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On the Origin of the Lunar and Solar Periods in Babylonian Lunar Theory

Summary

In this investigation, I sketch the way in which Babylonian astronomers may have derived the basic parameters of their lunar theory. I propose that the lunar velocity period of 6247 synodic months which underlies the construction of functions Φ and F of system A is derived by fitting a multiple of the Saros period of 223 synodic months within an integer number of solar years using the 27-year Sirius period relation. I further suggest that the lunar velocity period of 251 synodic months used to construct function F of system B is a direct derivative of the 6247-month period. I also briefly discuss the origin of the periods of the solar velocity function B (of system A) and of the solar longitude function A (of system B) suggesting that the periods of these functions may have been derived from a refined version of the 27-year Sirius period. I finally discuss the timeframe of the possible stepwise development of these early lunar and solar functions.

Keywords: History of science; history of astronomy; Babylonian astronomy; Babylonian lunar theory; Babylonian lunar and solar periods.

In dieser Untersuchung skizziere ich, auf welche Weise babylonische Astronomen die grundlegenden Parameter ihrer Mondtheorie möglicherweise abgeleitet haben. Die Mondgeschwindigkeitsperiode von 6247 synodischen Monaten, die der Konstruktion der Funktionen Φ und F des Systems A unterliegt, sind dadurch abzuleiten, dass man ein Vielfaches der Sarosperiode von 223 synodischen Monaten unter Verwendung der 27-jährigen Siriusperiode in eine ganzzahlige Anzahl von Sonnenjahren einpasst. Des Weiteren schlage ich vor, dass die Mondgeschwindigkeitsperiode von 251 synodischen Monaten, die für die Konstruktion von Funktion F des Systems B genutzt wird, ein direktes Ergebnis der Periode von 6247 Monaten ist. In aller Kürze diskutiere ich auch die Ursprünge der Perioden der Sonnengeschwindigkeitsfunktion B (des Systems A) und der Sonnenlängenfunktion A (des Systems B) und schlage vor, dass die Perioden dieser Funktionen eventuell aus einer präzisierten Version der 27-jährigen Siriusperiode hervorgehen. Abschließend wird der

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Zeitrahmen der möglicherweise schrittweisen Entwicklung der frühen Mond- und Sonnenfunktionen diskutiert.

Keywords: Wissenschaftsgeschichte; Geschichte der Astronomie; Babylonische Astronomie; Babylonische Mondtheorie; Babylonische Mond- und Sonnenperioden.

I have profited from discussions with and from critical remarks of many colleagues of whom I wish to mention here Lis Brack-Bernsen, the late John Britton, Alex Jones, Mathieu Ossendrijver and John Steele.

1 Introduction

One of the most basic questions in the field of Babylonian astronomy, "How did the scholars get from the observations as recorded in the *Astronomical Diaries*[1] to the theoretical computations as we know them from the ACT-type[2] texts", is still incompletely answered. What we do know is that the development of Babylonian lunar and planetary theory is based on periodicities in the orbital motion of the Sun, Moon, and planets. From the observed periods longer theoretical 'great' periods (of order several centuries up to about one millennium) were constructed by linear combination, and using these 'great' periods the observed variations in orbital velocity were cast in strictly periodic step functions and/or zigzag functions. These functions are based on linear difference schemes and involve extensive computation. The specific choice of the parameters characterizing these functions appears often to have been based on arithmetic convenience with the purpose of simplifying the calculations.[3]

In this paper I will limit myself to Babylonian lunar theory and I will concentrate on an investigation into the observational basis of the derivation of the basic periods used in the computation of the angular velocity of the Moon (column F in the ephemerides of systems A and B) and of the Saros function Φ (system A). It was through the work of Lis Brack-Bernsen[4] and several of her lectures that I became initially interested in – and after a while fascinated by – function Φ and its secrets and intricacies. This paper

1 Sachs and Hunger 1988–2001; henceforth referred to as the Diaries.

2 *Astronomical Cuneiform Texts* (Neugebauer 1955); henceforth referred to as ACT.

3 This is most obvious in the choice of the values for the angular velocities of the Sun and the planets in the ephemerides of system A. There we find that the angular velocities have different values in different sections of the Zodiac which are related by simple ratios. For instance in ephemerides where the 360° zodiac is divided into two sections we find for Saturn angular velocities in the proportion 21 : 25, for the Sun 15 : 16, and for Jupiter 5 : 6 (see Aaboe 2001, Tab. 3).

4 Brack-Bernsen 1997.

may be considered as a progress report of an investigation into the early development of Babylonian lunar theory which grew out of this fascination.

The time frame for the early development of Babylonian lunar theory is constrained by the lunar text BM 36737 + 47912 and its duplicate BM 36599 discussed by Aaboe and Sachs.[5] This text contains full-fledged versions of functions F_1 and Φ_1 of lunar system A (the index 1 refers to functions evaluated at New Moon, while index 2 refers to Full Moon) for the years 474–457 BC and may have been written shortly afterwards. Functions Φ and F of system A are zigzag functions based on the same long period of 6247 synodic months. Lunar system A contains one more function with this same period: function G which gives a first approximation to the excess in days of the synodic lunar month over 29 days.

The other text discussed by Aaboe and Sachs is BM 36822 (+ 27022).[6] It also contains fully developed versions of functions F_1 and Φ_1 computed for 398 BC and in addition a crude system A like function for the solar longitude, as well as primitive versions of functions G, C (length of daylight) and M (time between syzygy[7] and sunset/sunrise).

So it seems that functions F (lunar velocity) and Φ (excess time of one Saros of 223 months over 6585 days) were fully developed by the middle of the fifth century BC and that the solar longitude function B of system A was still under development around 400 BC. The earliest lunar ephemeris known so far (of system A) dates from 319 BC,[8] while the last known Babylonian lunar ephemeris (also of system A) dates from 49 BC (ACT 18).

Even a cursory treatment of Babylonian lunar theory is outside the scope of this paper but a short summary of its main features seems appropriate. For a detailed treatment the reader may be referred to Neugebauer's *History of Ancient Mathematical Astronomy*.[9] Lunar ephemerides come in two varieties, called system A and system B. In system A the lunar (solar) longitude at syzygy (function[10] B) is represented by a step function; all other functions are represented by (modified) zigzag functions. In system B all functions are represented by (modified) zigzag functions; the periods adopted for the construction of these functions in system B differ from those adopted in system A.

