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Publication date
2018

Document Version
Final published version

Published in
Proceedings of Sinn und Bedeutung 21

The distributive ignorance puzzle[1]

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Abstract. We observe that verbs like *wonder* do not just imply that their subject does not know the answer to the embedded question, but a stronger form of ignorance, which we call *distributive ignorance*. This is not predicted by existing work on the semantics of *wonder*, and we argue that it cannot be straightforwardly derived as a pragmatic inference either. We consider two possible semantic accounts, and conclude in favor of one on which the lexical semantics of *wonder* involves exhaustification w.r.t. structural alternatives as well as sub-domain alternatives of its complement.

Keywords: *wonder*, ignorance, inquisitive semantics, exhaustivity.

1. Introduction

This paper is concerned with clause-embedding predicates such as *wonder*, *investigate* and *be curious*. These predicates have two things in common. First, in terms of selectional restrictions, they only take interrogative clauses as their complement, not declarative ones:

(1) The doctor **is wondering** what the patient ate / *that the patient ate.

(2) The doctor **is investigating** what the patient ate / *that the patient ate.

(3) The doctor **is curious** what the patient ate / *that the patient ate.

Second, in semantic terms, they each imply, roughly, that their subject is *ignorant* with respect to the issue expressed by the complement and *interested* in resolving this issue.

We will refer to predicates with these two properties as INQUISITIVE PREDICATES,[2] and we will be concerned in this paper with the kind of ignorance that they imply on the part of their subject. Our starting point is a simple but novel empirical observation: when an inquisitive predicate takes an alternative question as its complement, it implies ignorance about all the alternatives introduced. For instance, *John wonders whether Ann, Bill, or Carol arrived* implies that John is ignorant as to whether Ann arrived, as to whether Bill arrived, and as to whether Carol arrived. We will show that this DISTRIBUTIVE IGNORANCE implication is not predicted by existing work on the semantics of *wonder*, even if we take pragmatic strengthening into account. We will then consider two ways of accounting for distributive ignorance: one directly encodes it in the lexical entry for *wonder*, the other derives it as a consequence of a lexicalized

[1]We are grateful to Ivano Ciardelli, Alexandre Cremers, Clemens Mayr, Yael Sharvit, Benjamin Spector, and two anonymous *Sinn und Bedeutung* reviewers for very helpful comments and discussion. We gratefully acknowledge financial support from the Netherlands Organisation for Scientific Research (NWO), the Japan Society for the Promotion of Science (JSPS), and the European Research Council (ERC, grant agreement number 680220).

[2]For Karttunen (1977), the class of inquisitive predicates also includes speech act predicates such as *ask* and *inquire*. We will leave such predicates out of consideration here since they do not semantically imply that the subject is ignorant (although this is often pragmatically implicated). In particular, in examples where the subject is a teacher or a quiz master, it is clear that she need not be ignorant about the answer to the embedded question.

exhaustive inference. We will argue that the exhaustivity-based account is preferable, since it better accounts for distributive ignorance when the complement is a *wh*-question or a polar disjunctive question rather than an alternative question. Throughout, we will focus on the case of *wonder*, but the arguments apply to other inquisitive predicates as well.

The paper is structured as follows: §2 briefly reviews existing work on the semantics of *wonder*; §3 introduces the distributive ignorance puzzle; §4 considers a *pragmatic* account, and the challenges it faces; §5 specifies two *semantic* approaches, and §6 attempts to tease these two apart with additional empirical observations.

2. Background on the semantics of *wonder*

Our theoretical point of departure here is the semantics for *wonder* proposed by Ciardelli and Roelofsen (2015), henceforth C&R.[3] Informally, the idea behind this account is that *wondering* φ amounts to (i) not knowing an answer to the issue expressed by φ, and (ii) entertaining the issue expressed by φ. To make this idea more precise, C&R develop a formal framework called *inquisitive epistemic logic* (IEL), combining notions from standard epistemic logic and inquisitive semantics. We will briefly review the relevant features of the framework, and then spell out the proposed semantics for *wonder*.

Information states and sentence meanings An INFORMATION STATE is modeled in epistemic logic as a set of possible worlds, namely those worlds that are compatible with the information available in the state. The MEANING OF A SENTENCE, whether declarative or interrogative, is modeled in inquisitive semantics as a set of information states, those states where (i) the information conveyed by the sentence is established, and (ii) the issue raised by the sentence is resolved. For instance:

- $[\![\text{Bill left}]\!] = \{s \mid \forall w \in s: \text{ Bill left in } w\}$
- $[\![\text{Did Bill leave}]\!] = \{s \mid \forall w \in s: \text{ Bill left in } w\} \cup \{s \mid \forall w \in s: \text{ Bill did not leave in } w\}$
- $[\![\text{Who left}]\!] = \{s \mid \exists d \in D: \forall w \in s: d \text{ left in } w\}$ [mention-some]

The meaning of a sentence in inquisitive semantics is always non-empty and closed under subsets: if $s \in [\![\varphi]\!]$ and $s' \subset s$ then $s' \in [\![\varphi]\!]$ as well (for motivation of these constraints on sentence meanings, see Ciardelli et al., 2015). The maximal elements of $[\![\varphi]\!]$ are called the ALTERNATIVES in $[\![\varphi]\!]$, denoted $\text{ALT}(\varphi)$. For instance:

- $\text{ALT}(\text{Bill left}) = \{\{w \mid \text{ Bill left in } w\}\}$
- $\text{ALT}(\text{Did Bill leave}) = \{\{w \mid \text{ Bill left in } w\}, \{w \mid \text{ Bill didn't leave in } w\}\}$
- $\text{ALT}(\text{Who left}) = \{\{w \mid d \text{ left in } w\} \mid d \in D\}$ [mention-some]

[3] For a closely related account see Uegaki (2015), and for comparison of the two see Theiler et al. (2016). For earlier informal discussions of the semantics of *wonder*, see Karttunen (1977) and Guerzoni and Sharvit (2007).

