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CONSTRAINTS**

Roel Beetsma and Ed Westerhout

***INTERNATIONAL MACROECONOMICS
AND FINANCE***



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A COMPARISON OF NOMINAL AND INDEXED DEBT UNDER FISCAL CONSTRAINTS

Abstract

This paper makes a welfare comparison between the issuance of price-indexed and nominal public debt in the presence of fiscal constraints, viz. a debt constraint, a deficit constraint and a combination of both. Distortionary taxes or public consumption are regulated to avoid the violation of the relevant fiscal constraint(s). Under a debt constraint indexed debt is generally preferred, while under a deficit constraint the results are more mixed. Introducing inflation persistence and raising the maturity of the debt tends to increase the magnitude of the welfare differences between the two types of debt. Welfare differences are further affected by the degree to which public consumption and tax revenues are indexed to actual versus structural nominal GDP.

JEL Classification: E62, H62, H63

Keywords: indexed debt, nominal debt, deficit, fiscal constraints, public consumption, tax revenues, welfare, inflation (persistence), wage growth, debt maturity.

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1. Introduction

Although inflation is currently low among high-income countries, over the longer run inflation uncertainty is non-negligible as history has repeatedly shown. The balance lengthening operations of central banks in response to the recent crisis create additional inflation uncertainty, as little is known about the long-run consequences of these operations. Eventually, what the issuers and the buyers of public debt are interested in, is the real rather than the nominal return on the debt. With a predetermined nominal interest rate on the debt, unanticipated changes in inflation can have substantial consequences for the real debt return, and this may have serious financial implications for both the issuer and the buyer of the debt. This creates a potential market for issuing debt indexed to inflation, which is what a number of countries now indeed do. These include the majority of the largest economies, like the U.S., the U.K., Germany, France, Canada, Italy, Japan and Spain (Krämer, 2013).

This paper compares the issuance of price-indexed and nominal public debt in the presence of fiscal constraints. Examples are ceilings on the public debt ratio or the deficit ratio,⁴ like in the European Union. Most of the outstanding debt of high-income countries is nominal. Yet, issuing indexed instead of nominal debt may be a more attractive strategy if this succeeds in delivering more stable debt or deficit ratios.⁵ More stability in those ratios is not beneficial for its own sake. However, it may reduce the chance that a fiscal constraint becomes binding, which would force the government to change taxes or public consumption, variables that do affect utility either directly (in the case of public consumption) or indirectly (in the case of taxes) via their effect on lifetime income available for private consumption.

Our paper is novel in a number of dimensions. First, to the best of our knowledge, it is the first paper to formally explore the trade-off between indexed and non-indexed debt in the presence of fiscal constraints. Second, in contrast to most of the literature, our analysis takes place under the assumption that inflation is exogenous. This implies that the government cannot use inflation as an instrument to stabilize the public finances. This case corresponds to that of a monetary union in which monetary policy is beyond the control of national governments.⁶ Third, we explore explicitly

⁴ In the sequel, the "debt ratio" refers to the ratio of the public debt over GDP, while the "deficit ratio" refers to the ratio of the public deficit over GDP.

⁵ These potential advantages of indexed debt neglect the microeconomic benefits of holding indexed debt in asset portfolios. For example, pension funds and insurance companies may desire protection against inflation uncertainty, and holding indexed debt may provide a natural way to achieve this.

⁶ In principle, a country could influence monetary policy through the decisions of the union's common central bank. However, taking the case of the Eurozone, each country's representative on the Governing Council (which takes the

the role of the debt maturity and how this combines with the persistence in the inflation process in affecting the tradeoff between the two types of public debt. Finally, we set up a model that is quite different from the models in the literature, by linking the growth of public consumption to a mixture of actual and structural nominal GDP growth. This is a realistic feature in a world in which part of public consumption grows automatically with actual output, while another part is fixed in nominal terms at the start of the year. We demonstrate the quantitative importance for the tradeoff between nominal and indexed debt of linking public consumption to actual versus structural nominal GDP growth.

Our micro-founded model features representative households who value private consumption, public consumption and leisure. The instrument of the government is either distortionary taxation or public consumption. Moreover, the government can issue either nominal or indexed debt. We distinguish between two risk factors, namely shocks to price inflation and productivity growth, and we allow these factors to be potentially correlated. In addition, we consider two possible fiscal constraints, namely a ceiling on the debt ratio and a ceiling on the deficit ratio. We also consider the case in which both constraints are imposed simultaneously. As long as the relevant ceilings are not hit, taxes and public consumption follow exogenous processes, which implies a partial link to structural nominal GDP and a partial link to actual nominal GDP. Such a formulation captures the essential movements of taxes and public consumption processes in reality. When a constraint threatens to become violated, the government adjusts its instrument such that the constraint is still obeyed.

Our main results are the following. In the absence of any fiscal constraint, the debt ratio fluctuates less under indexed debt, because inflation shocks move the gross nominal return on debt and nominal output into the same direction. This suggests that indexed debt is preferred to nominal debt in the presence of a ceiling on the debt ratio (a “debt ceiling”), as under indexed debt there is less need to use the government’s instrument. By contrast, the results are more mixed in the presence of a ceiling on the deficit ratio (a “deficit ceiling”), because under nominal debt the deficit-GDP ratio is more stable in the beginning of the projection period, although over time its variability rises relative to that under indexed debt. This is caused by the, over time, rising relative variance of the debt ratio under nominal debt and, hence, the rising relative variance of the interest burden as part of the deficit ratio. Under our baseline parameter setting, as well as most of the variations on the baseline, indexed debt does better also under a deficit ceiling. This shows the

monetary policy decisions) is supposed to act in the interest of the union as a whole and has only a small part of the total vote, while the ECB is explicitly modelled as an independent central bank. On top of this there is a substantial degree of uncertainty in the transmission from the basic monetary policy instrument to the inflation rate.

importance of analyzing the trade-off between the two types of debt in a dynamic rather than a static framework. Our simulations provide an indication of the order of magnitude of the welfare effects. Expressed in terms of a permanent change in public consumption, under the benchmark parameter setting switching from nominal to indexed debt implies a gain of 0.17 per cent under a debt constraint and a gain of 0.04 per cent under a deficit constraint. We also explore a number of variations on the baseline parameter setting, varying the variances of the shocks, their correlation, the initial debt ratio, the initial tax revenues as a share of GDP and the interest rate. The welfare ranking of the two types of debt is virtually always the same, though the magnitude of the welfare differences varies.

The impact of inflation and productivity shocks also depends on the fractions of tax revenues and public consumption related to structural rather than actual nominal output growth. With a full link of tax revenues and public consumption to actual nominal output growth, inflation shocks do not affect the primary deficit ratio of output.⁷ However, once a link to structural growth is introduced, the primary deficit ratio will be affected by inflation shocks. Since the primary deficit ratio is mostly negative (there is a primary surplus), with nominal debt the impact of inflation shocks on gross or net debt-servicing costs as a ratio of GDP will be partly offset through this link to structural growth, while under indexed debt this new effect has a destabilizing effect on the debt and deficit ratios. Hence, a stronger link of public consumption and revenues to structural rather than actual nominal output growth reduces the attractiveness of indexed debt relative to nominal debt.

While our baseline set-up assumes independently and identically distributed shocks and a one-year debt maturity, we extend the model to allow for persistence in inflation and a longer debt maturity. Allowing for an autoregressive component in inflation increases the welfare differences between nominal and price-indexed bonds, because it raises the long-run variability of inflation. Under the benchmark parameter combination, the welfare difference increases to a permanent consumption difference of roughly 0.36 per cent for the debt constraint and 0.09 percent for the deficit constraint. Welfare differences also increase when the debt maturity increases, because an innovation in inflation affects the ex-post real return on the debt over a longer period. The welfare differences are largest for a combination of longer-maturity debt and persistence in inflation – both strengthen the impact of inflation shocks and therefore widen the welfare gap between nominal and price-indexed debt. Under the benchmark parameter combination, the welfare difference rises to a

⁷ The numerator and denominator of the ratio are affected in the same proportion by inflation shocks.

permanent consumption difference of roughly 0.48 per cent under the debt constraint and 0.09 per cent under the deficit constraint.

This paper is connected to different strands of the literature. There is a substantial literature on optimal public debt management dating back to Lucas and Stokey (1983) – see Alfaro and Kanczuk (2006) for a non-technical review. The choice between indexed and nominal debt is part of this optimization problem.⁸ Several contributions focus on the potential for tax smoothing over time and states of nature through the use of nominal debt. Examples are Bohn (1990), Calvo and Guidotti (1993) and Chari and Kehoe (1999).⁹ These articles abstract from time inconsistency issues arising from the desire of the authorities to alleviate the real debt burden through the opportunistic use of inflation. Taking this issue into account, Van der Ploeg (1995) shows that when it is forced to issue nominal debt, the government will over time accumulate sufficient nominal assets to eliminate the time inconsistency problem. Other articles study the trade-off between nominal and indexed debt by combining tax smoothing and incentive effects into a single framework. Bohn (1988) finds that it is optimal for a government to issue at least some nominal debt, while in Díaz-Giménez et al. (2008) indexed debt welfare dominates nominal debt under full commitment or with log-utility of consumption. Otherwise, the comparison depends on the initial indebtedness. Barro (2003) argues for issuing indexed debt in the presence of incentive problems. Alfaro and Kanczuk (2010) find that indexed debt is preferable in a quantitative framework with incomplete information about whether the government is impatient or patient. Calvo and Guidotti (1990, 1992) and Blanchard and Missale (1994) investigate the role of the maturity of nominal debt when the authorities have an incentive to inflate away some of its real value. Persson *et al.* (1987, 2006) and Alvarez *et al.* (2004) show how proper management of nominal and indexed debt of various maturities can remove time inconsistencies associated with monetary and fiscal policy.

Importantly, the current paper is also related to the literature that investigates the costs and benefits of constraints on fiscal policy, in particular the literature that explores deficit and debt constraints of the type employed in the Eurozone. A substantial number of papers have studied these constraints. Some examples are Beetsma and Uhlig (1999), Buiters and Grafe (2004), Chari and Kehoe (2007), Pappa and Vassilatos (2007), Krogstrup and Wyplosz (2010) and Wyplosz (2012). Empirical work on the effectiveness of such constraints is found in, for example, Bohn and Inman (1996) and Debrun *et al.* (2008).

⁸ Fischer (1983) and Campbell and Shiller (1996) give an overview of the main trade-offs between nominal and indexed debt.

⁹ Another way in which public debt management can be deployed to stabilise the economy is through excusable debt default justified by the state of the economy – see Grossman and Van Huyck (1988).

The sequel of this paper is structured as follows. Section 2 presents the model, while Section 3 introduces the different fiscal constraints. In Sections 4, 5 and 6 we discuss, respectively, the simulation set-up, the calibration and the simulation results when distortionary taxes are used as the government's instrument. Section 7 extends the model to longer debt maturities and inflation persistence. Section 8 switches to public consumption as the government's instrument. Finally, Section 9 concludes the main body of the paper.

2. The model

The private sector of the economy consists of a continuum of infinitely-lived representative individuals with total mass normalized to unity. Each agent derives utility from consuming private goods, public goods and leisure. Private individuals have access to a storage technology that allows them to (dis)save against a constant real rate of interest. Firms produce using labor as their only input. The economy features two sources of fundamental risk, namely in inflation and labor productivity growth. Hence, inflation is exogenous and, therefore, beyond the control of the government. This corresponds to the case of a monetary union, like the Eurozone, for example, in which an independent common central bank controls monetary policy. The inflation uncertainty may arise from monetary policy having to respond to uncertain developments in the monetary union, from the central bank having imperfect control over the money supply or because the link between the money supply and inflation is stochastic. Stochastic productivity growth translates into stochastic growth of real wages. Further, there is a government that issues debt to risk-neutral investors on the international capital markets. The debt can be either nominal or indexed to price inflation. Because the investors are risk-neutral, they want to be compensated for expected inflation, but they do not require an inflation risk premium. The debt ratio, the deficit ratio or both may be subject to a ceiling, which, if hit, forces the government to intervene with its policy instrument. Finally, variables are measured at the end of the period.

2.1. Utility and individual decisions

For the representative individual we adopt the following inter-temporal utility function based on the Greenwood, Hercowitz and Huffman (1988) utility specification:

$$U_t = \sum_{j=t}^{\infty} (1+\delta)^{t-j} E_t \left[\frac{1}{1-\zeta} \left(c_j + \chi_j \frac{v_j^{1-\gamma}}{1-\gamma} \right)^{1-\zeta} + \varphi \frac{(G_j/P_j)^{1-\beta}}{1-\beta} \right], \quad \beta, \gamma, \delta, \zeta, \varphi, \chi_j > 0. \quad (1)$$

Parameter δ denotes the individual's rate of time preference, c and v denote real private consumption and leisure respectively, while G is public consumption in nominal terms and P the price level.¹⁰ We define public consumption explicitly in nominal terms for reasons that become clear below. Given the normalization of the population size, real public consumption G/P at the individual level equals aggregate public consumption. Further, χ denotes the preference for leisure relative to that for private and public consumption. It increases at a constant rate over time: $\chi_t = (1 + \bar{g}_w)\chi_{t-1}$, where $\chi_0 = 1$ and \bar{g}_w denotes the mean rate of growth of the wage rate, to be discussed further below. In other words, the utility of a given amount of leisure increases over time. This formulation avoids a trend in which leisure becomes an ever-falling fraction of available time, because it becomes more and more expensive in terms of foregone consumption when the real wage is on average growing at a positive rate. Such a negative time trend in leisure is clearly unrealistic. The parameter φ is a weighting factor that we will use to bring the utilities attached to private and public consumption on an equal footing. Finally, β and ζ are relative risk aversion parameters capturing the aversion to fluctuations in real public consumption and “full” consumption defined as the composite of private consumption and leisure given by the term in brackets in (1).

