standard labor-supply equation in a competitive equilibrium where individuals optimally choose their working hours and the costs of opening a vacancy are zero (in which case free entry implies $z = y$).

In addition, individuals face a trade-off between the wage and the probability of finding

[7] To see why, let $w = z/(y/n)$ denote the wage per unit of effort (say, hours). For given ability and output, a high profit margin $y - z = y(1 - w/n)$ then corresponds to a low wage.

[8] This condition is obtained by combining the first-order conditions of maximization problem (3) with respect to z and y. I assume the second-order conditions are satisfied. In the absence of taxes and benefits, the conditions on the utility and matching function guarantee this is the case.

employment. The relevant first-order condition is:

$$eu'(z - T(z))(1 - T'(z)) = \frac{e}{(\theta'(e) - \theta(e)/e)k}\left[u(z - T(z)) - v\left(\frac{y}{n}\right) - u(b)\right]. \tag{7}$$

The left-hand side multiplies the employment rate by the marginal utility of income. Hence, it equals the marginal benefits of applying for a job which pays a higher wage. The right-hand side captures the marginal costs of doing so. It equals the product of two terms. The first term measures by how much the job-finding probability decreases if an individual applies for a job which specifies higher earnings (i.e., a higher wage). This reduction is multiplied by the (opportunity) costs of not finding employment, as given by the utility difference between employment and unemployment.

It is worth pointing out that both terms on the right-hand side reflect an externality associated with the decision to post a vacancy. If a firm posts a vacancy, it does not take into account that it becomes more difficult for other firms to fill theirs. This business-stealing externality is captured by the difference between the social and private costs of getting a vacancy filled.[9] It shows up in the denominator of the first term on the right-hand side, which – as mentioned before – measures by how much the job finding (i.e., employment) rate changes if an individual applies for a job which pays a higher wage. Intuitively, if firms impose a larger business-stealing externality on each other vacancy posting (and hence employment) becomes less responsive to changes in the wage. Second, firms also do not internalize the utility gain a potential employee experiences if he finds a job, as captured by the second term. From a social perspective, the first (second) effect causes excessive (too little) vacancy creation. I discuss the implications for efficiency in the next section.

Combined, the two first-order conditions (6)-(7) and the zero-profit condition (2) constitute a system of three equations in three unknowns. Because individuals with the same ability but different participation costs make the same choices (conditional on participation), I denote the solution by $z(n)$, $y(n)$ and $e(n)$, respectively. The next section discusses in detail how these outcomes are affected by the tax-benefit system. For now, I complete the characterization of the equilibrium by requiring the government budget constraint is satisfied:

$$\int_{n_0}^{n_1}\int_{\varphi_0}^{\varphi(n)} e(n)(T(z(n)) + b)dF(\varphi, n) = b + G. \tag{8}$$

Total spending (on the right-hand side) equals total tax revenue collected from all employed individuals (on the left-hand side). The latter consists of both the tax bill and the foregone payments in unemployment benefits. The term $T(z(n))+b$ measures the increase in government revenue if an individual with ability n moves from unemployment to employment. In the remainder I refer to this term as the 'employment tax'.

[9] The social costs is given by $\theta'(e)k$ and the private costs by $\theta(e)k/e$.

3 Efficiency and comparative statics

This section discusses the efficiency properties of the equilibrium and analyzes how the labor-market outcomes (including unemployment) are affected by the tax-benefit system. Both turn out to be crucial for understanding how unemployment should be taken into account when the government optimally designs the tax-benefit system.

3.1 Efficiency

In a directed search equilibrium the allocation of resources is typically efficient in the absence of government intervention (see, for instance, Moen (1997)). Intuitively, this is because vacancies are posted in advance and hence play an allocative role similar to that of prices in a competitive equilibrium. In Appendix A.1 I show that this result also holds in my model where workers are heterogeneous in two dimensions and supply labor on the intensive and extensive margin, *provided* they are risk-neutral (i.e., $u(c) = c$). In that case, condition (7) simplifies to:

$$\left(\theta'(e(n)) - \frac{\theta(e(n))}{e(n)}\right) k = z(n) - v\left(\frac{y(n)}{n}\right). \tag{9}$$

In words, the business-stealing externality (on the left-hand side) exactly off-sets the utility gain of finding a job (on the right-hand side). In models with random search, this property only holds if the bargaining power of workers and firms equals the elasticity of the matching function with respect to their input (see Hosios (1990)). By contrast, if search is directed the condition for efficiency is endogenously satisfied. Equation (9) can be simplified further by combining it with the zero-profit condition (2):

$$\theta'(e(n))k = y(n) - v\left(\frac{y(n)}{n}\right). \tag{10}$$

This is another way to interpret the efficiency result. In equilibrium employment is raised to the point where the additional vacancy creation costs (on the left-hand side) equals the increase in output net of the utility costs to produce it (on the right-hand side).

In Appendix A.1 I also show that the allocation is Pareto inefficient if individuals are risk-averse. This is because unemployment risk is not privately insurable. Consequently, if individuals are risk-averse it is possible to increase all individuals' *expected* utilities by transferring income from the state of employment to the state of unemployment. However, it is not possible to increase an individual's *realized* utility without lowering that of someone else. The allocation without government intervention is therefore said to be constrained efficient, meaning there is no scope for an *ex post* Pareto improvement. As will be made clear in Section 4, the missing insurance market has important implications for the optimal design of the tax-benefit system.

3.2 Comparative statics

I now turn to study how changes in the tax-benefit system affect labor-market outcomes. Unlike the unemployment benefit b, tracing out the effects of income taxes requires changing a function $T(z)$ as opposed to a parameter. In a recent paper, Golosov et al. (2014) propose a method for doing so. I follow their approach and define a function

$$T^{*}(z,\kappa) = T(z) + \kappa R(z). \tag{11}$$

Here, $T^{*}(z,\kappa)$ is the tax function individuals face if the tax function $T(z)$ is perturbed "in the direction" $R(z)$ by a magnitude κ. Hence, $R(z)$ can be interpreted as a reform to the current system. The impact of a tax reform $R(z)$ on the labor-market outcomes can then be derived in two steps. The first is to characterize the equilibrium for a given tax system $T^{*}(z,\kappa)$. This simply requires replacing $T(z)$ by $T^{*}(z,\kappa)$ in the individual optimization problem (3). The second step is to determine the impact of the reform parameter κ on the equilibrium outcomes (e.g., through implicit differentiation) and evaluate the result at the reform of interest (see also Gerritsen (2016) and Jacquet and Lehmann (2017)).

As can be seen from equations (5)-(7), earnings, participation and unemployment are affected by changes in the marginal tax rate $T'(z)$, the tax liability $T(z)$ and the benefit b. I therefore consider a local increase in each of these (i.e., holding the other fixed). This can be done by setting $R(z) = z - z(n)$ and $R(z) = 1$. The first reform raises the marginal tax rate while leaving the tax liability (and hence the average tax rate) at income level $z(n)$ unaffected. The second reform increases the tax liability (and hence, the average tax rate) while holding the marginal tax rate fixed. The next Proposition summarizes how these reforms and an increase in the unemployment affect the labor-market outcomes.

Proposition 1. *Starting from the laissez-faire equilibrium, Table 1 shows how changes in the tax-benefit system affect labor-market outcomes (for fixed ability n). How the outcomes vary with abillity n is discussed separately in Appendix A.2.*

Table 1: Comparative statics

	Participation	Earnings	Unemployment
Marginal tax rate	=	−	−
Tax liability	−	+	+
Unemployment benefit	−	+	+

Note: A +/=/− indicates that the row variable has a positive/zero/negative impact on the column variable.

The first row in Table 1 shows the impact of locally increasing the marginal tax rate on participation, earnings and unemployment of individuals with ability n. Because the reform only increases the marginal tax rate (and not the tax liability), the participation rate remains unaffected. Earnings decline in response to the reform, for two reasons. First, a higher marginal tax rate reduces effort – the classic distortion which gives rise to the trade-off between equity and efficiency in Mirrlees (1971). Second, a higher marginal tax rate also reduces the wage. As a result, unemployment decreases as well. Intuitively, this is because individuals face a trade-off not only between consumption and leisure but also between high wages and low unemployment risk. An increase in the marginal tax rate makes individuals care less about a higher wage. In response, they optimally apply for a job which specifies a lower wage. Free entry then ensures that firms post more vacancies, which raises employment and hence, reduces unemployment. I label this the employment-enhancing (EEE) effect of taxation:[10]

Definition 1. ***Employment-enhancing effect (EEE)*** *For a given average tax rate, an increase in the marginal tax rate raises employment (and hence reduces unemployment).*

At this point, it is worthwhile to point out that the EEE has important implications for the elasticity of taxable income (ETI) – a statistic of central interest in the public economics literature. See Gruber and Saez (2002) and Saez et al. (2012) for extensive reviews. The ETI measures the percentage increase in earnings following a one-percent increase in the net-of-tax rate. It serves as a sufficient statistic for calculating the the efficiency costs of taxation in a wide class of models. As already pointed out by Hungerbühler et al. (2006), both the canonical labor-supply model and models with matching frictions are consistent with a positive ETI. However, the mechanisms which drive the ETI are very different. In the labor-supply model earnings decline because a higher marginal tax rate lowers the incentives to exert effort. By contrast, in models with matching frictions and wage bargaining a higher marginal tax rate lowers the wage. The model presented here captures both these mechanisms: a higher marginal tax rate reduces both effort as well as the wages posted by firms. While the focus of the optimal tax literature has been almost exclusively on the first of these, Blomquist and Selin (2010) find that a substantial share of the ETI can be attributed to wage (as opposed to hours) responses – at least in the short run.[11] As will be made clear below, if both these forces are present the ETI is no longer a sufficient statistic to calculate the efficiency costs of taxation.

Turning to the second row of Table 1, consider an increase in the tax liability which leaves the marginal tax rate unaffected (i.e., $R(z) = 1$). Such a tax reform reduces the benefits of working and hence, lowers participation. Earnings increase in response to the reform, again for two reasons. First, if there are income effects in labor supply a higher tax bill makes individuals work harder.[12] Second, the reform reduces the utility gain of finding a job. In response individuals apply for a job which pays a higher wage, thereby accepting an increase in the probability of remaining unemployed. I label this effect the employment-reducing effect (ERE) of taxation:[13]

Definition 2. ***Employment-reducing effect (ERE)*** *For a given marginal tax rate, an increase in the average tax rate reduces employment (and hence raises unemployment).*

The last row of Table 1 shows the impact of increasing the unemployment benefit. By raising the value of non-employment, this reform lowers participation. In addition, conditional on participation a higher unemployment benefit reduces the utility gain of finding employment. Following an increase in the unemployment benefit, individuals apply for higher-wage jobs which reduces their matching probability. Hence, both earnings and unemployment increase. This second effect is very similar to the ERE discussed above.

[10] An alternative way to understand the EEE is to reason from the firms' perspective. Firms post vacancies in order to attract applicants. An increase in the marginal tax rate, in turn, makes wages a less effective tool to attract applicants. Wages go down, which induces firms to post more vacancies. As a result, employment increases.

[11] The figures are around 70% for males and 40% for females, although the latter is estimated with less precision.

[12] This effect is only present if the marginal utility of consumption is decreasing. For details, see Appendix A.2.

[13] Again, the ERE can be understood through the lens of firms as well. A higher tax liability makes individuals care less about finding a job. Consequently, wages become a more effective tool to attract applicants. This leads to fewer vacancies being posted, and hence a higher unemployment rate.

4 Optimal taxation

I now turn to study how unemployment affects the optimal design of the tax-benefit system. To do so, I assume the government maximizes a standard (utilitarian) social welfare function:

$$\mathcal{W} = \int_{n_0}^{n_1} \int_{\varphi(n)}^{\varphi_1} \Psi(u(b)) dF(\varphi, n) + \int_{n_0}^{n_1} \int_{\varphi_0}^{\varphi(n)} \Psi(\mathcal{U}(n) - \varphi) dF(\varphi, n). \tag{12}$$

Here, $\Psi(\cdot)$ is strictly increasing and weakly concave, and $\mathcal{U}(n)$ and $\varphi(n)$ are as defined in (3) and (4). Note that the government maximizes a concave transformation of expected – as opposed to realized – utilities. As such, it respects individual preferences. Strict concavity in either $\Psi(\cdot)$ or $u(\cdot)$ generates a motive for redistribution, which is absent only if the marginal utility of consumption is constant (i.e., $u''(\cdot) = 0$) and if the government attaches equal weight to each individual's expected utility (i.e., $\Psi''(\cdot) = 0$).

Variational approach

The government chooses the tax function $T(\cdot)$ and the unemployment benefit b to maximize social welfare (12) subject to the budget constraint (8), taking into account the behavioral responses as summarized in Proposition 1. I solve this problem using the variational approach (see, e.g., Saez (2001), Golosov et al. (2014) and Gerritsen (2016)). This approach differs from the classic mechanism-design approach introduced in Mirrlees (1971) by relying directly on perturbations (i.e., reforms) of the tax system.[14] These reforms generate welfare-relevant effects. Optimal policy rules are then derived from the notion that if the tax-benefit system is optimal, these welfare-relevant effects must sum to zero. The reforms I consider are exactly the ones analyzed in Proposition 1: a (local) increase in (i) the marginal tax rate, (ii) the tax liability and (iii) the unemployment benefit.

Figure 1 graphically illustrates how the variational approach can be used to derive optimal policy rules. It shows the welfare-relevant effects associated with an increase in the marginal tax rate in the (small) interval $[Z, Z + \omega]$. The dashed and red line show the tax schedule before and after the reform. The increase in the marginal tax rate generates behavioral responses. According to Proposition 1, it reduces earnings and – through the EEE – unemployment for individuals with earnings potential between Z and $Z + \omega$.[15] Because a local change in the marginal tax rate does not affect the expected utility of individuals who are affected, I label these substitution (or compensated) effects. By the Envelope theorem, these behavioral responses have no first-order effect on individuals' expected utility. However, changes in earnings and employment do affect government finances. These so-called fiscal externalities are relevant for welfare and should be taken when considering such a reform.

The reform increases the tax liability for individuals with earnings above $Z + \omega$. This generates three types of welfare-relevant effects. First, there is a mechanical welfare effect

[14] Instead, the classic approach proceeds by first deriving the allocation which maximizes welfare subject to resource and incentive constraints, and then deriving the tax system which implements the allocation. See Jacquet and Lehmann (2017) for a rigorous proof that both methods yield the same outcomes.

[15] The individuals who are affected by the marginal tax rate are those who apply for jobs which pay an income in the interval $[Z, Z + \omega]$. Since not all applicants are successful, I refer to these workers as those with earnings potential (rather than those with earnings) between Z and $Z + \omega$.

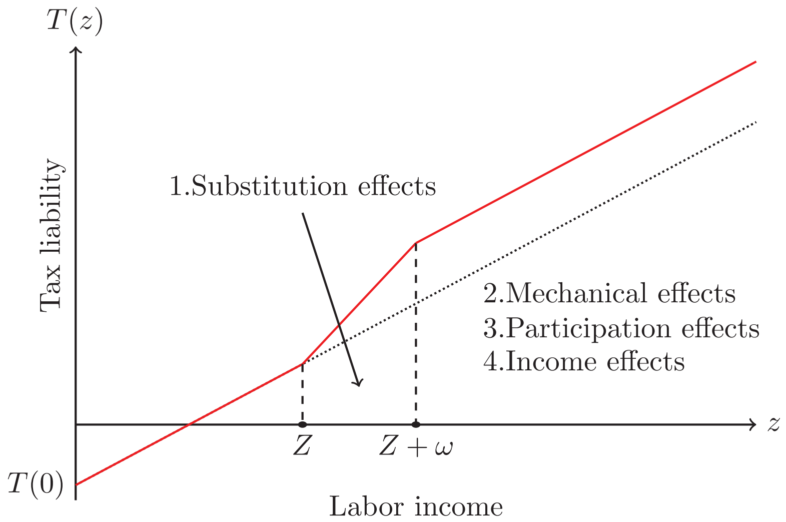

Figure 1: Variational approach

as the reform transfers income from these individuals to the government budget. Second, a higher tax liability reduces participation among individuals with earnings potential above $Z+\omega$. Because only individuals who are indifferent between participation and non-participation change their participation decisions following an increase in the tax liability, these responses have no direct utility effect. However, they do affect government revenue. Third, a higher tax liability also generates income effects in earnings and unemployment (see Proposition 1). In particular, individuals with earnings potential above $Z+\omega$ decide to search for higher-wage jobs which – through the ERE – raises unemployment. Again, by the Envelope theorem these behavioral responses do not affect individuals' expected utilities. However, because earnings and employment are taxed they do generate (welfare-relevant) fiscal externalities.