Inquisitive states The INQUISITIVE STATE of an agent a, Σ_a, is represented in IEL as a set of information states, namely those information states that resolve all the issues that a entertains. Just like sentence meanings, inquisitive states are also assumed to be non-empty and closed under subsets. Moreover, it is assumed that Σ_a always forms a *cover* of a's information state, denoted as σ_a: $\bigcup\Sigma_a = \sigma_a$ (for motivation of these constraints on Σ_a, see Ciardelli and Roelofsen, 2015). This means that σ_a can always be determined on the basis of Σ_a.

Modal operators, informally IEL has two basic modal operators. Informally, (i) an agent *knows* φ iff her current information state σ_a is a member of $[\![\varphi]\!]$, i.e., one where the issue expressed by φ is resolved, and (ii) an agent *entertains* the issue expressed by φ iff every information state in her inquisitive state is a member of $[\![\varphi]\!]$, i.e., every state that resolves the issues that she entertains is one where the issue expressed by φ is resolved. The semantics of *wonder* is defined in terms of these two basic modal operators. Informally, an agent wonders about φ iff she doesn't know φ but does entertain the issue expressed by φ.

Models and semantics An IEL MODEL for a given set of agents $\mathscr{A}$ and a set of atomic sentences $\mathscr{P}$ is a triple $M = \langle \mathscr{W}, V, \Sigma_{\mathscr{A}} \rangle$, where (i) $\mathscr{W}$ is a set of POSSIBLE WORLDS, (ii) $V : \mathscr{W} \mapsto \wp(\mathscr{P})$ is a VALUATION MAP, and (iii) $\Sigma_{\mathscr{A}} = \{\Sigma_a \mid a \in \mathscr{A}\}$ is a set of INQUISITIVE STATE MAPS, one for each agent $a \in \mathscr{A}$, mapping every world $w \in \mathscr{W}$ to the inquisitive state of a in w, $\Sigma_a(w)$. The information state of a in w, $\sigma_a(w)$, is defined as $\bigcup\Sigma_a(w)$. The semantics for the non-modal fragment of IEL is just that of basic inquisitive semantics. For the knowledge and entertain modality, the semantics is as described informally above:

(4) a. $[\![A]\!] := \{s \mid \forall w \in s : A \in V(w)\}$ for any atomic sentence $A \in \mathscr{P}$
b. $[\![\neg\varphi]\!] := \{s \mid \forall t \in [\![\varphi]\!] : s \cap t = \varnothing\}$
c. $[\![\varphi \wedge \psi]\!] := [\![\varphi]\!] \cap [\![\psi]\!]$
d. $[\![\varphi \vee \psi]\!] := [\![\varphi]\!] \cup [\![\psi]\!]$
e. $[\![K_a\varphi]\!] := \{s \mid \forall w \in s : \sigma_a(w) \in [\![\varphi]\!]\}$
f. $[\![E_a\varphi]\!] := \{s \mid \forall w \in s : \Sigma_a(w) \subseteq [\![\varphi]\!]\}$

If $s \in [\![\varphi]\!]$ we say that *s supports* φ. If φ is supported by a state of complete information $\{w\}$, then we say that φ is *true* in w, notation $w \models \varphi$:

(5) $w \models \varphi \iff \{w\} \in [\![\varphi]\!]$

It follows from the semantics given in (4) that modal statements of the form $K_a\varphi$ and $E_a\varphi$ are supported by a state s if and only if they are true in every world in s. Thus, in the case of modal statements, support is fully determined by truth (this does not hold in general, in particular not for disjunctions). To simplify the exposition, we may therefore just as well focus on the truth-conditions of such statements, which are as follows:

(6) a. $w \models K_a\varphi \iff \sigma_a(w) \in [\![\varphi]\!]$
b. $w \models E_a\varphi \iff \Sigma_a(w) \subseteq [\![\varphi]\!]$

As anticipated above the *wonder* modality in IEL, W, is defined in terms of the basic modal operators K and E, as in (7). This means that it has the truth-conditions specified in (8):

(7) $W_a\varphi := \neg K_a\varphi \wedge E_a\varphi$

(8) $w \models W_a\varphi \iff \sigma_a(w) \notin [\![\varphi]\!] \;\wedge\; \Sigma_a(w) \subseteq [\![\varphi]\!]$

IEL can be extended into a compositional, type-theoretic framework (cf., Ciardelli et al., 2017). English *wonder* can then be translated into the formal language of this framework as follows:

(9) $\ulcorner\text{wonder}\urcorner = \lambda Q_{\langle \mathtt{st},\mathtt{t}\rangle}\lambda x_{\mathtt{e}}.W_x(Q)$

For a full sentence involving *wonder* we then get the following:[4]

(10) a. $\ulcorner$John wonders whether Ann or Bill arrived.$\urcorner = W_j(A \vee B)$

b. $w \models W_j(A \vee B) \iff \sigma_j(w) \notin [\![A \vee B]\!] \;\wedge\; \Sigma_j(w) \subseteq [\![A \vee B]\!]$

c. John's current information state doesn't resolve the question whether A or B, but every information state that resolves the issues that he entertains is one in which the question whether A or B is resolved.

Before closing this background section it is worth mentioning that C&R's semantics predicts that *wonder* does not license declarative complements, assuming that the meaning of a declarative complement always contains a single alternative. Namely, whenever W applies to a sentence whose meaning contains a single alternative, it yields a contradiction.