The inter-temporal budget constraint of the representative agent is:

$$\sum_{j=t}^{\infty} (1+r)^{t-j} c_j = \sum_{j=t}^{\infty} (1+r)^{t-j} \left[(1-\tau_j)w_j l_j - (S_j / P_j) \right] + A_t. \quad (2)$$

Here, $r > 0$ is the exogenous and constant real interest rate, τ is the tax rate on labor income, l is hours worked, w is the wage rate and S denotes lump-sum taxes in nominal terms, *i.e.*, taxes that are unrelated to individual labour income. We normalize available time to one. Hence, $l = 1 - v$. Finally, A_t denotes financial wealth at the start of year t .

The agent's optimization problem is to maximize utility in equation (1) over the instruments c_j and v_j , $j \geq t$, subject to the inter-temporal budget constraint in (2) and initial financial wealth. Private consumption and leisure in period t are selected after the period- t stochastic shocks have occurred.

The separability between private consumption and leisure in equation (1) implies that leisure is a function of the contemporaneous wage rate only:

¹⁰ Since there is only one type of public spending in this model, in the sequel we will use the terms public consumption and public spending interchangeably.

$$v_t = 1 - l_t = \left(\frac{(1 - \tau_t) w_t}{\chi_t} \right)^{\frac{-1}{\gamma}}. \quad (3)$$

Hence, the dynamics assumed for the preference for leisure imply that, absent any shocks to the wage rate and any changes to the tax rate, leisure will remain constant through time. Above-average wage growth reduces leisure and increases the labor supply, while the opposite is true when the rate of wage growth is below average.

The derivation of private consumption is rather involved and is provided in Appendix A. The solution is a function of current financial wealth A_t , the net wage rate $(1 - \tau_t)w_t$ and lump-sum taxes S_t / P_t :

$$c_t = -d_t + \left[1 - \frac{1}{1+r} \left(\frac{1+r}{1+\delta} \right)^{1/\zeta} \right] \left\{ \begin{array}{l} A_t + \left[\frac{1+r}{r - \bar{g}_w} \right] (1 - \tau_t) w_t + \gamma \left[\frac{1+r}{1+r - (1 + \bar{g}_w) q_1} \right] d_t \\ - E_t \sum_{j=t}^{\infty} (1+r)^{t-j} (S_j / P_j) \end{array} \right\}, \quad (4)$$

where

$$d_t = \frac{\chi_t}{1-\gamma} \left(\frac{(1-\tau_t)w_t}{\chi_t} \right)^{\frac{\gamma-1}{\gamma}}, \quad q_1 = E_t \left[\left(\frac{1 + \bar{g}_w + \varepsilon_w}{1 + \bar{g}_w} \right)^{\frac{\gamma-1}{\gamma}} \right], \quad (5)$$

where ε_w is a mean-zero shock to real wage growth, which is discussed further below. The accumulation of financial wealth A_t follows from subtracting consumption and lump-sum taxes from after-tax labor income:

$$A_t = (1+r) \left[A_{t-1} + (1 - \tau_{t-1}) w_{t-1} l_{t-1} - (S_{t-1} / P_{t-1}) - c_{t-1} \right]. \quad (6)$$

The final term in curly brackets in (4) is calculated numerically, as explained in Appendix A.

Apart from the two terms involving d_t , equation (4) reflects the life-cycle model of consumption. Indeed, in the term between curly brackets, the first term refers to financial wealth, the second one to the discounted value of all net wage income flows and the last one to the discounted value of the lump-sum tax payments. The two terms involving d_t in equation (4) refer to the take-up of leisure. An increase in the current amount of leisure corresponds to lower consumption (labor supply and consumption are complements), as captured by the first term at the

right-hand side of equation (4), while more leisure in the future implies higher current consumption, as captured by the third term between curly brackets.

2.2. Production

We assume that real output y_t is linear in labor and that labor gets its marginal product paid. Hence, $y_t = w_t l_t$. Using (3) we have:

$$y_t = \left[1 - \chi_t^{1/\gamma} (1 - \tau_t)^{-1/\gamma} w_t^{-1/\gamma} \right] w_t. \quad (7)$$

2.3. The risk factors

There are two risk factors, $g_{w,t}$ and π_t , where the former is the rate of growth of the real wage (or, equivalently, the rate of growth of productivity) and the latter is the rate of price inflation. Our baseline assumes that these risk factors are stochastic variables driven by the following processes:

$$\begin{aligned} g_{w,t} &= \bar{g}_w + \varepsilon_{w,t}, \\ \pi_t &= \bar{\pi} + \varepsilon_{\pi,t}, \end{aligned} \quad (8)$$

where $\bar{\pi}$ is the average inflation rate, and $\varepsilon_{w,t}$ and $\varepsilon_{\pi,t}$ are identically and independently distributed zero-mean shocks with respective standard deviations $\sigma_w > 0$ and $\sigma_\pi > 0$ and contemporaneous correlation coefficient ρ . The link between the price level and the rate of inflation is given by $P_{t+1} = (1 + \pi_{t+1})P_t$.

Appendix B shows that, because the preference for leisure χ increases with the average real wage growth rate \bar{g}_w , the actual real output growth rate g_t can be approximated as $g_t = \bar{g}_w + \varepsilon_{g,t}$, where $\varepsilon_{g,t} = K_{t-1} \varepsilon_{w,t}$. Here, K_{t-1} is a function of the previous period's wage rate w_{t-1} . Hence, the average growth rates of real output and the real wage are equal, while the shock to real output growth is linked to the shock to real wage growth. Given that the labour supply is increasing in the wage rate, K_{t-1} exceeds unity. This implies that real output is more volatile than the wage rate. The intuition is that a positive wage shock produces an increase in the labour supply, implying that the

effect on GDP growth is magnified. We will simulate the real output growth rate between t and $t+1$ by drawing a shock $\varepsilon_{w,t+1}$ and calculating it as $g_{t+1} = (y_{t+1} - y_t) / y_t$.

2.4. The dynamics of public consumption and tax revenues in the absence of fiscal constraints

When none of the fiscal constraints are binding in period $t+1$, nominal public consumption and nominal tax revenues T evolve exogenously as

$$\tilde{G}_{t+1} = \alpha_t^G (1 + \bar{n}) G_t + (1 - \alpha_t^G) (1 + n_{t+1}) G_t = (1 + \bar{n}) G_t + (1 - \alpha_t^G) \varepsilon_{n,t+1} G_t, \quad (9)$$

$$\tilde{T}_{t+1} = S_{t+1} + \tau_{t+1} w_{t+1} l_{t+1} P_{t+1} = (1 + \bar{n}) T_t + (1 - \alpha_t^T) \varepsilon_{n,t+1} T_t, \quad (10)$$

where $1 + n_{t+1} = (1 + g_{t+1})(1 + \pi_{t+1})$ is the gross rate of nominal GDP growth. Tax revenues \tilde{T}_{t+1} carry a tilde, which we will use here and in the sequel to indicate that a process evolves without interruption, because none of the fiscal constraints are binding. Further, T_t are *actual* tax revenues in period t . Hence, T_t includes the effect of potential intervention in period t in case a fiscal constraint is binding in that period. Notice that the labour income tax rate τ_{t+1} equals its value in the previous period, because the government does not intervene in period $t+1$ if this is not necessary to ensure that the fiscal constraints are met.

We can write the (net) rate of nominal GDP growth as $n_{t+1} = \bar{n} + \varepsilon_{n,t+1}$, where $\bar{n} \equiv \bar{g} + \bar{\pi} + \bar{g}\bar{\pi} + Cov(\varepsilon_{g,t+1}, \varepsilon_{\pi,t+1})$, where $Cov(\varepsilon_{g,t+1}, \varepsilon_{\pi,t+1})$ is the unconditional covariance between $\varepsilon_{g,t+1}$ and $\varepsilon_{\pi,t+1}$, and $\varepsilon_{n,t+1} \equiv (1 + \bar{\pi})\varepsilon_{g,t+1} + (1 + \bar{g})\varepsilon_{\pi,t+1} + \varepsilon_{g,t+1}\varepsilon_{\pi,t+1} - Cov(\varepsilon_{g,t+1}, \varepsilon_{\pi,t+1})$. Hence, \bar{n} is equal to expected nominal GDP growth, while $\varepsilon_{n,t+1}$ is a shock to nominal GDP growth with an unconditional mean of zero.

Equation (9) assumes that public consumption, G , features two components. The first, with share α_t^G , grows in line with expected or structural nominal GDP, hence with growth rate \bar{n} . The second, with share $1 - \alpha_t^G$, grows in line with actual nominal GDP, hence with growth rate n_{t+1} . Hence, as the far right-hand side of equation (9) shows, only a fraction $1 - \alpha_t^G$ of nominal public consumption is affected by the productivity and inflation shocks in period $t+1$. This way of modeling nominal public consumption may be quite uncommon, but it is realistic, because in reality

public consumption tends to grow in line with the trend in economic activity, while on an annual basis some of its components grow with actual nominal GDP, e.g. as a result of a mid-term budget revision or a mid-term outcome of a new public wage negotiation, while other components are set before the start of the new fiscal year in line with structural nominal GDP growth. Our formulation allows for the maximum possible flexibility as regards to the nominal level of public consumption that is fixed in advance for the year.

Equation (10) shows that tax revenues feature two components as well. The first are the revenues from lump-sum taxation. We assume these to grow in line with structural nominal GDP. The second are the revenues from distortionary labor income taxation. This component grows proportionally to actual nominal GDP. Appendix C derives the far right-hand side of (10) and, hence, shows that a fraction $1 - \alpha_t^T$ of total tax revenues is affected by productivity and inflation shocks, where α_t^T is the fraction of total tax revenues that is lump sum.

Appendix C also shows that, under the assumption that the labor income tax rate τ is constant and there is no binding fiscal constraint, the updating rules for the components of public consumption and tax revenues linked to structural nominal GDP growth follow endogenously as:

$$\alpha_{t+1}^G = \frac{\alpha_t^G (1 + \bar{n}) G_t}{G_{t+1}}, \quad \alpha_{t+1}^T = \frac{\alpha_t^T (1 + \bar{n}) T_t}{T_{t+1}}. \quad (11)$$

2.5. The dynamics of debt and deficit in the absence of fiscal constraints

In our baseline set up, all debt is renewed after one year. To derive the required adjustments in the government's instrument in the presence of fiscal constraints, we first present the debt and deficit ratios for the various cases. In the following, we will use B and Y to denote public debt, respectively GDP, both in nominal value terms. We use the superscripts N and I to denote the cases in which debt is nominal, i.e. not indexed, respectively indexed. For some generic variable Z we define $\Delta Z_{t+1} = Z_{t+1} - Z_t$.

2.5.1. Nominal debt

When debt is nominal, the debt ratio evolves as:

$$\frac{B_{t+1}^N}{Y_{t+1}} = \frac{G_{t+1} - T_{t+1}}{Y_{t+1}} + \frac{1 + i_{t+1}}{(1 + g_{t+1})(1 + \pi_{t+1})} \frac{B_t^N}{Y_t}, \quad (12)$$

where i denotes the nominal interest rate. Substitution of (9) and (10) into (12) yields the debt ratio when no fiscal constraint is binding in period $t+1$. Because investors are risk neutral, the nominal interest rate compensates one-for-one for expected inflation, so that

$$1 + i_{t+1} = (1 + r)(1 + \pi_{t+1}^e), \quad (13)$$

where $\pi_{t+1}^e = \bar{\pi}$ is the expected rate of inflation in period $t+1$. In our simulations, we will assume that $r > 0$ and $\bar{\pi} > 0$, so that $i_{t+1} > 0$. The deficit ratio is:

$$\frac{D_{t+1}^N}{Y_{t+1}} = \frac{G_{t+1} - T_{t+1}}{Y_{t+1}} + \left[\frac{(1 + r)(1 + \pi_{t+1}^e) - 1}{(1 + g_{t+1})(1 + \pi_{t+1})} \right] \frac{B_t^N}{Y_t}. \quad (14)$$

Substituting (9) and (10) into this equation, we obtain the deficit ratio when no fiscal constraint is binding in period $t+1$.

2.5.2. Indexed debt

In the case of indexed debt, it is not the nominal, but the real return on the debt that is fixed at the moment that the debt is issued. Hence, the interest paid on the debt is adjusted to actual inflation, implying a gross nominal debt return of $(1 + r)(1 + \pi_{t+1})$, while the gross real return is $1 + r$.

