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SUPPLEMENTARY INFORMATION

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Ecological changes with minor effect initiate evolution to delayed regime shifts

P. Catalina Chaparro-Pedraza[1,2*] and André M. de Roos[1,3]

[1]Institute for Biodiversity and Ecosystem Dynamics, University of Amsterdam, Amsterdam, the Netherlands. [2]Eawag—Swiss Federal Institute of Aquatic Science and Technology, Dübendorf, Switzerland. [3]The Santa Fe Institute, Santa Fe, NM, USA. *e-mail: Catalina.Chaparro@eawag.ch

Supplementary information for:

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This supplementary note presents the following sections:

1. Non-dimensionalization of the size-structured population model
2. Eco-evolutionary model formulation
3. Effects of trait variation in the population

1. Non-dimensionalization of the model

The population model equations (5, 6, 7, 8, 9, 10) can be made non-dimensional by scaling the body size range, the time variable t, and the food resource densities F_1 and F_2.

- *Scaling the body size range*

Define

$$w = \frac{s - s_b}{s_m - s_b}$$

and

$$w_s = \frac{s_s - s_b}{s_m - s_b}$$

Thus, the density functions $n_1(t,s)$ and $n_2(t,s)$ can be rescaled to density functions $m_1(t,w)$ and $m_2(t,w)$ by the following transformation:

$$m_i(t,w) = (s_m - s_b)n_i(t, s_b + (s_m - s_b)w) \tag{18}$$

for $i = 1,2$. These transformations lead to the following identity:

$$\int_{w_1}^{w_2} m_i(t,w)\,dw = \int_{w_1}^{w_2} (s_m - s_b)n_i(t, s_b + (s_m - s_b)w)\,d\left(\frac{s - s_b}{s_m - s_b}\right) = \int_{s_b+(s_m-s_b)w_1}^{s_b+(s_m-s_b)w_2} n_i(t,s)\,ds$$

Using these identities we can derive PDEs for $m_1(t,w)$:

$$\begin{aligned}
\frac{\partial m_1(t,w)}{\partial t} &= (s_m - s_b)\frac{\partial n_1(t, s_b + (s_m - s_b)w)}{\partial t} \\
&= -(s_m - s_b)g_1(F_1)\frac{\partial n_1(t, s_b + (s_m - s_b)w)}{\partial s} - \mu_1 m_1(t,w) \\
&= -g_1(F_1)\frac{\partial m_1\left(t, \frac{s - s_b}{s_m - s_b}\right)}{\partial s} - \mu_1 m_1(t,w) \\
&= -\frac{g_1(F_1)}{s_m - s_b}\frac{\partial m_1(t,w)}{\partial w} - \mu_1 m_1(t,w)
\end{aligned}$$

and similarly for $m_2(t,w)$:

$$\frac{\partial m_2(t,w)}{\partial t} = -\frac{g_2\,(F_2)}{s_m - s_b}\frac{\partial m_2(t,w)}{\partial w} - \mu_2 m_2(t,w)$$

Defining

$$\tilde{g}_1\,(F_1) = \frac{\epsilon_g}{s_m - s_b} a_1\, F_1$$

and

$$\tilde{g}_2\,(F_2) = \frac{\epsilon_g}{s_m - s_b} a_2\, F_2$$

The rescaled model equations for the dynamics of the food resource density in either habitat are then given by

$$\frac{dF_1}{dt} = D_1 - \theta\, F_1 - a_1\, F_1 \int_0^{w_s} m_1(t,w)\, dw \tag{19a}$$

$$\frac{dF_2}{dt} = D_2 - \theta\, F_2 - a_2 F_2 \int_{w_s}^{\infty} m_2(t,w)\, dw \tag{19b}$$

and for the dynamics of the size-dependent density functions of the population are given by:

$$\begin{cases} \dfrac{\partial m_1(t,w)}{\partial t} + \tilde{g}_1\,(F_1)\dfrac{\partial\, m_1(t,w)}{\partial w} = -\,\mu_1 m_1(t,w) \\ \tilde{g}_1\,(F_1) m_1(t,0) = b(F_2)\displaystyle\int_1^{\infty} m_2(t,w)\, dw \end{cases} \tag{19c}$$

$$\begin{cases} \dfrac{\partial m_2(t,w)}{\partial t} + \tilde{g}_2\,(F_2)\dfrac{\partial\, m_2(t,w)}{\partial w} = -\,\mu_2 m_2(t,w) \\ \tilde{g}_2(F_2) m_2(t,w_s) = \tilde{g}_1(F_1) m_1(t,w_s) \end{cases} \tag{19d}$$