3. Problem: distributive ignorance

C&R's semantics of *wonder* faces an empirical problem. Consider the following example:

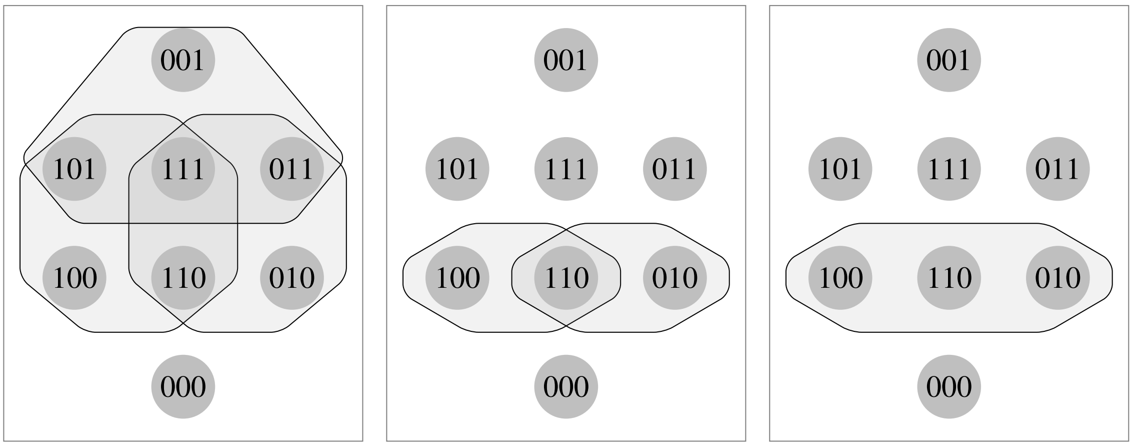
(11) **Situation** John has three students, Ann, Bill and Carol. He is waiting for all of them to arrive at a lab meeting. Someone knocks at the door, but John knows that it can't be Carol because she has just emailed him that she will be late.

Example John wonders whether Ann, Bill or Carol arrived. (Judgment: *False*)

The above example is judged false in the given situation. The sentence is true only if John's information state is compatible with *every alternative* expressed by the complement, i.e., the sets of worlds in which Ann arrived, Bill arrived, and Carol arrived, respectively. Moreover, John's information state should not entail any of these alternatives. Together, this means that the truth of (11) requires that, for each alternative, John is *ignorant* as to whether it holds.

[4] For simplicity, we assume here that the embedded alternative question *whether Ann or Bill arrived* is translated into IEL as $A \vee B$, disregarding the fact that alternative questions presuppose that exactly one of the disjuncts holds. This simplification does not affect the arguments that we will make.

Figure 1: The meaning of the complement, John's inquisitive state Σ_j, and John's information state σ_j in (11). Recall that Σ_j and $[\![A \vee B \vee C]\!]$ are downward-closed; the figures only depict their maximal elements.



(a) $[\![A \vee B \vee C]\!]$ (b) Σ_j (c) σ_j

More generally, for any individual-denoting DP x and any alternative question φ, the sentence x *wonders* φ implies that x is ignorant about each of the alternative answers to φ. We refer to this requirement in the meaning of *wonder* as the DISTRIBUTIVE IGNORANCE requirement. Later, we will discuss in detail what this requirement looks like in examples with *wh*-questions and polar questions. For now, however, let us focus on explaining why the distributive ignorance requirement that arises with alternative questions is a problem for C&R's account.

To see this, consider the example in (11) again. C&R's semantics incorrectly predicts that (11) is *true* in the given situation. This is so since John's current information state does not resolve the question of whether A, B or C, but every information state that resolves the issues that he is entertaining, i.e., every element of his inquisitive state, is one that does resolve the question whether A, B or C. Formally, we have the following:

(12) $\ulcorner$John wonders whether Ann, Bill or Carol arrived.$\urcorner = W_j(A \vee B \vee C)$

(13) $w \models W_j(A \vee B \vee C) \iff \sigma_j(w) \notin [\![A \vee B \vee C]\!]$ and $\Sigma_j(w) \subseteq [\![A \vee B \vee C]\!]$

The meaning of the complement, John's inquisitive state Σ_j, and his information state σ_j are as follows in the given situation (see Figure 1 for graphical representations):

(14) a. $[\![A \vee B \vee C]\!] = [\![A]\!] \cup [\![B]\!] \cup [\![C]\!]$ (meaning of the complement)
b. $\Sigma_j = \{s \mid s \in [\![A]\!] \cup [\![B]\!] \text{ and } \forall t \in [\![C]\!] : s \cap t = \varnothing\}$ (John's inquisitive state)
c. $\sigma_j = \{w \mid (w \in \bigcup [\![A]\!] \text{ or } w \in \bigcup [\![B]\!]) \text{ and } w \notin \bigcup [\![C]\!]\}$ (John's information state)

Thus, the truth conditions in (13) are indeed met in the given situation, since (14c) is not a member of (14a) while (14b) is a subset of (14a). This prediction is incorrect.

4. A pragmatic account and its challenges

4.1. The distributive ignorance requirement as a conversational implicature

Prima facie, one may think that the distributive ignorance requirement could be explained pragmatically, while retaining C&R's semantics for *wonder*. A possible pragmatic derivation of the requirement would go as follows. The speaker of (11) could have uttered another sentence with a *shorter disjunction*, such as the one in (15):

(11) John wonders whether Ann, Bill or Chris arrived.

(15) John wonders whether Ann or Bill arrived.