Therefore, the debt ratio is:

$$\frac{B_{t+1}^I}{Y_{t+1}} = \frac{G_{t+1} - T_{t+1}}{Y_{t+1}} + \frac{1 + r}{1 + g_{t+1}} \frac{B_t^I}{Y_t}, \quad (15)$$

while the deficit ratio is given by:

$$\frac{D_{t+1}^I}{Y_{t+1}} = \frac{G_{t+1} - T_{t+1}}{Y_{t+1}} + \left[\frac{(1+r)(1+\pi_{t+1})-1}{(1+g_{t+1})(1+\pi_{t+1})} \right] \frac{B_t^I}{Y_t}. \quad (16)$$

2.5.3. Marginal effects of shocks

For future use, we give the expressions for the effect of an inflation shock on the debt and deficit ratios when none of the fiscal constraints are binding and assuming that $\alpha_t^G = \alpha_t^T \equiv \alpha_t$:

$$\frac{\partial (B_{t+1}^N / Y_{t+1})}{\partial \varepsilon_{\pi,t+1}} = - \left[\frac{\alpha_t (1+\bar{n})(G_t - T_t) + (1+i_{t+1})B_t^N}{(1+\pi_{t+1})Y_{t+1}} \right], \quad \frac{\partial (B_{t+1}^I / Y_{t+1})}{\partial \varepsilon_{\pi,t+1}} = - \left[\frac{\alpha_t (1+\bar{n})(G_t - T_t)}{(1+\pi_{t+1})Y_{t+1}} \right], \quad (17)$$

$$\frac{\partial (D_{t+1}^N / Y_{t+1})}{\partial \varepsilon_{\pi,t+1}} = - \left[\frac{\alpha_t (1+\bar{n})(G_t - T_t) + i_{t+1}B_t^N}{(1+\pi_{t+1})Y_{t+1}} \right], \quad \frac{\partial (D_{t+1}^I / Y_{t+1})}{\partial \varepsilon_{\pi,t+1}} = - \left[\frac{\alpha_t (1+\bar{n})(G_t - T_t) - B_t^I}{(1+\pi_{t+1})Y_{t+1}} \right]. \quad (18)$$

These expressions are derived in Appendix D. The same appendix also derives the expressions for the marginal effects of a positive productivity shock.

Consider the special case of $\alpha_t = 0$, so that by (9) and (10) the primary deficit ratio when the fiscal constraints are not binding simplifies to $(\tilde{G}_{t+1} - T_{t+1})/Y_{t+1} = (G_t - T_t)/Y_t$ and, hence, it is not affected by any of the period $t+1$ shocks. Therefore, these shocks can only affect the debt and deficit ratios through the final terms in (12) and (14) – (16). Because the nominal interest rate i_{t+1} is predetermined, a positive inflation shock in period $t+1$ unambiguously reduces the absolute value of the debt ratio when debt is nominal, while the debt ratio is unaffected when debt is indexed. These results suggest that the debt ratio is more stable under indexed than under nominal debt. Further, because $i_{t+1} > 0$, when debt is nominal and positive in the previous period, a positive inflation surprise reduces the deficit ratio. By contrast, with indexed debt, a positive inflation surprise raises the deficit ratio if the debt in period t is positive. Moreover, assuming that $i_{t+1} < 1$, which is typically the case in reality, for a given debt ratio in period t the effect of an inflation shock on the deficit ratio is larger in absolute magnitude when debt is indexed than when debt is nominal. This suggests that for a given debt ratio the deficit ratio is more volatile under indexed debt than under nominal debt.

3. The fiscal constraints

We will consider six possible policy regimes in total: nominal debt combined with only a debt ceiling, only a deficit ceiling or both ceilings imposed simultaneously, and indexed debt combined with only a debt ceiling, only a deficit ceiling or both ceilings imposed simultaneously. These are all relevant cases for the EU countries, which are required to observe a 60% of GDP ceiling on the debt ratio and a 3% of GDP ceiling on the deficit ratio. So far, the monitoring of the Stability and Growth Pact (SGP) has mostly focused on deficit ratios (e.g., see Beetsma and Giuliadori, 2010). However, recent reforms of the SGP have introduced more emphasis on the debt ceiling (see European Commission, 2014).

The ceiling that we impose on the debt or deficit ratio prevents these ratios from growing without bound. In order to avoid that the debt ratio can fall to arbitrarily low levels we could also impose a lower bound of, say, zero on the debt ratio. This can be motivated by noting that a lack of outstanding public debt instruments may have undesirable broader consequences, such as disruptions of the proper functioning of financial markets and an expansion of the shadow banking sector, because of a shortage of safe and liquid assets (e.g., see Chadka et al., 2013, Footnote 12). We do not do so, as in reality such a lower bound would hardly ever be relevant. Debt ratios are generally substantially larger than zero and the usual worry is that the debt ratio grows too large for debt to remain sustainable. Moreover, given that the SGP does not state a minimum debt ratio, the choice of a minimum debt ratio would be somewhat arbitrary. In our welfare comparisons below, we will correct for differences in debt levels at the end of the evaluation period.

Whenever a constraint threatens to be violated, the government will adjust its instrument, such that the constraint is exactly obeyed. Our base case is when the government uses the labor income tax rate for this purpose. We use a single expression for both indexed and nominal debt for the *realized* nominal interest rate i_{t+1}^R . Indeed, $i_{t+1}^R = (1+r)(1+\pi_{t+1}^e) - 1$ in case of nominal bonds¹¹ and $i_{t+1}^R = (1+r)(1+\pi_{t+1}) - 1$ in case of price-index bonds.

Two matters deserve specific attention. First, as hours worked depend negatively on the labour income tax rate, our model features a Laffer curve, *i.e.* there is an upper bound to revenues from labour income taxation. Hence, if a fiscal constraint is binding, it may happen that the government cannot satisfy the constraint by adjusting the labour income tax rate alone. In this case, the tax rate will be set at the revenue-maximizing level and the remainder of the financing need will be met by

¹¹ In the case of autoregressive inflation or debt maturities exceeding one period, the expression changes accordingly, as explained in Subsection 7.1.

increasing lumpsum taxes. In our simulations this will happen only rarely, since the initial labor income tax rate is substantially to the left of the revenue-maximizing rate. Nevertheless, it is conceivable that a long string of negative inflation or productivity surprises pushes the debt ratio to such a high level that an increase in the labour tax rate is insufficient to prevent the fiscal constraint from being violated. Second, we exclude negative values for the labour income tax rate. Hence, it may happen that if the debt ratio threatens to become negative, the constraint cannot be met by adjusting the labor income tax rate alone. In this case, the tax rate will be given a zero value and the remainder of the financing surplus will be used to reduce lump-sum taxes.

We need to distinguish three cases. In the first case, no fiscal constraint is violated. Hence, the labour income tax rate keeps the value of the previous period, $\tau_{t+1} = \tau_t$, while lump-sum taxes increase with the rate of structural economic growth, $S_{t+1} = (1 + \bar{n})\alpha_t^T T_t$.

In the second case, a fiscal constraint is violated if the tax rate is not adjusted, while the Laffer curve constraint is not binding, i.e. and $X_{t+1} \leq \bar{X}_{t+1}$. We use X_{t+1} to denote the revenues from labour income taxation and \bar{X}_{t+1} to denote the upper bound on this variable. In this second case, the labor income tax rate is set at the value that ensures that the fiscal constraint equals its threshold level, $\tau_{t+1} = \tau_{\max,t+1}$, while lump-sum taxes increase with the rate of structural economic growth, $S_{t+1} = (1 + \bar{n})\alpha_t^T T_t$. Note that not only the labour supply, but also GDP, tax revenues and government spending are a function of the labor income tax rate. To be specific, let us assume that the debt ceiling is the constraint that threatens to be violated. The derivations for the deficit ceiling or the combinations of both ceilings are completely analogous. Hence, the value of $\tau_{\max,t+1}$ is determined by solving an implicit equation:

$$(1 + i_{t+1}^R)B_t + (\alpha_t^G G_t - \alpha_t^T T_t)(1 + \bar{n}) = \left[\left(\frac{B}{Y} \right)_{\max} + \tau_{\max,t+1} - (1 - \alpha_t^G) \left(\frac{G_t}{Y_t} \right) \right] \left\{ w_{t+1} P_{t+1} \left[1 - \left(\frac{(1 - \tau_{\max,t+1}) w_{t+1}}{\chi_{t+1}} \right)^{-1/\gamma} \right] \right\} \quad (19)$$

This equation is the standard budget accumulation equation in which the resulting nominal amount of debt has been set equal to the ceiling on the debt ratio times nominal output in period $t+1$, which is the term in curly brackets and which follows from the optimal labor-leisure trade-off. The unusual form of equation (19) is explained by the fact that we have brought all the terms related to realized nominal output to the right-hand side. In deriving (19) we have also made use of (9).

The third case is the same as the second case, except that now the Laffer curve constraint is binding, i.e. $X_{t+1} > \bar{X}_{t+1}$. In this case, the tax rate is set at the revenue-maximizing value, $\tau_{t+1} = \bar{\tau}_{t+1}$, while lump-sum taxes are adjusted to prevent the debt ratio from crossing its threshold level:¹²

$$S_{t+1} = \left[(1+i_{t+1}^R)B_t + \alpha_t^G G_t (1+\bar{n}) \right] - \left[\left(\frac{B}{Y} \right)_{\max} + \bar{\tau}_{t+1} - (1-\alpha_t^G) \left(\frac{G_t}{Y_t} \right) \right] \left\{ w_{t+1} P_{t+1} \left[1 - \left(\frac{(1-\bar{\tau}_{t+1})w_{t+1}}{\chi_{t+1}} \right)^{-1/\gamma} \right] \right\} \quad (20)$$

where the revenue-maximizing tax rate $\bar{\tau}_{t+1}$ is obtained by setting the derivative of $X_{t+1} = \tau_{t+1} Y_{t+1}$ with respect to τ_{t+1} to zero:

$$1 - \left(\frac{(1-\bar{\tau}_{t+1})w_{t+1}}{\chi_{t+1}} \right)^{-1/\gamma} \left(1 - \frac{1}{\gamma} \left(\frac{\bar{\tau}_{t+1}}{1-\bar{\tau}_{t+1}} \right) \right) = 0.$$

4. The simulation set up

We simulate 500 paths of 100 years. We simulate all six regimes with the same paths for the fundamental shocks, although this condition is irrelevant if the number of paths is sufficiently large (we have checked that 500 is a sufficiently large number in our case). For each simulated path, in year 1, the model draws an inflation shock and a productivity growth shock and calculates the effects on the contemporaneous values for public consumption, tax revenues and GDP. The resulting deficit and debt ratios are compared with the relevant constraints and, if necessary, are adjusted by changing the labour income tax rate as described in the previous section. This yields the outcomes for the budgetary variables in period 1. These form the starting point for a similar set of steps in year 2, and so on.

Unfortunately, not all simulated paths can be used. In particular, the trajectory of the wage rate can be so low that the consumption-leisure combination $c_j + \chi_j v_j^{1-\gamma} / (1-\gamma)$ turns negative in some years, so that the marginal utilities from private consumption and leisure becomes undefined. To handle this problem, we had to chose initial financial wealth relatively high. In those cases where this proved to be insufficient, we eliminated the entire path from the simulation sample. It

¹² The derivations for the deficit ceiling or the combinations of both ceilings are completely analogous.

turns out that in the majority of simulations, there is no need to eliminate any of the simulation paths. In the 12 percent of the simulations in which we had to throw away one or more simulation paths, we had to eliminate at most 0.2 percent of the simulation paths. Hence, the consequences for the results are at most very minor.

Having simulated all these paths, we can calculate the median paths and the paths corresponding to the 5% and 95% percentiles. We can also calculate the variance of each variable for each year into the simulation run. We measure the welfare effect of switching from nominal to indexed debt under a specific regime by the constant percent increase in the volume of public consumption, G_j / P_j , in all years and all states of nature that would lift utility under nominal debt to utility under indexed debt. We denote this measure by EQV . Appendix F provides its exact expression. $EQV > 0$ implies a welfare gain of switching from nominal to indexed debt, while $EQV < 0$ implies a welfare loss of such a switch.

There is one more issue that warrants discussion. The debt ratio can and will generally be different under the scenarios with nominal debt and indexed debt. In particular, the debt ratios in the two cases may differ at the end of the simulation horizon. This raises the question whether the two scenarios are comparable, because the case in which the debt ratio ends highest would seem unduly favourable, as it implies that over the simulation horizon more resources must have been borrowed on the international market, *ceteris paribus*, that have been allocated to public and/or private consumption. Therefore, we correct the EQV for the difference in the end-of-horizon public debt levels, which ensures that the two scenarios are fully comparable. Appendix F explains the details of the adjustment procedure.

5. The calibration

Table 1 summarizes the benchmark values of the parameters in our model and the assumed initial values of the endogenous variables. The benchmark parameter values are in line with values commonly assumed in the literature. This is the case for the curvature parameters β , ζ and γ in the utility function, the real interest rate r , average real-wage growth \bar{g}_w and average inflation $\bar{\pi}$. The latter is in line with the ECB objective of keeping the core inflation rate in the Eurozone under, but close to, 2%.

The initial public consumption ratio is 25% of GDP. Public consumption in our model corresponds to purchases of goods and services by the government, which on average is around a quarter of GDP for rich economies. The initial tax ratio is set such that in a scenario with

deterministic wage growth ($\sigma_w = 0$) and public consumption and tax revenues fully proportional to nominal GDP ($\alpha_0^G = \alpha_0^T = 0$), the debt ratio stabilizes at its initial level of 30% when debt is indexed. Hence, under this calibration there is no tendency of the debt ratio to systematically rise or fall and, hence, to deviate from the middle of its band when the 60% debt ceiling is imposed.

The initial gross wage rate is set such that the time endowment is initially divided equally between labor and leisure, i.e. $l=0.5$. This corresponds to a full working day of eight hours, assuming that an individual is forced to have eight hours of sleep, so that 16 hours remain for other activities. We treat private and public consumption symmetrically. Hence, we take equal values for β and ζ . Furthermore, we calibrate φ such that the average values of the marginal utility of full consumption and the marginal utility of public consumption are equal in the first year of the specific scenario just discussed ($\alpha_0^G = \alpha_0^T = \sigma_w = 0$).

