Essays in competition economics

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Competition policy (mededingingsbeleid) aims to preserve well-functioning market competition by restricting the acquisition or abuse of corporate dominance. Competition economics (mededingingseconomie) looks at the economics behind this. This dissertation consists of four separate essays in competition economics. The first two essays use microeconomic theory and simulations to look at two novel competition concerns: algorithmic collusion and financial benchmark rate collusion. The third essay revisits classic cartel theory to show how results on cartel stability change when assuming firms require a margin before colluding. The last essay reviews event studies in merger analysis and provides a novel application using Hoberg-Phillips TNIC data to argue that U.S. merger control may have been too lenient.

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Essays in Competition Economics

ACADEMISCH PROEFSCHRIFT

ter verkrijging van de graad van doctor aan de Universiteit van Amsterdam op gezag van de Rector Magnificus

prof. dr. ir. K.I.J. Maex

ten overstaan van een door het College voor Promoties ingestelde commissie, in het openbaar te verdedigen in de Aula der Universiteit

op vrijdag 2 oktober 2020, te 14:00 uur

door

Timo Klein

geboren te Apeldoorn
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Universität Hohenheim
prof. dr. J. Tuinstra
Universiteit van Amsterdam

Faculteit: Faculteit Economie en Bedrijfskunde
Preface

When I was growing up, there must have been little indication that I would be any good at Economics. Each year on April 30, my brother and I would be trying to sell our old stuff at the local Queensday children’s flee market in Ugchelen—the hometown where I grew up. Unlike my brother however, I somehow left with less money than I came, buying more useless stuff from others than actually managing to sell anything myself.

Also during my secondary school years, it took me a long time to warm up to Economics as a school subject. I was comfortable with Mathematics, but ended up going for what in the Netherlands are known as “Culture and Society” courses: History, Geography, English, Art History and Drama. Yet, when it came to choosing a university specialization in 2009, it was the practical value of a more quantitative social science that nevertheless convinced me start an academic journey in Economics.

In 2016, after a BSc in Economics and Business, an MSc in Political Economy and an MPhil in Economics, my parents would probably have been happy to finally see me getting a proper job, making a contribution to society and start earning some income. After all, these years did leave behind a bit of a student debt as well. But then I decided to add on another three or four years of PhD... Of course I would point out to my parents that in the Netherlands, PhD students are actually employees of the university, being paid a decent salary. And I would finally start making a contributing to society through research and teaching! Being a ‘PhD student’ is actually not at all the same as being a ‘student’, I would say. I am not sure whether I really managed to convince them (or myself), but they have nevertheless always been extremely supportive.

Pursuing a PhD (as academia in general) can be very stressful: academic research can take a very long time with many disappointments along the way, you are often not even sure whether what you are doing makes much sense and you are always surrounded by people that are at least as smart as you, but often a lot smarter. Nevertheless, I have always seen my time as a PhD student (as for my entire educational career) as one big
privilege. It has given me the opportunity to really delve into things that I am really interested in, without directly being asked to give something in return—which is something that I will always be thankful for.

The road from the Queensday flee market in Ugelhelen to a PhD in Economics has been long, with a lot of hard work, but I would have definitely ended up nowhere if not for so many amazing family and friends.

First of all, I would like to express an infinite gratitude to Maarten Pieter Schinkel for his professional and personal supervision over the past four years. I think that you may have once described me as a stubborn adolescent, but even though I did not always agree with your guidance, I cannot deny that it has been exactly the wisdom and patient and persistent guidance that has brought me to where I am today. My gratitude also goes to Maurice Bun for the last-minute but very valuable co-supervision, and of course to all members of the PhD committee—Arnoud den Boer, Joe Harrington, Sander Onderstal, Ulrich Schwalbe and Jan Tuinstra—for taking the time to actually read through and assess my work, and in many cases already providing me with the comments, feedback and guidance along the way that have proved invaluable.

For this ‘final stretch’ of my academic education, I am of course thankful for the many fellow PhD students and friends that were there along the way, including in particular (but definitely not limited to) Magda, Pim, Maria, Benji, Huaiping, David, Marc, Radu, Robin, Laura, James, Sarah, Leonard, Nuria, Simon, Emilie, Andrej and Luisa. Thanks for the trips, dinners, Kingsdays and Friday drinks at the Krater—looking forward to more of these!

I am also grateful for all the other amazing people that I have got to know during the course of my eleven year studies—including from my time at the student council, partij mei, Sefa, the LSE European Society, De Kleine Consultant and the Blue Book traineeship. A lot more friends deserve one, but a particular shout-out here goes to Nikolai, Maarten, Carlo and Jasper.

Finally, I would like to thank all my ‘hometown’ friends and all other friends that I have been and keep on seeing—whether at Soenda, Awaken-
ings, DGTL, Lowlands or another festival, at our kookclubje or elsewhere—including Ilse, Geert, Niek, Maurice, Floor, Victor, Mies, Lotte, Raffi, Nino, Jorick, Suus, Emile, Kees, Tanja, Wietse, Lieke, Nick, Mo and Ruben. Looking forward to the post-lockdown festivals and other events!

To close off, I would like to thank my family, Dick, parents and of course Laura. In particular I would like to thank my parents and Dick for their unconditional love and support over the past three decades and Laura for hers over the past three years. Laura, thank you for your love and company, and your sometimes impressive patience with me. Mom and dad, thank you for everything. You are the single most important reason that I am where I am, that I get to do what I do and that I have been able to achieve whatever I may have achieved.

Timo Klein
Amsterdam, 19 July 2020
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Chapter 1

Introduction

“The quest for the common good involves constructing institutions to reconcile, as far as possible, the interests of the individual with the general interest. [...] Once a definition of the common good has been agreed upon, economics can help develop tools that contribute to achieving it.”
– Jean Tirole, in: Economics for the Common Good (2017, pp. 3-5)

How can we ensure that market competition leads to desirable outcomes? On the one hand, the prospect of profit incentivizes firms and entrepreneurs to produce goods and services that consumers want to buy. The presence of competitors then generally pressures firms further to decrease price, increase quality and innovate. Moreover, market competition may protect citizens against corruption or incompetence in the public production and allocation of goods and services. However, market competition can also produce socially undesirables outcomes, such as harmful externalities, unfair distributions or an abuse of corporate dominance. The presence of these market failures may mandate public policy interventions.

Competition policy is the type of public policy that deals mostly with one type of market failure: corporate dominance. Firms have an incentive to acquire and abuse a dominant market position, for instance by coordinating or merging with competitors, or by imposing terms on suppliers or customers that push competing firms out of the market. Competition policy aims to prevent detrimental restrictions to competition. Competition economics looks at the economics behind this.
Chapter 1. Introduction

This dissertation consists of four separate essays in the field of competition economics. The first two essays use microeconomic theory and simulations to look at two novel competition concerns: algorithmic collusion and collusion on financial benchmark rate setting. The third essay revisits classic cartel theory to show how comparative statics on cartel stability change when assuming firms require a margin before colluding. The last essay reviews event studies in merger analysis and provides a novel application using Hoberg-Phillips TNIC data to argue that U.S. merger control may have been too lenient.

The remainder of this introductory chapter is organized as follows. Section 1.1 provides a background introduction to competition policy. Section 1.2 introduces the main contemporary topics in the field of competition economics and how some of the essays in this dissertation relate to them. Section 1.3 outlines the methodologies used. Finally, Section 1.4 provides the motivation, approach and key findings of each of the four essays.

1.1 Competition Policy

Competition policy is defined as “the set of policies and laws which ensure that competition in the marketplace is not restricted in a way that is detrimental to society” (Motta, 2004, p. 31). Commonly, competition policy is divided into four different areas: horizontal and vertical agreements, abuse of dominance, merger control and state aid. Although this dissertation mostly deals with concerns around collusion, with only the last chapter dealing also with merger control, I briefly review all four areas of competition policy. Notable textbook treatments on the economics of competition policy are provided by Motta (2004), Bishop and Walker (2010) and Niels, Jenkins and Kavanagh (2016). Davis and Garcés (2010) provides a textbook treatment of the quantitative techniques for competition analysis. Whish and Bailey (2018) and Jones, Sufrin and Dunne (2019) are the established EU competition law textbooks.
1.1.1 Horizontal and Vertical Agreements

Agreements between undertakings may inhibit competition. This is particularly a concern when there are agreements between horizontally related firms that are supposed to compete with each other. Although there may also be competition concerns in the case of certain agreements between vertically related firms (in other words, firms at different stages of a supply chain), there is also a recognition that in many cases these can have pro-competitive justifications—such as ensuring sufficient effort by and reward to the different firms at the different stages of the supply chain. Here I focus on competition policy in the context of horizontal agreements.

Generally, competition involves a prisoner’s dilemma: all firms would be best off if they coordinated on high prices, but each individual firm can improve its outcome by decreasing its price. The presence of this prisoner’s dilemma is what pressures firms to decrease price and compete. Table 1.1 provides a stylized illustration of this: if both firms maintain a high price they both receive a payoff of 10, but irrespective of what the other firm does, each firm can always improve its own situation by decreasing its price. This dynamic causes both firms to change low prices, leading to a lower payoff of 5 each. Although firms are worse off, consumers benefit from lower prices. Generally, similar dynamics apply to other dimensions of competition as well, such as quality and innovation.¹

Table 1.1: Illustration of a Prisoner’s Dilemma Under Price Competition

<table>
<thead>
<tr>
<th></th>
<th>Firm B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High Price</td>
</tr>
<tr>
<td>Firm A</td>
<td></td>
</tr>
<tr>
<td>High Price</td>
<td>10,10</td>
</tr>
<tr>
<td>Low Price</td>
<td>15,0</td>
</tr>
</tbody>
</table>

Notes: The first payoff is the payoff of Firm A and the second of Firm B.

¹Shapiro (2012) and Gilbert (2020), among others, discuss the literature on how competition affects innovation—which is more nuanced than suggested here.
Firms can improve their outcome if they coordinate. Such collusive agreements can be on price, but also on output, innovation or the allocation of different market segments. A cartel is the group of independent undertakings that join together to make a collusive agreements. Generally, collusive agreements benefit firms but hurt consumers and wider society and reduce total welfare. Article 101 of the Treaty on the Functioning of the European Union (TFEU) and Section 1 of the U.S. Sherman Act therefore generally prohibit any collusive agreement that has as its object or effect the restriction of competition (although some exemptions are possible). But despite the fact that collusive agreements are generally prohibited and prisoner’s dilemma dynamics incentivize firms to compete, illegal cartels do occur and can be successful (Levenstein and Suslow, 2006). Figure 1.1 shows the number of firms and amount of fines involved in recent EU-wide cartel cases (which only includes cartels that are actually detected and successfully prosecuted). Table 1.2 lists the top cases by their size of fines.

Figure 1.1: European Commission Cartel Statistics 2015-2019

Notes: 2019 figures are until 27 September. Fines are nominal and adjusted for possible court decisions. Source: European Commission.

But if cartels are illegal and supposedly unstable because of prisoner’s dilemma dynamics, how can they be explained and analyzed? Economic theory defines collusion as a situation in which firms use reward-punishment
1.1. Competition Policy

Table 1.2: Top 10 European Commission Cartel Cases by Fines

<table>
<thead>
<tr>
<th>Cartel Case</th>
<th>Fines in Euro</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Trucks</td>
<td>3,870,022,000</td>
<td>2016</td>
</tr>
<tr>
<td>2. TV and Computer Monitor Tubes</td>
<td>1,409,588,000</td>
<td>2012</td>
</tr>
<tr>
<td>3. Euro Interest Rate Derivatives</td>
<td>1,276,433,000</td>
<td>2013</td>
</tr>
<tr>
<td>4. Carglass</td>
<td>1,185,500,000</td>
<td>2008</td>
</tr>
<tr>
<td>5. Foreign Exchange Rates (Forex)</td>
<td>1,068,879,000</td>
<td>2019</td>
</tr>
<tr>
<td>7. Elevators and Escalators</td>
<td>832,422,250</td>
<td>2007</td>
</tr>
<tr>
<td>8. Vitamins</td>
<td>790,515,000</td>
<td>2001</td>
</tr>
<tr>
<td>9. Airfreight</td>
<td>785,345,000</td>
<td>2010</td>
</tr>
<tr>
<td>10. Yen Interest Rate Derivatives</td>
<td>669,719,000</td>
<td>2013</td>
</tr>
</tbody>
</table>

Notes: Figures are until 27 September 2019. Fines are nominal and adjusted for possible court decisions. Source: European Commission.

strategies to support an outcome above the competitive level (Harrington, 2019). These reward-punishment strategies can be theorized and analyzed. For instance, in case of Table 1.1 one possible reward-punishment strategy is that firms agree to set a high price as long as the other firm does, but set a low price forever after whenever one has deviated. If firms stick to the agreement, they will receive a payoff of 10 forever (reward), but if they deviate from the collusive agreement they will receive a one-off payoff of 15, but a lower payoff of 5 forever after (punishment). The payoffs in each of these two cases are shown in Table 1.3. As long as firms put enough weight on their future payoffs, they will prefer to stick to the collusive agreement—despite the per-period prisoner’s dilemma dynamics and the absence of a legally binding agreement.²

Note that the economic definition of collusion is silent on whether collusion is achieved through explicit communication or through some tacit or implicit understanding. Legally however, this distinction does matter. Generally speaking, explicit collusion is illegal while tacit collusion is not.

²Ivaldi, Jullien, Rey, Seabright and Tirole (2007) provide an overview on how this approach is used to analyse theoretically how market characteristics affect the stability of collusion.
Chapter 1. Introduction

Table 1.3: Payoffs Under a Possible Reward-Punishment Strategy

<table>
<thead>
<tr>
<th></th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
<th>Period 4</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Collusion</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>...</td>
</tr>
<tr>
<td>Deviation</td>
<td>15</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>...</td>
</tr>
</tbody>
</table>

Notes: Payoffs in case of collusion and deviation when firms agree to collude on a high price as long as the other firm does, but revert to a low price forever otherwise. Figures relate to the illustration in Table 1.1.

The main reason is that proving the existence of a collusion in the absence of explicit communication or signalling is very difficult. Moreover, it may not even be reasonable to mandate firms to compete when there is a profitable tacit understanding not to. As argued by U.S. Judge Stephen Breyer,

“[T]hat is not because [tacit collusion] is desirable (it is not), but because it is close to impossible to devise a judicially enforceable remedy for “interdependent” pricing. How does one order a firm to set its prices without regard to the likely reactions of its competitors?”

Hence, it is the act of communication or signalling with the object or effect of establishing a collusive agreement that is generally found to be illegal—not so much any collusive agreement itself. This distinction between the economic and legal definition of collusion is generally not considered a major practical concern, given the presumption that tacit collusion is inherently difficult to achieve and maintain. A textbook treatment on competition policy and collusion is provided by Kaplow (2013) and Harrington (2017).

Chapter 2 in this dissertation looks at the question whether self-learning pricing algorithms can learn to adopt reward-punishment strategies that support a collusive strategy, even in the absence of any illegal communication or explicit instructions to collude. Chapter 3 looks at collusion in the

\[^3\] Clamp-all Corporation v. Cast Iron Soil Pipe Institute, et al., 851 F.2d 478, 484 (1st Cir. 1988).
setting of financial benchmark rates—which is different from conventional cartels because of varying and often opposing interests. Note that of the top 10 cases in Table 1.2, three cases actually relate to financial benchmark rates: Euro Interest Rate Derivatives, Forex and Yen Interest Rate Derivatives. Finally, Chapter 4 revisits classic cartel theory to show how comparative statics on cartel stability change when you assume that firms require a margin before colluding.

1.1.2 Abuse of Dominance

A second area of competition policy looks at the unilateral abuse of market power by a dominant firm. Whenever a firm has substantial market power, it may by itself act in ways that restrict the process or outcome of competition. Generally, such types of abuse can be divided into exclusionary and exploitative abuses. Although the essays in this dissertation do not have a direct connection to abuse of dominance, it is an important area within competition policy. I discuss the two different types of abuse below.

1.1.2.1 Exclusionary Abuses

Under exclusionary abuses, a dominant firm imposes conditions on customers or suppliers that are aimed at deterring entry or forcing exit of competing firms. These can be pricing practices such as below-cost pricing or certain forms of price discrimination, but also non-pricing practices such as exclusive dealing agreements, tying or bundling, incompatibility choices or a refusal to deal. A recent textbook specifically on exclusionary abuse is provided by Fumagalli, Motta and Calcagno (2018).

Exclusionary abuses of dominance are in principle prohibited under Article 102 TFEU and Section 2 of the U.S. Sherman Act. However, many cases of apparent exclusionary conduct may also have one or more pro-competitive justifications. For instance, exclusive dealing agreements may incentivize firms to make necessary relationship-specific investments. The possibility of pro-competitive justifications for apparent exclusionary conduct means that competition authorities and courts often have to rely on
detailed and contentious effects-based analyses in their judgement.

Recent prominent cases of exclusionary abuse are the three Google cases by the European Commission. The Commission concluded in June 2017 that Google abused its online search dominance to exclude competitors in the market for shopping comparison websites, in July 2018 that it abused its dominance as a mobile operating system to protect its internet search market and in March 2019 that it used its online advertising dominance to exclude competing advertising services (European Commission, 2017; 2018; 2019). The Commission fined Google 2.42 billion, 4.34 billion and 1.49 billion euro for these case. All cases are currently under appeal in front of the courts.

1.1.2.2 Exploitative Abuses

Under exploitative abuses, a dominant firm does not aim to restrict the process of competition, but instead imposes conditions on customers or suppliers that directly extract a surplus from the transaction excessively higher than what the firm would receive under competitive conditions. These can relate to price, but also to non-price aspects of a product or service. For example, the German competition authority is currently prosecuting Facebook on the allegation that it exploits users by excessively infringing on their privacy (Bundeskartellamt, 2019).

The main challenge in any prosecution of exploitative abuse is to establish that the conduct is excessive relative to both the relevant costs and some reasonable competitive benchmark—both of which are generally contentious. Additionally, the prosecution of exploitative abuse implies an expectation that firms behave below their profit-maximizing level, which risks changing competition authorities into regulatory agencies. Although the EU does have an explicit provision against exploitative abuse in Article 102 TFEU, this provision is rarely invoked. The U.S. does not have provisions against exploitative abuse under its competition laws. Instead, there is much more reliance on market dynamics and entry to solve apparent cases of exploitative abuse. Larouche and Schinkel (2013) provide a more general discussion on the difference in the treatment of dominant firms in
the U.S. and the EU. The textbook by O’Donoghue and Padilla (2013) on the law and economics Article 102 TFEU includes a critical discussion on excessive pricing and other cases of exploitative abuse.

1.1.3 Merger Control

From a competition perspective, mergers and acquisitions involve a trade-off. On the one hand, mergers and acquisitions may enable synergies and efficiencies that improve the product or service offerings of firms. On the other hand, they may also lead to adverse unilateral or coordinated effects: unilateral effects occur when a merger increases the possibility and incentive of the firm to abuse a dominant market position; and coordinated effects occur when the reduction in the amount of firms active in the market enables collusive behavior between the remaining firms. Additionally, mergers may also affect other public policy objectives not directly related to competition policy, such as national security, industrial policy or employment.

Although not always mutually exclusive, mergers can generally be classified as horizontal, vertical or conglomerate mergers: horizontal mergers involve the combination of competing firms that produce substitute goods; vertical mergers involve the combination of firms that operate at different levels in the supply chain; and conglomerate mergers involve firms that operate in different markets but generally produce complementary goods. As a general proposition, vertical and conglomerate mergers involve less concerns to competition, because they generally take away the incentive of previously separate firms to each charge a markup over their separate products—known as double marginalization. However, adverse unilateral and coordinated effects may occur in each type of merger.

In the EU, the 2004 Merger Regulation prescribes that mergers involving a certain turnover and a significant EU dimension are notified to the European Commission. If an initial Phase I investigation of at most 25 working days gives an indication of possible competition concerns, the Commission may initiate an in-depth Phase II investigation. A Phase II investigation generally lasts up to 90 working days. During these inves-
tigations, the Commission may ask the merging parties for remedies or even prohibit the merger. In case of a prohibition, the merging parties can challenge the prohibition in front of the courts.

In the U.S., the 1976 Hart-Scott-Rodino Premerger Notification Act similarly requires firms of a certain size to notify their intended merger to the competition authorities, which in the U.S. are the Federal Trade Commission and the Department of Justice. This notification allows the competition authorities to decide whether to formally investigate a merger or not, possibly asking the merging parties for remedies or arguing for a prohibition in front of the courts.

Although merger investigations are a large part of the work of competition authorities, only very few mergers are actually prohibited. For instance, Figure 1.2 shows that from the approximately 375 yearly mergers notified to the Commission in the past five years, only a few dozen have been subjected to a Phase II investigation and only six have been formally prohibited—listed in Table 1.4. However, this does not account for the fact that the possibility of prohibition is likely to have deterred firms from even attempting certain mergers in the first place. Additionally, the vast majority of the Phase II merger investigations are cleared subject to remedies aimed at reducing the negative competitive effects of mergers (for instance by mandating divestitures or certain commitments to access) and some mergers have been revoked prior to a decision.

Table 1.4: Merger and Acquisition Prohibitions by European Commission

<table>
<thead>
<tr>
<th>Case</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hutchison – Telefónica UK</td>
<td>2016</td>
</tr>
<tr>
<td>Deutsche Börse – London Stock Exchange</td>
<td>2017</td>
</tr>
<tr>
<td>HeidelbergCement and Schwenk – Cemex Croatia</td>
<td>2017</td>
</tr>
<tr>
<td>Siemens – Alstom</td>
<td>2019</td>
</tr>
<tr>
<td>Wieland – Aurubis Rolled Products and Schwermetall</td>
<td>2019</td>
</tr>
<tr>
<td>Tata Steel – ThyssenKrupp</td>
<td>2019</td>
</tr>
</tbody>
</table>

Notes: Includes all Commission prohibitions between 1 January 2015 and 31 December 2019. Source: European Commission.
Notes: Source: European Commission.

Mergers can be evaluated from both an ex ante or an ex post perspective. From an ex ante perspective, mergers can be evaluated based on market structure characteristics (such as amount of firms, concentration levels and entry conditions) and their presumed predictive power on competition. Additionally, in case of potentially anti-competitive mergers, competition authorities often use so-called merger simulations, in which a theorized model of competitive interaction is formulated and estimated using sales data, with the aim of predicting the counterfactual outcome under which the merger would have been approved (Davis and Garcés, 2010). From an ex post perspective, the actual effects of past mergers can in principle be estimated based on observed market developments and an estimation of the counterfactual of a prohibition.

It is generally difficult however to establish whether competition authorities have struck the right balance in prohibiting anti-competitive mergers and clearing pro-competitive mergers. For instance, ex ante merger analyses are often uncertain and contentious and ex post analyses remain relatively limited because of their complexity or limited data availability—in addition to the fact that competition authorities may not have sufficient resources (or incentives) for a continuous ex post review their own decisions. In a European Commission commissioned review of ex post studies
on EU mergers, Ormosi, Mariuzzo, Havell, Fletcher and Lyons (2015) do generally find moderate price increases, although they emphasize the non-random sample selection and other limitations. The key reference on U.S. merger analyses is Kwoka (2015), who concludes based on several case studies that U.S. merger control has generally been too lenient.

Chapter 5 of this dissertation reviews the use of event studies as an alternative to acquire empirical insights into the competitive effects of mergers. It also provides a novel application to argue that U.S. merger control may indeed have been too lenient. Chapter 4 also has some relation to merger control. It shows theoretically how the presence of merger efficiencies may increase the risk of coordinated effects.

1.1.4 State Aid

A final area of competition policy relates to public restrictions of competition. Governments themselves may also restrict the process of competition, for instance through licensing rules or the provision of state aid in the form of subsidies or tax exceptions to specific firms. Such restrictions can inhibit competition on the merit by providing certain firms with an unfair competitive advantage, at the expense of other firms and, ultimately, consumers.

In response, some jurisdictions have policies in place that scrutinize public restrictions of competition by different levels of government. In particular, Articles 106-109 TFEU prohibit EU Member States or local governments from providing ineffective or disproportionately distortionary state aid to firms. Whenever Member States or local governments want to implement public interventions in the process of competition, they may therefore be required to provide economic justifications towards the European Commission and acquire approval.

State aid is sometimes considered to be outside the core of competition policy, because it concerns an intergovernmental policy rather than public policies towards firms. Additionally, different jurisdictions treat state aid concerns very differently. State aid also does not have a direct connection to this dissertation, and is therefore not discussed any further. A key reference on the economics and policy of EU state aid is Quigley (2015).
1.2 Contemporary Topics

Several topics are dominating current competition policy debates. Below I discuss what I think are the three main topics: (i) digitalization and competition, (ii) the increase of markups, concentration and market power and (iii) the increase of public interest advocacy within competition policy. The first two of these topics are in some way related to some of the essays in this dissertation. The third is not directly related and is therefore only briefly discussed.

1.2.1 Digitalization and Competition

Recently, several prominent reports have been written around the topic of digitalization and competition policy. These include in particular the January 2019 special advisors’ report *Competition Policy for the Digital Era* for the European Commission (Cremer, de Montjoye and Schweitzer, 2019), the March 2019 competition expert panel report *Unlocking Digital Competition* for the British government (Digital Competition Expert Panel, 2019) and the September 2019 *Stigler Committee on Digital Platforms* by the Stigler Center at the University of Chicago (Stigler Committee on Digital Platforms, 2019). Broadly, the concerns to competition as a consequence of digitalization relate to the market power of ‘Big Tech’ firms, the role of data as a strategic input and the rise of algorithmic decision-making.

1.2.1.1 The Market Power of ‘Big Tech’

On the market power by digital platforms and Big Tech firms such as Apple, Amazon, Facebook, Google and others, the prominent concern is that network effects lead to a self-reinforcing ‘winner-takes-all’ or ‘winner-takes-most’ dynamic, in which a single firm becomes the preferred option for all or most consumers. Rather than competition *in* the market, there is then competition *for* the market, in which innovative or first-mover firms are rewarded with a dominant market position.

Competition for the market may not be a concern, provided potential
Chapter 1. Introduction

competitor entry or other disruption remains as a competitive threat. However, the fear is that once dominant, these firms can reinforce their own dominance. For instance, they may exclude potential competitors or preemptively acquire innovative start-ups before they become a serious threat (often referred to as ‘killer acquisitions’).\(^4\) Additionally, consumers may be discouraged from properly considering potential alternatives because of the exploitation of behavioral biases, such as default or immediacy bias. Once dominant, firms may be able to abuse their dominance by increasing the price of their services, self-preferencing their own products at the expense of competitors or monetizing on an excessive deterioration of privacy or quality dimensions.

There is a broad range of possible measures that have been proposed in response to these Big Tech concerns—for instance by the various expert reports. Without discussing their relative merits, these include stricter enforcement of existing competition law, more attention to non-price exploitation, stricter merger control in case of potential competitors, a balance of harms test instead of a balance of probably test in merger cases, reversing the burden of proof in mergers or abuse cases, interim measures in case of allegation of exclusion or exploitation, a digital regulator, data portability and interoperability between competing firms, a code of conduct against self-preferencing behavior, more transparency in case of business disputes, regulation in response to the abuse of behavioral biases, the facilitation of micro-payments in return for user-generated data, stricter privacy laws and a forced separation of different activities within firms. It will be interesting to see exactly which proposals will be adopted by which jurisdictions. This dissertation does not have a direct connection to these questions around Big Tech, however.

\(^4\)The key references on this concept of killer acquisitions is Cunningham, Ederer and Ma (2020), which provides a theoretical model and empirical results in the context of pharmaceutical markets. Cabral (forthcoming) provides a critical discussion on the concerns of killer acquisitions in the context of digital markets.
1.2. Contemporary Topics

1.2.1.2 Algorithmic Decision-Making

On algorithmic decision-making, the main concern is that algorithms may be used to facilitate or initiate collusion between competing firms. In Klein (2020), I make the distinction between three ways in which pricing algorithms may facilitate collusion. First, pricing algorithms can be used to facilitate tacit collusion by humans by automating a rapid response in case of a competitor price reduction. Second, pricing algorithms can be very effective in the implementation of various collusive strategies by firms that are explicitly colluding. And third, self-learning pricing algorithms may be able to learn by themselves—through trial-and-error—to adopt collusive pricing strategies. Chapter 2 in this dissertation relates specifically to this last concern, by looking at the collusive capacity of self-learning algorithms in a simulated environment.

Other concerns related to algorithmic decision-making relate to non-price algorithms (such as search or ranking algorithms) and the use of algorithms to price discriminate between consumers. For more on the concerns around algorithmic decision-making, see in particular report by the British competition authority (CMA, 2018) and the joint report by the French and German competition authorities (Autorité de la Concurrence and Bundeskartellamt, 2019) on algorithms and competition.

1.2.2 Markups, Concentration and Market Power

Several recent and prominent studies have documented an increase in market power, as measured by markups (price over marginal cost) and industry concentration. For instance, using micro-panel data on accounting profits and structural econometric modelling, De Loecker, Eeckhout and Unger (forthcoming) estimate that the average markup of U.S. publicly-traded firms increased from slightly above 20 percent in the three decades prior to 1980 to around 61 percent by 2016. They also find that this increase is driven by firms in the top of the markup distribution, especially by those in the top 10 percent. Although they also find an increase in fixed costs, the markup increase is in excess of these costs. They therefore ascribe the
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increase in markups to an increase in market power. De Loecker and Eeckhout (2018) find similar results for Europe, but not for emerging countries in Latin America and China.

Autor, Dorn, Katz, Patterson and Van Reenen (forthcoming) show that many U.S. industries have also become more concentrated. They argue that the dominant factor is the rise of so-called “superstar firms”, in which superior firms with lower costs or better innovations have gained market share by outperforming their competitors. Although increased concentration and market power may therefore be the result of virtuous competition on the merit, Autor et al. do point out that this concentration may lead to a competition concern when dominant firms are able to abuse their dominance going forward. Gutiérrez and Philippon (2018) and Philippon (2019) similarly document an increase in industry concentration. They suggest however that an insufficient deterrence of anticompetitive mergers in the U.S. is one of the main drivers behind the observed concentration.

Chapter 5 in this paper relates to the suggestion by Gutiérrez and Philippon that U.S. merger control has been too lenient. Using stock market data and a novel dataset from Finance, I document how merger announcements affect competitor stock market performance. I then use this information to make inferences on the anticipated competitive effects of mergers and their relation to market power concerns.

1.2.3 Public Interest Advocacy

A third dominant competition policy debate concerns an increased advocacy of other interests than competition and consumer welfare in competition policy. Such ‘public interest’ advocacy can take place both in the defense (for instance by firms attempting to justify collusive behavior as a means to achieve public interest objectives such as sustainability or animal welfare) and in the offence (for instance by social advocacy groups calling for a more interventionist competition policy that is not just based on consumer welfare). Public interest advocacy in the offence is often referred to as ‘neo-Brandeisian’—a reference to early twentieth century U.S. Supreme Court Justice Louis Brandeis, who is recognized for his progressive social
1.3. Methodology

Broadly, the different methodologies used in this dissertation can be classified as oligopoly theory, machine learning and econometrics. I briefly review each of these methodologies in turn.

1.3.1 Oligopoly Theory

Competition concerns often arise in oligopolistic markets, which are markets with a limited number of firms. Such markets are characterized by strategic interdependencies, in which the behavior of any one firm depends on the behavior of other firms.

The theoretical analysis of oligopolistic markets in competition economics is often done using game theory, which involves the mathematical modelling of optimal behavior under strategic interdependencies. Key within game theory is the concept of a Nash equilibrium, which is an outcome in which no decision-maker can do better given what the others are doing. Theoretical predictions of behavior are then derived by formu-
lating a game-theoretic model and deriving its Nash equilibria (or some refinement thereof). Oligopoly theory is the application of game theory to oligopoly settings.

### 1.3.1.1 Cournot Competition

Two common specifications within oligopoly theory are Cournot and Bertrand competition: under *Cournot competition*, competing firms have to decide on the optimal quantity of products to produce, letting their price be determined by the market; while under *Bertrand competition*, competing firms instead decide on the optimal price, letting their quantities be determined by the market. Chapter 4 uses both these specifications.

To introduce oligopoly theory in general and Cournot competition more specifically, assume that there are two firms \( i \in \{1,2\} \) that each set a quantity \( q_i \geq 0 \). Market price \( p \geq 0 \) is determined based on the following linear function:

\[
p = a - b(q_1 + q_2),
\]

where \( a \) and \( b \) are the intercept and slope parameters. Profit \( \pi_i \) of each firm \( i \) is determined as the market price minus marginal cost \( c \), times quantity \( q_i \), so \( \pi_i = (p - c)q_i \). Plugging in for market price \( p \) provides

\[
\pi_i(q_i, q_j) = (a - b(q_i + q_j) - c)q_i,
\]

where \( j \in \{1,2\} \setminus i \). Provided certain conditions on \( a, b \) and \( c \), this function has a unique maximum \( q_i^* \) as a function of \( q_j \). This maximum is found by setting the first-order derivative of \( \pi_i \) with respect to \( q_i \) equal to zero and solving for \( q_i \), so

\[
\frac{\partial \pi_i(q_i, q_j)}{\partial q_i} = a - 2bq_i - bq_j - c = 0 \quad \Rightarrow \quad q_i^*(q_j) = \frac{a - bq_j - c}{2b}.
\]

Setting the first-order derivative equal to zero is known as the first-order condition, necessary for finding a maximum. The second order condition
(assumed to hold here) is that the second-order derivative is negative, so as to ensure that the solution is a maximum as opposed to a minimum or saddle point.

The solution $q_i^*(q_j)$ is known as the \textit{best-response function} of firm $i$: it shows what is the optimal quantity of firm $i$ given the quantity of firm $j$. Finally, the Nash equilibrium is derived by plugging the best-response function for firm $j$ into that of firm $i$ and solving for $q_i$, so

$$q_i = \left( a - b \left( \frac{a - bq_i - c}{2b} \right) - c \right) \frac{1}{2b} \Rightarrow q_i^* = \frac{a - c}{3b}, \quad (1.4)$$

which is then the theoretical prediction of firm behavior as a function of the different parameters. It shows that the optimal quantity increases in demand intercept $a$, but decreases in demand slope $b$ and marginal cost $c$.

This stylized illustration can be readily extended to account for more firms, product differentiation, capacity constraints, demand fluctuations or consumer search costs, among many other things. An extension of this model is used in Chapter 4.

\subsection*{1.3.1.2 The Use of Oligopoly Theory}

Oligopoly theory does not provide universal conclusions. Instead, its theoretical predictions very much depend on the underlying modelling assumptions and different settings require different modelling assumptions. This generally includes assumptions on things like the objective function, choice variables, information availability and game play.

For instance, in Chapter 3 of this dissertation we develop a new game-theoretic model specific to the process of benchmark rate setting by banks. The choice variables in this case are not price or quantity, but an adjustment of the exposure by banks and a manipulation of transactions that are eligible for the benchmark rate. And in Chapter 2, I assess the capacity of autonomous pricing algorithms to collude by simulating price competition based on the oligopoly model of sequential price competition of Maskin and Tirole (1988) in which firms take turns setting their price—which I argue to be suitable for the specific context of pricing algorithms.
The main contribution of oligopoly theory (and mathematical modelling of social relations more generally) is that it allows for hypothetical experimentation. Based on the underlying assumptions, a theoretical prediction can be made on the outcome in alternative, unobserved worlds by changing the underlying parameters. This is for instance how competition authorities try to predict the consequences of a merger on prices and output. Furthermore, mathematical modelling (ideally) makes the underlying assumptions of a theory explicit and forces internal consistency.

There are two main points of attention when using mathematical modelling in economic theory. First, it may make research a lot less accessible to a general audience. Non-technical translations of theoretical work may therefore be required to get a general audience engaged and subject the research to more outside scrutiny. Second, mathematical modelling may give a false sense of certainty. A model is only as strong as its underlying assumptions, and it may depend very much on the context whether the assumptions are reasonable or necessary for the results of the model. The legitimacy of theoretical modelling therefore requires sufficient scrutiny on whether the underlying assumptions are reasonable or necessary, ideally combined with empirically testing the theoretical predictions of the model.