The alternative in (15) would have been a simpler, presumably still relevant, way to describe John's state. Thus, we could derive the negation of (15) as an implicature, which has the following truth conditions:

(16) $w \models \neg W_j(A \vee B) \iff \sigma_j \in [\![A]\!] \cup [\![B]\!] \text{ or } \Sigma_j \not\subseteq [\![A]\!] \cup [\![B]\!]$

Distributive ignorance can be derived from this implicature, together with the assumed literal meaning of the sentence. Namely, the ignorance condition in the literal meaning implies that $\sigma_j \notin [\![A]\!] \cup [\![B]\!]$. Therefore, for the implicature to hold it must be the case that $\Sigma_j \not\subseteq [\![A]\!] \cup [\![B]\!]$. But the 'entertainment' condition in the literal meaning says that $\Sigma_j \subseteq [\![A]\!] \cup [\![B]\!] \cup [\![C]\!]$. This allows us to conclude that $\Sigma_j \cap [\![C]\!] \neq \{\varnothing\}$, which in turn implies that John's information state, σ_j must be compatible with $\bigcup[\![C]\!]$. Following the same line of reasoning for the other disjuncts, we can derive that σ_j has to be compatible with $\bigcup[\![A]\!]$ and with $\bigcup[\![B]\!]$ as well. At the same time, σ_j cannot entail any of $\bigcup[\![A]\!]$, $\bigcup[\![B]\!]$ and $\bigcup[\![C]\!]$, due to the ignorance condition in the literal meaning. Thus, we derive that John has to be ignorant w.r.t. $\bigcup[\![A]\!]$, $\bigcup[\![B]\!]$, and $\bigcup[\![C]\!]$.

Thus, we see that the distributive ignorance requirement of (11) can be derived as a pragmatic inference. However, there are empirical features of the distributive ignorance requirement which, at face value, do not seem to be in line with the traditional Gricean conception of pragmatic implicatures. Below, we describe three such empirical features.

4.2. Challenges for the pragmatic account

4.2.1. Non-monotonicity of *wonder*

First of all, due to the non-monotonicity of C&R's semantics for *wonder*, the sentence in (15) is actually not stronger than the original sentence in (11). For instance, (15) is true while (11) is false when John's inquisitive state is as follows:

(17) $\Sigma_j = \{s \mid s \in [\![A]\!] \cup [\![B]\!] \text{ and } s \in [\![C]\!]\}$

Since quantity implicatures under the traditional Gricean conception arise only with respect to *strictly stronger* alternatives, the fact that (15) is not strictly stronger than (11) suggests that the inference cannot be treated as a traditional quantity implicature.

One might suggest that the ignorance condition in the semantics of *wonder* could be treated as a presupposition, and that (15) would then indeed asymmetrically entail (11) if we only consider their assertive component, which would consist in the entertainment condition. However, we submit that the ignorance condition of *wonder* is part of its assertive meaning, given that it does not exhibit the projection behavior of presuppositions. The following examples show that it does not project out of (a) negation, (b) polar questions, and (c) the attitude predicate *doubt*, as evidenced by the fact that the underlined continuations are perfectly felicitous.

(18) a. The detectives are not wondering whether Ann stole the jewels.
They already know that she did.
b. A: Are the detectives wondering whether Ann stole the jewels?
B: No, they already know that she did.
c. Bill doubts that the detectives are wondering whether Ann stole the jewels.
He believes that they already know that she stole them.

This contrasts with the behavior of attitude-verb meanings whose presuppositional status is undisputed. For example, the factive presupposition of *know* does exhibit the typical projection behavior, as shown by the oddness of the contradicting continuations in the following examples (in order to become felicitous, these continuations need a marker of presupposition denial such as *actually* or *in fact*).

(19) a. The detectives don't know that Ann stole the jewels.
#She didn't steal them.
b. A: Do the detectives know that Ann stole the jewels?
B: No, they don't. #She didn't steal them.
c. Bill doubts that the detectives know that Ann stole the jewels.
#He doesn't believe that she stole them.

4.2.2. Obligatoriness

The second challenge for a pragmatic account of distributive ignorance is its *obligatory* nature. The following example indicates that distributive ignorance cannot be canceled, contrary to what would be expected under a traditional Gricean approach.

(20) John wonders whether Ann, Bill or Carol arrived.
In fact, he already knows that Carol is still at home, but he doesn't know yet whether Ann or Bill arrived.

At this point, it should be mentioned that not all Gricean analyses of implicatures predict cancellability as a necessary feature. In particular, Lauer (2014) points out that Gricean pragmatics predicts that an obligatory implicature arises if an utterance of an expression *necessarily* makes a more preferred expression salient. Lauer (2014) argues that this is exactly the case with the ignorance implicature of unembedded disjunctions since an utterance of the form α *or* β necessarily makes each disjunct salient (see also Westera, 2017).

It is conceivable that Lauer's analysis can be applied to the distributive ignorance requirement of *wonder*, correctly capturing its obligatory nature within a pragmatic approach. However, such an analysis would still face the non-monotonicity issue discussed above, and would also have difficulty capturing the *locality* of distributive ignorance, to which we turn next.

4.2.3. Locality

The distributive ignorance requirement of *wonder* is local, in the sense that it takes scope below operators that are syntactically above *wonder*. The following example illustrates this:

(21) **Situation** There is a crime with three suspects, Ann, Bill, and Carol. There are five detectives investigating the case; one has already ruled out Carol but is still wondering whether it was Ann or Bill. The others don't know anything yet.

Example Exactly four detectives are wondering whether it was Ann, Bill, or Carol. (Judgment: *true*)

The judgment that this example is *true* can only be accounted for if the distributive ignorance requirement takes scope below the subject quantifier *exactly four detectives*. If the distributive ignorance requirement is derived as a global pragmatic implicature, the sentence would be predicted to be *false*. Here's why: the literal meaning of the given sentence would be that exactly four detectives are such that (i) they don't know whether it was Ann, Bill, or Carol, and (ii) every information state they want to be in resolves the issue of whether it was Ann, Bill, or Carol. This is *false* in the situation above since all five detectives meet these conditions. Adding implicatures to the literal meaning of the sentence could only strengthen it and could thus not make it true in the given situation.