Table 1: Benchmark parameter values and initial values of endogenous variables

δ	γ	β	r	\bar{g}_w	σ_w	$\bar{\pi}$	σ_π	φ
2.0%	10.0	2.0	3.0%	1.0%	1.0%	2.0%	2.0%	0.0533
ρ	ς	α_0^T	α_0^G	P_0	$(A/Y)_0$	$(T/Y)_0$	$(G/Y)_0$	$(B/Y)_0$
0	2.0	0%	0%	1.0	10.0	25.59%	25%	30%

6. The simulation results

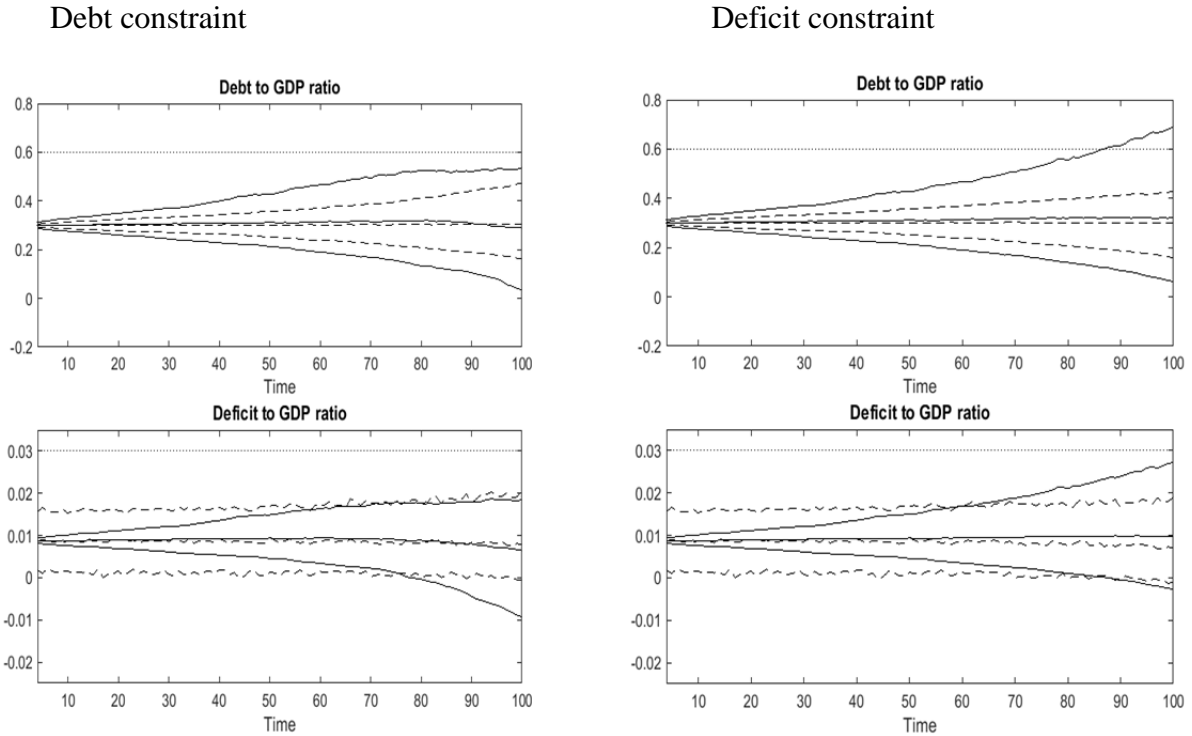
This section presents and discusses the simulation results. The discussion above suggested that, for $\alpha_t^G = \alpha_t^T = 0$, the debt ratio is more stable when debt is indexed, while the deficit ratio is more stable in the beginning of the projection period when debt is nominal. Table 2 reports for the benchmark parameter combination the results of the welfare comparison, confirming that indexed debt performs better under a debt constraint. However, this is also the case under a deficit constraint and when both constraints are combined. To see why, see Figure 1, which displays the dynamics of the debt and deficit ratios under both a debt and a deficit constraint for the benchmark parameter combination. The figure shows that, even though the deficit ratio is initially more stable under nominal debt, in the longer run it is more stable under indexed debt. The reason is that the rising variance of the debt ratio under nominal debt relative to that under indexed debt causes over time a rise in the relative variance of the interest burden, which is a component of the deficit.

Table 2: EQV (in %)

	Debt constraint	Deficit constraint	Both constraints simultaneously
Benchmark	0.17	0.04	0.15
$\sigma_\pi = 0.015$	0.09	0.03	0.08
$\sigma_\pi = 0.025$	0.30	0.05	0.25
$\sigma_w = 0.005$	0.11	0.06	0.11
$\sigma_w = 0.015$	0.22	0.01	0.21
$\rho = -0.5$	0.09	0.03	0.07
$\rho = 0.5$	0.28	0.07	0.27
$(B/Y)_0 = 20\%$	0.02	0.02	0.02
$(B/Y)_0 = 40\%$	0.84	-0.49	0.87
$(T/Y)_0 = 25.3\%$	0.72	-0.42	0.85
$(T/Y)_0 = 25.9\%$	0.04	0.04	0.04
$r = 2.5\%$	0.05	0.04	0.04
$r = 3.5\%$	0.38	-0.14	0.38

Note: EQV is the percent change in public consumption in all years and states of nature that makes utility under nominal debt equal to utility under indexed debt. A positive EQV means that indexed debt performs better in welfare terms. All cases are based on the benchmark parameter combination, except for the parameter specifically given.

Figure 1: Comparisons for a debt and a deficit constraint under the benchmark



Notes: the solid lines in the top panels refer to the median, 10th percentile and 90th percentile of the debt ratio in the case of nominal debt. The dashed lines refer to the analogues for indexed debt. The solid (dashed) lines in the bottom panels of the figure refer to the analogues for the deficit ratio under nominal (indexed) debt.

Table 2 also reports welfare comparisons for variations on the baseline parameter combination. Our simulations confirm that nominal and indexed debt perform identically (i.e., $EQV=0$) when we set inflation uncertainty to zero (not shown to preserve space). In all cases indexed debt performs better under a debt constraint, while the same holds true in most cases under a deficit constraint, and it always holds true when both constraints are simultaneously imposed.

Not surprisingly, raising inflation uncertainty σ_π relative to the benchmark always increases the magnitude of the welfare differences, while reducing inflation uncertainty has the opposite effect. Increasing productivity uncertainty σ_w also magnifies the welfare difference under a debt constraint. To understand this finding, notice that the two types of shocks interact with each other via the term $(1+g_{t+1})(1+\pi_{t+1})$ in expression (12) for the debt ratio. Hence, an increase in the uncertainty about productivity magnifies the effects of inflation shocks and raises the volatility of the debt ratio under nominal relative to indexed debt. Reducing the correlation ρ between the productivity and inflation shocks dampens the variance of the term $(1+g_{t+1})(1+\pi_{t+1})$, thereby reducing the fluctuations in the final term of (12) relative to that in (15) and in the final term of (14) relative to that in (16). Hence, the relative performance of indexed debt is reduced both under a debt and a deficit constraint.

Under the debt constraint, a higher starting value of the debt ratio has a strong magnifying effect on the welfare difference between the two types of debt. The higher the initial debt ratio, the smaller the expected time to hitting the debt ceiling for the first time. The shortening of the expected time before the first hit weighs relatively heavily because of the discounting applied to future outcomes. Hence, under a debt constraint the relative performance of indexed debt becomes better. Further, given the starting value of the ratio of taxes over GDP, a higher initial debt ratio implies a larger average deficit, because of the larger interest payments. Given the larger variability of the deficit ratio under indexed debt, the expected time to hitting the deficit ceiling for the first time is shorter under indexed debt, which reduces the relative attractiveness of indexed debt under a deficit constraint.

A reduction in the initial tax revenues ratio relative to the benchmark raises the relative attractiveness of indexed debt under a debt constraint. We consider a rather small reduction, as the effects are quite substantial. It implies an upward trend in the debt ratio immediately after the start of a simulation run and, for the same reason as starting with a higher initial debt ratio than under the benchmark, this reduces further the relative attractiveness of nominal debt under a debt constraint. It also implies a higher initial deficit ratio, which makes indexed debt relatively less attractive under a deficit ceiling. An increase in the real interest rate r also makes indexed debt relatively more

attractive under a debt constraint and relatively less attractive under a deficit constraint for the same reason as the reduction in initial tax revenues. Immediately after the start of the simulation, due to the higher interest payments on the outstanding public debt, the deficit is higher than under the benchmark and debt is put on an upward trend.

Table 3: EQV (in %)

	Debt constraint	Deficit constraint	Both constraints simultaneously
Benchmark	0.17	0.04	0.15
$\alpha_0^T = \alpha_0^G = 0.25$	0.16	0.04	0.14
$\alpha_0^T = \alpha_0^G = 0.5$	0.13	0.03	0.12
$\alpha_0^T = \alpha_0^G = 0.75$	0.08	0.03	0.07
$\alpha_0^T = \alpha_0^G = 1$	0.04	0.02	0.02

Note: See notes to Table 2.

Inspection of the derivatives in (17) and (18) shows that, for $\alpha_t > 0$ and a positive initial debt ratio, under nominal debt the original effects of inflation shocks through $(1+i_{t+1})B_t^N$ and $i_{t+1}B_t^N$ on the debt and deficit ratios are dampened, because the effects of the shocks through the primary surplus work in the opposite direction.¹³ Moreover, in contrast to the benchmark case, now, with indexed debt, the debt ratio is affected by inflation shocks, while the effect of a given inflation shock on the deficit ratio is strengthened. This suggest that an increase in $\alpha_0 > 0$ weakens the relative performance of indexed debt in the presence of a debt or a deficit ceiling. This is indeed borne out by the figures reported in Table 3.

7. Extensions: shock persistence and longer-maturity debt

This section extends our model into two directions by introducing shock persistence and by allowing for longer-maturity debt. When inflation is persistent, as it is in reality, a positive inflation shock this year raises expected inflation in ensuing years. Further, while with one-period debt the effect of an inflation shock on expected future inflation will be fully factored into the nominal interest rate when the debt is renewed after one period, with longer-maturity debt nominal interest rates will only gradually adjust as the existing stock of outstanding debt is gradually phased out. Therefore, with longer-maturity debt an inflation shock not only reduces the current real value of the nominal debt, but also its real value in future periods until it is redeemed. This discussion also

¹³ Notice that, since the average debt in the simulations is positive, while the average deficit is close to zero, on average there is a primary surplus.

suggests that it is the *combination* of inflation persistence and longer-maturity debt that may affect the welfare comparison between nominal and indexed debt.

In this section the vector of variables (g_{t+1}^w, π_{t+1}) follows a vector auto-regressive (VAR) system, of which (8) is a special case. All the debt has maturity $L \geq 1$ at the moment of its issuance. We start by deriving the debt and deficit dynamics under nominal and indexed debt, after which we turn to discussing the simulation results for this more general setting.

7.1. Nominal debt

When moving from period $t-1$ to period t , debt that was issued in $t-L$ is replaced with new debt of length L . Define F_t^N as the amount of nominal debt issued in period t and to be redeemed in period $t+L$. We can write total debt in period t as:

$$B_t^N = \sum_{j=0}^{L-1} F_{t-j}^N, \quad (21)$$

i.e., the sum of the current and past $L-1$ debt issues, and the dynamics of the total debt as:

$$\Delta B_{t+1}^N = G_{t+1} - T_{t+1} + \sum_{j=0}^{L-1} i_{t+1-j} F_{t-j}^N, \quad (22)$$

where i_{t+1-j} is the nominal interest rate on the debt issued in period $t-j$. It is constant over the life of the debt.

Using (21) and (22), the debt ratio is given as the counterpart to equation (12) as

$$\frac{B_{t+1}^N}{Y_{t+1}} = \frac{G_{t+1} - T_{t+1} + \sum_{j=0}^{L-1} (1 + i_{t+1-j}) F_{t-j}^N}{Y_{t+1}},$$

while the deficit ratio equals the change in the total debt (22) divided by Y_{t+1} , which yields the counterpart to equation (14). When the fiscal constraints are not binding, the deficit and debt dynamics are obtained by substituting (9) and (10) into the new expressions for the debt and deficit ratios.

The nominal interest rate i_{t+1-j} should compensate for the expected inflation over the life of the debt issued in period $t-j$. Hence,

$$(1+i_{t+1-j})^L = \prod_{k=1}^L [(1+r)(1+\pi_{t+k-j}^{e|t-j})] \Rightarrow i_{t+1-j} = \left(\prod_{k=1}^L [(1+r)(1+\pi_{t+k-j}^{e|t-j})] \right)^{1/L} - 1,$$

where $\pi_{t+\tau-j}^{e|t-j}$ is the inflation rate expected in period $t-j$ for period $t+\tau-j$.