Income distribution

In Section 4.1 I use the variational approach to derive optimal tax formulas. These formulas are expressed in terms of the (observable) income distribution rather than the (unobservable) distribution of types (i.e., skills and participation costs). I denote the income distribution by $H(z)$, and the corresponding density by $h(z)$. Moreover, let $z_0 = z(n_0)$ and $z_1 = z(n_1)$ denote the lowest and highest level of positive earnings. The income and type distribution are related via:

$$H(z(n)) = \int_{n_0}^{n_1}\int_{\varphi(m)}^{\varphi_1} f(\varphi,m)d\varphi dm + \int_{n_0}^{n_1}\int_{\varphi_0}^{\varphi(m)} (1-e(m))f(\varphi,m)d\varphi dm + \int_{n_0}^{n}\int_{\varphi_0}^{\varphi(m)} e(m)f(\varphi,m)d\varphi dm. \tag{13}$$

Here, $f(\cdot)$ denotes the density of the type distribution, $e(m)$ the employment rate for individuals with ability m and the participation thresholds $\varphi(m)$ are as defined in (4). The fraction of individuals with income below $z(n)$ equals the sum of the non-participants (first term), the unemployed (second term) and the employed individuals with ability below n (third term).

Because there is non-participation and unemployment, the income distribution has a mass point at zero. The fraction of individuals with zero income equals $H(z_0)$.

Elasticity concepts

In addition to the income distribution, a second main ingredient in the optimal tax formulas are the behavioral responses with respect to changes in the tax-benefit system. With some abuse of notation, I denote by $x_{T'} = \mathrm{d}x/\mathrm{d}T'$ and $x_T = \mathrm{d}x/\mathrm{d}T$ the equilibrium changes in $x \in \{\pi, z, e\}$ following a (local) increase in the marginal tax rate and the tax liability, respectively.[16] For instance, $\mathrm{d}z/\mathrm{d}T'$ measures the impact of a change in the marginal tax rate on income, holding the tax liability (i.e., the average tax rate) fixed. Conversely, $\mathrm{d}z/\mathrm{d}T$ refers to the change in earnings following an increase in the tax liability, holding the marginal tax rate fixed.

For future references I introduce the following elasticities:

$$\varepsilon_{zT'} = -\frac{\mathrm{d}z}{\mathrm{d}T'}\frac{1-T'}{z}, \quad \varepsilon_{eT'} = \frac{\mathrm{d}e}{\mathrm{d}T'}\frac{1-T'}{e}. \tag{14}$$

Both elasticities are positive according to Proposition 1. The first is the elasticity of taxable income (ETI). As discussed before, the ETI captures both labor-supply and wage responses. The second measures the percentage decrease in the employment rate following a one-percent increase in the net-of-tax rate. This elasticity therefore quantifies the employment-enhancing effect (EEE) of taxation.

4.1 Optimal tax formulas

The next Proposition characterizes the optimal tax-benefit system.

Proposition 2. *If the tax-benefit system is optimal, the following condition must hold for all* $z \in [z_0, z_1]$*:*

$$\begin{aligned}\frac{T'(z)}{1-T'(z)} = \frac{1}{\varepsilon_{zT'}}\int_z^{z_1}\left[1-g(z')+\frac{\pi_T}{\pi}(T(z')+b)+z_T T'(z')\right]\frac{dH(z')}{zh(z)} \\ +\frac{\varepsilon_{eT'}}{\varepsilon_{zT'}}\frac{(T(z)+b)/z}{1-T'(z)}+\frac{1}{\varepsilon_{zT'}}\int_z^{z_1}\frac{e_T}{e}(T(z')+b)\frac{dH(z')}{zh(z)}.\end{aligned} \tag{15}$$

In addition, the optimum satisfies the following conditions:

$$\int_{z_0}^{z_1}\left[1-g(z)+\frac{\pi_T}{\pi}(T(z)+b)+z_T T'(z)\right]dH(z)+\int_{z_0}^{z_1}\frac{e_T}{e}(T(z)+b)dH(z)=0, \tag{16}$$

$$(g(0)-1)H(z_0)+\int_{z_0}^{z_1}\frac{\pi_b}{\pi}(T(z)+b)dH(z)+\int_{z_0}^{z_1}\left[z_b T'(z)+\frac{e_b}{e}(T(z)+b)\right]dH(z)=0. \tag{17}$$

Here, $g(z')$ denotes the average social welfare weight of individuals with earnings z' and $g(0)$ the average social welfare weight of the non-participants and involuntarily unemployed. Both

[16] These behavioral responses capture the (total) equilibrium changes along the actual budget curve, and not along a linearized budget curve (as in Saez, 2001). Hence, they account for the non-linearity of the tax schedule. See Jacquet et al. (2013) for a discussion of this issue.

are formally defined in Appendix A.3.

Proposition 2 characterizes the optimal tax-benefit system in terms of three sufficient statistics (Chetty (2009)): the income distribution, behavioral responses, and social welfare weights. The welfare weight $g(z)$ measures by how much social welfare increases if individuals with income z (possibly zero) receive an additional unit of consumption. These weights fully summarize the redistributive preferences of the government (see also Saez and Stantcheva (2016)).

Optimal marginal tax rate

Equation (15) gives the optimality condition from considering a local increase in the marginal tax rate around income level $z \in [z_0, z_1]$, as graphically illustrated in Figure 1. The formula clearly demonstrates how unemployment affects the optimal marginal tax rate. Without unemployment responses (i.e., $\varepsilon_{eT'} = e_T = 0$), both terms in the second line would cancel. The resulting optimal tax formula is then closely related to those derived in Saez (2002), Jacquet et al. (2013) and Jacobs et al. (2017), who analyze a model with labor-supply responses on the intensive and extensive margin but abstract from unemployment. For a detailed explanation of this formula, see their papers. It should be pointed out that although my optimal tax formula without unemployment responses is similar to theirs when written in sufficient statistics, the mechanisms which drive these statistics are quite different. In particular, the elasticity of taxable income (ETI) $\varepsilon_{zT'}$ captures both labor-supply and wage responses. The same is true for the effect of the tax liability on earnings (as captured by z_T). By contrast, in the aforementioned studies these statistics only capture labor-supply responses.

When matching frictions generate unemployment, the expression for the optimal marginal tax rate is modified in two ways: see the second line of equation (15). The first modification results from the employment-enhancing effect (EEE) of taxation. Intuitively, an increase in the marginal tax rate at income level z reduces wage pressure around this income level, as the reform makes individuals care less about higher wages. This leads firms to hire more workers, which increases the employment rate of individuals with earnings potential z. By how much depends on the elasticity of employment with respect to the net-of-tax rate, as captured by $\varepsilon_{eT'}$. This term is multiplied by the fiscal externality of reducing unemployment, as given by the employment tax $T(z) + b$. In the typical case that the employment tax is positive, the EEE calls for a *higher* optimal marginal tax rate.

The second modification occurs because an increase in the marginal tax rate at income level z mechanically raises tax liabilities further up in the income distribution (see Figure 1). In response, individuals with higher earnings potential choose to apply for higher-wage jobs and accept a decrease in the probability of finding employment. The magnitude of this employment-reducing effect (ERE) of taxation is captured by the responsiveness of the employment rate with respect to the tax liability, as given by e_T. As before, the behavioral response is multiplied by the employment tax. If the product of these terms averaged over all individuals with earnings above z is negative (which is to be expected, as $e_T < 0$ and typically $T(z') + b > 0$), the ERE calls for a *lower* the optimal marginal tax rate.

Which of these forces dominates depends critically on two types of statistics: (i) the responsiveness of the employment (or unemployment) rate with respect to the marginal and average tax rate (holding the other fixed) and (ii) the (relative) hazard rate of the income distribution.[17] To see this, multiply the expression for the optimal marginal tax rate (15) by $\varepsilon_{zT'}$. The last two terms on the right-hand side can then be written as:

$$\frac{e_{T'}}{e}\left(\frac{T(z)+b}{z}\right)+\mathbb{E}\left[\frac{e_t}{e}\left(\frac{T(z')+b}{z'}\right)\middle|z'>z\right]\frac{1-H(z)}{zh(z)}. \tag{18}$$

Here, e_t measures the impact of a change in the average tax rate $t = T(z)/z$ on the employment rate, holding the marginal tax rate fixed. In addition to the fiscal externalities associated with changes in unemployment (as captured by $(T(z) + b)/z$), equation (18) depends on the semi-elasticity of the employment rate with respect to the marginal and average tax rate (as captured by $e_{T'}/e$ and e_t/e) and the relative hazard rate $zh(z)/(1 - H(z))$ of the income distribution. The latter is critical for quantifying the EEE and ERE as it captures how many people are affected by an increase in the marginal tax rate at income level z compared to those who see their tax liability increase, i.e., those with earnings above z. If employment taxes are positive, unemployment is therefore more likely to reduce optimal marginal tax rates if the hazard rate of the income distribution is low – as is the case at low levels of income.

Many studies find that at high levels of income, the relative hazard rate is approximately constant (see, e.g., Saez (2001)). This implies the top of the income distribution is well approximated by a Pareto distribution. Equation (15) can then readily be manipulated to obtain an expression for the optimal top rate (see Appendix A.4 for details).

Corollary 1. *If incomes at the top are Pareto distributed with tail parameter a and if the elasticities of earnings, employment and participation with respect to (one minus) the marginal and average tax rate converge, the optimal top rate is given by:*

$$T'(z)=\frac{1-g(z)}{1-g(z)+a(\varepsilon_{zT'}-\varepsilon_{eT'})+\varepsilon_{\pi t}+\varepsilon_{zt}+\varepsilon_{et}}, \tag{19}$$

where ε_{xt} denotes the elasticity of $x \in \{\pi, z, e\}$ with respect to one minus the average tax rate.

Equation (19) generalizes the results of Jacquet et al. (2013) and Jacobs et al. (2017), who abstract from unemployment responses and who assume away income effects for high-income earners (in which case $\varepsilon_{zt} = 0$). Whether the optimal top rate is higher or lower if unemployment is taken into account depends on a very simple condition. In particular, for a given welfare weight, earnings and participation elasticities for top-income earners, the optimal top rate is higher compared to a setting without unemployment if and only if the following condition holds:

$$a\varepsilon_{eT'} > \varepsilon_{et}. \tag{20}$$

This condition is intuitive. If the employment tax is positive (as is always the case at the top), the employment-enhancing effect raises the optimal top rate, whereas the employment-reducing effect does the opposite. The optimal top rate with unemployment is therefore higher if the responsiveness of employment with respect to the marginal tax rate is high relative to the

[17] Note that the employment elasticities can easily be transformed into *un*employment elasticities by multiplying it with $e/(1-e)$.

average tax rate (i.e., if $\varepsilon_{eT'}$ is high relative to ε_{et}) and if the tail of the income distribution is thin (i.e., if a is high). A high Pareto parameter a implies that an increase in the marginal tax rate reduces the employment prospects of only only a few people further up in the income distribution. While the presence of unemployment leads to an intuitive adjustment of the expression for the optimal top rate, it should be noted that the quantitative implications are likely to be small if unemployment is not an important margin for individuals with high ability (as one might expect). This will be confirmed in Section 5.

Employment taxes and the optimality of an EITC

Equation (16) gives the optimality condition from considering a uniform increase in the tax liability. Compared to a setting without unemployment, the only modification of this condition is due to the employment-reducing effect (ERE), as captured by the second term. For a given distribution of income, welfare weights, and behavioral responses, the optimal tax liability (and hence the employment tax) is therefore lower if unemployment is taken into account. Intuitively, a lower tax liability reduces the incentives for individuals who participate to look for higher-wage jobs. In response, firms post more vacancies and employment increases. Provided the employment tax is positive on average, the reduction in the tax liability generates a positive fiscal externality which would be absent if there are no unemployment responses.

Combined, equations (15) and (16) have an important implication for the optimal design of employment subsidies (such as the EITC).

Proposition 3. *If employment is subsidized for low-income workers, the optimal marginal tax rate at the bottom is negative. Hence, it is optimal to let employment subsidies (such as the EITC) phase in with income.*

In two influential papers, Diamond (1980) and Saez (2002) show that employment for low-income workers is optimally subsidized if labor-supply responses are concentrated (mostly) along the extensive margin and if the government cares sufficiently about the working poor. These papers thus explain why the *level* of the employment tax can be negative for low-income workers. Proposition 3 complements this result by showing that if employment is optimally subsidized, the optimal *marginal* tax rate for low-income workers is negative as well.[18] Intuitively, a negative marginal tax rate off-sets the upward distortion in employment generated by employment subsidies. This distortion occurs because employment subsidies (i.e., high in-work benefits) induce workers to apply for low-wage jobs. The associated increase in employment generates a negative fiscal externality if employment is subsidized. A negative marginal tax rate is then optimal as it makes applying for high-wage jobs more attractive. The associated reduction in employment positively affect government finances.

To see how Proposition 3 and the results from Diamond (1980) and Saez (2002) are linked, consider a reform which decreases the marginal tax rate at a low income level z combined with an increase in the intercept of the tax function which ensures the net income of individuals with earnings above z is unaffected. The optimality condition associated with this reform (which is

[18] In a model without unemployment, Hansen (2017) shows that the optimal EITC may also feature a phase-in region if participation elasticities are decreasing in the skill dimension.

obtained by combining equations (15) and (16)) is:

$$\varepsilon_{zT'} \frac{T'(z)}{1-T'(z)} z h(z) = \int_{z_0}^{z} \left[g(z') - 1 - \frac{\pi_T}{\pi}(T(z')+b) + z_T T'(z') \right] dH(z')$$

$$+ \varepsilon_{eT'} \frac{T(z)+b}{1-T'(z)} h(z) - \int_{z_0}^{z} \frac{e_T}{e}(T(z')+b) dH(z'). \tag{21}$$

From equation (21), it is clear that it is optimal to subsidize employment (i.e., to set $T(z')+b < 0$) if the government cares a lot about the working poor (i.e., if $g(z') > 1$ at low levels of income) and if earnings and unemployment responses are absent (i.e., if $\varepsilon_{zT'} = z_T = \varepsilon_{eT'} = e_T = 0$). This finding goes back to Diamond (1980), who was the first to provide a rationale for employment subsidies (such as the EITC) in an optimal-tax framework. The terms in the second line of equation (21) highlight why it might be optimal to let the EITC phase in with income if unemployment is taken into account. First, a negative marginal tax rate raises wages and reduces employment around income level z through the EEE. If employment is subsidized, this generates a positive fiscal externality. Second, the reduction in the marginal tax rate allows the government to increase the tax liability. Through the ERE, this lowers employment among individuals with earnings potential below z.[19] Again, this generates a positive fiscal externality. Hence, if employment is subsidized, the fiscal externalities of the EEE and ERE go hand in hand and the optimal marginal tax rate for low-income workers is unambiguously lower if unemployment is taken into account. When evaluated at $z = z_0$, equation (21) immediately implies that the optimal marginal tax rate is negative at the bottom if employment is subsidized (confirming the result from Proposition 3).

Optimal unemployment insurance

The results from Proposition 2 are related to those obtained in Baily (1978) and Chetty (2006). They study the optimal provision of unemployment insurance in a model where (identical) risk-averse individuals face an uninsurable risk of becoming unemployed. The government optimally provides UI payments, which are financed through a lump-sum or proportional tax on labor income. The optimal benefit trades off the insurance gains against the distortionary costs of UI on job search. To see how my results are related to theirs, assume all individuals are identical and participate (i.e., $n_0 = n_1$ and $\varphi_0 = \varphi_1$ sufficiently low). Equations (16) and (17) then simplify to:[20]

$$e(1-g(z)) + e\left[z_T T'(z) + \frac{e_T}{e}(T(z)+b) \right] = 0, \tag{22}$$

$$(1-e)(g(0)-1) + e\left[z_b T'(z) + \frac{e_b}{e}(T(z)+b) \right] = 0. \tag{23}$$

If individuals are risk-averse, consumption in the state of unemployment is valued more than in the state of unemployment. As a result, $g(z) > g(0)$ and the government optimally provides

[19] Recall: the joint reduction in the marginal tax rate around income z and the reduction in the tax liability leaves individuals with earnings potential above z unaffected.