The following example further strengthens our claim that distributive ignorance scopes locally:

(22) **Situation** There is a crime with three suspects, Ann, Bill, and Carol. There are three detectives investigating the case.

- Detective 1 has ruled out Ann but still wonders whether it was Bill or Carol.
- Detective 2 has ruled out Bill but still wonders whether it was Ann or Carol.
- Detective 3 has ruled out Carol but still wonders whether it was Ann or Bill.[5]

[5] We thank Benjamin Spector for drawing our attention to this type of situations.

Example Every detective is wondering whether it was Ann, Bill, or Carol.
(Judgment: *false*)

The judgment that this example is *false* makes sense if the distributive ignorance requirement takes scope under *every detective*. On the other hand, a global derivation of the pragmatic inference would predict the sentence to be *true*. Here's why: the predicted implicatures would be as follows:

(23) a. It is not the case that every detective wonders whether it was Ann or Bill.
b. It is not the case that every detective wonders whether it was Bill or Carol.
c. It is not the case that every detective wonders whether it was Carol or Ann.
and so on...

These implicatures are all *true* in the given situation. The presence of detectives 1 and 2 makes (23a) true. The presence of detectives 2 and 3 makes (23b) true. The presence of detectives 1 and 3 makes (23c) true.

The locality of distributive ignorance is a challenge for the pragmatic approach since pragmatic maxims are traditionally assumed to apply globally, i.e., to the sentence as a whole.[6] Certain apparently local implicatures have been explained within pragmatic approaches that are essentially Gricean in nature (see, e.g., Franke, 2009). It is possible that such pragmatic theories could ultimately derive local distributive ignorance implicatures with *wonder* as well. However, it is not immediately clear, to us, how this may be achieved.

We have seen, then, that a 'conservative' approach, which maintains C&R's semantics for *wonder* and tries to derive the distributive ignorance requirement pragmatically, encounters a number of challenges. However, this is of course not the only possible approach. Another option is to reconsider C&R's semantics of *wonder* and see if it could be adapted so as to derive the distributive ignorance requirement directly, without pragmatics. Clearly, such an approach would directly predict the obligatory and local nature of the distributive ignorance requirement. Moreover, it would steer clear of the non-monotonicity issue that the pragmatic approach faces. This is the route that we will take in the remainder of this paper.

[6] Locality also distinguishes *wonder* from other predicates, such as *believe*. Although *believe* also implies distributive ignorance when taking a referential subject, as illustrated in (i) below, examples with quantificational subjects reveal that this implication is not local, as illustrated in (ii) and (iii).

(i) John believes that it was Ann, Bill, or Carol. (implies distributive ignorance)
(ii) Exactly four detectives believe that it was Ann, Bill, or Carol. (*false* in (21))
(iii) Every detective believes that it was Ann, Bill, or Carol. (*true* in (22))

This contrast between *wonder* and *believe* suggests that distributive ignorance with *wonder* is a local implication associated with the semantics of the predicate itself while distributive ignorance with *believe* is a global pragmatic implicature that arises regardless of the embedding predicate.

5. Two semantic accounts

We will consider two ways to adapt C&R's semantics. The first (§5.1) directly strengthens the ignorance requirement in the lexical semantics of *wonder*. The second (§5.2) leaves C&R's basic entry for *wonder* intact, but additionally assumes that the semantics of the verb involves an *exhaustivity operator*, just like the semantics of *only*.[7] The main difference between these two accounts is that on the former, *wonder* remains sensitive only to the semantic content of the clause that it combines with, while on the second, due to the exhaustivity operator, it becomes sensitive to the formal structure of its complement clause as well. In §6 we will attempt to tease these two accounts apart based on data involving polar questions and *wh*-questions.

5.1. Directly encoding strong ignorance

In C&R's semantics of *wonder*, the ignorance condition is encoded as $\neg K_a\varphi$. That is, the subject's information state must not be contained in any alternative in $[\![\varphi]\!]$. This is a relatively weak notion of ignorance. A natural way to strengthen it would be to require that, in addition, the subject's information state should be compatible with every alternative in $[\![\varphi]\!]$. Let us introduce a new modal operator, I, which expresses this strong form of ignorance:

(24) $w \models I_a\varphi \iff \forall\alpha \in \textsc{alt}(\varphi) : \sigma_a(w) \not\subseteq \alpha$ and $\sigma_a(w) \cap \alpha \neq \varnothing$

Using this strong ignorance operator, we can then re-define the *wonder* modality in IEL:

(25) $W_a\varphi := I_a\varphi \wedge E_a\varphi$

This analysis directly encodes the distributive ignorance requirement in the lexical semantics of *wonder*. Clearly, the local and obligatory nature of the requirement are straightforwardly captured in this way.

5.2. Strong ignorance via exhaustivity

We now consider a more indirect account, which supplements C&R's entry for *wonder* with a built-in exhaustivity operator. We will assume that this exhaustivity operator is sensitive to the formal structure of the complement clause, rather than just its semantic content, because an account involving a purely semantic exhaustivity operator would be difficult to distinguish from the account specified above.

For any two natural language expressions φ and φ', we write $\varphi' \lesssim \varphi$ iff φ' is formally simpler

[7] Yet another possible analysis would be one involving an exhaustivity operator in the syntax (Chierchia et al., 2012) rather than in the lexical semantics of *wonder*. However, on such an approach additional assumptions would be needed about the distribution of this operator, in order to account for the locality and obligatory nature of the distributive ignorance requirement. For now, we leave this possible analysis out of consideration.

than φ in the sense of Katzir (2007), i.e., iff φ' can be obtained from φ by deleting constituents or replacing them with other constituents of the same syntactic category, taken either from the lexicon or from φ itself.