7.2. Indexed debt

Define F_t^I as the nominal amount of indexed debt issued in period t and to be redeemed in period $t+L$. Because debt holders are compensated for actual inflation, the dynamics of the nominal amount of the total debt are:

$$\Delta B_{t+1}^I = G_{t+1} - T_{t+1} + \sum_{j=0}^{L-1} [(1+r)(1+\pi_{t+1}) - 1] F_{t-j}^I.$$

Using $B_t^I = \sum_{j=0}^{L-1} F_{t-j}^I$, we can now write the debt ratio as:

$$\frac{B_{t+1}^I}{Y_{t+1}} = \frac{G_{t+1} - T_{t+1} + \sum_{j=0}^{L-1} [(1+r)(1+\pi_{t+1})] F_{t-j}^I}{Y_{t+1}}.$$

The deficit ratio follows directly by dividing the above expression for ΔB_{t+1}^I by Y_{t+1} .

7.3. Simulation results

We consider the same set of fiscal regimes as before. For each regime we can derive the settings for the labour income tax rate in the same manner as we did before under one-period debt. Table 4 presents the simulation results. Compared to the simulations in Section 6, in the columns under $L = 1$ we add inflation persistence to the model, while still keeping the debt maturity at its original length $L = 1$. In particular, while for labor productivity growth we continue to use the i.i.d. specification in (8), for price inflation we adopt the following autoregressive process:

$$\pi_{t+1} = \bar{\pi} + 0.7(\pi_t - \bar{\pi}) + \varepsilon_{\pi,t+1}. \quad (23)$$

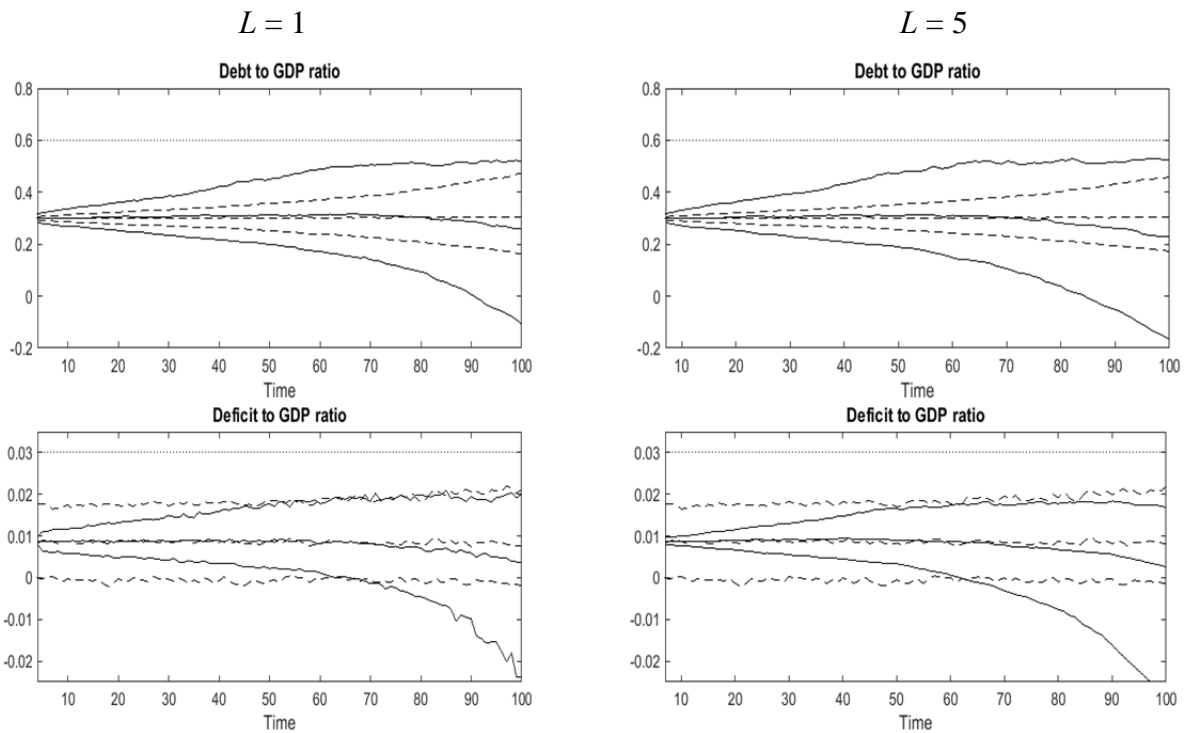
This specification is inspired by Whelan (2007), who analyses autoregressive price inflation in both the U.S. and the euro area. It is also backed by Campbell and Mankiw (1987). Inflation and productivity shocks continue to be independent of each other (at all leads and lags). The columns under the header $L = 5$ use the wage growth and inflation specifications in (8) and (23), but in addition assume that the debt maturity at issuance date is $L = 5$ years.

Table 4: EQV (in %)

	Debt constraint	Debt constraint	Deficit Constraint	Deficit constraint	Both Constraints simultaneously	Both Constraints simultaneously
	$L=1$	$L=5$	$L=1$	$L=5$	$L=1$	$L=5$
Benchmark	0.36	0.48	0.09	0.09	0.22	0.08
$\sigma_\pi = 0.015$	0.14	0.21	0.05	0.06	0.11	0.19
$\sigma_\pi = 0.025$	0.65	0.81	0.10	0.09	0.34	0.65
$\sigma_w = 0.005$	0.25	0.39	0.10	0.12	0.17	0.37
$\sigma_w = 0.015$	0.40	0.58	0.07	0.06	0.30	0.51
$\rho = -0.5$	0.17	0.27	0.03	0.05	0.07	0.21
$\rho = 0.5$	0.49	0.69	0.14	0.14	0.38	0.64
$(B/Y)_0 = 20\%$	0.03	0.01	0.03	0.04	0.03	0.04
$(B/Y)_0 = 40\%$	1.56	1.51	-0.31	-0.56	0.93	1.28
$(T/Y)_0 = 25.3\%$	1.38	1.29	-0.15	-0.42	0.83	1.30
$(T/Y)_0 = 25.9\%$	0.05	0.06	0.05	0.06	0.05	0.06
$r = 2.5\%$	0.06	0.12	0.06	0.08	0.06	0.12
$r = 3.5\%$	1.47	0.73	-0.01	-0.09	0.39	0.65
$\alpha_0^T = \alpha_0^G = 0.25$	0.28	0.44	0.06	0.05	0.20	0.40
$\alpha_0^T = \alpha_0^G = 0.5$	0.30	0.47	0.05	0.04	0.19	0.43
$\alpha_0^T = \alpha_0^G = 0.75$	0.26	0.49	0.03	0.03	0.19	0.45
$\alpha_0^T = \alpha_0^G = 1$	0.23	0.61	0.01	0.01	0.10	0.57

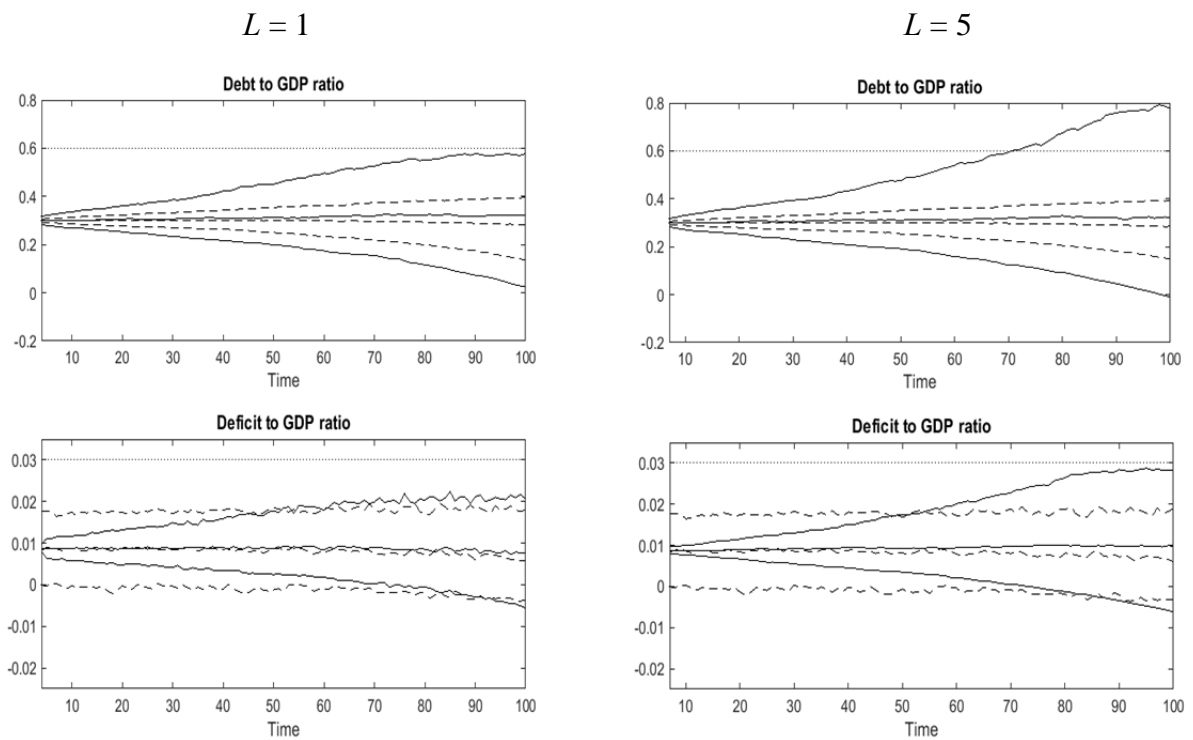
Note: See notes to Table 2.

Figure 2: Comparisons for a debt constraint under the benchmark



Note: Solid (dashed) lines refer to the case of nominal (indexed) debt. Further, see notes to Figure 1.

Figure 3: Comparisons for a deficit constraint under the benchmark



Note: Solid (dashed) lines refer to the case of nominal (indexed) debt. Further, see notes to Figure 1.

The effect of adding inflation persistence is to increase the variability of inflation. Average price inflation remains unchanged at 2 percent. The resulting higher inflation variance widens the gap in performance between indexed and nominal debt in most instances with a debt constraint and in most instances with a deficit constraint or when both constraints are combined (columns under $L = 1$).

The effect of moving from a one-year to a five-year maturity is to widen the gap in performance further in most instances. The reason is that a shock in price inflation now affects the servicing costs of the public debt over a period of five years rather than one year. For the baseline parameter setting, Figures 2 and 3 display the dynamics of the debt and deficit ratios under both a debt and a deficit ceiling.

8. Public consumption as instrument

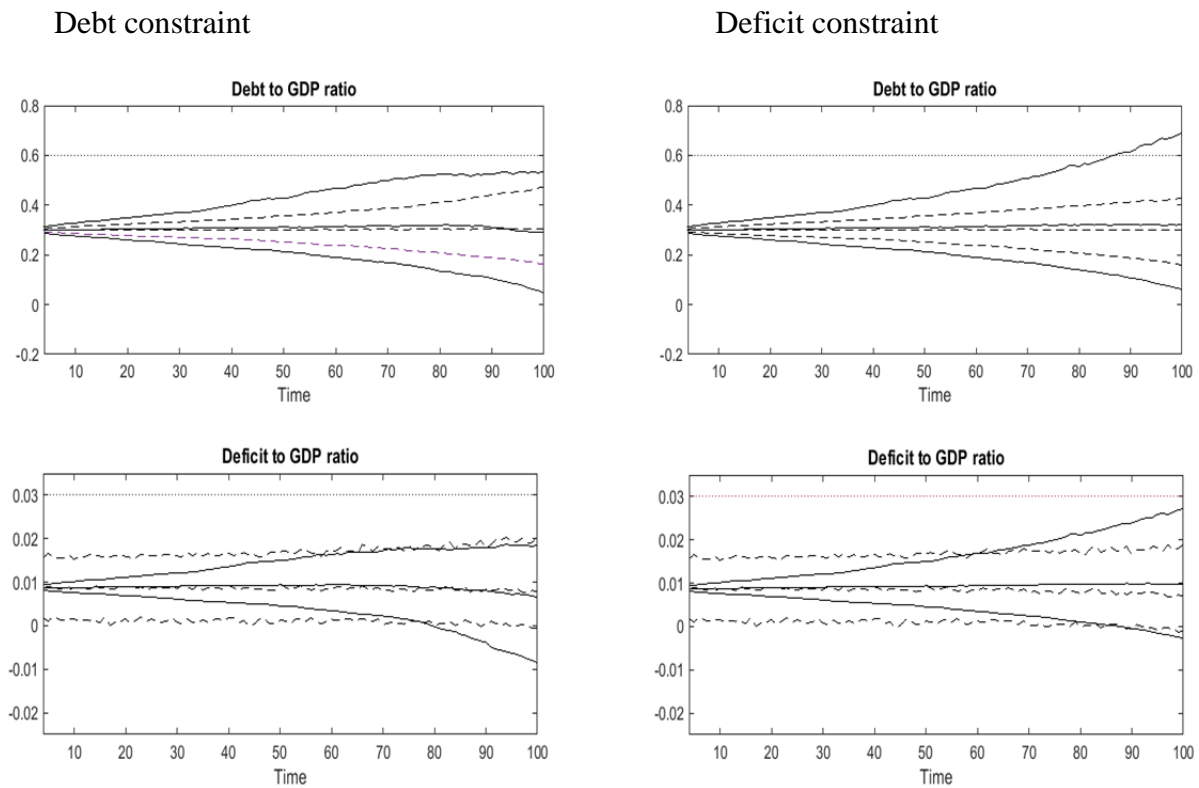
Up till now, the government used the labour income tax rate to ensure that the debt and deficit ratios obeyed their constraints. In this section, public consumption takes up this role instead. This is an interesting variation as, unlike in the case in which the labour income tax rate is adjusted, changes in public consumption do not affect the labour-leisure decision.