Representative examples of lunar ephemerides are ACT 5 (New Moons for S.E. 146–148 according to system A) and ACT 122 (New Moons for S.E. 208–210 according to

5 Aaboe and Sachs 1969.

6 Aaboe and Sachs 1969.

7 The term syzygy refers to the conjunction or opposition (Full Moon) of the Sun and Moon.

8 Aaboe 1969.

9 Neugebauer 1975, 474–540; henceforth referred to as HAMA.

10 In this paper I will often not discriminate between functions and columns. For example function B stands for the mathematical function reproducing the arithmetical sequence of numbers displayed in column B of the ephemeris.

system B). The ultimate goal of the construction of lunar ephemerides was to predict a number of lunar phenomena:

- length of the lunar month (columns G–K)
- lunar eclipse magnitudes (columns E and Ψ)
- date and time of syzygy (columns L–N)
- duration of first and last visibility of the Moon (columns O–P)

Prerequisites for the computation of these quantities are the function Φ which serves as an auxiliary function for the computation of function G in system A, the longitude of the Sun/Moon at syzygy (function B in both systems), the orbital velocity of the Moon expressed as its daily displacement (function F in both systems) and the length of daylight (function C in both systems). The fact that these basic functions occupy the first few columns in both systems may be related to the stepwise character of the computation of the ephemeris but it may also reflect the gradual development in time of the theoretical framework on which the computation is based.

In a recent series of papers Britton has argued that the construction of system A lunar theory was a singular creative act by an unknown author rather than a gradual development where a limited number of different scholars during several succeeding generations contributed to its final form as we know it from the surviving ephemerides of the Seleucid and Arsacid era.[11] He dates the invention of system A lunar theory to within a few years of 400 BC and the derivative system B theory about one century later.

In his study Britton emphasizes that the invention of the Babylonian zodiac of 360° must predate – or have been invented simultaneously with – the construction of system A lunar theory.[12] He suggests that the invention of the Babylonian zodiac must have taken place between 409 and 398 BC, consistent with his dating of system A lunar theory. His dating is somewhat late compared to the more generally accepted view that the Babylonian zodiac was introduced into Babylonian astronomy sometime during the second half of the fifth century BC.[13]

The parameters on the basis of which the ephemerides are constructed must have been derived from lunar observations, extensively and routinely carried out during centuries, probably those recorded in the Diaries from about 750 BC onward. As we have seen above about 10 to 20 different functions are needed to build a full-fledged lunar ephemeris. I will concentrate here on the periods of functions Φ and F of system A. Based on the textual evidence mentioned above it seems that these functions were the first ones developed by the Babylonian scholars and that both were fully developed by

11 Britton 2007b; Britton 2009; Britton 2010.

12 Britton 2010.

13 See e.g. Steele 2007, 301.

	System	Column	Function	Π	Z	Years	Period
Moon	A	Φ, F, G	zigzag	6247	448	505	13.9442
Sun	A	B	step	2783	225	225	12.3689
Moon	B	F, G	zigzag	251	18	20.3	13.9444
Sun	B	A	zigzag	10019	810	810	12.3691

Tab. 1 Parameters of basic functions in lunar ephemerides.

the middle of the fifth century BC. In addition I will also briefly discuss the period of function F of system B as a derivative of function F of system A, and the periods of functions B (system A) and A (system B) which (are needed to) determine the position of the Moon and/or Sun at the moment of conjunction or opposition (syzygy). As I will argue later the development of these early basic functions was a gradual process taking place within the community of Babylonian astronomers during the late sixth and fifth century BC. The 'great' periods Π and wave numbers Z of these basic functions are summarized in Tab. 1.

Several important properties and features of these functions and the parameters defining them may be noted:

- The values adopted for the 'great' periods Π (in synodic months) and the wave numbers Z are generally so large that they must have been constructed from shorter (presumably observed) periods. This is also suggested by the fact that 6247 and 251 are prime numbers. To a lesser extent this also holds for the periods 2783 ($= 11^2 \times 23$) and 10 019 ($= 43 \times 233$) which can be factorized but only into products involving fairly awkward prime numbers. The construction of 'great' periods by linear combination of shorter observed periods in Babylonian astronomy is also known from planetary theory. If lunar theory was developed first this technique may have been pioneered in the construction of these early lunar functions.

- The specific choice of the parameter values may also have been influenced by arithmetical convenience. This is suggested by the factorization of the wave numbers Z into nice low prime integers: $448 = 2^6 \times 7$, $225 = 3^2 \times 5^2$, $810 = 2 \times 3^4 \times 5$, and $18 = 2 \times 3^2$. Notice that several of these wave numbers show nice behavior in sexagesimal arithmetic ($60 = 2^2 \times 3 \times 5$).

– Another important aspect of the construction process appears to be that the 'great' period Π generally spans an integer number of solar years. This is also known from planetary theory and is generally thought to be introduced to eliminate the effect of variable solar velocity (solar anomaly). The numbers in Tab. 1 suggest that the 27-year Sirius period (334 synodic months = 27 solar years) was used in the construction of the period of functions F and Φ because 6247 synodic months span almost exactly 505 years (minus 1 day) while according to the 19-year cycle (235 synodic months = 19 solar years) 6247 synodic months are equivalent to 505 years plus 28 days. On the other hand, the 19-year cycle does result in a better – albeit far from perfect – approximation to the periods of the solar functions B and A (errors of 3 and 17 days, respectively). Since the 19-year cycle was recognized as superior to the 27-year Sirius period by the end of the sixth century BC,[14] this suggests that the 6247-month period is the oldest period in the lunar theory and that its derivation dates from before 500 BC.

– The reason why the system B lunar period of 251 months does not fit an integer number of years may be related to the way in which that period was derived as will be discussed later in this paper.

– If function Φ was originally meant to represent the time difference between two eclipses one Saros apart as first suggested by Neugebauer,[15] it involved Full Moon dates only (function Φ_2), running from one Full Moon to the next one with a time step of one synodic month. The new moon function Φ_1 was probably derived later from an intermediate daily variant function Φ^* by applying a phase shift of 15 tithis's with respect to the Φ_2-values.[16] It is consistent with this scenario that the original function Φ_2 – and not Φ_1 – contains the 'nice value' 2,0,0,0,0,0 which may have been adopted as initial value. The values of function Φ are generally 'dirty' sexagesimal numbers with 5 'decimal' places.[17] Of all 6247 entries of function Φ_2 only two are 'nice' numbers, both having the value 2,0,0,0,0,0, one on an ascending branch and one on a descending branch. Note that 2 'large hours' correspond to 2,0 UŠ,[18] equivalent to 480 minutes of time or 8 equinoctial hours, indeed about the average time interval between eclipse times of two eclipses one Saros apart. The average value of function Φ_2 equals 2;7,26,26,20,0 'large hours', equivalent to 8 ½ equinoctial hours.

14 Britton 2002, 30.
15 Neugebauer 1957.
16 See HAMA, 499–502.
17 This may be illustrated by listing an arbitrary set of five consecutive values of function Φ: 2,2,45,55,33,20; 2,5,31,51,6,40; 2,8,17,46,40,00; 2,11,3,42,13,20; 2,13,49,37,46,40; etc.
18 See HAMA, 367.