The exhaustivity operator that we will assume takes an expression φ and a set of formal alternatives A, and strengthens φ by negating every $\psi \in A$ that is not entailed by φ:

(26) $\mathbf{EXH}_A(\varphi) := \varphi \wedge \bigwedge\{\neg\psi \mid \psi \in A \text{ and } \varphi \not\models \psi\}$

Using this exhaustivity operator, the semantics of *wonder* can be formulated as follows:[8]

(27) $\ulcorner\text{wonder } Q\urcorner = \lambda x.\mathbf{EXH}_{\{W_x(\ulcorner Q'\urcorner) \mid Q' \lesssim Q\}}\, W_x(\ulcorner Q\urcorner)$

The formal alternatives for exhaustification are expressions of the form $W_x\ulcorner Q'\urcorner$, where Q' is grammatically simpler than the original complement Q. As a result of exhaustification, *x wonders Q* negates those formal alternatives that are not entailed by $W_x(\ulcorner Q\urcorner)$.[9] The obligatory and local nature of the distributive ignorance requirement follow from this analysis as well.

To see the account at work, consider the following sentence:

(28) Every detective wonders whether A, B or C.

For concreteness, let us assume that the complement in this sentence has the following structure (as far as we can see, all other reasonable assumptions about the structure of alternative questions are compatible with our argument as well):

(29) [whether [A or [B or C]]]

Further assume that *or* is translated as $\vee$ and that *whether* is semantically vacuous but syntactically requires at least one occurrence of *or* in its scope—if no such occurrence is overtly present, as in a polar question, there must be a covert *or not* (cf., Guerzoni and Sharvit, 2014; though again, other assumptions about the syntax-semantics interface of alternative questions would be compatible with our argument as well, as far as we can tell).

This structure can be simplified by eliminating either one or two of the disjuncts; in the latter case *whether* has to be eliminated as well. Such simplifications yield the following structures:

[8] Since the exhaustivity operator is structure-sensitive, our entry for *wonder* has to be syncategorematic.

[9] We could assume that formal alternatives are only negated if they are innocently excludable (IE), in order to avoid potential contradictions arising from exhaustification (Fox, 2007):

(i) $\mathbf{EXH}_A(\varphi) := \varphi \wedge \bigwedge\{\psi \mid \psi \in \mathbf{IE}(\varphi, A)\}$

(ii) $\mathbf{IE}(\varphi, A) := \bigcap\{A' \subseteq A \mid A' \text{ is a maximal subset of } A \text{ s.t.} \{\neg\varphi' | \varphi' \in A'\} \cup \{\varphi\} \text{ is consistent}\}$

However, we keep the simpler formulation in (26) since there would be no contradiction arising from the negation of non-weaker alternatives in the examples under consideration.

(30) [whether [A or B]] [whether [B or C]] [whether [A or C]] A B C

Thus, the sentence in (28) receives the interpretation in (32), which means that it is correctly predicted to be false in the scenario described in (22).[10]

(31) $\ulcorner$Every detective wonders whether A, B or $C\urcorner$

$= \forall x : \textbf{detective}(x) \rightarrow \textbf{EXH}_{\{W_x(\ulcorner Q' \urcorner) \mid Q' \lesssim \text{whether } A, B \text{ or } C\}} W_x(A \vee B \vee C)$

(32) $$= \forall x : \textbf{detective}(x) \rightarrow \begin{pmatrix} W_x(A \vee B \vee C) \\ \wedge \neg W_x(A \vee B) \\ \wedge \neg W_x(B \vee C) \\ \wedge \neg W_x(C \vee A) \\ \wedge \neg W_x(A) \\ \wedge \neg W_x(B) \\ \wedge \neg W_x(C) \end{pmatrix}$$

6. Teasing the two semantic accounts apart

We will now try to tease the two semantic accounts apart, by considering cases where *wonder* takes a polar question or a *wh*-question as its complement, rather than an alternative question. We will consider polar disjunctive questions like *whether-or-not Ann or Bill arrived* (§6.1), plain *wh*-questions like *which of the students arrived* (§6.2), ones where the domain of quantification is explicitly listed, as in *which of Ann, Bill, and Carol arrived* (§6.3), and ones where the domain specification involves a numeral, as in *which of John's three students arrived* (§6.4).[11]

6.1. Polar questions involving a disjunction

Distributive ignorance is observed also when *wonder* takes a polar disjunctive question complement, as in the following example:[12]

(33) John wonders whether-or-not Ann or Bill arrived.

That is, (33) implies that John is ignorant as to whether Ann arrived and as to whether Bill arrived. This is expected under the exhaustivity-based account, since the following complements are structurally simpler alternatives for the complement in (33):

[10] The last three conjuncts in (32) are tautologous, so they could in principle we left out. We do display them here just for transparency.

[11] We should note that the judgments reported in this section are based on introspection and discussion with a small number of native speakers. More systematic empirical work will be needed to obtain a more reliable picture of the relevant data.

[12] We use *whether-or-not* here, rather than just *whether*, in order to make sure that the complement is read as a polar question, and not as an alternative question. The relevant observation, however, applies just as well to complements headed by plain *whether*, interpreted as polar questions.

(34) a. whether-or-not Ann arrived.
b. whether-or-not Bill arrived.

Thus, (33) is predicted to imply, through exhaustivity, that $\neg W_j(A \vee \neg A)$ and $\neg W_j(B \vee \neg B)$. It follows from this that John must be ignorant as to whether Ann arrived. To see this, first suppose that John believes that Ann did arrive. This is incompatible with the basic ignorance requirement of (33). Now suppose that John believes that Ann did not arrive. Then, the basic ignorance and entertain conditions of (33) are satisfied only if John is wondering whether Bill arrived. But this is incompatible with the implication $\neg W_j(B \vee \neg B)$. So, John must be ignorant as to whether Ann arrived. Similarly, ignorance as to whether Bill arrived follows as well.