[20] The income distribution has a mass point at zero. Consequently, $H(z_0) = 1 - e$ and the income distribution between z_0 and z_1 integrates to e: $H(z_1) - H(z_0) = e$.

unemployment insurance. As in the Baily-Chetty framework, the government balances the insurance gains against the distortionary costs of UI on employment (as captured by e_b and e_T). I show in Appendix A.6 how the Baily-Chetty formula (see Proposition 1 in Chetty (2006)) is recovered if UI payments are financed by lump-sum taxes (i.e., $T'(z) = 0$). If this is not the case and if UI also affects wages (as is the case in my model), the Baily-Chetty formula is modified to take into account the fiscal externalities associated with earnings responses.

The fact that I do not restrict UI payments to be financed by lump-sum or proportional taxes on labor income has another important implication.

Proposition 4. *Suppose all individuals are identical and decide to participate (i.e., $n_0 = n_1$ and $\varphi_0 = \varphi_1$ sufficiently low). The optimal marginal tax rate then satisfies the following inverse-elasticity rule:*

$$\frac{T'(z)}{(T(z)+b)/z} = \frac{\varepsilon_{eT'}}{\varepsilon_{zT'}}. \tag{24}$$

Moreover, financing UI payments through lump-sum or proportional taxes on labor income is generally sub-optimal.

Proposition 4 shows there exists a close link between the optimal provision of unemployment insurance and the shape of the tax schedule – even without income heterogeneity. I am not aware of any other paper which highlights this link.[21] Instead the vast majority of the literature on the optimal provision of UI assumes benefit payments are either financed by lump-sum or proportional taxes on labor income (see, e.g., Baily (1978), Flemming (1978), Shavell and Weiss (1979), Hopenhayn and Nicolini (1997), Acemoglu and Shimer (1999), Chetty (2006, 2008)). Proposition 4 states that doing so is generally sub-optimal. Intuitively, the government wants to use the marginal tax rate to partially off-set the upward distortion in unemployment generated by UI. If the government provides such insurance (which is optimal if individuals are risk-averse), the employment tax is positive. This reduces the utility difference between employment and unemployment and thereby induces individuals to apply for high-wage jobs. Through the ERE, employment decreases which generates a negative fiscal externality. It is then optimal for the government to set a positive *marginal* tax rate as well. Doing so reduces the attractiveness of applying for a job which specifies a high wage. Consequently wages drop and – through the EEE – employment increases. The optimal marginal tax rate satisfies a simple inverse-elasticity rule (see equation (24)). Naturally, it increases in the fiscal externality of raising employment and the responsiveness of employment with respect to the marginal tax rate and decreases in the elasticity of taxable income (ETI). This result immediately implies that lump-sum taxes are sub-optimal. The same is generally true for proportional taxes, except in special cases.[22]

[21] Golosov et al. (2013) also study the link between unemployment insurance and the shape of the tax schedule. However, they do so in a framework where search frictions generate heterogeneity in wages among equally skilled workers. By contrast, in my model there is no heterogeneity in wages if individuals are identical.

[22] Intuitively, this is because the *level* of the tax function serves to finance UI payments whereas the *slope* is used to partially off-set the distortions of UI on unemployment. However, if the tax function is proportional (i.e., $T(z) = tz$) the level and the slope cannot be set separately.

5 Quantitative analysis

This section explores the quantitative implications of unemployment for the (optimal) design of the tax-benefit system. The purpose is twofold. The first is to get a sense of the importance of unemployment considerations (i.e., the EEE and ERE) in the *current* tax-benefit system. The second is to analyze the *optimal* tax-benefit system if unemployment is taken into account.

5.1 Calibration

The model is calibrated to the US economy. The data source I use is the March release of the 2016 Current Population Survey (CPS). This data set provides detailed information on earnings, taxes and benefits for a large sample of individuals. Importantly, it also provides information on individuals' employment status (i.e., employment, unemployment, or not in the labor force). The participation rate is 86.2% and the unemployment rate (conditional on participation) equals 5.1%. For the individuals with positive earnings, I focus on full-time employees who earn at least the federal hourly minimum wage of \$7.25. A detailed description of the sample selection procedure can be found in Appendix A.8.

In the model, earnings, participation and unemployment all vary with ability. Naturally, in the data I only observe individual earnings and their employment status, but not their ability or probability of having found employment. To get an estimate of these, I invert the first-order conditions (6) and (7) for each individual with positive earnings. This gives a distribution of abilities which is consistent with the empirical income distribution. Doing so requires specifying the current tax-benefit system, functional forms for the utility and matching function, and a value for the costs of opening a vacancy k. Moreover, to get an estimate of the participation rate at different ability levels (as given by equation (5)), I require an empirical counterpart of the distribution of participation costs. I discuss each of these inputs in turn.

Tax-benefit system

As in Saez (2001) and Sleet and Yazici (2017), I approximate the current US tax schedule by regressing total taxes paid on taxable income. This gives an estimate of a (constant) marginal tax rate of 34.3% and a negative intercept of \$3,663. These numbers imply that individuals start paying taxes if their annual income exceeds $\$10,679$. The value for the benefit is set at \$4,605, which equals the average income from unemployment compensation for individuals who received such income and whose reported labor force status is unemployment.

Functional forms and the costs of opening a vacancy

The utility function takes a simple quasi-linear and iso-elastic form:

$$u(c) - v\left(\frac{y}{n}\right) = c - \frac{\left(\frac{y}{n}\right)^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}}. \tag{25}$$

The value for the labor-supply elasticity is set at $\varepsilon = 0.33$ (Chetty, 2012).[23] For the matching technology, I use the following specification:

$$e(\theta) = (\theta^{\gamma} + 1)^{\frac{1}{\gamma}}, \tag{26}$$

which can be inverted to get $\theta(e)$. Equation (26) can be derived from an aggregate matching function where the elasticity of substitution between vacancies and unemployment is constant, see also Den Haan et al. (2000) and Hagedorn and Manovskii (2008).[24] The main advantage of this functional form over the often-used Cobb-Douglas specification is that the employment rate is bounded between 0 and 1, provided $\gamma < 0$. This is important in the current application, because in my model the employment rates vary along the income distribution (i.e., vary with ability) and because I consider potentially large policy changes (in particular from the current tax-benefit system to the optimal one). The value of γ is used to target a matching elasticity of 0.3 at the current aggregate rate of unemployment (Petrongolo and Pissarides (2001)).

The costs of opening a vacancy k is calibrated to match the unemployment rate of 5.1% in the data. The corresponding value is $\$10,250$. While I refer to these as the costs of opening a vacancy, in the model they can also be thought to include more indirect costs associated with hiring a new worker, such as those related to recruitment and training. The reason why these costs are fairly substantial, is because the calibrated value of $\gamma = -6.81$ implies a low degree of substitutability between vacancies and unemployment.[25] If this is the case, vacancies are very efficient in generating matches. The costs of opening one must therefore be high in order to match the unemployment rate of 5.1%. I analyze an example with a higher elasticity of substitution and lower vacancy creation costs in Section 5.4.

Distribution of ability and participation costs

Ability and participation costs are assumed to be independently distributed. The joint distribution $F(\varphi, n)$ is then obtained in two steps. First, I assume ability follows a log-normal distribution up to the level associated with \$250,000 in annual earnings, above which I append a Pareto tail. The parameters μ and σ are estimated using maximum likelihood on the (censored) ability distribution. The latter is obtained jointly with the employment rates from the empirical income distribution by inverting the first-order conditions (6) and (7). The value of the Pareto coefficient a^* of the *ability* distribution is set in such a way that the Pareto coefficient of the *income* distribution is $a = 1.5$ in the absence of frictions (Piketty et al. (2014), Saez and Stantcheva (2016)).[26] This implies a value of $a^* = 2$. The scale parameter of the Pareto distribution ensures the density is continuous at the point where the Pareto tail is pasted.

[23] Without frictions, the absence of income effects ensures ε corresponds to both the compensated and uncompensated elasticity of labor supply.

[24] To see this, let $m(b, s) = (b^{\gamma} + s^{\gamma})^{1/\gamma}$ denote the number of matches for a given number of buyers b (i.e., vacancies) and sellers s (i.e., job-seekers). The elasticity of substitution between buyers and sellers is constant and equal to $1/(1-\gamma)$. The probability that a seller is matched (i.e., the employment rate) is then given by $m(b, s)/s = ((b/s)^{\gamma} + 1)^{1/\gamma}$ or $e = (\theta^{\gamma} + 1)^{1/\gamma}$, where θ equals the buyer-seller ratio (i.e., the ratio of vacancies to job-seekers).

[25] The constant elasticity of substitution equals approximately 0.13.

[26] If there are no frictions, no income effects (i.e., quasi-linear utility) and if the tax system is proportional, the tail parameter of the income distribution is $a = a^*/(1 + \varepsilon)$.

Second, I assume the distribution of participation costs is such that the participation rate is iso-elastic with respect to the *utility* difference $\varphi(n) = \mathcal{U}(n) - u(b)$:[27]

$$\pi(n) = A\varphi(n)^{\eta}. \tag{27}$$

The latter is capped at a maximum value of one. Besides its simplicity, an additional benefit of using this functional form is that, in line with the empirical evidence, it generates a decreasing pattern of participation elasticities (i.e., the percentage increase in the participation rate if the *consumption* difference increases by 1%). See, for instance, Meyer and Rosenbaum (2001) and Meghir and Phillips (2006). I calculate A to target an aggregate participation rate of 86.2% in the data, and η to target a participation elasticity of 0.15 at the median of the income distribution (corresponding to $50,000 in annual earnings). This value is below the one reported in Chetty et al. (2011), who base their estimate of 0.25 on an extensive meta analysis. However, in the analysis I focus on full-time employees, who are typically found to be less responsive. Moreover, the value of 0.15 masks substantial heterogeneity: the implied participation elasticity is around 0.21 at the bottom of the income distribution, and 0 at income levels above $192,000.

Participation and employment rates by income

Figure 2 plots the employment rates for different income levels up to $100,000. As mentioned before the employment rates are obtained from inverting the first-order conditions (6) and (7) for each level of positive earnings observed in the data. Because the model predicts that individuals with higher ability earn more and are less likely to be unemployed, the relationship is increasing. In particular, the current calibration suggests that around 15% of the participants with the lowest earnings ability are unemployed, whereas the unemployment rate of individuals with an earnings potential of $100,000 is only 2.5%, and drops further to 1.4% for individuals with earnings potential $200,000 (not plotted). Figure 2 also shows the (non-targeted) distribution of average earnings and employment rates for individuals with different educational attainments. The categories are the following: less than high school, high school, some college, college, advanced. In line with the model, the data shows an increasing relationship.

Figure 3 plots the participation rates at different levels of income, which are obtained from equation (27). As before, the model predicts an increasing relationship: individuals with higher ability earn more and are more likely to participate. In the current calibration, the participation rate among individuals with the lowest skill level is around 70% and it increases monotonically to 100% for individuals with earnings potential above $192,000 (not plotted). The predictions from the model are contrasted with the (non-targeted) profile of earnings and participation rates of individuals with different educational backgrounds. Again, in line with the model, the data shows an increasing relationship.

[27]This expression for the participation rate is obtained if the lowest participation costs is $\varphi_0 = 0$ and the (conditional) density is $\eta A \varphi^{\eta-1}$.

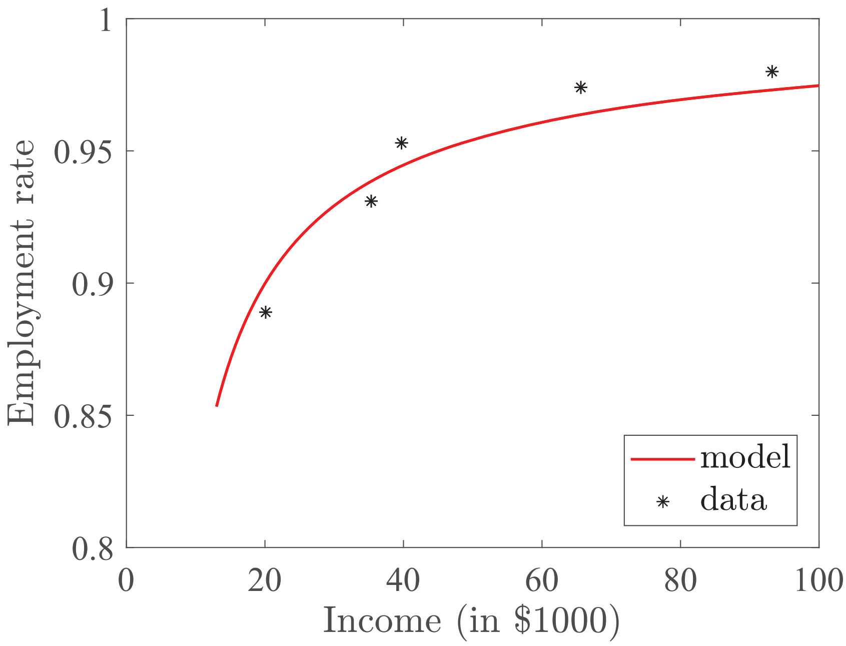

Figure 2: Employment rates by income

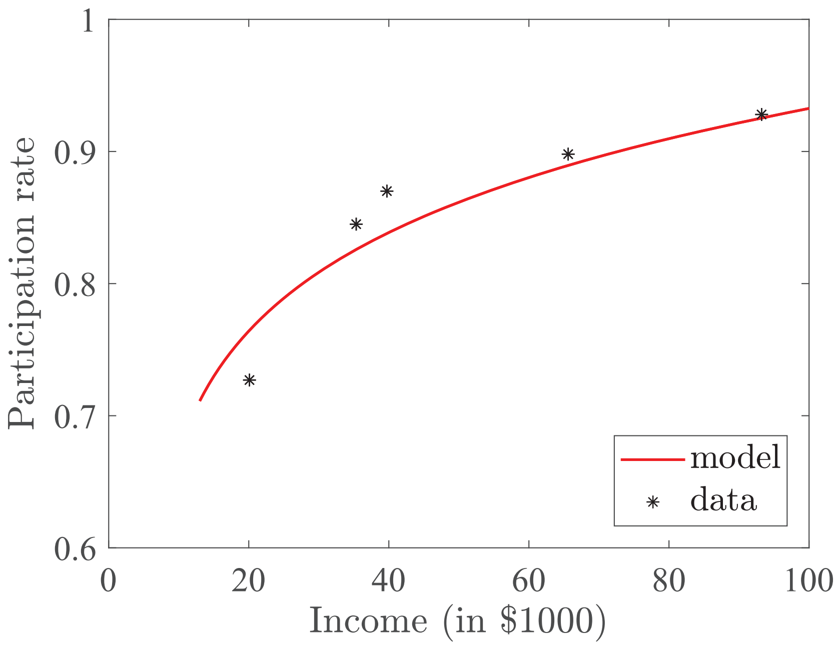

Figure 3: Participation rates by income

5.2 Unemployment responses in the current tax-benefit system

To get a sense of the quantitative importance of the employment-enhancing effect (EEE) and the employment-reducing effect (ERE), Figure 4 plots the employment responses to taxation at the current tax-benefit system. To facilitate the comparison, I plot the elasticity of employment with respect to one minus the marginal and one minus the average tax rate, both in absolute value (i.e., $\varepsilon_{eT'}$ and ε_{et} as defined in equation (14) and Corollary 1). Two points are worth mentioning.

First, in the calibrated model employment (or unemployment) is more responsive to changes in the average tax rate than to changes in the marginal tax rate. Second, the elasticities are decreasing in income. In particular, the numbers suggest that a one percentage point increase in the marginal tax rate (holding the average tax rate fixed) reduces the unemployment rate by 0.11 percentage points for the lowest skill type and by 0.05 percentage points around the median of the income distribution.[28] The corresponding figures for a one percentage point increase in the average tax rate (holding the marginal tax rate fixed) are a 0.25 and 0.07 percentage points increase in the unemployment rate.