1.3.1.3 Unrealistic Assumptions in Economic Theory

Economic theory (including oligopoly theory) builds on assumptions to simplify the analysis and achieve clarity on the underlying mechanisms. At times, these assumptions may appear highly unrealistic—for instance when they involve fully rational economic actors that are capable of calculating their optimal behavior using complex differential calculus. Chapter 3 in this dissertation is an example of this. It derives profit-maximizing behavior of banks in an environment of financial benchmark rate setting. Although the environment is already stylized, finding the optimal behavior is still complicated. Assuming that traders within banks are explicitly deriving their optimal behavior the actual benchmark rate setting may therefore not be fully realistic. To what degree is a lack of realism in the assumptions underlying economic theory problematic?
1.3. Methodology

An influential claim by Nobel-laureate Milton Friedman (1953) is that the degree of realism in theoretical assumptions is irrelevant, as long as the theory provides correct predictions. As he puts it,

“[t]ruly important and significant hypotheses will be found to have “assumptions” that are wildly inaccurate descriptive representations of reality, and, in general, the more significant the theory, the more unrealistic the assumptions (in this sense). The reason is simple. A hypothesis is important if it “explains” much by little, that is, if it abstracts the common and crucial elements from the mass of complex and detailed circumstances surrounding the phenomena to be explained and permits valid predictions on the basis of them alone.” (p. 14)

In other words, the lack of realism in the assumptions underlying economic theory may actually allow for the clear identification of relevant mechanisms, as long as it still provides valid predictions.

The limitation of this justification for unrealistic assumptions, however, is that it requires the clear empirical verification of theoretical hypotheses, which may not always be possible. Additionally, as argued by Rodrik (2015, pp. 25-29), for a model to be useful the assumptions that are critical for the derivation of the conclusions still have to track reality sufficiently closely—in other words, they cannot be considered too strong. Put differently, if the modification of a certain assumption in an arguably more realistic direction would lead to substantially different theoretical conclusions, the reliability of the theoretical model can be questioned—irrespective of its predictive accuracy.

The potentially strong assumptions underlying the benchmark rate collusion model in Chapter 3 can be justified based on these arguments. First, we discuss how its theoretical conclusions track the observed behavior in financial benchmark rates. Second, we discuss how we believe that the critical assumptions are not very strong. And assumptions that are strong but are still used for simplification purposes, we argue are unlikely to be critical for the conclusions derived.
1.3.2 Machine Learning

A second type of methodology used in this dissertation comes from machine learning. Machine learning is a set of methods that aim to automatically detect patterns in data, with the goal of generalizing to new observations. Although not mutually exclusive, machine learning is generally divided into three broad categories: supervised learning, unsupervised learning and reinforcement learning. Chapter 2 in this dissertation uses reinforcement learning techniques to see whether self-learning pricing algorithms can learn collusive strategies. For completeness, I briefly review each category of machine learning below. A stylized schematic overview of each category of machine learning is provided in Figure 1.3. An accessible overview of machine learning practice is provided by Domingos (2012) and McKinsey & Company (2018).

1.3.2.1 Supervised, Unsupervised and Reinforcement Learning

Supervised learning uses training data to learn a function that predicts an output variable given a set of input variables. The training data consists of examples that have been provided by an external supervisor. The objective of the supervised learning algorithm is generally to maximize the expected accuracy of out-of-sample predictions, often in a ‘black-box’ optimization approach. Common supervised learning methods include linear and logistic regression, regularized estimation, non-parametric regression (such as nearest-neighbor identification), additive models, decision trees, random forests, gradient boosting, support vector machines and neural networks.

Unsupervised learning generally aims to identify structure hidden in a dataset. Unlike with supervised learning, the training data in case of unsupervised learning only includes the input variables and does not include output labels. A main approach within unsupervised learning is cluster analysis, which aims to identify commonalities in data by categorizing the observations into previously unclassified groups with shared characteristics. The established textbook on supervised and unsupervised machine learning is Hastie, Tibshirani and Friedman (2009).
Finally, reinforcement learning is learning how to map situations to actions so as to maximize some numerical reward signal. It generally does not learn for existing data, but generates its own training data through autonomous trial-and-error interaction with its environment. The most interesting and challenging reinforcement learning cases are those in which actions do not only affect the immediate reward signal, but also the state transition and hence possible subsequent rewards. As such, reinforcement learning often has to take into account the delayed rewards as a consequence of its current actions. In choosing its action, the reinforcement learning algorithm also has to make a continuous trade-off between exploration and exploitation: under exploration it experiments with different actions to see what happens; while under exploitation it takes the perceived optimal action. This trade-off is generally exogenously programmed into the algorithm. The established textbook on reinforcement learning is Sutton and Barto (2018).

**Figure 1.3: Schematic Overview of Machine Learning Categories**

1.3.2.2 Q-Learning

Chapter 2 of this dissertation develops an adaptation of Q-learning, which is a simple and well-established type of reinforcement learning algorithm. Intuitively, Q-learning works as follows: after choosing an action given the current state, it observes an intermediate reward and new state and updates
an estimate of the highest possible sum of future rewards following its action. This allows Q-learning to become better informed over time about the long-run consequences of its actions. In choosing its action, Q-learning makes a probabilistic trade-off between exploration and exploitation.

Like any reinforcement learning algorithm, Q-learning consists of two interacting modules: a learning module that processes the observed information and an action-selection module that decides which action to take in which state of the world, taking into account the trade-off between exploration and exploitation.

In the learning module, Q-learning estimates a Q-function $Q(a, s)$, which maps action $a \in A$ into its estimated optimal long-run value given current state $s \in S$. Assuming a discrete action and state set, $Q$ is a $|A| \times |S|$ matrix. After choosing action $a$ in state $s$, it observes new state $s'$ (which may be a stochastic function of action $a$) and intermediate reward signal $R(a, s, s')$. The algorithm then updates entry $Q(a, s)$ according to the following recursive relationship

$$Q(a, s) \leftarrow (1 - \alpha) \cdot Q(a, s) + \alpha \cdot \left( R(a, s, s') + \delta \max_{a'} Q(a', s') \right)$$

where $\alpha \in (0, 1)$ is a stepsize parameter that determines the weight ascribed to the observed estimate relative to its old value and $\delta \in [0, 1)$ a discount factor.

In the action-selection module, the algorithm makes a probabilistic trade-off between exploration and exploitation. Often, a straightforward procedure called $\epsilon$-greedy exploration is used: with probability $\epsilon \in [0, 1]$ it selects an action randomly (exploration) and with probability $1 - \epsilon$ it selects the currently perceived optimal action (exploitation).

Q-learning is guaranteed to converge to the strategy that maximizes the expected sum of discounted future reward signals, provided that the environment is stationary and all action-state pairs continue to be updated (Watkins and Dayan, 1992; Tsitsiklis, 1994). This puts mild conditions on stepsize parameter $\alpha$ and the rate of exploration $\epsilon$. Its relative simplicity and attractive theoretical properties mean that Q-learning underlies many...
of the more advanced reinforcement learning algorithms.

Chapter 2 in this dissertation adapts the basic Q-learning algorithm to a situation of sequential price competition, in which two competing firms learn their optimal pricing strategy at the same time. Because the environment is no longer stationary, convergence guarantees that exist for single-agent Q-learning no longer hold. In absence of theoretical guarantees I provide an empirical understanding through simulations instead. Q-learning is first proposed by Watkins (1989). A textbook treatment is provided by Sutton and Barto (2018, pp. 119-134).

1.3.3 Econometrics

Econometrics is a collection of statistical tools that can be used to summarize relations between economic variables and analyze the possible effects of changes in these variables, based on observed data. Theoretical econometrics aims to draw inferences on the theoretical properties of existing statistical tests and develop new tests that are valid or robust given certain data characteristics. Applied econometrics, on the other hand, applies these statistical tests to real-world data to test hypotheses based on economic theory. Chapter 5 in this dissertation is the result of an applied econometrics project.

1.3.3.1 Applied Econometrics and Causal Inference

A large part of applied econometrics focuses on causal inference: how does a change in one variable lead to a change in another variable? This approach uses statistical methods to separate causal relations from simple correlations, relying generally on observational data only. Common methods include ordinary least squares with control variables, instrumental variable regressions, difference-in-difference estimation, panel data regression and regression discontinuity design. A prominent and accessible introduction into these methods is provided by Angrist and Pischke (2009).

Chapter 5 in this dissertation combines applied econometrics with stock market data to see whether U.S. merger control may have been too lenient.
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More specifically, it uses event studies to estimate the abnormal stock market return of competitors as a consequence of merger announcements and uses this information to draw an inference on the anticipated causal effect of mergers on future competitiveness. The details on the specific methodology used are provided in Chapter 5. Although Chapter 3 on benchmark rate collusion is mostly based on oligopoly theory and simulations, it also includes a discussion on how econometric methods may be used to identify collusive benchmark rate fixing.

1.3.3.2 Econometrics and Machine Learning

Note that there is some overlap between econometrics and supervised machine learning. For instance, both methodologies generally use data to estimate the parameters of pre-specified models. Once estimated, these models can be used to produce out-of-sample predictions. The main distinction between econometrics and supervised machine learning is that econometrics has developed a lot of tools that focus specifically on causality. Supervised machine learning, on the other hand, is mostly focused on out-of-sample predictive accuracy, often in a ‘black-box’ manner and without much regard for how results may change if input variables are adjusted. An interesting discussion on this distinction and on the relevance of machine learning for economics and econometrics (and vice versa) is provided by Athey and Imbens (2019).

1.4 Outline

The remainder of this dissertation consists of four separate essays in competition economics. Below, a more detailed explanation of the motivation, approach and findings of each of the four essays is provided. Chapter 6 of this dissertation provides a final recapitulation of and reflection on each chapter.

Chapter 2. Autonomous Algorithmic Collusion: Q-Learning Under Sequential Competition The growing prominence of digitaliza-
tion, big data and artificial intelligence in commercial activities has given rise to several novel concerns. One such concern is that intelligent, self-learning pricing algorithms may work out how to ensure high prices (Ezrachi and Stucke, 2016; 2017). If so, the outcome may be collusive, but without any overt act of communication required to establish a competition law infringement (Harrington, 2019). Although this concern has received a lot of attention from policymakers and practitioners, the question whether autonomous algorithms can even learn to tacitly collude—and what practical limitations may be—is still open.

With this essay, I aim to show more formally whether and how autonomous algorithms could collude. By simulating the sequential price competition environment of Maskin and Tirole (1988), I show that Q-learning (a simple and well-established reinforcement learning algorithm) can indeed converge to strategies that sustain supra-competitive fixed-price equilibria—at least when the number of discrete prices is limited. When the number of discrete prices increases, Q-learning increasingly converges on profitable Edgeworth price cycles. This outcome is achieved without any communication or objectives other than own profit maximization.

I use the sequential move environment of Maskin and Tirole, as opposed to a more conventional simultaneous move environment, because it may be very unlikely that competing algorithms update their prices at exactly the same time (or, alternatively, that they are unaware of the current competitor price and hence act ‘as if’ prices are set simultaneously). A parallel project by Calvano, Calzolari, Denicolò and Pastorello (2019b) shows that Q-learning also learns collusive strategies in the more conventional simultaneous move framework, although this does require history-dependent strategies and much longer learning. By using the sequential move environment, I also establish that Q-learning can learn to approximate the Markov perfect equilibria theorized by Maskin and Tirole.

Although I show that autonomous algorithmic collusion is in principle possible, practical limitations remain: it requires many periods to learn, it assumes a stationary environment and it does not show a consistent convergence to mutually optimal behavior. However, these limitations do
not seem to be principled obstacles to autonomous algorithmic collusion in practice. For instance, Calvano et al. already argue that algorithms could be trained in an offline, simulated environment before being put into the real world. This greatly increases the scope for learning (although it does provide new challenges in defining the simulated environment and ensuring that independently trained algorithms can properly adapt to each other once put to use). Another solution would be to impose more structure on the learning algorithm, for instance by approximating some function of the environment using techniques from supervised machine learning. Additionally, firms may transfer learning from one product or period to a new product or period to guide or speed up learning—instead of starting each time from scratch, which is what I assume. Finally, extensions using much more state-of-the-art learning algorithms could be developed, and I discuss some of these.

Chapter 3. Collusive Benchmark Rate Fixing

On 29 October 2013, Rabobank received a fine of 774 million euro for the manipulation of Libor and Euribor. This was until then the largest fine ever within the Netherlands. It also led to the resignation of Rabobank Chairman Piet Moerland. But Rabobank was not the only bank: many of the largest global banks received fines and convictions in the wake of the Libor scandal and similar benchmark rate scandals. These scandals also led to a large-scale reform of prominent financial benchmarks.

The London Interbank Offered Rate (Libor) and the Euro Interbank Offered Rate (Euribor) are a collection of financial benchmarks that are supposed to reflect the borrowing capacity of major banks. They are key variables in portfolio and risk management decisions, as well as barometers on financial sector health. Moreover, between 370 and 400 trillion dollar worth of interest rate derivatives were estimated to directly derive their value from these rates (Financial Stability Board, 2014, p. 6). The problem with these financial benchmarks is that they are determined on the basis of contributions by a small set of large banks, who themselves trade in the financial products that are valued on these rates. The participating banks
therefore have an incentive to distort the benchmarks in a direction that is favorable to their financial interests. The majority of the manipulation cases that came to light focused on individuals who had fraudulently tried to direct the rate for their own gain. However, benchmark manipulation is a lot more effective when done cooperatively, and there have been several cartel proceedings.

In this essay, which is joint work with Nuria Boot and Maarten Pieter Schinkel, we develop a theory behind collusion on benchmark rates. Benchmark rate collusion is challenging due to varying and often opposing trading interests of the participating banks: sometimes banks may want the rate to go up, sometimes they want the rate to go down. Our theory is based on two reinforcing mechanisms. We define ‘front-running’ as information sharing that allows cartel members to partially adjust their exposure to the benchmark ahead of the market. To support a joint-profit maximizing rate, designated traders engage in costly ‘eligible transactions rigging’, in which they manipulate the transactions that are eligible for submission for the benchmark. We find that observed episodic recourse to independent behavior is part of a feasible continuous collusion equilibrium and that all panel members would want to participate in the scheme. Simulations show how collusion could be indicated by high rate volatility or the correlation between exposure adjustments and the new benchmark rate. Additionally, the simulations show how collusion could unintentionally be facilitated by the proposed reforms to broaden the class of transactions eligible for submission and to average over fewer submissions.

Our theory on benchmark rate collusion applies to both the original Libor and Euribor settings (in which manipulation is known to have taken place) and their post-reform setting. In the original settings, banks did not have to support their submissions with actual transactions. Eligible transactions rigging simply involved the misreporting of borrowing cost. Post-reform, banks need to support their submissions with eligible transactions. However, banks may still have the opportunity to support particular submissions by engaging in transaction manipulation. Moreover, they may have the incentive to do so, because the portfolio of financial prod-
ucts valued on these benchmarks is many times larger than the portfolio of interbank borrowing transactions that are eligible for submission. As long as commercial entities are both involved in the setting of financial benchmarks and trade in products that are valued on them, the vulnerability to manipulation may remain.

Chapter 4. Cartel Stability by a Margin  Firms may try to restrict market competition by colluding with each other. However, collusion is not straightforward. Collusive agreements are generally illegal, and economic incentives may have to be used to stabilize a collusive understanding.

Microeconomic theory provides two opposing mechanisms affecting the stability of collusion: on the one hand, collusion generally provides higher profit than competition, which incentivizes firms to continue to collude; on the other hand, deviating from a collusive agreement generally provides a one-off higher deviation profits. The relative profitability of these two mechanisms depends on the market structure as well as institutional settings, such as competition policy enforcement or leniency programs. Ever since Friedman (1971) and Abreu (1986, 1988), it is common in cartel theory to derive a critical discount factor above which firms are better off sticking to the collusive agreement than deviating from the agreement and receiving the one-off higher deviation profit, at the expense of long-run collusive profits. Any change in market structure characteristics that lowers the (binding) critical discount factor is then assumed to increase the scope for cartels in the market, in the sense that cartels are sustainable for a wider range of actual discount factors.

This essay, which is joint work with Maarten Pieter Schinkel, revisits this classic cartel stability theory by introducing the concept of a cartel margin. We show how comparative statics on the critical discount factor change when firms require a margin before colluding. Such a cartel margin can have different sources. It may be needed to compensate for moral disutility from participating in the illegal conspiracy, or to insure against uncertain events. Other reasons can be liabilities for fines, damages and reputation loss in the event of break-down. We show that the presence
of a cartel margin may provide new and unambiguous comparative statics for a wide class of market structure characteristics, for which we provide the conditions—including conditions on how the cartel margin itself may depend on the market structure characteristics.

Driving our result is the following: the presence of a cartel margin increases the relevance of gains from collusion relative to the gains from deviation in the comparative statics on the critical discount factor. When the cartel margin is sufficiently large, the effect on the gains from collusion has to dominate. Specifically, we find that lower marginal cost and less product differentiation generally increases the scope for collusion in the presence of a cartel margin. Implications for competition policy may include a focus in enforcement on standardized product and low input cost industries. Results also suggest that merger efficiencies may increase the risk of coordinated effects.

Chapter 5. Event Studies in Merger Analysis: Review and an Application Using U.S. TNIC Data  
Recently, a broad discussion has emerged on the observation of increased industry concentration, markups and market power (Autor, Dorn, Katz, Patterson and Van Reenen, forthcoming; Basu, 2019; De Loecker and Eeckhout, 2018; De Loecker, Eeckhout and Unger, forthcoming; Grullon, Larkin and Michaely, 2019; Syverson, 2019). One concern is that concentration and market power may have increased as a consequence of an insufficient deterrence of anti-competitive mergers, especially in the U.S. (Baker, 2019, p. 15; Gutiérrez and Philippon, 2018; Kwoka, 2015; Philippon, 2019; Shapiro, 2018; 2019; Wollmann, 2019). However, empirical evidence remains limited, mostly because of the limited data availability necessary for proper ex post merger reviews.

Event studies have been proposed as a simple alternative in the search for empirical insights into the competitive effects of mergers. By estimating the abnormal stock returns around an event date, event studies aim to identify the anticipated effect of this event on future firm performance. In the context of mergers, existing event studies generally use as identifying assumption that an abnormal increase (decrease) in competitor stock
price following a merger announcement indicates that financial markets anticipate the merger to be anti-competitive (pro-competitive). Although grounded in microeconomic theory, this inverse relationship between competitor stock price and consumer welfare is not guaranteed, as it may be weakened by the presence of other mechanisms and stock market noise.

This essay includes a general discussion on the use of event studies in merger analysis. I identify that existing studies generally suffer from three limitations. First, the identifying assumption that abnormal competitor returns and consumer welfare are inversely related may fail to hold for various reasons and would have to be tested if used. Second, event studies in merger analysis require the reliable identification of competitors, which is often not obvious. And third, they involve the identification of small effects in very noisy data. This necessitates a sufficiently large dataset.

I am able to overcome these challenges by using a novel application of Hoberg-Phillips (2010, 2016) Text-Based Network Industry Classification (TNIC) data. Hoberg-Phillips TNIC data derives a yearly pair-wise similarity proxy for all publicly traded U.S. firms, based on their formal business description. These scores allow me to readily identify likely competitors to 1,751 of the largest U.S. mergers and acquisitions involving a publicly-traded target, going back to 1997. I document that likely competitors experience on average a positive and statistically significant abnormal return. I also find that abnormal returns show a positive association with a TNIC-based Herfindahl-Hirschman Index (HHI). Because HHI serves as an indicator of market power concerns, this association suggests that results are at least in part driven by an anticipation of anti-competitive market power effects, and hence an insufficient deterrence of anti-competitive mergers.

There are several limitations to my approach. First, the Hoberg-Phillips TNIC data is only a proxy, and only includes publicly-traded U.S. firms. Although any erroneous competitor identification will generally bias results towards zero (making the estimates more conservative), the sample selection means that results are not necessarily externally valid in case of privately-owned or foreign-traded target firms. Second, by looking only at
the average effects, it is not possible to identify exactly which mergers have been anticompetitive. This essay only argues that U.S. merger control has been too lenient on average. Stock market data is noisy and market expectations may be incorrect in individual cases. This means that case-specific ex ante or ex post merger reviews remain necessary—perhaps even more so in light of the growing concern on market concentration and the concern that U.S. merger review has indeed been too lenient.
Chapter 2

Autonomous Algorithmic Collusion: Q-Learning Under Sequential Competition*

“It’s true that the idea of automated systems getting together and reaching a meeting of minds is still science fiction. [...] But we do need to keep a close eye on how algorithms are developing. [...] So that when science fiction becomes reality, we’re ready to deal with it.”
– EU Competition Commissioner Margrethe Vestager (16 March 2017)

The growing prominence of digitalization, big data and artificial intelligence in commercial activities has given rise to several novel concerns. One such concern is that intelligent, self-learning pricing algorithms may work out how to ensure high prices (Ezrachi and Stucke, 2016; 2017). This would be akin to collusion, but without any overt act of communication required

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to establish a competition law infringement (Gal, 2019; Harrington, 2019). The debate has received extensive press coverage\(^1\) and increasing interest from competition authorities\(^2\), the OECD\(^3\) and economic consultancy\(^4\). However, the concern on tacit algorithmic collusion is for the most part still based on intuition and many remain skeptical that this is a problem.

To show more formally whether and how autonomous algorithms could collude, we discuss the collusive capacity of Q-learning—a simple and canonical reinforcement learning algorithm. We show in a simulated environment of sequential competition that Q-learning can indeed converge to strategies that sustain supra-competitive fixed-price equilibria—at least when the number of discrete prices is limited. When the number of discrete prices increases, Q-learning increasingly converges on profitable Edgeworth price cycles, in which periodic upward price jumps reset a gradual price decline.\(^5\) Coordination occurs even though the algorithm does not communicate and is only instructed to maximize its own profits. We show that results are generally robust to changes to the learning parameters and timing and discuss how more advanced algorithms could speed up learning or deal with less stylized environments.


\(^2\)See for instance speeches by EU Commissioner Vestager (“Algorithms and Competition”, March 2017) and Acting FTC Chairman Ohlhausen (“Should We Fear The Things That Go Beep In the Night?”, May 2017), as well as a report by the British competition authority entitled “Pricing Algorithms: Economic Working Paper on the Use of Algorithms to Facilitate Collusion and Personalised Pricing” (October 2018) and a joint report by the French and German competition authorities entitled “Algorithms and Competition” (November 2019).

\(^3\)OECD (2017)

\(^4\)Including Oxera (2017, 2018) and RBB Economics (2018).

\(^5\)Such asymmetric cycles are similarly observed in other markets often suspected of tacit collusion—most prominently gasoline markets (Noel, 2011; Eckert, 2013; Byrne and de Roos, 2019).
Self-learning algorithms are programmed in discrete time. It may be very unlikely, however, that competing algorithms update their prices at exactly the same time (or, alternatively, that they are unaware of the current competitor prices and hence act ‘as if’ prices are set simultaneously). We therefore deviate from the conventional infinitely repeated simultaneous move framework and use the sequential move framework of Maskin and Tirole (1988) instead, in which firms take turns setting prices. Additionally, firms are restricted to Markov strategies: pricing is only conditional on the current competitor price and not any pricing history. This framework has many Nash equilibria, including pricing at marginal cost. As a refinement, Maskin and Tirole define the Markov perfect equilibrium (MPE): any Nash equilibrium with Markov strategies and subgame perfection. They show that when the discount factor is sufficiently large, MPE are characterized by either price cycles or a fixed high price. This paper establishes that Q-learning can learn to approximate these MPE—including collusive fixed-price equilibria.

The learning algorithm applied is a novel adaptation of Q-learning to sequential interaction. Q-learning is a well-established type of reinforcement learning, which is the class of machine learning in which agents learn through autonomous trial-and-error behavior. Q-learning uses recursive value-function estimation to maximize the net present value of future rewards: after choosing an action given the current state, it observes a reward and new state and updates an estimate of the highest possible sum of future rewards following its action. This allows Q-learning to become better informed over time about the long-run consequences of its actions. In choosing its action, Q-learning makes a continuous trade-off between experimenting with different actions (exploration) and taking the perceived optimal action (exploitation).

Recently, reinforcement learning provided major breakthroughs in artificial intelligence by self-learning superhuman play in complex board games like Go and Chess (Silver et al., 2016; 2017; 2018) as well as Atari video games (Mnih et al., 2015). For a discussion in the context of pricing algorithms, see Calvano, Calzolari, Denicolo and Pastorello (2019a), Harrington (2019) and Schwalbe (2019). For a textbook treatment of reinforcement learning and Q-learning, see Sutton and Barto (2018).
In single-agent environments, Q-learning is guaranteed to converge to optimal behavior under mild conditions. However, this guarantee is absent when multiple interacting Q-learning algorithms are learning simultaneously. In absence of theoretical guarantees we therefore provide an empirical understanding through simulations.

The main criticism against the concerns of autonomous collusion is that self-learning algorithms would be similarly ill-equipped as humans to tacitly coordinate on one out of many possible equilibria, at least in reasonably realistic environments (Kühn and Tadelis, 2017; Schwalbe, 2019). This criticism is often supported with references to the experimental economics literature, where tacit collusion by humans fails to occur in moderately complex oligopoly settings. Such a comparison does suggest similar cognitive processes and limits in humans and artificial intelligence, which may not be the case.\footnote{That intelligence matters for achieving human cooperation in repeated games has been shown for instance by Jones (2008) and more recently by Proto, Rustichini and Sofianos (2019).} Additionally, it is pointed out that Milgrom and Roberts (1990)—in their seminal work on learning in games—prove for a wide class of games that no adaptive learning process can lead to a tacit collusive outcome. However, Milgrom and Roberts take that players aim to optimize only their static best response, while reinforcement learning aims to optimize the net present value of all future payoffs. Finally, a criticism is the lack of any known cases of truly autonomous algorithmic collusion. Although true, this does not guarantee that such cases cannot become feasible in the near future.\footnote{One prominent cartel involving algorithms is the Topkins (US) / GB Eye-Trod (UK) case in 2015 and 2016, where online poster retailers used algorithms to coordinate differentiated product prices. Another is the allegation that Accenture (a management consultancy) provided competing car manufacturers in the EU with an algorithm that allowed them to coordinate prices of spare parts (see “Software and Stealth: How Carmakers Hike Spare Parts Prices”, by Reuters, 3 June 2018). Neither is a case of algorithms learning autonomously to collude, however. Another illustration is “The Making of a Fly”, a biology textbook sold on Amazon. In 2011, one seller used an algorithm that each day priced 25% above its competitor, while its competitor used a price-matching algorithm. This caused prices to escalate (up to 23 million dollar per copy). This is again no autonomous collusion.}

Research on autonomous collusion in oligopoly settings is limited.
vanò, Calzolari, Denicolò and Pastorello (2019b) are showing how Q-learning is able to learn collusive strategies when competing algorithms update their prices at exactly the same time. Their results generally align with what we find. The main difference is that they use the conventional model of simultaneous competition. As said however, it may be very unlikely that competing pricing algorithms update their prices simultaneously (or have to act ‘as if’). Additionally, they require conditioning on (own) past prices for collusion to occur, which increases the state space at least quadratically and greatly increases the required learning duration.

Looking at some form of sequential price competition, Tesauro and Kephart (2002) show how dynamic programming techniques based on Q-learning can converge to profitable asymmetric price cycles—with cycles becoming shorter and profits increasing if products are more differentiated or consumers less informed. They require full knowledge of the environment and rival learning however, which we do not. Similarly, Noel (2008) shows how dynamic programming can be used to identify and analyze likely MPE in various extensions to Maskin and Tirole (1988), given some technical conditions on fluctuating marginal cost. He only provides an equilibrium analysis however, and does not deal with coordination on any equilibrium.

Taking quantity instead of price competition, Huck, Normann and Oechssler (2003) find that a “win-continue-lose-reverse” rule provides joint-profit maximizing convergence and Waltman and Kaymak (2008) that Q-learning may collude on lower quantities. However, these results are either not robust to small fluctuations in the payoff function (Izquierdo and Izquierdo, 2015) or do not seem to be based on equilibrium behavior (Calvano et al., 2019b).

Looking instead at pricing algorithms used within operations research and management science, Cooper, Homem-de-Mello and Kleywegt (2015) show that the convention of estimating and optimizing monopoly models—ignoring in effect strategic considerations—may unknowingly lead to cooperative outcomes. However, such model misspecifications are not based on equilibrium behavior either.9

9See Den Boer (2015) for a survey on dynamic pricing in operations research and
Finally, Salcedo (2015) shows that under certain sufficient conditions collusion is inevitable, provided firms adopt a fixed-strategy pricing algorithm that periodically decodes the other algorithm and subsequently adjusts. The periodic decoding may however be framed as illegal explicit collusion by communicating your pricing strategies through decoding (Kühn and Tadelis, 2017; Schwalbe, 2019).

The remainder of this paper is organized as follows. Section 2.1 defines the competitive environment, algorithm and performance metrics. Section 2.2 discusses the empirical results. We look at the case where a Q-learning algorithm faces a basic tit-for-tat strategy and where two Q-learning algorithms face each other. Section 2.3 discusses several robustness checks and extensions. Finally, Section 2.4 provides a discussion, in particular on the main practical limitations, and Section 2.5 concludes.

### 2.1 Environment and Learning Algorithm

This section discusses the economic environment (Section 2.1.1), the algorithm used (Section 2.1.2) and the performance metrics considered (Section 2.1.3).

#### 2.1.1 Sequential Pricing Duopoly

To capture the dynamics of sequential pricing we take the infinitely repeated sequential move pricing duopoly environment of Maskin and Tirole (1988). Below we describe this environment, our baseline setting and its equilibrium behavior.

Competition between two firms $i \in \{1, 2\}$ takes place in infinitely repeated discrete time indexed by $t \in \{0, 1, 2, \ldots\}$. Adjustments in price occur sequentially: in turn, each firm adjusts its price $p^i_t \in P$, where in odd-numbered periods firm 1 adjusts its price and in even-numbered periods firm 2. Price is a discrete variable scaled between 0 and 1 and with $k$ equally
sized intervals—so prices are taken from a discrete set \( P = \{0, \frac{1}{k}, \frac{2}{k}, ..., 1\} \).

At time \( t \), firm \( i \) profit is derived as

\[
\pi^i(p^i_t, p^j_t) = (p^i_t - c^i) D^i(p^i_t, p^j_t),
\]

where \( c^i \) is its marginal cost and \( D^i(p^i_t, p^j_t) \) its demand as function of own price \( p^i_t \) and competitor price \( p^j_t \), with \( j \in \{1, 2\} \setminus i \). There are no fixed costs and demand is such that profit is strictly concave. Firms discount future profits with a discount factor \( \delta \in [0, 1) \), where each firm has as objective to maximize at time \( t \) its cumulative stream of discounted future profits, so

\[
\max \sum_{s=0}^{\infty} \delta^s \pi^i(p^i_{t+s}, p^j_{t+s}).
\]

In our baseline case, we follow Maskin and Tirole in imposing the Markov assumption: strategies only depend on variables that are directly payoff relevant, which in this case is limited to the previous competitor price \( p^j_{t-1} \) and does not include, for instance, communication or the history of prices. The strategy of firm \( i \) is therefore a (possibly randomizing) dynamic reaction function \( R^i(\cdot) \), where in its turn \( p^i_t = R^i(p^j_{t-1}) \). In extensions we also allow for conditioning on own past price.

In showing whether and to what degree autonomous collusion using Q-learning is possible, we restrict ourselves to the simple setting of homogeneous goods with linear demand and zero marginal cost, which is also the baseline case of Maskin and Tirole. Demand has an intercept and slope equal to 1 such that

\[
D^i(p^i_t, p^j_t) = \begin{cases} 
1 - p^i_t & \text{if } p^i_t < p^j_t \\
0.5(1 - p^i_t) & \text{if } p^i_t = p^j_t \\
0 & \text{if } p^i_t > p^j_t 
\end{cases}
\]

Assuming zero marginal cost \( c^i \), we have as monopoly or joint-profit maximizing collusive price \( p^C = 0.5 \). A discussion on these and other underlying
modelling assumptions is provided in Section 2.4.

A strategy pair \((R^1, R^2)\) is a Nash equilibrium if for all prices along the equilibrium path the following value-function condition holds for both firms:

\[
V^i(p^j) = \max_p \left[ \pi^i(p, p^j) + E_{p^j'} \left[ \delta \pi^i(p, p^j') + \delta^2 V^i(p^j') \right] \right]
\]  

(2.4)

where reaction function \(R^i(p^j)\) is a maximizing choice of firm \(i\) and the expectation over competitor response \(p^j'\) is taken with respect to the distribution of \(R^j(p)\).

One Nash equilibrium here is the static Nash outcome in which firms always price at or one increment above marginal cost, although more equilibria exist for a sufficiently high discount factor.\(^{10}\)

As a refinement of the Nash equilibrium, Maskin and Tirole define the concept of a Markov perfect equilibrium (MPE), which is a subgame perfect Nash equilibrium under the Markov assumption. A strategy pair \((R^1, R^2)\) is a MPE if Condition (2.4) holds for both firms and for all prices, including off-equilibrium prices. They show that if firms value future profits sufficiently high there are two sets of MPE: focal price equilibria and Edgeworth price cycle equilibria.

In focal price equilibria both firms sustain a fixed price with the common belief that the other firm would undercut if it were to decrease its price and not follow if it were to increase it. Such beliefs are sustained by off-equilibrium price wars in case any firm undercuts, in which case prices drop and firms mix between staying at lower prices and returning to the fixed price.

In Edgeworth price cycle equilibria firms gradually undercut each other. When further price cuts become too costly, both firms have an incentive to raise their price and reset the gradual downward spiral but prefer the other firm to do so. They therefore mix between maintaining lower prices

\(^{10}\)To see that one increment above marginal cost is a Nash equilibrium, assume that \(R^2(p^1) = \frac{1}{k}\), such that firm 2 always prices one increment above zero marginal cost. Condition (2.4) then simplifies to \(V^1(\frac{1}{k}) = \frac{1+\delta}{1+\delta^2} \max_p \pi^1(p, \frac{1}{k})\). A maximizing choice of firm 1 is then similarly \(R^1(p^2) = \frac{1}{k}\), which, by symmetry, is a Nash equilibrium.
(to punish the other firm for not resetting the price cycle) and resetting itself.

2.1.2 Sequential Q-Learning

The learning algorithm applied here is a novel adaptation of Q-learning to sequential interaction. Q-learning is a simple and well-established reinforcement learning model that aims to maximize the net present value of expected future rewards for unknown environments with repeated interaction. It was originally proposed by Watkins (1989) to solve unknown Markov decision processes, which are discrete time stochastic processes in which actions affect both current reward and the next state in an otherwise stationary environment. Below the specification is discussed in detail, followed by a note on its theoretical limitations and challenges in our context.

2.1.2.1 Specification

Reinforcement learning algorithms consist of two interacting modules: a learning module that processes the observed information and an action-selection module that balances exploitation (choosing the currently perceived optimal price) with exploration (choosing perhaps another price, to learn what happens).

Learning Module Q-learning estimates a Q-function $Q^i(p^i, s)$, which maps for firm $i \in \{1, 2\}$ action $p^i$ (new own price) into its estimated optimal long-run value given current state $s \in S$. Assuming a discrete state set, $Q^i$ is a $|P| \times |S|$ matrix in our case. After observing own profits and new state $s'$, the algorithm updates entry $Q^i(p^i, s)$ according to the following recursive relationship

$$Q^i(p^i, s) \leftarrow (1 - \alpha) \cdot \text{old estimate} + \alpha \cdot \text{observed estimate}, \quad (2.5)$$

old estimate = $Q^i(p^i, s)$
observed estimate = $\pi(p^i, s) + \delta \pi(p^i, s') + \delta^2 \max_p Q^i(p, s')$
where $\alpha \in (0, 1)$ is a stepsize parameter that determines the weight ascribed to the observed estimate relative to its old value and $\delta \in [0, 1)$ is again a discount factor.

Note that the observed estimate consists of three components: direct profit $\pi(p^i, s)$, next period profit $\pi(p^i, s')$ when new state $s'$ realizes (discounted for one period), and the highest possible Q-value $\max_p Q^i(p, s')$ in this new state $s'$ (discounted for two periods). Initially, Q-values are imprecise, but over time they become better estimates of the long-run consequences of choosing $p^i$ in state $s$, allowing for convergence.

Under the Markov assumption current and new state $s$ and $s'$ are equivalent to current and new competitor price. Also note the parallel between Condition (2.4) and Equation (2.5). This comes from the fact that through recursive updating, Q-learning aims to solve for a dynamic programming condition.

**Action-Selection Module** In balancing exploration and exploitation, the algorithm adopts a probabilistic action-selection policy. We use a straightforward procedure called $\varepsilon$-greedy exploration: with probability $\varepsilon_t \in [0, 1]$ it selects a price randomly (exploration) and with probability $1 - \varepsilon_t$ it selects the currently perceived optimal price (exploitation), so

$$
p_i^t \begin{cases} 
    \sim U\{P\} \quad & \text{with probability } \varepsilon_t \\
    = \arg\max_p Q^i(p, s_t) \quad & \text{with probability } 1 - \varepsilon_t
\end{cases}
$$

(2.6)

where $U\{P\}$ is a discrete uniform distribution over action set $P$. In case of ties under exploitation, the algorithm randomizes over all perceived optimal actions.

