Dynamic inquisitive semantics: anaphora and questions

Dotlačil, J.; Roelofsen, F.

DOI
10.18148/sub/2019.v23i1.538

Publication date
2019

Document Version
Final published version

Published in
Proceedings of Sinn und Bedeutung 23

Dynamic inquisitive semantics: Anaphora and questions[1]

Jakub DOTLAČIL — *ILLC, University of Amsterdam*
Floris ROELOFSEN — *ILLC, University of Amsterdam*

Abstract. This paper develops a dynamic inquisitive semantics and illustrates its potential to capture interactions between anaphora and questions.

Keywords: dynamic semantics, inquisitive semantics, anaphora, questions.

1. Introduction

This paper develops a new logical framework for the analysis of questions, bringing together insights from dynamic semantics (Kamp, 1981; Heim, 1982; Groenendijk and Stokhof, 1991) and inquisitive semantics (Ciardelli et al., 2018).

One important advantage of dynamic approaches to questions over static ones is that they allow for a straightforward account of discourse anaphora with wh-antecedents, exemplified in (1):[2,3]

(1) Which^x of your new dresses did you wear today? Did Peter like it$_x$?

However, existing dynamic theories of questions which capture such cases of anaphora (Groenendijk, 1998; van Rooij, 1998; Haida, 2007) are all built on the idea that questions induce a *partition* on the set of all possible worlds (Groenendijk and Stokhof, 1984), which is known to have certain shortcomings. In particular, while it is designed to capture the *exhaustive* interpretation of questions like (2), whose resolution requires identifying all participants that ordered a vegetarian lunch, it does not straightforwardly capture the *non-exhaustive* interpretation of questions like (3), whose resolution only requires identifying one person who has a bike to borrow.

(2) Which participants have ordered a vegetarian lunch?

(3) Who has a bike that I could borrow for 15 minutes?

This and other limitations of partition semantics have been addressed in recent work on *inquisitive semantics* (Ciardelli et al., 2018). In particular, exhaustive and non-exhaustive question interpretations can both be captured straightforwardly in this framework.

The aim of the present paper is to integrate the main insights from dynamic and inquisitive semantics in a way that preserves the benefits of both. We will not, however, develop a full-fledged compositional dynamic inquisitive semantics here. Rather, we will present a simple, first-order system, which is intended to serve as the dynamic inquisitive counterpart of standard first-order logic. While for detailed analysis of certain linguistic phenomena such a first-order

[1]We thank Ivano Ciardelli, Alexandre Cremers, Thom van Gessel, Matthew Gotham, Jeroen Groenendijk, Rick Nouwen, Paolo Santorio, Nadine Theiler and audiences at InqBnB 1, PALLMYR 11, and Sinn und Bedeutung 23 for helpful discussion. We are also grateful to the Netherlands Organization for Scientific Research (NWO) for financial support.

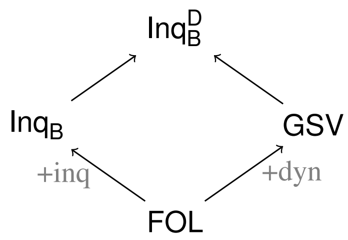
[2]Throughout, relevant antecedents are superscripted with an index corresponding to the discourse referent they introduce and anaphora are subscripted with an index corresponding to the discourse referent they pick up.

[3]Other advantages of a dynamic semantic treatment of questions are discussed in Aloni and van Rooij (2002), Isaacs and Rawlins (2008), and Haida (2007). We will focus here on anaphora.

Proceedings of Sinn und Bedeutung 23, vol. 1, pp. 365–382. Universitat Autònoma de Barcelona, Bellaterra (Cerdanyola del Vallès).

system is certainly not sufficient, it is an important first step in the direction of a full-fledged framework, and its relative simplicity will make it easier to explain the main underlying ideas.

The system to be presented here combines the basic first-order inquisitive system $\mathsf{Inq_B}$ (Ciardelli et al., 2018) with the first-order dynamic system of Groenendijk, Stokhof, and Veltman (1996), which we will refer to as GSV. We will refer to the resulting system as $\mathsf{Inq_B^D}$.[4] The various systems are depicted to the right, together with standard first-order logic (FOL). In this figure, an arrow from one system to another indicates that the latter is an extension of the former (modulo modality, see fn. 4).



The structure of the paper is as follows. Sect. 2 discusses how conversational contexts are modelled in $\mathsf{Inq_B^D}$, Sect. 3 discusses notions related to context update, Sect. 4 provides a dynamic inquisitive semantics for a first-order language, and Sect. 5 illustrates how the framework can be used for the semantic analysis of declarative and interrogative sentences in English, showing in particular that it can capture anaphora with wh-antecedents in non-exhaustive wh-questions, as well as anaphora with indefinite antecedents in polar questions. Sect. 6 concludes.

2. How to model contexts?

In dynamic semantics the meaning of a sentence is viewed as its *context change potential*. One common way to formalize this idea is to take the semantic value of a sentence to be a function that maps any input context (the context in which the sentence is uttered) to a corresponding output context (the new context after the utterance).

2.1. Contexts in GSV

In GSV contexts are modelled in such a way that they capture two types of information that the conversational participants have established as common knowledge in the conversation so far: (i) information about the *world* (e.g., that it's Tuesday), and (ii) information about the *discourse referents* that have been introduced so far (e.g., that the first discourse referent denotes Alice and the second a friend of hers).

Formally, this is achieved by modelling a context as a set of pairs $\langle w, g\rangle$, where w is a possible world and g an assignment function mapping every discourse referent introduced so far to an individual. Such pairs are called *possibilities*. A context C formally represented as a set of possibilities captures information about the world, namely the information that the actual world coincides with some w such that $\langle w, g\rangle \in C$, as well as information about discourse referents, namely the information that the individuals they refer to must be as specified by some g such that $\langle w, g\rangle \in C$. Finally, it also captures possible *dependencies* between these two types of information. For example, it may be known that one discourse referent denotes either Alice or Kim, and that the actual world is one in which this discourse referent denotes the tallest woman on earth. Further, it may be known that the second discourse referent is a friend of the woman

[4] GSV in turn combines the dynamic predicate logic of Groenendijk and Stokhof (1991) with the update semantics of Veltman (1996) and its main focus is to account for the dynamic properties of indefinites and modals within one framework. We will leave out the analysis of modals from $\mathsf{Inq_B^D}$.

denoted by the first discourse referent (Sue for Alice; Mary for Kim). In this context, the value of the second discourse referent depends on that of the first, and the value of the first discourse referent depends on what the world is like.

While in GSV contexts are modelled in such a way as to represent both information about the world and about discourse referents, they do not represent the *issues* that may have been raised about the world and about the discourse referents. Raising such issues is the primary conversational role of questions. Thus, in a dynamic framework for question semantics, the notion of contexts must be richer than in GSV: it has to represent both the information that has been established and the issues that have been raised.