In contrast, on the direct account it is wrongly predicted that (33) may be true even if John already knows that Ann did not arrive, or similarly, if he already knows that Bill did not arrive.

6.2. Plain *wh*-questions

When *wonder* takes a plain *wh*-question as its complement, as in (35-37), the distributive ignorance requirement seems to be absent.

(35) *Situation: as in (11).*
John wonders which of his students arrived. (*true*)

(36) *Situation: as in (21).*
Exactly four detectives are wondering which of the suspects did it. (*false*)

(37) *Situation: as in (22).*
Every detective is wondering which of the suspects did it. (*true*)

If the distributive ignorance requirement were present in the same way as in our earlier examples involving alternative questions, (35) should be judged false, (36) true, and (37) false. The actual judgments, however, seem to be the opposite.

This contrast is expected under the view that the distributive ignorance requirement is a result of exhaustification w.r.t. structurally determined alternatives. After all, complements like *whether Ann or Bill arrived* count as structural alternatives for *whether Ann, Bill or Carol arrived*, but not for *which of his students arrived.* As a consequence, strong ignorance is predicted to arise with alternative questions but not with *wh*-questions.

On the other hand, the contrast is puzzling if the mechanism responsible for the distributive ignorance requirement is only sensitive to the semantic properties of the complement, as on the direct account. After all, it is hard to distinguish between alternative questions and *wh*-questions in terms of purely semantic properties.

One may attempt to derive the contrast on this approach from the assumption that *wh*-questions, unlike alternative questions, may involve *implicit domain restriction*. That is, (35-37) could be taken to involve the following implicit domain restrictions:

(38) John wonders which of his students [except Carol] arrived.

(39) Exactly four detectives are wondering which of the suspects [that they are still suspecting] did it.

(40) Every detective is wondering which of the suspects [that he is still suspecting] did it.

If this is indeed what is going on, the data is compatible with the direct account. For instance, it would then be predicted that (39) is false in the given scenario because all five detectives satisfy the distributive ignorance requirement with respect to the suspects that they are still suspecting. And similarly for the other examples.

However, on such an account it remains to be explained why the assumed implicit domain restrictions seem to be *obligatory*. Prima facie, one would expect that implicit domain restriction is optional, and that various restrictions would be possible, not just the ones explicated in (38-40). The case of (36)/(39) would be especially puzzling, because this sentence is judged *false* in the given context. Under the assumption that implicit domain restriction is optional, the Charity Principle would favor an interpretation *without* domain restriction in this case, because under such an interpretation the sentence would be *true* in the given scenario. However, the only interpretation that seems to be available is one on which the sentence is false.

6.3. *Wh*-questions with listed domain of quantification

Now consider the following variants of (35-37), where the quantificational domain is explicitly listed.

(41) *Situation: as in (35).*
John wonders which of Ann, Bill and Carol arrived. (*false*)

(42) *Situation: as in (36).*
Exactly four detectives are wondering which of Ann, Bill and Carol did it. (*true*)

(43) *Situation: as in (37).*
Every detective is wondering which of Ann, Bill and Carol did it. (*false*)

We see that 'list *wh*-questions' pattern with alternative questions rather than with plain *wh*-questions. This is again expected on the structure-sensitive exhaustivity-based account, because we have that *which of Ann and Bill arrived* $\lesssim$ *which of Ann, Bill and Carol arrived*.

Under the direct account, 'list *wh*-questions' are also expected to pattern with alternative questions under the assumption that they do not permit implicit domain restriction. This assumption seems quite natural, so the examples in (35-37) arguably do not present any new challenges for the direct account. However, as discussed above, it remains puzzling why plain *wh*-questions would obligatorily involve a specific kind of domain restriction.

An interesting hybrid between plain *wh*-questions and list *wh*-questions, as suggested to us by Brian Buccola, is the following:

(44) **Situation** A certain linguistics department has one full professor (Jones), three associate professors (A, B, and C), and three assistant professors (X, Y, and Z). Jones is a phonologist and has a question about semantics, which he thinks only a real semanticist could possibly answer. The only semanticists in the department are A and C (both associate professors).

Example Jones wonders which assistant or associate professor could answer his question. (*false*)

What is required for (44) to be true is that Jones considers at least one assistant professor and at least one associate professor possibly capable of answering his question (for instance, the sentence would be true if, in addition to A and C, Y were a semanticist as well). On the other hand, it is not required that Jones considers *all* assistant and associate professors possibly capable of doing so.

This is straightforwardly accounted for on the exhaustivity-based account, because we have that *which assistant professor* and *which associate professor* are structural alternatives for *which assistant or associate professor*. On the other hand, it is not so clear that the direct account could generate this prediction in a principled way. If we assume that implicit domain restriction is blocked in (44), just as in alternative questions and list *wh*-questions, then the predicted ignorance requirement would be too strong: Jones would have to be ignorant about all assistant and associate professors. If we assume that implicit domain restriction can apply freely, then the predicted ignorance requirement would be too weak: it would be predicted that the sentence is true even if Jones knows that none of the assistant professors are semanticists, as in the given context. So, some intermediate constraint on domain restriction would be needed, and it is not clear how such a constraint could be justified.

6.4. *Wh*-questions with numerical domain specification

Now consider the following variants of (35-37), where the domain of quantification is specified using a numeral. This has been argued in previous work to strongly disfavor, or even completely block, implicit domain restriction (Chemla, 2009; Geurts and van Tiel, 2016).

(45) *Situation: as in (35).*
John wonders which of his three students arrived. (*false*)

(46) *Situation: as in (36).*
Exactly four detectives are wondering which of the three suspects did it. (*true*)

(47) *Situation: as in (37).*
Every detective is wondering which of the three suspects did it. (*false*)

We see that 'numerical *wh*-questions' pattern like alternative questions and list *wh*-questions, and unlike plain *wh*-questions. This is predicted by the direct account, under the assumption that numerical *wh*-questions don't allow for implicit domain restriction, just like alternative questions and list *wh*-questions.