Note that in the learning module, each time only one entry within the Q-matrix is updated. Such tabular learning leads to a slow learning process. To speed up learning, and allow for continuous state and action spaces, function approximations could be used. This would however increase the amount of parameters and modelling assumptions and is left for future
research. Additionally note that ε-greedy exploration is very untargeted: when exploring, it selects any price randomly. As with the learning module, the action-selection module could be improved by using more sophisticated techniques. A pseudocode of the algorithm as used in the simulations is provided below.

<table>
<thead>
<tr>
<th>Pseudocode Sequential Q-Learning (Simulation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Set demand and learning parameters; Initiate Q-functions</td>
</tr>
<tr>
<td>2 Initialize ( {p_1^1, p_1^2} ) for ( t = {1, 2} ) randomly</td>
</tr>
<tr>
<td>3 Initialize ( t = 3, i = 1 ) and ( j = 2 )</td>
</tr>
<tr>
<td>4 Loop over each period</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>8 Until ( t = T ) (specified number of periods)</td>
</tr>
</tbody>
</table>

2.1.2.2 Theoretical Limitations and Challenges

When a single Q-learning agent faces a fixed-strategy competitor, it is guaranteed to converge to the optimal (rational, best-response) strategy, given mild conditions on stepsize parameter \( \alpha \) and the rate of exploration \( \varepsilon_t \) (Watkins and Dayan, 1992; Tsitsiklis, 1994). The sequential Q-learning algorithm developed here could therefore converge to the optimal strategy if the opponent maintains a fixed strategy.

However, the Q-learning algorithm may not converge to optimal behavior in our environment, because it remains vulnerable to adaptation and experimentation by its opponent. More generally, agents that are simultaneously adapting to the behavior of others face a moving-target learning problem (Busoniu, Babuska and De Schutter, 2008; Tuyls and Weiss, 2012), in which their best response changes as others change their strategies. Convergence guarantees that exist for single-agent reinforcement learning algorithms then no longer hold.

Additionally, Q-learning is restricted to playing pure strategies, but the MPE identified by Maskin and Tirole require mixing strategies—either off-
equilibrium (in case of the focal price) or along the equilibrium path (in case of the Edgeworth price cycles). Although it is incapable of learning any of the subgame perfect equilibria, subgame imperfect Nash equilibria do remain feasible.

Despite these challenges the algorithm does not have to perform badly in practice. It only means that theory is unable to say how well it is expected to behave. In absence of theoretical guarantees we provide an empirical understanding through simulations.

2.1.3 Performance Metrics

In assessing the performance of the algorithm, we look at how profitable it is and how optimal it is relative to best-response behavior.

2.1.3.1 Profitability

Values in the Q-matrix are estimates of discounted future profits when setting price \( p^i \) given current competitor price \( p^j \). Although initially very imprecise, estimated and actual discounted future profits converge if Q-values converge. We can therefore simply take the observed Q-values to capture the degree of profitability:

\[
\text{Profitability: } Q^i_t(p^i_t, p^j_t) \approx \sum_{s=0}^{\infty} \delta^s \pi(p^i_{t+s}, p^j_{t+s}),
\]

(2.7)

where \( p^i_t = R^i(p^j_{t-1}) \) and \( p^j_t = R^j(p^i_{t-1}) \) when firm \( i \) or firm \( j \) adjusts its price, with \( R^i \) and \( R^j \) the dynamic reaction functions based on Equation (2.6) in the action-selection module. In the simulation, we compare this profitability to the net present value of all future profits under joint-profit maximization and under some competitive benchmark.

The joint-profit maximizing benchmark is derived straightforwardly in our case by taking monopoly price \( p^C = 0.5 \), providing each firm with a per-period profit of \( \pi^i = 0.125 \) and a net present value of all future profits of \( 0.125/(1 - \delta) \). A competitive benchmark is less straightforward, how-
ever. An obvious candidate may seem to be the static Nash outcome of prices equal to (or one increment above) marginal cost. However, the sequential environment makes pricing at (or one increment above) marginal cost not subgame optimal: if any firm would deviate and charge higher prices, the other firm would be better off deviating from low prices as well. Although marginal cost is still an interesting benchmark for practical purposes, we take the more conservative (higher) competitive benchmark that approximates the most competitive Edgeworth price cycle MNE identified by Maskin and Tirole (1988): firms undercut each other by one increment until prices reach their lower bound, after which one firm resets prices to one increment above monopoly price and the cycle restarts. It is taken that the first firm that observes the lower-bound price resets the price cycle. This provides in our case an average per-period of $\pi^i \approx 0.0611$ for $k = 6$, with a net present value of all future profits of approximately $0.0611/(1-\delta)$. This increases in the limit of $k$ to approximately $0.0833/(1-\delta)$ per period.

### 2.1.3.2 Optimality

An outcome is a Nash equilibrium if and only if both firms are behaving optimally, in the sense that neither firm can adopt a better strategy given the strategy of the other firm. To capture this degree of optimality, we define $\Gamma^i_t$ as the ratio of estimated and best-response discounted future profits:

$$\text{Optimality: } \Gamma^i_t \equiv \frac{Q^i_t(p^i_t, p^j_t)}{\max_p Q^i_t(p, p^j_t)}$$

where $Q^i_t$ is the optimal Q-function given current competitor strategy at period $t$. $Q^i_t$ is not observed by the algorithm. However, because we control the simulation we can derive it using dynamic programming: keeping the competitor Q-function fixed, we loop over all action-state pairs until Equation (2.5) converges.

$\Gamma^i_t$ has the following interpretation: it shows how much the estimated discounted future profits are below the discounted future profits under best-
response behavior. When the algorithm learned a best-response strategy it therefore produces $\Gamma^i_t = 1$. An outcome is a Nash equilibrium if and only if $\Gamma^i_t = 1$ holds for both algorithms.

Note that $\Gamma^i_t$ does not only take into account the next period best-response behavior, but also possible off-equilibrium exploitation of its competitor in states that are otherwise never visited. For $\Gamma^i_t$ to be reliable, stepsize parameter $\alpha$ or the rate of exploration $\varepsilon_t$ has to decrease sufficiently. This allows $Q^i_t$ to converge and become a reliable estimate of actual discounted future profits.

## 2.2 Simulation Results

For the simulation exercise, we look at price intervals $k = \{6, 12, 24\}$, where $k = 6$ is the illustrating example in Maskin and Tirole (1988) and the lowest price interval at which both fixed-price and price cycle MPE exist. To assess the distribution of performance, we simulate for each price interval 1,000 runs. In our baseline simulation, stepsize parameter $\alpha = 0.3$ and discount factor $\delta = 0.95$ are set arbitrarily. Each run lasts $T = 1,000(k + 1)^2$ periods, which is 1,000 times the size of the Q-matrix. Probability of exploration $\varepsilon_t = (1 - \theta)^t$, where $\theta$ is set such that the probability of exploration gradually decreases from 100% to 0.1% halfway the run, reaching 0.0001% at the end to allow for convergence. Finally, the Q-values are initiated with all zeros (results are insensitive to initialization). Section 2.3 discusses robustness checks and extensions that deviate from these baseline settings.

We first consider the case where the Q-learning algorithm faces a basic tit-for-tat strategy in which the opponent sets the monopoly price if the Q-learner does as well, but undercuts otherwise (Section 2.2.1). Secondly, we consider the case where two Q-learning algorithms are set to face each other (Section 2.2.2).
2.2. Simulation Results

2.2.1 Q-Learning Versus Fixed-Strategy Tit-For-Tat

The fixed-strategy competitor behaves according to the following fixed strategy: it sets the joint-profit maximizing price if the Q-learner has a price at or above this price, but undercuts with one increment otherwise, towards one increment above zero marginal cost.

Figure 2.1 shows for $k = 6$ that Q-learning (firm 1) quickly learns to adhere to the joint-profit maximizing strategy, with final profitability $Q_i$ converging to the joint-profit maximizing level for both firms in all runs. It also learns to behave optimally, with $\Gamma^1 = 1$ in all runs.

This outcome is not an equilibrium, however. This is because the Q-learning algorithm does not learn the off-equilibrium punishment strategy necessary for the fixed-strategy competitor to behave optimally. Keeping the final Q-learning strategy fixed, the fixed-strategy competitor has an average final optimality of $\Gamma^2 \approx 0.75$. Alternative fixed strategies may however ensure that the Q-learning algorithm does learn a stabilizing punishment strategy.

Figure 2.1: Average Firm 1 Profitability (Q-Learning Versus Tit-For-Tat)

Notes: Firm 1 (Q-learning) average profitability $Q_1^t$ over time, averaged over all runs. Q-learning versus tit-for tat, $k = 6$, baseline settings.
2.2.2 Q-Learning Versus Q-Learning

More interestingly, we find that when two Q-learning algorithms face each other sequentially, they manage to converge to strategies that sustain supra-competitive fixed-price outcomes or profitable asymmetric price cycles. Both outcomes occur when \( k \) is low, but when \( k \) increases competing Q-learning algorithms increasingly coordinate only on asymmetric price cycles. When \( k \) is low, the supra-competitive outcome are often Nash equilibria, in which neither algorithm can adopt a better strategy given its competitor strategy.

2.2.2.1 Outcomes

Figure 2.2 shows the Q-learning profitability over time, as averaged over all 1,000 runs, for \( k \) equal to 6, 12 and 24. After an initial learning phase (in which \( Q^i \) is still an imprecise estimate of future discounted rewards), Q-learning algorithms eventually converge to an average estimated profitability of around 2.1—below the joint-profit maximizing level of 2.5 but above the competitive benchmark of 2.2 (which approximates the most competitive Edgeworth price cycle MPE identified by Maskin and Tirole) and well above zero marginal cost. This holds for different price intervals.

Figure 2.3 top panel shows the underlying joint distributions of profitabilities \( Q^i \) for both firms at the end of each run. When \( k \) increases, runs becomes more concentrated around the joint-profit maximizing level of \( Q^i = 2.5 \) for both algorithms (indicated with the dotted square). However, exactly this joint-profit maximizing outcome is reached less often. Figure 2.3 middle panel shows the underlying joint distribution of optimality \( \Gamma^i \) for both algorithms at the end of each run. In many cases the algorithm learns to behave optimally, at least for one of the firms. When the price set is limited, it still happens often that both algorithms are playing mutual best response, with \( \Gamma^i = 1 \) for both algorithms—in other words, are playing a Nash equilibrium. When the number of discrete prices increases however, it is more often the case that only one of the two Q-learners is behaving optimally. For those runs where both algorithms are playing best
response to each other, Figure 2.3 lower panel shows the joint distribution of profitabilities $Q^i$. It shows that a Nash equilibrium is reached in 312 runs for $k = 6$, 79 runs for $k = 12$ and only 10 runs for $k = 24$. It also shows that for this subset of runs with mutual best response, profits are high.

![Figure 2.2: Average Firm 1 Profitability](image)

**Notes:** Firm 1 average profitability $Q^1_t$ over time, averaged over all runs, for different amounts of $k$ price intervals. Q- versus Q-learning, baseline settings.
Figure 2.3: Joint Distribution of Outcomes

Joint Distribution of Profitability $Q^i$

Joint Distribution of Optimality $\Gamma^i$

Joint Distribution of Profitability $Q^i$ if Nash equilibrium

**Notes:** Joint distribution of final profitabilities $Q^i$ (top), optimalities $\Gamma^i$ (middle) and profitabilities $Q^i$ conditional on mutual optimality $\Gamma^i = 1$ for both firms (lower). Dotted squares indicate joint-profit maximization (profitability) or Nash equilibrium (optimality). Q- versus Q-learning, baseline settings.
Finally, Figure 2.4 shows the underlying pricing dynamics. The upper panel shows the distribution of final market prices in those runs that converged to a single, fixed price. A single, fixed price occurs in 475 runs for $k = 6$, 115 runs for $k = 12$ and only 26 runs for $k = 24$. The lower panel in Figure 2.4 shows for all other runs the distribution of changes in market price in the final 100 periods. There is a clear asymmetric pricing pattern: price decreases occur much more often but are smaller in size than price increases. This produces the asymmetric price cycles, as illustrated in Figure 2.5 for three individual runs for $k = 24$. Interestingly, this illustration

![Figure 2.4: Final Price Dynamics](image)

**Notes:** Distribution of final market price in all fixed-price runs (upper) and of changes in market price in the final 100 periods of all non-fixed price runs (lower). Q- versus Q-learning, baseline settings.
shows price cycles that are reset before they reach their lower bound and to price levels that are above the monopoly level. This pushes prices and profits above the most competitive Edgeworth price cycle MPE identified by Maskin and Tirole. However, this may be the consequence of the algorithm learning suboptimal responses, as optimality is generally below one for at least one of the firms.

Figure 2.5: Illustration of Final Market Prices ($k = 24$)

Notes: Market prices in the final 40 periods of three individual runs, Q-learning versus Q-learning, $k = 24$, baseline settings.

2.2.2.2 Reward-Punishment Strategies

Economic theory defines collusion as a situation in which firms use reward-punishment strategies to support a supra-competitive outcome (Harrington, 2019). This is irrespective of whether the strategy is achieved through
explicit or tacit coordination. This section illustrates the kind of reward-punishment strategies that Q-learning has learned. This is done by forcing a deviation and tracking how market price and profit develop. Results are shown for those runs in which the algorithms managed to coordinate on a Nash equilibrium with joint-profit maximizing prices in case of \( k = 6 \) (135 runs) and with at most one price increment away from joint-profit maximizing prices in case of \( k = 12 \) (19 runs).

Figure 2.6 shows that if one algorithm deviates, the other punishes by continuing a downward spiral in which both keep undercutting each other. At some point however, one of the algorithms resets prices back to their supra-competitive level. Punishment is therefore temporary. Figure 2.7 illustrates the reward-punishment consequences: if one algorithm deviates

![Figure 2.6: Average Market Price After Forced Deviation](image)

**Figure 2.6:** Average Market Price After Forced Deviation

\( k = 6 \)

\( k = 12 \)

**Notes:** Average market price in those runs with a (near) joint-profit maximizing Nash equilibrium (135 runs for \( k = 6 \) and 19 runs for \( k = 12 \)), following a forced deviation at period 0.
(in this case firm 2 at $t = 0$) it may receive a temporarily higher profit, but the subsequent price response by the other algorithm ensures that on balance firm 2 is worse off from the deviation. In other words, the Q-learning algorithm has learned a strategy that punishes deviation. Both algorithms are rewarded with higher profits if they stick to the supra-competitive prices. Although average prices and profits gradually return, this is the consequence of runs jumping up in price at different times.

Figure 2.7: Average Two-Period Profit After Forced Deviation

Notes: Average two-period profit in those runs with a (near) joint-profit maximizing Nash equilibrium (135 runs for $k = 6$ and 19 runs for $k = 12$), following a forced deviation at period 0.
2.3 Robustness and Extensions

Below we show that results are generally robust to changes to and asymmetries in the learning parameters (Sections 2.3.1 and 2.3.2). We also show that collusion only occurs when the discount factor is set sufficiently large but not too close to 1 (Section 2.3.3). Finally, we show that results are generally robust to asymmetric timing (Section 2.3.4) and that self-reactive conditioning does not improve results materially (Section 2.3.5).

2.3.1 Stepsize Parameter and Learning Duration

Rate of exploration $\varepsilon_t$ is a function of total duration $T$ following $\varepsilon_t = (1-\theta)^t$, where $\theta$ is such that $\varepsilon_t$ decreases to 0.1% halfway, reaching 0.0001% at the end. We have set $\alpha = 0.3$ and $T = 1,000(k + 1)^2$. Figure 2.8 shows for $k = 6$ that performance can be improved by increasing the duration. This is also shown in Figure 2.9, which shows average profitability, average optimality and the share of equilibrium runs as a function of $T$, given $\alpha = 0.3$. Increasing $T$ pushes average profitability close to 2.2, average optimality to 97% and the share of mutually optimal runs to above 67%.

Figure 2.8: Average Outcomes For Different Parameters

![Figure 2.8: Average Outcomes For Different Parameters](image)

Notes: Average profitability $Q^i$ (left), average optimality $\Gamma^i$ (middle) and share of Nash equilibrium runs (right) for different values of stepsize parameter $\alpha$ and total learning duration $T$. 

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Figure 2.9: Average Outcomes For Different Learning Duration

Notes: Average profitability $Q^i$ (top), average optimality $\Gamma^i$ (middle) and share of Nash equilibrium runs (bottom) as a function of total learning duration $T$.

2.3.2 Asymmetries in Learning

The simulations up until now have assumed that two competing Q-learning algorithms are set identically. Figure 2.10 shows for $k = 6$ what happens to the average firm 1 profitability and optimality and the share of equilibrium
runs when firm 1 and firm 2 set different stepsize parameters $\alpha^1$ and $\alpha^2$. Generally results are robust to asymmetries in $\alpha$, unless $\alpha^1$ and $\alpha^2$ are set too high or their difference is too large.

As previously shown, increasing learning duration $T$ is individually rational, with decreasing marginal gains. In case of asymmetric duration however, firms have the additional incentive to increase $T^i$ given any competitor duration $T^j$. This follows from the fact that a comparatively longer learning period allows you to better adapt your strategy. Finally note that Q-learning does not require that its competitor uses Q-learning as well. Although not considered in this paper, Q-learning can similarly be used against other strategies or algorithms.

Figure 2.10: Average Outcomes For Asymmetric Stepsize Parameters

![Figure 2.10: Average Outcomes For Asymmetric Stepsize Parameters](image)

Notes: Average firm 1 profitability $Q^1_{\text{Prof.}}$ (left), average firm 1 optimality $\Gamma^1_{\text{Opt.}}$ (middle) and share of Nash equilibrium runs (right) for different values of stepsize parameters $\alpha^1$ and $\alpha^2$.

### 2.3.3 Discount Factor

The discount factor was set arbitrarily at $\delta = 0.95$. In case of very short periods the actual discount factor of a firm would be much closer to 1. However, when setting $\delta$ very close to 1, sufficient learning may fail because old Q-value estimates will get too much weight. It may then be required to set a lower $\delta$. Figure 2.11 shows what happens when adjusting $\delta$. When $\delta$ is low, it consistently learns to coordinate on a static Nash outcome. When $\delta$ increases, average profitability increases, while average optimality
Chapter 2. Autonomous Algorithmic Collusion

decreases—as it happens more often that it is no longer optimal for both firms. When $\delta$ is set too close to 1, it fails to learn properly.

Figure 2.11: Average Outcomes For Different Discount Factors

![Figure 2.11: Average Outcomes For Different Discount Factors](image)

Notes: Average per-period profitability $\pi^i = Q^i(1 - \delta)$ (top), average optimality $\Gamma^i$ (middle) and share of Nash equilibrium runs (bottom) as a function of discount factor $\delta$. 

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2.3. Robustness and Extensions

2.3.4 Asymmetric Sequential Timing

The sequential pricing framework of Maskin and Tirole (1988) assumes perfect symmetry in the timing between updates. However, similarly as it is very unlikely that competing algorithms update their prices at exactly the same time, it is unlikely they update their prices exactly sequentially—with equal time between responses.

Timing could instead also be seen as a spectrum: at one extreme firm 1 updates only just before firm 2; at the middle, the timing between updates is the same (as considered up until here); and at the other extreme firm 1 updates right after firm 2.

Take $\rho^i \in (-1, 1)$ as a parameter capturing this: if $\rho^i < 0$ firm $j$ updates its price more quickly after firm $i$ than vice versa, with $\rho^i = -1$ as the extreme where firm $j$ updates directly after firm $i$. The converse holds if $\rho^i > 0$. It follows that $\rho^i = -\rho^j$. Equation (2.5) now adjusted to

$$Q^i(p^i, p^j) \leftarrow (1 - \alpha)Q^i(p^i, p^j) + \alpha \left( \pi_1(\cdot) + \delta \pi_2(\cdot) + \delta^2 \max_{\bar{p}} Q^i(p, \bar{p}) \right),$$

(2.9)

where the first observed profit $\pi_1(\cdot)$ and second observed profit $\pi_2(\cdot)$ are derived as weighted averages depending on parameter $\rho^i$ as follows:

$$\pi_1(p^i, p^j, p^{j'} | \rho^i) = \begin{cases} (1 - |\rho^i|)\pi(p^i, p^j) + |\rho^i|\pi(p^i, p^{j'}) & \text{if } \rho^i \in [-1, 0] \\ \pi(p^i, p^j) & \text{if } \rho^i \in [0, 1] \end{cases}$$

$$\pi_2(p^i, p^j, p^{j'} | \rho^i) = \begin{cases} \pi(p^i, p^{j'}) & \text{if } \rho^i \in [-1, 0] \\ \rho^i \pi(p^i, p^j) + (1 - \rho^i)\pi(p^i, p^{j'}) & \text{if } \rho^i \in [0, 1] \end{cases}$$

Although formally the weighting should take into account intermediate discounting as well, this does not affect results qualitatively and is omitted for simplicity. Figure 2.12 shows that results are generally robust to asymmetric sequential timing, unless the other firm prices much more quickly in response ($\rho^i$ close to $-1$). As expected, average profitability is lower when the competitor responds more quickly ($\rho^i < 0$), which reduces the
opportunity to learn optimal behavior.

Figure 2.12: Average Outcomes For Different Timing Parameters

Notes: Average firm 1 profitability $Q^1$ (top), average firm 1 optimality $\Gamma^1$ (middle) and share of Nash equilibrium runs (bottom) as a function of timing parameter $\rho^1$. 
2.3.5 Self-Reactive Conditioning

Calvano et al. (2019b) allow for and requires (at least) one-period memory. Figure 2.13 shows that in our case, this increases average profitability moderately, but with longer learning and less (mutual) optimality.

Figure 2.13: Average Outcomes For Different Learning Durations

Notes: Average profitability $Q^i$ (top), average optimality $\Gamma^i$ (middle) and share of Nash equilibrium runs (bottom) as a function of total learning duration $T$. Stepsize parameter $\alpha = 0.3$. 

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2.4 Discussion

We find that competing Q-learning algorithms can coordinate on collusive strategies when firms alternate in updating their price. This occurs even though the algorithms are not communicating with each other and each algorithm only aims to maximize the net present value of its own profits. This section provides a discussion on the main practical limitations (Section 2.4.1), the appropriate benchmark to which to assess these results (Section 2.4.2) and possible changes to the underlying modelling assumptions (Section 2.4.3).

2.4.1 Practical Limitations

Q-learning is a popular and well-established reinforcement learning algorithm. However, it is unclear to what degree Q-learning is used in real-world pricing applications. Based on the above analysis, it is possible to identify three main limitations to the practical application of Q-learning in pricing.

First, sequential Q-learning requires many periods to learn, which can be very costly in practice. Assuming a less stylized environment would amplify this problem. Second, we have assumed a stationary environment. If, however, there are structural changes in the environment—such as entry, exit or shifts in cost or demand—sequential Q-learning may have to adapt its learned behavior. And third, sequential Q-learning does not show a consistent convergence to mutually optimal behavior. Optimality is not guaranteed and runs end up with different outcomes. Before implementing an autonomous reinforcement learning algorithm, firms may require more guarantees on convergence.

There are several possible ways to deal with these practical limitations. Calvano et al. (2019b) argue that Q-learning algorithms could be trained in an offline, simulated environment before being put to use in the real world. This would be similar to how reinforcement learning algorithms are trained in for instance board games and autonomous driving. Using offline
learning greatly increases the scope for learning, although it does provide other challenges in defining the simulated environment and ensuring that independently trained algorithms can properly adapt to each other once put to use in the real world.

Another solution would be to impose more structure on the learning algorithm. The sequential Q-learning algorithm discussed here is model free in the sense that it does not require any demand specification and does not model competitor behavior. However, it may be valuable to approximate some function of the environment instead, using observed data and techniques from supervised machine learning. This would also allow Q-learning to extend beyond its tabular specification (Sutton and Barto, 2018; Romero and Rosokha, 2019). Additionally, the specification discussed here learns from scratch. In practice, however, firms may transfer learning from one product or period to a new product or period to guide or speed up learning (Pan and Yang, 2010; Griffith, Subramanian, Scholz, Isbell and Thomaz, 2013).

Finally, extensions using much more state-of-the-art learning algorithm could be developed. For instance, Crandall et al. (2018) already show how state-of-the-art reinforcement learning algorithms are capable of cooperating with both other algorithms as well as humans in very simple repeated matrix games and Mnih et al. (2015), Leibo et al. (2017) and Peysakhovich and Lerer (2017) show how deep reinforcement learning could lead to coordination even in more complex repeated matrix games. Additionally, theoretical developments within multi-agent reinforcement learning (or joint learning) may be able to deal with the challenges that remain in guaranteeing convergence to mutually optimal collusive behavior. However, key developments in multi-agent reinforcement learning still lack practical applicability to oligopoly environments.\(^\text{11}\)

2.4.2 Experimental Economics Benchmark

A good test on whether autonomous algorithmic collusion is not only in principle possible, but also an immediate practical concern would be a direct comparisons between algorithms and humans in an otherwise identical experimental environment. For the environment considered in this paper, humans are expected to show a superior collusive performance because tacit collusion is relatively straightforward. For instance, Leufkens and Peeters (2011) test experimentally whether humans are capable of coordinating on either a focal price or an Edgeworth price cycle in case of sequential interaction. Taking the environment of Maskin and Tirole (1988) and $k = 6$, they find that under a random ending rule, 13 out of 15 human pairs end up coordinating on the joint-profit maximizing fixed price. This is also achieved without requiring many periods to learn.

In other environments, humans have more difficulty to collude tacitly. For instance, Huck, Normann and Oechssler (2004) and Oechssler, Roomets and Roth (2016) show that in simple quantity-setting games, humans fail to tacitly collude when extending the amount of players to four, even when given many periods to learn (up to 1200 periods). Autonomous algorithmic collusion would be more of an immediate concern if they can be shown to outperform humans in these environments.

One problem with comparing algorithmic performance with the experimental economics literature however, is that experiments with human subjects cannot replicate the speed and scalability of algorithms. That this is relevant follows for instance from Friedman, Huck, Oprea and Weidenholzer (2015). They show that human tacit collusion in quantity-setting games becomes more feasible provided that subjects are given enough time to learn. Algorithms have the advantage over humans that they can experiment with different strategies much more quickly. This means that in many environments in which algorithms are used, a direct human benchmark may not be available.

Finally, it may be interesting to explore human-machine interaction as well. Crandall et al. (2018) show in very simple social dilemma games that frontier reinforcement learning algorithms are able to coordinate on
cooperative outcomes—both in self-play in when interacting with humans. To what extend these results extend to oligopoly settings is an interesting open question.

### 2.4.3 Modelling Assumptions

The ambition of this paper is to show whether and to what degree autonomous algorithmic collusion is in principle possible. We have therefore restricted ourselves to a simple stationary setting of a homogeneous goods duopoly with linear demand and zero marginal cost. It may nevertheless be valuable to develop a better understanding on the sensitivity of the results to the modelling assumptions.

Specifically, it may be insightful to allow for more firms, horizontal or vertical product differentiation, stochastic demand or cost, firm asymmetries in demand, cost or price set, incomplete consumer information, switching cost, implementation or pricing frictions or other dimensions of competition (such as quantity instead of price). Such changes to the modelling assumptions may also be informative on the question whether collusion stability and coordination is affected differently by structural factors in case of algorithms than humans.

Additionally, we have assumed throughout that the environment remains stationary. As discussed, sequential Q-learning may have to adapt its learned behavior if instead there are structural changes in the environment, such as entry, exit or shifts in the profit function. Before considering robustness of sequential Q-learning to such non-stationarities, it may be more valuable to first develop an algorithm that can explicitly deal with non-stationarities, as discussed above.

### 2.5 Concluding Remarks

There is a growing concern that increasingly more sophisticated pricing algorithms may at some point, inevitably, be able to learn to avoid competitive pressures and achieve higher profits—at the expense of consumers.
However, this concern is often based on a loose and intuitive interpretation of artificial intelligence only.

In trying to shed some light on this, we show how autonomous algorithmic collusion could indeed be possible. In a stylized duopoly environment with repeated sequential price competition, Q-learning algorithms can learn reward-punishment strategies that sustain supra-competitive prices—at least when the amount of discrete prices is limited. If the number of discrete prices increases, Q-learning increasingly converges to profitable asymmetric price cycles.

Although this shows that autonomous algorithmic collusion is in principle possible, practical limitations remain (in particular long learning and required stationarity). However, rapid progress made within artificial intelligence means that these limitations may in reasonable time be resolved. This provides the ground for academics to develop a further understanding and for authorities to remain vigilant when observing the rise of autonomous pricing algorithms in the market place.
Chapter 3

Collusive Benchmark Rate Fixing*

Trader RBS: “It’s just amazing how Libor fixing can make you that much money or lose if opposite. It’s a cartel now in London.”

Trader Deutsche Bank: “Must be damn difficult to trade man, especially if you are not in the loop.”

Benchmarks such as the Libor and Euribor, silver and gold fixes and foreign exchange (forex) rates have been proven vulnerable to manipulation. They are determined on the basis of contributions (or ‘quotes’) by a small set

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of larger market participants, who also trade in the financial products that are valued on these rates and therefore have incentives to distort the benchmarks in a direction that is favorable to their financial interests.\footnote{See Duffie and Stein (2015), who highlight the problem of a small volume of transactions used to determine a rate on which a much larger volume of transactions is benchmarked.} In the majority of the manipulation cases that came to light, the focus of investigation was on individuals who had fraudulently tried to direct the rate for gain on their own trading book, primarily within their own bank, or incidentally as a favor among a few rogue traders. However, benchmark manipulation is a lot more effective when done cooperatively and there have been several cartel proceedings.

The growing literature on benchmark rate manipulation focuses mostly on individual manipulation, based on two incentives that banks could have to manipulate the rate. The first academic paper (Abrantes-Metz, Kraten, Metz and Seow, 2012) extends a Wall Street Journal analysis in which it was shown that Libor submissions were lower than those implied by prevailing credit default swap (CDS) spreads, conjecturing that banks understated their borrowing costs to signal financial health. Snider and Youle (2014) include banks’ trading incentives to manipulate the rate—due to financial products benchmarked on the rates—and find empirical evidence for this phenomenon. The theoretical literature investigates how, or under which conditions or benchmark design, banks in equilibrium would reveal their true borrowing costs, focusing on individual manipulation incentives (Coulter, Shapiro and Zimmerman, 2018; Duffie and Dworczak, 2018; Hernando-Veciana and Tröge, 2019). Although the possibility of collusion is raised in some papers (Eisl, Jankowitsch and Subrahmanyam, 2017; Fouquau and Spieser, 2015), it is not modelled.

In this paper we model benchmark rate collusion, in particular the tension between banks’ diverging and time-varying individual interests and the effect of the institutional settings. Contrary to conventional cartels, in which all members want to increase product prices, the interests of the panel banks setting the rates are typically not sufficiently aligned for them to agree even on the direction in which to manipulate a rate. Influencing a
benchmark is complicated by the fact that most of them are based on only a subset of contributions—for example a trimmed average or a median of observations during a given time window.

We show how a cartel in the fixing of benchmark rates can work, despite conflicting and time-varying interests. In our model, panel members have a daily private information baseline contribution and portfolio exposure, which they can adjust at a cost. When in a cartel, they exchange inside information on their private information, which is then used to jointly agree on a target rate and corresponding optimal contributions for each cartel member. We refer to the manipulation of these contributions as ‘eligible transactions rigging’. Subsequently, all participants can partially adjust their own exposure positions to the new rate, which they know ahead of the market.\footnote{See also Abrantes-Metz (2012), who emphasizes this incentive that banks have to communicate about their future submission in order to eliminate risk and increase trading profits.} We refer to this as ‘front-running’. The costs of manipulation are minimized and shared over time, so that each cartel member has a strictly positive expected payoff from participating in the scheme. Contrary to Athey and Bagwell (2001), in our model there is no history-dependent favoring of certain players, which in their model is based on productive efficiency.

Some colluding panel members still face an incentive to deviate from the cartel agreement, particularly when they have an extreme baseline contribution or exposure draw that conflicts with the joint optimal benchmark. We find that observed episodic recourse to independent quoting is part of a feasible continuous collusion equilibrium that deals with these extreme draws. This equilibrium with episodic break-up is similar to that used by Fershtman and Pakes (2000). We also find that all panel members have an incentive to participate in the scheme, as not participating makes a panel member strictly worse off for two reasons. One is that the non-member would miss out on the information necessary to front run, which generates profits that are higher in expectation than the costs of collusion. In addition, the cartel would not take the outsider’s exposure into account when determining the rate.
Simulations of our model suggest that broadening the class of transactions eligible for submission and averaging over fewer middle quotes, although reducing the extend to which the rate is manipulated, can unintentionally make collusion more sustainable. The simulations also suggest patterns that can be used for screening. We find that benchmark collusion creates higher rate volatility in the benchmark over time, as movements in the rate are inside information to the members that a potential for cartel trading profits. Additionally, a high positive correlation between panel banks’ exposure adjustments in the front-running window and the subsequent change in the published rate is indicative of coordinated manipulation.

The model is tailored to the interest rate derivatives cartel infringements that the European Commission found. The Libor and Euribor manipulations have also been extensively investigated by the American and British authorities, albeit predominantly as fraud cases for misreporting in breach of the rates’ code of conduct. Reforms to the rate setting protocols have since been proposed. In particular, the submissions are to be based on a specific and relatively small subset of actual trades, so-called ‘eligible’ transactions. Our cartel theory applies to both the original rate setting procedure and the implemented reforms. Whereas before, manipulation entailed misreporting borrowing costs, in the revised procedures it would be necessary to manipulate eligible transactions. We therefore refer to it as eligible transactions rigging.

The paper is organized as follows. Section 3.1 sets out mechanisms and evidence of collusive Libor and Euribor fixing. In Section 3.2, related literature is reviewed. Section 3.3 lays out the model and presents existence and stability results. In Section 3.4, simulation exercises illustrate collusive rate patterns. In Section 3.5, we discuss several extensions of our model. Section 3.6 concludes, briefly discussing how the theory also applies to other benchmark fixings. The source code of a software that calculates

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5European Commission, Case A.39914 – Euro Interest Rate Derivatives and Case AT.39861 – Yen Interest Rate Derivatives. See Section 3.1 for details.

6For example, Financial Services Authority, “Final Notice: Barclays Bank PLC,” 27 June 2012.
3.1 Collusion on the Libors and Euribors

The London Interbank Offered Rate (Libor) and the Euro Interbank Offered Rate (Euribor) are financial benchmarks that globally underlie enormous transaction values. They are key variables in portfolio and risk management decisions, as well as barometers of the financial sector’s health. Between 370 and 400 trillion dollar worth of interest rate derivatives, consumer credit and commercial loans—or over four times global GDP—are estimated to directly derive their value from these rates.\(^7\)

The rates are calculated daily for numerous currencies and maturities, ranging from overnight to 12 months, as the trimmed average of submissions by a set panel of banks.\(^8\) A member bank’s submission (or ‘quote’) is meant to reflect its opportunity costs of unsecured funds in the interbank market.\(^9\) Each trading day morning, quotes are submitted to a central administrator who discards the extremes, averages the middle range and, publishes the new rates at a given time.\(^10\) All the individual submissions are also published.\(^11\)

Suspicion of manipulation of the fixings rose when, in the run-up to the global financial crisis, the Libors appeared to periodically diverge from other proxies of bank borrowing costs and risk, in particular credit default

\(^8\)The Libor panels are consistently formed by 11 to 16 banks. The Euribor panel consisted of 44 banks before the crisis, after which over half of them withdrew.
\(^10\)The Libor quotes are submitted before 11:00 a.m. GMT. Of the middle 50% of the quotes, the average is taken, which is published at 11:45 a.m. For the Euribor, this is 10:45 a.m. CET, 70% and 11:00 a.m. The Libors used to be produced by the British Banking Association (BBA), but the process was transferred to ICE Benchmark Administration (IBA) in February 2014. The Euribor is published by the European Money Markets Institute (EMMI), formerly the Euribor-EBF.
\(^11\)As of 2013, individual Libor quotes are no longer published simultaneously with the final rate, but with a 3-month delay. HM Treasury, “The Wheatley Review of Libor: Final Report,” 2012. Euribor submissions are still published simultaneously with the rates.
swaps (CDS) spreads. Subsequent investigations focussed on the incentives of individual contributing banks to appear more creditworthy during the 2007-2009 financial crisis by underreporting their true borrowing cost—so-called ‘low-balling’.

The panel banks also have strong incentives to manipulate submissions in order to enhance their portfolio results. The British Bankers Association (BBA), responsible for overseeing the rate setting process, knew that:

“Many institutions set their Libors based on their derivative reset positions.”

Traders requested submissions aimed at benefiting their trading positions, illustrating how a bank with a net lending (borrowing) position would profit from a higher (lower) Libor or Euribor. Money market desks are in a position to know their banks’ overall net exposure to the various rates and how they would gain or lose from changes in the rates. The potential trading gains from even a small move in the rates are large.