2.2. Contexts in $\mathsf{Inq_B}$

In $\mathsf{Inq_B}$, contexts represent information and issues about the world. Formally, this is achieved by modelling contexts as *sets of information states*, where each information state in turn is a set of possible worlds. The information states that make up a context C are precisely those that (i) contain enough information to resolve the issues that have been raised in the conversation so far and that (ii) do not contain any possible worlds that are already ruled out by the information established in the conversation so far. The union of all the elements of C, $\bigcup C$, is precisely the set of worlds which are compatible with the information available in C. This union is denoted as $\text{INFO}(C)$.

Not just any set of information states constitutes a proper context representation in $\mathsf{Inq_B}$. Rather, this only holds for sets of information states that are *downward closed*: if they contain a certain information state s, they must also contain all stronger information states $s' \subset s$. This requirement follows from how contexts are construed. To see this, suppose that a context C contains an information state s. This means (i) that s contains enough information to resolve the contextual issues and (ii) that s does not contain any possible worlds that are ruled out by the information available in C. But then, the same goes for any stronger information state $s' \subset s$, which in turn means that these stronger information states must also be in C.

Furthermore, it is assumed in $\mathsf{Inq_B}$ that the inconsistent information state, $\emptyset$, trivially resolves any issue and is therefore contained in any context representation. Given the requirement that contexts are downward closed, the additional requirement that any context contains $\emptyset$ is equivalent to the requirement that any context be non-empty.

In sum, contexts are modelled in $\mathsf{Inq_B}$ as non-empty, downward closed sets of information states, which in turn are sets of possible worlds. This allows us to represent both information and issues about the world. However, information and issues about discourse referents are not represented.

2.3. Contexts in $\mathsf{Inq_B^D}$

The context representations used in GSV and in $\mathsf{Inq_B}$ need to be integrated in order to arrive at a notion of context that comprises both information and issues about the world as well as about the discourse referents introduced so far.

How should this be done? As a starting point, note that the formal objects that are used to represent contexts in GSV, i.e., sets of world-assignment pairs (possibilities), can also be thought of as representing information states, comprising both information about the world and about the discourse referents. Using this as our notion of information states, we can construe a context exactly as in $\mathsf{Inq_B}$, i.e., as a non-empty, downward-closed set of information states. We think of these information states, just like in $\mathsf{Inq_B}$, as those that (i) contain enough information to resolve the contextual issues, and (ii) do not contain any possibilities that are ruled out by the contextually established information. Only now, the contextual issues and the contextually established information may not only pertain to what the world is like but also to the values of the discourse referents that have been introduced. This way, our new notion of context encompasses both information and issues about the world and about discourse referents.

As in $\mathsf{Inq_B}$, the union of all the information states in a context C, $\bigcup C$, is the set of possibilities which are compatible with the information available in C. We can thus still write $\textsc{info}(C)$ for $\bigcup C$.

2.4. Formal definitions

We now provide explicit formal definitions of the notions discussed above. In order to do this, we consider a first-order logical language L with individual constants, variables, relation symbols, and the standard connectives and quantifiers (some non-standard operators will be added later). We use V to denote the set of variables in L, and we define a model for L as a triple $M = \langle W, D, I \rangle$, where W is a set whose elements are called possible worlds, D a set which is referred to as the domain of the model, and I a world-dependent interpretation function, i.e., for every world $w \in W$, $I(w)$ is a function that maps every individual constant c in L to an individual in D, denoted as $[c]^{M,w}$, and every n-ary relation symbol R in L to a set of n-tuples of individuals in D, denoted as $[R]^{M,w}$. Throughout the discussion below we assume a particular model M and suppress M-indices on $[c]^{M,w}$ and $[R]^{M,w}$. Finally, we define an assignment function g with domain $r \subseteq V$ as a function that maps every variable $x \in r$ to some individual in D, denoted as $[x]^g$.

Definition 1 (Possibilities)**.** For any set of variables $r \subseteq V$, thought of as a set of active discourse referents, we define a *possibility* with domain r as a pair $\langle w, g \rangle$, where $w \in W$ is a possible world and g an assignment function with domain r.

Definition 2 (Information states)**.** For any $r \subseteq V$, an *information state s* with domain r is a set of possibilities such that the union of the domains of all the possibilities in s is r.

Definition 3 (Downward closed)**.** A set of information states S is *downward closed* just in case for every $s \in S$, every subset of s is also in S.

Definition 4 (Contexts)**.** For any $r \subseteq V$, a *context C* with domain r is a non-empty, downward closed set of information states such that the union of the domains of all the information states in C is r.

Definition 5 (The information available in a context)**.** For any context C, $\textsc{info}(C) := \bigcup C$.

2.5. Informed and inquisitive contexts

A context with domain r contains non-trivial information just in case $\textsc{info}(C)$ excludes at least one possibility with domain r.

Definition 6 (Informed contexts)**.** A context C with domain r is *informed* just in case there is a possibility i with domain r such that $i \notin \text{INFO}(C)$.

Resolving the open issues in a context C requires extending the contextually available information, represented by $\text{INFO}(C)$, in such a way as to reach one of the information states in C. In case $\text{INFO}(C)$ is itself an element of C, all the contextual issues are already resolved, i.e., there are no *open* issues in C. In this case, we say that C is *non-inquisitive*. On the other hand, if $\text{INFO}(C) \notin C$ we say that C is *inquisitive*.

Definition 7 (Inquisitive contexts)**.** A context C is *inquisitive* just in case $\text{INFO}(C) \notin C$.

Given a context C, we can always construct a context $!C$ which contains exactly the same information as C, both about the world and about the discourse referents introduced so far, but is not inquisitive. This is achieved by letting $!C$ consist of the information state $\text{INFO}(C)$ plus all subsets thereof. We will refer to this context as the *non-inquisitive closure* of C. In the definition below we use a downarrow to represent closure under subsets, i.e., for any set of information states S, $S^{\downarrow} := \{s' \mid s' \subseteq s \text{ for some } s \in S\}$.

Definition 8 (Non-inquisitive closure)**.** For any context C, $!C := \{\text{INFO}(C)\}^{\downarrow}$

Since contexts are always downward closed they are often fully determined by their *maximal elements*.[5] These elements are information states which contain *just enough* information to resolve the contextual issues. Non-maximal elements also contain enough information to resolve these issues, but they contain more information than is strictly needed to do so. The maximal elements of a context are referred to as the *alternatives* in that context.

Definition 9 (Alternatives)**.** An *alternative* in a context C is an information state $s \in C$ which is such that C does not contain any strictly weaker information state $t \supset s$.

It generally holds that if a context C contains more than one alternative, it is inquisitive. Vice versa, if a context is non-inquisitive, then it contains only one alternative.