The second line in equations 19c and 19d correspond to the rescaled boundary condition at the birth size s_b in habitat 1, and at the switching size s_s from habitat 1 to habitat 2.

- *Scaling time, food and population densities*

Define the following scaled variables:

$$t' = t/t^*$$

$$F_1' = F_1/F_1^*$$

$$F_2' = F_2/F_2^*$$

$$m_1'(t,w) = m_1(t,w)/m^*$$

$$m_2'(t,w) = m_2(t,w)/m^*$$

Substituting these in the model equations yields:

$$\frac{dF_1'}{dt'} = \frac{t^*}{F_1^*} D_1 - t^*\theta\, F_1' - t^* m^* a_1\, F_1' \int_0^{w_s} m_1'(t,w)\, dw$$

$$\frac{dF_2'}{dt'} = \frac{t^*}{F_2^*} D_2 - t^*\theta\, F_2' - t^* m^* a_2\, F_2' \int_{w_s}^{\infty} m_2'(t,w)\, dw$$

$$\begin{cases} \dfrac{\partial m_1'(t,w)}{\partial t'} = -t^* \tilde{g}_1\,(F_1) \dfrac{\partial\, m_1'(t,w)}{\partial w} - t^* \mu_1 m_1'(t,w) \\ t^* \tilde{g}_1(F_1) m_1'(t,0) = t^* b(F_2) \displaystyle\int_1^{\infty} m_2'(t,w)\, dw \end{cases}$$

$$\begin{cases} \dfrac{\partial m_2'(t,w)}{\partial t} = -t^* \tilde{g}_2\,(F_2) \dfrac{\partial\, m_2'(t,w)}{\partial w} - \ t^* \mu_2 m_2'(t,w) \\ t^* \tilde{g}_2(F_2) m_2'(t,w_s) = t^* \tilde{g}_1(F_1) m_1'(t,w_s) \end{cases}$$

where

$$t^* \tilde{g}_i(F_i) = t^* F_i^* \frac{\epsilon_g}{s_m - s_b} a_i\, F_i'$$

and

$$t^* b(F_2) = t^* F_2^* \epsilon_b a_2\, F_2'$$

Scale such that

$$\frac{t^*}{F_1^*} = \frac{t^*}{F_2^*} = \frac{1}{D_1} \quad \Rightarrow \quad F_1^* = F_2^* = D_1 t^*$$

$$t^* m^* = \frac{1}{a_1} \quad \Rightarrow \quad m^* = \frac{1}{a_1\, t^*}$$

$$t^* F_1^* = t^* F_2^* = \frac{s_m - s_b}{\epsilon_g a_1}$$

Then

$$D_1 (t^*)^2 = \frac{s_m - s_b}{\epsilon_g a_1}$$

and hence

$$t^* = \sqrt{\frac{s_m - s_b}{\epsilon_g a_1 D_1}}$$

(20a)

$$F_1^* = F_2^* = \sqrt{\frac{(s_m - s_b) D_1}{\epsilon_g a_1}}$$

(20b)

$$m^* = \sqrt{\frac{\epsilon_g D_1}{a_1 (s_m - s_b)}}$$

(20c)