The design of the Libor and Euribor setting processes is conducive to collusion. The trimming of the highest and the lowest submissions allows an individual bank only a very limited effect on the rate, so that manipulation

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12C. Mollenkamp and M. Whitehouse, “Study Casts Doubt on Key Rate; WSJ Suggests Banks may have Reported Flawed Interest Rate Data for Libor,” Wall Street Journal, 29 May 2008.


14Bank of Scotland trader in an email to the BBA’s Libor Director quoted in Vaughan and Finch (2017), page 163.

15UBS instructed its traders to base submissions on the bank’s derivatives position, for which spreadsheets were kept that calculated the exact effects of a change in Libor in each currency and maturity on trading profits. Tom Hayes, a convicted derivatives trader for UBS and later Citigroup, stated at his trial that he had acted on the instructions of his employer. An internal document titled ‘Publishing Libor Rates’, containing such instructions, was recovered from the communal drive at UBS. Vaughan and Finch (2017), pages 23 and 154.

16Internal documents from Deutsche Bank, for example, show that on 30 September 2008 Deutsche Bank tallied that it could gain up to 68 million euro for each basis point change in Euribor and Libor. “Bank Made Huge Bet, and Profit, on Libor,” Wall Street Journal, 10 January 2013.
3.1. Collusion on the Libors and Euribors

is most effective when done in cooperation between the panel banks.\textsuperscript{17} The same known banks form the panels for long periods of time and follow each others’ submissions closely. Monitoring adherence to a collusive agreement is easy from the published rates alone, which facilitates the implementation of punishment strategies to stabilize against unilateral defection.

The manipulation cases gave ample indication of more widespread communication and coordination of for-profit manipulation strategies.\textsuperscript{18} The U.K. Financial Service Authority (FSA) concluded that Barclays had acted in concert with other banks.\textsuperscript{19} The U.S. Commodity Futures Trading Commission (CFTC) found that:

“Libor was routinely being gamed by the banks that set it.”\textsuperscript{20}

Several antitrust cases have been brought for benchmark rate collusion. The European Commission established cartel violations in breach of Article 101 TFEU in interest rate derivatives against nine of the panel banks for record fines.\textsuperscript{21} The U.S. Department of Justice’s Antitrust Division, which

\textsuperscript{17}In theory, any group that is strictly larger than the fraction trimmed on either side can have an unbounded effect on the rate. For example, a group of 5 in a 16-bank Libor panel, or a group of 4 in the current 20-bank Euribor panel. In practice, extreme quoting will raise suspicion of manipulation, whereas a larger group in coordination can influence the rate more smoothly.


\textsuperscript{20}CFTC head of enforcement Greg Mocek quoted in Vaughan and Finch (2017), page 76.

\textsuperscript{21}See European Commission, \textit{Case AT.39914 – Euro Interest Rate Derivatives} and European Commission, \textit{Case AT.39861 – Yen Interest Rate Derivatives}, two hybrid settlements of 4 December 2013, involving Barclays, Deutsche Bank, Société Générale, RBS, UBS, JP Morgan, Citigroup and RP Martin (broker); prohibition decisions in both cases, of respectively 7 December 2016 against Crédit Agricole, HSBC and JPMorgan Chase, and 4 February 2015 against broker ICAP for facilitating collusion (later on, this decision was partially annulled by the European Court of Justice); European Commission, \textit{Case AT.39924 – Swiss Franc Interest Rate Derivatives}, two prohibition decisions on 21 October 2014, one against RBS and JP Morgan on derivatives based on the Swiss franc Libor and one against RBS, UBS, JP Morgan and Crédit Suisse for bid-ask spreads charged on Swiss Franc interest rate derivatives.
was involved in the fraud investigations, did not prosecute for collusion. However, several private antitrust damages actions have been brought. Seminarily, the Court of Appeals for the Second Circuit in Manhattan ruled that Libor manipulation could constitute price-fixing as a per se antitrust violation under Section 1 of the Sherman Act.22

The workings of such financial benchmark rate cartels are not obvious, however. Due to often opposite exposure positions, some banks gain from an increase in the rate, although the others gain from a decrease. Moreover, the position a trader or bank faces on any given day is uncertain and largely stochastic. For a bank, it is the sum total of a vast number of transactions done by various trading desks worldwide. Around a kernel of longer-term contracted money in- and outflows, exposure positions are largely driven by positions in over-the-counter (OTC) derivatives that are highly volatile. This means banks’ exposure positions regularly flip back and forth between negative and positive.

Furthermore, rate manipulation is costly, especially after the reforms. Previously, there was no clearly prescribed method for panel members to determine their Libor and Euribor submissions. The misreporting of their true borrowing costs was a form of cheap lying, with really only the risk of too unusual quotes raising suspicion with clients or the authorities. The reforms prescribe that a submission is to be the volume weighted average rate of eligible transactions executed during the last day.23 In the case of Libor, transactions closer to 11 a.m. are also given a higher weight in the quote.24 If a cartel were to attempt to move the rate, it would need to do substantial and timed actual transactions in line with the submitted rate rather than the going rate, which is potentially suboptimal.

3.1. Collusion on the Libors and Euribors

Two complementary mechanisms facilitate collusion. First, designated traders engage in eligible transactions rigging: they submit cartel quotes supported by manipulated transactions. By having only some of its members manipulate, in turns, the cartel minimizes and spreads the costs of its manipulation. Also, eligible transactions could (partly) be matched between cartel members, with no net cost to the cooperative. Obviously, prior to the reforms, when the rates were not transaction-based, there was no need for panel banks to engage in eligible transactions rigging. Hence, no direct evidence of this mechanism can be expected from the cases investigated under the old regime. However, as detailed above, the Libor and Euribor panel banks regularly misreported their true borrowing costs when submitting quotes, and with information and objectives that were in line with eligible transactions rigging.

In the second mechanism, all cartel members benefit from front-running. The cartel creates inside information for its members on what the future rate will be, before it is published. This information allows cartel members not only to increase their trading profits at the expense of other market participants, but also to better align their interests by creating more beneficial portfolio positions. Front-running involves some direct transaction costs, trade risks and liquidity constraints, but is mostly lucrative.

There is ample evidence of front-running.\(^{25}\) The European Commission describes how:

\[
\text{“On occasion, certain traders also explored possibilities to align their EIRD trading positions on the basis of ... communicated preferences for an unchanged, low or high fixing of certain EURIBORS tenors [which] depended on their trading positions/exposures ... [and] ... detailed not publicly known/available information on the trading positions.”}\(^{26}\)
\]

Further findings of traders’ strategies to adjust trading exposure on behalf

\(^{25}\)Vaughan and Finch (2017), on page 114 quote Hayes explaining to another submitter by email: “If we know ahead of time we can position and scalp the market.”

\(^{26}\)European Commission, Case A.39914 – Euro Interest Rate Derivatives, settlement decision of 4 December 2013, recital 32.
of their banks with an eye to the cash flows expected to be received are given in the Commission’s prohibition decisions. Without OTC data, which is proprietary to banks, it is not possible to examine the extent or magnitude of this trading position alignment.

A third characteristic of benchmark rate collusion is that the cartel may alternate days of collusive quoting with days of individual quoting. The panel banks regularly agreed not to coordinate behavior when interests were too diverging. One example of a failed attempt to coordinate submissions is that of a Euribor submitter who was unable to accommodate another trader’s request due to opposing interests. In another, a Lloyds submitter explained to two new colleagues making the Yen Libor submissions that:

“We usually try and help each other out. .. but only if it suits!”

There was consensus that although coordination would not be possible in every period, the longer-term collusive arrangement was valid and valuable. A submitter preemptively contacted a trader at another bank with “Submitter-4: ‘morning skip - [Trader-5] has asked me to set high libors today - gave me levels of 1m 82, 3m 94...6m 1.02’, in effect to excuse that the trader could not follow in the manipulation of the rate that day:

“Trader-B: ‘sry mate cant oblige today...i need em lower!!!’

Submitter-4: ‘yes was told by [a third party]...just thought i’d let you know why mine will be higher...and you don’t get cross with me.”

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27See the EIRD prohibition decision of 7 December 2016, at recitals 130 and 384 amongst others; and the YIRD prohibition decision of 4 February 2015, recital 89.


3.2 Related Literature

Despite the cost of collusion, the potential for cartel profits is large. Currently, the class of eligible transactions is only a small subset of all trades benchmarked against the rate.\textsuperscript{31} The volume of OTC derivatives trades alone, which are not eligible, is a factor that is 10 higher than all the other asset classes that make up the panel banks’ total exposure positions to the benchmark rates combined.\textsuperscript{32} Basing submissions on more actual trades would increase the cost of manipulation. A further reform considered is to discard more of the highest and lowest quotes.\textsuperscript{33}

3.2 Related Literature

The emerging literature on benchmark rate manipulation focuses almost exclusively on manipulation by one or a few rates-setters. Abrantes-Metz, Kraten, Metz and Seow (2012) point at episodes of low variation in Libor submissions by individual banks before August 2007 as suspicious of collusion, yet do not find that the rate is significantly different from its predicted level in comparison to the federal fund effective rate and 1-month T-Bill rates. Using a revealed preference approach, Youle (2014) identifies unobserved bank exposures and finds evidence suggesting that Libor was downward-biased during the financial crisis.

Abrantes-Metz and Sokol (2012) suggest that screens could have detected interbank rate manipulation and collusion earlier. Monticini and

\textsuperscript{31}Libor quotes are to be supported by transactions in unsecured deposits, commercial paper and certificates of deposit, where the submitting bank received funding from specified counterparties. ICE Benchmark Administration, “ICE LIBOR Evolution Report,” 25 April 2018, pages 15-17. For Euribor, only transactions of unsecured cash deposits from specified counterparties traded in the wholesale unsecured money markets are eligible. European Money Markets Institute, “The Path Forward to Transaction-based Euribor,” 21 June 2016, pages 4-6.

\textsuperscript{32}The Financial Stability Board (FSB) reported in 2014 that over 170 trillion dollar in OTC derivatives were tied to the USD Libor and over 197 trillion dollar to the Euribor. Financial Stability Board, “Market Participants Group on Reference Rate Reform, Final report,” March 2014, page 243.

\textsuperscript{33}EMMI reform proposals include that Euribor be calculated as the average of only the middle 4 or 5 of all quotes. European Money Markets Institute, “Consultative Paper on the Evolution of Euribor,” 30 October 2015, page 14. The Libors are still based on the average of the middle 50% of the quotes.
Thornton (2013) find more material anomalous patterns for the same period when using the relationship between Libor and a large, unsecured certificate of deposit rates.

Kuo, Skeie, Vickery and Youle (2014) develop an algorithm to infer information about individual term dollar interbank loans settled through the bank payment system operated by the Federal Reserve Banks. Kuo, Skeie and Vickery (2018) use among others that method and different measures of bank borrowing costs to find robust evidence of underreporting of the Libor. Snider and Youle (2014) focus on a different incentive for manipulation: banks’ portfolios of products that reference benchmark rates. They develop a simple model of portfolio-driven manipulation and find empirical evidence supporting its predictions. Gandhi, Golez, Jackwerth and Plazzi (2019) estimate monthly Libor-related positions and find a relation between the positions and banks’ submissions, which is initially stronger for banks that were sanctioned by the regulators.

A few papers raise the possibility of agreements between two or several panel members, but none model how collusion could work. Eisl, Jankowitsch and Subrahmanyam (2017) calculate how Libor misreporting by one or several banks together could have moved the average, but do not analyze incentives. Using a time-varying threshold regression model, Fouquau and Spieser (2015) argue that the breaks they find are not consistent with exogenous money market shocks, suggesting manipulation by small groups of panel banks, which they propose identifying using a hierarchical clustering method.

The theoretical literature on benchmark rate manipulation looks at how the benchmarks perform under different types of behavior and designs. Hernando-Veciana and Tröge (2019) focus on banks’ incentives to manipulate the rate downwards. They show that under normal market conditions there exists an equilibrium in which banks submit their true borrowing costs, while in times of financial stress there are no equilibria in which banks submit their true borrowing costs. Chen (2017) studies Libor as a standard survey and finds that the bias in Libor decreases in panel size, but that the expected benchmark bias is no longer distribution-free.
3.2. Related Literature

under collusion or signaling. Diehl (2013) models portfolio and reputation incentives and compares the performance of different aggregates, such as the mean and the median, under individual manipulation. Coulter, Shapiro and Zimmerman (2018) investigate the optimal design of fines, in case a submission is different from a comparison rate, in a “revealed preference mechanism” and show how it is more efficient and accurate than a purely transaction-based Libor. Duffie and Dworzack (2018) study the actual design of the optimal rate, against portfolio-driven manipulation, ignoring collusion.

Our paper relates to the literature on collusion with heterogeneity in players and market conditions in repeated games. In benchmark rate setting, heterogeneity between banks stems from their time-varying interests, both in their exposure position and true borrowing costs, which is different from other types of heterogeneity that have been modeled in the cartel literature. As a result, some cartel members prefer a higher rate and others a lower one. When firms have different costs, capacity constraints or product varieties, they may prefer different levels of the cartel price increase, but they never want a decrease.\textsuperscript{34} Heterogeneity in discount factors affects firms’ ability to collude on higher prices.\textsuperscript{35}

As during booms in Rotemberg and Saloner (1986), extreme portfolio positions or true borrowing costs in our model may give one or more panel banks incentives to deviate. However, in our model there is no fallback strategy from which no cartel member has an incentive to deviate, as competitive pricing is in Rotemberg and Saloner (1986). In their model, by getting sufficiently close to competitive pricing levels, continuous collusion can be assured.

We show the stability of an equilibrium as in Fershtman and Pakes (2000), which is broadly consistent with the evidence of the panel tem-

\textsuperscript{34}Heterogeneity in costs is studied by, among others, Harrington (1991) and Rothschild (1999); in capacities in Davidson and Deneckere (1990) and Compte, Jenny and Rey (2002); and in product differentiation in Ross (1992) and Osterdal (2003). Although there may not be a common collusive price when the firms differ widely, Harrington (2016) establishes that a minimum price always exists.

\textsuperscript{35}See Harrington (1989) and Obara and Zincenko (2017).
porarily reverting to independent quoting if agreeing on a collusive submission is not possible for the period. Whenever at least one bank has an incentive to deviate, there is episodic recourse to non-cooperative quoting. Such ‘price wars’ are unprofitable in the short run, as in Green and Porter (1984), but they are an integral part of the collusive strategy and not punishment. In our model, each cartel member incurs occasional losses as part of the cartel strategy, but randomly and not through the history-dependent favoring of certain players based on productive efficiency, as in Athey and Bagwell (2001).

Whereas in a classic cartel, the attraction of defecting is to steal the full cartel profit, deviation from a benchmark cartel only affects the final rate to the extent of the deviator’s submission—and not the demand or portfolio exposure position of the other banks. As a result, when the number of banks in the panel is larger, it is harder for each individual bank to move the rate and, thus, less attractive to deviate, which makes the benchmark cartel easier to sustain. A similar mechanism also makes average bid auctions, where the winning bid is the one closest to a trimmed average bid, more receptive to collusion, as found in Conley and Decarolis (2016). A benchmark cartel creates negative externalities for non-members, which induces the grand coalition, as in Yi (1997). That is, the cartel is externally stable in the sense of D’Aspremont, Jacquemin, Gabszewicz and Weymark (1983) only if all banks in the panel participate.

### 3.3 A Model of Benchmark Rate Collusion

We develop a model for one Libor, representative for different benchmark rates that are set for different maturities on a daily basis. Consider a panel of $N$ banks indexed $i \in \{1, ..., N\}$ that play an infinitely repeated simultaneous-move game. Let $\bar{v}_{it}$ be the baseline exposure of bank $i$ to changes in the interbank rate on trading day $t$ and $\bar{c}_{it}$ the baseline rate.

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36 We are indebted to Joe Harrington for suggesting this equilibrium concept.

37 We are indebted to Richard Gilbert for suggesting this property of a standard setting cartel.
of its transactions eligible for submission to the benchmark—which equals the borrowing cost of a bank. Both $\bar{v}_{it}$ and $\bar{c}_{it}$ are assumed to be private information daily draws from a common and known distribution.

New interbank rate $L_t (c_t)$ is a function of submissions $c_t = \{c_{1t}, ..., c_{Nt}\}$. If a bank intends to submit a rate $c_{it}$ that is different from its baseline rate $\bar{c}_{it}$, it would have to engage in transactions against this intended rate. The eligible transactions rate submission of bank $i$ is $c_{it} = \bar{c}_{it} + \Delta c_{it}$. We will refer to choice variable $\Delta c_{it}$ as 'eligible transactions rigging'. Similarly, bank $i$ may adjust its exposure to changes in the interbank rate. The realized exposure of bank $i$ at the time of the new benchmark rate is $v_{it} = \bar{v}_{it} + \Delta v_{it}$, where we will refer to choice variable $\Delta v_{it}$ as 'front-running'.

The gains of bank $i$ from changes in the interbank rate are modelled as

$$\pi_{it} = v_{it} (L_t (c_t) - L_{t-1}) - C (\Delta c_{it}, \Delta v_{it})$$

(3.1)

where $C (\Delta c_{it}, \Delta v_{it})$ are the costs banks incur from eligible transactions rigging and front running. These costs are assumed to be additively separable and strictly convex in both $\Delta c_{it}$ and $\Delta v_{it}$, constraining extreme adjustments as the risk of raising suspicion of manipulation is likely increasing in the degree of front-running and eligible transactions rigging.

Both $\bar{v}_{it}$ and $\bar{c}_{it}$ are assumed to be independent and identically distributed, each according to a symmetric, constant and commonly known continuous distribution, with $E [\bar{v}_{it}] = 0$ and $E [\bar{c}_{it}] = L_{t-1}$. The zero mean assumption captures that large part of the exposure stems from zero-sum

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38 It is reasonable to assume that panel banks can always find counterparties for their intended trades in these vast and liquid markets. Front-running takes place at the going prices, while for eligible transactions rigging a panel bank proposes terms that would be preferred by the unsuspecting outsider. Members could also carry out offsetting transactions within the cartel, either to generate free eligible transactions at the desired rate or to bring portfolio exposure positions more in line, although the latter would not increase overall cartel profits.

39 This assumption assures that a global maximum for each bank $i$’s objective function $\pi_{it}$ exists and is also unique if $C''_{\Delta v_{it}\Delta c_{it}}$ is small enough, which is a mild assumption as the two manipulation mechanisms relate to very different classes of transactions. As the first part of $\pi_{i}$ is linear in both $\Delta v_{it}$ and $\Delta c_{it}$, together with positive and increasing marginal costs, a necessary and sufficient condition for global maximum is that $C''_{\Delta v_{it}\Delta v_{it}} C_{\Delta c_{it}\Delta c_{it}} - C''_{\Delta v_{it}\Delta c_{it}}^2 > 0$. 

83
and volatile OTC derivative markets. $E[\bar{c}_{it}] = L_{t-1}$ reflects that the Libor is a main signal to creditors, who would not know about any manipulation. The mean is assumed equal across panel banks, which are all global systemically important banks. Shocks to the borrowing costs of banks are assumed to be non-persistent. Note, importantly, that under these assumptions the optimization problem of banks is static, because only the difference to the current rate matters and not its absolute level.

Figure 3.1 illustrates the timing in our model. Each bank $i$ receive at the start of day $t$ their baseline true borrowing costs $\bar{c}_{it}$ and initial exposure position $\bar{v}_{it}$. After determining their strategies (either manipulating individually or after exchanging information in a cartel), they have until the submission deadline to manipulate their eligible transactions through $\Delta c_{it}$. This deadline is at 11 a.m. in case of Libor. Front-running $\Delta v_{it}$ can be done until new rate $L_t$ is published and information about the future rate is no longer private, which is at 11.45 a.m. in case of Libor.

Section 3.3.1 outlines a simplified version to provide insight on behavior under different strategies. We also provide the first-best collusive strategy as in Rotemberg and Saloner (1986) and a more straightforward episodic break-up strategy as in Fershtman and Pakes (2000). We show that even for this simplified model, a closed-form analytical solution cannot be derived. Section 3.3.2 outlines the full model, for which we prove existence of the episodic break-up strategy. Section 3.3.3 discusses several benchmark cartel properties.
3.3. A Model of Benchmark Rate Collusion

3.3.1 Simple Model

In this simple model, we assume that daily baseline exposure draws $\bar{v}_{it}$ and borrowing cost draws $\bar{c}_{it}$ are commonly known to the banks, that banks can only engage in eligible transactions rigging $\Delta c_{it}$ and that the new interbank rate $L_t$ is computed as a simple average of final eligible rates $c_t$. Furthermore, we assume as cost function $C(\Delta c_{it}) = \alpha (\Delta c_{it})^2$, with $\alpha \geq 0$. This provides the following objective function:

$$\pi_{it} = \bar{v}_{it} \left( \frac{1}{N} \sum_{j=1}^{N} (\bar{c}_{jt} + \Delta c_{jt}) - L_{t-1} \right) - \alpha (\Delta c_{it})^2. \quad (3.2)$$

Below the different strategies in the daily stage games are outlined, followed by strategies that use repeated interaction to stabilize collusion.

3.3.1.1 Stage Game Strategies

On each trading day $t$, banks can decide to manipulate individually or collusively, or deviate from a collusive agreement. Below the optimal strategies for each are outlined.

**Individual manipulation** Under honest behavior, banks would refrain from both front-running and eligible transactions rigging. However, this is not individually optimal. If a bank manipulates the rate individually, it maximizes its profits with respect to eligible transactions rigging $\Delta c_{it}$. For each bank $i$, the optimal eligible transactions rigging is given by

$$\Delta c_{it}^* = \frac{\bar{v}_{it}}{2\alpha N}. \quad (3.3)$$

Take $\pi_{it}^*$ as the profit for bank $i$ under $\Delta c_{it}^*$, which can be derived explicitly.

**Collusive quoting** Through $L_t (c_t)$, the payoff function of each bank depends not only on its own exposure and eligible transactions, but also on the submissions of other banks. This provides an incentive to coordinate.
Chapter 3. Collusive Benchmark Rate Fixing

Under joint-profit maximization, the optimal eligible transactions rigging for each bank \( i \) would be given by

\[
\Delta c^C_{it} = \sum_{j=1}^{N} \frac{\bar{v}_{jt}}{2\alpha N}.
\] (3.4)

Take \( \pi^C_{it} \) as the profit for bank \( i \) when each bank sticks to the joint-profit maximizing strategy, which can again be derived explicitly.

**Deviation**  A cartel member may want to deviate from collusive quoting if its trading interests are not sufficiently aligned with that of the cartel. A deviating bank chooses its individual profit-maximizing \( \Delta c^D_{it} \), assuming all other banks play the collusively optimal strategy \( \Delta c^C_{it} \). In this simplified case, note that the optimal deviation is simply the same as under individual manipulation, as a bank by construction contributes a fraction \( 1/N \):

\[
\Delta c^D_{it} = \frac{\bar{v}_{it}}{2\alpha N}.
\] (3.5)

Take \( \pi^D_{it} \) as the profit for bank \( i \) under \( \Delta c^D_{it} \), which can again be derived explicitly.

### 3.3.1.2 Cartel Stability

Because the per-period profit under deviation is higher than under collusive quoting, the cartel would need to stabilize adherence to any collusive agreement using repeated interactions. Specifically, in case any bank deviates from the collusive agreement, the other banks can punish by reverting to individual optimization. Banks will then continue colluding as long as the expected value of collusion \( V^C_{it} \) is at least as high as the expected value of deviation \( V^D_{it} \).

Using discount rate \( \delta \in (0,1) \) and \( \pi^*_it, \pi^C_{it} \) and \( \pi^D_{it} \) as defined above, we can specify for bank \( i \) in period \( t \) the expected value of collusion and
deviation as

\[ V_{it}^C = \pi_{it}^C + \delta E [V^C] \quad \text{and} \quad V_{it}^D = \pi_{it}^D + \delta E [V^P], \]  

(3.6)

where \( E [V^C] = \sum_{t=0}^{\infty} \delta^t E [\pi^C] \) is the expected discounted continuation value of collusion and \( E [V^P] \) the expected discounted value under punishment following a deviation. For every punishment strategy in which deviation triggers \( T \geq 0 \) periods of reversion to individual manipulation, the off-equilibrium occurrence of punishment means that increasing \( T \) only increases cartel stability. Therefore, it is optimal to set \( T \to \infty \), so that \( E [V^P] = \sum_{t=0}^{\infty} \delta^t E [\pi^*] \).

Below we outline the first best solution in which joint profits are maximized subject to the constraint that each bank prefers continuous collusion over deviation—as in Rotemberg and Saloner (1986). We also outline a more straightforward episodic break-up strategy in which banks maximize joint profit as long as each bank prefers collusion over deviation, but revert to one-period individual optimization otherwise—as in Fershtman and Pakes (2000).

**Continuous Collusion** Optimally, banks maximize joint profits subject to the constraints that each bank \( i \) has a higher expected value of collusion than deviation:

\[
\max_{\Delta c_{it}} \sum_{j=1}^{N} \pi_{jt} \quad \text{subject to} \quad V_{it}^C \geq V_{it}^D \quad \forall i. \tag{3.7}
\]

Solving for constraint \( V_{it}^C \geq V_{it}^D \) provides \( \pi_{it}^D - \pi_{it}^C \leq \frac{\delta}{1-\delta} (E [\pi^C] - E [\pi^*]) \), where the left-hand side are stochastic draws based on baseline values \( \bar{v}_{it} \) and \( \bar{c}_{it} \) and the right-hand side is a fixed value.

Although a solution exists in this simplified model, a closed-form analytical expression of the optimal strategies and associated profits cannot be derived.\(^{40}\) This is because the joint-profit maximizing assumption for \( \pi_{it}^C \)

\(^{40}\)A solution exists because \( \Delta c_{it}^C = \Delta c_{it}^D \) in this simplified model. This means that colluding banks can always set \( \Delta c_{it}^C \) sufficiently close to \( \Delta c_{it}^* \) for each bank \( i \) such that no
no longer holds (as it may be required for banks to set collusive rates differently to satisfy the constraints) and the above optimization is a fixed-point equation for which continuation value $E[\pi^C]$ is unknown.

**Episodic Break-Up** In line with observed behavior discussed in Section 3.1, banks could also adopt a more straightforward episodic break-up strategy, in which they coordinate only if all banks prefer collusion over deviation. The optimization now looks as follows

$$\max_{\Delta c_t} \sum_{j=1}^{N} \pi_{jt} \quad \text{if} \quad V_{it}^C \geq V_{it}^D \quad \forall i$$  

(3.8)

and $\max_{c_t} \pi_{it}$ for all $i$, as under individual manipulation, otherwise. Solving for constraint $V_{it}^C \geq V_{it}^D$ now provides

$$\pi_{it}^D - \pi_{it}^C \leq \frac{\delta}{1-\delta} \left( (1-\rho) E[\pi^C|NB] + \rho E[\pi^*|B] - E[\pi^*] \right)$$  

(3.9)

for all $i$, where $\rho \in [0,1]$ is the probability at which one-period break-up occurs, $E[\pi^C|NB]$ the expected profit of coordination on joint-profit maximization conditional on no break-up and $E[\pi^*|B]$ the expected profit of individual manipulation conditional on break-up.

Note that the left-hand side of the constraints are the payoff differentials, which are stochastic draws based on baseline values $\bar{v}_{it}$ and $\bar{c}_{it}$. The right-hand side is a fixed cut-off value. Using $\Psi (\delta, \rho)$ to indicate this right-hand side cut-off value, break-up probability $\rho$ is determined from the following fixed-point equation

$$\rho (\delta) = 1 - \Pr \left[ \max_{i \in \{1, \ldots, N\}} \left( \pi_{it}^D - \pi_{it}^C \right) \leq \Psi (\delta, \rho) \right].$$  

(3.10)

This fixed-point equation is illustrated in Figure 3.2, which shows break-up probability $\rho$ as equal to the area under the distribution of the highest bank is better off deviating. Put differently, banks can always set $\Delta c_{it}^C = \Delta c_{it}^*$ to ensure that a bank weakly prefers collusion over deviation.
3.3. A Model of Benchmark Rate Collusion

payoff differential and to the right of cut-off value $\Psi (\delta, \rho)$.

Figure 3.2: Distribution Highest Payoff Differential

![Distribution Highest Payoff Differential](image)

Notes: Illustration of the distribution of the highest payoff differential and how break-up probability $\rho$ is determined as a function of cut-off value $\Psi (\delta, \rho)$.

Although a unique solution again exists (proven below for the extended model), a closed-form analytical expression of the optimal strategies and associated profits can again not be derived. This is in spite of the fact that we do have a closed-form expression for $\pi_i^C$ as the joint-profit maximizing outcome now, which we did not have under continuous collusion. The limitation here is that we do not have a closed-form expression for the distribution of the highest payoff differential and the optimization is again a fixed-point equation, for which continuation values $E [\pi^C | NB]$ and $E [\pi^* | B]$ are unknown.

3.3.2 Full Model

The absence of an analytical solution for either continuous collusion or episodic break-up under even the simplified model motivates an extension that includes more of the unique features of benchmark rate setting. The main aim here is to prove at least existence. Additionally, subsequent
simulations using the full model are used to illustrate comparative statics as well as possible screens to detect benchmark collusion.

We now assume that $\bar{v}_{it}$ and $\bar{c}_{it}$ are private information, that banks can engage in both eligible transactions rigging $\Delta c_{it}$ and front-running $\Delta v_{it}$ and that $L_t(c_t)$ is computed as a trimmed average of $c_t = \{c_{1t}, ..., c_{Nt}\}$. We call the set of submissions from which the upper and lower share of ranked quotes are discarded the ‘trimmed range’ $\mathcal{T}$ consisting of $n$ banks. Hence,

$$L_t(c_t) = \frac{1}{n} \sum_{j \in \mathcal{T}} c_{jt}. \quad (3.11)$$

Below the different strategies in the daily stage games are outlined, followed by strategies that use repeated interaction to stabilize collusion.\(^{41}\)

### 3.3.2.1 Stage Game Strategies

As in the simple model, banks can decide on each trading day $t$ to manipulate individually or collusively, or deviate from a collusive agreement. Below the optimal strategies for each are outlined.

**Independent manipulation** Unlike in the simple model, banks are now uninformed of the baseline exposure and borrowing cost of the other panel banks. Under independent manipulation, each bank is therefore induced to independently engage in some front-running and eligible transactions rigging by maximizing own expected gains as follows

$$\pi_{it}^{BN} : \max_{\Delta v_{it}, \Delta c_{it}} E[\pi_{it}], \quad (3.12)$$

where $\pi_{it}^{BN}$ is used to indicate the payoff of bank $i$ in period $t$ in this static Bayesian-Nash equilibrium. The equilibrium need not be in pure strategies.

\(^{41}\)Note that we do still abstract from some of the details of the reformed rates, such as the assignment of higher weights to transactions closer to the submission deadline and the administrator’s discretion to discard contributions, which are not essential to the analysis and are straightforward to include in practical collusion.
3.3. A Model of Benchmark Rate Collusion

**Collusive quoting** When banks collude, they share their baseline information in order to coordinate behavior. To allow for tractability, we assume that banks share their baseline information truthfully. Although not guaranteed, this assumption is unlikely to be critical. Firstly, lying about eligible transactions rates would be revealed when the final rates are published. Secondly, lying about baseline exposure remains constrained: (i) by overstating its exposure, a bank may itself be assigned even higher (and more costly) eligible transactions rigging and (ii) in case of the episodic breakup strategy, more extreme positions makes the cartel less stable.

Under joint-profit maximization, banks would optimize as follows

$$
\pi_{it}^C : \max_{\Delta v_t, \Delta c_t} \sum_{j=1}^{N} \pi_{jt},
$$

(3.13)

where $\Delta v_t$ and $\Delta c_t$ indicate the set of choice variables for all banks. Let $\pi_{it}^C$ indicate the payoff of bank $i$ in period $t$ in case of joint-profit maximization. We can offer the following result.

**Proposition 1** There exists a per-period unique globally optimal cartel strategy.

**Proof.** See Appendix.

Joint-profit maximization is efficient in the sense that the order of the baseline transaction rates is preserved in the submissions that are asked of the members, as it minimizes the total eligible transactions rigging costs. Banks outside $T$, even though their submissions are discarded in the determination of the interbank rate, may also be called upon to engage in eligible transactions rigging in order to move over and accommodate the rigging by banks within $T$. Figure 3.3 illustrates such a situation in the case of four panel banks, the middle two of which are in the trimmed range.
Chapter 3. Collusive Benchmark Rate Fixing

Figure 3.3: Illustration Coordinated Eligible Transactions Rigging

Notes: Illustration of eligible transactions rigging under coordinated quoting in $N = 4$. In this case, panel bank 1 is not in $\mathcal{T}$ but still engages in transaction rigging.

Deviation  A cartel member may again want to deviate from the collusive strategies if its trading interests are not sufficiently aligned with that of the cartel. A deviating bank chooses its individual strategies assuming other banks stick to the collusive strategies, so as follows

$$
\pi^D_{it} : \max_{\Delta v_{it}, \Delta c_{it}} \pi_{it} \{ \Delta v^C_{-it}, \Delta c^C_{-it} \}, \quad (3.14)
$$

where $\Delta v^C_{-it}$ and $\Delta c^C_{-it}$ are the choice variables of under banks under joint-profit maximization. $\pi^D_{it}$ is used to indicate the payoff of bank $i$ in period $t$ in case it optimally deviates from the cartel agreement.

3.3.2.2 Cartel Stability

Similar as in the simple model, the cartel needs to stabilize against the incentive to deviate by using repeated interaction. Take $\pi^B_{it}$, $\pi^C_{it}$ and $\pi^D_{it}$ as the profits under Bayesian-Nash individual manipulation, collusion and deviation as defined above and $V^C$ and $V^D$ as the expected value of collusion and deviation.

Below we again outline the first best solution in which joint profits are
maximized subject to the constraint that each bank prefers collusion over deviation—as in Rotemberg and Saloner (1986). Although a solution exists in the simple model, finding a solution in the full model is considerably more complex and computationally demanding. For the more straightforward episodic break-up strategy—as in Fershtman and Pakes (2000)—we can however prove existence and uniqueness. Subsequent simulations also show how the optimal strategy can be computed.

**Continuous Collusion** Optimally, banks maximize joint profit subject to the constraints that each bank $i$ has a higher expected value of collusion than deviation:

\[
\max_{\Delta v_t, \Delta c_t} \sum_{j=1}^{N} \pi_{jt} \quad \text{subject to} \quad V_{it}^{C} \geq V_{it}^{D} \quad \forall i. \quad (3.15)
\]

Solving for $V_{it}^{C} \geq V_{it}^{D}$ now provides $\pi_{it}^{D} - \pi_{it}^{C} \leq \frac{\delta}{1-\delta} \left( E[\pi^{C}] - E[\pi^{BN}] \right)$ for all $i$, where the left-hand side are again stochastic draws based on baseline values $\bar{v}_{it}$ and $\bar{c}_{it}$ and the right-hand side a fixed value.

Note that continuous collusion in benchmark rates is much more complex than in conventional markets, because of asymmetric payoff functions and $N$ different inequality constraints. Finding common ground is also computationally demanding. Firstly, solving (3.15) requires knowing the expected collusion payoff $E[\pi^{C}]$, which is not known a priori. Additionally, both the optimization and its constraints are endogenous, as $\pi_{it}^{C}$ and $\pi_{it}^{D}$ both follow from the solution and are part of the constraints. Furthermore, it is not guaranteed that ranking of submissions remains unchanged—as is used in the proof of Proposition 1. Finally, note that brute force calculations on (3.15) are not feasible, as the number of discretized and restricted strategies that would need to be checked is equal to a necessarily high number of small bins to the power $2N$.\[^{42}\] As it seems prohibitively com-

\[^{42}\text{For example, with 16 banks choosing the two choice variables bounded and discretized (somewhat arbitrarily) to 300 bins, the cartel algorithm would still need to check } 300^{2^{16}}, \text{ or approximately } 10^{80} \text{ cases—which is of the same order of magnitude as the number of atoms that are in the universe.}\]
plex to determine the continuous collusion strategy, we revert to the more straightforward episodic break-up strategy.

**Episodic Break-Up** In line with observed behavior discussed in Section 3.1, banks can also collude by maximizing joint profit as long as no bank has an incentive to deviate, but revert to one-period individual optimization otherwise.