- Using the known relation of function Φ_2 to the Babylonian lunar calendar,[19] one finds that Φ_2 attained the value 2,0,0,0,0,0 (on an ascending branch) on the Full Moon date of month VIII in year 1 of Cambyses, corresponding to Julian date 17 November 529 BC, a date listed in the Early Saros Scheme[20]. On this date a partial lunar eclipse took place in Babylon with first contact occurring 45 UŠ after sunset. The observation of this eclipse is recorded in the lunar eclipse text BM 36879.[21] The date associated with the other value 2,0,0,0,0,0 of Φ_2 (on a descending branch) is the Full Moon date of month VII of year 25 of Artaxerxes I, corresponding to 26 October 440 BC. This date is not associated with a lunar eclipse.
- From ephemerides of system A one finds that functions Φ and F do not only have the same period but also have the same phase. This is somewhat counter-intuitive because function Φ is supposed to model the time elapsed between two lunar events a whole number of lunar anomaly periods apart while function F models orbital velocity (the lunar anomaly itself) so that one might a priori expect them to be 180° out of phase rather than in phase. Following a suggestion by John Britton, Aaboe provides an explanation for this.[22]
- Finally I note that the accuracy of the astronomical parameters implicit in the periods and wave numbers displayed in Tab. 1 is remarkably good. Dividing the periods Π by the wave numbers Z we display in the last column of Tab. 1 the length of the anomalistic lunar period (the number of synodic months after which the Full Moon returns to the perigee of the lunar orbit, the point of closest approach of the Moon to the Earth and – by definition – the position of maximum lunar velocity) and the length of the (sidereal) solar year expressed in synodic months (the basic Babylonian unit of time). Apparently Babylonian astronomers managed to determine these parameters with an accuracy of about 10^{-5} and 10^{-4}, respectively.

2 The 27-year Sirius period and the solar year

The first visibility of the bright star Sirius has played an important role in Babylonian calendar regulation from early times onward. This is attested by several passages in the astronomical compendium MUL.APIN[23] dating from the late second millennium BC.[24] It ultimately resulted in the intercalation pattern of the 19-year calendar cycle adopted in Babylonia shortly after 500 BC.[25]

19 See HAMA, 484.
20 Steele 2000.
21 See Huber and De Meis 2004, 94.
22 Aaboe 1968, 10–11.
23 Hunger and Pingree 1999, 57–83.
24 De Jong 2007.
25 Sachs 1952; Britton 2007a.

The early text BM 45728 containing Babylonian period relations includes a 27-year Sirius period. This text was first discussed by Kugler and dated by Britton to around 600 BC.[26] Use of the 27-year Sirius period is attested in the early text BM 36731+ in which rising and setting dates of Sirius are computed for the years 627–562 BC.[27]

Observations of the first visibility of Sirius show that after 27 years Sirius rises again on about the same date in the Babylonian lunar calendar. This implies that 27 solar (sidereal) years correspond to 334 synodic months. This period relation is not very accurate because the dates shift backward by about 1.5 days in the lunar calendar after each cycle. Due to variations in the atmospheric extinction (weather) the dates of first visibility of Sirius may vary by up to about 3 days around the nominal date so that it may have taken the Babylonian astronomers about one century before they found out about the limited accuracy of the 27-year period.

One interesting aspect of the 27-year cycle is the implicit existence of the 8-year and 19-year cycles. A period of 8 years corresponds to $8/27 \times 334 = 98;57,46,40$ synodic months and 19 years corresponds to 235;02,13,30 months. Thus according to the 27-year cycle 99 months is 1;06,40 tithi longer than 8 years, while 235 months is 1;06,40 tithi shorter than 19 years. Around 500 BC when the 19-year cycle was adopted as the fundamental calendar cycle, the Babylonian scholars had apparently realized that the 27-year cycle was about 1–2 tithis short and that the 19-year cycle was of superior accuracy.

3 Lunar Four observations and the Saros

The velocity of the Moon varies during its course through the heavens. Thanks to Johannes Kepler (1571–1630) we know now that this variability is due to the ellipse form of the lunar orbit. The Moon reaches its largest velocity ($\sim 16°$ per day) at perigee (minimum distance to the Earth) and its lowest velocity ($\sim 12°$ per day) at apogee (maximum distance to the Earth). The perigee progresses about 3° per synodic month so that it takes the Moon longer to return to its perigee (27.55 days) than to return to the same position in the sky (27.32 days). Since the Sun moves about 30° per month it takes even longer for the Moon to move from one Full Moon to go the next one (29.53 days). The deviation from circularity of the lunar orbit is known as its anomaly (after Ptolemy) and the time it takes for a Full Moon at perigee to return to the next Full Moon at perigee (13.94 synodic months) is called the anomalistic period of the Moon. After one anomalistic period the Moon has completed 15 orbital revolutions and an additional 24° in the sky. The ellipse form of the lunar orbit is a modern notion; Babylonian astronomers were thinking in terms of variable velocity of the Moon.

26 Kugler 1907, 45–48; Britton 2002, 26.

27 Britton 2002.

Early Babylonian awareness of a roughly 14-month velocity period of the Moon is attested in Atypical Text C, first discussed by Neugebauer and Sachs and most recently by Brack-Bernsen and Steele.[28] This awareness probably originates from inspection of long sequences of so-called Lunar Four data,[29] which were routinely recorded in the Diaries. The Lunar Four consist of the set of four observations of the time elapsed between sunrise/-set and moonrise/-set on days around full moon: ŠU (sunrise to moonset around sunrise), NA (moonset to sunrise around sunrise), ME (moonrise to sunset around sunset), and GE_6 (sunset to moonrise around sunset).

The earliest collection of Lunar Four observations dates from the late seventh century BC (BM 38414).[30] The well-preserved text Strassmaier Cambyses 400 (BM 33066) contains Lunar Four observations for the seventh year of Cambyses II (523/522 BC). The fact that the data set in Cambyses 400 is virtually complete implies that missing observations (e.g. due to bad weather) must have been filled in by some predictive method. Britton suggests that most probably previous data – one or more Saroi back – were used for this.[31]

Brack-Bernsen and Schmidt have shown that the Lunar Four observations play an important role in the early development of Babylonian lunar theory.[32] They realized that the sum of the observed values of the Lunar Four, a quantity they called Σ, provides a good approximation to twice the lunar velocity at full Moon. Thus the availability of long sequences of Lunar Four observations enabled the Babylonian scholars to study the variability of the lunar velocity. In this way they must have first discovered the crude 14-month period in the return of the Full Moon to maximum (or minimum) lunar velocity and later the refinement of this period to $223/16 = 13.9375$ synodic months based on the Saros period of 223 synodic months. The latter is based on the realization that 16 lunar velocity periods are more accurately approximated by 223 than by 224 synodic months.