Finally, we define trivial contexts, the initial context, and the inconsistent context.

Definition 10 (Trivial / initial / inconsistent contexts)**.** A context is *trivial* just in case it is neither informed nor inquisitive. The *initial* context $C_{\top}$ is the trivial context whose domain is empty. The inconsistent context $C_{\bot} := \{\emptyset\}$ is one in which all possible worlds have been excluded.

3. Context update

Now that we have spelled how contexts are modelled in $\mathsf{Inq}_\mathsf{B}^\mathsf{D}$, we turn to notions pertaining to context update.

3.1. Extension and subsistence

Updating a context normally leads to an *extension* of that context. When exactly does one context count as an extension of another? In Inq_B, where information states are sets of worlds, the answer to this question is simple, namely, C' is an extension of C if and only if $C' \subseteq C$. This

[5]In particular, a context is *always* fully determined by its maximal elements in case the set of all possibilities is *finite*, which can be assumed in all our examples.

guarantees not only that C' contains at least as much information as C but also that the open issues in C' subsume those in C, in the sense that any piece of information that resolves the open issues in C' also resolves those in C.

In $\mathsf{Inq}_\mathsf{B}^\mathsf{D}$, however, where information states are sets of possibilities, set inclusion is not the right notion of context extension. To see this, consider two contexts, $C = \{\{\langle w, g\rangle\}\}^{\downarrow}$ and $C' = \{\{\langle w, g'\rangle\}\}^{\downarrow}$, where the domain of g is $\{x\}$, that of g' is $\{x, y\}$, and g' agrees with g on the value of x, i.e., $g'(x) = g(x)$. Both C and C' carry the same information about the world. The only difference is that C pertains to a situation in which there is a single discourse referent, x, whereas C' pertains to a situation in which there is another discourse referent, y, as well. In this case we would like to say that C' is an extension of C. After all, all the information available in C is also available in C', together with additional information about the discourse referent y. Yet, $C' \not\subseteq C$. This shows that set inclusion is not the right notion of context extension in $\mathsf{Inq}_\mathsf{B}^\mathsf{D}$.

Groenendijk et al. (1996) specify a natural notion of extension which fits their notion of information states as sets of possibilities. This can readily be adapted to our purposes.

Definition 11 (Extending possibilities)**.** A possibility $\langle w', g'\rangle$ is an *extension* of another possibility $\langle w, g\rangle$ if and only if $w' = w$ and $g' \supseteq g$. In this case we write $\langle w', g'\rangle \geq \langle w, g\rangle$.

Definition 12 (Extending information states)**.** An information state s' is an *extension* of another information state s if and only if every possibility in s' is an extension of some possibility in s. In this case we write $s' \geq s$.

Definition 13 (Extending contexts)**.** A context C' is an *extension* of a context C if and only if every state in C' is an extension of some state in C. In this case we write $C' \geq C$.

It is useful to also define a specific kind of context extension, one which only involves the addition of new discourse referents. Obviously, this type of extension has no counterpart in Inq_B. In GSV it is called *subsistence* and we will use the same term here.

Definition 14 (Subsistence of one state in another)**.** Let s, s' be information states such that $s' \geq s$. Then we say that s *subsists* in s' if and only if every possibility in s has an extension in s'.

Definition 15 (Subsistence of a state in a context)**.** Let s be a state and C a context. We say that s subsists in C if and only if there is at least one $s' \in C$ such that s subsists in s'. We call every s' in C that satisfies this condition a *descendant* of s in C.

Definition 16 (Subsistence of contexts)**.** Let C, C' be two contexts such that $C' \geq C$. Then we say that C *subsists* in C' if and only if every state in C subsists in C'.

3.2. Support, consistency, and entailment

The semantic value of a sentence in $\mathsf{Inq}_\mathsf{B}^\mathsf{D}$ is a function from contexts to contexts, as is common in dynamic semantics. Given any context C and sentence φ, we will write $C[\varphi]$ to denote the context that results from updating C with φ. $C[\varphi][\psi]$ denotes the result of first updating C with φ, and then updating the output context with ψ.

Update functions can be partial. If a context C contains an information state that has a possibility with domain r and an atomic sentence φ contains a variable that is not in r, then the update of C with φ is undefined. This undefinedness percolates up to sentences containing φ.[6]

Under which circumstances are the information conveyed and the issues raised by a sentence φ consistent with or already supported by a given context C? Following GSV, we say that C supports φ just in case updating C with φ does not have any effect beyond the potential addition of discourse referents, and that φ is consistent with C just in case updating C with φ does not lead to the inconsistent context, $\{\emptyset\}$.

Definition 17 (Support)**.** A context C supports φ if and only if $C[\varphi]$ is well-defined and C subsists in $C[\varphi]$.

Definition 18 (Consistency)**.** A sentence φ is consistent with a context C if and only if $C[\varphi]$ is well-defined and $C[\varphi] \neq \{\emptyset\}$.

Finally, we specify when one sentence entails another. In static semantics, entailment is defined in terms of set-inclusion. In dynamic semantics, this is not the right notion, for reasons similar to those discussed in Sect. 3.1 pertaining to context extension. Rather, following Groenendijk et al. (1996) and much other work in dynamic semantics, we define entailment in terms of support.

Definition 19 (Entailment)**.** A sentence φ entails another sentence ψ, written $\varphi \models \psi$, if and only if for every context such that $C[\varphi][\psi]$ is well-defined, $C[\varphi]$ supports ψ.

3.3. Informative and inquisitive sentences, contradictions and tautologies

A sentence is informative just in case it has the potential to turn an uninformed context into an informed one. Similarly, a sentence is inquisitive just in case it has the potential to turn a non-inquisitive context into an inquisitive one.

Definition 20 (Informative and inquisitive sentences)**.**

- A sentence φ is informative if and only if there exists an uninformed context C such that $C[\varphi]$ is well-defined and informed.
- A sentence φ is inquisitive if and only if there exists a non-inquisitive context C such that $C[\varphi]$ is well-defined and inquisitive.

A sentence is contradictory if updating any context with it leads to the contradictory context. On the other hand, a sentence is tautologous if updating a context with it never has any effect.

Definition 21 (Contradictions and tautologies)**.**

- A sentence φ is a contradiction if and only if for any context C: $C[\varphi] = \{\emptyset\}$.
- A sentence φ is a tautology if and only if for any context C: $C[\varphi] = C$.

4. Semantics for a first-order language

We now turn to the semantics of $\mathsf{Inq}^{\mathsf{D}}_{\mathsf{B}}$, i.e., a recursive definition of the context change potential of all sentences in our logical language.

[6]This condition resembles presupposition, but it cannot be expressed in the object language. Groenendijk et al. (1996) call it ‘meta-presupposition’.