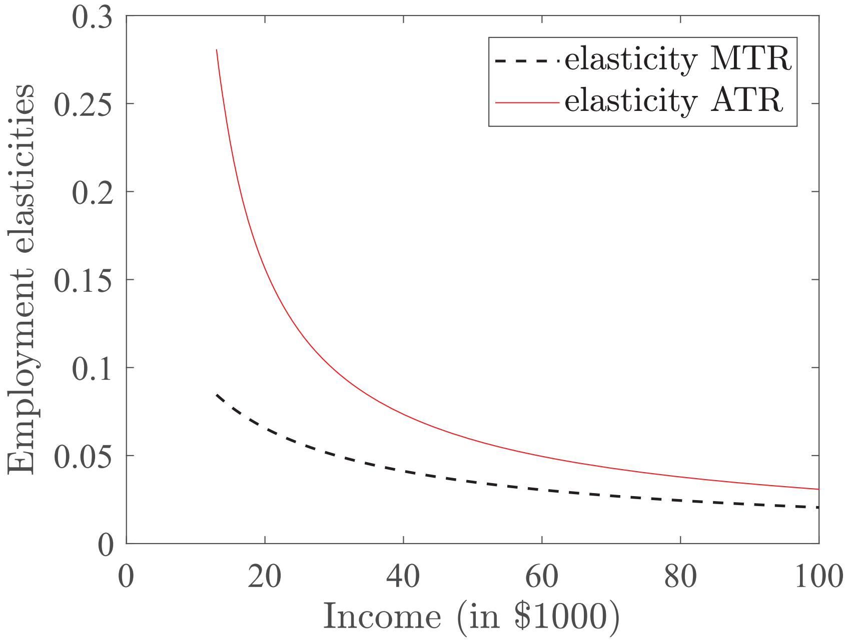

Figure 4: Employment elasticities

How the government should take into account unemployment responses ultimately depends on the fiscal externalities associated with the EEE and the ERE: see equation (15). Figure 5 plots these, as well as the total effect. In particular, it shows – for different levels of income z – the budgetary effects due to the unemployment responses following a one percentage point increase in the marginal tax rate in the interval $[z, z + \Delta]$. For ease of interpretation, I set $\Delta = \$100$, so that the tax reform raises the tax liability of individuals with earnings above $z + \Delta$ by exactly \$1. On the one hand, the reform raises tax revenue because it reduces wage pressure and hence raises employment in the interval $[z, z+\Delta]$ (through the EEE). The top line shows the budgetary effect. Since the effect is proportional to the density, the shape is similar to that of the income distribution (which is approximately log-normal). On the other hand, the implied rise in the average tax rates for incomes above $z + \Delta$ raises wage pressure and hence unemployment for individuals with higher earnings potential (through the ERE). The bottom line shows by how much government revenue is affected. As can be seen from the figure, the revenue effect is larger at low levels of income. This happens for two reasons. First, an increase

[28]Note: these numbers are obtained from, but not equal to, the ones shown in Figure 4. The latter plots the percentage decrease (increase) in the employment rate for a one percent increase in one minus the marginal (average) tax rate.

in the marginal tax rate at some income level raises only the average tax rates of individuals with higher income. This also explains why the shape is more similar to that of the cumulative income distribution. Second, employment responses to the average tax rates are declining in income: see Figure 4.

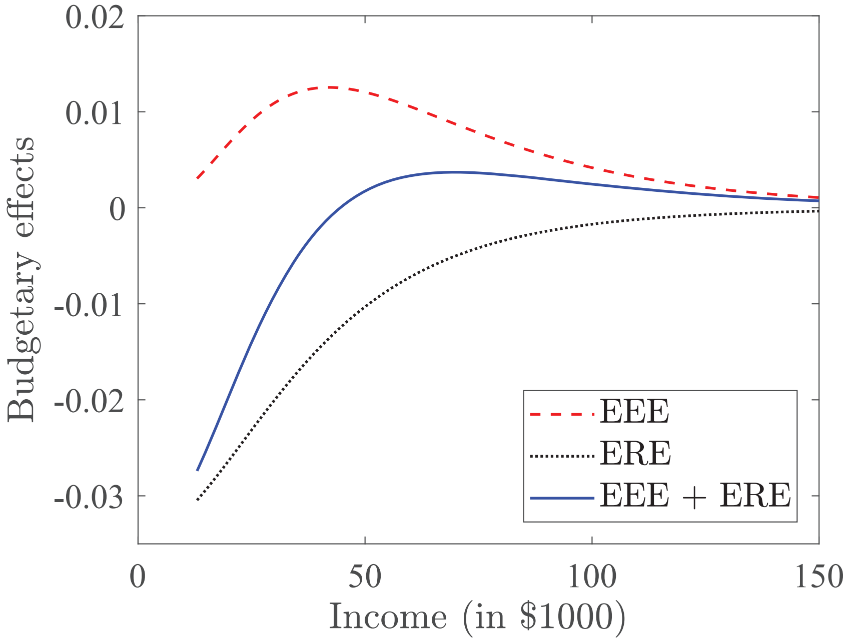

Figure 5: Budgetary effects

Figure 5 also plots the sum of both effects. Two points are noteworthy. First, the net revenue effect is virtually zero for reforms which (locally) increase the marginal tax rate at income levels above the median (i.e., $50,000). Unemployment responses to taxation therefore only appear important to consider when changing tax rates for low-income workers. Second, the negative revenue effect of the ERE is much stronger than the positive effect of the EEE at the bottom of the income distribution. In particular, the government loses close to $0.03 due to unemployment responses if it aims to collect $1 more from (almost) all employed individuals by raising the marginal tax rate at the bottom. The reason why the ERE dominates the EEE is threefold. First, unemployment is more responsive to changes in the average tax rate than to changes in the marginal tax rate (see Figure 4). Second, the hazard rate is low at low levels of income. Consequently, an increase in the marginal tax rate improves the employment prospects of only a few individuals, whereas the implied increase in the average tax rate raises the unemployment rate at virtually all other income levels. Third, the fiscal externality due to unemployment responses increases in income. As a result, the increase in unemployment at higher income levels generates a larger revenue effect than the reduction in unemployment at low levels of income.

Figure 6 compares the net revenue effects due to unemployment responses (i.e., the sum of EEE and ERE) to those associated with earnings responses (the typical focus in the optimal tax literature). On the one hand, an increase in the marginal tax rate at some income level reduces both effort and wages around that income level. This leads to a reduction in revenue. On the other hand, the associated increase in the average tax rate raises wages further up in the income distribution, which increases tax revenue. As can be seen from Figure 6, the net revenue effect is negative and follows the shape of the income distribution. This suggests the first of these effects dominates. Moreover, the revenue effects due to earnings responses are significantly larger (in absolute value) than those associated with unemployment responses, *except* at low levels of income. For instance, raising the marginal tax rate around the median income level (so as to generate \$1 dollar of revenue from individuals with higher income) costs the government close to \$0.14. At the bottom, however, the revenue effect due to earnings responses is negligible, whereas the revenue effect due to the EEE and ERE is close to \$0.03 (see also Figure 5). Until roughly \$18,000 in annual earnings, the net revenue effect due to unemployment responses is larger (in absolute value) than the effect due to earnings responses.

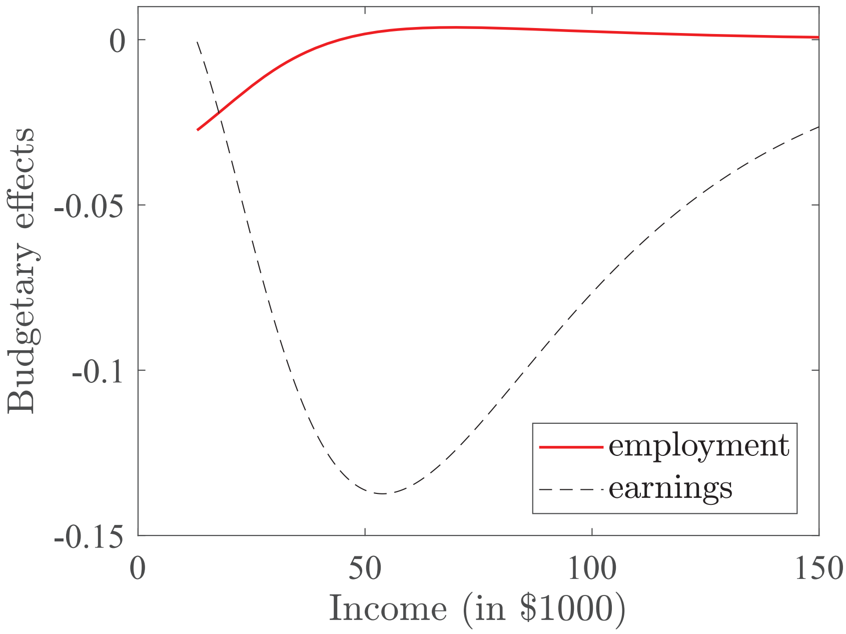

Figure 6: Budgetary effects

5.3 A quantitative analysis of optimal taxes

How does the optimal tax-benefit system look like and how does it compare to a setting where unemployment is not taken into account? Answering these questions requires a specification of the government's objective function as well as the revenue requirement G. The latter is calibrated to ensure the budget constraint (8) holds at the current tax-benefit system. The implied value equals $G = \$15,157$, which corresponds to roughly 21.2% of average annual earnings. For the welfare function (12), I use the following:

$$\Psi(U) = \frac{U^{1-\sigma} - 1}{1 - \sigma}, \tag{28}$$

where $\sigma > 0$ reflects the degree of inequality aversion. In the baseline calibration, I set $\sigma = 0.5$.[29] I solve the government's optimization problem by maximizing social welfare (12) subject to the budget constraint (8), taking into account the behavioral responses as summarized in Table 1. Further details on the numerical optimization procedures can be found in Appendix A.8.

Figure 7 plots the optimal marginal tax rates $T'(z)$ up to \$300,000 in annual earnings. The tax rates clearly follow the conventional U-shape pattern (Diamond (1998), Saez (2001)). They are quickly decreasing until modal income, stay roughly constant up to \$150,000, and start increasing afterwards. The optimal top rate is around 58% in the current calibration. The latter applies at income levels above \$215,000, which is the point where the Pareto tail starts at the optimal allocation.[30] The reason why optimal tax rates follow a U-shape pattern is largely due to the behavior of the hazard rate of the income distribution: see Saez (2001).

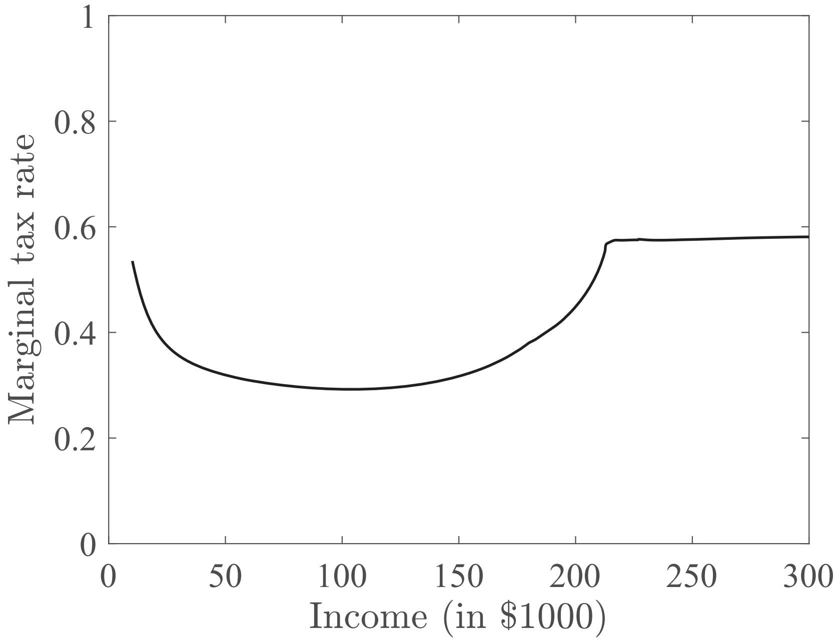

Figure 7: Optimal marginal tax rates

Figure 8 plots the optimal average tax rates $T(z)/z$, again for income levels up to \$300,000. The average tax rates are negative at low income levels, remain more or less flat between \$50,000 and \$215,000 (i.e., the region where the marginal tax rates are approximately constant) and start increasing again at the point where the Pareto tail starts. Given the degree of inequality aversion (i.e., given $\sigma = 0.5$), the optimal tax-benefit system is more redistributive than the current one. In particular, average tax rates are lower than in the current system up to roughly \$222,000 in annual earnings (not plotted). Moreover, the average tax rate remains negative up to approximately \$15,000 in annual earnings at the optimal tax-benefit system, whereas the corresponding figure in the current tax-benefit system is \$10,679. Also the optimal unemployment benefit is higher than in the current system (i.e., \$6,086 compared to \$4,605).

[29]This value implies the government is indifferent between giving \$1 to an individual whose net income is x and giving \$0.71 to an individual whose net income is $x/2$.

[30]Note that the Pareto tail starts at \$250,000 in annual earnings at the *current* tax-benefit system. At the *optimal* tax-benefit system, the earnings for individuals with that ability are approximately \$35.000 lower.

As a result, the optimal employment tax (as given by the sum of $T(z) + b$) is always positive in the current calibration. Hence, subsidizing employment for low-income workers (through an EITC type of policy) is not optimal given the current specification of social welfare.

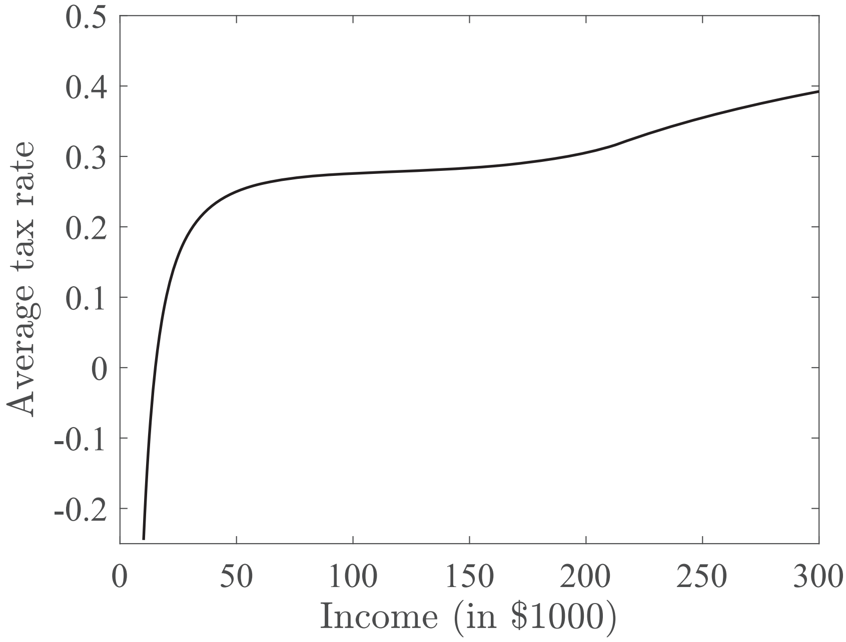

Figure 8: Optimal average tax rates

Comparison to the tax schedule without unemployment

How does the optimal tax-benefit system compare to the one that would be obtained if unemployment is not taken into account? To answer that question, I also calculate the optimal tax-benefit system assuming there are no frictions. In this case, the costs of opening a vacancy are zero and all individuals who participate find employment (i.e., $k = 0$ and $e(n) = 1$ for all n). I recalibrate the ability distribution to make it consistent with the empirical income distribution. The current tax-benefit system, utility function and procedures to calibrate the distribution of participation costs and the revenue requirement are the same as before.

Figure 9 compares the optimal marginal tax rates with and without unemployment. In both cases, the optimal tax rates follow a very similar U-shape pattern and converge to virtually the same top rate. This suggests unemployment responses at the top are quantitatively unimportant. For incomes up to $200,000, the optimal tax rates are somewhat lower if unemployment is taken into account. The largest discrepancies are found at very low levels of income (where the optimal tax rate is considerably higher if there are matching frictions), and at income levels between $100,000 and $200,000. These results should be interpreted with caution, for at least two reasons. First, the differences in optimal tax rates cannot simply be attributed to the fiscal externalities associated with unemployment responses. The reason is that the condition for the optimal tax rate (15) is expressed in terms of endogenous objects. The income distribution, fiscal externalities, welfare weights, and elasticities all vary with the tax-benefit system. In addition, the ability distribution and revenue requirement are recalibrated to match the empirical income distribution and ensure the government's budget constraint holds. Second, as will be shown below the difference between the optimal marginal tax rates with and without unemployment are much smaller under an alternative calibration of the matching function.