During a break-up, the panel banks determine their contributions non-cooperatively with complete information as

\[
\pi^N_{it} : \max_{\Delta v_{it}, \Delta c_{it}} \pi_{it},
\]  

(3.16)

where \(\pi^N_{it}\) is used to indicate the payoff of bank \(i\) in period \(t\) in this Nash equilibrium. Note that this involves individual manipulation, but now on the basis of full information exchange. The optimization now looks as follows

\[
\max_{\Delta v_{it}, \Delta c_{it}} \sum_{j=1}^{N} \pi_{jt} \quad \text{if} \quad V^C_{it} \geq V^D_{it} \quad \forall i
\]  

(3.17)

and individual optimization otherwise. In case of deviation, banks still revert to Bayesian-Nash optimization for each subsequent period. Solving for constraint \(V^C_{it} \geq V^D_{it}\) now provides

\[
\pi^D_{it} - \pi^C_{it} \leq \frac{\delta}{1-\delta} \left( (1 - \rho) E [\pi^C|NB] + \rho E [\pi^N|B] - E [\pi^{BN}] \right)
\]  

(3.18)

for all \(i\), where \(\rho \in [0, 1]\) is the probability at which one-period break-up occurs, \(E [\pi^C|NB]\) the expected profit of coordination (joint profit maximization) conditional on no break-up and \(E [\pi^N|B]\) the expected profit of individual manipulation conditional on break-up.

As before, the left-hand side of the constraints are the payoff differentials, which are stochastic draws based on baseline values \(\bar{v}_{it}\) and \(\bar{c}_{it}\), and the right-hand side a fixed cut-off value. Using \(\Psi(\delta, \rho)\) again to indicate this right-hand side cut-off value, break-up probability \(\rho\) is determined
from the same fixed-point equation

$$\rho (\delta) = 1 - \Pr \left[ \max_{i \in \{1, \ldots, N\}} \left( \pi_{it}^D - \pi_{it}^C \right) \leq \Psi (\delta, \rho) \right]. \quad (3.19)$$

We can now establish the following proposition on existence and uniqueness in case of collusion with episodic break-up.

**Proposition 2** For a continuous and sufficiently widely supported distribution of $\max_i (\pi_{it}^D - \pi_{it}^C)$, there exists a unique $\rho \in (0, 1)$ that maximizes cartel profits.

**Proof.** See Appendix. ■

For reasonable assumptions on the underlying stochastics, the cartel always exists to at least share information, regularly quotes collusively ($\rho < 1$), but occasionally reverts back to non-coordinated quoting with inside information ($\rho > 0$) to deal with extreme value exposure and eligible transactions rate drawings. Note that although switches between collusion and break-up are discrete, the cartel agreement itself is continuous in that actual deviation is off-equilibrium.

We say the cartel is more ‘steady’ if $\rho$ is closer to 0, so that it breaks up less regularly. Equation (3.19) does not yield a closed-form solution for the effect of the discount factor $\delta$ or manipulation costs $C(\Delta c_{it}, \Delta v_{it})$ on cartel steadiness $\rho$ in general, which is probability distribution-specific. However, a negative relationship between $\delta$ and $\rho$ is to be expected, as the more patient the panel banks are, the less tempted they are to deviate with a more extreme position.

### 3.3.2.3 Collusion of a coalition

A panel bank benefits from participating in the benchmark cartel in two ways. It shares in the information on all other cartel members’ strategies and the new rate prior to becoming public knowledge, which allows it to front run. In addition, by submitting its own private information, it ensures that the cartel takes the bank’s trading interest into account in
formulating the collusive manipulation strategy. The cost is that a bank may be required to rig its eligible transactions more than if it behaved independently, but in expectation it is better off participating in the scheme.

Suppose that all \( N \) panel banks know that there exists a benchmark cartel that consists of a coalition of \( M \leq N \) of the panel banks. Not participating in the partial benchmark cartel means not sharing in the information that all cartel members exchange. The (partial) cartel maximizes the joint profits of its members. We call a coalition of panel banks colluding on the benchmark \emph{internally} stable if no coalition member has an incentive to leave the coalition to act on its own. A coalition is \emph{externally} stable if no individual panel bank outside the partial cartel has an incentive to join the coalition. Maintaining perfect monitoring, we then have the following result.

\begin{proposition}
Only a full-panel cartel is both internally and externally stable.
\end{proposition}

\begin{proof}
See Appendix. \end{proof}

Ex ante, no cartel member has an incentive to unilaterally leave the cartel, because it would lose both its interests being taken into account by the (remaining) cartel and inside information on the future rate needed to front run. Moreover, unless all panel banks are part of the cartel, there would always be an outsider willing to join. If there is collusive manipulation of the benchmark rates and all panel banks are aware of it, the grand coalition can be expected to be involved.

It is not necessarily the case that a cartel of \( N \) members also yields the highest expected cartel profits among all possible cartel sizes. Although the unconstrained expected per-period profit of colluding—conditional on there not being a break-up—is larger for larger cartel sizes, the effect of cartel size on the break-up probability \( \rho \) is ambiguous. How the overall expected profits of the cartel are affected by its number of members \( M \) depends therefore on a trade-off. In a smaller cartel, on the one hand, the interests of a member with a sizable exposure position get a relatively higher weight in determining the cartel strategy, so that a larger member is
more likely to benefit from the cartel strategy. On the other hand, collusive profits are likely to be lower with less information about the future rate. Although a partial cartel may or may not be better off excluding some banks, note that it cannot do so unilaterally—as any outside bank can threaten with reporting the cartel.

Note that for a partial cartel monitoring of the adherence to a partial-panel cartel from the published rate only is no longer straightforward, as it also depends on the submissions of the independent panel banks. Deviation is more constrained when more banks are in the cartel, because uncertainty around the collective behavior of independent banks decreases.\footnote{Note that it is possible to tell defection from the published rate with certainty only if there are at most \((N-n)/2\) panel members outside of the collusive coalition, so that a maximum possible rate—when all outsiders quote above the highest cartel submission—and a minimum possible rate—when all outsiders quote below the lowest cartel submission—can be established. With that many outsiders, the resulting rate without deviation will at least (at most) be an average of some of the lower (upper) quotes of the collusive coalition. A lower (higher) published rate would mean that at least one bank deviated.} In any case, defection will become apparent with the publication of individual submissions.

### 3.3.3 Benchmark Cartel Properties

As shown, a closed-form, analytical solution of the episodic break-up equilibrium using the primitives of the stage game cannot be formulated. However, it is still possible to discuss several properties specific to benchmark collusion.

One way to consider the original protocols is as having very low eligible transactions rigging costs. How this affects the cartel equilibrium depends on specifics of the case: in general, both the costs of collusion and of defection decrease, so that the net effect on cartel steadiness \((\rho)\) is ambiguous. If defection profits increase, it causes the cartel to break up constantly, to the point of being merely an exchange of information to play Nash rather than Bayesian-Nash.

The introduction of transaction-based submissions increases manipulation costs, which on the one hand make defection less attractive, so that
higher extreme value positions can be sustained without the cartel having to break up. On the other hand, higher manipulation costs also reduce cartel profits, thus making collusion less attractive. Yet, even if break-ups occur less often, the cartel would be manipulating the rates less extremely when manipulation costs are higher. Therefore, although likely reducing the extent of manipulation, broadening the class of eligible transactions can increase the frequency of collusive quoting.

Further reforms to decrease the trimmed range $\mathcal{T}$ by discarding more of the highest and lowest quotes also has opposing effects on incentives to collude. Although fewer banks can influence the rate by deviating from the collusive agreement, each one has a larger individual effect on the published rate, as a smaller number of quotes are averaged, so that the overall effect on defection incentives is ambiguous. The same is true for a lower number of panel banks $N$, which also increases the likelihood that extreme position drawings are of the same sign as the average portfolio, reducing the expected cost of collusion.

Under the assumptions made, the benchmark rate time-series would fluctuate around a fixed mean, as the probabilities of higher and lower drawings are equal. The theory does predict that the volatility under collusion is larger than under independent quoting. The cartel benefits from more volatility in the rates over time, as that allows the panel bank members to better exploit their inside information on the rates movements in advance by adjusting their portfolio exposures, against non-initiated financial institutions and investors. During break-ups, these benefits are much smaller, as cartel members no longer take into account the externality effects of their behavior. It can also cause them to pursue conflicting directional changes, reducing volatility.

In front-running, the theory also predicts a clear pattern. Periods of collusion would leave traces in transactions over time, as the banks involved change their exposure position in the same direction in which the rate is rigged. A high correlation between a bank’s transactions in the front-running window and the subsequent change in the published rate could be an indication of suspicious exposure alignment. A screen would flag
increases in these combined correlations over time or compared to non-panel banks. An advantage of correlations over variances is that they are not largely driven by larger banks with larger trading volumes or lower front-running costs. To be effective, however, a correlation screen requires a complete picture or an unbiased sample of all bank transactions in the window.

In the next section, we use simulations of our model to illustrate the possible comparative statics discussed in this section. We also show how empirical screens could be used to identify collusive benchmark rate fixing.

3.4 Collusive Rate Patterns

In this section we simulate the full model in order to provide illustrations of comparative statics, as well as the type of empirical trail it may leave. We assume the cost function

\[ C(\Delta c_{it}, \Delta v_{it}) = \alpha \Delta c_{it}^2 + \beta \Delta v_{it}^2, \] (3.20)

in which \( \alpha > 0 \) and \( \beta > 0 \) are cost parameters. The resulting linear-quadratic payoff function satisfies the conditions for a unique global maximum. Parameter values in the simulation are \( N = 16, n = 8, \alpha = \beta = 1, \bar{v}_{it} \sim N(0,0.1) \) and \( \bar{c}_{it} \sim N(L_{t-1},0.1) \), with starting value \( L_0 = 1 \).\(^{44}\)

First, using Monte Carlo simulations the implicit probability of breakup was calculated for different discount rates. Providing convergence in the sample distribution, we simulated 100,000 daily draws of baseline eligible transactions rates \( \bar{c}_{it} \) and baseline exposures \( \bar{v}_{it} \), derived payoffs in static Bayesian-Nash (\( \pi_{it}^{BN} \)), collusion (\( \pi_{it}^{C} \)), defection (\( \pi_{it}^{D} \)) and static Nash (\( \pi_{it}^{N} \)) in each draw, for each bank \( i \in \{1,...,N\} \), and determined the expected (conditional) payoffs \( E[\pi_{it}^{BN}], E[\pi_{it}^{C}|NB] \) and \( E[\pi_{it}^{N}|B] \). The simulated distribution of the largest payoff differential \( \max_i(\pi_{it}^D - \pi_{it}^C) \) is then used to identify fixed point \( \rho \) as a function of the discount rate \( \delta \). Second, with

\(^{44}\)Qualitatively similar results obtain for different values of \( \alpha, \beta \) and the variances—in particular for \( \alpha > \beta \) and the variance of \( \bar{v}_{it} \) of a higher order than that of \( \bar{c}_{it} \).
the elements obtained a 240-day time series of the interbank rate was generated, looking separately at honest, Bayesian-Nash and optimal collusive behavior. The MATLAB\textsuperscript{®} source code of the cartel routine, including advised positions and submission targets, for $N = 4$ is given as an appendix.

### 3.4.1 Payoffs and Break-Ups

Figure 3.4 gives the simulated payoff frequency distributions for independent Bayesian-Nash (blue) and collusive (orange) quoting. Under independent quoting, payoffs are more closely concentrated around zero—the mean is slightly positive because of the independent manipulation benefits, which are small.\footnote{$E \left[ \pi^{BN} \right] \approx 0.000024, \sigma_{BN} \approx 0.00274; E \left[ \pi^{C} \right] \approx 0.000898, \sigma_{C} \approx 0.00679; E \left[ \pi^{N} \right] \approx 0.000218, \sigma_{N} \approx 0.00283.$} Portfolio exposure adjustment is more than 20 times higher under the cartel. The cartel materializes higher profits more often but also losses: there are more instances in which cartel members take one for the team in the sense that they would have done better under independent quoting. Yet, in collusion, both losses and profits are more concentrated on the right side of their spectra: losses are more often closer to zero and profits are more often large. As a result, the average expected payoff is almost 40 times higher under collusion than under independent quoting. All panel banks gain in expectation from participating in the collusion.

For $\delta = 0.90$, the critical cut-off value below which collusion is stable is $\Psi \approx 0.0028$. Together with the conditional expected collusion payoff this implies $\rho \approx 0.38$, which is unique.\footnote{$E \left[ \pi^{C} | NB \right] \approx 0.00012, E \left[ \pi^{N} | B \right] \approx 0.00069$ and $\rho \approx 0.37901$.} Figure 3.5 plots the break-up probability $\rho$ as a function of $\delta$ for different cost levels of eligible transactions rigging ($\alpha$) and different trimmed ranges ($T$). For this specification, cartel steadiness increases in $\delta$ and the cost of manipulation, as monotonic increases in the cost of eligible transactions rigging decrease the probability of break-up for all discount factors. Although the higher manipulation costs reduce both defection and future cartel profits, the decrease is larger for defection profits, which results in more steady continuous collusion. The same is true for averaging over fewer middle quotes by discarding a
3.4. Collusive Rate Patterns

Figure 3.4: Frequency Distribution Monte Carlo Simulations

Notes: Payoff frequencies from Monte Carlo simulations under independent quoting (with $E[\pi^{BN}] \approx 0.000024$) and collusive quoting (with $E[\pi^C] \approx 0.000898$).

Figure 3.5: Comparative Statics Simulated Cartel Break-Up Probability

Notes: Simulated cartel break-up probability $\rho$ as a function of discount rate $\delta$ for different values of $\alpha$ (left) and trimmed range $T$ (right), where $n$ out of $N$ banks are included in the trimmed range.
larger part of extreme submissions. Although not shown here, the effect of
different panel sizes \( N \) on \( \rho \) is negligible.

Note that when eligible transactions rigging is punitively costly (\( \alpha \) very
high), the banks collude only to exchange information and front run indi-
vidually. The published benchmark rate is unaffected. There is no incentive
to deviate in that case, so that the cartel never breaks up. In the other
extreme case, when the costs of quote submission are very low, the cartel
breaks up almost all of the time but continues to exchange information for
individual front-running purposes (\( \rho \) approaches one when \( \alpha \) goes to zero).

\[ \text{3.4.2 Time Series} \]

With the fixed point determined, we simulate time series. Figure 3.6 dis-
plays an interbank rate over time for \( \delta = 0.90 \), first when banks determine
their submissions independently, respectively honest and individually op-
timal for 60 days each, and then in continuous collusion with episodic
break-ups for 120 days. In the collusion period, the vertical shaded areas
are episodes of non-cooperative quoting following an extreme value draw-
ing.\(^{47}\)

Although the rate pattern may seem somewhat different between the
collusive and non-collusive periods, it is not evident from the simulated
benchmark rates alone whether the banks quoted independently or collu-
sively, nor which cartel periods were break-ups. Any drift in the mean
is random hysteresis as the rate follows a random walk around 1 and the
effects on volatility are not obvious.

Although not illustrated here, the intraday variance patterns (or quote
dispersion) in the submissions are not statistically different between the
regimes, either for the full panel or the banks that determine the rate.\(^{48}\)

\(^{47}\)This happened on 48 out of the 120 days of collusion, which is in the neighborhood
of the 38% projected.

\(^{48}\)On average, the intraday variance is 0.0098 for the full panel and 0.0019 for the
(trimmed range during the 60 honest days and 0.0099 for the full panel and 0.0017 for the
(trimmed range during the 120 manipulation days. These differences are not statistically
significant. Also, within the 120 manipulation days there is not significant difference
between the collusion days and temporary break-up days.
Colluding banks may be expected to ‘bunch’ together around one of the boundaries of the trimmed range, which would decrease the intraday variance of bank quotes. However, for the full panel this intraday variance decreasing effect is partially offset by a larger distance between the manipulating banks and the share of trimmed banks on the other extreme that quote their true rates.\(^{49}\) Within the trimmed range, there is more bunching together around one of the pivotal quotes in the same direction than under independent quoting, so that it is more likely that a decreased intraday variance is found.

The interday variance (or volatility) of the interbank rate over a certain time window does result in distinct differences in some of the runs. Figure 3.7 shows the interday variance for a five-day rolling window. Clearly, the benchmark rate under collusion displays more extreme behavior than during independent quoting—although again it is not possible to tell optimal Bayesian-Nash apart from honest independent quoting. The average volatility under collusive quoting is about twice as high as under independent behavior. This difference is statistically significant for both windows.\(^{50}\) Note that the underlying cause of the volatility is eligible transactions rigging, not the occasional break-up in and of itself—as volatility is larger when no break-up occurs.

Furthermore, the average absolute change in the interbank rate is significantly different between the break-up and full collusion regimes.\(^{51}\) Moreover, within the collusion period it is significantly higher in no break-up than during break-up.\(^{52}\) These markers are robust against changes in the

\(^{49}\) Abrantes-Metz, Kraten, Metz and Seow (2012) conjectured that the reduced intraday variance they found was indicative of collusion, but we do not find evidence for it in our illustration.

\(^{50}\) Using a one-sided Wilcoxon ranked sum test, the \(p\)-value is below 0.00001. The average interday variance is 0.0006 during the 60 honest days and 0.0012 during the 120 manipulation days.

\(^{51}\) At the 1\% level. On average, the absolute change in the interbank rate is 0.0193 during the 60 honest days and 0.0304 during the 120 manipulation days.

\(^{52}\) Using a one-sided Wilcoxon rank-sum test, the null that the mean of the volatility is the same during no break-up and break-up within the collusion period is rejected with a \(p\)-value of 0.0301. The absolute change in the interbank rate is 0.0332 during collusion and 0.0262 during break-up.
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Figure 3.6: Simulated Benchmark Rate

Notes: Simulated benchmark rate under three different regimes: honest quoting, Bayesian-Nash independent quoting and collusion.

Figure 3.7: Volatility Screen

Notes: Five-day rolling window volatility in quotes (solid blue line) under the three different regimes, including results from the Bai-Perron structural break test (dashed orange line).
length of the rolling window. Nevertheless, although increased volatility is in line with our theory, in a substantial number of simulations, volatility patterns are not identifiably different.

### 3.4.3 Screening

The different benchmark rate patterns that our cartel theory predicts suggest empirical screens that can help identify signs of manipulation after the reforms and target deeper investigations by government agencies or private counterparties that are potentially affected. Fluctuations in the rate are more pronounced during periods of collusion (no break-up or break-up) than independent quoting (Bayesian-Nash or honest), which suggests non-standard variance screens. With the actual periods of collusive quoting unknown, Bai-Perron structural break tests can be used to identify cartel episodes more systematically. On the five-day volatility in Figure 3.7, the Bai-Perron test identifies one and only one break occurring on day 129, which is close to the actual break day 120.

For accurate application in practice, such collusion screens would need to be further calibrated and controlled for other drivers of volatility in benchmark rates, in order to avoid them falsely flagging as suspicious increased volatility between different days that is due to legitimate market events. However, banks’ quotes are difficult to rationalize with other measures of bank borrowing costs, as Snider and Youle (2014) show, even when including banks’ own quotes in other currency panels. Kuo, Skeie, Vickery and Youle (2014) list several reasons why comparable measures of bank borrowing costs would follow quite different paths than Libor. These volatility screens would require a considerable level of sophistication so as to be powerful in different circumstances.

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53Bai and Perron (1998, 2003) provide a collection of tests that allow for identifying structural changes, break dates and magnitudes of change in time series when both the number and the dates of the breaks are unknown. For an application to identify the begin and end dates of cartel effects, see Boswijk, Bun and Schinkel (2018).

54We note that the increased volatility under collusion in Figure 3.8 results in large part from the true borrowing costs following the published, manipulated Libor, in other words that \( E[\overline{c}_{it}] = L_{t-1} \). Upward (downwards) manipulation is followed by a higher
Figure 3.8 shows a simulation of the correlation between changes in the interbank rate ($L_t - L_{t-1}$) and daily positions ($\Delta v_{it}$ for all $i$) for a five-day rolling window. Under the assumptions in our model, in collusion banks adapt their portfolio exposure position perfectly in the same direction as the future rate, so that the correlation is 1, compared to 0 for honest quoting. The correlations are positive and fluctuate for Bayesian-Nash quoting, reflecting minor front-running based only on private information. The screen can also be applied to individual submissions $\Delta c_{it}$ instead of the rate, or to panel members individually.

**Figure 3.8: Correlation Screen**

Notes: Five-day rolling window correlation between changes in the interbank rate and changes in portfolio positions under the three different regimes.

Applied to real data, heterogeneity among banks and other trading during the day will make the correlation-screen distinction non-binary. For example, the exact moment of information exchange—that is, the opening of the front-running window—will not be known outside the cartel. Furthermore, other transactions that classify as eligible can take place simultaneously for non-collusive reasons. Panel banks may not involve all (lower) baseline value draw in the next period, which combines with the collusive variance. If we used a more stable mean for the daily rate drawing instead, volatility alone would remain a sufficient statistic to tell apart independent from coordinated quoting only rarely. However, this would require dynamic optimization.
of their trading activities worldwide—facing internal coordination issues or possibly lacking a complete picture themselves. Although correlations will be different in magnitude as a result, in general they can be markedly higher even with a somewhat shifted window, transaction set, or general noise in the transaction data. To complement, robustness checks for different length front-running windows up to the submission time can be used.

However, although data on the interbank rates and individual submissions of panel banks is readily available, the transaction data needed to reconstruct overall exposure positions is not. They are only starting to be systematically collected. Eligible transactions, although limited to interbank loan data, could to a certain extent be retrieved from the TARGET2 real-time gross settlements system, using a method such as the Furfine (1999) algorithm, for transactions within Europe. A similar data set, the Fedwire Funds Service, is the large-value bank payments system operated by the U.S. Federal Reserve banks. Kuo et al. (2014) develop a methodology to infer information about individual term dollar interbank loans settled through this system. However, the real challenge lies in identifying banks’ overall exposure positions to the rate, as these are largely driven by OTC derivatives transactions, which take place without an exchange. Data on those transactions is currently not publicly available.

Initiatives to construct Trade Repositories (TRs) aim at maintaining electronic records of all transactions data, including OTC derivatives transactions in which one of the counterparties is of the same nationality as the repository. If sufficiently developed in the future across different countries, these repositories could provide authorities with the necessary transactions data on a sufficiently detailed level to be useful in screening for collusive benchmark rates fixing. Our analysis advises on what data to collect for this purpose.

Finally, note that screening for increased intraday variance patterns may deter manipulation if panel banks are aware of this, also when imperfect. It would be hard to simultaneously circumvent these screens and gain from collusion, as the volatility the cartel generates to have inside information is an important source of cartel profits. Dodging the screens
thus undermines cartel stability. As an alternative to government oversight, counterparties to panel banks would have an interest in monitoring for rate manipulations, as they structurally lose out on their OTC trades with panel members that front run. Systematic differences in their rates of return on trades with panel and non-panel institutions may be indicative of collusive benchmark rates fixing.

### 3.5 Discussion and Extensions

Several of the assumptions we make warrant further discussion. We model portfolio positions as independently distributed around zero, so that there is no accumulation and expectations on future positions are unrelated to current positions. Although trade in OTC derivatives is fast-changing and vast in comparison, banks may have a relatively stable exposure profile of the same sign, such as long-term mortgage contracts with Libor-based rates. However, a steady bank-specific exposure profile, positive or negative, although still generating changing conflicts of interests with sufficiently large variances, introduces a drift in the rate manipulation in the direction of the sign of the panel’s overall mean. Our symmetric model can instead be interpreted as an approximation on the larger part of the portfolio or, alternatively, as being about desks or traders cartel-maximizing joint profits on their liquid trading books only, and not their employer banks’ overall exposures.

We assume that panel banks’ opportunity costs of funds fluctuate around a common mean and that shocks to baseline borrowing costs or exposure positions are not persistent. In practice, some banks may be able to borrow at lower rates than others, due to reputation, portfolio risk profile or scale, for example. The model can be extended to each bank $i$ having an idiosyncratic mean of the common distribution, such that $E_i[\tilde{c}_{it}] = L_{t-1} + \theta_i$. As long as the distribution of $\theta_i$ is symmetric around zero, this will still be consistent with the absence of drift in the (manipulated) published rate. If it additionally varies in time, such persistence in shocks to true borrowing costs introduces complex dynamic optimization, as do trade book building,
expectations about future demand, correlation of demand shocks and other features of business cycles, as analyzed in, among others, Haltiwanger and Harrington (1991), Kandori (1991) and Bagwell and Staiger (1997).

The daily rates can alternatively be assumed to be drawn from an unmanipulated mean, in particular a daily rate that follows from honest reporting only. If manipulation was indeed widespread and commonly known, possibly the initiated financial institutions would also account for an actual borrowing standard that would have followed from honest reporting only—like a ‘shadow Libor’. Yet, even if there were purer determinants for the panel banks’ true costs of borrowing, it seems reasonable to expect those to have been contaminated by the Libor manipulations. If such a shadow Libor existed and were the mean of the distribution of banks’ true borrowing costs, panel members’ manipulation today would affect their ability to profitably manipulate tomorrow, which would also introduce dynamic optimization.

We model manipulation costs equally across all panel banks, which is reasonable to assume for the main cost components, in particular raising suspicion and suboptimal transactions in eligible transactions rigging. In front-running, however, certain panel banks may face lower costs than others, depending on their core activities and size. Our proof of existence of a one-shot collusively optimal set of submissions relies on the fact that, with equal manipulation costs, banks with the highest baseline true cost parameter submit the highest quotes and that the cost parameters are equal across banks. With heterogeneous costs, this order may be broken and a certain set of collusive submissions may, in theory, be achieved in different ways: either by choosing the minimum amount of eligible transactions rigging or by letting banks with lower manipulation costs engage in more eligible transactions rigging than others. Given banks’ heterogeneous cost functions, the probability of a collusive outcome not being unique—which would require the exact occurrence of certain draws—is zero, so that a unique global cartel optimum remains, provided all individual cost functions satisfy the existence conditions.

No side payments are necessary for maintaining collusion as, due to the
symmetry of the model, all cartel members have the same positive expected profits from participating. Explicit transactions between the panel banks or more sophisticated forms of side payments, such as partially swapping positions internally, in which a cartel member with a major profitable position to the cartel strategy would trade with other members to mitigate their positions opposite to the general cartel interest, can further facilitate collusive manipulation through the internal alignment of interests. Note, however, that they do not increase cartel profits, as the panel’s overall net portfolio position generally revolves around zero—while incurring transaction costs. Wash trades or doing offsetting eligible transactions between cartel members can reduce manipulation costs, the effect of which on cartel stability is ambiguous. These, as well as skipping a quote, may be made part of a more sophisticated cartel strategy that comes closer to continuous collusion by periodically alleviating members with strong defection incentives.

We find that all panel banks in the know about the cartel would want to be part of it. The coordinated manipulation cases so far investigated suggest rather that smaller subsets of traders may have colluded, possibly unbeknownst to part of the panel. We find that all panel banks in the know about the cartel would want to be part of it. Outsiders of the cartel would not have taken their interests into account when determining submissions and would miss out on inside information. Monitoring adherence to agreements within smaller groups is also more difficult.

Alternatively, after the full-panel cartel shared information, collusion in (rotating) sub-coalitions could be part of the continuous collusion strategy. This would allow the cartel to avoid manipulation costs and break-ups, as banks with an unfavorable position that would otherwise have incentives to defect, could skip a period of cartel participation. Such extensions of the cartel strategy space, however, introduce incentives to falsely report and to freeride on a partial cartel, which we leave for further study.

Although sharing all private information allows the cartel members to determine collusive strategies and play Nash during break-ups, it also has the unattractive feature that it facilitates defection. To avoid this, the
colluding banks could employ an independent administrator who collects the information, runs the cartel software and only then provides personal instructions on quotes to be submitted and front-running. Organizing the cartel this way would reduce the incentives to deviate—break-up profits would be Bayesian-Nash instead of Nash. However, there is no hard evidence that such an administrator existed.\footnote{A broker from ICAP was nicknamed ‘Lord Libor’ for sending a daily email with Libor predictions at 7 a.m. GMT to more than a hundred traders and brokers, including representatives of almost all of the Libor panel banks. See Vaughan and Finch (2017), page 29. However, the EU General Court partly annulled the Commission’s decision that implicated ICAP in the EIRD case.}

The main mechanisms we model also apply to the collusive rigging of foreign exchange (forex) rates, which similarly relies on exchanging inside information, aligning exposure positions and planning eligible transactions. A small number of banks account for the bulk of transactions on the platform that are used to calculate the rates. ‘Banging the close’ is essentially eligible transactions rigging, as those trades in the window are eligible for the calculation of the rate and a cartel is able to exercise more influence on the rate jointly than any individual bank. Exchanging information on large client orders to be executed in the future and on manipulation strategies towards them, banks in the forex cartel were able to front run as they had inside information on the direction in which the rate would move in the future.

A key difference in forex is that the same set of transactions to manipulate the rate is also part of a bank’s exposure position. Evans (2018) models competitive forex trading around the fix and suggests that the anomalies found in the data could be explained by collusion. Other possible applications include front-running in benchmarks and price reference points in gold, energy and commodities markets—some of which have been subject to allegations of misconduct.
3.6 Concluding Remarks

Our cartel theory shows that it is possible to operate a benchmark rates cartel, despite interests typically not being aligned—and without a need for side payments. We tailor our model to Euribor and Libor and model two mechanisms that reinforce each other in facilitating benchmark collusion: front-running and eligible transactions rigging (or, pre-reform, simple mis-reporting of borrowing costs). By creating inside information, panel banks are in a position to take a more favorable exposure position to the upcoming rate, while reducing conflicting interests in their trading books. Some cartel banks need to engage in eligible transactions rigging, placing transactions at rates required to allow the cartel to justify the collusively optimal quotes.

Even though the cost of this may exceed the member’s cartel gains in those periods, the average expected collusion payoff is higher than under independent quoting. Consistent with the evidence, benchmark collusion can be characterized by episodic recourse to independent quoting. We explain these temporary break-ups as part of an ongoing collusive strategy, to which the cartel reverts in response to occasional extreme values that provide incentives to deviate.

The collusion leaves no obvious traces in either benchmark patterns over time, or in intraday variance in the quotes. It does markedly increase the volatility in quotes between trading days—as opposed to price variance decreases that are commonly found in regular cartels. On this basis, we suggest volatility screens to monitor submissions for periods of collusive manipulation. To the extent that sufficient transactions data is available, another test could look for suspiciously strong positive correlations between rate and portfolio changes.

The primary victims of the collusion are the counterparties on whom the cartel members rolled off their unwanted portfolio positions, including central banks, non-panel banks, institutional investors, pension funds, non-bank corporations and branches of the government.\textsuperscript{56} Although this may

\textsuperscript{56}See European Commission, \textit{Case A.39914 – Euro Interest Rate Derivatives}, prohibi-
largely have been rent shifting, the manipulations may also have induced
different borrowing behavior, impacting the efficient allocation of resources
in many underlying markets. Moreover, the manipulation scandals affected
the benchmarks’ trustworthiness as foundations of value and signals of un-
derlying risks. This will likely have had consequences for financial market
stability.\footnote{See Duffie and Stein (2015) and Bank of International Settlements, “Timothy Lane:
Financial benchmarks—a question of trust,” 24 March 2014.}

The benchmarks remain vulnerable to collusion, despite recent and pro-
posed reforms. Duffie, Skeie and Vickery (2013) and Duffie and Stein (2015)
show that widening the set of eligible transactions reduces the scope for
individual manipulation. However, with respect to collusion, we find that
it has an ambiguous effect. On the one hand, widening the set of eligible
transactions increases manipulation costs, lowering collusive profits. On
the other hand, it also reduces defection gains, which has a positive effect
on cartel stability. Calculating the rate on the basis of fewer quotes by re-
ducing the trimmed range, as proposed for Euribor, has similar ambiguous
effects. Both reforms may potentially lead to more steady collusion.

3.A Proofs

Proof of Proposition 1. As part of the equilibrium conditions, the
marginal bank-specific costs of changes in the eligible transactions rate are
assumed to increase in $\Delta c_{it}$, in other words $C''_{\Delta c_{it}, \Delta c_{it}} > 0$. Therefore, if
the cartel changed the ranking of the eligible transactions rates, the same
set of final rates $c_t$ could be achieved at lower total eligible transactions
rate rigging costs by retaining the ranking. This implies that the following
inequality constraints hold

$$\bar{c}_{(i+1)t} + \Delta c_{(i+1)t} \leq \bar{c}_{it} + \Delta c_{it} \quad \forall i \in \{1, ..., N - 1\}, \quad (3.21)$$

where bank indicator $i$ is now equal to its rank based on the baseline
eligible transactions rates $c_t$. As the baseline eligible transactions rates are
drawn from a continuous distribution, there exist different strategies where the ranking does not change and all constraints hold with inequality—an obvious candidate is the strategy of no manipulation, \((\Delta c_t, \Delta v_t) = (0, 0)\). These are Slater points, the existence of which is both necessary and sufficient for the existence of a global optimum in a non-linear optimization problem with inequality constraints.\(^{58}\) As the objective function is strictly convex, this optimum is unique. ■

**Proof of Proposition 2.** The implicit definition of \(\rho\) in Equation (3.19) is a continuous mapping from a nonempty, compact and convex set \(\rho \in [0, 1]\) onto itself, so that at least one fixed-point solution exists.

Note that the highest payoff differential is drawn from a continuous distribution over \([0, \infty)\). For \(\rho = 1\) to be a fixed-point solution, it must then hold that \(\Psi (\delta, \rho = 1) \leq 0\) and hence that \(E [\pi^N] \leq E [\pi^{BN}]\), which is not the case. Alternatively, for \(\rho = 0\) to be a fixed-point solution, it must hold that \(\Psi (\delta, \rho = 1) \to \infty\) and hence that \(E [\pi^C] - E [\pi^{BN}] \to \infty\), which is also not the case. Hence, any fixed point \(\rho\) must lie strictly between 0 and 1.

Although there may be more than one solution, there is a unique fixed-point that maximizes expected cartel profit, as the distribution from which the baseline values are drawn is continuous. ■

**Proof of Proposition 3.** Let \(E [\pi^{C,M}]\) be the expected profit of each member of an \(M\)-member cartel. A panel member that is not in the collusive coalition has no specific information about true rates and positions. The outsider therefore obtains \(E [\pi^{BN,M}]\). For any size \(M \geq 2\), the unconstrained per period profits of collusion satisfy:

\[
E [\pi^{C,M}] > E [\pi^{BN,M}] . \tag{3.22}
\]

A partial cartel of any size has more influence on the rate and is better at predicting it (and is therefore better at front running) than an individual bank.\(^{59}\)

\(^{58}\)See, for example, Brinkhuis and Tikhomirov (2005, pp. 210-211).

\(^{59}\)The per-period unconstrained profits from collusion are increasing in the number of
Similarly, because it improves banks’ accuracy in predicting the rate, the per-period expected profits of playing Nash with only the information of a subset of banks $M \geq 2$ are strictly larger than the Bayesian Nash profits. That is:

$$E\left[\pi^{N,M}\right] > E\left[\pi^{BN,M}\right],$$

where $E\left[\pi^{N,M}\right]$ is the expected profit for each member of an $M$-member cartel in case of a one-period break-up. Therefore, for any break-up probability $\rho \in [0, 1]$ and any $M \geq 2$ it holds that

$$E\left[\pi^{BN,M}\right] < (1 - \rho) E\left[\pi^{C,M} | NB\right] + \rho E\left[\pi^{N,M} | B\right],$$

where $E\left[\pi^{C,M} | NB\right]$ the expected profit of coordination (joint profit maximization) conditional on no break-up and $E\left[\pi^{N,M} | B\right]$ the expected profit of individual manipulation conditional on break-up.

From this, it follows that any cartel with more than one member is internally stable: The expected sum of discounted profits of being in an $M$-member cartel is larger than the expected sum of discounted profits of being outside an $(M - 1)$-member cartel for every bank $i$. Similarly, no cartel with less than $M = N$ members is externally stable: The expected sum of discounted profits of being inside an $(M + 1)$-cartel - or of joining a cartel of any size larger than 1 - is strictly larger than the expected sum of discounted profits of being outside an $M$-member cartel for every bank $i$ and for all $M < N$. 

members: the more members the cartel has, the larger the expected per-period profits, because a larger cartel has a larger impact on the rate and can more accurately predict the future rate and front run. This does not mean that a larger cartel is also more profitable, because it could be that a larger cartel breaks up more frequently and therefore has lower expected profits.
3.B A MATLAB® Cartel Routine

The following MATLAB® script calculates the optimal cartel strategies. Each bank inputs its daily baseline values, with which the software derives the optimal collusion and deviation strategies and their associated payoffs. The routine also determines whether all of the $N$ cartel stability conditions hold and dictates break-up as a strategy to all cartel members when one or more do not. The script provides all banks with the exact front-running and eligible transactions rigging strategies. The kernel is provided in Figure 3.9—for the condensed case of $N = 4$.