Lunar and solar eclipses are already mentioned in the Old-Babylonian omen series 'Enuma Anu Enlil' (second millennium BC). Reports and letters sent by Assyrian and Babylonian astronomers to the Assyrian kings Esarhaddon and Assurbanipal in the seventh century BC show awareness that lunar eclipse possibilities occurred at intervals of 6 and (occasionally) 5 months. Lunar eclipses were recorded routinely in the Diaries. The oldest preserved Diary dates from 652 BC.

From the available texts it appears that at the end of the seventh century BC, a detailed scheme to predict lunar eclipses based on an 18-year cycle (the so-called Saros) had been worked out.[33] These texts suggest that apparently a continuous lunar eclipse record was available from $\sim$750 BC onward.

28 Neugebauer and Sachs 1967; Brack-Bernsen and Steele 2011.

29 See Hunger and Pingree 1999, 196–198.

30 Huber and Steele 2007.

31 Britton 2008; see also Brack-Bernsen 2002.

32 Brack-Bernsen and Schmidt 1994.

33 Steele 2000.

The Saros consists of a sequence of 38 lunar eclipse possibilities distributed in a fixed pattern of groups of 8 or 7 eclipses at 6-months intervals, each group separated from the next one by a 5-month interval, altogether totaling 223 synodic months, equivalent to about 18 years. After one Saros the Sun, Moon, and Earth return to approximately the same relative geometry and a nearly identical lunar eclipse will occur. The Saros derives from the approximate equality: 223 synodic months ($29^d.530588$) $\approx$ 242 draconitic months ($27^d.212220$) $\approx$ 239 anomalistic months ($27^d.554550$) $\approx 6585\,{}^1/_3$ days $=$ 18 years $+$ 10 (or 11) days $+$ 8 hours. The Saros is referred to in the texts as '18 MU.MEŠ' ('18 years').

It turns out that all eclipses mentioned in Babylonian astronomical texts – either predicted or observed – between 750 and 300 BC are part of the so-called 'Early Saros Scheme'.[34] The Early Scheme breaks down around 300 BC because the resonances between the different periods on which it is based are not perfect. There is evidence that the Saros scheme was revised several times after 300 BC. The revision around 260 BC resulted in the so-called 'Saros Canon'.[35]

The Babylonian scholars must have discovered the Saros period by inspecting their large database of hundreds of lunar eclipses and recognizing that after one Saros lunar eclipses repeat with similar magnitude, occultation pattern and duration.[36] This similarity evolves quite slowly so that it typically persists for some hundred years for successive eclipses in one and the same line of the Saros scheme.

Measured in days the Saros period corresponds to $6585\,{}^1/_3$ days so that for lunar eclipses in one and the same line of the Saros scheme, we have:

- After three Saros periods (about 54 years) similar lunar eclipses occur at about the same time of night,
- Lunar eclipses in a Saros line often occur in pairs, separated by one or two unobservable day-time eclipses.

4 The 6247-month lunar period

After 223 synodic months the Sun has progressed $\sim 10°$ with respect to its position one Saros earlier so that the exact length of the Saros is affected by the variable velocity of the Sun (the solar anomaly). At maximum solar velocity 10 days (the excess of one Saros over 18 years) correspond to $\sim 10°$ and at minimum velocity to $\sim 9°$ so that the average time between two eclipses one Saros apart may differ by about 2 hours (the time for the Moon to traverse $1°$).

34 Steele 2000.

35 The Saros scheme discovered by Strassmaier in the 1890s; see Aaboe, Britton, et al. 1991.

36 Pannekoek 1918.

Thus an improved lunar velocity period can be derived by eliminating the effect of solar anomaly or – phrased in Babylonian language – the variable velocity of the Sun in its orbit. As we know from Babylonian planetary theory this is achieved by constructing a new 'great' period from a linear combination of shorter observed periods such that they span an integer number of solar years.

For this construction we need a relation between the length of the Saros, the period after which the Moon returns to its orbital velocity (the lunar anomalistic period), and the solar year after which the Sun returns to its orbital velocity (the solar anomalistic period), both expressed in synodic months. By definition the Saros is equivalent to 16 lunar velocity periods spanning 223 synodic months. The length of the solar year may be expressed in synodic months by using the Sirius period relation discussed above where we have seen that 27 solar years correspond to 334 synodic months. We then find that one solar year lasts $334/27 = 12.37$ months, corresponding to 12 months and 11 days, or approximately $12\,^1/_3$ months. The Sirius period relation further implies that three Saroi (669 synodic months) correspond to 54 solar years (668 synodic months) + 1 month so that one Saros of 223 synodic months lasts 18 years + $^1/_3$ month. Using these relations the Babylonian astronomers may have computed the smallest common multiple of the synodic month, the solar year and the Saros to find that

$$\begin{aligned} 37\text{ Saroi} &= 37 \times 223\text{ months} = 8251\text{ months} \\ &= 37 \times (18\text{ years} + {}^1/_3\text{ month}) = 666\text{ years} + 12\,{}^1/_3\text{ months} = 667\text{ years.} \end{aligned}$$

Thus 37 Saroi are equivalent to $37 \times 16 = 592$ lunar velocity periods or $37 \times 223 = 8251$ synodic months and last 667 solar years.

Using this relation as a starting point I display in Tab. 2 other relations that fit within an integer number of years by subtracting 54 years (= 3 Saroi − 1 month) in steps. The entries in Tab. 2 are the only linear combinations of an integer number of Saroi and at most 12 lunar months that result in periods of an integer number of solar years defined according to the 27-year Sirius period relation. These relations may also be considered as linear combinations between 223-month periods (Saroi) and the more primitive 14-month periods providing improved approximations to the lunar velocity period. Thus starting with $8251 = 37 \times 223 + 0 \times 14$ months making up 592 lunar velocity periods, we have $7583 = 33 \times 223 + 16 \times 14$ months making up 544 velocity periods, $6915 = 29 \times 223 + 32 \times 14$ months making up 496 velocity periods, $6247 = 25 \times 223 + 48 \times 14$ months making up 448 velocity periods, etc.

Period [years]	Saroi	Months	Months added	Π [months]	Largest factor	Z	Largest factor	Vel. per. [months]
667	37	8251	0	8251	223	592	37	13.937500
613	34	7582	1	7583	7583	544	17	13.939338
559	31	6913	2	6915	461	496	31	13.941532
505	28	6244	3	6247	6247	448	7	13.944196
451	25	5575	4	5579	797	400	5	13.947500
397	22	4906	5	4911	1637	352	11	13.951705
343	19	4237	6	4243	4243	304	19	13.957237
289	16	3568	7	3575	143	256	2	13.964844
235	13	2899	8	2907	17	208	13	13.975962
181	10	2230	9	2239	2239	160	5	13.993750
127	7	1561	10	1571	1571	112	7	14.026786
73	4	892	11	903	301	64	2	14.109375
19	1	223	12	235	47	16	2	14.687500
(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)

Tab. 2 Linear combinations of Saros periods and lunar months resulting in an integer number of solar years.