The rescaled model is:

$$\frac{dF_1'}{dt'} = 1 - \theta \sqrt{\frac{s_m - s_b}{\epsilon_g a_1 D_1}}\; F_1' - F_1' \int_0^{w_s} m_1'(t, w)\, dw$$

$$\frac{dF_2'}{dt'} = \frac{D_2}{D_1} - \theta \sqrt{\frac{s_m - s_b}{\epsilon_g a_1 D_1}} F_2' - \frac{a_2}{a_1} F_2' \int_{w_s}^{\infty} m_2'(t,w)\, dw$$

$$\begin{cases} \dfrac{\partial m_1'(t,w)}{\partial t'} = -F_1' \dfrac{\partial\, m_1'(t,w)}{\partial w} - \mu_1 \sqrt{\dfrac{s_m - s_b}{\epsilon_g a_1 D_1}}\, m_1'(t,w) \\ F_1' m_1'(t,0) = \dfrac{(s_m - s_b)\epsilon_b}{\epsilon_g} \dfrac{a_2}{a_1} F_2' \displaystyle\int_1^{\infty} m_2'(t,w)\, dw \end{cases}$$

$$\begin{cases} \dfrac{\partial m_2'(t,w)}{\partial t} = -\dfrac{a_2}{a_1} F_2' \dfrac{\partial\, m_2'(t,w)}{\partial w} - \mu_2 \sqrt{\dfrac{s_m - s_b}{\epsilon_g a_1 D_1}}\, m_2'(t,w) \\ \dfrac{a_2}{a_1} F_2'\, m_2'(t,w_s) = F_1'\, m_1'(t,w_s) \end{cases}$$

Defining scaled parameters as in table 1 (see Methods), $\gamma_1(F_1) = F_1$ and $\gamma_2(F_2) = qF_2$, and after substitution in the model equations yields:

$$\frac{dF_1}{dt} = 1 - \delta\, F_1 - \int_0^{w_s} \gamma_1(F_1)\, m_1(t,w)\, dw$$

(21)

$$\frac{dF_2}{dt} = \rho - \delta\, F_2 - \int_{w_s}^{\infty} \gamma_2(F_2)\, m_2(t,w)\, dw$$

(22)

$$\begin{cases} \dfrac{\partial m_1(t,w)}{\partial t} + \gamma_1(F_1) \dfrac{\partial\, m_1(t,w)}{\partial w} = -\,\eta_1 m_1(t,w) \\ \gamma_1(F_1) m_1(t,0) = \beta\, \gamma_2(F_2) \displaystyle\int_1^{\infty} m_2(t,w)\, dw \end{cases}$$

(23)

$$\begin{cases} \dfrac{\partial m_2(t,w)}{\partial t} + \gamma_2(F_2) \dfrac{\partial\, m_2(t,w)}{\partial w} = -\,\eta_2 m_2(t,w) \\ \gamma_2(F_2) m_2(t,w_s) = \gamma_1(F_1)\, m_1(t,w_s) \end{cases}$$

(24)

In this rescaled model equations 21 and 22 describe the dynamics of the food resource density in either habitat. Equations 23 and 24 describe the dynamics of the size-dependent density functions of the population, including the boundary condition at the birth size s_b in habitat 1 (in eq.23), and at the switching size s_s from habitat 1 to habitat 2 (in eq. 24).

2. Eco-evolutionary model formulation

To study evolutionary dynamics on ecological timescales, we use standard methods of quantitative genetics. We consider the body size at habitat switch to be a quantitative trait controlled by a number of loci of small effect. To capture both ecological and evolutionary dynamics the population is characterized by the density functions $m_1(t, w, \omega)$ and $m_2(t, w, \omega)$, in which ω refers to the trait value (body size at habitat switch) of the individuals. Hence, $m_1(t, w, \omega)$ and $m_2(t, w, \omega)$ refers to the density of individuals at time t with body size w and trait value ω, respectively. Since the trait value does not change during an individual's life, the dynamics of resource densities and population abundances are now described by

$$\frac{dF_1}{dt} = 1 - \delta\, F_1 - \int_0^1 \int_0^\omega \gamma_1\,(F_1)\, m_1(t, w, \omega\,)\, dw\, d\omega \tag{25}$$

$$\frac{dF_2}{dt} = \rho - \delta\, F_2 - \int_0^1 \int_\omega^\infty \gamma_2\,(F_2)\, m_2(t, w, \omega\,)\, dw\, d\omega \tag{26}$$

$$\frac{\partial m_1(t, w, \omega\,)}{\partial t} + \gamma_1\,(F_1) \frac{\partial\, m_1(t, w, \omega\,)}{\partial w} = -\,\eta_1 m_1(t, w, \omega\,) \tag{27}$$

$$\frac{\partial m_2(t, w, \omega\,)}{\partial t} + \gamma_2\,(F_2) \frac{\partial\, m_2(t, w, \omega\,)}{\partial w} = -\,\eta_2 m_2(t, w, \omega\,) \tag{28}$$