We assume that whenever a constraint threatens to get violated, public consumption will be adjusted such that the constraint is exactly obeyed, while the labour income tax rate is kept constant. The effects of this adjustment depend on its specific allocation over the two spending components, *i.e.* the component related to structural nominal GDP growth and the component related to actual nominal GDP growth. In order to avoid arbitrariness, the two components are adjusted such that their relative size remains unchanged. Recall that when the labour income tax rate was the policy instrument, changes in lumpsum taxes could occur, especially when otherwise the tax revenues from labour income taxation would be insufficiently large or the labour income tax rate would become negative. Now, with public consumption as the policy instrument, utility will be undefined if public consumption turns negative. Therefore, lump-sum taxes will be employed whenever public consumption threatens to fall below some positive lower bound $G_L > 0$, at which it is then held until shocks lift public consumption up again.

We present results for the same cases that we studied before. These cases differ along the following lines: they feature a debt constraint, a deficit constraint or both, and they feature a non-autoregressive inflation process or the autoregressive inflation process (23) and a debt maturity of one or five years. In Appendix E we derive the instrument settings for the various cases.

The discussion above suggested that, for $\alpha_t^G = \alpha_t^T = 0$, the debt ratio is more stable when debt is indexed, while in the beginning of the simulation period the deficit ratio is more stable when debt is nominal. Indeed, as shown in Figure 4 for the benchmark parameter setting, the confidence band on the debt ratio is wider with nominal than with indexed debt. Consistent with this result, though not depicted here in order to preserve space, the variance of the debt ratio increases faster for nominal debt than for indexed debt as time proceeds. In contrast, the confidence band on the deficit ratio is wider and its variance is larger under indexed debt than under nominal debt over the first half of the simulation period, both under a debt constraint and under a deficit constraint, while the opposite is true later in the simulation period. From Table 5 we observe that for the benchmark parameter setting indexed debt always outperforms nominal debt. The ranking of the welfare effects for variations on the benchmark parameter combination in the case where public consumption is the instrument to absorb shocks is quite similar to that in the case where the labour income tax is the instrument. The welfare effects for the simulations that use public consumption as the instrument are available upon request from the authors.

Figure 4: Comparisons for a debt and a deficit constraint under the benchmark



Note: Solid (dashed) lines refer to the case of nominal (indexed) debt. Further, see notes to Figure 1.

Table 5: EQV (in %) under benchmark for government consumption as instrument

	Debt constraint	Deficit constraint	Both constraints simultaneously
Inflation i.i.d., $L=1$	0.03	0.06	0.04
Inflation persistence, $L=1$	0.03	0.06	0.04
Inflation persistence, $L=5$	0.03	0.11	0.04

Note: See notes to Table 2.

9. Concluding remarks

This paper has made a welfare comparison between the issuance of price-indexed and nominal public debt in the presence of a debt constraint, a deficit constraint and a combination of the two constraints. Shocks originate in inflation and productivity growth. Public consumption is indexed to actual and structural nominal GDP, except when fiscal constraints threaten to be violated, in which case the government's instrument (the labour tax rate or public consumption) is adjusted to avoid such a violation. In the absence of any fiscal constraint, the debt ratio is more volatile under nominal debt, while the deficit ratio is initially more, and in the longer run less, volatile under indexed debt. Under our benchmark parameter combination indexed debt always outperforms non-indexed debt in terms of welfare, while this is also the case in most of the variations on the benchmark that we considered. Introducing inflation persistence and moving from one- to multi-period public debt increases the magnitude of the welfare differences between the two types of debt under both types of constraints, because both the variance of inflation and the period over which inflation shocks are felt increases.

Hence, our analysis suggests that from a macro-economic perspective it is likely a wise idea for governments to issue indexed debt. Our analysis ignores the additional micro-economic benefits of indexed debt in terms of making the bond market more complete and giving investors additional hedging possibilities. Since these benefits come on top of the benefits we detected, it seems puzzling why not more indexed debt is issued by governments, especially in countries that have lost control over their monetary instrument and, thereby, over the possibility to actively exploit nominal debt to hedge shocks affecting the public budget. A possible explanation may be the rather small size of the market in indexed debt, especially in the starting up phase, which leads to relatively high liquidity premia. In the case of the Eurozone this obstacle should become smaller once countries start to issue debt jointly. Overall, research taking an integral approach of analyzing the various costs and benefits of issuing indexed public debt would be highly desirable.

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Appendices

A: Derivations for the household's optimization problem

Utility is given by:

$$U_t = \sum_{j=t}^{\infty} (1+\delta)^{t-j} E_t \left[\frac{1}{1-\zeta} \left(c_j + \chi_j \frac{v_j^{1-\gamma}}{1-\gamma} \right)^{1-\zeta} + \varphi \frac{(G_j/P_j)^{1-\beta}}{1-\beta} \right].$$

Households maximize this utility function, subject to initial financial wealth, A_t , and the inter-temporal budget constraint:

$$\sum_{j=t}^{\infty} (1+r)^{t-j} c_j = \sum_{j=t}^{\infty} (1+r)^{t-j} \left[(1-\tau_j) w_j l_j - (S_j/P_j) \right] + A_t. \quad (\text{A.1})$$

The instruments are consumption and leisure in all years. The respective first-order conditions for period t are:

$$\begin{aligned} \left(c_t + \chi_t \frac{v_t^{1-\gamma}}{1-\gamma} \right)^{-\zeta} &= \lambda, \\ \left(c_t + \chi_t \frac{v_t^{1-\gamma}}{1-\gamma} \right)^{-\zeta} \chi_t v_t^{-\gamma} &= \lambda (1-\tau_t) w_t, \end{aligned}$$

where λ is the Lagrange multiplier associated with the inter-temporal budget constraint. Combining these conditions and eliminating yields leisure as a function of the contemporaneous wage rate only:

$$v_t = 1 - l_t = \left(\frac{(1 - \tau_t)w_t}{\chi_t} \right)^{\frac{-1}{\gamma}}. \quad (\text{A.2})$$

The first-order condition for $c_j, j > t$, is:

$$(1 + \delta)^{t-j} \mathbf{E}_t \left[z_j^{-\zeta} \right] = \lambda (1 + r)^{t-j}, \text{ where } z_j = c_j + \chi_j \frac{v_j^{1-\gamma}}{1-\gamma}.$$

Taking a first-order approximation of $z_j^{-\zeta}$ around $z_j = \mathbf{E}_t [z_j]$ yields

$$z_j^{-\zeta} \simeq \left[\mathbf{E}_t (z_j) \right]^{-\zeta} + [z_j - \mathbf{E}_t (z_j)] (-\zeta) \left[\mathbf{E}_t (z_j) \right]^{-\zeta-1} \Rightarrow \mathbf{E}_t [z_j^{-\zeta}] \simeq \left[\mathbf{E}_t (z_j) \right]^{-\zeta}.$$

Hence, the preceding expression can be written as:

$$(1 + \delta)^{t-j} \left[\mathbf{E}_t \left(c_j + \chi_j \frac{v_j^{1-\gamma}}{1-\gamma} \right) \right]^{-\zeta} = (1 + r)^{t-j} \left(c_t + \chi_t \frac{v_t^{1-\gamma}}{1-\gamma} \right)^{-\zeta}, \quad (\text{A.3})$$

where we have substituted away λ using the above first-order condition for c_t . Using (A.2) for period j , we can write:

$$\mathbf{E}_t \left(\chi_j \frac{v_j^{1-\gamma}}{1-\gamma} \right) = \frac{1}{1-\gamma} \mathbf{E}_t \left[\chi_j \left(\frac{(1-\tau_j)w_j}{\chi_j} \right)^{\frac{\gamma-1}{\gamma}} \right]. \quad (\text{A.4})$$

Substituting (A.2) and (A.4) into (A.3):

$$(1 + \delta)^{t-j} \left\{ \mathbf{E}_t (c_j) + \frac{1}{1-\gamma} \mathbf{E}_t \left[\chi_j \left(\frac{(1-\tau_j)w_j}{\chi_j} \right)^{\frac{\gamma-1}{\gamma}} \right] \right\}^{-\zeta} = (1 + r)^{t-j} \left(c_t + \frac{\chi_t}{1-\gamma} \left(\frac{(1-\tau_t)w_t}{\chi_t} \right)^{\frac{\gamma-1}{\gamma}} \right)^{-\zeta}.$$

$$\begin{aligned} \Rightarrow E_t(c_j) &= \left(\frac{1+\delta}{1+r} \right)^{\frac{t-j}{\zeta}} \left(c_t + \frac{\chi_t}{1-\gamma} \left(\frac{(1-\tau_t)w_t}{\chi_t} \right)^{\frac{\gamma-1}{\gamma}} \right) - \frac{1}{1-\gamma} E_t \left[\chi_j \left(\frac{(1-\tau_j)w_j}{\chi_j} \right)^{\frac{\gamma-1}{\gamma}} \right] \\ \Rightarrow E_t(c_j) &= \left(\frac{1+\delta}{1+r} \right)^{\frac{t-j}{\zeta}} (c_t + d_t) - E_t(d_j), \text{ where } d_j = \frac{\chi_j}{1-\gamma} \left(\frac{(1-\tau_j)w_j}{\chi_j} \right)^{\frac{\gamma-1}{\gamma}}. \end{aligned}$$

Take expectations of (A.1) and substitute this expression into it. Some rewriting yields:

$$c_t = \left[\sum_{j=t}^{\infty} (1+r)^{t-j} \left(\frac{1+\delta}{1+r} \right)^{\frac{t-j}{\zeta}} \right]^{-1} \left\{ \begin{aligned} & E_t \left[A_t + \sum_{j=t}^{\infty} (1+r)^{t-j} \left((1-\tau_j)w_j l_j - (S_j / P_j) \right) \right] \\ & - \sum_{j=t}^{\infty} (1+r)^{t-j} \left[\left(\frac{1+\delta}{1+r} \right)^{\frac{t-j}{\zeta}} d_t - E_t d_j \right] \end{aligned} \right\}. \quad (\text{A.5})$$

Assuming that $\frac{1}{1+r} \left(\frac{1+r}{1+\delta} \right)^{1/\zeta} < 1$, we can write:

$$\left[\sum_{j=t}^{\infty} (1+r)^{t-j} \left(\frac{1+\delta}{1+r} \right)^{\frac{t-j}{\zeta}} \right]^{-1} = 1 - \frac{1}{1+r} \left(\frac{1+r}{1+\delta} \right)^{1/\zeta}. \quad (\text{A.6})$$

Further, recall that $\chi_t = (1+\bar{g}_w) \chi_{t-1}$. Hence, $\chi_j = (1+\bar{g}_w)^{j-t} \chi_t$. Hence,

$$\begin{aligned} E_t(d_j) &= E_t \left[\frac{\chi_j}{1-\gamma} \left(\frac{(1-\tau_j)w_j}{\chi_j} \right)^{\frac{\gamma-1}{\gamma}} \right] = E_t \left[(1+\bar{g}_w)^{j-t} \frac{\chi_t}{1-\gamma} \left(\frac{(1-\tau_j)w_j}{(1+\bar{g}_w)^{j-t} \chi_t} \right)^{\frac{\gamma-1}{\gamma}} \right] \\ &= E_t \left[(1+\bar{g}_w)^{j-t} \frac{\chi_t}{1-\gamma} \left(\frac{(1-\tau_t)w_t}{\chi_t} \right)^{\frac{\gamma-1}{\gamma}} \left(\frac{(1+\bar{g}_w + \varepsilon_{w,t+1}) \dots (1+\bar{g}_w + \varepsilon_{w,j})}{(1+\bar{g}_w)^{j-t}} \right)^{\frac{\gamma-1}{\gamma}} \right] \\ &= (1+\bar{g}_w)^{j-t} d_t \prod_{i=t+1}^j E_t \left[\left(\frac{1+\bar{g}_w + \varepsilon_{w,i}}{1+\bar{g}_w} \right)^{\frac{\gamma-1}{\gamma}} \right] = [q_1 (1+\bar{g}_w)]^{j-t} d_t, \end{aligned} \quad (\text{A.7})$$

where

$$q_1 \equiv E_t \left[\left(\frac{1+\bar{g}_w + \varepsilon_{w,i}}{1+\bar{g}_w} \right)^{\frac{\gamma-1}{\gamma}} \right], \quad i > t,$$

and where we have used the assumption that the subsequent ε_w are i.i.d. and, hence, we can write the expectation of the product of functions of the subsequent shocks as the product of the expectations of the functions of the subsequent shocks. Using (A.2), we can work out:

$$\begin{aligned}
\mathbf{E}_t(1-\tau_j)w_j l_j &= \mathbf{E}_t \left\{ (1-\tau_j)w_j \left[1 - \left(\frac{(1-\tau_j)w_j}{\chi_j} \right)^{\frac{1}{\gamma}} \right] \right\} \\
&= \mathbf{E}_t(1-\tau_j)w_j - \mathbf{E}_t \left[(1-\tau_j)^{\frac{\gamma-1}{\gamma}} \chi_j \left(\frac{w_j}{\chi_j} \right)^{\frac{\gamma-1}{\gamma}} \right] \\
&= (1-\tau_t)w_t \prod_{i=t+1}^j \mathbf{E}_t(1+\bar{g}_w + \varepsilon_{w,i}) - (1-\tau_t)^{\frac{\gamma-1}{\gamma}} \chi_t (1+\bar{g}_w)^{j-t} \left(\frac{w_t}{\chi_t} \right)^{\frac{\gamma-1}{\gamma}} \prod_{i=t+1}^j \mathbf{E}_t \left[\left(\frac{1+\bar{g}_w + \varepsilon_{w,i}}{1+\bar{g}_w} \right)^{\frac{\gamma-1}{\gamma}} \right] \\
&= (1-\tau_t)w_t (1+\bar{g}_w)^{j-t} - (1-\gamma)d_t [(1+\bar{g}_w)q_1]^{j-t}
\end{aligned} \tag{A.8}$$