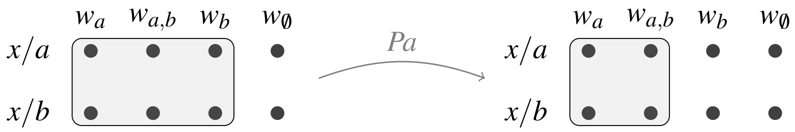


Figure 1: The update effect of an atomic sentence, *Pa*. Each black dot is a possibility and the shaded rectangles represent alternatives. The world component of each possibility is specified above it, and the assignment component is specified to the left. w_a is a world in which only a has the property P, and similarly for other worlds. x/a represents an assignment which maps x to a, and similarly for other assignments. The update function expressed by *Pa* is a function which maps the input context on the left to the output context on the right.

4.1. Atomic sentences and conjunction

We start with the two most basic cases: atomic sentences and conjunctions. The update function expressed by an atomic sentence $Rt_1 \dots t_n$ is defined as the function which, when applied to a context C, retains only those information states in C which are such that $Rt_1 \dots t_n$ is true across all possibilities in it. As usual, if t is a term, i.e., an individual constant or a variable, we write $[t]^{w,g}$ for the denotation of t relative to w and g. If t is an individual constant then $[t]^{w,g}$ amounts to $[t]^w$; if t is a variable it amounts to $[t]^g$.

(4) $C[R(t_1 \dots t_n)] = \{s \in C \mid \text{for every possibility } \langle w, g \rangle \in s :\ \langle [t_1]^{w,g}, \dots, [t_n]^{w,g} \rangle \in [R]^w\}$

We assume that our logical language contains a designated atomic sentence $\top$ with a trivial update effect.

(5) $C[\top] = C$

Conjunction is taken to express sequential update.

(6) $C[\varphi \wedge \psi] = C[\varphi][\psi]$

To illustrate the semantics we will make use of diagrams representing input and output contexts. Fig. 1 illustrates the interpretation of an atomic sentence.

4.2. Introducing new discourse referents: existential quantification

Existential quantifiers introduce new discourse referents. We define what this means incrementally, first specifying what it means for a possibility to be extended with a new discourse referent, and then doing the same for contexts. Intuitively, if C is a context with domain r then extending the context with x, $C[x]$, creates the largest context with domain $r \cup \{x\}$ such that every $s' \in C[x]$ is an extension of some $s \in C$ and every possibility in every $s' \in C[x]$ assigns a value to x.

Definition 22 (Adding discourse referents to possibilities)**.**
Let $i = \langle w, g \rangle$ be a possibility with domain r, and x a variable such that $x \notin r$.
Then $i[x/d] = \langle w, g' \rangle$, where $g'(x) = d$ and $g'(y) = g(y)$ for all $y \neq x$ in r.

Definition 23 (Extending contexts with discourse referents)**.**
Let C be a context with domain r, and let $x \notin r$. Then:

(7) $C[x] := \{\ \{i[x/d] \mid i \in s \text{ and } (i,d) \in \Delta\}\ \mid s \in C,\ \Delta \text{ a relation between } s \text{ and } D\}$

Given these definitions, an existentially quantified sentence $\exists x\varphi$ can be interpreted simply as introducing a new discourse referent x and then updating with φ.

(8) $C[\exists x\varphi] = C[x][\varphi]$

Existential quantifiers are not inquisitive in $\mathsf{Inq}_\mathsf{B}^\mathsf{D}$, unlike in Inq_B. The rationale behind this will become clear in Sect. 4.4. For now, let us illustrate with an example, $\exists xPx$. Suppose that D contains only two entities, a and b. In Inq_B, $\exists xPx$ is inquisitive: its semantic value contains two maximal states, one of which consists of all worlds in which Pa holds, and the other of all worlds in which Pb holds. In $\mathsf{Inq}_\mathsf{B}^\mathsf{D}$, the sentence maps an input context C to the output context $C[x][Px]$. The first update simply extends every state in C in such a way that possibilities are expanded with arbitrary values for x. The second update eliminates all states that contain a possibility in which Px does not hold. If C is the initial context, the output context contains just one maximal state, consisting of all possibilities $\langle w, g\rangle$ such that $[x]^{w,g}$ is in $[P]^w$. To make this more concrete, let us use subscripts on worlds to indicate the extension of P at that world and let us write $g_{x/d}$ for an assignment that assigns d to x. Then, as depicted in Fig. 2, the unique maximal state in the output context is $\{\langle w_{a,b}, g_{x/a}\rangle, \langle w_{a,b}, g_{x/b}\rangle, \langle w_a, g_{x/a}\rangle, \langle w_b, g_{x/b}\rangle\}$.

4.3. Raising issues: disjunction

Our treatment of disjunction stays very close to Inq_B. Namely, we assume that the result of updating a context C with a disjunction $\varphi \vee \psi$ is the union of $C[\varphi]$ and $C[\psi]$.

(9) $C[\varphi \vee \psi] = C[\varphi] \cup C[\psi]$

Just like in Inq_B, disjunctions can be *inquisitive*, i.e., they can turn a non-inquisitive context into an inquisitive one. This is illustrated in Fig. 3, where the input context contains a single alternative, but the output context contains two alternatives, each corresponding to one of the disjuncts, and is therefore inquisitive.[7]

Many dynamic semantic theories, including GSV, assume that disjunction is 'externally static', i.e., that discourse referents introduced by one of the disjuncts cannot be picked up by anaphoric expressions outside of the disjunction. Cases like (10) support this. However, Stone (1992) observes that disjunction in natural language is not always externally static, as witnessed by (11).

[7]A word of caution: this does not mean that we take declarative disjunctive sentences in English to be inquisitive. As will be discussed in Sect. 5, we assume that declarative sentences are headed by an operator which discharges any issues raised within its scope.

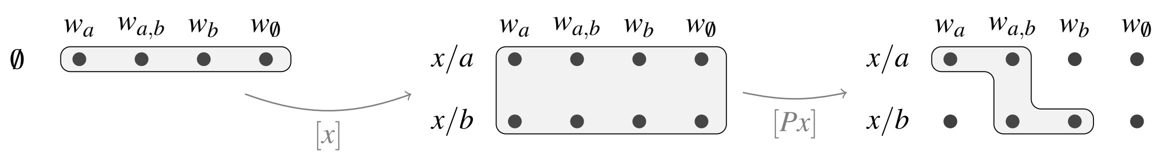


Figure 2: The two-step update effect of $\exists xPx$.

Figure 3: The update effect of a disjunctive sentence, $Pa \vee Pb$.