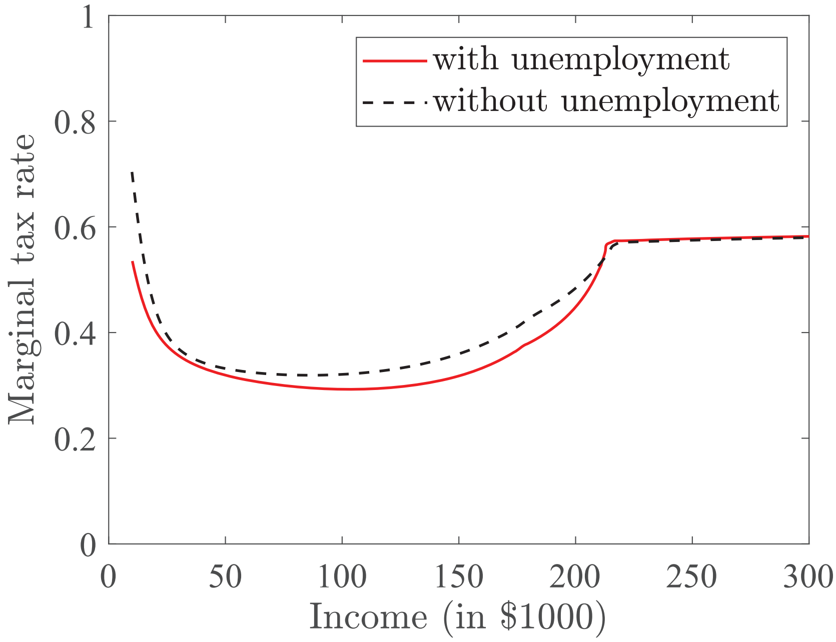

Figure 9: Comparison optimal marginal tax rates

5.4 An example with low vacancy costs

In the current calibration firms incur significant costs to get their vacancies filled. For the median worker, they amount to roughly 20% of output, leaving around 80% to be paid in wages (recall: firms make zero profits).[31] As mentioned before, this is because the low degree of substitutability between vacancies and unemployment implies vacancies are very efficient in generating matches. The vacancy costs must then be substantial to match the unemployment rate in the data. This is not the case if the elasticity of substitution between vacancies and unemployment is larger. Suppose, for instance, the latter is increased from around 0.13 to 0.44 (as in Den Haan et al., 2000).[32] In order to get an unemployment rate of 5.1%, the costs of opening a vacancy equals $k = \$255$. For the median worker, the costs of getting a vacancy filled then amount to approximately 4.4% (compared to 20% in the baseline calibration). I now repeat part of the analysis using these values. As before, the ability distribution is recalibrated and I use the same tax-benefit system, utility function and procedure to calibrate the distribution of participation costs and the revenue requirement.

Figure 10 shows the relationship between employment rates and earnings when vacancy costs are lower. For comparison, I also plot the relationship in the baseline calibration and the profile of employment rates and earnings for different education levels (see Figure 2). Employment

[31]This number differs from $k = \$10,250$ as a fraction of median earnings $50,000, because the expected costs of getting a vacancy filled equal $\theta(e)k/e$ and not k.

[32]They use a dynamic model and calibrate a value of $\gamma = -1.27$ to match statistics job separation and firm and worker match probabilities. The implied CES between vacancies and unemployment is approximately 0.44.

rates are increasing less quickly in earnings if vacancy costs are low. Moreover, the baseline calibration appears to do a better job in capturing the profile of earnings and employment for different education levels. In particular, the unemployment rate at the average earnings level of individuals with a low educational attainment is understated relative to their actual unemployment rate, and *vice versa* for high income levels.

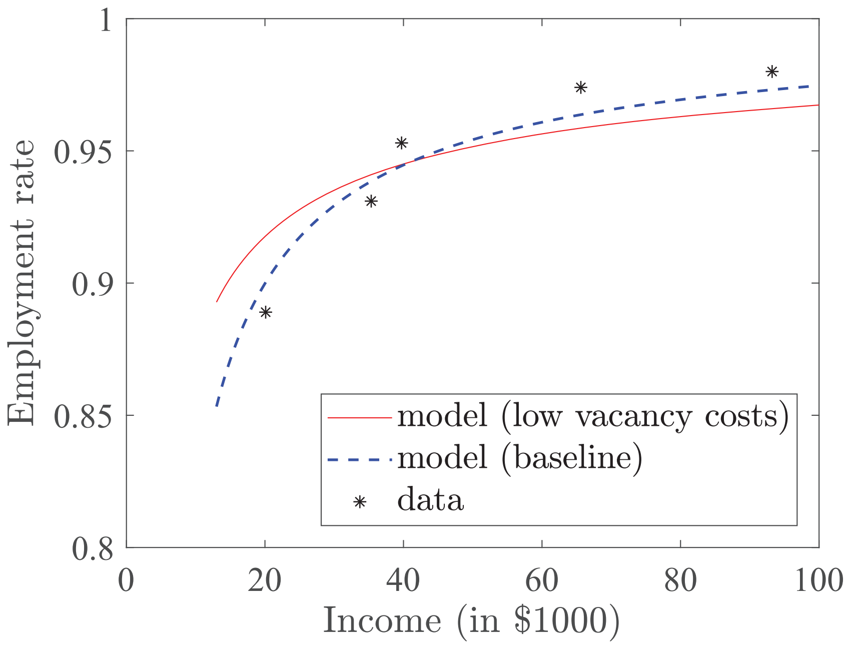

Figure 10: Employment rates by income

A direct implication is that unemployment is less responsive to changes in the tax-benefit system if vacancy costs are low. Consequently, the revenue effects associated with employment responses are smaller as well. This is graphically illustrated in Figure 11, which compares the sum of the EEE and ERE in the calibration with low vacancy costs and in the baseline calibration. If vacancy costs are low, raising the marginal tax rate at the bottom of the income distribution (so as to generate $1 from almost all working individuals) implies a reduction of $0.02 in revenue due to unemployment responses. The corresponding figure in the baseline calibration is close to $0.03. As before the net revenue effect is close to zero at higher income levels, suggesting again that unemployment responses to taxation are most relevant to consider when setting taxes at low income levels.

Finally, Figure 12 compares the optimal marginal tax rates with those obtained in the baseline calibration and assuming away frictions (i.e., abstracting from unemployment). The final two are also shown in Figure 9. If vacancy costs are low, the optimal tax rates are very similar to the ones that would be obtained if unemployment is not taken into account (except possibly at the very bottom). Hence, the finding that *optimal* tax rates are lower if unemployment is taken into account (as is suggested by Figure 9) is not particularly robust. However, this does not imply that unemployment responses should not be taken into account when considering reforms to the *current* tax-benefit system. In fact, the findings presented in Figures 5, 6 and 11 suggest unemployment responses to taxation are important to consider when

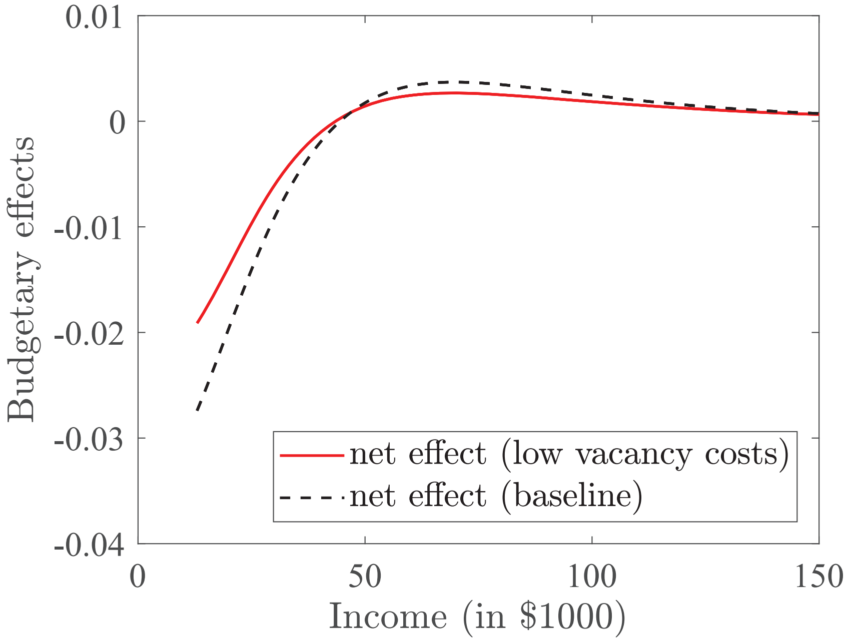

Figure 11: Budgetary effects

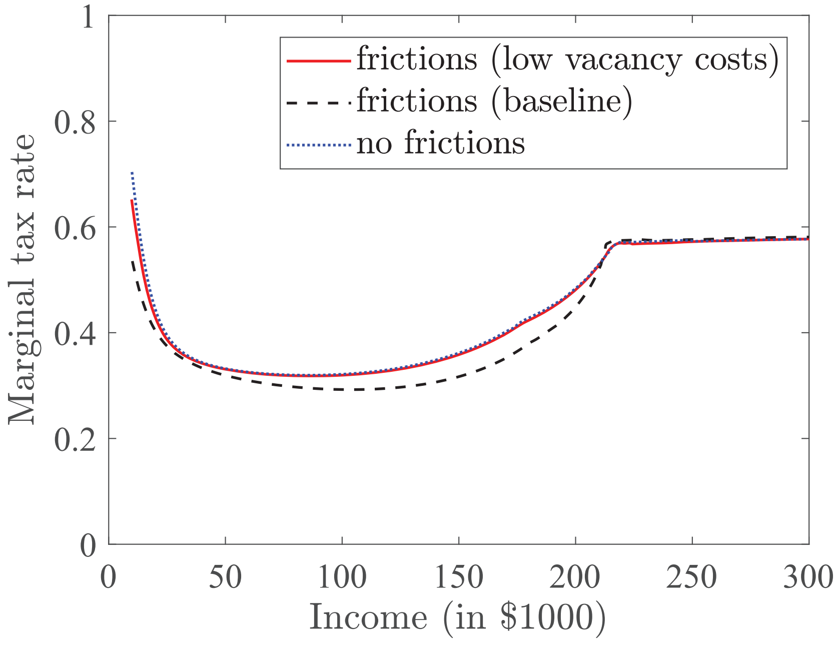

Figure 12: Comparison marginal tax rates

setting tax rates at low levels of income. The negative revenue effects due to unemployment considerations lower the revenue gain of raising tax rates at low income levels by \$0.03 in the baseline calibration and \$0.02 in the calibration with lower vacancy costs.

6 Conclusion

This paper characterizes optimal unemployment insurance and income redistribution in a directed search model where matching frictions generate uninsurable and heterogeneous unemployment risk. Individuals differ in terms of their ability and participation costs and supply labor on the intensive and extensive margin. In addition to the standard trade-off between consumption and leisure they face a trade-off between high wages and low unemployment risk. The government affects this trade-off and hence unemployment by altering the costs and benefits of searching. On the one hand, an increase in the marginal tax rate raises employment as it lowers the benefits of looking for higher-wage jobs. On the other hand, an increase in the tax burden or unemployment benefit reduces employment as it lowers the benefits of finding a job. I label the first of these the employment-enhancing effect (EEE) and the second the employment-reducing (ERE) effect of taxation.

Because an unemployed worker receives unemployment benefits and does not pay income taxes, changes in unemployment affect government finances. These fiscal externalities call for intuitive adjustments of standard optimal tax formulas. The latter are used to obtain the following insights. First, how unemployment affects optimal tax policy depends on two types of statistics: (i) the elasticity of unemployment with respect to the marginal and average tax rate and (ii) the hazard rate of the income distribution. Second, if it is optimal to subsidize employment, the optimal marginal tax rates are negative at the bottom of the income distribution. My model can therefore explain why it is optimal to let employment subsidies (such as the EITC) phase in with income. Third, the optimal provision of unemployment insurance is closely linked to the shape of the tax schedule. In particular, financing UI payments through lump-sum or proportional taxes on labor income – as is commonly assumed in the literature – is sub-optimal even in the absence of a motive for redistribution. Finally, a calibration of the model to the US economy reveals that unemployment is an important margin to consider when setting tax rates at low levels of income. I find that the government loses three cents on the dollar due to unemployment responses if – starting from the current tax-benefit system – it raises the marginal tax rate for low-income workers. Despite this, the quantitative impact of unemployment on the pattern of *optimal* taxes appears to be modest.

The analysis from this paper can be extended in a number of directions. First, in my model the missing insurance market is the only source of inefficiency and unemployment affects optimal tax policy only through fiscal externalities. It would be interesting to allow for further departures from efficiency. Second, I have assumed wages can freely adjust in response to changes in the tax-benefit system. This may not be the case if there is a binding minimum wage. How the tax-benefit system and minimum wages should jointly be optimized is an interesting and policy-relevant question. Third, I have abstracted from dynamic considerations. As a result, individuals cannot insure their unemployment risk through precautionary savings and UI payments cannot be conditioned on past earnings. Extending the model to include these features would both make it more realistic and significantly enrich the set of policy questions it can address. Finally, this paper can serve as a motivation for future empirical work. There is little evidence about the separate effects of marginal and average tax rates on unemployment. My analysis indicates that both responses are key for analyzing the welfare effects of tax reforms.

References

Acemoglu, D. and R. Shimer (1999). Efficient unemployment insurance. *Journal of Political Economy 107*(5), 893–928.

Baily, M. (1978). Some aspects of optimal unemployment insurance. *Journal of Public Economics 10*(3), 379–402.

Bassanini, A. and R. Duval (2009). Unemployment, institutions, and reform complementarities: re-assessing the aggregate evidence for OECD countries. *Oxford Review of Economic Policy 25*(1), 40–59.

Blanchard, O. and J. Wolfers (2000). The role of shocks and institutions in the rise of European unemployment: the aggregate evidence. *Economic Journal 110*(462), 1–33.

Blomquist, S. and H. Selin (2010). Hourly wage rate and taxable labor income responsiveness to changes in marginal tax rates. *Journal of Public Economics 94*(11-12), 878–889.

Boadway, R. and K. Cuff (1999). Monitoring job search as an instrument for targeting transfers. *International Tax and Public Finance 6*(3), 317–337.

Boadway, R., K. Cuff, and N. Marceau (2003). Redistribution and employment policies with endogenous unemployment. *Journal of Public Economics 87*(11), 2407–2430.

Boone, J. and L. Bovenberg (2004). The optimal taxation of unskilled labor with job search and social assistance. *Journal of Public Economics 88*(11), 2227–2258.

Boone, J. and L. Bovenberg (2006). Optimal welfare and in-work benefits with search unemployment and observable abilities. *Journal of Economic Theory 126*(1), 165–193.

Bovenberg, L. and F. van der Ploeg (1994). Effects of the tax and benefit system on wage formation and unemployment. mimeo, Tilburg University and University of Amsterdam.

Card, D., A. Johnston, P. Leung, A. Mas, and Z. Pei (2015). The effect of unemployment benefits on the duration of unemployment insurance receipt: new evidence from a regression kink design in Missouri, 2003-2013. *American Economic Review 105*(5), 126–30.

Chetty, R. (2006). A general formula for the optimal level of social insurance. *Journal of Public Economics 90*(10-11), 1879–1901.

Chetty, R. (2008). Moral hazard versus liquidity and optimal unemployment insurance. *Journal of Political Economy 116*(2), 173–234.

Chetty, R. (2009). Sufficient statistics for welfare analysis: a bridge between structural and reduced-form methods. *Annual Review of Economics 1*(1), 451–488.

Chetty, R. (2012). Bounds on elasticities with optimization frictions: a synthesis of micro and macro evidence on labor supply. *Econometrica 80*(3), 969–1018.

Chetty, R., A. Guren, D. Manoli, and A. Weber (2011). Are micro and macro labor supply elasticities consistent? A review of evidence on the intensive and extensive margins. *American Economic Review 101*(3), 471–75.

Daveri, F. and G. Tabellini (2000). Unemployment, growth and taxation in industrial countries. *Economic Policy 15*(30), 48–104.