Given the parameters of the rate setting process, the cut-off value $\Psi$ is found in routine $\text{fpsi}$ that simulates a sufficient amount of daily payoff values in the case of Bayesian-Nash, Collusion, Defection and Nash (100,000 times in the simulation in the paper), such that fixed point $\rho$ can be identified with sufficient precision. The Bayesian Nash strategies are found by calculating the expected baseline values of the other $N - 1$ banks and for each bank separately using the $\text{fminunc}$ function in MATLAB® under the assumption that the calculated expected baseline values hold for the other banks. Nash strategies following break-up are found by each bank consecutively maximizing its own payoff function, repeated for a sufficient number of rounds. Banks respond to each other for up to 24 rounds, after which either a non-cooperative equilibrium in pure strategies is reached or none is concluded to exist, in which case the outcome of round 24 is taken as the mixed-strategy equilibrium draws.

In Step 1, at $0_t$ in the morning, all cartel members report their baseline drawings, exposures $\tilde{v}_i$ and eligible transactions rate $\tilde{c}_i$, which are entered as inputs in the prompt as shown in the screens in Figure 3.10.

In Step 2, the script subsequently derives the optimal cartel strategies, using the $\text{fminunc}$ function. Taking $V_0$ as the $1 \times N$ vector of baseline exposures and $C_0$ as the $1 \times N$ vector of baseline eligible transactions rates, the code minimizes the objective function $\text{ObjFunc}$ along the $2 \times N$ choice matrix $DC$, which represents the front-running choice variables (first row) and eligible transactions rigging choice variables (second row). Note
3.B. A MATLAB® Cartel Routine

Figure 3.9: Cartel Routine in Case of $N = 4$

```matlab
% This script allows banks to input parameters and baseline values and % calculate whether collusion is sustainable and if so, what the optimal % collusion strategies are. Version: 21 Dec 2017

%% Clear worksheet set path
% CD('C:\Users\Code Cartel Calculator');
options = optimoptions('fminunc', 'Display', 'off'); warning('off', 'all')

%% Step 1: Set parametric values, prompt input for baseline values
N = 4; n = 2; a = 1; b = 1; sc = 0.1; sv = 0.1; delta = 0.9;

%% Derive critical cut-off level psi
psi = fpsi(N,a,n,b,sc,sv,delta);

%% Prompt input baseline values
dig_title = 'Enter previous interbank rate'; num_lines = 1;
prompt = 'Previous interbank rate';
defaults = {'1.039'};
L0 = str2double(inputdlg(prompt,dig_title,num_lines,defaults,'on'));

%% Enter baseline transaction rates
dig_title = 'Enter baseline transaction rates'; num_lines = 1;
prompt = {'Bank 1','Bank 2','Bank 3','Bank 4'};
defaults = {'1.075','1.065','1.035','1.031'};
C0 = str2double(inputdlg(prompt,dig_title,num_lines,defaults,'on'));

%% Enter baseline exposures
dig_title = 'Enter baseline exposures'; num_lines = 1;
prompt = {'Bank 1','Bank 2','Bank 3','Bank 4'};
defaults = {'0.012','0.102','0.032','0.061'};
VO = str2double(inputdlg(prompt,dig_title,num_lines,defaults,'on'));

%% Step 2: Calculate collusion and deviation payoffs
% Collusion payoffs
% PointProfit = 8*(DC1-(sum(numVO)+sum(numDC1,:)))/trimm(0.25)*DC2(:,:);
% n/M*1000-L0-0.28*(sum((DC1,:)-2)-b*(sum(numDC2,:)-2));
CStrat = fminunc(0PointProfit, zeros(2, N), options);
FCol = payoff(V0, CO, N, n, L0, a, b, CStrat);

%% Deviation payoffs
% For i = 1:2
% DC = CStrat; DC1 = DC(:,[1,1]=1); CO = CO; CO2 = CO2(:,[1,1]=1);
% fOwnProfit = f(DDC=DC1+(0V0,2,i)+(CO2(0i,:))=innDC1=((DDC,1,2)+a*(DDC,2,1,2)-b*(DDC,2,1,2));
% DCStrategy = fminunc(fOwnProfit, zeros(2, N), options);
% FDev(i,1) = payoff(V0, CO, N, n, L0, a, b, CStrat, DCStrategy, i);
end

%% Step 3: Check whether constraints hold and produce output
if FDev = FCol <= psi
    msgbox(['Break up: No.', '...']);
    'Inj Advised positions', ...
    'Inj Bank 1: Adjustment = 5.6f, New position = 4.6f', ...
    'Inj Bank 2: Adjustment = 4.6f, New position = 3.6f', ...
    'Inj Bank 3: Adjustment = 4.6f, New position = 3.6f', ...
    'Inj Bank 4: Adjustment = 4.6f, New position = 3.6f', ...
    'Inj Advised submission targets', ...
    'Inj Bank 1: Adjustment = 4.6f, New submission = 4.6f', ...
    'Inj Bank 2: Adjustment = 4.6f, New submission = 4.6f', ...
    'Inj Bank 3: Adjustment = 4.6f, New submission = 4.6f', ...
    'Inj Bank 4: Adjustment = 4.6f, New submission = 4.6f', ...
    CStrat(1,1), VO(1)=CStrat(1,1), CStrat(1,2), VO(2)=CStrat(1,2), ...
    CStrat(1,3), VO(3)+CStrat(1,3), CStrat(1,4), VO(4)=CStrat(1,4), ...
    CStrat(2,1), CO(1)=CStrat(2,1), CStrat(2,2), CO(2)=CStrat(2,2), ...
    CStrat(2,3), CO(3)=CStrat(2,3), CStrat(2,4), CO(4)=CStrat(2,4));
else
    msgbox('Break up: Yes.'); end
```
that $\text{ObjFunc}$ is specified as the negative of the sum of the individual payoff functions, which is subsequently minimized. Output $\text{CStrat}$ are the optimal cartel strategies. Plugging these into individual payoff functions provides each bank’s cartel profits. This is done in the routine $f\text{payoff}$. Similarly, defection payoffs are found by maximizing own payoffs given that other banks adhere to cartel strategy.

Finally, in Step 3 it is checked whether the difference between the defection payoff and the collusion payoff of each bank is below the critical cut-off value $\Psi$. The cartel instructions of all members are given to each, together with all shared information, as in the screen. Note that all banks optimally adjust their positions by the same amount, independent of their initial portfolio position, because manipulation costs are quadratic and profits linear, the same for all banks. If none of the banks have a payoff differential above $\Psi$, a collusive quote is scripted (‘Break-up: No.’), including which strategies each bank should implement. This is shown in Figure 3.11. If at least one bank has a payoff differential above $\Psi$, all banks receive the notification that collusive optimization is not stable (‘Break-up: Yes.’), instructing them to revert to one-period static Nash.
3.B. A MATLAB® Cartel Routine

Figure 3.10: Choice Prompts in Case of $N = 4$

Figure 3.11: Final Cartel Instructions in Case of $N = 4$
Chapter 4

Cartel Stability by a Margin*

“If you are going to kill, then kill an elephant. If you are going to steal, make sure it’s a treasure.”
– Indian Proverb

The conditions under which collusion can be sustainable are well known to depend on such characteristics as the number and relative market shares of the firms active in the market, their frequency of interaction and price adjustment, the threat of entry and prospective demand. In a framework of repeated interaction, Friedman (1971) and Abreu (1986, 1988) formalized that cartel members have a critical discount factor against which they value the future stream of cartel profits just equal to the instantaneous gains from defection, followed by the implementation of a punishment strategy. The common approach in cartel theory is to consider a cartel agreement internally stable when for each member its actual discount factor is above its critical value, and unstable below.1 Any change in market structure characteristics that lowers the (binding) critical discount factor increases the scope for cartels in that market, in the sense that they are sustainable for a wider range of actual discount factors.

*This paper is joint work with Maarten Pieter Schinkel. We thank Joe Harrington, Thomas Ross, Jeanine Miklos-Thal, Juan-Pablo Montero and participants of the 2018 CRESSE and EARIE conferences for discussion and comments on earlier versions of this paper. Opinions and errors remain ours. See SSRN for the latest public working paper version of this chapter.

1The approach is classic textbook at least since Tirole (1988). Deneckere (1982) is another early application of the measure.
In the knife-edge configuration of cartel stability, it is not made explicit by how much firms may be better off colluding than not. Yet it is reasonable to consider a cartel ‘more robust’ when its members’ incentives to adhere to the collusive agreement exceed those to defect by a larger margin. Changes in the circumstances of the conspiracy may affect the scope for stable collusion differently when a cartel is stable by a margin. In this paper, we study the comparative statics of cartel stability when firms require a margin before colluding. We bring the analysis under conventional cartel stability theory by incorporating the margin in the derivation of the critical discount factor. The premise is that firms maintain a cartel only if each period they are sufficiently better off colluding. The margin is therefore a threshold value that firms require on top of instantaneous cartel profits for participation in the cartel agreement.

The cartel margin can have different sources. It may be needed to compensate for moral disutility from participating in the illegal conspiracy, or to insure against uncertain events such as the sudden appearance of a maverick firm or sharpened exposure to antitrust damages actions, which fan the distrust between co-conspirators. More concretely, the cartel margin can cover the expected consequences of cartel participation. Thomadsen and Ree (2007) were the first to introduce the cartel margin, as a cover of the cost of colluding, such as monitoring, communication and managerial effort to repair internal tensions. Their paper and Colombo (2013) analyze its consequences for the effect of product differentiation on cartel. Other reasons for cartel members to require a cartel profit margin can be liabilities for fines, damages and reputation loss in the event of break-down.

The presence of a cartel margin may provide new and unambiguous comparative statics for a wide class of market structure characteristics, for which we provide the conditions. The reason is that an increase in the margin increases the effect that the gains from collusion has on cartel stability (which is positive) relative to the gains from deviation (which is negative). When the cartel margin is sufficiently large, the effect on the gains from collusion has to dominate. More specifically, we find that in the presence of a cartel margin, the scope for collusion generally increases under (i) lower
industry marginal cost and (ii) less product differentiation. Although there are common conjectures on the effects of both, these lack a solid theoretical foundation. In standard cartel theory, the overall level of marginal cost is immaterial to cartel stability, because common changes in marginal cost do not affect the classic critical discount factor(s)—as reiterated in Section 4.2. However for any positive margin the critical discount factor decreases in the marginal cost level, provided the margin itself does not decrease too strongly in marginal cost. On the effects of product differentiation on collusion there is no consensus in the theoretical literature—reviewed in Section 4.3. Results are highly case specific without, but with a sufficiently high cartel margin that does not itself increase too strongly in product similarity, the critical discount factor increases unambiguously in the level of product differentiation—provided competition is not too strong to begin with.

These findings have implications for competition policy. Ceteris paribus, cartels should be expected in lower marginal cost sectors, in response to industry-cost reducing innovations or a common input cost drop. For merger control, they suggest a new possible concern for coordinated effects when the parties claim merger-specific efficiencies in compensation for the unilateral anticompetitive effects of their merger. Such marginal cost reductions are generally seen as pro-competitive. For example, the 2010 U.S. Horizontal Merger Guidelines state that:

“In a coordinated effects context, incremental cost reductions may make coordination less likely or effective by enhancing the incentive of a maverick to lower price or by creating a new maverick firm.”

This is not obviously true if the merging firms happen to critically de-

\[^2\text{Op.cit., page 33. The 2004 European Guidelines on the Assessment of Horizontal Mergers under the Council Regulation on the Control of Concentrations between Undertakings similarly hold at recital 82 that: ”In the context of coordinated effects, efficiencies may increase the merged entity’s incentive to increase production and reduce prices, and thereby reduce its incentive to coordinate its market behaviour with other firms in the market. Efficiencies may therefore lead to a lower risk of coordinated effects in the relevant market.”}^\]
termine the binding critical discount factor for collusion that requires a margin.

Our results also suggest that there is more scope for collusion when products are more homogeneous. Competition authorities already work on this conjecture and are critical, for example, to industry standardization. Levenstein and Suslow (2006) report that of discovered cartels, a substantial part was active in fairly homogeneous goods industries. The Merger Guidelines hold on product differentiation that it diminishes business stealing effects, due to better anticipation of responses, to conclude that:

“A market typically is more vulnerable to coordinated conduct if [...] products in the relevant market are relatively homogeneous.”

We contribute that this is canonically (and only truly “typically”) true when firms require to gain from coordination by a sufficiently large margin, so that the effect of increased product similarity on the incentives of firms to compete is not generally dominated by the increased incentives to collude when products are more homogeneous.

In the main text, we analyze a general model specification, so as not to unnecessarily burden the notation. It can encompass various specific models that fit the motivations for the existence of a cartel margin given above. Section 4.4 shows how the cartel margin can be interpreted as the expected consequences of cartel participation when there is a risk of cartel break-down that leads to liabilities and costs of colluding as long as the cartel continues. In that setting, the exogenous cost of colluding introduced in Thomadsen and Rhee (2007) and Colombo (2013) to study the effect of product differentiation on cartel formation are a special case in the sense that we provide the necessary and sufficient conditions under which any cartel margin affects the sign of the comparative statics.

\[\text{Op.cit., page 26. The European Commission’s Horizontal Mergers Guidelines state the same at recital 45: “It is also easier to coordinate on a price for a single, homogeneous product, than on hundreds of prices in a market with many differentiated products.”}\]
4.1. Model and General Result

The remainder of this paper is organized as follows. Section 4.1 presents the general result on the unambiguity of the comparative statics of market structure changes when there is a proper cartel margin. The margin bounds generally depend on structural variables. We subsequently specify the commonly used Dixit-Vives market structure to apply the result to changes in industry marginal cost of production (Section 4.2) and product differentiation (Section 4.3). Analytical results are proven in the case of linear demand. Numerical results provide illustrations of the material effects and insight into the parameter ranges for which results are unambiguous, including when demand is non-linear. In Section 4.4 illustrates how the general model encompasses specific cartel models with enforcement. Section 4.5 concludes.

4.1 Model and General Result

Consider \( n \) identical firms in repeated interaction, each discounting with the common factor \( \delta \in [0,1) \). Let \( \pi^N(x) \) be the per-period competitive static Nash profit as a function of some generic market structure variables \( x \). A firm’s per-period profit if all firms collude is \( \pi^C(x) \). This does not have to be the joint-profit maximizing profit. The firm that deviates obtains as profit \( \pi^D(x) \). Assume that each profit function is continuous and differentiable in each market structure variable \( x \in x \), and has a unique maximum. Naturally, \( \pi^N(x) \leq \pi^C(x) \leq \pi^D(x) \).

The cartel margin is an absolute per-period required payoff \( M(x) \geq 0 \) that depends on \( x \). Each period, each cartel member requires at least that

\[
\pi^C(x) - \pi^N(x) \geq M(x) .
\]

(4.1)

Naturally, no cartel can ever be sustained if its members required a margin that is larger than the per-period gains from colluding. This puts upper bound \( \bar{M}(x) = \pi^C(x) - \pi^N(x) \) on the margin for collusion to even be feasible.

Firms play an infinitely repeated game with grim trigger punishment
strategy: upon defection by one, all firms revert back to competition forever after. Hence, for each firm the present discounted value of collusion and deviation are

\[
V_C^C (x) = \sum_{t=0}^{\infty} \delta^t (\pi^C (x) - M (x)), \quad (4.2)
\]

\[
V_D^D (x) = \pi^D (x) + \sum_{t=1}^{\infty} \delta^t \pi^N (x). \quad (4.3)
\]

Collusion is internally stable as long as

\[
V_C^C (x) \geq V_D^D (x),
\]

which holds if

\[
\delta \geq \delta^* (x, M (x)) = \frac{\pi^D (x) - \pi^C (x) + M (x)}{\pi^D (x) - \pi^N (x)}. \quad (4.4)
\]

The critical discount factor \( \delta^* (x, M (x)) \) is between 0 and 1 as long as \( M (x) < \bar{M} (x) \). Other things equal, the cartel margin negatively affects the scope for stable collusion: \( \partial \delta^*/\partial M > 0. \)

The effect of changes in one or more of the market structure variables \( x \) on collusion can now be evaluated based on how they affect \( \delta^* \). If \( \delta^* \) decreases, it increases the scope of discount factors for which collusion is stable (and vice versa). A change in \( x \) can affect the scope for collusion in two ways, by changing, relative to competition:

(i) the net gains from colluding \( \pi^C (x) - \pi^N (x) - M (x) = \Delta \pi^C (x) - M (x) \), which affects the scope positively, and

(ii) the gains from deviating \( \pi^D (x) - \pi^N (x) = \Delta \pi^D (x) \), which affects the scope negatively.

Only when the net gains from collusion and the gains from deviation move in different directions with a change in \( x \), the combined effect on the scope

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4See Section 4.4 for foundational specifications of this general model setup with costs of colluding, a probability of cartel break-down, liability for fines and antitrust damages. Note that although in the main specification the cartel margin is assumed not to be incurred in case of deviation, the results go through in that case by simply taking \( \delta M (x) \) instead of \( M (x) \).
of collusion is a priori unambiguous. This is, however, rarely the case (not even with the number of firms).

When $\Delta \pi^C (x) - M (x)$ and $\Delta \pi^D (x)$ instead move in the same direction with a change in $x$, the effect of the change in the market structure variable on the scope of collusion is a priori ambiguous. We find however that there exists a range for the cartel margin above a lower bound $\bar{M}(x)$ and upper bound $\bar{M}(x)$, which is well-defined and non-empty, but modelspecific, for which the scope for collusion changes unambiguously with the market structure.

**Proposition 1** When the net gains from collusion $\Delta \pi^C (x) - M (x)$ and the gains from deviation $\Delta \pi^D (x)$ both increase (decrease) in market structure variable $x \in \mathcal{X}$, then for any cartel margin $M (x)$ from non-empty set $(\bar{M}(x), \bar{M}(x))$ it holds that $\partial \delta^*/\partial x$ is strictly negative (positive).

**Proof.** See Appendix. ■

To see why the cartel margin can ensure unambiguous comparative statics, note that (4.4) can be rewritten as

$$\delta^* (x, M (x)) = \frac{\Delta \pi^D (x) - \Delta \pi^C (x) + M (x)}{\Delta \pi^D (x)},$$

so that

$$\frac{\partial \delta^* (\cdot)}{\partial \Delta \pi^C (x)} = -\frac{1}{\Delta \pi^D (x)},$$

$$\frac{\partial \delta^* (\cdot)}{\partial \Delta \pi^D (x)} = \frac{\Delta \pi^C (x) - M (x)}{\Delta \pi^D (x)^2}.$$  

The value of $M (x) > 0$ unambiguously decreases the comparative effect of $\Delta \pi^D (x)$ on $\delta^*$. For $M (x)$ sufficiently large, therefore, the comparative effect of $x$ on $\Delta \pi^C (x)$ will dominate the comparative effect of $x$ on $\Delta \pi^D (x)$ in determining the joint comparative effect of $x$ on $\delta^*$. In other words, an increase in $M (x)$ increases the effect that the gains from collusion has on the scope for cartels (which is positive) relative to the gains from deviation.
(which is negative).

In the proof, lower bound $M(x)$ is found as the cartel margin at which partial derivative $\partial \delta^*/\partial x$ is exactly zero. This lower bound does not have to be positive. However, when $M(x) > 0$ and $M(x)$ surpasses $M(x)$, there is a reversal in the comparative statics of $x$ on $\delta^*$. When $M(x) < 0$, no sign change to $\partial \delta^*/\partial x$ can occur for any $M(x) < M(x)$, because $M(x)$ is non-negative.

In a wide class of cases—including those considered in the next two sections—$\Delta \pi^C(x)$ and $\Delta \pi^D(x)$ move in the same direction. If $M(x)$ also changes in the market structure variable $x$ considered, $\Delta \pi^C(x) - M(x)$ and $\Delta \pi^D(x)$ continue to move in the same direction only when $M(x)$ either change is the opposite direction of $\Delta \pi^C(x)$ or less. In the first case, the fact that $M$ depends on $x$ simply amplifies the comparative statics. In the second case, $M$ also changing with $x$ dampens the comparative statics. The effect of changes in $x$ on $M$ should then not be too large for the proposition to hold, in the sense that

$$\left| \frac{\partial \pi^C(x)}{\partial x} - \frac{\partial \pi^N(x)}{\partial x} \right| > \left| \frac{\partial M(x)}{\partial x} \right|.$$  \hspace{1cm} (4.8)

When firms are heterogeneous so that static Nash, collusive and deviation payoffs differ between them, each firm $i$ has its own actual and critical discount factor, where each condition $\delta_i \geq \delta_i^*$ needs to hold in order for a complete cartel to be internally stable. As a result, the cartel member with the tightest incentive compatibility constraint determines the critical discount factor. The comparative statics result in the proposition holds for each firm individually, provided the conditions do, and therefore in particular also for the critical cartel member.

In the following two sections, we study for two a priori ambiguous variations (in marginal cost of production and product differentiation) how material the effects on comparative static are, and how restrictive lower and upper bounds $\bar{M}$ and $\tilde{M}$. 

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4.2 Changes in Marginal Cost

The marginal cost of production of firms is a key driver of prices and in many markets can vary considerably for all producers over time, for example due to seasonality in the availability of inputs, financial market fluctuations or price developments in commodity, energy and labour markets. The effects of heterogeneity in cost structures across firms on the stability of collusive agreements between them has been studied extensively, including in Bae (1987), Harrington (1991), Rothschild (1999) and Miklos-Thal (2011). For linear demand, the industry level of marginal costs has no effect on the classic critical discount factor. Although this does not mean that it is irrelevant for cartel stability, there are few studies on the subject. An exception is Lambertini and Sasaki (2001), who argue that an overall decrease in marginal cost decreases cartel stability—using a model with optimal punishment that relies upon the ability of firms to charge prices strictly below marginal cost. This argument applies only, however, in cases where full collusion on monopoly conduct cannot be sustained and only once marginal cost is so low that penal pricing becomes constrained by non-negativity.

4.2.1 Analytical Results Under Linear Demand

To obtain more insight in the effects of a cartel margin on the sustainability of collusion, consider the widely used specification of a market in which \( n \) symmetric firms labelled \( i \in \{1, ..., n\} \) produce horizontally differentiated products, firm \( i \) serving inverse demand

\[
p_i = (a - bq_i - \gamma b \sum_{j \neq i} q_j)^\rho,
\]

in which \( a \) and \( b \) are positive parameters, \( \gamma \in (0, 1] \) reflects product differentiation—where products are more homogeneous when \( \gamma \) is close to 1—and \( \rho \in (0, \infty] \) is a curvature parameter—where demand is concave...
for $\rho < 1$, convex for $\rho > 1$ and linear for $\rho = 1$.\(^5\)

All firms produce with the same constant returns to scale production technology, giving them each marginal cost $c$, and no fixed cost, so that the profits of firm $i$ are $\pi_i = (p_i - c)q_i$ for all $i = 1, \ldots, n$. In the following, we consider competition in quantities—qualitatively equivalent results obtain for price competition and are provided in an appendix. We impose that $(p_i, q_i) \geq 0$ for all $i$ and take $c < a^\rho$ to ensure that prices and quantities are positive in equilibrium. Initially, we consider $M$ independent of $c$.

For linear demand ($\rho = 1$), a change in marginal cost has an a priori ambiguous effect on the scope for stable collusion: both $\Delta \pi^C(x)$ and $\Delta \pi^D(x)$ decrease in $c$, for all relevant parameters values (taking the joint-profit maximizing level for $\pi^C(x)$). We find that the proposition applies with $M(x) = 0$, so that the smallest positive margin suffices to obtain new and unambiguous comparative statics.

**Corollary 1** When demand is linear, then for any cartel margin $M$ from non-empty set $(0, \bar{M})$ it holds that $\partial \delta^*/\partial c$ is strictly positive. For $M = 0$, $\partial \delta^*/\partial c = 0$.

**Proof.** See Appendix. □

The introduction of a positive margin breaks the classic finding that (in case of linear demand) cartel stability is independent of the overall level of marginal costs. With the cartel margin in the numerator, the marginal cost no longer divide out as a common scaling term that appears in both the numerator and denominator of Equation (4.4). The intuition is that the effect of changes in the marginal cost on the gains from collusion dominates that on the gains from deviation.

The corollary assumes $M$ to be invariant in marginal cost $c$. However, the cartel margin may itself vary with marginal cost. For instance, $M$ can increase in $c$: with a higher industry cost level implying higher prices and affected turnover, the basis on which public fines are typically determined

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4.2. Changes in Marginal Cost

Increases. M can decrease in c as well: for example, lower marginal cost may trigger a maverick firm to stir up competition, or a higher cartel overcharge may increase the probability of detection and antitrust damages. Including M(c) changes Condition (4.30) in the proof into

\[
\frac{\partial}{\partial c} \left( \pi^C - \pi^N - M(c) \right) = -\frac{2(a-c)}{b} \left( \tilde{\pi}^C(\cdot) - \tilde{\pi}^N(\cdot) \right) - \frac{\partial M(c)}{c},
\]

which is negative as long as \( \partial M/\partial c > -2(a-c)(\tilde{\pi}^C - \tilde{\pi}^N)/b \). That is, the net effect of changes in c on M can be of either sign but should not be too negative for the proposition to apply.

4.2.2 Numerical Results Under (Non-)Linear Demand

For non-linear demand (\( \rho \leq 1 \)), closed-form solutions for the profits and critical discount factor are not available. Numerical analyses show, however, that the effects of the cartel margin on the comparative statics remain and are material. Figure 4.1 provides \( \delta^* \) (left-side panels) and ranges for M and \( \hat{M} \) (right-side panels) for the case of symmetric Cournot duopoly (\( n = 2 \)) with homogeneous products (\( \gamma = 1 \)) and concave (\( \rho = 0.5 \)), linear (\( \rho = 1 \)) and convex (\( \rho = 2 \)) demand, with \( a = 2, b = 0.5 \) and \( c \in (0, 1] \).

The lower continuous lines in the panels on the left confirm that in the absence of a cartel margin, the critical discount factor is not affected by marginal cost in case of linear demand (middle panel) or only marginally in case of concave (upper panel) or convex (lower panel) demand. When there is a cartel margin (\( M > 0 \)), the critical discount factor unambiguously increases in marginal cost—apart from the extreme case in the upper-left panel where demand is highly concave (\( \rho = 0.5 \)) and M and c small.

\[\text{United States Sentencing Commission (2016), United States Sentencing Guidelines, §2R1.1(d)(1); Commission of the European Communities (2006), Guidelines on the method of setting fines imposed pursuant to Article 23(2)(a) of Regulation No 1/2003, recital 13.}\]

\[\text{This role of the maverick firm is relied upon in the U.S. Merger Guidelines quoted in the introduction. The cartel overcharge for linear demand (4.9) is } p^C - p^N = (a-c)(1+\gamma n)(n-1)\gamma/((2+2(n-1)\gamma)(2+(n-1)\gamma)), \text{ which decreases in } c.\]

\[\text{Calculations are programmed in MATLAB® using grid-search algorithms.}\]
Figure 4.1: Illustration of Comparative Statics in Marginal Cost

Notes: Critical discount factor (left-side panels) and bounds on the cartel margin (right-side panels) as a function of marginal cost in case of: (top to bottom) concave ($\rho = 0.5$), linear ($\rho = 1$) and convex ($\rho = 2$) demand.
The relationship remains for higher numbers of firms \((n)\), more product differentiation (lower \(\gamma\)), and any of the other parameters. Changes in the slope of \(\delta^*\) are major, which implies that the effect on comparative statics of \(M\) are material.

In the panels on the right in Figure 4.1, the dark-blue area where \(M \in (\bar{M}, \bar{M})\) is the relevant region where the proposition holds. The lower bound \(M\) is almost always negative or (near) zero, and so does not bind, except from the extreme case mentioned: in the upper-right panel, in the lower-left corner, in which case \(M\) is still very close to zero. These panels also show that the margins chosen in the comparative statics \((M \in (0, 0.03])\) are low relative to upper bound \(\bar{M}\), particularly at low levels of \(c\) and more convex demand (higher values of \(\rho\)). These remain for higher \(n\), lower \(\gamma\) and other variations.

### 4.3 Changes in Product Differentiation

Theoretical work on the effects of product differentiation on cartel stability is more developed than on industry cost, but has not generated consensus. When products are closer substitutes, competition is stronger, which means that firms have more to gain from colluding, making adherence to the cartel agreement more attractive, but also are able to more easily attract customers away from rival firms when deviating, which undermines cartel stability.

Deneckere (1983; 1984) finds in a duopoly model with multi-product demand and grim trigger punishment strategies that increased product substitutability decreases cartel stability in case of quantity competition, and has ambiguous effects in case of price competition. Wernerfelt (1989) shows how in this setting the effect of product substitutability becomes ambiguous when firms use the optimal carrot-and-stick punishment strategy proposed by Abreu (1986, 1988)—although Osterdal (2003) argues that Deneckere’s result remains for the number of firms sufficiently large. Tyagi (1999) shows that Deneckere’s quantity competition results crucially depend on the curvature of demand—where increased product substitutabil-
ity instead increases cartel stability when demand is sufficiently convex.

Similarly ambiguous results obtain for spatial product differentiation. Ross (1992) shows for duopoly price competition and grim trigger punishment that when transportation costs are linear, product substitutability increases cartel stability. Chang (1991) finds the opposite when transportation costs are quadratic. Häckner (1996) shows in this setting that when firms use the optimal carrot-and-stick punishment, the collusive price decreases when product substitutability increases. Miklos-Thal (2008) establishes for duopoly price competition that conclusions crucially depend on which punishment strategy is used: in case of optimal carrot-and-stick punishment, product substitutability tends to undermine collusion, while in case of grim trigger punishment, there may be more scope for collusion if products are more homogeneous. Extensions beyond duopoly price competition are not readily available for spatial specifications.

4.3.1 Analytical Results Under Linear Demand

We analyze the effects of product differentiation in the model introduced in the previous section, which is a generalization of Deneckere (1983) that allows for \( n \)-firms and a varying demand curvature. Again we consider \( M \) independent of \( \gamma \) first. For the case of linear demand we establish that Proposition 1 applies, provided that competition is not too strong to begin with.\(^9\)

**Corollary 2** When demand is linear, the amount of firms \( n \) sufficiently low and the degree of product similarly \( \gamma \) sufficiently high, then for any cartel margin \( M \) from non-empty set \( (\bar{M}, \bar{M}) \) it holds that \( \partial \delta^* / \partial \gamma \) is strictly negative.

**Proof.** See Appendix. ■

The reason why there should not be too many firms in the market—with the upper bound increasing in the degree of product heterogeneity—is that collusion should not be too attractive a proposition to make it more

\(^9\)Colombo (2013) offers this result, without the bounds.
interesting, relative to defection. Exactly how restrictive the lower and upper bounds on the cartel margin are, we study numerically.

Again, the corollary assumes $M$ to be invariant in the market structure parameter—in this case product similarity $\gamma$. However, there are opposite possible effects of changes in product substitutability on the cartel margin. Arguably, it is easier to coordinate a cartel and monitor deviations when products are more homogeneous—which are the mechanisms behind the statements quoted from the horizontal merger guidelines in the introduction. If $\partial M/\partial \gamma < 0$, Condition (4.34) holds for more values of $n$ and $\gamma$, further increasing the scope for collusion. Yet, expected liabilities may increase in product similarity as well, for example when agencies or plaintiffs find it easier to discover and successfully prosecute collusion when products are more similar. $\partial M/\partial \gamma > 0$ goes against derivative (4.34) being positive, but changes the comparative statics only for a very strong net increase of $M$ in $\gamma$. The proposition applies for all net effects of changes in $\gamma$ on $M$, except too large an increase.

4.3.2 Numerical Results Under (Non-)Linear Demand

For non-linear demand ($\rho \leq 1$), closed-form solutions for the profits and critical discount factor are again not available. Figure 4.2 provides $\delta^*$ (left-hand panels) and ranges for $\bar{M}$ and $\tilde{M}$ (right-hand panels) for the case of symmetric Cournot duopoly ($n = 2$) and different demand curvatures, with $a = 2$, $b = 0.5$ and $c = 0$.

The panels on the left in Figure 4.2 show that without a cartel margin, an increase in product substitutability decreases the scope for collusion, as found by Deneckere (1983). However, when demand is convex the opposite may hold, as shown by Tyagi (1999), although the effect is marginal for the specifications here analyzed. When there is a sufficiently large margin, an increase in product substitutability unambiguously increases collusion. The effect on the slope also appears to be material. From the panels on the right, it shows that the lower bound $M$ is not very restrictive for any degree of product differentiation, nor is the upper bound $\tilde{M}$ often binding, at least not for low degrees of product differentiation.
Figure 4.2: Illustration of Comparative Statics in Product Similarity

Notes: Critical discount factor (left-side panels) and bounds on the cartel margin (right-side panels) as a function of product similarity in case of: (top to bottom) concave ($\rho = 0.5$), linear ($\rho = 1$) and convex ($\rho = 2$) demand.
4.3. Changes in Product Differentiation

These conclusions remain when adjusting any parameter—apart from number of firms. In the numerical exercises, for a larger number of firms than four, there are values of $\gamma$ sufficiently close to 1 for which lower bound $\bar{M}$ exceeds upper bound $\bar{M}$, and there exists no margin effect. Figure 4.3 shows for linear demand ($\rho = 1$) how sufficiently low values of $\gamma$ maintain a space for $M$ for which increased product substitutability increases the scope for collusion when $n = 5$. The right-side panel shows that $M$ increases more than $\bar{M}$, so that the lower bound at some point surpasses the upper for $\gamma$ sufficiently close to 1. The dark-blue area is nevertheless substantial, which remains the case when the number of firms becomes larger, even though the area shrinks in $n$.

Figure 4.3: Illustration of Comparative Statics in Product Similarity, $n = 5$

Notes: Critical discount rate (left-side panel) and bounds on the cartel margin (right-side panel) as a functions of product similarity for linear demand ($\rho = 1$) and $n = 5$. 

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4.4 A Model of Cartel Enforcement

In this section, we illustrate how the cartel margin requirement can be founded in commonly considered elements of competition law enforcement.\textsuperscript{10} We consider a stylized enforcement model in which we base $M(x)$ on first principles. Let there be a per-period risk of cartel break-down $\mu \in [0, 1)$, followed by liabilities of size $L$, while as long as the cartel continues, with probability $1 - \mu$, there are costs of colluding $C$. Suppressing $x$ in the notation, the values of the discounted sums of future profits in case of collusion and deviation are respectively:

$$V^C = (1 - \mu) (\pi^C - C + \delta V^C) + \mu (\pi^C - L + \delta V^N), \quad (4.11)$$

$$V^D = \pi^D + \frac{\delta}{1 - \delta} \pi^N, \quad (4.12)$$

where $V^N$ is the continuation value of static Nash competition.

Using the closed-form solution for $V^C$ and rearranging terms to solve for $\delta$ provides $V^C \geq V^D$ if

$$\delta \geq \delta^* = \frac{\pi^D - \pi^C + (1 - \mu) C + \mu L}{\pi^D - \pi^N} \frac{1}{1 - \mu}, \quad (4.13)$$

which is identical to (4.4) for

$$M = C + (\pi^D - \pi^C + L) \frac{\mu}{1 - \mu}. \quad (4.14)$$

Hence, changes in $M$ and its lower and upper bounds depend on market structure variables $x$ through $\pi^D$ and $\pi^C$, although obviously also $C$, $L$ and $\mu$ can be functions of $x$. Exactly how will depend on the level of these variables, specifics of demand and the collusive strategy. Note that $C$ simply replaces $M$ if it were the only aspect of cartel enforcement modelled—that is if $\mu = L = 0$, which replicates Thomadsen and Rhee (2007) and Colombo

\textsuperscript{10}Seminal contributions to the literature on the effects of cartel enforcement on collusion include Block, Nold and Sidak (1981) and Harrington (2004).
4.4. A Model of Cartel Enforcement

(2013) as a special case. For the fuller models, simulations with demand (4.9) for all reasonable parameter values show that variations in $\mu$ and $L$ keep $M$ well within bounds. In fact, the upper and lower bound almost perfectly track $M$ and $\tilde{M}$ in Figure 4.1 for marginal cost changes and in Figure 4.2 for product differentiation.

The proposition applies only when, following a change in market structure variable $x$, $M$ moves either in the opposite direction of $\Delta \pi^C(x) = \pi^C(x) - \pi^N(x)$ or less in the same direction—see Condition (4.8) in Section 4.1. In this model

$$\frac{\partial M}{\partial x} = \frac{\partial C}{\partial x} + \left( \frac{\partial \pi^D}{\partial x} - \frac{\partial \pi^C}{\partial x} + \frac{\partial L}{\partial x} \right) \frac{\mu}{1 - \mu} \frac{\pi^D - \pi^C + L \partial \mu}{(1 - \mu)^2} \frac{\partial \mu}{\partial x}. \quad (4.15)$$

In case of varying marginal cost, that is, taking $x = c$, it generally holds that $\Delta \pi^C$ decreases in $x$ and the proposition applies as long as

$$\frac{1}{1 - \mu} \frac{\partial \Delta \pi^C}{\partial c} < \frac{\partial C}{\partial c} + \left( \frac{\partial \Delta \pi^D}{\partial c} + \frac{\partial L}{\partial c} \right) \frac{\mu}{1 - \mu} \frac{\pi^D - \pi^C + L \partial \mu}{(1 - \mu)^2} \frac{\partial \mu}{\partial c}, \quad (4.16)$$

where the left-hand side is negative. As long as $C$, $L$ and $\mu$ are unaffected by $c$, this condition simplifies to

$$\frac{\partial \Delta \pi^C}{\partial c} < \mu \frac{\partial \Delta \pi^D}{\partial c}, \quad (4.17)$$

which are both negative. It holds as long as $\mu$ is sufficiently small, as generally the gains from deviation decreases quicker in $c$ than the gains from collusion. If additionally $C$, $L$ and/or $\mu$ are affected by $c$, the comparative statics may go out of bounds. However for the predominant such dependence, which is that $M$ increases in $c$ through affected turn-over increasing fines and antitrust damages $L$, the condition is reinforced. It can accommodate a higher probability of entry by a maverick firm in low cost industries.