On the other hand, the contrast between numerical *wh*-questions and plain *wh*-questions is not immediately accounted for under the exhaustivity-based account, because the structural alternatives for numerical *wh*-questions do not include ones such as *whether Ann or Bill arrived*, which would be necessary to derive the distributive ignorance requirement from exhaustivity.

There is, however, a possible extension of the exhaustivity-based account formulated above, which does capture the contrast between numerical *wh*-questions and plain *wh*-questions. This extension incorporates a number of ideas from Chierchia's (2013) work on different types of indefinites (plain, NPI, free choice) and their interaction with exhaustification. We will briefly review these ideas and then return to the case of numerical *wh*-questions.

First, Chierchia assumes that indefinites and other quantifiers always come with a domain variable D, whose value is contextually determined. For instance, a sentence like *Some students arrived* is analyzed as $\exists x \in D.S(x) \wedge A(x)$, involving existential quantification over a contextually given domain D. This assumption is quite widespread, though not everyone agrees that implicit domains should be represented syntactically.

Second, Chierchia proposes that sentences involving a quantifier with domain variable D generate so-called *sub-domain alternatives*, which involve quantification over a subdomain $D' \subseteq D$. For instance, the sub-domain alternatives of *Some students arrived* are $\exists x \in D'.S(x) \wedge A(x)$, for any $D' \subseteq D$. Crucially for us, Chierchia proposes that these sub-domain alternatives can serve as input for exhaustivity operators, alongside structurally determined alternatives. Whether they actually play this role depends on whether they are *activated*. Chierchia proposes that the sub-domain alternatives generated by plain indefinites are only optionally activated. On the other hand, the sub-domain alternatives of specially marked indefinites (such as *any* and *ever*) are obligatorily activated, and therefore must always serve as input for an exhaustivity operator. This has certain interpretational consequences, and also accounts for the restricted distribution of such marked indefinites. For instance, the ungrammaticality of a sentence like *Any students arrived* is accounted for by the fact that exhaustification w.r.t. the sub-domain alternatives generated by *any* yields a contradiction in this case. On the other hand, in *John didn't expect any students*, exhaustification w.r.t. sub-domain alternatives does not yield a contradiction.

Now let us return to the contrast between plain *wh*-phrases (e.g., *which of his students*) and numerical *wh*-phrases (e.g., *which of his three students*). Extending Chierchia's ideas to *wh*-

phrases, it would be natural to assume that (i) *wh*-phrases always come with a domain variable and generate sub-domain alternatives, (ii) the sub-domain alternatives generated by plain *wh*-phrases are only optionally activated, but (iii) the sub-domain alternatives generated by specially marked *wh*-phrases, such as numerical ones, are obligatorily activated and therefore must serve as the input for an exhaustivity operator.

Given this, it is natural to assume that the exhaustivity operator in the lexical semantics of *wonder* is not only sensitive to structurally determined alternatives for the complement clause, but also to its sub-domain alternatives, when activated. Using $\textsc{sda}(Q)$ to denote the set of activated sub-domain alternatives generated by Q, this can be implemented as follows:

(48) $$\ulcorner \text{wonder } Q \urcorner = \lambda x.\mathbf{EXH}_{\underbrace{\{W_x(\ulcorner Q' \urcorner) \mid Q' \lesssim Q\}}_{\text{structural alternatives}} \cup \underbrace{\{W_x(\varphi) \mid \varphi \in \textsc{sda}(Q)\}}_{\text{sub-domain alternatives}}} W_x(\ulcorner Q \urcorner)$$

Under this treatment of *wonder*, we get exactly the desired predictions. For instance, (45) now implies, due to exhaustification w.r.t. sub-domain alternatives, that John is *not* wondering which of Ann and Bill arrived. This implication is false in the given context, so the sentence as a whole comes out false as well. And similarly for (46) and (47).

Thus, while the exhaustivity-based account needs to be further worked out and tested on a broader range of empirical data, it seems to be able to capture some interesting contrasts, and to have an advantage over the direct account in that it does not rely on particular constraints on implicit domain restriction which would seem difficult to motivate independently.

7. Conclusion

If an inquisitive predicate like *wonder* takes an alternative question as its complement, it implies that its subject is ignorant about each of the alternatives introduced by the question. This implication, which we call the distributive ignorance requirement, is not predicted by previous work on the semantics of *wonder*. Furthermore, a pragmatic approach to deriving distributive ignorance faces a number of challenges (non-monotonicity, obligatoriness, and locality). We thus considered two semantic accounts: one that directly encodes a strong form of ignorance in the meaning of *wonder* and the other based on a built-in exhaustivity operator.

To tease apart these two semantic accounts, we looked at four types of questions as the complement of *wonder*: polar questions involving a disjunction, plain *wh*-questions, *wh*-questions with a listed domain of quantification and *wh*-questions with a numerical domain specification. The fact that the distributive ignorance requirement is absent with 'plain' *wh*-questions but present with polar questions involving a disjunction and 'list' *wh*-questions suggests that the distributive ignorance requirement is sensitive to the structure of the complement. This is expected on the exhaustivity-based account, which comes with a structure-sensitive notion of alternatives (Katzir, 2007). Furthermore, although at first sight the presence of distributive ignorance with 'numerical' *wh*-questions seemed unexpected under the exhaustivity-based account, we proposed that it could be accounted for by incorporating exhaustification w.r.t.

sub-domain alternatives (Chierchia, 2013). On the other hand, in order to capture the contrasts between the different types of *wh*-questions, the 'direct' account would have to invoke particular constraints on implicit domain restriction. Given that such constraints seem difficult to motivate independently, we conclude that the exhaustivity-based approach is more promising.

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