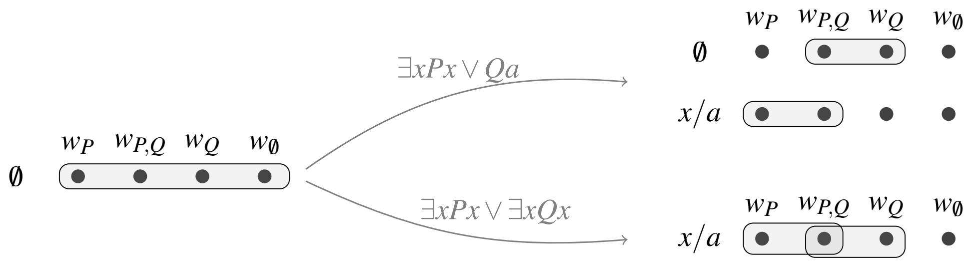


Figure 4: Update effect of $\exists x Px \vee \exists x Qx$ and $\exists x Px \vee Qa$ on $C_\top$. For this example, it is assumed that D has only one element, a. w_P is a world in which a has property P but not Q, w_Q a world in which a has property Q but not P, and similarly for $w_{P,Q}$ and $w_\emptyset$.

(10) Bill either rented a^x car or hitchhiked. *It$_x$ was probably a cabriolet.

(11) Bill either rented a^x blue car or a^x red car. It$_x$ was probably a cabriolet.

This contrast is captured in $\mathsf{Inq}_\mathsf{B}^\mathsf{D}$. To see this, consider the following two sentences in our logical language: $\exists x Px \vee Qa$ and $\exists x Px \vee \exists x Qx$. The effects of updating the initial context with these two sentences are depicted in Fig. 4. In the case of $\exists x Px \vee Qa$, only the first disjunct introduces a new discourse referent, while in the case of $\exists x Px \vee \exists x Qx$, both disjuncts introduce the same new discourse referent. This affects the binding potential of the two sentences. The discourse referent introduced by the first disjunct in $\exists x Px \vee Qa$ cannot be picked up by subsequent anaphora. For instance, a subsequent update with Qx would be undefined. However, the discourse referent introduced by both disjuncts in $\exists x Px \vee \exists x Qx$ *can* be picked up by subsequent anaphora.

In general, if only one of the disjuncts, call it dis_1, introduces a discourse referent, this referent is not accessible outside of the disjunction, at least not immediately. However, if further updates confirm disjunct dis_1, then the discourse referent introduced by it does become accessible. This captures the felicity of the following mini-dialogue:

(12) **A:** Bill either rented a^x car or hitchhiked.
B: The former of course. It$_x$ was a cabriolet.

We will see that discourse referents introduced in polar questions behave similarly.

4.4. Raising issues about the identity of a discourse referent

We introduce a new operator, $?x$, which we intuitively understand as raising an issue about the identity of the discourse referent x. We refer to it as the *identification operator*. Formally, when $?x$ is applied to a context C, it reduces that context to a new context in which the possibilities in each state agree on the entity assigned to x.

(13) $C[?x] := \{ s \in C \mid \text{for all } \langle w,g \rangle, \langle w',g' \rangle \in s : g(x) = g'(x) \}$

An illustration is given in Fig. 5. Note that $?x$ ensures that states in the output context never include possibilities from two different rows in the diagram. Also note that the conjunction $\exists x Px \wedge ?x$ achieves, modulo the introduction of a new discourse referent, exactly what the inquisitive existential quantifier in $\mathsf{Inq_B}$ does: it raises an issue whose resolution requires establishing of some individual d that it has the property P. Thus, inquisitive existential quantification is decomposed in $\mathsf{Inq_B^D}$ into two operations: one which just introduces a discourse referent, and one which raises an issue about the identity of this discourse referent. We believe that this decomposition is useful in analyzing the semantics of wh-words and indefinites in natural languages, as will be illustrated in Sect. 5.

4.5. Discharging issues and discourse referents: negation

Our treatment of negation, given in (14), is very close to both GSV and $\mathsf{Inq_B}$.

(14) $C[\neg\varphi] = \{ s \in C \mid \text{no consistent state } t \subseteq s \text{ subsists in } C[\varphi] \}$

In computing $C[\neg\varphi]$ we first compute $C[\varphi]$. In this process new discourse referents may be introduced. However, the states that end up in $C[\neg\varphi]$ are ones that were already in C and which moreover have no consistent substate that subsists in $C[\varphi]$. So discourse referents that are introduced within the scope of a negation are disregarded outside of that scope. In other words, negation is 'externally static', as is standardly assumed in dynamic semantics. Moreover, as in $\mathsf{Inq_B}$, negation also discharges any issues that are raised within its scope. That is, even if φ is inquisitive, $\neg\varphi$ never is.

Let us illustrate this with a few examples: $\neg Pa$, $\neg(Pa \vee Pb)$ and $\neg\exists x Px$, see Fig. 6. Note that when the domain of the model only contains two atomic individuals, a and b, $\neg(Pa \vee Pb)$ and $\neg\exists x Px$ are equivalent, even though $Pa \vee Pb$ and $\exists x Px$ are not: only the former is inquisitive, and only the latter introduces a discourse referent. The equivalence of $\neg(Pa \vee Pb)$ and $\neg\exists x Px$ arises because negation discharges issues as well as discourse referents that are introduced within its scope.

One particular consequence of this is that the double negation of a sentence φ, $\neg\neg\varphi$, while always conveying exactly the same information as φ itself, never raises any issues and never introduces any discourse referents. This is illustrated in Fig. 6 for $\neg\neg(Pa \vee Pb)$ and $\neg\neg\exists x Px$ (compare with Fig. 2 and Fig. 3).

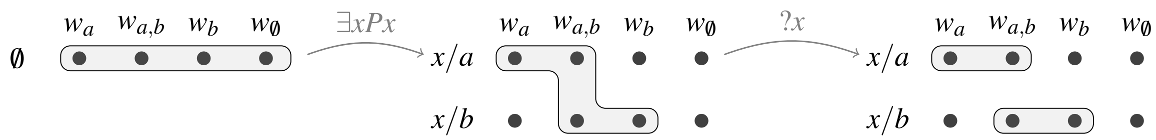


Figure 5: Update effect of $\exists x Px \wedge ?x$ on $C_\top$.

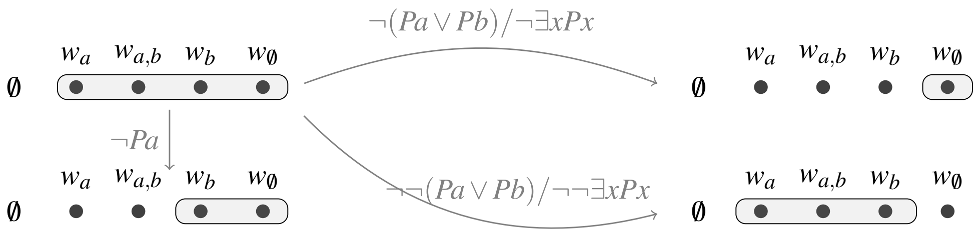


Figure 6: Update effect of $\neg Pa$, $\neg(Pa \vee Pb)$, $\neg\exists xPx$, $\neg\neg(Pa \vee Pb)$, $\neg\neg\exists xPx$ on $C_\top$.