The boundary condition of the last PDE is given by:

$$\gamma_2\,(F_2) m_2(t, \omega\,, \omega\,) = \gamma_1\,(F_1)\, m_1(t, \omega\,, \omega\,) \tag{29}$$

Which expresses that individuals with trait value ω switch habitat when they reach body size $w = \omega$. The boundary condition of PDE (20) is given by:

$$\gamma_1\,(F_1) m_1(t, 0, \omega_0) = \Phi(\omega_0, \overline{\omega_a}, \sigma)\;\beta\;\gamma_2\,(F_2) \int_0^1 \int_1^\infty m_2(t, w, \omega\,)\;dw\;d\omega$$

(30)

The left-hand side of this equation represents the birth rate of individuals at body size $w = 0$ with a trait value equal to $\omega = \omega_0$. The function $\Phi(\omega_0, \overline{\omega_a}, \sigma)$ models the relation between the trait distribution of the offspring and the average trait value of the reproducing population. Analogous to the quantitative genetics formulation of Lande for structured-populations (*7*) (eq. A2) we assume that the average value of the produced offspring equals the average breeding value of parents reproducing at time t. Since individual fecundity is independent of both body size and trait value, selection results of differential survival from birth to the adulthood stage among individuals with different trait values. Therefore, the average breeding value of the parents equals their average trait value:

$$\overline{\omega_a} = \int_0^1 \int_1^\infty \omega\; m_2(t, w, \omega\,)\;dw\;d\omega \bigg/ \int_0^1 \int_1^\infty m_2(t, w, \omega\,)\;dw\;d\omega$$

(31)

We assume the distribution of offspring trait values to follow a truncated, approximately normal distribution (specifically, a Bates distribution of degree 3) around the mean $\overline{\omega_a}$. In particular $\Phi(\omega_0, \overline{\omega_a}, \sigma)$ is defined as:

$$\Phi(\omega_0, \overline{\omega_a}, \sigma) = \begin{cases} \dfrac{27}{2} x^2, & \text{if } \overline{\omega_a}\,(1-\sigma) \le \omega_0 \le \overline{\omega_a}\,(1 - \dfrac{1}{3}\sigma) \\ -27x^2 + 27x - \dfrac{9}{2}, & \text{if } \overline{\omega_a}\left(1 - \dfrac{1}{3}\sigma\right) < \omega_0 \le \overline{\omega_a}\,(1 + \dfrac{1}{3}\sigma) \\ \dfrac{27}{2} x^2 - 27x + \dfrac{27}{2}, & \text{if } \overline{\omega_a}\left(1 + \dfrac{1}{3}\sigma\right) < \omega_0 \le \overline{\omega_a}\,(1+\sigma) \\ 0 & \text{otherwise} \end{cases}$$

(32)

where $x = \frac{1}{2}\left(\frac{\omega_0 - \overline{\omega_a}\,(1-\sigma)}{\overline{\omega_a}\,\sigma}\right)$.

3. Effects of trait variation in the population

The amount of trait variation in the population influences the eco-evolutionary dynamics following the change in conditions. As expected, lower trait variation causes the evolutionary dynamics to slow down. A reduction of variation in the population of the body size at habitat switch by 50% increases the time to the regime shift after the perturbation by more than 3 times (regime shift at time 390 when minimum and maximum trait values are 80% and 120% of the mean trait value and at time 940 when they are 90% and 110% of the mean trait value in figure S2). Although the amount of trait variation affects the time it takes for natural selection to drive the population to the critical trait value at which the regime shift occurs, once the mean trait value is equal to the critical value, the ecological trajectory followed by the population and its resource is not affected by the amount of trait variation in the population.

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