Based on the data in Tab. 2 one can make the following observations:

- The 6247-month period is among the constructed periods Π listed in column (v).
- The values of the 'great' periods Π are often prime numbers and not reducible to products of nice numbers as follows from the largest factors in column (vi).
- The wave numbers Z result from multiplication of the number of Saroi in column (ii) by 16 (the number of anomaly periods contained in one Saros)
- Only about half of the wave numbers Z are reducible to factors smaller than 10 (column (viii)).
- The relation 28 Saroi + 3 months = 6247 months which underlies the derivation of the 6247-month period naturally explains the ratio 3/28 which plays a central role in the arithmetical structure of function Φ and in the computation of function G.[37]

37 See HAMA, 484–488 and 497–499.

- The 6247-month period provides the best approximation to the modern value of the anomalistic period of 13.943355 synodic months (column (ix)).

Why did the Babylonian scholars select the 6247-month period from the possible periods listed in Tab. 2? I propose that the answer to this question may be sought in a combination of astronomical considerations and numerical convenience. It is clear that a Saros of 223 months provides a much better approximation to 16 lunar velocity periods than 14 months to one velocity period. This implies that the best 'great' period Π must be chosen from the candidate periods in the upper half of Tab. 2 because they may be considered as linear combinations of 223-month and 14-month periods which are most strongly dominated by the 223-month period. Among those candidate periods the 6247-month period is the only one for which all 6247 function values are different because 6247 is a prime number *and* for which the wave number Z is reducible to a small fairly 'nice' integer number. The fact that the 6247-period also provides the most accurate approximation to the value of the lunar velocity period must then be considered as accidental.

5 Function F of system A

Brack-Bernsen and Schmidt were the first to realize that the sum of the Lunar Four (designated Σ by them) provided a good approximation to twice the lunar velocity around Full Moon.[38] That this must have provided the basis for the choice of the other parameters characterizing function F of system A (the amplitude and the average value) can best be demonstrated by showing how remarkably well the lunar velocity function F of system A reproduces the observed $\Sigma/2$ values. This is done in Fig. 1 where I have plotted $\Sigma/2$-values during 10 years in the middle of the sixth century BC together with function F values computed from its defining parameters: $\Pi = 6247$, $Z = 448$, $d = 0°;42$, and $\mu = 13°;30,30$.[39] Notice that 42 is a multiple of 7, the largest factor in the wave number Z (column (viii) in Tab. 2). In view of the excellent fit of function F to the variable lunar velocity I believe that the 6247-month period was first and foremost constructed to provide an improved lunar velocity period and that function F was the first lunar function developed.

38 Brack-Bernsen and Schmidt 1994.

39 See HAMA, 478–479.

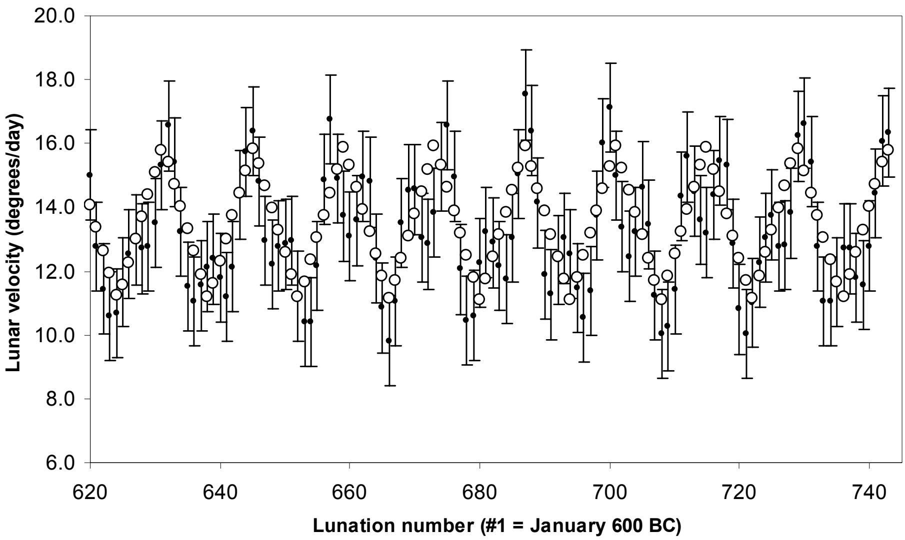

Fig. 1 Synthetic lunar velocity data (small dots with error bars) and function F_2 of system A (large open dots) for the years 550–540 BC. The lunar velocity data are computed from synthetic Lunar Four data taken from a database generated for the period 750–0 BC. Error bars in the (synthetic) observational data are estimated by comparing the Cambyses 400 Lunar Four data (Britton 2008) with synthetic data and noting that the errors in Σ (the sum of the Lunar Four) are twice those in the individual Lunar Four values and that $\Sigma/2$ is displayed.

6 Function Φ of system A

According to the text BM 36705+ function Φ was meant to represent the magnitude of the change in the time difference (of about 8 hours) between two lunar eclipses one Saros apart,[40] and thus originally applied to Full Moon dates only (designated Φ_2).[41] The text mentions the small number 0;17,46,40 as the magnitude of this change. Since 0;17,46,40 UŠ corresponds to 1;11,6,40 minutes of time it follows that function Φ predicts that the time difference between two eclipses one Saros apart changes only very slowly. This is qualitatively in agreement with observation but quantitatively too small because in reality eclipse time differences change by up to about 3 UŠ between eclipses one Saros

40 Neugebauer 1957.

41 An alternative interpretation of function Φ was suggested by Brack-Bernsen (Brack-Bernsen 1990; Brack-Bernsen 1997). Struck by the fact that function Φ provided a remarkably good fit to the sum Σ of the Lunar-Four with 100 UŠ added, she suggested that function Φ was meant to represent the quantity Σ + 100 UŠ. Her suggestion was recently criticized and refuted by Britton (2009, 415–416). However, Brack-Bernsen's basic observation that function Φ and the quantity Σ are in phase is correct. Instead I have argued above that the fact that the quantity Σ provides a good approximation to twice the daily lunar velocity at Full Moon (see Fig. 1) may have been used by the Babylonian scholars to construct function F rather than function Φ.

apart.[42] In view of the limited accuracy of their measurement of eclipse times,[43] it is amazing that the Babylonian scholars managed to observe this small gradual change at all. They correctly concluded that this slow change must be due to the variable lunar velocity and could therefore be modeled by a zigzag function with the same period and wave number as the lunar velocity function F. The fact that they also realized that functions Φ and F have the same phase is more miraculous because that is far from obvious as argued earlier in this paper.[44]

I will show elsewhere[45] that function Φ_2 indeed provides a fairly satisfactory fit to the differences of eclipse times between successive lunar eclipses for all 38 Saros lines in the Saros scheme. This fit is superior to the early zigzag function in BM 45861[46] in the sense that one and the same function Φ_2 fits eclipse time differences both for odd *and* even Saros lines and that it models the slow change in the eclipse time difference with time but it is inferior in the sense that the accuracy with which it fits the eclipse time differences as a whole is less than that of the fits of the zigzag function in BM 45861 for odd and even Saros lines separately.