4.6. Discharging issues while projecting discourse referents

$\mathsf{Inq_B}$ comes with a ! operator, which behaves just like double negation: when applied to a sentence φ, it discharges the issues raised by φ. That is, $!\varphi$ conveys the same information as φ itself but does not raise any issues. In $\mathsf{Inq_B^D}$, we define ! in such a way that it has these two properties as well. However, in this setting, unlike in $\mathsf{Inq_B}$, we also have a choice as to whether ! should be externally dynamic or static, i.e., as to whether the discourse referents introduced in its scope can be picked up outside by external anaphoric expressions or not. We define it in such a way that it is externally dynamic. Thus, ! differs from double negation in $\mathsf{Inq_B^D}$, while in $\mathsf{Inq_B}$ the two operations are indistinguishable. This choice is motivated by the fact that, as we will see in Sect. 5, sentences in English plausibly involve an operator which discharges inquisitiveness but is externally dynamic. We thus define the update effect of ! as follows:

(15) $C[!\varphi] = \{\, s' \in !C[\varphi] \mid s' \text{ is an extension of some } s \in C\}$

Let us break this down. First, every state s' in $C[!\varphi]$ must be an element of $!C[\varphi]$, the context that is obtained from C by updating with φ and then removing any open contextual issues. This means that s' has to support the information conveyed by φ, but does not need to resolve the issues that φ introduces. Crucially, however, s' still has to resolve the issues that were present in the old context C, the context preceding the update with φ. This is ensured by requiring that s' is an *extension* of some $s \in C$.

The workings of ! are illustrated in Fig. 7. On the left, we see that when ! is applied to an inquisitive disjunction, $Pa \vee Pb$, it eliminates inquisitiveness. In this case it has the same effect as double negation would have. On the other hand, when ! is applied to an existentially quantified sentence, $\exists xPx$, it does not eliminate the discourse referent that this sentence introduces. So in this case it behaves differently from double negation.

4.7. Ensuring inquisitiveness

Besides the ! operator, which eliminates inquisitiveness as we have just seen, $\mathsf{Inq_B}$ also comes with a ? operator, which *ensures inquisitiveness*. That is, when ? is applied to a sentence φ, the resulting sentence $?\varphi$ is always inquisitive (unless φ is a contradiction or a tautology, in which case $?\varphi$ is also a tautology). This is achieved in $\mathsf{Inq_B}$ by postulating that a state supports $?\varphi$ just in case it either supports φ or $\neg\varphi$. This means that $?\varphi$ is equivalent in $\mathsf{Inq_B}$ to the disjunction $\varphi \vee \neg\varphi$. We adopt this treatment of $?\varphi$ in $\mathsf{Inq_B^D}$:

(16) $C[?\varphi] := C[\varphi \vee \neg\varphi]$

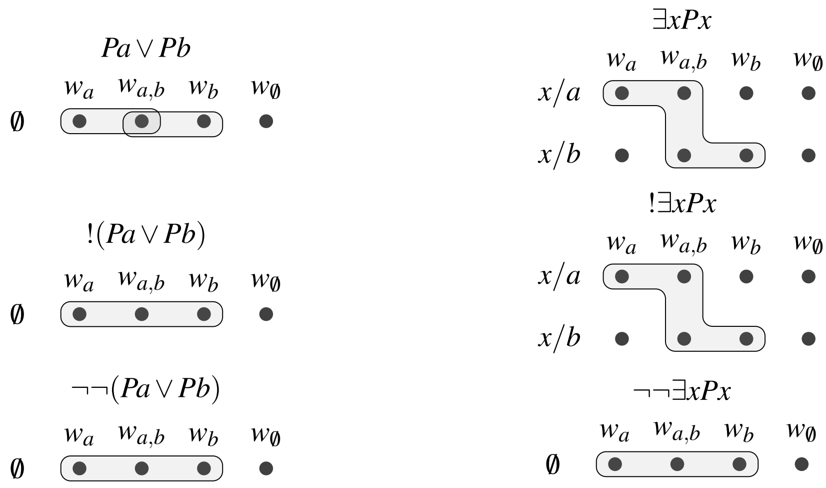


Figure 7: Workings of ! and double negation in $\mathsf{Inq}_\mathsf{B}^\mathsf{D}$. Each subfigure shows the result of updating the initial context $C_\top$ with the given sentence.

4.8. Implication and universal quantification

In our treatment of implication and universal quantification we stay very close to GSV. For reasons of space, we cannot discuss or illustrate this treatment here in much detail. The update effect of an implication is given in (17).

(17) $C[\varphi \to \psi] = \{s \in C \mid \text{for all } t \subseteq s, \text{ all descendants of } t \text{ in } C[\varphi] \text{ subsist in } C[\varphi][\psi]\}$

In computing $C[\varphi \to \psi]$ we first compute $C[\varphi]$ and $C[\varphi][\psi]$. The states that end up in $C[\varphi \to \psi]$ are ones that are already in C and moreover have no substate which has a descendant in $C[\varphi]$ that fails to subsist in $C[\varphi][\psi]$. Thus, implications are externally static, just like negated sentences. However, just like in Inq_B, implication does project inquisitiveness. In particular, if the consequent of an implication is inquisitive, the implication as a whole is typically inquisitive as well (unless the antecedent resolves the issue expressed by the consequent).

The update effect of a universally quantified sentence is given in (18).[8]

(18) $C[\forall x\varphi] = \{\, s \in C \mid \text{for all } t \subseteq s, \text{ all descendants of } t \text{ in } C[x] \text{ subsist in } C[x][\varphi]\}$

Just like implication, universal quantification is externally static but projects inquisitiveness. Moreover, the equivalence in (19) holds, as is common in dynamic semantics.

(19) $C[\forall x\varphi] = C[\exists x\top \to \varphi]$

5. Illustrations

We will now briefly illustrate how $\mathsf{Inq}_\mathsf{B}^\mathsf{D}$ can be used to analyze declarative and interrogative sentences in English, paying particular attention to certain non-trivial interactions between anaphora and inquisitiveness.

[8]The universal quantier as defined here and in GSV is an example of an unselective quantifier (cf. Kadmon, 1987; Heim, 1990; Brasoveanu and Dotlačil, 2016 for discussion). This approach would not be desirable if we want to go beyond universal quantification and define generalized quantifiers, but it is sufficient for our purposes here.