Den Haan, W., G. Ramey, and J. Watson (2000). Job destruction and propagation of shocks. *American Economic Review 90*(3), 482–498.

Diamond, P. (1980). Income taxation with fixed hours of work. *Journal of Public Economics 13*(1), 101–110.

Diamond, P. (1998). Optimal income taxation: an example with a U-shaped pattern of optimal marginal tax rates. *American Economic Review 88*(1), 83–95.

Flemming, J. (1978). Aspects of optimal unemployment insurance: search, leisure, savings and capital market imperfections. *Journal of Public Economics 10*(3), 403–425.

Fredriksson, P. and B. Holmlund (2001). Optimal unemployment insurance in search equilibrium. *Journal of Labor Economics 19*(2), 370–399.

Gerritsen, A. (2016). Optimal nonlinear taxation: the dual approach. mimeo, Erasmus University Rotterdam.

Golosov, M., P. Maziero, and G. Menzio (2013). Taxation and redistribution of residual income inequality. *Journal of Political Economy 121*(6), 1160–1204.

Golosov, M., A. Tsyvisnki, and N. Werquin (2014). A variational approach to the analysis of tax systems. NBER Working Paper No. 20780, Cambridge-MA: NBER.

Griffith, R., R. Harrison, and G. Macartney (2007). Product market reforms, labour market institutions and unemployment. *Economic Journal 117*(519), C142–C166.

Gruber, J. and E. Saez (2002). The elasticity of taxable income: evidence and implications. *Journal of Public Economics 84*(1), 1–32.

Hagedorn, M. and I. Manovskii (2008). The cyclical behavior of equilibrium unemployment and vacancies revisited. *American Economic Review 98*(4), 1692–1706.

Hansen, E. (2017). Optimal income taxation with labor supply responses at two margins: when is an Earned Income Tax Credit optimal? mimeo, University of Cologne.

Hersoug, T. (1984). Union wage responses to tax changes. *Oxford Economic Papers 36*(1), 37–51.

Holmlund, B. and A. Kolm (1995). Progressive taxation, wage setting and unemployment: theory and Swedish evidence. *Swedish Economic Policy Review 2*, 423–460.

Hopenhayn, H. and J. Nicolini (1997). Optimal unemployment insurance. *Journal of Political Economy 105*(2), 412–438.

Hosios, A. (1990). On the efficiency of matching and related models of search and unemployment. *Review of Economic Studies 57*(2), 279–298.

Hummel, A. and B. Jacobs (2018). Optimal income taxation in unionized labor markets. CESifo Working Paper No. 7188, Munich: CESifo.

Hungerbühler, M. and E. Lehmann (2009). On the optimality of a minimum wage: new insights from optimal tax theory. *Journal of Public Economics 93*(3), 464–481.

Hungerbühler, M., E. Lehmann, A. Parmentier, and B. Van der Linden (2006). Optimal redistributive taxation in a search equilibrium model. *Review of Economic Studies 73*(3), 743–767.

Jacobs, B., E. Jongen, and F. Zoutman (2017). Revealed social preferences of Dutch political parties. *Journal of Public Economics 156*, 81–100.

Jacquet, L. and E. Lehmann (2017). Optimal income taxation with composition effects. IZA Discussion Paper No. 11019, Bonn: IZA.

Jacquet, L., E. Lehmann, and B. Van der Linden (2013). Optimal redistributive taxation with both extensive and intensive responses. *Journal of Economic Theory 148*(5), 1770–1805.

Jacquet, L., E. Lehmann, and B. Van der Linden (2014). Optimal income taxation with Kalai wage bargaining and endogenous participation. *Social Choice and Welfare 42*(2), 381–402.

Kassenboehmer, S. and J. Haisken-DeNew (2009). You're fired! The causal negative effect of entry unemployment on life satisfaction. *Economic Journal 119*(536), 448–462.

Koskela, E. and J. Vilmunen (1996). Tax progression is good for employment in popular models of trade union behaviour. *Labour Economics 3*(1), 65–80.

Kroft, K., K. Kucko, E. Lehmann, and J. Schmieder (2017). Optimal income taxation with unemployment and wage responses: a sufficient statistics approach. CEPR Discussion Paper No. 12328, London: CEPR.

Lehmann, E., C. Lucifora, S. Moriconi, and B. Van der Linden (2016). Beyond the labour income tax wedge: the unemployment-reducing effect of tax progressivity. *International Tax and Public Finance 23*(3), 454–489.

Lehmann, E., A. Parmentier, and B. Van der Linden (2011). Optimal income taxation with endogenous participation and search unemployment. *Journal of Public Economics 95*(11), 1523–1537.

Lockwood, B. and A. Manning (1993). Wage setting and the tax system: theory and evidence for the United Kingdom. *Journal of Public Economics 52*(1), 1–29.

Malcomson, J. and N. Sartor (1987). Tax push inflation in a unionized labour market. *European Economic Review 31*(8), 1581–1596.

Manning, A. (1993). Wage bargaining and the Phillips curve: the identification and specification of aggregate wage equations. *Economic Journal 103*(416), 98–118.

Meghir, C. and D. Phillips (2006). Labour supply and taxes. In S. Adam, T. Besley, R. Blundell, S. Bond, R. Chote, M. Gammie, P. Johnson, G. Myles, J. Mirrlees, and J. Poterba (Eds.), *Dimensions of tax design: the Mirrlees review*, Chapter 3, pp. 202–274. Oxford: Oxford University Press.

Meyer, B. (1990). Unemployment insurance and unemployment spells. *Econometrica 58*(4), 757–82.

Meyer, B. and D. Rosenbaum (2001). Welfare, the Earned Income Tax Credit, and the labor supply of single mothers. *Quarterly Journal of Economics 116*(3), 1063–1114.

Mirrlees, J. (1971). An exploration in the theory of optimum income taxation. *Review of Economic Studies 38*(114), 175–208.

Moen, E. (1997). Competitive search equilibrium. *Journal of Political Economy 105*(2), 385–411.

Nickell, S. and R. Layard (1999). Labor market institutions and economic performance. In O. Ashenfelter and D. Card (Eds.), *Handbook of Labor Economics*, Chapter 46, pp. 3029–3084. Amsterdam: Elsevier.

OECD (2018). Education at a glance: OECD indicators. `http://www.oecd.org/education/education-at-a-glance/`.

Petrongolo, B. and C. Pissarides (2001). Looking into the black box: a survey of the matching function. *Journal of Economic Literature 39*(2), 390–431.

Picard, P. and E. Toulemonde (2003). Taxation and labor markets. *Journal of Economics 78*(1), 29–56.

Piketty, T., E. Saez, and S. Stantcheva (2014). Optimal taxation of top labor incomes: a tale of three elasticities. *American Economic Journal: Economic Policy 6*(1), 230–271.

Pisauro, G. (1991). The effect of taxes on labour in efficiency wage models. *Journal of Public Economics 46*(3), 329–345.

Pissarides, C. (1985). Taxes, subsidies and equilibrium unemployment. *Review of Economic Studies 52*(1), 121–133.

Pissarides, C. (1998). The impact of employment tax cuts on unemployment and wages: the role of unemployment benefits and tax structure. *European Economic Review 42*(1), 155–183.

Rattenhuber, P. (2017). Marginal taxes: a good or a bad for wages? The incidence of the structure of income and labor taxes on wages. DIW Discussion Paper No. 1193, Berlin: DIW.

Saez, E. (2001). Using elasticities to derive optimal income tax rates. *Review of Economic Studies 68*(1), 205–229.

Saez, E. (2002). Optimal income transfer programs: intensive versus extensive labor supply responses. *Quarterly Journal of Economics 117*(3), 1039–1073.

Saez, E., J. Slemrod, and S. Giertz (2012). The elasticity of taxable income with respect to marginal tax rates: a critical review. *Journal of Economic Literature 50*(1), 3–50.

Saez, E. and S. Stantcheva (2016). Generalized social marginal welfare weights for optimal tax theory. *American Economic Review 106*(1), 24–45.

Schneider, K. (2005). Union wage setting and progressive income taxation with heterogeneous labor: theory and evidence from the German income tax reforms 1986–1990. *Labour Economics 12*(2), 205–222.

Shavell, S. and L. Weiss (1979). The optimal payment of unemployment insurance benefits over time. *Journal of Political Economy 87*(6), 1347–1362.

Sleet, C. and H. Yazici (2017). Taxation, redistribution and frictional labor supply. mimeo, Carnegie Mellon University and Sabanci Üniversitesi.

Stantcheva, S. (2014). Optimal income taxation with adverse selection in the labour market. *Review of Economic Studies 81*(3), 1296–1329.

Winkelmann, L. and R. Winkelmann (1998). Why are the unemployed so unhappy? Evidence from panel data. *Economica 65*(257), 1–15.

Wright, R., P. Kircher, B. Julien, and V. Guerrieri (2017). Directed search: a guided tour. NBER Working Paper No. 23884, Cambridge-MA: NBER.

A Appendices

A.1 Constrained efficiency

This appendix formally demonstrates that the allocation of resources in the absence of government intervention is Pareto efficient (from an *ex ante* perspective) if and only if individuals are risk-neutral. To do so, I first show that if individuals are risk-neutral the *laissez-faire* allocation maximizes the sum of expected utilities subject to the aggregate resource constraint. Then, I show that if individuals are risk-averse there exists a Pareto-improving and resource-feasible perturbation of the equilibrium allocation.

If individuals are risk-neutral (i.e., $u(c) = c$) and if there are no taxes and benefits (i.e., $T(z) = b = 0$), the equilibrium satisfies the following conditions:[33]

$$\theta(e(n))k = e(n)(y(n) - z(n)), \tag{29}$$

$$\varphi(n) = e(n)\left(z(n) - v\left(y(n)/n\right)\right), \tag{30}$$

$$n = v'\left(y(n)/n\right), \tag{31}$$

$$\big(\theta'(e(n)) - \theta(e(n))/e(n)\big)\, k = z(n) - v\left(y(n)/n\right). \tag{32}$$

[33] The government budget constraint (8) then requires $G = 0$.

These correspond to the zero-profit condition (2), the participation decision (4) and the first-order conditions (6) and (7). To see why the implied allocation $[z(n), y(n), e(n), \varphi(n)]_{n_0}^{n_1}$ is Pareto efficient, suppose the government chooses the allocation which maximizes the sum of all individuals' expected utilities subject to the resource constraint. The Lagrangian is given by:

$$\begin{aligned}\mathcal{L} = &\int_{n_0}^{n_1}\int_{\varphi_0}^{\varphi(n)} \left[e(n)\left(z(n) - v\left(\frac{y(n)}{n}\right)\right) - \varphi\right] f(\varphi, n) d\varphi dn \\ &+ \lambda\left(\int_{n_0}^{n_1}\int_{\varphi_0}^{\varphi(n)} \Big[e(n)(y(n) - z(n)) - \theta(e(n))k\Big] f(\varphi, n) d\varphi dn\right).\end{aligned} \tag{33}$$

Here, $\mathcal{U}(n) - \varphi = e(n)(z(n) - v(y(n)/n)) - \varphi$ denotes the expected utility of an individual of type (n, φ) who participates. Utility of the non-participants equals $u(0) = 0$. Moreover, λ denotes the multiplier on the aggregate resource constraint. The resource constraint is obtained by integrating the zero-profit condition over all participants. The first-order conditions associated with the optimization problem (33) are:

$$z(n): \quad (1-\lambda)\int_{\varphi_0}^{\varphi(n)} e(n) f(\varphi, n) d\varphi = 0, \tag{34}$$

$$y(n): \quad \int_{\varphi_0}^{\varphi(n)} e(n)\left(-\frac{v'(y(n)/n)}{n} + \lambda\right) f(\varphi, n) d\varphi = 0, \tag{35}$$

$$e(n): \quad \int_{\varphi_0}^{\varphi(n)} \Big[z(n) - v(y(n)/n) + \lambda(y(n) - z(n) - \theta'(e(n))k)\Big] f(\varphi, n) d\varphi = 0, \tag{36}$$

$$\varphi(n): \quad \left(e(n)\left(z(n) - v\left(\frac{y(n)}{n}\right)\right) - \varphi(n)\right) + \lambda\Big(e(n)(y(n) - z(n)) - \theta(e(n))k\Big) = 0. \tag{37}$$

To verify that the equilibrium allocation (as implicitly defined by (29)-(32)) satisfies these conditions, first observe that (34) implies $\lambda = 1$. Then, equations (31) and (35) coincide. Similarly, setting $\lambda = 1$ and using (29) to substitute out for $z(n)$ in (32) implies (36) holds as well. Finally, (37) is equivalent to (29) because the zero-profit condition (30) implies the second term in (37) is zero. The *laissez-faire* allocation thus maximizes the sum of expected utilities subject to the aggregate resource constraint. This implies there exists no resource-feasible Pareto improvement if individuals are risk-neutral.

Conversely, if individuals are risk-averse there exists a Pareto-improving and resource-feasible perturbation of the equilibrium allocation. To see why, note that – in the absence of taxes and benefits – an individual with ability n who decides to participate, solves:

$$\mathcal{U}(n) \equiv \max_{y,z,e}\left\{e\left[u(z) - v\left(\frac{y}{n}\right)\right] + (1-e)u(0) \ \text{ s.t. } \ k = \frac{e}{\theta(e)}(y - z)\right\}. \tag{38}$$

Denote the solution of the above maximization problem by $(y(n), z(n), e(n))$. Hence, a fraction $e(n)$ of the participants with ability n becomes employed, produces $y(n)$ and consumes $z(n)$, whereas a fraction $1 - e(n)$ remains unemployed and does not consume at all. Now, consider a perturbation where the consumption of individuals with ability n in the state of unemployment is marginally raised by $\mathrm{d}c_u > 0$ and consumption in the state of employment is reduced by $\mathrm{d}c_u(1 - e(n))/e(n)$. Such a perturbation is resource feasible as it does not affect aggregate consumption of individuals with ability n. The impact on expected utility is:

$$d\mathcal{U}(n) = (1 - e(n))(u'(0) - u'(z(n)))dc_u. \tag{39}$$

The latter is strictly positive whenever $u(\cdot)$ is strictly concave. Since the perturbation raises the expected utility of type n individuals without decreasing the expected utility of any other type, the laissez-faire equilibrium is not Pareto efficient if individuals are risk-averse.[34]

A.2 Comparative statics

This appendix derives how the labor-market equilibrium outcomes are affected by the tax-benefit system and how they vary with ability (see Proposition 1). To do so, I use the variational approach introduced in Golosov et al. (2014). As a first step, I characterize the equilibrium outcomes for an individual with ability n who is confronted with the tax schedule $T^*(z,\kappa)$. The equilibrium earnings, output and employment rate are determined by:

$$e(y - z) - \theta(e)k = 0, \tag{40}$$

$$u'(z - T(z) - \kappa R(z))n(1 - T'(z) - \kappa R'(z)) - v'(y/n) = 0, \tag{41}$$

$$(u(z - T(z) - \kappa R(z)) - v(y/n) - u(b))n - v'(y/n)(\theta'(e) - \theta(e)/e)k = 0. \tag{42}$$

These correspond with the equilibrium conditions (2), (6) and (7) under the assumption that the tax schedule is given by $T^*(z,\kappa) = T(z) + \kappa R(z)$. Denote the above system by $\Lambda(\mathbf{x};\mathbf{t}) = 0$, which implicitly defines the equilibrium outcomes $\mathbf{x} = (z, y, e)'$ as a function of parameters $\mathbf{t} = (\kappa, b, n)'$. The comparative statics can be determined via the implicit function theorem:

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}\mathbf{t}} = -\left(\frac{\partial\Lambda(\mathbf{x};\mathbf{t})}{\partial\mathbf{x}}\right)^{-1}\frac{\partial\Lambda(\mathbf{x};\mathbf{t})}{\partial\mathbf{t}} = -\Lambda_{\mathbf{x}}^{-1}\Lambda_{\mathbf{t}}. \tag{43}$$

Working out (43) using co-factor expansion yields:

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}\mathbf{t}} = \frac{-1}{|\Lambda_{\mathbf{x}}|}\begin{pmatrix} \Lambda_y^2\Lambda_p^3 - \Lambda_p^2\Lambda_y^3 & \Lambda_p^1\Lambda_y^3 - \Lambda_y^1\Lambda_p^3 & \Lambda_y^1\Lambda_p^2 - \Lambda_p^1\Lambda_y^2 \\ \Lambda_p^2\Lambda_z^3 - \Lambda_z^2\Lambda_p^3 & \Lambda_z^1\Lambda_p^3 - \Lambda_p^1\Lambda_z^3 & \Lambda_p^1\Lambda_z^2 - \Lambda_z^1\Lambda_p^2 \\ \Lambda_z^2\Lambda_y^3 - \Lambda_y^2\Lambda_z^3 & \Lambda_y^1\Lambda_z^3 - \Lambda_z^1\Lambda_y^3 & \Lambda_z^1\Lambda_y^2 - \Lambda_y^1\Lambda_z^2 \end{pmatrix}\begin{pmatrix} \Lambda_\kappa^1 & \Lambda_b^1 & \Lambda_n^1 \\ \Lambda_\kappa^2 & \Lambda_b^2 & \Lambda_n^2 \\ \Lambda_\kappa^3 & \Lambda_b^3 & \Lambda_n^3 \end{pmatrix}. \tag{44}$$

The superscripts correspond to the rows in $\Lambda(\mathbf{x};\mathbf{t})$. The elements of $\Lambda_{\mathbf{x}}$ are (ignoring function arguments for notational convenience):

$$\begin{aligned} &\Lambda_z^1 = -e, \quad \Lambda_y^1 = e, \quad \Lambda_e^1 = -\chi, \\ &\Lambda_z^2 = u''n(1 - T' - \kappa R')^2 - u'n(T'' + \kappa R''), \quad \Lambda_y^2 = -v''/n, \quad \Lambda_e^2 = 0, \\ &\Lambda_z^3 = v', \quad \Lambda_y^3 = -v' - \chi v''/n, \quad \Lambda_e^3 = -v'(\theta''k - \chi/e). \end{aligned} \tag{45}$$

[34] Note however that the reform only raises the *expected* (i.e., *ex ante*) utility of individuals. It does not generate an increase in *realized* (i.e., *ex post*) utilities, because the reform lowers the utility in the state of employment.