Substituting (A.7) – (A.8) into (A.5), we obtain:

$$c_t = \left[1 - \frac{1}{1+r} \left(\frac{1+r}{1+\delta} \right)^{1/\zeta} \right] \left\{ \begin{aligned} & A_t + \sum_{j=t}^{\infty} \left[\frac{(1+\bar{g}_w)}{1+r} \right]^{j-t} [(1-\tau_t)w_t - q_1^{j-t} (1-\gamma)d_t] \\ & - \sum_{j=t}^{\infty} (1+r)^{t-j} \left(\frac{1+\delta}{1+r} \right)^{\frac{t-j}{\zeta}} d_t + \sum_{j=t}^{\infty} \left[\frac{(1+\bar{g}_w)q_1}{1+r} \right]^{j-t} d_t \\ & - \mathbf{E}_t \sum_{j=t}^{\infty} (1+r)^{t-j} \left(\frac{S_j}{P_j} \right) \end{aligned} \right\}.$$

Simplifying:

$$c_t = -d_t + \left[1 - \frac{1}{1+r} \left(\frac{1+r}{1+\delta} \right)^{1/\zeta} \right] \left\{ \begin{aligned} & A_t + \left[\frac{1+r}{r-\bar{g}_w} \right] (1-\tau_t)w_t + \gamma \left[\frac{1+r}{1+r-(1+\bar{g}_w)q_1} \right] d_t \\ & - \mathbf{E}_t \sum_{j=t}^{\infty} (1+r)^{t-j} \left(\frac{S_j}{P_j} \right) \end{aligned} \right\},$$

where we have made use of (A.6).

We still need to handle the term involving the lump-sum taxes. Recall that we can write:

$$\mathbf{E}_t \left(\frac{S_j}{P_j} \right) = \frac{S_t}{P_t} \mathbf{E}_t \left[\left(\frac{1+\bar{n}}{1+\pi_{t+1}} \right) \dots \left(\frac{1+\bar{n}}{1+\pi_j} \right) \right].$$

If inflation is given by $\pi_{t+1} = \bar{\pi} + \varepsilon_{\pi,t+1}$, we can write the expectations term as the product of expectations terms and, thus, we can further write:

$$E_t \left(\frac{S_j}{P_j} \right) = \frac{S_t}{P_t} \prod_{i=t+1}^j E_t \left[\left(\frac{1 + \bar{n}}{1 + \pi_i} \right) \right] = \frac{S_t}{P_t} q_2^{j-t}, \text{ where } q_2 \equiv E_t \left[\left(\frac{1 + \bar{n}}{1 + \pi_i} \right) \right].$$

For more general inflation processes, we have to evaluate $E_t (S_j / P_j)$ numerically. We do this as follows.

1. We divide the allowable range for inflation of $[\bar{\pi} - 3\sigma_\pi, \bar{\pi} + 3\sigma_\pi]$ into a grid of 100 points. Call the set of grid points $[a_1, \dots, a_{100}]$.
2. We draw $Z=1000$ paths of 25 inflation shocks: $[\varepsilon_{z,t+1}, \dots, \varepsilon_{z,t+25}]$, where z denotes the path.
3. Starting at each grid point $\pi_t = a_g$ and using the postulated process for inflation, we calculate for each path z the vector $[\pi_{g,z,t+1}, \dots, \pi_{g,z,t+25}]$.

4. We calculate $E \left(\frac{S_{t+j}}{P_{t+j}} \mid \pi_t = a_g \right) \approx \frac{1}{Z} \sum_{z=1}^Z \left[\prod_{i=1}^j \left(\frac{1 + \bar{n}}{1 + \pi_{g,z,t+i}} \right) \right]$ for $j=1, \dots, 25$, and

$$E \left(\frac{S_{t+j}}{P_{t+j}} \mid \pi_t = a_g \right) \approx q_1^{j-25} \frac{1}{Z} \sum_{z=1}^Z \left[\prod_{i=1}^{25} \left(\frac{1 + \bar{n}}{1 + \pi_{g,z,t+i}} \right) \right] \text{ for } j > 25.$$

5. Now that we have values for $E_t (S_j / P_j)$ at each grid-point for inflation, we can by means of a simple linear interpolation calculate

$$E (S_{t+j} / P_{t+j} \mid \hat{\pi}_t) \approx \frac{a_{g+1} - \hat{\pi}_t}{a_{g+1} - a_g} E (S_{t+j} / P_{t+j} \mid a_g) + \frac{\hat{\pi}_t - a_g}{a_{g+1} - a_g} E (S_{t+j} / P_{t+j} \mid a_{g+1}), \quad a_g < \hat{\pi}_t < a_{g+1}.$$

B: Derivation of relationship between g_t and $g_{w,t}$

This appendix shows that the output growth rate g_t can be approximated as $g_t \approx \bar{g} + \varepsilon_{g,t}$, where $\varepsilon_{g,t} = K_{t-1} \varepsilon_{w,t}$. Define $\kappa_t = \chi_t^{1/\gamma} (1 - \tau)^{-1/\gamma} w_t^{-1/\gamma}$. Hence, $y_t = (1 - \kappa_t) w_t$. In order to find an expression for the real output growth rate, we elaborate its continuous-time equivalent:

$$g_t = (dy_t / dt) / y_t = (\partial y_t / \partial w_t)(dw_t / dt) / y_t + (\partial y_t / \partial \chi_t)(d\chi_t / dt) / y_t =$$

$$(\partial y_t / \partial w_t)(w_t / y_t) g_{w,t} + (\partial y_t / \partial \chi_t)(\chi_t / y_t) \bar{g}_w.$$

The derivative $\partial y_t / \partial w_t$ can be elaborated as follows:

$$\partial y_t / \partial w_t = 1 - \kappa_t - w_t (\partial \kappa_t / \partial w_t) = 1 - \kappa_t - w_t (-1/\gamma) \kappa_t / w_t = 1 - (1 - 1/\gamma) \kappa_t = 1 - ((\gamma - 1) / \gamma) \kappa_t.$$

The derivative $\partial y_t / \partial \chi_t$ can be elaborated as follows:

$$\partial y_t / \partial \chi_t = -w_t \kappa_t / (\gamma \chi_t).$$

Hence,

$$g_t = \bar{g}_w + \left(\frac{1 - ((\gamma - 1) / \gamma) \kappa_t}{1 - \kappa_t} \right) \varepsilon_{w,t} \equiv \bar{g}_w + K_t \varepsilon_{w,t},$$

in continuous time. The equivalent expression in discrete time reads as follows:

$$g_t = \bar{g}_w + K_{t-1} \varepsilon_{w,t}.$$

Hence, $\bar{g} = \bar{g}_w$ and $\varepsilon_{g,t} = K_{t-1} \varepsilon_{w,t}$. The distribution of $\varepsilon_{g,t}$ is thus the same as that for $\varepsilon_{w,t}$, up to a time-dependent proportionality factor K_{t-1} . Hence, the standard deviation of output growth is $\sigma_{g,t} = K_{t-1} \sigma_w$. Therefore, unlike the standard deviations of inflation and productivity growth, the standard deviation of GDP growth will generally vary over time.

C: Derivation of the tax revenues dynamics (10) and recursion (11)

We assume that the labor income tax rate is constant at τ as long as none of the fiscal ceilings is hit or always if public consumption is the policy instrument. This implies that labor-income taxes grow with actual GDP. Hence, given that $Y_t = w_t l_t P_t$, we must have $\tau w_{t+1} l_{t+1} P_{t+1} = (1 + n_{t+1}) \tau w_t l_t P_t$. If we further assume that nominal lump-sum taxes grow with expected nominal GDP, hence $S_{t+1} = (1 + \bar{n}) S_t$, we can write total tax revenues as:

$$\begin{aligned}
T_{t+1} &= S_{t+1} + \tau w_{t+1} l_{t+1} P_{t+1} \\
&= (1 + \bar{n})S_t + \tau(1 + n_{t+1})w_t l_t P_t \\
&= (1 + \bar{n})T_t + \tau \varepsilon_{n,t+1} w_t l_t P_t.
\end{aligned}$$

Define $1 - \alpha_t^T$ as the share of labor-income tax revenues in total tax revenues. (This share must obviously vary with the shocks $\varepsilon_{n,t+1}$.) Hence, $\tau Y_t = \tau w_t l_t P_t = (1 - \alpha_t^T)T_t$, so that we can write the expression for total tax revenues further as (10):

$$\begin{aligned}
T_{t+1} &= (1 + \bar{n})T_t + (1 - \alpha_t^T)T_t \varepsilon_{n,t+1} \\
\Rightarrow T_{t+1} &= (1 + \bar{n})\alpha_t^T T_t + (1 + n_{t+1})(1 - \alpha_t^T)T_t \\
\Rightarrow (1 + n_{t+1})(1 - \alpha_t^T)T_t &= T_{t+1} - (1 + \bar{n})\alpha_t^T T_t.
\end{aligned}$$

Using that τ is constant, we must also have that $(1 - \alpha_t^T)T_t / Y_t = (1 - \alpha_{t+1}^T)T_{t+1} / Y_{t+1}$, which can be rewritten as:

$$(1 - \alpha_t^T)(1 + n_{t+1})T_t = (1 - \alpha_{t+1}^T)T_{t+1}.$$

Combining this with the preceding expression, we get:

$$(1 - \alpha_{t+1}^T)T_{t+1} = T_{t+1} - (1 + \bar{n})\alpha_t^T T_t,$$

which can be written as (11).

In our simulation analysis, we start by choosing a value for α_0^T and a value for T_0 / Y_0 . This then implies via the relationship $\tau Y_t = (1 - \alpha_t^T)T_t$ a value for τ . Expressions (10), (11) and the shocks determine that paths for total tax revenues and α_t^T .

D: Derivatives of debt and deficit ratios with respect to shocks

The derivatives are taken under the assumption $\alpha_t^G = \alpha_t^T \equiv \alpha_t$. They make use of

$$\frac{\partial(\tilde{G}_{t+1} - T_{t+1})}{\partial \varepsilon_{\pi,t+1}} = (1 - \alpha_t)(1 + g_{t+1})(G_t - T_t), \quad \frac{\partial(1 + n_{t+1})}{\partial \varepsilon_{\pi,t+1}} = 1 + g_{t+1}, \quad \frac{\partial Y_{t+1}}{\partial \varepsilon_{\pi,t+1}} = Y_t(1 + g_{t+1}) = \frac{Y_{t+1}}{1 + n_{t+1}}.$$

The marginal effects of an inflation shock are reported in the main text. The marginal effects of a productivity shock are:

$$\begin{aligned} \frac{\partial(B_{t+1}^N / Y_{t+1})}{\partial \varepsilon_{g,t+1}} &= - \left[\frac{\alpha_t(1 + \bar{n})(G_t - T_t) + (1 + i_{t+1})B_t^N}{(1 + g_{t+1})Y_{t+1}} \right], \\ \frac{\partial(B_{t+1}^I / Y_{t+1})}{\partial \varepsilon_{g,t+1}} &= - \left[\frac{\alpha_t(1 + \bar{n})(G_t - T_t) + (1 + r)(1 + \pi_{t+1})B_t^I}{(1 + g_{t+1})Y_{t+1}} \right], \\ \frac{\partial(D_{t+1}^N / Y_{t+1})}{\partial \varepsilon_{g,t+1}} &= - \left[\frac{\alpha_t(1 + \bar{n})(G_t - T_t) + i_{t+1}B_t^N}{(1 + g_{t+1})Y_{t+1}} \right], \\ \frac{\partial(D_{t+1}^I / Y_{t+1})}{\partial \varepsilon_{g,t+1}} &= - \left[\frac{\alpha_t(1 + \bar{n})(G_t - T_t) + ((1 + r)(1 + \pi_{t+1}) - 1)B_t^I}{(1 + g_{t+1})Y_{t+1}} \right] \end{aligned}$$

E: Instrument settings when government consumption is used as instrument

For convenience, we will denote by superscript R the specific policy regime under consideration. The regimes are nominal debt combined with only a debt ceiling ($R=NB$), only a deficit ceiling ($R=ND$) or both ceilings imposed simultaneously ($R=NBD$), and indexed debt combined with only a debt ceiling ($R=IB$), only a deficit ceiling ($R=ID$) or both ceilings imposed simultaneously ($R=IBD$).