In the early texts in which full-fledged versions of functions Φ and F are encountered they are given in their truncated form,[47] while in the later ephemerides we only find the pure versions. I think that this may have to do with the fact that initially Φ_2 was meant to model eclipse time differences but that later its use in the ephemerides was limited to the chronological connection of ephemerides and to its application as auxiliary function for the computation of function G.[48]

7 Function F of system B

Lunar velocity periods of 251 and 223 lunar months are very hard to detect in Lunar Four observations because their effect is drowned in the more crude but fairly obvious 14-month period. The fact that 251 synodic months, the period chosen for function F of system B, equals $223 + 2 \times 14$ synodic months is not of much help, because it is not clear why this particular linear combination of 223 and 14 synodic months may have been chosen. I believe that the most straightforward way by which Babylonian astronomers

42 See Appendix C1 of Britton 2007b.

43 In Mesopotamia eclipse times were measured with respect to sunrise and sunset presumably with the aid of water clocks. Steele, Stephenson, and Morrison (1997) have shown that the random errors in the Babylonian measurements of eclipse times amount to about 2 UŠ while systematic errors of about 10% are expected due to clock drift.

44 See again Aaboe 1968, 10–11.

45 This paper is a progress report of a more extensive study on the early development of Babylonian lunar theory that I intend to publish separately.

46 Discussed by Steele 2002 and Brack-Bernsen and Steele 2005.

47 Aaboe and Sachs 1969.

48 See HAMA, 505–513.

were able to single out the velocity period of 251 synodic months is based on its superior accuracy as derived by continuing function F of system A.

We have seen above that after one Saros of 223 synodic months function Φ_2 returns to a value that differs from its previous value by the small amount of 0;17,46,40 UŠ. Similarly, according to function F of system A (which has the same amplitude and phase as function Φ but a different amplitude and initial value), after 223 months the lunar velocity attains a value that differs only $0°$;4,30 per day from its value 223 months before. While this is quite a small difference it is not the smallest one possible. It is easy to show by numerically continuing function F of system A that the smallest difference between all possible pairs of its 6247 function values is $0°$;0,5,37,30 per day for a pair separation of 2998 synodic months. The next one up has a velocity difference of $0°$;0,11,15 per day, twice larger than the smallest value, at a pair separation of 251 months. This 'period' is the first one found by continuing function F beyond 223 months, and the one apparently chosen for function F of system B.

In view of the algorithms developed by Babylonian astronomers to numerically check some of the computations in their ephemerides, one would expect that they should also have been able to find the larger more accurate velocity period of 2998 months but that the 251-months period was chosen because of numerical convenience. The fact that 251 synodic months does not correspond to an integer number of solar years might indicate that it was indeed not found from a linear combination of smaller periods as is the case for most other Babylonian lunar and planetary periods. The argument presented here for the choice of the 251-month period for function F of system B suggests that system B must have been developed after system A.

8 Early solar models

The most obvious starting point for early solar models is the 27-year Sirius period because it defines a period after which the Sun returns exactly to its position in the sky expressed in synodic months, the time unit of Babylonian astronomy. Now, as we have seen before, the accuracy of the 27-year period is limited because the lunar calendar date of first visibility of Sirius regresses ~ 1 day after 27 years. Thus a better approximation is provided by a modified 27-year period: 334 synodic months $-$ 1 day $=$ 27 solar years. Making use of the identity 30 days (tithis) $=$ 1 month, and multiplying both sides of this relation by 30 we immediately find the system B solar period relation: 10 020 $-$ 1 month $=$ 10 019 months $=$ 810 years (see Tab. 1). Cast in Babylonian sexagesimal notation we find that after a period of $\Pi = 2,46,59$ months the Sun has completed $Z = 13,30$ revolutions. This period relation is used in system B to construct the zigzag function A for the solar velocity. It yields a year length of $10019/810 = 12;22,8,53,20$ synodic months.

Since year-length enters a lot of astronomical computations it is convenient to use a truncated or rounded-off value, i.e. 12;22,08 or 12;22,09 synodic months. Both values can be translated into period relations. We find: $\Pi = 2783$ (46,23), $Z = 225$ (4,45) for a year-length of 12;22,08 months, and $\Pi = 14843$ (4,7,23), $Z = 1200$ (20,0) for 12;22,09 months. Apparently Babylonian astronomers chose the smaller period for their system A function B, possibly because an 'epact'[49] of 11;04 tithis is more attractive for computational purposes than 11;04,30 tithis.

9 Discussion

I begin this discussion about the time frame and evolution of the early phase of Babylonian lunar theory by noting that function Φ was originally constructed to represent eclipse time differences and thus by definition applied to full moon dates only (designated Φ_2). Using the known relation of Φ_2 to the Babylonian calendar, we have seen that Φ_2 attains the value 2,0,0,0,0,0 on day 13, month VIII in year 1 of Cambyses, corresponding to Julian date 17 November 529 BC, the date of an attested lunar eclipse listed in the Early Saros Scheme.[50] I suggest that this nice sexagesimal number was chosen as initial value of function Φ_2. Notice that Brack-Bernsen and Steele in their analysis of the early attempt to fit eclipse time differences by zigzag functions in BM 45861 suggest that these functions were constructed around 530 BC,[51] surprisingly close to the eclipse date of the initial value of the more sophisticated function Φ_2.

The lunar eclipse one Saros after the one of 17 November 529 BC took place in the morning of day 13, month VIII in year 11 of Darius, corresponding to Julian date 29 November 511 BC. This lunar eclipse was visible in Babylon with first contact occurring at 40 UŠ before sunrise,[52] but there is no record of this eclipse in presently known astronomical cuneiform texts. The eclipse time difference between these two eclipses is 2,01 UŠ, within the measurement error identical to the initial value adopted for function Φ_2. This does not only hold for this eclipse pair but it can be shown that the average eclipse time difference between all lunar eclipses in this Saros line during the sixth and the first half of the fifth century BC equals $2{,}0 \pm 0{,}01$ UŠ.[53]

49 The 'epact' is defined as the excess of a solar year over the lunar year of 12 synodic months. An epact of 11;04 tithis is also used in Babylonian planetary theories, both of system A and B (Neugebauer 1975, 395–396).