The elements in $\Lambda_{\mathbf{t}}$ are:

$$\Lambda_\kappa^1 = 0, \quad \Lambda_b^1 = 0, \quad \Lambda_n^1 = 0,$$
$$\Lambda_\kappa^2 = -u''n(1 - T' - \kappa R')R - u'nR', \quad \Lambda_b^2 = 0, \quad \Lambda_n^2 = v'/n + v''y/n^2,$$
$$\Lambda_\kappa^3 = -u'nR, \quad \Lambda_b^3 = -u_0'n, \quad \Lambda_n^3 = v'\chi/n + v'y/n + \chi v''y/n^2. \tag{46}$$

Equations (45)-(46) are simplified somewhat using the conditions (40)-(42). In addition, I denote by $\chi = (\theta'(e) - \theta(e)/e)k > 0$ the difference between the marginal and average costs of getting a vacancy filled and by $u_0' \equiv u'(b)$ the marginal utility of consumption of the unemployed.

The impact of income taxes on labor-market outcomes can be determined by calculating the partial effect of the reform parameter κ, evaluated at the reform of interest (see, e.g., Gerritsen (2016), Jacquet and Lehmann (2017)). The reforms I consider are the following:

$$R(z) = z - z(n) \tag{47}$$
$$R(z) = 1 \tag{48}$$

The first of these increases the marginal tax rate while leaving the average tax rate at income level $z(n)$ unaffected. The second generates an increase in the tax liability but does not affect the marginal tax rate. In order to simplify the exposition and to sign the partial effects, I perturb the tax functions starting from the *laissez-faire* equilibrium: $T(\cdot) = b = \kappa = 0$. The impact of the first reform is obtained by working out the first column of (44) evaluated at the reform (47). With a slight abuse of notation, I denote the results by:

$$\frac{\mathrm{d}z}{\mathrm{d}T'} = \frac{u'n(\chi^2 v''/n + pkv'\theta'')}{u''n(\chi^2 v''/n + pkv'\theta'') - pkv'\theta''v''/n} < 0 \tag{49}$$
$$\frac{\mathrm{d}y}{\mathrm{d}T'} = \frac{u'npkv'\theta''}{u''n(\chi^2 v''/n + pkv'\theta'') - pkv'\theta''v''/n} < 0 \tag{50}$$
$$\frac{\mathrm{d}e}{\mathrm{d}T'} = \frac{-\chi u'npv''/n}{u''n(\chi^2 v''/n + pkv'\theta'') - pkv'\theta''v''/n} > 0 \tag{51}$$

Similarly, for the reform (48):

$$\frac{\mathrm{d}z}{\mathrm{d}T} = \frac{u''n(\chi^2 v''/n + pkv'\theta'') - \chi u'v''}{u''n(\chi^2 v''/n + pkv'\theta'') - pkv'\theta''v''/n} > 0 \tag{52}$$
$$\frac{\mathrm{d}y}{\mathrm{d}T} = \frac{u''n(pkv'\theta'' - \chi u'n)}{u''n(\chi^2 v''/n + pkv'\theta'') - pkv'\theta''v''/n} \geq 0 \tag{53}$$
$$\frac{\mathrm{d}e}{\mathrm{d}T} = \frac{p(u'v'' - u''n(\chi v''/n + u'n))}{u''n(\chi^2 v''/n + pkv'\theta'') - pkv'\theta''v''/n} < 0 \tag{54}$$

The effects of changing the unemployment benefit b are given by:

$$\frac{\mathrm{d}z}{\mathrm{d}b} = \frac{-\chi v''u_0'}{u''n(\chi^2 v''/n + pkv'\theta'') - pkv'\theta''v''/n} > 0 \tag{55}$$
$$\frac{\mathrm{d}y}{\mathrm{d}b} = \frac{-\chi u''n^2 u_0'}{u''n(\chi^2 v''/n + pkv'\theta'') - pkv'\theta''v''/n} \leq 0 \tag{56}$$

$$\frac{\mathrm{d}e}{\mathrm{d}b} = \frac{pnu_0'(v''/n - u''n)}{u''n(\chi^2 v''/n + pkv'\theta'') - pkv'\theta''v''/n} < 0 \tag{57}$$

Finally, how labor-market outcomes vary with ability n is determined by:

$$\frac{\mathrm{d}z}{\mathrm{d}n} = \frac{-v'(p(\theta''k - \chi/p)v''y/n^2 + pv'\theta''k/n)}{u''n(\chi^2 v''/n + pkv'\theta'') - pkv'\theta''v''/n} > 0 \tag{58}$$

$$\frac{\mathrm{d}y}{\mathrm{d}n} = \frac{-((v'/n + v''y/n^2)(pv'\theta''k - \chi^2 u''n) - \chi v' y u'')}{u''n(\chi^2 v''/n + pkv'\theta'') - pkv'\theta''v''/n} > 0 \tag{59}$$

$$\frac{\mathrm{d}e}{\mathrm{d}n} = \frac{p((v'/n + v''y/n^2)\chi u''n - v'y(v''/n - u''n)/n)}{u''n(\chi^2 v''/n + pkv'\theta'') - pkv'\theta''v''/n} > 0 \tag{60}$$

The signs of all these effects follow from the assumptions on $u(\cdot)$, $v(\cdot)$ and $\theta(\cdot)$ combined with the observation that $u'(\cdot)n = v'(\cdot)$ in the absence of taxes.[35] Since unemployment is given by $1 - e(n)$, unemployment is decreasing in ability n and the marginal tax rate T' and increasing in the tax liability T and the unemployment benefit b.

Finally, to analyze the impact of the tax-benefit system on the participation rate, note that the participation threshold satisfies:

$$\varphi(n) = \mathcal{U}(n) - u(b) = \max_{z,e} \left\{ e \left[u(z - T(z) - \kappa R(z)) - v\left(\frac{1}{n}\left(z + \frac{\theta(e)}{e}k\right)\right) - u(b) \right] \right\}, \tag{61}$$

where the last step follows from using the zero-profit condition (2) to substitute out for z in (3). Differentiating (61) with respect to the parameters κ and b, evaluated at the reform of interest, gives:

$$\frac{\mathrm{d}\varphi}{\mathrm{d}T'} = 0, \qquad \frac{\mathrm{d}\varphi}{\mathrm{d}T} = -eu' < 0, \qquad \frac{\mathrm{d}\varphi}{\mathrm{d}b} = -eu_0' < 0. \tag{62}$$

Since the participation rate (5) increases in the threshold (61), the participation rate is decreasing in the tax liability and the unemployment benefit and unaffected by a local change in the marginal tax rate.

A.3 Optimal tax-benefit system

To derive the optimal policy rules from Proposition 2, write the Lagrangian associated with the government's optimization problem as follows:

$$\begin{aligned} \mathcal{L} = & \int_{n_0}^{n_1} \int_{\varphi_0}^{\varphi(n,\kappa,b)} \Psi\left(\mathcal{U}(n,\kappa,b) - \varphi\right) f(\varphi,n) d\varphi dn + \int_{n_0}^{n_1} \int_{\varphi(n,\kappa,b)}^{\varphi_1} \Psi(u(b)) f(\varphi,n) d\varphi dn \\ & + \lambda \left[\int_{n_0}^{n_1} \int_{\varphi_0}^{\varphi(n,\kappa,b)} e(n,\kappa,b) \left(T(z(n,\kappa,b)) + \kappa R(z(n,\kappa,b)) + b\right) f(\varphi,n) d\varphi dn - b - G \right]. \end{aligned} \tag{63}$$

Here, λ is the multiplier on the government's budget constraint and the tax function is given by $T(z) + \kappa R(z)$. The notation (n, κ, b) is used to denote which variables depend on ability n, the tax reform parameter κ and the benefit b. The expected utility of an individual with ability

[35] The assumption that the elasticity of $\theta(\cdot)$ is non-decreasing implies $\theta''k - \chi/p > 0$, which is the only term of which the sign might appear ambiguous.

n who participates is given by:

$$\mathcal{U}(n,\kappa,b)=\max_{z,e}\left\{u(b)+e\left[u(z-T(z)-\kappa R(z))-v\left(\frac{1}{n}\left(z+\frac{\theta(e)}{e}k\right)\right)u(b)\right]\right\}, \tag{64}$$

where I used the zero-profit condition (2) to substitute out for $y=z+\theta(e)k/e$. Moreover, the participation threshold satisfies:

$$\varphi(n,\kappa,b)=\mathcal{U}(n,\kappa,b)-u(b) \tag{65}$$

To derive equation (17), differentiate the Lagrangian with respect to b, taking into account the impact on $\mathcal{U}(n,\kappa,b)$ and $\varphi(n,\kappa,b)$. The first-order condition is given by:

$$\begin{aligned}\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}b} &= \int_{n_0}^{n_1}\int_{\varphi_0}^{\varphi(n,\kappa,b)}(1-e(n,\kappa,b))\left[\Psi'(\mathcal{U}(n,\kappa,b)-\varphi)u'(b)-\lambda\right]f(\varphi,n)d\varphi dn\\ &+\int_{n_0}^{n_1}\int_{\varphi(n,\kappa,b)}^{\varphi_1}\left[\Psi'(u(b))u'(b)-\lambda\right]f(\varphi,n)d\varphi dn\\ &+\ \lambda\int_{n_0}^{n_1}\int_{\varphi_0}^{\varphi(n,\kappa,b)}\frac{\mathrm{d}z}{\mathrm{d}b}\left[e(n,\kappa,b)(T'(z(n,\kappa,b))+\kappa R'(z(n,\kappa,b)))\right]f(\varphi,n)d\varphi dn\\ &+\ \lambda\int_{n_0}^{n_1}\int_{\varphi_0}^{\varphi(n,\kappa,b)}\frac{\mathrm{d}e}{\mathrm{d}b}\left[T(z(n,\kappa,b))+\kappa R(z(n,\kappa,b))+b\right]f(\varphi,n)d\varphi dn\\ &+\ \lambda\int_{n_0}^{n_1}\frac{\mathrm{d}\varphi}{\mathrm{d}b}\left[e(n,\kappa,b)(T(z(n,\kappa,b))+\kappa R(z(n,\kappa,b))+b)\right]f(\varphi(n),n)dn=0.\end{aligned} \tag{66}$$

The first two lines give the mechanical welfare effects of transferring income to the unemployed (first line) and the non-participants (second line). The mass of these individuals equals the share of the population with zero income:

$$H(z_0)=\int_{n_0}^{n_1}\int_{\varphi_0}^{\varphi(n,\kappa,b)}(1-e(n,\kappa,b))f(\varphi,n)d\varphi dn+\int_{n_0}^{n_1}\int_{\varphi(n,\kappa,b)}^{\varphi_1}f(\varphi,n)d\varphi dn. \tag{67}$$

Denote their average welfare weight by:

$$\begin{aligned}g(0)=\ &\frac{1}{\lambda H(z_0)}\left[\int_{n_0}^{n_1}\int_{\varphi_0}^{\varphi(n,\kappa,b)}(1-e(n,\kappa,b))\Psi'(\mathcal{U}(n,\kappa,b)-\varphi)u'(b)f(\varphi,n)d\varphi dn\right.\\ &\left.+\int_{n_0}^{n_1}\int_{\varphi(n,\kappa,b)}^{\varphi_1}\Psi'(u(b))u'(b)f(\varphi,n)d\varphi dn\right],\end{aligned} \tag{68}$$

which measures the monetized increase in social welfare if individuals with zero income receive an additional unit of consumption. Using this notation, the first two terms of (66) can be written as:

$$\lambda(g(0)-1)H(z_0). \tag{69}$$

To write the remaining terms of (66) also in terms of the income distribution, note that the relationship between the income and type distribution implies:

$$h(z(n))z'(n) = \int_{\varphi_0}^{\varphi(n)} e(n)f(\varphi, n)d\varphi, \tag{70}$$

which is obtained by differentiating (13) with respect to n. Upon changing variables and evaluating the reform at $\kappa = 0$, the second and third line from (66) can be written as:

$$\lambda \int_{z_0}^{z_1} \left[z_b T'(z) + \frac{e_b}{e}(T(z) + b) \right] h(z)dz. \tag{71}$$

The final term can be simplified as follows. First, note that (5) implies:

$$\frac{\mathrm{d}\varphi}{\mathrm{d}b} f(\varphi(n), n) = \frac{\mathrm{d}\pi/\pi}{\mathrm{d}b} \int_{\varphi_0}^{\varphi(n)} f(\varphi, n)d\varphi. \tag{72}$$

The last line of (66) then simplifies to (using the same change of variables):

$$\int_{z_0}^{z_1} \frac{\pi_b}{\pi}(T(z) + b)h(z)dz. \tag{73}$$

Equation (17) is obtained by setting the sum of (69), (71) and (73) equal to zero, divide the resulting expression by λ and rearrange.

The procedure for deriving equation (16) is very similar. As a first step, maximize the Lagrangian with respect to κ, evaluated at the reform $R(z) = 1$. The first-order condition is:

$$\begin{aligned}
\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}T} &= \int_{n_0}^{n_1} \int_{\varphi_0}^{\varphi(n,\kappa,b)} e(n, \kappa, b) \\
&\quad \times \left[\lambda - \Psi'(\mathcal{U}(n, \kappa, b) - \varphi)u'(z(n, \kappa, b) - T(z(n, \kappa, b)) - \kappa R(z(n, \kappa, b))) \right] f(\varphi, n)d\varphi dn \\
&\quad + \lambda \int_{n_0}^{n_1} \int_{\varphi_0}^{\varphi(n,\kappa,b)} \frac{\mathrm{d}z}{\mathrm{d}T} \left[e(n, \kappa, b)(T'(z(n, \kappa, b)) + \kappa R'(z(n, \kappa, b))) \right] f(\varphi, n)d\varphi dn \\
&\quad + \lambda \int_{n_0}^{n_1} \int_{\varphi_0}^{\varphi(n,\kappa,b)} \frac{\mathrm{d}e}{\mathrm{d}T} \left[T(z(n, \kappa, b)) + \kappa R(z(n, \kappa, b)) + b \right] f(\varphi, n)d\varphi dn \\
&\quad + \lambda \int_{n_0}^{n_1} \frac{\mathrm{d}\varphi}{\mathrm{d}T} \left[e(n, \kappa, b)(T(z(n, \kappa, b)) + \kappa R(z(n, \kappa, b)) + b) \right] f(\varphi(n), n)dn = 0.
\end{aligned} \tag{74}$$

The first two lines capture the mechanical welfare effect of transferring income from all employed individuals to the government budget. To write this in terms of the income distribution and welfare weights, denote the average welfare weight of all individuals with earnings $z(n)$ as:

$$g(z(n)) = \frac{\int_{\varphi_0}^{\varphi(n)} e(n)\Psi'(\mathcal{U}(n) - \varphi)u'(z(n) - T(z(n)))f(\varphi, n)d\varphi}{\int_{\varphi_0}^{\varphi(n)} e(n)f(\varphi, n)d\varphi}. \tag{75}$$

Here, the welfare weight is evaluated at $\kappa = 0$ and I suppress the dependency on the policy parameters (κ, b). The first two lines of (74) can now be written as:

$$\lambda \int_{z_0}^{z_1} (1 - g(z))h(z)dz. \tag{76}$$

The last three lines of (74) can be simplified in exactly the same way as was done with the first-order condition with respect to the benefit b (66). The welfare effect associated with the participation, earnings and unemployment responses is:

$$\lambda \int_{z_0}^{z_1} \left[\frac{\pi_T}{\pi}(T(z') + b) + z_T T'(z') + \frac{e_T}{e}(T(z') + b)\right] dH(z'). \tag{77}$$

Equation (16) from Proposition 2 is then obtained by setting the sum of (76) and (77) equal to zero, and divide by λ.