In case of varying product differentiation, that is, taking $x = \gamma$, $\Delta \pi^C$
generally increases in $x$ and the proposition applies as long as

$$\frac{1}{1-\mu} \frac{\partial \Delta \pi^C}{\partial \gamma} > \frac{\partial C}{\partial \gamma} + \left( \frac{\partial \Delta \pi^D}{\partial \gamma} + \frac{\partial L}{\partial \gamma} \right) \frac{\mu}{1-\mu} + \frac{\pi^D - \pi^C + L \partial \mu}{(1-\mu)^2} \frac{\partial \mu}{\partial \gamma},$$

(4.18)

where the left-hand side is positive. Again, with $C$, $L$ and $\mu$ unaffected by $\gamma$, it holds for $\mu$ small enough. If indeed coordination and monitoring is easier when the goods are more homogeneous, so that $C$ decreases in $\gamma$, the condition is reinforced. It can accommodate that cartel detection may be more likely in sectors in which products are more alike. In a wide set of the most reasonable circumstances, our unambiguous comparative statics results apply.

4.5 Concluding Remarks

We generalize the imperative of accounting for a required payoff beyond the common collusive gains in cartel stability analysis. Specifically, when the net gains from collusion and the gains from deviation both increase (decrease) in a market structure variable—so that unambiguous comparative statics results are not a priori available—then a sufficiently large margin assures that the scope for stable collusion unambiguously increases (decreases) in this market structure variable.

In particular, we find that both lower marginal cost and reduced product differentiation generally increase the scope for collusion when cartel members require a margin. These comparative statics effects are material in common (non-)linear demand systems, both in quantity competition and price competition. We show that beliefs that collusion may be easier for more homogeneous goods and low cost industries can be theoretically based on even a small margin. That lower cost and more homogeneous goods industries are more prone to collusion may be relevant for focus in competition policy. In merger control, invoking the efficiency defense may imply an increased risk of coordinated effects. However, the circumstances under which this is the case would need to be studied fully in a model with marginal cost asymmetries between groups of companies.
4.5. Concluding Remarks

Empirical implications are suggested by the general break-down of effects in Condition (4.8) for a specific foundational model. Although we found it to hold in our simple models in Section 4.4, we note that many more specific models can become quite involved. For example in a model with stochastic demand, the margin can be expressed as compensation for the uncertain fluctuations, including a buffer to reduce the change of break-down, or a risk premium if cartel members are risk averse. However, the point of these models is that cartel prices are to be adjusted below monopoly levels in order to stabilize the cartel when the incentive compatibility constraint binds, as in Rotemberg and Saloner (1986). The critical discount factor is effectively equated to the actual discount factor continuously by adjusting prices in that case, so that the cartel is always stable. Instead of on the critical discount factor, comparative statics may then focus on price development over time, in correlation with demand shock controls—with the added complication that the boundary values vary with the collusive price. We leave this for further research.

Possible extensions of the general model include asymmetry between firms and alternative punishment strategies, such as temporary Nash reversion or optimal carrot-and-stick combinations. Although our qualitative findings hold if firms have heterogeneous discount factors or profits, the comparative statics will have different magnitudes. Different demand systems, for example iso-elastic demand or spatial product differentiation, also change the trade-offs quantitatively.

Finally, note that our results apply only for market characteristics that directly affect profits. Many relevant variables do, including the number of firms, cost and demand characteristics. However, some circumstances that are known to affect the scope for stable collusion in markets are not readily incorporated in the profit function of firms, such as the frequency of interaction or entry barriers. Instead these factors affect the calculation of the present discounted values of the payoff streams from collusion and deviation. Combinations of these two types of relevant market structure variables will have their own comparative statics results, depending on case specifics.
4.A Proofs

Proof of Proposition. In the proof we suppress $x$ in the notation where possible. Solving for the derivative of $\delta^*$ with respect to $x$ provides

$$\frac{\partial \delta^*}{\partial x} = \frac{\partial}{\partial x} \left( \frac{\pi^D - \pi^C + M}{\pi^D - \pi^N} \right)$$  \hspace{1cm} (4.19)$$

if $M < \bar{M}$ and $\partial \delta^*/\partial x = 0$ if $M \geq \bar{M}$. Define $M(x)$ as the lower bound value of $M$ at which $\partial \delta^*/\partial x = 0$ where $M < \bar{M}$, which solves as

$$M(x) = (\pi^D - \pi^N) \left( \frac{\partial (\pi^D - \pi^C + M)}{\partial x} \right)^{-1} \left( \frac{\partial (\pi^D - \pi^N)}{\partial x} \right)^{-1} - (\pi^D - \pi^C).$$  \hspace{1cm} (4.20)$$

Whenever the gains from deviation increases in $x$, that is $\partial \Delta \pi^D/\partial x > 0$, it follows that $\partial \delta^*/\partial x > 0$ when $M < \bar{M}$ and $\partial \delta^*/\partial x < 0$ when $M > \bar{M}$. And when the gains from deviation decreases in $x$, that is $\partial \Delta \pi^D/\partial x < 0$, it follows that $\partial \delta^*/\partial x < 0$ when $M < \bar{M}$ and $\partial \delta^*/\partial x > 0$ when $M > \bar{M}$. From the definitions of $M$ and $\bar{M}$ it subsequently follows that

$$\frac{\partial \delta^*}{\partial x} \begin{cases} > 0 & \text{if } M < \{M, \bar{M}\} \\ = 0 & \text{if } M = M < \bar{M} \\ < 0 & \text{if } M \in (M, \bar{M}) \\ = 0 & \text{if } M \geq \bar{M} \end{cases}$$  \hspace{1cm} (4.21)$$

if $\partial \Delta \pi^D/\partial x > 0$ and

$$\frac{\partial \delta^*}{\partial x} \begin{cases} < 0 & \text{if } M < \{M, \bar{M}\} \\ = 0 & \text{if } M = M < \bar{M} \\ > 0 & \text{if } M \in (M, \bar{M}) \\ = 0 & \text{if } M \geq \bar{M} \end{cases}$$  \hspace{1cm} (4.22)$$

if $\partial \Delta \pi^D/\partial x < 0$. The proposition describes the situation where $M \in (M, \bar{M})$. What remains to be shown is therefore that $\bar{M} < \bar{M}$, so that
this situation can actually occur. The necessary and sufficient condition for this can be derived as follows

\[ M < \tilde{M} \]

\[ \pi^C - \pi^N > (\pi^D - \pi^N) \frac{\partial (\pi^D - \pi^C + M)}{\partial x} \left( \frac{\partial (\pi^D - \pi^N)}{\partial x} \right)^{-1} - (\pi^D - \pi^C) \]

\[ \pi^D - \pi^N > (\pi^D - \pi^N) \frac{\partial (\pi^D - \pi^C + M)}{\partial x} \left( \frac{\partial (\pi^D - \pi^N)}{\partial x} \right)^{-1} \]

\[ 1 > \left( \frac{\partial (\pi^D - \pi^N)}{\partial x} - \frac{\partial (\pi^C - \pi^N - M)}{\partial x} \right) \left( \frac{\partial (\pi^D - \pi^N)}{\partial x} \right)^{-1} \]

\[ 0 < \frac{\partial (\Delta \pi^C - M)}{\partial x} \left( \frac{\partial \Delta \pi^D}{\partial x} \right)^{-1}, \quad (4.23) \]

which holds whenever \( \Delta \pi^C - M \) and \( \Delta \pi^D \) both in- or decrease in \( x \).

**Proof of Corollary 1.** The quantities produced in static Nash equilibrium, collusion and optimal deviation respectively are

\[ q_i^N = \frac{a - c}{b (2 + (n - 1)\gamma)}, \quad (4.24) \]

\[ q_i^C = \frac{a - c}{2b (1 + (n - 1)\gamma)}, \quad (4.25) \]

\[ q_i^D = \frac{(a - c) (2 + (n - 1)\gamma)}{4b (1 + (n - 1)\gamma)}, \quad (4.26) \]

The associated profits are multiplicatively separable into the common scaling term \( (a - c)^2/b \) and a profit term depending only on \( \gamma \) and \( n \), indicated with tilde:

\[ \pi^N = \frac{(a - c)^2}{b} \frac{1}{(2 + (n - 1)\gamma)^2} \]

\[ = \frac{(a - c)^2}{b} \tilde{\pi}^N(\gamma, n), \quad (4.27) \]
\begin{align*}
\pi^C &= \frac{(a - c)^2}{b} \frac{1}{4(1 + (n-1)\gamma)} \\
&= \frac{(a - c)^2}{b} \bar{\pi}^C(\gamma, n), \quad (4.28) \\
\pi^D &= \frac{(a - c)^2}{b} \frac{(2 + (n-1)\gamma)^2}{16(1 + (n-1)\gamma)^2} \\
&= \frac{(a - c)^2}{b} \bar{\pi}^D(\gamma, n). \quad (4.29)
\end{align*}

Scaled profits rank strictly as $\bar{\pi}^N(\gamma, n) < \bar{\pi}^C(\gamma, n) < \bar{\pi}^D(\gamma, n)$. Note that:
\begin{align*}
\frac{\partial}{\partial c} \left( \pi^C - \pi^N - M \right) &= -\frac{2(a - c)}{b} \left( \bar{\pi}^C(\cdot) - \bar{\pi}^N(\cdot) \right) < 0, \quad (4.30) \\
\frac{\partial}{\partial c} \left( \pi^D - \pi^N \right) &= -\frac{2(a - c)}{b} \left( \bar{\pi}^D(\cdot) - \bar{\pi}^N(\cdot) \right) < 0, \quad (4.31)
\end{align*}
so that the proposition applies. Combining the profit functions in (4.4),
\begin{align*}
\delta^* &= \frac{\bar{\pi}^D(\cdot) - \bar{\pi}^C(\cdot)}{\bar{\pi}^D(\cdot) - \bar{\pi}^N(\cdot)} + \frac{b}{(a - c)^2} \frac{M}{\bar{\pi}^D(\cdot) - \bar{\pi}^N(\cdot)}, \quad (4.32)
\end{align*}
so that
\begin{align*}
\frac{\partial \delta^*}{\partial c} &= \frac{2b}{(a - c)^3} \frac{M}{\bar{\pi}^D(\cdot) - \bar{\pi}^N(\cdot)} > 0 \quad \text{for any } M \in (0, \bar{M}), \quad (4.33)
\end{align*}
where $\bar{M} = (a - c)^2 \left( \bar{\pi}^C - \bar{\pi}^N \right) / b$ is strictly positive and decreasing in $c$, as established. From (5) it is clear that $\partial \delta^*/\partial c = 0$ for $M = 0$. ■

**Proof of Corollary 2.** From the proposition, it is required to show that the gains from collusion and deviation both increase in $\gamma$. Using the solutions in the proof of Corollary 1,
\begin{align*}
\frac{\partial}{\partial \gamma} \left( \pi^C - \pi^N - M(\gamma) \right) &= -\frac{(a - c)^2}{b} \left( \frac{n - 1}{4(1 + (n-1)\gamma)^2} - \frac{2(n-1)}{(2 + (n-1)\gamma)^3} \right)
\end{align*}
\[
- \frac{\partial M(\gamma)}{\partial \gamma},
\]

which, as \(\partial M/\partial \gamma = 0\), is positive when \((2 + (n - 1)\gamma)^3 < 8 (1 + (n - 1)\gamma)^2\), which solves as \(n < (1 + \sqrt{5} + \gamma)/\gamma\). This condition holds for values of \(n\) for \(\gamma\) sufficiently small, from \(n \leq 4\) when \(\gamma \to 1\) to unbounded \(n\) for \(\gamma \to 0\).

Analogously,

\[
\frac{\partial (\pi^D - \pi^N)}{\partial \gamma} = -\frac{(a - c)^2}{b} \left( \frac{(2 + (n - 1)\gamma)(n - 1)}{8 (1 + (n - 1)\gamma)^3} - \frac{2(n - 1)}{(2 + (n - 1)\gamma)^3} \right)
\]

is positive when \((2 + (n - 1)\gamma)^4 < 16 (1 + (n - 1)\gamma)^3\), which holds for values of \(n\) for \(\gamma\) sufficiently small, from \(n \leq 11\) when \(\gamma \to 1\) to unbounded \(n\) for \(\gamma \to 0\).

The expression of the lower bound is

\[
M = \frac{128b (1 + (n - 1)\gamma) (2 + (n - 1)\gamma)^2 X}{(a - c)^2\gamma^2 \left( 8 + 8(n - 1)\gamma + (n - 1)^2 \gamma^2 \right)^4},
\]

where \(X = \left( 16 + 24(n - 1)\gamma + 8(n - 1)^2 \gamma^2 - (n - 1)^3 \gamma^3 \right)\). \(M\) increases in \(\gamma\) and \(n\) for \(\{n, \gamma\}\) small enough: from \(n \leq 8\) when \(\gamma \to 1\) to unbounded \(n\) for \(\gamma \to 0\). As established, the upper bound \(\tilde{M}\) increases in \(\gamma\) under the same condition that \(n < (1 + \sqrt{5} + \gamma)/\gamma\). ■

4.B Price Competition

All results reported in the text hold qualitatively the same for Bertrand competition. Demand from (4.9) of firm \(i\) is

\[
q_i = \alpha - \beta_1 p_i^{\frac{1}{2}} + \beta_2 \sum_{j \neq i} p_j^{\frac{1}{2}},
\]
Chapter 4. Cartel Stability by a Margin

in which

\[ \alpha = \frac{a}{b(1 + (n - 1)\gamma)} \]  
(4.37)

\[ \beta_1 = \frac{1 - (n - 2)\gamma}{b(1 - \gamma)(1 + (n - 1)\gamma)} \]  
(4.38)

\[ \beta_2 = \frac{\gamma}{b(1 - \gamma)(1 + (n - 1)\gamma)} \]  
(4.39)

This specification allows for proofs of both corollaries in case of price competition, as well as similar numerical illustrations.

**Proof of Corollary 1 (Price Competition).** Profit maximizing prices for static Nash, collusion and deviation in case of Bertrand are

\[ p_{N,i} = \frac{(a + c)(1 - \gamma) + (n - 1)\gamma c}{2 + (n - 3)\gamma}, \]  
(4.40)

\[ p_{C,i} = \frac{a + c}{2}, \]  
(4.41)

\[ p_{D,\text{nb},i} = \frac{(a + c)(2 + (n - 3)\gamma) + 2(n - 1)\gamma c}{4(1 + (n - 2)\gamma)}, \]  
(4.42)

\[ p_{D,b,i} = \frac{(2\gamma - 1)a + c}{2\gamma}, \]  
(4.43)

where \( p_{D,\text{nb},i} \) is the optimal deviation price in case the non-negativity constraint \( q_{-i} \geq 0 \) does not bind and \( p_{D,b,i} \) in case it does bind. The non-negativity constraint binds for \( \gamma > \left( n - 3 + \sqrt{n^2 - 1} \right) / (3n - 5) \), which is found by solving for \( \gamma \) in \( p_{D,\text{nb},i} = p_{D,b,i} \). Associated profits are again multiplicatively separable into the common scaling term \( (a - c)^2/b \) and a profit term depending only on \( \gamma \) and \( n \):

\[ \pi_N = \frac{(a - c)^2}{b} \frac{(1 - \gamma)(1 + (n - 2)\gamma)}{(1 + (n - 1)\gamma)(2 + (n - 3)\gamma)^2} \]

\[ = \frac{(a - c)^2}{b} \tilde{\pi}^N(\gamma, n), \]  
(4.44)
\[ \pi^C = \frac{(a - c)^2}{b} \frac{1}{4(1 + (n - 1)\gamma)} \]
\[ = \frac{(a - c)^2}{b} \tilde{\pi}^C(\gamma, n), \quad (4.45) \]

\[ \pi^{D, nb} = \frac{(a - c)^2}{b} \frac{(2 + (n - 3)\gamma)^2}{16(1 - \gamma)(1 + (n - 1)\gamma)(1 + (n - 2)\gamma)} \]
\[ = \frac{(a - c)^2}{b} \tilde{\pi}^{D, nb}(\gamma, n), \quad (4.46) \]

\[ \pi^{D, b} = \frac{(a - c)^2}{b} \frac{2\gamma - 1}{4\gamma^2} \]
\[ = \frac{(a - c)^2}{b} \tilde{\pi}^{D, b}(\gamma, n), \quad (4.47) \]

for which \( \tilde{\pi}^N(\gamma, n) < \tilde{\pi}^C(\gamma, n) < \{ \tilde{\pi}^{D, nb}(\gamma, n), \tilde{\pi}^{D, b}(\gamma, n) \} \). (4.4) provides

\[ \delta^* = \frac{\tilde{\pi}^D(\gamma, n) - \tilde{\pi}^C(\gamma, n)}{\tilde{\pi}^D(\gamma, n) - \tilde{\pi}^N(\gamma, n)} + \frac{b}{(a - c)^2} \frac{M}{\tilde{\pi}^D(\gamma, n) - \tilde{\pi}^N(\gamma, n)}, \quad (4.48) \]

so that

\[ \frac{\partial \delta^*}{\partial c} = \frac{2b}{(a - c)^3} \frac{M}{\tilde{\pi}^D(\gamma, n) - \tilde{\pi}^N(\gamma, n)} > 0 \quad \text{for any } 0 < M < \bar{M}. \quad (4.49) \]

We omit the figures of numerical exercises under Bertrand competition, but these provide conclusions that are similar to those under Cournot competition. The critical discount factor still unambiguously decreases when marginal cost decreases, with the exception of extreme case where demand is highly concave and \( c \) and \( M \) are very low. The relationship remains for higher numbers of firms \( n \), more product differentiation (lower \( \gamma \)), and any of the other parameters. Changes in the slope of \( \delta^* \) are major, which implies that the effect on comparative statics of \( M \) are material. Lower bound \( M \) is also regularly non-positive, so that it does not bind.
There are two differences with the numerical results under Cournot competition. On the one hand is the upper bound $\bar{M}$ around eight times higher in price competition, which means that the permissible area of $M$ is larger. This results from the fact that static Nash competitive profits are zero under Bertrand competition with homogeneous goods, whereas they are positive under Cournot competition. On the other hand, however, the material effects on the comparative statics for any cartel margin are around five times lower under Bertrand than Cournot competition. This means that a higher cartel margin is required to achieve the same material effect on comparative statics. This is because also the gains from deviation is a lot larger under price than quantity competition, which requires a larger margin to offset.

**Proof of Corollary 2 (Price Competition).** From the proposition it is required to show that the gains from collusion and deviation both increase in $\gamma$. Using the derivations in the proof of Corollary 1 in case of price competition, the gains from collusion is

$$\pi^C - \pi^N = \frac{(a - c)^2}{b} \frac{(n - 1)^2 \gamma^2}{4(1 + (n - 1) \gamma)(2 + (n - 3) \gamma)^2}, \quad (4.50)$$

and its derivative with respect to $\gamma$

$$\frac{\partial (\pi^C - \pi^N)}{\partial \gamma} = -\frac{(a - c)^2}{b} \frac{(n - 1)^2 \gamma ((n - 1)(n - 3) \gamma^2 - 2(n - 1) \gamma - 4)}{4(1 + (n - 1) \gamma)^2(2 + (n - 3) \gamma)^3}, \quad (4.51)$$

which is positive when $(n - 1)(n - 3) \gamma^2 - 2(n - 1) \gamma - 4 < 0$, which solves to $n < \left(1 + 2\gamma + \sqrt{5 + 2\gamma + \gamma^2}\right)/\gamma$. This condition holds for values of $n$ for $\gamma$ sufficiently small, from $n \leq 5$ when $\gamma \to 1$ to unbounded $n$ for $\gamma \to 0$. The gains from deviation are:

$$\pi^{D, nb} - \pi^N =$$

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\[
\pi_{D,b} - \pi_N = \frac{(a - c)^2 (2\gamma - 1) (1 + (n - 1)\gamma) (2 + (n - 3)\gamma)^2 - 4\gamma^2(1 - \gamma) (1 + (n - 2)\gamma)}{b 4\gamma^2 (1 + (n - 1)\gamma) (2 + (n - 3)\gamma)^2}.
\]

(4.53)

Taking the derivatives of these functions with respect to \(\gamma\) provides, in case lower bound \(q_{-i} \geq 0\) non-binding

\[
\frac{\partial (\pi_{D,nb} - \pi_N)}{\partial \gamma} = \frac{(a - c)^2 (n - 1)^2 \gamma (32 + \gamma F_1 + \gamma F_2 + \gamma^3 F_3 + \gamma^4 F_4 + \gamma^5 F_5 + \gamma^6 F_6)}{b 16 (1 - \gamma)^2 (1 + (n - 1)\gamma)^2 (1 + (n - 3)\gamma)^2 (2 + (n - 3)\gamma)^3}.
\]

(4.54)

in which

\[
\begin{align*}
F_1 &= 80n - 208 \\
F_2 &= 64n^2 - 368n + 496 \\
F_3 &= 14n^3 - 154n^2 + 490n - 478 \\
F_4 &= -2n^4 + 22n^3 - 62n^2 + 34n + 40 \\
F_5 &= 14n^4 - 118n^3 + 366n^2 - 482n + 220 \\
F_6 &= n^5 - 16n^4 + 88n^3 - 218n^2 + 247n - 102.
\end{align*}
\]

As \(32 + \sum_{i=1}^{6}\gamma_i F_i > 0\) for all values of \(n \geq 2\) and \(\gamma \in (0, 1]\), the derivative is always positive.

Similarly, in case lower bound \(q_{-i} \geq 0\) binding

\[
\frac{\partial (\pi_{D,b} - \pi_N)}{\partial \gamma} = \frac{(a - c)^2 8 + \gamma F_1 + \gamma^2 F_2 + \gamma^3 F_3 + \gamma^4 F_4 + \gamma^5 F_5 + \gamma^6 F_6}{b 2\gamma^3 (1 + (n - 1)\gamma)^2 (2 + (n - 3)\gamma)^3},
\]

(4.55)
in which

\[
\begin{align*}
F_1 &= 28n - 60 \\
F_2 &= 38n^2 - 176n + 186 \\
F_3 &= 25n^3 - 191n^2 + 443n - 309 \\
F_4 &= 8n^4 - 93n^3 + 365n^2 - 571n + 299 \\
F_5 &= n^5 - 19n^4 + 118n^3 - 318n^2 + 381n - 163 \\
F_6 &= -n^5 + 11n^4 - 48n^3 + 102n^2 - 103n + 39.
\end{align*}
\]

As \(8 + \sum_{i=1}^{6}\gamma^i F_i > 0\) for all values of \(n \geq 2\) and \(\gamma \in (0, 1]\), this derivative is also always positive. ■

Numerical analyses of various demand variations again lead to results comparable to those under Cournot competition. Without a cartel margin, we obtain the ambiguous relationship between product differentiation and cartel stability found in Deneckere (1983), where cartel stability initially decreases but at some point increases when products become more homogeneous. However, introducing a sufficiently large cartel margin makes that product substitutability unambiguously increases the scope for stable collusion. The effects on the slope again appear to be material, as in the case of Cournot competition.

The same two material differences between quantity and price competition as in the case of changing marginal cost are found. The upper bound on \(M\) lies about eight times higher in case of Bertrand than Cournot competition, resulting from the much lower static Nash competitive profit. The required size of the margin to obtain similar effects on the comparative statics is about five times higher. Also in case of product differentiation, the relationship is non-monotonous with no cartel margin, as in Deneckere (1983), but becomes monotonous once the cartel margin is sufficiently large. The results require also that competition is not too strong to begin with.
Chapter 5

Event Studies in Merger Analysis: Review and an Application Using U.S. TNIC Data*

A broad discussion has emerged on the observation of increased industry concentration, markups and market power (Autor, Dorn, Katz, Patterson and Van Reenen, forthcoming; Basu, 2019; De Loecker and Eeckhout, 2018; De Loecker, Eeckhout and Unger, forthcoming; Grullon, Larkin and Michaely, 2019; Syverson, 2019). One concern is that concentration and market power may have increased as a consequence of an insufficient deterrence of anti-competitive mergers, especially in the U.S. (Baker, 2019, p. 15; Gutiérrez and Philippon, 2018; Kwoka, 2015; Philippon, 2019; Shapiro, 2018; 2019; Wollmann, 2019). Despite the prominence of this concern, empirical evidence remains limited. In particular, ex post reviews of merger decisions remain complex or costly (and often politically unfavorable), and hence scarce.¹

Event studies have been proposed as a simple alternative to acquire

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¹For a review of existing work, see in particular Kwoka (2015). The relative absence of empirical evidence within Industrial Organization on the competitive effects of mergers is prominently criticized by Angrist and Pischke (2010, pp. 20-22).
empirical insights into the anticipated competitive effects of mergers. By estimating the abnormal stock returns around an event date, event studies aim to identify the anticipated effect of this event on future firm performance. In the context of mergers, existing event studies look at the abnormal returns of competitors around a merger announcement. This follows from seminal microeconomic theory that predicts that competitors to a merger benefit if anti-competitive effects from increased market concentration dominate, but lose out if pro-competitive effects from merger efficiencies dominate (Farrell and Shapiro, 1990). Existing studies then often use as identifying assumption that an abnormal increase (decrease) in competitor stock price following a merger announcement indicates that financial markets anticipate the merger to be anti-competitive (pro-competitive). However, this inverse relationship between competitor stock price and consumer welfare is not guaranteed, as it may be weakened by the presence of other mechanisms as well as stock market noise.

This paper starts with a discussion on the use of event studies in merger analysis. Existing studies generally suffer from the following limitations. First, the identifying assumption that abnormal competitor returns and consumer welfare are inversely related may fail to hold for various reasons and would have to be tested if used. Second, event studies in merger analysis require the reliable identification of competitors, which is often not obvious. And third, they involve the identification of small effects in very noisy data. This necessitates a sufficiently large dataset.

In a novel application of Hoberg-Phillips (2010, 2016) Text-Based Network Industry Classification (TNIC) data I am able to readily proxy a ranking of likely competitors to 1,751 U.S. mergers and acquisitions between 1997 and 2017 with a real transaction value above one billion dollar and a publicly-traded target. I document that likely competitors experience on average a positive and statistically significant abnormal return—with an estimated effect of around one percent. I also find that abnormal returns show a positive association with a TNIC-based Herfindahl-Hirschman Index (HHI). Because HHI serves as an indicator of market power concerns, this association suggests that results are at least in part driven by an antic-
ipation of anti-competitive market power effects, and hence an insufficient deterrence of anti-competitive mergers.

The remainder of this paper is as follows. Section 5.1 provides an overview of possible mechanisms through which a merger announcement may affect stock prices, reviews event studies in merger analysis and identifies and discusses the main challenges in using event studies in merger analysis. Sections 5.2 discusses the data collection and cleaning. Section 5.3 outlines the methodology used and Sections 5.4 and 5.5 discuss the results and sensitivity checks. Section 5.6 concludes.

5.1 Review of Merger Event Studies

Event studies are a well-developed method within finance and economics that aims to identify the anticipated effect of an event on firm performance (Fama, Fisher, Jensen and Roll, 1969; MacKinlay, 1997). Event studies are based on the efficient market hypothesis, which states that stock prices reflect all publicly available information on future profits. By estimating the abnormal stock returns around an event date, event studies aim to identify the anticipated effect of this event on future firm performance.

The estimation of abnormal return for a particular stock is generally done in the following three steps (MacKinlay, 1997). First, a linear relationship between stock and market return (the ‘market model’) is estimated during some estimation window prior to the event date. Usually, the estimation window lasts for around 250 days and stops several days before the event, so as to exclude any event effects in the estimation. Second, the abnormal return for each day around the event date is derived as the difference between actual stock return and the return as predicted by the market model. Third, the cumulative abnormal return from the event is derived as the sum of all daily abnormal returns during some event window—which is the range of days in which the new information has likely become public. This cumulative abnormal return is supposed to capture the anticipated effect from the event on future firm performance. Different event window specifications are often used for robustness.
Event studies have been used to analyze the anticipated effects of mergers by looking at the abnormal returns that occur around a merger announcement date. Below I first discuss the mechanisms through which a merger announcement may affect stock prices and how they relate to the anticipated competitive effects of the merger. I then review existing literature that aims to identify these mechanisms and close off with a review of the main challenges in using event studies in merger analysis.

5.1.1 Mechanisms

When using event studies in merger analysis, the identifying assumption that is often used is that a positive (negative) abnormal competitor return at the time of the merger announcement indicates an anticipation of anti-competitive (pro-competitive) effects (Cichello and Lamdin, 2006). This inverse relationship follows from two opposing mechanisms within seminal microeconomic theory (Farrell and Shapiro, 1990): on the one hand, mergers generate cost efficiencies and other synergies that benefit the merging parties and consumers, but hurts competitors through reduced relative competitiveness; on the other hand, mergers increase unilateral market power of all market participants through a reduction in amount of firms active in the market. This benefits all firms, but hurts consumers. Although merging firms benefit from both mechanisms, competitors lose out if the pro-competitive efficiency effect dominates and benefit if the anti-competitive market power effect dominates.

The inverse relationship between consumer welfare and competitor performance, as measured by its stock price, may however be weakened by the presence of other mechanisms. First, as also noted by Kwoka (2015, p. 42), a merger may enable the merged entity to foreclose its competitor through predatory behavior or other exclusionary practices. An increased expectation of such behavior would lead to a negative abnormal competitor return, but driven by an anticipation of anti-competitive effects.

Second, a merger may signal that similar firms are “in play” and hence have an increased probability of being acquired in the future (Servaes and Tamayo, 2014; Song and Walkling, 2000). Because being acquired often
involves a stock price premium, any increase in the acquisition probability would already cause the current stock price to increase. In principle, this mechanism can be either anti- or pro-competitive, as any future merger may again generate both anti- and pro-competitive effects. Contrasting this “in-play” effect is also a possible “out-of-play” effect, in which a merger announcement signals that competitors have lost a race to acquire the target (Fridolfsson and Stennek, 2010), or perhaps be acquired themselves.

Finally, a merger announcement may reveal positive information on market fundamentals, industry prospects or a general scope for efficiencies that was previously private. For instance, Derrien, Frésard, Slabik and Valta (2019) show that positive abnormal returns for industry peers occur in merger announcements when the target is a public firm, but not when it is a private firm. They argue that an acquirer—who is assumed to be better informed on current and future industry performance—will prefer a public over a private firm when public firms are undervalued, all else equal. The acquisition of a public firm may therefore involve a signal that similar public firms are undervalued. This may cause competitors to experience a positive abnormal return even in the absence of any anticipated anti-competitive effects or “in-play” effects.

Table 5.1 summarizes these mechanisms and the way in which they affect stock prices. It shows that the effect on the merging firms jointly is unambiguously positive. It also shows that, a priori, the effect on competitors is ambiguous: different mechanisms affect the competitor returns differently, and these mechanisms can be anti-competitive, pro-competitive or competitively neutral.

Note that each mechanism may already be present prior to an official merger announcement when the merger is to some degree anticipated. In as far as the different mechanisms are affected differently by any anticipation, estimates of the abnormal returns may be biased when looking at returns too close to the official announcement. This requires a careful consideration of the event window. Additionally, when financial markets believe that there is a probability that competition authorities will object to the merger, the effect of mechanisms that are conditional on the merging
Table 5.1: How Merger Announcements May Affect Stock Prices

<table>
<thead>
<tr>
<th>Mechanisms</th>
<th>Merging Firms</th>
<th>Competitor Firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anticipated efficiencies</td>
<td>+</td>
<td>−</td>
</tr>
<tr>
<td>Anticipated market power effects</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Anticipated exclusion effects</td>
<td>+</td>
<td>−</td>
</tr>
<tr>
<td>“In-play” effects</td>
<td>·</td>
<td>+</td>
</tr>
<tr>
<td>“Out-of-play” effects</td>
<td>·</td>
<td>−</td>
</tr>
<tr>
<td>Signalling on industry health, prospects</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

actually occurring may be discounted. Finally note that empirically, these mechanisms are generally obscured by the presence of stock market noise. This noise is amplified by the fact that firms are often large and diversified and the merger may only relate to one part of the business.

5.1.2 Existing Literature

Pioneering work on the use of event studies in merger analysis includes Stillman (1983), Eckbo (1983) and Eckbo and Wier (1985). Stillman (1983) looks at 11 U.S. horizontal mergers that occurred between 1964 and 1972 and were challenged by the competition authorities. Competitors are identified using opinions published in litigations or fact memoranda prepared by the competition authorities. Using a variation to the event study methodology mentioned above, he observes that in only one of these challenged mergers there is a positive abnormal competitor return around the time of the merger announcement, which he says suggests that U.S. competition authorities have challenged too many mergers.

Eckbo (1983) and Eckbo and Wier (1989) instead look at the average effect of a larger sample of up to 82 challenged U.S. horizontal mergers between 1963 and up to 1981. They use both SIC codes and public case summaries to identify competitors. Using event windows of up to 20 days prior and 10 days after the announcement, both studies find a positive and statistically significant average abnormal competitor return. How-
ever, they reject the hypothesis that this is driven by any anticipation of anti-competitive market power effects, because they do not observe the opposite result at the time of a merger challenge, which would have reduced the likelihood of merger approval. They suggest that the positive average abnormal competitor returns are instead driven by some signalling on the scope for competitors to improve performance. Fee and Thomas (2004) and Shahrur (2005) replicate these results for later periods, looking also at the effects on customers and suppliers.

These earlier studies have been criticized on several other grounds. First, their datasets are often limited to a few dozen mergers. Because using stock prices for merger analysis involves detecting small effects in noisy data, event studies have a very low precision in classifying individual mergers, or even small samples, as anti-competitive (Kwoka and Gu, 2015; McAfee and Williams, 1988; Werden and Williams, 1989a; 1989b). Second, they generally rely on industry codes such as SIC to identify competitors. This is problematic, because industry codes do a poor job at identifying markets from a competition policy perspective (Werden, 1988; Hoberg and Phillips, 2016). Third, their event windows—often only a few days around the announcement—may be too restrictive, because an anticipation of a merger announcement may already occur much earlier (Duso, Gugler and Yurtoglu, 2010). And finally, these papers fail to test properly for the different possible mechanisms through which competitor stock prices are affected. For instance, they reject an anticipation of anti-competitive effects (despite the positive average abnormal competitor returns at the time of announcement) solely on the basis of an absence of statistically negative returns at the time of a merger challenge—which may simply be the consequence of low statistical power. Alternative explanations are suggested but not subjected to scrutiny.

Several more recent papers use event studies to analyze EU instead of U.S. merger control. These have the advantage over the earlier criticized work that they can reliably identify competitors to a large subset of EU mergers by using the published decisions by the European Commission—which is generally not available in the case of U.S. mergers. Instead of
looking at the average abnormal competitor return (which these papers generally find is not statistically different from zero), they use the estimated abnormal returns to inform other policy questions.

More specifically, Duso, Neven and Röller (2007) look at 167 EU mergers between 1990 and 2002 and classify 46 as anti-competitive and 121 as pro-competitive—using the inverse relationship between competitor stock prices and consumer welfare as identifying assumption and an event window of up to five days prior and after the announcement. They go on to explain how the EU institutional and political environment can explain false positive and negative. Duso, Gugler and Yurtoglu (2010) additionally show that a positive and statistically significant pairwise correlation coefficient exists between the estimated abnormal returns and ex post accounting profit, provided a sufficiently long event window of up to 50 days prior to the event is used. Duso, Gugler and Yurtoglu (2011), using an event window of 50 days prior and five days after the announcement, go on to show that EU merger control has been partially effective at reversing positive abnormal returns when they are observed—correcting additionally for an estimated probability of competition policy interference. Finally, Duso, Gugler and Szücs (2013) extends these papers by looking at 368 EU mergers between 1990 and 2007. They also show how the 2004 merger reforms have partially improved EU merger control.²

For their approach, these papers require the classification of individual cases as either anti- or pro-competitive based on whether abnormal returns are positive or negative. Because of stock market noise, this approach may have a very low precision. Additionally, the existence of possible alternative mechanisms affecting the abnormal returns is acknowledged, but generally ignored in the analysis. For instance, Duso, Neven and Röller (2007) accept that merger announcements may signal a scope for competitor efficiencies as well as “in-play” or “out-of-play” effects. However, they argue that none

²Although not using event studies, Stiebale and Szücs (2019) also look at EU competitor performance—using micro panel data instead. They show for 194 EU mergers between 1999 and 2007 that competitors experience a statistically significant increase in estimated markups relative to a control, with larger estimates when market power concerns are more likely.
of these mechanisms have a convincing empirical or theoretical basis and hence ignore them (pp. 462-464).