We make three assumptions about how English sentences are translated into our logical language. First, we assume that declarative and interrogative sentences consist of a TP clause as well as a number of syntactic heads in the left periphery (see, e.g., Rizzi, 1997), and that in both sentence types ! applies to the TP clause, discharging any issues that are raised within its scope (as proposed and motivated in some detail in Roelofsen and Farkas, 2015; Farkas and Roelofsen, 2017). Second, we propose that the interpretation of both indefinites and wh-phrases involves (i) an existential quantifier, which introduces a discourse referent, and (ii) an identification operator, which raises an issue about the identity of this discourse referent. The difference is that in the case of indefinites, the identification operator enters the semantic composition locally, within the TP clause boundary, while in the case of a wh-phrase, the identification operator scopes above the ! operator that closes off the TP clause (this proposal can be made more precise in terms of Agreement between wh-phrases and an operator in the left periphery). This captures the fact that issues raised by wh-phrases about the identity of the discourse referents that they introduce, unlike those raised by indefinites, surface in the overall interpretation of the containing sentence and need to be addressed in the proceeding discourse. Our last assumption concerns polar questions: we assume that such questions involve a ? operator, which is introduced in the left periphery above !.[9]

We will first illustrate these assumptions with three simple examples. After that, we will turn to somewhat more complex cases involving anaphora.

First consider the simple declarative sentence in (20), which contains an indefinite. The presence of the ! operator in the translation follows from our first assumption above, and the translation of the indefinite as an existential quantifier followed by the identification operator follows from our second assumption.

(20) A man left. $\rightsquigarrow$ $!\exists x(?x \wedge Mx \wedge Lx)$

Suppose that (20) is uttered in a non-inquisitive context which does not have x in its domain. Then, the output context is still non-inquisitive, but has x as an active discourse referent. The context contains one maximal state, consisting of all possibilities $\langle w, g\rangle$ which are extensions of a possibility in the input context and which are such that $[x]^g$ is a man that left in w. This captures the intuitively correct meaning of the sentence, including the fact that (20) does not raise an issue that has to be resolved but does introduce a discourse referent that can be picked up in the subsequent discourse.

Now consider the simple wh-question in (21). The presence of the ! operator in the translation follows from our first assumption, and the fact that the identification operator appears outside its scope follows from our second assumption.

(21) Which man left? $\rightsquigarrow$ $!\exists x(Mx \wedge Lx) \wedge ?x$

This question introduces a discourse referent x, conveys that x is a man who left, and then raises an issue about the identity of x, i.e., it asks who x is. The ! operator is vacuous in (21) since its scope is non-inquisitive (see Sect. 4.6). However, it is non-vacuous in wh-questions which, besides wh-phrases, host other inquisitive elements as well (see also Champollion et al.,

[9]This last assumption suffices for our purposes here, but needs to be refined in order to deal with alternative questions. We are hopeful that the analysis of such questions in $\mathsf{Inq_B}$ (see, e.g., Roelofsen and Farkas, 2015; Roelofsen, 2015) can be transferred to $\mathsf{Inq_B^D}$, but must leave this for future work.

2017). For example, (22) introduces a discourse referent x, conveys that x is a man who sang or danced, and asks who x is. Crucially, a proper answer to this question does not need to resolve whether x sang or whether x danced, it only needs to resolve the identity of x. This is correctly captured.

(22) Which man sang or danced? $\rightsquigarrow$ $!\exists x(Mx \wedge (Sx \vee Dx)) \wedge ?x$

Given our current assumptions, we derive non-exhaustive interpretations for wh-questions. To see this, consider (21) again. It is predicted that this question can be resolved by identitying just one man who left. It is not necessary to identify all men who left. In inquisitive semantics, it is common to view non-exhaustive question interpretations as basic, and to derive exhaustive interpretations by means of an additional operation in the composition process (see, e.g., Theiler, 2014; Champollion et al., 2017). Explicitly incorporating such an operator into $\mathsf{Inq}_\mathsf{B}^\mathsf{D}$ is left for future work.

We now turn to cases involving anaphora. In (23), the indefinite in the first sentence introduces a discourse referent, and the pronoun in the second sentence refers back to it. Since ! projects discourse referents introduced in its scope and conjunctions are dynamic, we correctly predict that this case of anaphora is licensed.

(23) A^x man left. He$_x$ was angry. $\rightsquigarrow$ $!\exists x(?x \wedge Mx \wedge Lx) \wedge !Ax$

A parallel question-answer pair is given in (24). As long as the input context allows for the possibility that more than one man left, the question will lead to an inquisitive output context. Crucially, both $?x$ and Ax can pick up the discourse referent introduced by the existential quantifier because no operator blocks the projection of discourse referents, just as in (23).

(24) **A:** Which^x man left?
B: (I don't know but) he$_x$ was angry.
$\rightsquigarrow$ $!\exists x(Mx \wedge Lx) \wedge ?x \wedge !Ax$

Now consider the more complex case in (25), involving a sequence of two questions.

(25) Which^x man read a^y book? And did he$_x$ like it$_y$?
$\rightsquigarrow$ $!\exists x \exists y(Mx \wedge By \wedge Rxy) \wedge ?x \wedge ?!Lxy$

Suppose that the input context is non-inquisitive, there are two men, Adam and Bill, two books that could be read, and it is unknown who read what and who liked what. Then the output context contains four alternatives, corresponding to 'Adam read a book and he liked it', 'Adam read a book and he did not like it', 'Bill read a book and he liked it', and 'Bill read a book and he did not like it'. Crucially, $\mathsf{Inq}_\mathsf{B}^\mathsf{D}$ allows us to capture the fact that the dependency between x and y created in the first question is anaphorically accessed in the second question.

The system can also deal with conditional questions involving donkey anaphora, as exemplified in (26). In the translation, we assume that the antecedent of the conditional is not closed off by the ! operator (for independent evidence that inquisitiveness is not discharged in conditional antecedents, see Ciardelli et al., 2018).

(26) If a^x farmer owns a^y donkey, does he$_x$ beat it$_y$?
$\rightsquigarrow$ $\exists x \exists y(?x \wedge ?y \wedge Fx \wedge Dy \wedge Oxy) \rightarrow ?!Bxy$

Suppose that the input context is non-inquisitive, there are two farmers, Adam and Bill, two donkeys, d_1 and d_2, and it is unclear who owns and beats what. Then, it is predicted that the question leads to an output context containing two alternatives, one corresponding to the positive response 'yes, farmers beat the donkeys they own' and one to the negative response 'no, farmers don't beat the donkeys they own'. Crucially, the discourse referents introduced by the indefinites in the conditional antecedent can be picked up by the pronouns in the consequent. As far as we know, this is the first account of donkey anaphora in conditional questions. As mentioned above, earlier dynamic semantic theories of questions that capture basic anaphoric patterns (Groenendijk, 1998; van Rooij, 1998; Haida, 2007) are built on partition semantics, and conditional questions are not within the immediate reach of partition semantics. Isaacs and Rawlins (2008) do provide an account of conditional questions in a partition semantics extended with hypothetical updates. However, their theory does not account for anaphora.