50 Steele 2000.

51 Brack-Bernsen and Steele 2005.

52 Huber and De Meis 2004, 188.

53 See Britton 2007b, Appendix C1.

If the 529 BC lunar eclipse is indeed associated with the initial value of function Φ_2 it provides a 'terminus post quem' for the construction of functions Φ and F. If the full-fledged versions of both functions for the years 475–457 BC in BM 36737+ may be considered as a 'terminus ante quem' functions Φ and F were conceived before 450 BC. Of course, the conception date may have been later if the computations in the text were carried out for comparison with older data. A strong argument in favor of an early date for the derivation of the 6247-month period is provided by the fact that it is partly based on the 27-year Sirius cycle which was used for calendar purposes in the sixth century BC but was superseded by the 19-year cycle around 500 BC.[54] This suggests that functions F and Φ were conceived in the late sixth century, consistent with an initial value for Φ_2 associated with the lunar eclipse of November 529 BC.

Recently Britton has published a detailed study of Babylonian lunar theory in which he suggests that it dates from shortly after 404 BC and that its creation may be attributed to one single author.[55] While the final version of the theory as we know it from the lunar ephemerides of the Seleucid and Arsacid era may well have been formulated by one single Babylonian scholar I prefer to think that it is the end product of a more gradual process to which several generations of Babylonian scholars have contributed. As argued here this gradual process may have started with the construction of the improved lunar anomaly period of 6247 synodic months on which functions F and Φ of system A are built.

Britton anchors function Φ_2 in time by using the shortest 6-month time interval between two lunar eclipses since systematic records were maintained (about 750 BC) and by assigning the associated Φ-value of 2,8,53,20 to the syzygy corresponding to the eclipse of 18 August 404 BC at the end of this interval.[56] He assumes that the final formulation of Babylonian lunar theory was completed shortly afterwards and he uses this as constraints for dating the invention of the Babylonian theoretical zodiac.[57] I must confess that I find his reasoning far from convincing. One – but not the only – reason for this is that dating the minimum 6-month eclipse time interval by Babylonian observers is doomed to be extremely uncertain because the accuracy with which they could determine this interval from observed eclipse times is of the order of 1 hour (see note 43 above).

If my suggestion for the construction of the solar periods for system A and B is correct they both derive from the same refined 27-year Sirius relation. In system B the period relation obtained was directly used for the construction of function A. In system A the original period relation was modified to obtain a numerically suitable value

54 Britton 2002; Britton 2007a.

55 Britton 2007b; Britton 2009; Britton 2010.

56 Britton 2009, 404–405.

57 Britton 2010.

of the year length, associated with the derivation of function B. This does not provide a direct clue about the priority of the two functions.

Given the fact that the lunar function F (system A) may already have been fully developed by about 450 BC and that a primitive version of the solar function B (system A) with a year length of 12;23 months was used in a text from about 400 BC,[58] we may conclude that the system A solar model (function B) was developed after the lunar velocity model (function F) and that it took at least half a century, and probably longer, to reach its canonical form.

In summary, I propose that the development of Babylonian lunar theory was a gradual process. It started in the late sixth century BC with the derivation of the 'great' period of 6247 synodic months for the lunar velocity variation. Based on this period the lunar velocity function F and the eclipse time difference function Φ of system A were constructed shortly afterwards. The next step was to model the position of the Moon at syzygy and during eclipses. Therefore the position of the Sun at syzygy was needed, as well as a theoretical coordinate system. Several early attempts of system A-type solar functions are textually attested (BM 36737+ and BM 36822+). The Babylonian 360° zodiac may have been introduced around 450 BC while it took until the early fourth century BC before the solar longitude function B of system A reached its canonical form. System A lunar theory was apparently finished by 320 BC. System B lunar theory may have been a later invention, possibly dating from around 300 BC.

58 Aaboe and Sachs 1969, Text A.

Bibliography

Aaboe 1968
Asger Hartvig Aaboe. *Some Lunar Auxiliary Tables and Related Texts from the Late Babylonian Period*. Det Kongelige Danske Videnskabernes Selskab, Matematisk-fysiske Meddelelser 36.12. København: Munksgaard, 1968.

Aaboe 1969
Asger Hartvig Aaboe. *A Computed List of New Moons for 319 B.C. to 316 B.C. from Babylon: B.M. 40094*. Det Kongelige Danske Videnskabernes Selskab, Matematisk-fysiske Meddelelser 37.3. København: Munksgaard, 1969.

Aaboe 2001
Asger Hartvig Aaboe. *Episodes from the Early History of Astronomy*. New York, Berlin, and Heidelberg: Springer Science, 2001. DOI: 10.1007/978-1-4613-0109-7.

Aaboe, Britton, et al. 1991
Asger Hartvig Aaboe, John Phillips Britton, Janice Adrienne Henderson, Otto Eduard Neugebauer, and Abraham Joseph Sachs. “Saros Cycle Dates and Related Babylonian Astronomical Texts”. *Transactions of the American Philosophical Society* 81.6 (1991). DOI: 10.2307/1006543.

Aaboe and Sachs 1969
Asger Hartvig Aaboe and Abraham J. Sachs. “Two Lunar Texts of the Achaemenid Period from Babylon”. *Centaurus* 14.1 (1969), 1–22. DOI: 10.1111/j.1600-0498.1969.tb00130.x.

Brack-Bernsen 1990
Lis Brack-Bernsen. “On the Babylonian Lunar Theory: A Construction of Column Φ from Horizontal Observations”. *Centaurus* 33.1 (1990), 39–56. DOI: 10.1111/j.1600-0498.1990.tb00719.x.

Brack-Bernsen 1997
Lis Brack-Bernsen. *Zur Entstehung der Babylonischen Mondtheorie: Beobachtung und theoretische Berechnung von Mondphasen*. Boethius 40. Stuttgart: Franz Steiner, 1997.

Brack-Bernsen 2002
Lis Brack-Bernsen. “Predictions of Lunar Phenomena in Babylonian Astronomy”. In *Under One Sky: Astronomy and Mathematics in the Ancient Near East*. Ed. by J. M. Steele and A. Imhausen. Alter Orient und Altes Testament 297. Münster: Ugarit-Verlag, 2002, 5–19.

Brack-Bernsen and Schmidt 1994
Lis Brack-Bernsen and Olaf Schmidt. “On the Foundations of the Babylonian Column Φ: Astronomical Significance of Partial Sums of the Lunar Four”. *Centaurus* 37.3 (1994), 183–209. DOI: 10.1111/j.1600-0498.1994.tb00008.x.

Brack-Bernsen and Steele 2005
Lis Brack-Bernsen and John M. Steele. “Eclipse Prediction and the Length of the Saros in Babylonian Astronomy”. *Centaurus* 47.3 (2005), 181–206. DOI: 10.1111/j.1600-0498.2005.470301.x.

Brack-Bernsen and Steele 2011
Lis Brack-Bernsen and John M. Steele. “14-Month Intervals of Lunar Velocity and Column Φ in Babylonian Astronomy: Atypical Text C”. In *The Empirical Dimension of Ancient Near Eastern Studies – Die empirische Dimension altorientalischer Forschungen*. Ed. by G. J. Selz and K. Wagensonner. Wiener Offene Orientalistik 6. Vienna: LIT-Verlag, 2011, 111–130.