Finally, two recent papers use event studies to explicitly test for two of the other mechanisms. Firstly, Bernile and Lyandres (2019) show for 480 U.S. horizontal mergers between 1996 and 2005 a negative association between announced efficiencies and abnormal competitor returns—in line with an anticipation of pro-competitive efficiencies. Competitors are identified using the SIC3-granularity TNIC provided by Hoberg and Phillips (2010, 2016), providing on average 12.5 competitors per merger, and they use an event window of up to 20 days prior and after the event. Although they do report positive average abnormal competitor returns overall, they do not explore the possible underlying mechanisms.

Secondly, Derrien, Frésard, Slabik and Valta (2019) observe that positive average abnormal competitor returns occur when the target firm is public, but not when it is private—looking at 984 horizontal U.S. mergers involving a public target and 7,010 involving a private target occurring between 1990 and 2015 and with a deal value above 10 million dollar. They identify competitors using four-digit SIC codes and an event window of at most five days prior and after the event. They argue that the difference in case of a public or private target is driven by a signal on industry health: an acquirer—who is assumed to be better informed on industry performance—would prefer a public firm when public firms are undervalued. The acquisition of a public firm therefore involves a signal that similar public firms are undervalued. They reject an anticipation of anti-competitive market power effects or “in-play” effects, based on an absence of statistical significance with SIC-based HHI and future competitor acquisition.

### 5.1.3 Challenges

The review above identifies the following main challenges to the application of event studies in merger analysis. First, any assumed mechanism would have to be tested explicitly. For instance, the identifying assumption that a positive (negative) abnormal competitor return implies an anticipation of
anti-competitive (pro-competitive) effects may fail to hold. When testing any mechanism, good practice dictates that a lack of statistical significance on itself does not prove an absence of effect.

Second, the use of event studies in merger analysis requires the reliable identification of competitors. This is not straightforward. As mentioned, the often-used method of relying on industry codes such as SIC does a poor job at identifying markets from a competition policy perspective (Werden and Williams, 1988; Hoberg and Phillips, 2016). This is because they classify firms based on the supply instead of demand side, are often too broad, hardly reclassify firms as markets evolve and their binary nature imposes transitivity (in which the set of competitors to any two firms has to be identical).

And third, event studies in merger analysis involves the identification of small effects in very noisy data (Werden and Williams, 1989a). This means that event studies have a very low precision when looking at individual mergers or even small samples. In other words, they require a sufficiently large dataset to achieve sufficient statistical power, which many of the existing studies do not have.

In the remainder of this paper I use event studies to look at a large subset of major U.S. mergers that occurred between 1997 and 2017. I deal with the second and third challenges by using a novel application of Hoberg-Phillips data, which can be used to proxy a ranking of competitors to many mergers. I deal with the first challenge by testing whether the cumulative abnormal returns are associated with indicators of market power concerns. I remain agnostic on whether results are additionally driven by any of the other mechanisms.

5.2 Data Collection and Cleaning

I use the Refinitiv SDC Platinum database to identify all major U.S. mergers between January 1997 and December 2017 with a real transaction value above one billion dollar and a publicly-traded target. Using the Hoberg-Phillips TNIC database, I am able to proxy a ranking of competitors to
1,751 of the largest U.S. mergers with a public target. Daily stock market returns of all relevant firms and the S&P 500 as benchmark stock market index are collected using Compustat. Below the data collection and cleaning is discussed in more detail.

5.2.1 Subset of Mergers

A dataset of all 98,123 transactions between January 1997 and December 2017 with a U.S. firm as target is extracted from the Refinitiv SDC Platinum database. The collected variables include target firm name, acquiring firm name, date of official announcement, transaction value, increase in shareholding, post-transaction shareholding and whether or not the transaction has been completed.

From this dataset, 3,794 transactions are identified that (i) had a transaction value of at least one billion dollar (in December 2017 value), (ii) have been completed, (iii) involved a share acquisition of at least 50 percent and (iv) did not have the acquirer listed as ‘shareholders’. Of these, 1,661 transactions had to be dropped because the name of the target firm could not be identified in the Hoberg-Phillips database. In most cases, this is because the target firm is not publicly listed and hence not even included in the Hoberg-Phillips database (which only includes publicly-traded U.S. firms, as discussed below). In some cases however, an absence of identification may be because the names in the Refinitiv SDC Platinum database were not properly matched with those in the Hoberg-Phillips database. This has been checked manually, but with no guarantee that all matches have been correct.

Finally, 382 mergers were dropped because the merging parties had no TNIC score despite both being identified in the Hoberg-Phillips database. This could be because Hoberg and Phillips drop scores if firms are classified as vertically related, discussed below. Excluding these is justified because we are interested in horizontal mergers. Another reason could be an erroneous matching of firm names. Such observations are essentially random noise, which justifies excluding them.

Figure 5.1 shows the distribution of mergers over the years and real
transaction values—separately for all 3,794 transactions and for the final dataset of 1,751 transactions. The left panel illustrates the well-known merger waves: before 2000, before 2008 and until recently. The middle and right panels show that the distribution over the real transaction value is skewed. In both the full and the final dataset, about 50% of mergers have a real transaction value between one and 2.5 billion dollar, 25% between 2.5 and five billion dollar, 20% between five and 20 billion dollar and only the remaining five percent above 20 billion dollar. Although the distributions of the full and final dataset are relatively similar, the final distribution does have more observations with higher transaction value. This is most likely because the final dataset only includes transactions with a target firm that is publicly traded and higher-valuation firms are more often publicly traded.

Figure 5.1: Distribution of Mergers

![Graph showing the distribution of mergers over years and real transaction value for all U.S. mergers with a real transaction value of at least one billion dollar (in December 2017 value) and for the final dataset used.]

Notes: Distribution of mergers over years and real transaction value, for all U.S. mergers with a real transaction value of at least one billion dollar (in December 2017 value) and for the final dataset used.

5.2.2 Hoberg-Phillips TNIC Scores

The Hoberg-Phillips (2010, 2016) Text-Based Network Industry Classification (TNIC) database consists of yearly matrices with pair-wise firm differentiation scores for all 13,808 publicly-traded U.S. firms from 1996 to 2017. The scores are derived from a text-based analysis of their 10-K
5.2. Data Collection and Cleaning

business descriptions, which firms submit yearly to the Securities and Exchange Commission and which is legally required to represent a concise and accurate summary of their product offerings. Hoberg and Phillips claim that the TNIC scores capture the degree of competition between two firms, based on the premise that firms with more common vocabulary in their 10-K product descriptions are nearer competitors. As such, the TNIC scores can be interpreted as a Hotelling-like product differentiation score: firms with higher TNIC scores have more similar 10-K product descriptions and are therefore closer competitors.\(^3\)

For the purpose of this paper, I identify the most likely competitors as those firms with the highest TNIC score with the target firm. I also vary the lowest admissible rank, to see for which ranks an average effect may be found. An alternative approach would select all firms with a TNIC score above a certain cut-off value. This approach is omitted for two reasons. First, it requires the assumption that TNIC scores are also cardinal instead of only ordinal—which may not hold across industries or years. Second, empirically this approach leads to many mergers with no competitors at all, while at the same time leading to many other mergers with unreasonably many competitors.

TNIC scores have several advantages over industry codes such as SIC. Most importantly, TNIC scores are a continuous measure between zero and one, instead of a binary in-out classification. This allows for a much finer selection of potential competitors than industry codes, which often include many hundreds of firms. Additionally, they are based on product descriptions (demand side) instead of the production process (supply side). Furthermore, because TNIC scores are pair-wise, they are not restricted to transitivity—in which two competitors have to have the same set of other competitors. Finally, TNIC scores are updated yearly, which accommodates changing industry relations following from innovation, dynamic product differentiation or mergers.

It may be that vertically related firms use similar vocabulary for the

\(^3\)All data is available open-source on www.hobergphillips.tuck.dartmouth.edu. This website also includes further explanations, as well as instructions on how to use and interpret the data.
Chapter 5. Event Studies in Merger Analysis

10-K product descriptions as well, without competing with each other. Hoberg and Phillips purge the TNIC scores for possible vertical relations as follows. Using Benchmark Input-Output Accounts of the U.S. Economy, they calculate the fraction of inputs that flow between the four-digit SIC codes of each firm pair. If this fraction exceeds one percent of all inputs, it is assumed that the firms have a vertical relation and their TNIC scores are excluded. This occurs in four percent of all firm pairs. Although this approach is not exhaustive in excluding all vertical relations, note that any remaining vertically related firms identified as competitors will bias results downwards, making any positive estimates more conservative.

The Hoberg-Phillips TNIC database provides a convenient differentiation proxy without requiring any market definition or detailed price and quantity data. The main limitation is that it remains a proxy. Because we use the TNIC scores for the ranking of most likely competitors, we risk including weak competitors while excluding strong competitors. Note however, that this cause our estimates to be biased towards zero, making any estimate more conservative. Additionally, the TNIC database only includes publicly-traded U.S. firms. Results are therefore not necessarily externally valid in case of privately-owned or foreign-traded target firms.

5.2.3 Stock Market Returns

The daily closing prices of all 13,808 firms in the Hoberg-Phillips TNIC database are collected from Compustat for 1 January 1995 to 31 December 2018. In case a company has multiple issue IDs trading at the same time, the oldest issue ID is maintained. The first observation of any new issue ID is also dropped, because it often involves a different stock price level and hence a discontinuous jump in stock return. The S&P 500 is used as the market index.

The daily stock market returns are dropped when they are larger than 30% or smaller than 30%. This occurs in 1.4% of all daily stock market returns. Dropping these observations is done to exclude outliers: the remaining dataset includes several inexplicably large stock price movements that have a large impact on the final results. One possible explanation
for such major stock market movements are (reverse) stock splits. These cause sudden jumps in stock prices but are unfortunately not flagged in the Compustat stock market data. Sensitivity checks show that results are robust when dropping daily stock market returns beyond bounds of 20% and 20% or of 50% and 50%—which occurs in 2.4% and 0.7% of all cases, respectively. In the event study estimation, I also only include stocks that have at most 20 days without an observation during the estimation and event window.

5.3 Event Study Methodology

Using event study methodology, the cumulative abnormal return of each of the proxied competitors is calculated around the official merger announcement date. The event study methodology is used as follows, which is in line with existing literature (Cichello and Lamdin, 2006). For an estimation window of 290 to 51 days prior to the official announcement date of each merger, a linear relationship between stock return $R_{it}$ of competitor $i$ to this specific merger on day $t$ and the market index $R_{mt}$ is estimated:

$$R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it}, \quad (5.1)$$

where we omit a subscript indicating the merger. For an event window of 50 days prior and 30 days after the official announcement date, abnormal return $AR_{it}$ is calculated for each separate day as the actual return minus the return as predicted by the above market model:

$$AR_{it} = R_{it} - \hat{\alpha}_i - \hat{\beta}_i R_{mt}, \quad (5.2)$$

where $\hat{\alpha}_i$ and $\hat{\beta}_i$ are the coefficient estimates from the market model. Finally, we derive cumulative abnormal return $CAR_{it}$ for each competitor $i$ up to day $t$ as the sum of the average abnormal returns in window $[t_0, t]$:

$$CAR_{it} = \sum_{s=t_0}^{t} AR_{is}, \quad (5.3)$$
with starting date $t_0 \geq 50$. CAR$_{it}$ has a straightforward interpretation: it shows for each competitor $i$ to a merger how much more cumulative return it has gotten since $t_0$ than what would be expected based on its previous performance.

CAR$_{it}$ is derived for those firms proxied as closest competitors to each of the 1,751 mergers, based on a TNIC-based ranking. The amount of observations per day is therefore 1,751 times the lowest admissible rank. This allows us to estimate the average and confidence interval of the cumulative abnormal returns of competitors for each day around the official merger announcement.

Note that market awareness of the upcoming merger may occur already in the run-up to the official merger announcement. News of the merger may leak out or markets may anticipate such a move. This is why the estimation window runs until 51 days prior to the official announcement and the event window starts at most 50 days prior to the announcement. This is in line with the literature discussed. The trade-off is that setting the cut-off earlier reduces the risk of excluding merger effects from the event window, but increases noise and vulnerability to structural breaks in the estimated market model—and vice versa when setting the cut-off later.

### 5.4 Results

Estimates are derived using robust regression (Berk, 1990; Hamilton, 1991), with standard errors clustered per merger. Robust regression has the benefit over ordinary least squares (OLS) in that it reduces the weight of observations that disproportionately affect the estimation, hence reducing the vulnerability to outliers. While not (yet) common within event studies, Sorokina, Booth and Thorton (2013) show how in event studies the accuracy of cumulative abnormal returns generally improves under robust regression. Additionally, sensitivity checks in the next section suggest that results are conservative relative to OLS.

Figure 5.2 plots the average and 95% confidence interval for all CAR$_{it}$ for the three closest competitors, taking $t_0 = 50$ as the start of the event
window. Results clearly show a positive average cumulative abnormal competitor return of around one percent. In other words, financial markets anticipate on average a higher discounted future cash flow for competitors following a merger announcement. Figure 5.3 shows that the average cumulative abnormal competitor returns are only statistically significantly different from zero for firms ranked more closely to the merger target.

Figure 5.2: Estimated Average CAR\textsubscript{it} Around Event Date

![Graph showing estimated average CAR\textsubscript{it} around event date](image_url)

**Notes:** Average cumulative abnormal returns for three closest competitors, for event window starting at \( t_0 = 50 \). Estimate and 95% confidence interval are based on robust regression with merger-clustered standard errors.

Existing work often interprets such results as an anticipation of anti-competitive effects, based on the assumed inverse relationship between competitor returns prices and consumer welfare. To test this, I regress the cumulative abnormal returns on a TNIC-based Herfindahl-Hirschman Index (HHI) for market concentration for each competitor, which serves as an indication of market power concerns. This TNIC-based HHI is provided by Hoberg and Phillips and is derived as a conventional HHI by summing the squared market shares of all firms within a market.\(^4\)

\(^4\)Hoberg and Phillips define the relevant market for each firm as all other firms with a
Notes: Average cumulative abnormal returns for different ranks, for event window \( t \in [50, 5] \). Estimate and 95% confidence interval are based on robust regression with merger-clustered standard errors.

Table 5.2 shows that there is indeed an association between the TNIC-based SIC3-granularity HHI of each competitor, at least for a sufficiently wide event window. This suggests empirically that competitors in markets where there is potentially a bigger market power concern experience a larger abnormal return following a merger announcement. While this result does not exclude additional mechanisms, it does suggest that results are at least in part driven by an anticipation of anti-competitive market power effects.

It is unclear exactly why there is no significance for the shorter event window starting 10 days prior to the official announcement. One reason could be that mergers with a larger market power concern are generally anticipated earlier, such that the stock price may have already increased TNIC score above a certain threshold. They set this threshold such that the granularity is equivalent to three-digit SIC codes. This means that if you pick two random firms, they have the same probability of being in the same market as under three-digit SIC codes. See www.hobergphillips.tuck.dartmouth.edu for this data and further use-instructions.
Table 5.2: Results Regressing CAR_{it} on Market Concentration Proxy

<table>
<thead>
<tr>
<th>Event Window</th>
<th>[50, 5]</th>
<th>[40, 5]</th>
<th>[30, 5]</th>
<th>[20, 5]</th>
<th>[10, 5]</th>
</tr>
</thead>
<tbody>
<tr>
<td>TNIC3 HHI</td>
<td>0.0345**</td>
<td>0.0273**</td>
<td>0.0454***</td>
<td>0.0291***</td>
<td>0.0106</td>
</tr>
<tr>
<td></td>
<td>(0.0152)</td>
<td>(0.0131)</td>
<td>(0.0116)</td>
<td>(0.0093)</td>
<td>(0.0069)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.0054*</td>
<td>0.0058*</td>
<td>0.0031</td>
<td>0.0034*</td>
<td>0.0059***</td>
</tr>
<tr>
<td></td>
<td>(0.0031)</td>
<td>(0.0028)</td>
<td>(0.0024)</td>
<td>(0.0019)</td>
<td>(0.0014)</td>
</tr>
<tr>
<td>Observations</td>
<td>5,160</td>
<td>5,155</td>
<td>5,169</td>
<td>5,163</td>
<td>5,141</td>
</tr>
</tbody>
</table>

Notes: Robust regression estimates for cumulative abnormal return for three closest competitors for different event windows, controlling for competitor TNIC-based SIC3-granularity HHI. Merger-clustered standard errors are in parentheses. ***, ** and * indicate statistical significance at 1%, 5% and 10% respectively.

prior to this shorter window. Note finally that the difference in amount of observations is driven by the fact that robust regression drops observations from the total of 5,253 observations that have a particularly disproportionate effect on the estimation.

An additional possible mechanism is the “in-play” effect, in which the announcement signals that similar firms are more likely to be acquired in the future. Tables 5.3 and 5.4 show the association between the cumulative abnormal returns and whether the competitor itself or any of the six closest competitors becomes a target in the subsequent 12 or 60 months (and in this sample). There is a general absence of statistically significant results, apart from the narrow event window of 10 days prior and five days after the announcement date. It therefore remains unclear whether this mechanism is very prominent. Note again that the difference in amount of observations is driven by the fact that robust regression drops apparent outliers, from the total of 5,025 in case of 12 months and 4,026 in case of 60 months (which are also less than the previous 5,253 observations, because of less available data on the future).
Note that the proxy for the anticipation of anti-competitive effects or “in-play” effects may still be insufficient to fully capture these mechanisms. Specifically, the TNIC-based HHI has the limitation that it only considers publicly-traded U.S. firms, although many markets also include privately-owned or foreign-traded firms. This is only a concern however when abnormal competitor returns are somehow lower in markets with more privately-owned or foreign firms (and hence an artificially lower TNIC-based HHI). Additionally, any abnormal competitor returns may still, at least in part, be driven by the signalling on industry health and prospects as well as countervailing mechanisms.

<table>
<thead>
<tr>
<th>Event Window</th>
<th>[50, 5]</th>
<th>[30, 5]</th>
<th>[10, 5]</th>
<th>[50, 5]</th>
<th>[30, 5]</th>
<th>[10, 5]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Competitor Merges 12M</td>
<td>0.0051 (0.0091)</td>
<td>0.0059 (0.0072)</td>
<td>0.0114** (0.0047)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Competitor Merges 60M</td>
<td>0.0045 (0.0066)</td>
<td>0.0084 (0.0053)</td>
<td>0.0068** (0.0031)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.0118*** (0.0024)</td>
<td>0.0096*** (0.0019)</td>
<td>0.0075*** (0.0011)</td>
<td>0.0112*** (0.0029)</td>
<td>0.0088*** (0.0023)</td>
<td>0.0072*** (0.0013)</td>
</tr>
<tr>
<td>Observations</td>
<td>4,934</td>
<td>4,947</td>
<td>4,920</td>
<td>3,952</td>
<td>3,961</td>
<td>3,943</td>
</tr>
</tbody>
</table>

Notes: Robust regression estimates for cumulative abnormal return for three closest competitors for different event windows, controlling for whether the competitor mergers within the next 12 or 60 months. Merger-clustered standard errors are in parentheses. *** and ** indicate statistical significance at 1% and 5% respectively.
### 5.5 Sensitivity Checks

In the empirical strategy used, there are two choices in particular that may warrant attention. First, the choice for robust regression instead of OLS reduces the vulnerability of the estimates to outliers during the event window, but is at the same time much less conventional. Second, daily stock market returns in excess of 30% or 30% have been dropped to deal with outliers in the estimation window, but this cut-off has been chosen arbitrarily.

Figure 5.4 shows the estimates when using OLS instead of robust regression. Note that the estimated average cumulative abnormal return is now higher at around 1.5% instead of one percent. Although not statistically significant, the estimate now also increases in the run-up to the

#### Table 5.4: Results Regressing CAR$_{it}$ on Any Future Competitor Mergers

<table>
<thead>
<tr>
<th>Event Window</th>
<th>[50, 5]</th>
<th>[30, 5]</th>
<th>[10, 5]</th>
<th>[50, 5]</th>
<th>[30, 5]</th>
<th>[10, 5]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Competitor Merges 12M</td>
<td>0.0012</td>
<td>0.0037</td>
<td>0.0039</td>
<td>(0.0059)</td>
<td>(0.0046)</td>
<td>(0.0027)</td>
</tr>
<tr>
<td>A Competitor Merges 60M</td>
<td>0.0028</td>
<td>0.0027</td>
<td>0.0012</td>
<td>(0.0055)</td>
<td>(0.0044)</td>
<td>(0.0025)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.0112***</td>
<td>0.0091***</td>
<td>0.0072***</td>
<td>0.0134***</td>
<td>0.0115***</td>
<td>0.0076***</td>
</tr>
</tbody>
</table>

**Notes:** Robust regression estimates for cumulative abnormal return for three closest competitors for different event windows, controlling for whether any of the six closest competitors mergers within the next 12 or 60 months. Merger-clustered standard errors are in parentheses. ***, ** and * indicate statistical significance at 1%, 5% and 10% respectively.
official announcement date. Additionally, the standard errors under OLS are larger than under robust regression. The difference is driven by the fact that robust regression reduces the weight of observations that heavily affect the estimation—in the most extreme case dropping the observation. This makes OLS much more vulnerable to outliers than robust regression.

Figure 5.5 shows the estimated average cumulative abnormal return, with 95% confidence interval, separately for when daily returns are dropped if they are in excess of 50% and 50%, or in excess of 20% and 20%. Results are not materially affected relative to the benchmark of 30% and 30%. Although not shown here, the qualitative conclusions from regressing \( \text{CAR}_{it} \) on the market concentration proxy and future competitor merger are also similar. Although not shown here, results are robust to reasonable variations in the estimation and event window.

**Figure 5.4: Estimated Average \( \text{CAR}_{it} \) using OLS**

![Graph showing estimated average cumulative abnormal returns](image)

**Notes:** Average cumulative abnormal returns for three closest competitors, for event window starting at \( t_0 = 50 \). Estimate and 95% confidence interval are based on ordinary least squares regression with merger-clustered standard errors.
5.6 Concluding Remarks

Figure 5.5: Estimated Average CAR$_{it}$ Around Event Date

Notes: Average cumulative abnormal returns for three closest competitors, for event window starting at $t_0 = 50$. Estimate and 95% confidence interval are based on robust regression with merger-clustered standard errors.

5.6 Concluding Remarks

There is a growing concern that U.S. merger control may have been too lenient. Empirical evidence remains limited however, mostly because of the limited data availability necessary for proper ex post merger reviews. Event studies have been proposed as a simple alternative to acquire at least some empirical insights into the anticipated competitive effects of mergers.

This paper shows how event studies can be used as supplementary or circumstantial empirical evidence on the concern of insufficient merger control—at least in the aggregate. Existing studies do generally suffer from strong identifying assumptions, unreliable competitor identification or small samples. I am able to overcome these challenges by using a novel application of Hoberg-Phillips TNIC data, which allows me to readily proxy the most likely competitors to a large subset of the largest U.S. mergers between 1997 and 2017. I find that the most likely competitors benefit on
average from a major merger announcement.

The association with a TNIC-based HHI further suggests that results are at least in part driven by an anticipation of anti-competitive effects. This provides the circumstantial evidence that U.S. merger control has been too lenient—even though event studies cannot say anything about individual mergers, for which case-specific ex ante or ex post merger reviews remain necessary.
Chapter 6

Conclusions and Reflections

This dissertation is a collection of four separate essays in the field of competition economics. It shows in four different cases (with different methods and at different levels of abstraction) how the absence of sound competition policy may lead to adverse market outcomes. In this chapter, I provide a final recapitulation and reflection on each chapter.

Chapter 2. Autonomous Algorithmic Collusion: Q-Learning Under Sequential Competition

This essays shows that in principle, autonomous pricing algorithms could learn collusive strategies, even in the absence of illegal communication or explicit human instructions to collude. Specifically, I show in a stylized simulated environment of sequential price competition how Q-learning can converge to collusive Markov perfect equilibria. Many practical limitations remain when applying such pricing algorithms to the real world, related for instance to long learning and uncertain outcomes. At the same time, advances in artificial intelligence may be able to deal with these, including simulated offline learning, function approximation, transfer learning or the use of other more state-of-the-art learning algorithms.

I see current research on this topic going into four directions. First, there is a lot of scope for improvement upon basic Q-learning in pricing applications. The adaptation of Q-learning used here is a very basic version, which learns very slowly and has a very untargeted exploration policy. Improvements upon this original algorithm can see whether the practical
limitations can indeed be overcome—and if so, what kind of policy inter-
nentions may be required to prevent autonomous algorithmic collusion in
practice.

Second, a good test on whether autonomous algorithmic collusion is
not only in principle possible, but also a practical concern would be a di-
rect comparison between algorithms and humans in an otherwise identical
experimental environment. In the absence of a good experimental bench-
mark in the field, laboratory studies are an attractive alternative to test the
plausibility of both human and algorithmic collusion. To date, the superi-
ority of truly autonomous algorithmic collusion in laboratory experiments
has not yet been shown. At the same time, it may be difficult to obtain
a direct human benchmark when considering the high-speed, broad-scope
environments in which pricing algorithms are more likely to be used.

Third, current academic work is still mostly a hypothetical exercise,
aimed at identifying potential concerns. Empirical research, market stud-
ies and competition cases can provide a valuable idea on exactly which
kind of algorithms are used in practice. This includes non-price algorithms
such as search or ranking algorithms. Similarly, it is important to remem-
ber that autonomous algorithmic collusion is only one concern in the rise
of pricing algorithms, and to-date mostly a hypothetical one. A more im-
mediate concern may be the use of pre-programmed pricing algorithms to
efficiently implement collusive strategies designed by humans. I provide a
more general discussion on the potential risks of different pricing algorithms
to competition in Klein (2020).

Finally, it may be interesting to use methods from machine learning
to combat concerns around collusion more generally. Empirical screening
techniques such as price-variance screens or other break tests have already
been proposed to flag suspicious behavior (Harrington, 2006). These are
basically supervised machine learning methods applied to a particular mar-
ket using a specified model of the economic environment. But it may also
be possible to train other kinds of supervised machine learning algorithms
to predict possible cases of collusion—akin to how supervised machine
learning algorithms are used to identify anomalous conditions on medi-
cal images, for instance. For example, Huber and Imhof (2018) train a prediction algorithm on a dataset of Swiss construction procurement that correctly classifies 80 percent of out-of-sample observations as collusive or non-collusive. Finally, unsupervised machine learning may be used to classify suspiciously similar behavior, without requiring a properly labelled training dataset of known collusive behavior.

Chapter 3. Collusive Benchmark Rate Fixing Benchmark rate collusion has emerged as a novel competition policy concern that separates itself from the more conventional collusion. In this essay, we show how front running and eligible transactions rigging can be used as complementary mechanisms to stabilize a collusive arrangement around benchmark rate setting, despite varying and often conflicting interests. Simulations also show how recent reforms may unintentionally make collusion more stable, and how empirical screens may be used to identify anomalous behavior.

There are several reforms to financial benchmarks that do make (individual or collusive) benchmark rate manipulation more difficult. This includes most notably the strict internal separation between the traders and those submitting the contributions. However, it remains unclear how effective these internal ‘Chinese walls’ will be, given the major financial incentives involved. If the characteristics of these financial benchmarks make it impossible the truly separate the rate-setting from those with an interest in them, regulators may do well to consider continuous empirical screening mechanisms as a second line of defense. In this essay, we propose two of them: a volatility screen and a correlation screen. Further developing these screens, together with the necessary data collection, seems like a valuable undertaking—also for the application to other benchmark rate settings that have not yet be scrutinized as much.

Chapter 4. Cartel Stability by a Margin This essay revisits classic cartel stability theory by introducing the concept of a cartel margin. We show that comparative statics on the critical discount factor change when firms require a margin before colluding. What drives our result is that the
presence of a cartel margin increases the relevance of gains from collusion relative to the gains from deviation in the comparative statics on the critical discount factor. When the cartel margin is sufficiently large, the effect on the gains from collusion has to dominate. We show specifically that lower marginal cost and less product differentiation generally increases the scope for collusion in the presence of a cartel margin—which may have implications for competition policy.

Although our model is general, we also provide a simple foundational justification based on the probability of cartel break-down and antitrust liabilities. Future research may try to provide alternative foundational justifications for the margin—for instance based on stochastic demand or risk aversion. Additionally, it is possible to extend the general model to include asymmetries between firms as well as alternative punishment strategies—such as temporary Nash reversion or optimal carrot-and-stock combinations.

Chapter 5. Event Studies in Merger Analysis: Review and an Application Using U.S. TNIC Data

There appears to be a growing acknowledgement that merger control has been too lenient, especially related to U.S. merger control. Acquiring empirical insights into the effectiveness of merger control is not straightforward. In particular, ex ante merger analyses are often uncertain and contentious and ex post analyses remain relatively limited because of their complexity or limited data availability. In this essay I discuss how event studies may be used to acquire at least some empirical insights into the anticipated competitive effects of mergers. Using a novel application of Hoberg-Phillips TNIC data, I also show that U.S. merger control may have been too lenient: likely competitors on average benefit from merger announcements and this benefit is positively associated with market power concerns.

I discuss several limitations to my approach. First, the Hoberg-Phillips TNIC data is only a proxy and only includes publicly-traded U.S. firms. Second, my approach does not allow for the identification of individual merger cases as anti- or pro-competitive, as it only looks at average effects.
Event studies could in theory be used as an ex ante merger review method, as it only requires data that is available prior to a possible merger prohibition. However, as also argued by Kwoka (2015, pp. 41-44), using event studies for the classification of individual cases involves a lot of noise. And as also discussed in this essay, there may be other mechanisms that affect stock prices, unrelated to the anticipated competitiveness of a merger.

Irrespective of the limitations to my approach, its conclusions remain in line with the growing concern of an insufficient deterrence of anti-competitive mergers. This should give support for more extensive ex ante and ex post merger reviews—not with the aim of replacing apparent under-enforcement with overzealous over-enforcement, but with the aim of better identifying exactly which mergers may have been or are likely to be anti- or pro-competitive. One way of achieving this could be the obligation of an independent (ex post) merger review in case of major mergers, possibly financed by the merging parties but commissioned by the competition authority.
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Summary

The presence of competition generally pressures profit-seeking firms to decrease price, increase quality and innovate. However, market competition can also produce socially undesirable outcomes, such as harmful externalities, unfair distributions and the abuse of corporate dominance. Containing these market failures may mandate public policy interventions. Competition policy is the type of public policy specifically aimed at preserving well-functioning market competition, by restricting the acquisition or abuse of corporate dominance. Competition economics in turn looks at the economic theory and empirics behind competition policy. This dissertation consists of four separate essays in the field of competition economics.

The first essay contributes to recent discussions on whether self-learning pricing algorithms may learn to coordinate on high prices. It combines microeconomic theory with simulations to show how under sequential competition, Q-learning (a simple and well-established self-learning algorithm) can indeed learn collusive strategies. Although practical limitations remain, this essay discusses how advances in artificial intelligence may be able to deal with these.

The second essay proposes a theoretical model of collusion on financial benchmark rates such as Libor and Euribor. Recent scandals have shown that such benchmarks are vulnerable to manipulation by the banks that set them. Theoretically, however, benchmark collusion is challenging because of varying and often opposing interests. This essay shows how ‘front-running’ (where banks adjust their exposure ahead of the market) and ‘eligible transactions rigging’ (where banks manipulate the transactions that are part of the benchmark) can enable collusion. Episodic break-up as observed in recent cases can be part of an ongoing coordination. Simula-
tions show how recent reforms could even facilitate collusion and provide suggestions for empirical screens.

The third essay revisits classic cartel theory to show how comparative statics on cartel stability change when firms require a margin before colluding. Colluding firms may require such a margin to compensate, for instance, for the cost of colluding, antitrust liabilities or the risk of cartel breakdown. This essay shows that a cartel margin increases the relevance of gains from collusion, relative to the gains from deviation—which may provide new and unambiguous comparative statics. Specifically, it shows how in the presence of a cartel margin lower marginal cost and less product differentiation generally increases the scope for collusion.

The last essay uses event study methodology to see whether U.S. merger control may have been too lenient. After reviewing the limitations of existing event studies in merger analysis, this essay uses a novel application of Hoberg-Phillips TNIC data to show that close competitors experience a positive average abnormal stock market return following a merger announcement. This abnormal return is also associated with market power concerns. This suggests that results are at least in part driven by an anticipation of anti-competitive effects, and hence an insufficient deterrence of anti-competitive mergers.
Samenvatting

Over het algemeen zet de aanwezigheid van concurrentie winstzoekende bedrijven onder druk om de prijs te verlagen, de kwaliteit te verhogen en te innoveren. Marktwerking kan echter ook tot sociaal onwenselijke uitkomsten leiden, zoals schadelijke externe effecten, oneerlijke verdelen gen en een misbruik van een dominante bedrijfspositie. Het imperken van dit soort marktfalen kan overheidsingrijpen vereisen. Mededingingsbeleid is het type overheidsingrijpen dat specifiek gericht is op het behoud van goed functionerende marktwerking, door middel van het imperken van de acquisitie of misbruik van een dominante bedrijfspositie. Mededingings economie kijkt vervolgens naar de economische theorie en empirie achter mededingingsbeleid. Dit proefschrift bestaat uit vier afzonderlijke essays op het gebied van mededingingseconomie.

Het eerste essay draagt bij aan recente discussies over de vraag of zelflerende prijsalgoritmen kunnen leren om te coördineren op hoge prijzen. Het combineert micro-economische theorie met simulaties om te laten zien hoe Q-leren (een eenvoudig en gevestigd zelflerend algoritme) in opeenvolgende prijscompetitie inderdaad kartelstrategieën kan leren. Hoewel er nog praktische beperkingen zijn, wordt in dit essay besproken hoe ontwikkelingen in kunstmatige intelligentie hier wellicht mee om kunnen gaan.

Het tweede essay stelt een theoretisch kartelmodel voor met betrekking tot financiële benchmarkrentes zoals Libor en Euribor. Recente schandalen hebben aangetoond dat dergelijke benchmarks kwetsbaar zijn voor manipulatie door de banken die ze vaststellen. Theoretisch is een benchmarkkartel echter uitdagend vanwege de uiteenlopende en vaak tegengestelde belangen. Dit essay laat zien hoe ‘front-running’ (waar banken hun blootstelling voortijdig aanpassen) en ‘eligible transactions rigging’ (waarbij
Samenvatting

banken de transacties manipuleren die deel uitmaken van de benchmark) samenzwering mogelijk maken. Het episodisch uiteenvallen van het kartel, zoals waargenomen in de recente schandalen, kan onderdeel zijn van een voortdurende samenzwering. Simulaties laten zien hoe recente hervormingen samenzwering juist kunnen vergemakkelijken en bieden suggesties voor empirische controles.

Het derde essay herziet klassieke karteltheorie om te laten zien hoe theoretische resultaten rond kartelstabiliteit veranderen wanneer bedrijven een marge nodig hebben voordat ze gaan samenzweren. Samenzwerende bedrijven kunnen een dergelijke marge verlangen ter compensatie voor bijvoorbeeld de kosten van samenzwering, toezichtrisico’s of de kans dat het kartel opbreekt. Dit essay laat zien dat een kartelmarge de relevantie van winsten bij samenzwering vergroten in vergelijking met winsten bij afwijking van samenzwering—wat nieuwe en ondubbelzinnige comparatieve statica kan opleveren. Specifiek laat het zien hoe in de aanwezigheid van een kartelmarge lagere marginale kosten en minder productdifferentiatie in het algemeen de mogelijkheden voor samenzwering vergroten.

Het laatste essay gebruikt event studies methodologie om te zien of Amerikaanse fusiecontrole wellicht te soepel is geweest. Na het onderzoeken van de beperkingen van bestaande event studies in fusieanalyse, laat dit essay met behulp van een nieuwe toepassing van Hoberg-Phillips TNIC-data zien dat naaste concurrenten een positief gemiddeld abnormaal rendement op de aandelenmarkt ervaren na een fusieaankondiging. Dit abnormaal rendement hangt tevens samen met zorgen over marktmacht. Dit suggerereert dat de resultaten ten minste gedeeltelijk worden aangedreven door een verwachting van concurrentiebeperkende effecten en dus een onvoldoende controle op concurrentiebeperkende fusies.
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