E.1. One-period debt

Consider first the case of a *debt ceiling* $(B/Y)_{\max}$, but no ceiling on the deficit ratio. Under *nominal* debt, hence regime $R=NB$, we have

$$G_{t+1}^{NB} = \begin{cases} \tilde{G}_{t+1}^{NB}, & \text{if } (\tilde{B}^{NB} / Y)_{t+1} \leq (B / Y)_{\max}, \\ \max(G_{\max,t+1}^{NB}, G_L), & \text{if } (\tilde{B}^{NB} / Y)_{t+1} > (B / Y)_{\max}, \end{cases} \quad (\text{A.9})$$

where

$$\frac{G_{\max,t+1}^{NB}}{Y_{t+1}} = \left(\frac{B}{Y}\right)_{\max} + \left(\frac{T_{t+1}}{Y_{t+1}}\right) - \left(\frac{1+i_{t+1}}{1+n_{t+1}}\right) \left(\frac{B^{NB}}{Y}\right)_t, \quad (\text{A.10})$$

A positive inflation surprise reduces the absolute size of the final term of (A.10) and, assuming a positive debt ratio in period t , it raises the public consumption ratio that can be sustained when the debt ratio is at its upper bound. This effect will be absent with indexed debt. Nominal lump-sum taxes are given by

$$S_{t+1}^{NB} = \begin{cases} (1+\bar{n})\alpha_t^T T_t, & \text{if } (\tilde{B}^{NB} / Y)_{t+1} \leq (B / Y)_{\max}, \\ (1+\bar{n})\alpha_t^T T_t - \min(0, G_{\max,t+1}^{NB} - G_L), & \text{if } (\tilde{B}^{NB} / Y)_{t+1} > (B / Y)_{\max}. \end{cases}$$

If lump-sum taxes are adjusted, because public consumption is at its lower bound, the share of tax revenues linked to structural nominal GDP growth is adjusted accordingly: $\alpha_{t+1}^T = S_{t+1}^{NB} / T_{t+1}$. Substituting the preceding expressions back into (12), we see that the kinked nature of the equation for public consumption carries over to that for the debt ratio:

$$\left(\frac{B^{NB}}{Y}\right)_{t+1} = \begin{cases} (\tilde{B}^{NB} / Y)_{t+1} & \text{if } (\tilde{B}^{NB} / Y)_{t+1} \leq (B / Y)_{\max}, \\ (B / Y)_{\max} & \text{if } (\tilde{B}^{NB} / Y)_{t+1} > (B / Y)_{\max}. \end{cases}$$

With *indexed debt* we have:

$$G_{t+1}^{IB} = \begin{cases} \tilde{G}_{t+1}^{IB}, & \text{if } (\tilde{B}^{IB} / Y)_{t+1} \leq (B / Y)_{\max}, \\ \max(G_{\max,t+1}^{IB}, G_L), & \text{if } (\tilde{B}^{IB} / Y)_{t+1} > (B / Y)_{\max}, \end{cases}$$

where

$$\frac{G_{\max,t+1}^{IB}}{Y_{t+1}} = \left(\frac{B}{Y}\right)_{\max} + \left(\frac{T_{t+1}}{Y_{t+1}}\right) - \left(\frac{1+r}{1+g_{t+1}}\right) \left(\frac{B^{IB}}{Y}\right)_t.$$

Lump-sum taxes are given by

$$S_{t+1}^{IB} = \begin{cases} (1 + \bar{n})T_t, & \text{if } (\tilde{B}^{IB} / Y)_{t+1} \leq (B / Y)_{\max}, \\ (1 + \bar{n})T_t - \min(0, G_{\max, t+1}^{IB} - G_L), & \text{if } (\tilde{B}^{IB} / Y)_{t+1} > (B / Y)_{\max}. \end{cases}$$

Now, consider a *deficit ceiling* $(D/Y)_{\max}$. If the deficit ratio threatens to exceed its ceiling, public consumption is corrected so as to equate the deficit ratio to its ceiling. Hence, in the case of *nominal debt* we have:

$$G_{t+1}^{ND} = \begin{cases} \tilde{G}_{t+1}^{ND}, & \text{if } (\tilde{D}^{ND} / Y)_{t+1} \leq (D / Y)_{\max}, \\ \max(G_{\max, t+1}^{ND}, G_L), & \text{if } (\tilde{D}^{ND} / Y)_{t+1} > (D / Y)_{\max}. \end{cases}$$

where

$$\frac{G_{\max, t+1}^{ND}}{Y_{t+1}} = \left(\frac{D}{Y} \right)_{\max} + \left(\frac{T_{t+1}}{Y_{t+1}} \right) - i_{t+1} \frac{B_t^{ND}}{Y_{t+1}}.$$

Lump-sum taxes are given by

$$S_{t+1}^{ND} = \begin{cases} (1 + \bar{n})T_t, & \text{if } (\tilde{D}^{ND} / Y)_{t+1} \leq (D / Y)_{\max}, \\ (1 + \bar{n})T_t - \min(0, G_{\max, t+1}^{ND} - G_L), & \text{if } (\tilde{D}^{ND} / Y)_{t+1} > (D / Y)_{\max}. \end{cases}$$

In the case of *indexed debt* we have:

$$G_{t+1}^{ID} = \begin{cases} \tilde{G}_{t+1}^{ID}, & \text{if } (\tilde{D}^{ID} / Y)_{t+1} \leq (D / Y)_{\max}, \\ \max(G_{\max, t+1}^{ID}, G_L), & \text{if } (\tilde{D}^{ID} / Y)_{t+1} > (D / Y)_{\max}. \end{cases}$$

where

$$\frac{G_{\max, t+1}^{ID}}{Y_{t+1}} = \left(\frac{D}{Y} \right)_{\max} + \left(\frac{T_{t+1}}{Y_{t+1}} \right) - [(1+r)(1+\pi_{t+1})-1] \frac{B_t^{ID}}{Y_{t+1}}.$$

Lump-sum taxes are given by

$$S_{t+1}^{ID} = \begin{cases} (1 + \bar{n})T_t, & \text{if } (\tilde{D}^{ID} / Y)_{t+1} \leq (D / Y)_{\max}, \\ (1 + \bar{n})T_t - \min(0, G_{\max, t+1}^{ID} - G_L), & \text{if } (\tilde{D}^{ID} / Y)_{t+1} > (D / Y)_{\max}. \end{cases}$$

When debt and deficit ceilings are simultaneously imposed, and intervention is necessary, the instrument is always set so that the most binding constraint is exactly obeyed.

E.2. Maturity-L debt

Consider a *debt ceiling*. If $(\tilde{B}^R / Y)_{t+1} \leq (B / Y)_{\max}$, public consumption is given by (9), while if $(\tilde{B}^R / Y)_{t+1} > (B / Y)_{\max}$, we adjust public consumption such that $(B^R / Y)_{t+1} = (B / Y)_{\max}$, so that for *nominal*, respectively *indexed*, debt we have:

$$\frac{G_{\max, t+1}^{NB}}{Y_t} = \left(\frac{B}{Y}\right)_{\max} + \frac{T_{t+1}}{Y_{t+1}} - \frac{\sum_{j=0}^{L-1} (1 + i_{t+1-j}) F_{t-j}^{NB}}{Y_{t+1}},$$

$$\frac{G_{\max, t+1}^{IB}}{Y_{t+1}} = \left(\frac{B}{Y}\right)_{\max} + \frac{T_{t+1}}{Y_{t+1}} - \frac{\sum_{j=0}^{L-1} [(1+r)(1+\pi_{t+1})] F_{t-j}^{IB}}{Y_{t+1}},$$

while lump-sum taxes are given by the corresponding expressions for the case of one-period debt.

Consider now a *deficit ceiling*. When the deficit ceiling is not hit, public consumption is again given by (9). If the deficit ceiling is hit, we have for *nominal*, respectively *indexed*, debt:

$$\frac{G_{\max, t+1}^{ND}}{Y_{t+1}} = \left(\frac{D}{Y}\right)_{\max} + \left(\frac{T_{t+1}}{Y_{t+1}}\right) - \frac{\sum_{j=0}^{L-1} i_{t+1-j} F_{t-j}^{ND}}{Y_{t+1}},$$

$$\frac{G_{\max, t+1}^{ID}}{Y_{t+1}} = \left(\frac{D}{Y}\right)_{\max} + \left(\frac{T_{t+1}}{Y_{t+1}}\right) - \frac{\sum_{j=0}^{L-1} [(1+r)(1+\pi_{t+1}) - 1] F_{t-j}^{ID}}{Y_{t+1}},$$

while lump-sum taxes are given by the corresponding expressions for the case of one-period debt.

F: Derivation of the equivalent variation EQV

Consider the case of a debt constraint. The cases of a deficit constraint and simultaneously-imposed debt and deficit constraints are analogous. Welfare is defined as follows for the cases of indexed, respectively nominal debt:

$$U_1^R = \sum_{j=1}^J (1+\delta)^{1-j} E_1 \left[\frac{1}{1-\zeta} \left(c_j^R + \chi_j \frac{(v_j^R)^{1-\gamma}}{1-\gamma} \right)^{1-\zeta} + \varphi \frac{(G_j^R / P_j)^{1-\beta}}{1-\beta} \right], \quad (\text{A.11})$$

where U_1^R is utility under the specific regime R (for example, $R = IB$ in the case of indexed debt and a debt constraint and $R = NB$ in the case of nominal debt and a debt constraint) and J is the length of the simulation path. We measure the welfare effect, denoted as EQV , of switching from nominal to indexed debt as that constant per cent change in the volume of public consumption in all time periods and along all simulation paths that raises welfare in the case of nominal debt to welfare in the case of indexed debt. Hence, in the case of a debt constraint EQV is the unique solution to the following equation:

$$U_1^{IB} = \sum_{j=1}^J (1+\delta)^{1-j} E_1 \left[\frac{1}{1-\zeta} \left(c_j^{NB} + \chi_j \frac{(v_j^{NB})^{1-\gamma}}{1-\gamma} \right)^{1-\zeta} + \varphi \frac{((1+EQV)G_j^{NB} / P_j)^{1-\beta}}{1-\beta} \right]$$

with U_1^{IB} calculate according to the above expression for $R = IB$. Equivalently, this implies the following expression for EQV :

$$EQV = \left(\frac{U_1^{IB} - \sum_{j=1}^J (1+\delta)^{1-j} E_1 \left[\frac{1}{1-\zeta} \left(c_j^{NB} + \chi_j \frac{(v_j^{NB})^{1-\gamma}}{1-\gamma} \right)^{1-\zeta} \right]}{U_1^{NB} - \sum_{j=1}^J (1+\delta)^{1-j} E_1 \left[\frac{1}{1-\zeta} \left(c_j^{NB} + \chi_j \frac{(v_j^{NB})^{1-\gamma}}{1-\gamma} \right)^{1-\zeta} \right]} \right)^{\frac{1}{1-\beta}} - 1.$$

The values in the expressions for U_1^R and EQV follow from taking averages across the simulation runs of the relevant discounted sums associated with each simulation path. For example, we approximate

$$\sum_{j=1}^J (1+\delta)^{1-j} E_1 \left[\frac{1}{1-\zeta} \left(c_j^{NB} + \chi_j \frac{(v_j^{NB})^{1-\gamma}}{1-\gamma} \right)^{1-\zeta} \right] \approx \frac{1}{Q} \sum_{q=1}^Q \sum_{j=1}^J (1+\delta)^{1-j} \left[\frac{1}{1-\zeta} \left(c_{q,j}^{NB} + \chi_j \frac{(v_{q,j}^{NB})^{1-\gamma}}{1-\gamma} \right)^{1-\zeta} \right]$$

where q is the number of the simulation run, Q is the total number of simulation runs and $c_{q,j}^{NB}$ and $v_{q,j}^{NB}$ are the values of c_j^{NB} and v_j^{NB} in the q -th simulation run.

As mentioned above, we correct our calculation of the equivalent variation in order to make sure that the scenarios of nominal and indexed debt are fully comparable. Our approach is as follows. Denote the amount of public wealth at the end of the simulation period as $W_J^R = -B_J^R$ (public wealth is the negative of debt; it can be positive or negative). We now calculate the corresponding annuity \tilde{g}^R , *i.e.* the annual volume of public consumption that, when corrected for structural real growth \bar{g}_w and when applied to all years of the simulation period, is equivalent to W_J^R :

$$\tilde{g}^R \left[(1+r)^{J-1} + (1+\bar{g}_w)(1+r)^{J-2} + \dots + (1+\bar{g}_w)^{J-1} \right] = W_J^R / P_J$$

Rewriting, and recalling our assumption that $r > \bar{g}_w$, gives the following expression for \tilde{g}^R :

$$\tilde{g}^R \equiv \frac{1}{(1+r)^{J-1}} \frac{1}{(1-(1+\bar{g}_w)^{-J})} \left(\frac{q_3}{1+q_3} \right) (W_J^R / P_J), \text{ where } q_3 \equiv \frac{1+r}{1+\bar{g}_w}.$$

The last step is to correct the flows of public consumption throughout the simulation period for this annuity factor and recalculate utility:

$$U_1^R = \sum_{j=1}^J (1+\delta)^{1-j} E_1 \left[\frac{1}{1-\zeta} \left(c_j^R + \chi_j \frac{(v_j^R)^{1-\gamma}}{1-\gamma} \right)^{1-\zeta} + \varphi \frac{\left(G_j^R / P_j + \tilde{g}^R (1+\bar{g}_w)^{j-1} \right)^{1-\beta}}{1-\beta} \right],$$

which replaces (A.11). In the case of a debt constraint, for example, EQV becomes the unique solution to:

$$U_1^{IB} = \sum_{j=1}^J (1+\delta)^{1-j} E_1 \left[\frac{1}{1-\zeta} \left(c_j^{NB} + \chi_j \frac{(v_j^{NB})^{1-\gamma}}{1-\gamma} \right)^{1-\zeta} + \varphi \frac{\left((1+EQV)(G_j^{NB} / P_j + \tilde{g}^{NB} (1+\bar{g}_w)^{j-1}) \right)^{1-\beta}}{1-\beta} \right]$$