Finally, to derive equation (15) consider the following reform:

$$R(z) = \begin{cases} 0 & \text{if } z \leq z(m), \\ z - z(m) & \text{if } z \in (z(m), z(m) + \omega], \\ \omega & \text{if } z > z(m) + \omega. \end{cases} \tag{78}$$

For ω small, this reform corresponds to a local increase in the *marginal* tax rate around income level $z(m)$ (see Figure 1). Clearly, the reform does not affect individuals with ability below m. For individuals with earnings potential above $z(m) + \omega$, the reform increases their (expected) tax liability. The welfare effects are therefore the same as before (see (76) and (77)), and given by:

$$\omega \times \lambda \int_{z(m)+\omega}^{z_1} \left[1 - g(z') + \frac{\pi_T}{\pi}(T(z') + b) + z_T T'(z') + \frac{e_T}{e}(T(z') + b)\right] h(z')dz' \tag{79}$$

This equation is obtained by summing (76) and (77) and replacing the lower bound of the integral by $z(m) + \omega$. The multiplication with ω is because the increase in the tax liability for individuals with earnings above $z(m) + \omega$ is proportional to ω.

The reform also affects individuals with earnings potential in the interval $[z(m), z(m) + \omega]$. The corresponding interval of the ability distribution is $[m, m + \omega/z'(m)]$. The welfare effect is obtained by differentiating the Lagrangian (63) with respect to κ, evaluated at the reform $R(z) = z - z(m)$:

$$\begin{aligned}
\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}T'} &= \int_m^{m+\frac{\omega}{z'(m)}} \int_{\varphi_0}^{\varphi(n,\kappa,b)} e(n,\kappa,b) \times \\
&\left[\lambda - \Psi'(\mathcal{U}(n,\kappa,b) - \varphi)u'(z(n,\kappa,b) - T(z(n,\kappa,b)) - \kappa R(z(n,\kappa,b)))\right](z(n) - z(m))f(\varphi,n)d\varphi dn \\
&+ \lambda \int_m^{m+\frac{\omega}{z'(m)}} \int_{\varphi_0}^{\varphi(n,\kappa,b)} \frac{\mathrm{d}z}{\mathrm{d}T'}\left[e(n,\kappa,b)(T'(z(n,\kappa,b)) + \kappa R'(z(n,\kappa,b)))\right] f(\varphi,n)d\varphi dn \\
&+ \lambda \int_m^{m+\frac{\omega}{z'(m)}} \int_{\varphi_0}^{\varphi(n,\kappa,b)} \frac{\mathrm{d}e}{\mathrm{d}T'}\left[T(z(n,\kappa,b)) + \kappa R(z(n,\kappa,b)) + b\right] f(\varphi,n)d\varphi dn
\end{aligned}$$

$$+\ \lambda \int_{m}^{m+\frac{\omega}{z'(m)}} \frac{\mathrm{d}\varphi}{\mathrm{d}T'}\Big[e(n,\kappa,b)(T(z(n,\kappa,b)) + \kappa R(z(n,\kappa,b)) + b)\Big] f(\varphi(n),n)dn \tag{80}$$

To proceed, divide the resulting expression by ω and take the limit as $\omega \to 0$. The mechanical welfare effect then cancels (see the first line).[36] Moreover, by Proposition 1, $\mathrm{d}\varphi/\mathrm{d}T' = 0$ and hence the final term cancels as well. Finally, using the property (70) we can simplify equation (80) to:

$$\lambda\Big[z_{T'}T'(z(m)) + \frac{e_{T'}}{e}(T(z(m)+b)\Big]h(z(m)). \tag{81}$$

To obtain equation (15), also divide equation (79) by ω and take the limit as $\omega \to 0$. Add the resulting expression to (81) and set the sum equal to zero. Finally, use the definitions of the elasticities (14) and replace the point where the marginal tax rate is increased $z(m)$ by z. Rearranging gives (15).

A.4 Optimal top rate

The expression for the optimal top rate (19) is obtained in a number of steps. First, define by

$$\varepsilon_{\pi t} = -\frac{\mathrm{d}\pi}{\mathrm{d}t}\frac{1-t}{\pi}, \quad \varepsilon_{zt} = -\frac{\mathrm{d}z}{\mathrm{d}t}\frac{1-t}{z}, \quad \varepsilon_{et} = -\frac{\mathrm{d}e}{\mathrm{d}t}\frac{1-t}{e} \tag{82}$$

the elasticity of participation, earnings, and employment with respect to one minus the *average* tax rate (i.e., holding the marginal tax rate fixed). The term $\frac{\mathrm{d}x}{\mathrm{d}t}$ is related to $\frac{\mathrm{d}x}{\mathrm{d}T}$ through $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}x}{\mathrm{d}(T/z)} = \frac{\mathrm{d}x}{\mathrm{d}T}z$ for $x \in \{\pi, z, e\}$.

Next, consider equation (15). If the marginal tax rate converges, $\lim_{z\to\infty}(T(z)+b)/z = T'(z)$. For high levels of income, equation (15) can then be rewritten as:

$$\frac{T'(z)}{1-T'(z)}(\varepsilon_{zT'} - \varepsilon_{eT'}) = \mathbb{E}\left[1 - g(z') + \frac{T'(z')}{1-T'(z')}\left(\varepsilon_{\pi t} + \varepsilon_{zt} + \varepsilon_{et}\right)\middle| z' > z\right]\frac{1-H(z)}{zh(z)}. \tag{83}$$

If incomes at the top are Paeto distributed (with tail parameter a), $zh(z)/(1-H(z)) = a$. Moreover, if the welfare weight and elasticities for top-income earners converge, condition (83) simplifies to:

$$\frac{T'(z)}{1-T'(z)}a(\varepsilon_{zT'} - \varepsilon_{eT'}) = 1 - g(z) + \frac{T'(z)}{1-T'(z)}\left(\varepsilon_{\pi t} + \varepsilon_{zt} + \varepsilon_{et}\right), \tag{84}$$

Equation (19) is then obtained by collecting terms and solving the above expression for $T'(z)$.

A.5 Proof Proposition 3

The result from Proposition 3 follows immediately from Propositions 1 and 2. To see this, evaluate equation (15) in Proposition 2 at $z = z_0$ and combine the result with equation (16) to

[36] To see why, note that ω shows up in the upper bound of the integral and $z(n) - z(m) \leq \omega$.

find:

$$\frac{T'(z_0)}{(T(z_0)+b)/z_0} = \frac{\varepsilon_{eT'}}{\varepsilon_{zT'}}. \tag{85}$$

Since $\varepsilon_{eT'}, \varepsilon_{zT'} > 0$ (see Proposition 1), the marginal tax rate is negative at the bottom (i.e., $T'(z_0) < 0$) if and only if the employment tax for individuals with the lowest skills is negative (i.e., $T(z_0) + b < 0$). Hence, it is optimal to let employment subsidies (such as the EITC) phase in with income.

A.6 Relation to Baily-Chetty formula

This appendix demonstrates the link between my results for the optimal provision of unemployment insurance and those from Baily (1978) and Chetty (2006). To do so, suppose all individuals are identical and participate (i.e., $n_0 = n_1$ and $\varphi_0 = \varphi_1$ sufficiently low). Moreover, assume unemployment benefits are financed by lump-sum taxes on labor and set the revenue requirement $G = 0$. The government's budget constraint then reads $eT = (1-e)b$.

For a given tax-benefit system (T, b), individuals solve:

$$V(T,b) = \max_{z,e} \left\{ u(b) + e\left(u(z-T) - v\left(\frac{1}{n}\left(z + \frac{\theta(e)}{e}k\right)\right) - u(b)\right)\right\}. \tag{86}$$

The government then optimally chooses T and b to maximize $V(T,b)$ subject to the budget constraint. Upon substituting $T = (1-e)b/e$, the first-order condition is:

$$\frac{\partial V}{\partial b} = (1-e)(u'(b) - u'(z-T)) - \varepsilon_{eb} u'(z-T) = 0, \tag{87}$$

where ε_{eb} is the elasticity of the employment rate with respect to a budget-neutral increase in the unemployment benefit.[37] Next, divide the above equation by $u'(z-T)$ and use a first-order Taylor approximation to write:

$$\frac{u'(b) - u'(z-T)}{u'(z-T)} = -\left(\frac{u''(z-T)(z-T)}{u'(z-T)}\right)\left(\frac{z-T-b}{z-T}\right). \tag{88}$$

The first term on the right-hand side is the coefficient of relative risk aversion and the second measures the percentage drop in consumption due to unemployment. Substituting in (87) gives the same result as in Chetty (2006), Proposition 1. The main difference between this result and equations (22)-(23) is that the latter are obtained from considering a separate perturbation of the tax liability $T(z)$ and the benefit b. By contrast, Baily (1978) and Chetty (2006) consider a joint (budget-neutral) increase in the benefit and the tax liability. Moreover, I do not assume UI payments are financed by lump-sum taxes on labor. As a result, also the fiscal externalities due to the wage responses show up in (22) and (23) (which are absent in Baily (1978) and Chetty (2006)).

[37] The employment rate corresponds to one minus the unemployment duration in the Baily-Chetty framework.

A.7 Derivation inverse-elasticity rule (24)

This appendix derives the inverse-elasticity rule from Proposition 4. To do so, assume there is no heterogeneity and all individuals decide to participate (i.e., $n_0 = n_1$ and $\varphi_0 = \varphi_1$ sufficiently low). The government chooses the tax function $T(z)$ and b to maximize social welfare. The Lagrangian is given by:

$$\mathcal{L} = \mathcal{U}(\kappa, b) + \lambda\Big[e(\kappa, b)(T(z(\kappa, b)) + \kappa R(z(\kappa, b)) + b) - b - G\Big]. \tag{89}$$

Here, the expected utility solves:

$$\mathcal{U}(\kappa, b) = \max_{z,e}\left\{u(b) + e\left(u(z - T(z) - \kappa R(z)) - v\left(\frac{1}{n}\left(z + \frac{\theta(e)}{e}k\right)\right) - u(b)\right)\right\}. \tag{90}$$

Now, consider a local increase in the marginal tax rate. This can be done by setting $R(z) = z - z(\kappa, b)$, where $z(\kappa, b)$ denotes equilibrium income. A local increase in the marginal tax rate does not affect individual's expected utility (see Appendix A.3). Hence, an increase in the marginal tax rate only affects welfare through fiscal externalities. The first-order condition is:

$$\frac{\partial \mathcal{L}}{\partial T'} = \lambda\left[\frac{\mathrm{d}e}{\mathrm{d}T'}(T(z(\kappa, b) + \kappa R(z(\kappa, b)) + b) + \frac{\mathrm{d}z}{\mathrm{d}T'}e(\kappa, b)(T'(z(\kappa, b)) + \kappa R'(z(\kappa, b)))\right] = 0. \tag{91}$$

To obtain the inverse-elasticity rule (24), divide the resulting expression by λ. Next, evaluate (91) at $\kappa = 0$ and use the definitions of $\varepsilon_{zT'}$ and $\varepsilon_{eT'}$. Rearranging gives (24).

The above result immediately implies that financing UI payments through lump-sum taxes is sub-optimal. Moreover, financing them through proportional taxes (i.e., $T(z) = tz$) is also generally sub-optimal, as it requires marginal and average tax rates to be the same. The reason is that the *marginal* tax rate is set to maximize $e(T(z) + b)$ in (89). The *average* tax rate, on the other hand, is set to balance the insurance gains of higher UI payments against the distortionary effects of income taxes on unemployment. The optimal marginal and average tax rate therefore coincide only in special cases.

A.8 Quantitative analysis

Sample selection

The data source I use in the quantitative analysis is the 2016 March release of the Current Population Survey (CPS). These can be freely downloaded in Stata-format from `http://ceprdata.org/cps-uniform-data-extracts/march-cps-supplement/march-cps-data/`. The unemployment rate is calculated as the number of individuals whose reported labor-force status is unemployed as a fraction of all individuals whose reported labor-force status is either employed or unemployed. This gives an (aggregate) unemployment rate of 5.1%. The unemployment rates are also calculated separately for five different educational attainments: less than high school, high school, some college, college, advanced (see Figure 2). To obtain the participation rates, I keep individuals who listed inability to find work or taking care of home and family as reasons for not working and drop those who listed disability, going to school, retirement or other reasons.[38] The aggregate participation rate equals 86.2% and the participation rates by educational attainment are shown in Figure 3.

I measure labor earnings as the income from wage and salary payments. The averages by education level are shown in Figures 2 and 3 for individuals who received such income. To align the model with the data, in the final sample I focus on individuals between 25 and 65 years who worked full-time (i.e., who were working at least 45 weeks in 2015 for on average 35 hours per week or more). Moreover, I drop all individuals who received an hourly wage below the Federal minimum wage of $7.25. Finally, I multiply the incomes of individuals who are top-coded by a factor of 3, which is consistent with a Pareto parameter of $a = 1.5$.[39] The number of observations I use in the final analysis is 55,425. For each individual, I calculate the tax liability as the sum of state and federal taxes after credits. The latter is regressed linearly on taxable income to approximate the current US tax schedule (see also Saez (2001) and Sleet and Yazici (2017)). The unemployment benefit is calculated as the average income from unemployment compensation for individuals who received such income and whose reported labor force status is unemployment.

Numerical optimization

To numerically solve the government's optimization problem, I formulate it as an optimal control problem where the government directly optimizes over the allocation variables $\mathcal{U}(n)$, $y(n)$, $e(n)$ and the benefit b. The objective function is given by (12). The differential equation which serves as a constraint in the optimization problem is obtained by differentiating expected utility (3) with respect to n:

$$\mathcal{U}'(n) = e(n)v'\left(\frac{y(n)}{n}\right)\frac{y(n)}{n^2}. \tag{92}$$

Moreover, combining the household's first-order conditions (6) and (7) leads to the following implementability constraint:

$$n\left(\mathcal{U}(n) - u(b)\right) = v'\left(\frac{y(n)}{n}\right)\left(\theta'(e(n))e(n) - \theta(e(n))\right)k, \tag{93}$$

which must hold for all n. Finally, the aggregate resource constraint is obtained by using the definition of expected utility (3) and the zero-profit condition (2) to substitute out for the tax liability in the government budget constraint.

For the numerical optimization I use the GPOPS-II software package. The parameterization of the utility function, matching function, social welfare function and the joint distribution of ability and participation costs are as described in the main text. The values for n_0 and n_1 are set at the ability level of the individual with the lowest and highest earnings, respectively. Finally, I assume there is a 5% share of non-participants whose utility is bounded from above by the expected utility of the individuals with the lowest ability.

[38] The reason for doing so is that in my model ability is fixed and individuals can always choose to participate.

[39] In particular, if incomes at the top are Pareto distributed the average income for individuals above some threshold z^* equals $\mathbb{E}[z|z \geq z^*] = \frac{a}{a-1}z^*$.