Britton 2002
John Phillips Britton. “Treatments of Annual Phenomena in Cuneiform Sources”. In *Under One Sky: Astronomy and Mathematics in the Ancient Near East*. Ed. by J. M. Steele and A. Imhausen. Alter Orient und Altes Testament 297. Münster: Ugarit-Verlag, 2002, 21–78.

Britton 2007a
John Phillips Britton. “Calendars, Intercalations and Year-Lengths in Mesopotamian Astronomy”. In *Calendars and Years 1: Astronomy and Time in the Ancient Near East*. Ed. by J. M. Steele. Oxford: Oxbow Books, 2007, 115–132.

Britton 2007b
John Phillips Britton. "Studies in Babylonian Lunar Theory: Pt. I. Empirical Elements for Modeling Lunar and Solar Anomalies". *Archive for History of Exact Sciences* 61.2 (2007), 83–145. DOI: 10.1007/s00407-006-0121-9.

Britton 2008
John Phillips Britton. "Remarks on Strassmaier Cambyses 400". In *From the Banks of the Euphrates: Studies in Honor of Alice Louise Slotsky*. Ed. by M. Ross. Winona Lake: Eisenbrauns, 2008, 8–34.

Britton 2009
John Phillips Britton. "Studies in Babylonian Lunar Theory: Pt. II. Treatments of Lunar Anomaly". *Archive for History of Exact Sciences* 63.4 (2009), 357–431. DOI: 10.1007/s00407-009-0041-6.

Britton 2010
John Phillips Britton. "Studies in Babylonian Lunar Theory: Pt. III. The Introduction of the Uniform Zodiac". *Archive for History of Exact Sciences* 64.6 (2010), 617–663. DOI: 10.1007/s00407-010-0064-z.

Huber and De Meis 2004
Peter J. Huber and Salvo De Meis. *Babylonian Eclipse Observations from 750 BC to 1 BC*. Milano and Roma: Mimesis and Istituto Italiano per l'Africa e l'Oriente, 2004.

Huber and Steele 2007
Peter J. Huber and John M. Steele. "Babylonian Lunar Six Tablets". *SCIAMVS* 8 (2007), 3–36.

Hunger and Pingree 1999
Hermann Hunger and David Edwin Pingree. *Astral Sciences in Mesopotamia*. Handbuch der Orientalistik, 1. Abt., Nr. 44. Leiden, Boston, and Köln: Brill, 1999. DOI: 10.1163/9789004294134.

de Jong 2007
Teije de Jong. "Astronomical Dating of the Rising Star List in MUL.APIN". *Wiener Zeitschrift für die Kunde des Morgenlandes* 97 (2007): *Festschrift für Hermann Hunger zum 65. Geburtstag*. Ed. by M. Köhlbach, S. Prochazka, G. J. Selz, and R. Lohlker, 107–120.

Kugler 1907
Franz Xaver Kugler. *Sternkunde und Sterndienst in Babel, 1. Buch: Babylonische Planetenkunde – Entwicklung der babylonischen Planetenkunde von ihren Anfängen bis auf Christus*. Münster: Aschendorffsche Verlagsbuchhandlung, 1907. URL: http://openlibrary.org/works/OL13121420W (visited on 17/7/2017).

Neugebauer 1955
Otto E. Neugebauer. *Astronomical Cuneiform Texts: Babylonian Ephemerides of the Seleucid Period for the Motion of the Sun, the Moon, and the Planets*. [ACT.] 3 vols. Sources in the History of Mathematics and Physical Sciences 5. London and New York: Lund Humphries and Springer Science, 1955. DOI: 10.1007/978-1-4612-5507-9.

Neugebauer 1957
Otto E. Neugebauer. *'Saros' and Lunar Velocity in Babylonian Astronomy*. Det Kongelige Danske Videnskabernes Selskab, Matematisk-fysiske Meddelelser 31.4. København: Munksgaard, 1957.

Neugebauer 1975
Otto E. Neugebauer. *A History of Ancient Mathematical Astronomy*. [HAMA.] 3 vols. Studies in the History of Mathematics and Physical Sciences 1. New York, Berlin, and Heidelberg: Springer Science, 1975. DOI: 10.1007/978-3-642-61910-6.

Neugebauer and Sachs 1967
Otto E. Neugebauer and Abraham J. Sachs. "Some Atypical Astronomical Cuneiform Texts, Pt. I". *Journal of Cuneiform Studies* 21 (1967): *Special Volume Honoring Professor Albrecht Goetze*, 183–218. DOI: 10.2307/1359371.

Pannekoek 1918
Antonie Pannekoek. "The Origin of the Saros". *Koninklijke Nederlandsche Akademie van Wetenschappen, Proceedings: Wis- en natuurkundige afdeeling* 20.2 (1918), 943–955. URL: http://www.dwc.knaw.nl/DL/publications/PU00012295.pdf (visited on 17/7/2017).

Sachs 1952
Abraham J. Sachs. "Sirius Dates in Babylonian Astronomical Texts of the Seleucid Period". *Journal of Cuneiform Studies* 6.3 (1952), 105–114. DOI: 10.2307/1359081.

Sachs and Hunger 1988–2001
Abraham J. Sachs and Hermann Hunger. *Astronomical Diaries and Related Texts from Babylonia, Vols. I–III; Vol. V*. Denkschriften der österreichischen Akademie der Wissenschaften, philosophisch-historische Klasse 195; 210; 247; 299. Vienna: Verlag der Österreichischen Akademie der Wissenschaften, 1988–2001.

Steele 2000
John M. Steele. "Eclipse Prediction in Mesopotamia". *Archive for History of Exact Sciences* 54.5 (2000), 421–454. URL: http://www.jstor.org/stable/41134091 (visited on 17/7/2017).

Steele 2002
John M. Steele. "A Simple Function for the Length of the Saros in Babylonian Astronomy". In *Under One Sky: Astronomy and Mathematics in the Ancient Near East*. Ed. by J. M. Steele and A. Imhausen. Alter Orient und Altes Testament 297. Münster: Ugarit-Verlag, 2002, 405–420.

Steele 2007
John M. Steele. "Celestial Measurement in Babylonian Astronomy". *Annals of Science* 64.3 (2007), 293–325. DOI: 10.1080/00033790701245711.

Steele, Stephenson, and Morrison 1997
John M. Steele, Francis Richard Stephenson, and Leslie V. Morrison. "The Accuracy of Eclipse Times Measured by the Babylonians". *Journal for the History of Astronomy* 28 (1997), 337–345. URL: http://adsabs.harvard.edu/abs/1997JHA....28..337S (visited on 17/7/2017).

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