Finally, $\mathsf{Inq}_\mathsf{B}^\mathsf{D}$ also allows us to capture the fact that indefinites in polar questions can license subsequent anaphora, though only to a limited extent. To see this, consider (27).

(27) Do you see a^x man?
$\rightsquigarrow$ $?!\exists x(?x \wedge Mx \wedge Syx)$
$=$ $!\exists x(?x \wedge Mx \wedge Syx) \vee \neg !\exists x(?x \wedge Mx \wedge Syx)$

Suppose that this question is uttered in the initial context. Then, given the interpretation of disjunction (see Sect. 4.3) the output context contains information states contributed by an update with the first disjunct, as well as information states contributed by an update with the second disjunct. Only the former will consist of possibilities that assign a value to x. Since a subsequent update is well-defined only if all the terms in the sentence receive an interpretation in all the states in the context, we predict that the question in (27) on its own does not license anaphora to the indefinite *a man*. However, when the question is resolved affirmatively, states that were contributed by the update with the second disjunct are eliminated, and the discourse referent introduced by the indefinite should become accessible. On the other hand, resolving the question negatively should block anaphora. These predictions are correct, as shown in (28).

(28) a. **A:** Do you see a^x man? **B:** Yes, he$_x$ is behind the tree.
b. **A:** Do you see a^x man? **B:** #No, he$_x$ is behind the tree.

The fact that indefinites inside polar questions can license anaphora, as in (28), is particularly problematic for Haida (2007), who has to treat polar questions as externally static. Other existing dynamic accounts of anaphora in questions, van Rooij (1998) and Groenendijk (1998), do not account for this phenomenon either but may possibly be extended to do so. In particular, Groenendijk (1998) suggests that in cases like (29), the binding possibilities of the indefinite may be captured by the same mechanism that is used to capture modal subordination.

(29) Did you see a^x man? And was he$_x$ angry?

The question operator, ?, would be present in both questions. We could assume, then, that the context created under the first ? operator might be accessible for the second ? operator, as a form of subordination. This strategy, however, does not work for (28).

6. Conclusion

We have presented a basic dynamic inquisitive semantic framework, Inq_B^D, and illustrated its potential to capture certain non-trivial interactions between anaphora and inquisitiveness in English. Unlike previous dynamic accounts of questions and anaphora (Groenendijk, 1998; van Rooij, 1998; Haida, 2007), Inq_B^D can straightforwardly derive non-exhaustive question interpretations and deal with conditional questions, just like its static counterpart Inq_B. In ongoing work, we are exploring further potential benefits of a dynamic inquisitive approach to questions. In particular, we are pursuing an account of intervention effects and of the cross-linguistic morphological affinity between wh-words and indefinites (both previously analyzed by Haida, 2007 in a dynamic partition theory of questions).

References

Aloni, M. and R. van Rooij (2002). The dynamics of questions and focus. In B. Jackson (Ed.), *Proceedings of SALT 12*. Cornell University, CLC Publications.

Brasoveanu, A. and J. Dotlačil (2016). Donkey anaphora: Farmers and bishops. In L. Matthewson, C. Meier, H. Rullmann, and T. E. Zimmerman (Eds.), *To appear in the Blackwell Companion to Semantics*. Wiley.

Champollion, L., I. Ciardelli, and F. Roelofsen (2017). On questions and presuppositions in typed inquisitive semantics. Handout for the talk given at the 2nd workshop on Inquisitiveness Below and Beyond the Sentence Boundary (InqBnB).

Ciardelli, I., J. Groenendijk, and F. Roelofsen (2018). *Inquisitive Semantics*. Oxford: Oxford University Press.

Ciardelli, I., L. Zhang, and L. Champollion (2018). Two switches in the theory of counterfactuals. *Linguistics and Philosophy 41*(6), 577–621.

Farkas, D. F. and F. Roelofsen (2017). Division of labor in the interpretation of declaratives and interrogatives. *Journal of Semantics 34*(2), 237–289.

Groenendijk, J. (1998). Questions in update semantics. In J. Hulstijn and A. Nijholt (Eds.), *Formal semantics and pragmatics of dialogue (Twendial '98)*, pp. 125–137.

Groenendijk, J. and M. Stokhof (1984). *Studies on the Semantics of Questions and the Pragmatics of Answers*. Ph. D. thesis, University of Amsterdam.

Groenendijk, J. and M. Stokhof (1991). Dynamic predicate logic. *Linguistics and Philosophy 14*, 39–100.

Groenendijk, J., M. Stokhof, and F. Veltman (1996). Coreference and modality. In S. Lappin (Ed.), *Handbook of Contemporary Semantic Theory*, pp. 179–216. Oxford: Blackwell.

Haida, A. (2007). *The Indefiniteness and Focusing of Wh-Words*. Ph. D. thesis, Humboldt University, Berlin.

Heim, I. (1982). *The Semantics of Definite and Indefinite Noun Phrases*. Ph. D. thesis, University of Massachusetts, Amherst.

Heim, I. (1990). E-type pronouns and donkey anaphora. *Linguistics and Philosophy 13*, 137–177.

Isaacs, J. and K. Rawlins (2008). Conditional questions. *Journal of Semantics 25*, 269–319.

Kadmon, N. (1987). *On unique and non-unique reference and asymmetric quantification (published 1992, New York: Garland)*. Ph. D. thesis, UMass Amherst, Amherst, MA.

Kamp, H. (1981). A theory of truth and semantic representation. In J. Groenendijk, T. Janssen, and M. Stokhof (Eds.), *Formal Methods in the Study of Language*, pp. 277–322. Amsterdam: Mathematical Centre.

Rizzi, L. (1997). The fine structure of the left periphery. In L. Haegeman (Ed.), *Elements of Grammar*, pp. 281–337. Dordrecht: Kluwer.

Roelofsen, F. (2015). The semantics of declarative and interrogative lists. Manuscript, ILLC, University of Amsterdam.

Roelofsen, F. and D. F. Farkas (2015). Polarity particle responses as a window onto the interpretation of questions and assertions. *Language 91*(2), 359–414.

van Rooij, R. (1998). Modal subordination in questions. In *Proceedings of Twendial*, pp. 237–248.

Stone, M. (1992). Or and anaphora. In *Proceedings of Semantics and Linguistic Theory (SALT 2)*, pp. 367 – 385. Linguistics Society of America.

Theiler, N. (2014). A multitude of answers: Embedded questions in typed inquisitive semantics. University of Amsterdam, MSc thesis.

Veltman, F. (1996). Defaults in update semantics. *Journal of Philosophical Logic 25*(3), 221–261.