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Indicative Conditionals

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1 Introduction

This article is about a particular class of conditional sentences: indicative conditionals. For English these are sentences like (1-a) and (1-b) below.

(1) a. If you drop this glass, it will break.
b. If Peter got your message, then he will be here this afternoon.

Defining this class of conditionals is notoriously difficult. Ideally, we would like to offer a semantic definition of indicatives. But understanding what exactly these sentences mean is the central problem that we will address in this article. This leaves us with the option to offer a definition in terms of their form. But then we have to deal with substantial cross linguistic variation. For instance, indicative conditionals are named after the indicative mood. However, English, which is the language mostly used when it comes to discussing the semantics of indicative conditionals, doesn't have a systematic mood distinction. For the purpose of this paper we will focus on English indicative conditionals. Given this restriction we can define indicative conditionals as those that do not contain a past modal as finite verb-form in the main clause. This is meant to distinguish these conditionals from so called subjunctive conditionals, like (2-a) and (2-b). Subjunctive conditionals are the subject of a different chapter in this handbook.

(2) a. If you were in Paris next week, we could meet.
b. If you had prepared for the exam, you would have passed.

We will furthermore narrow down our target by focusing on what Haegeman [2003] calls event-conditionals (see (1-a) and (1-b)) and exclude relevance conditionals (see (3-a)). It is generally accepted that relevance conditionals, though in form very similar, differ in their semantics, and Haegeman [2003] argues convincingly that relevance conditionals also have different syntactic properties. For the same reasons we also exclude biscuit conditionals like (3-b). A reader interested in relevance or biscuit conditionals is referred to the relevant chapters in this handbook.

(3) a. If we are so short of teachers, why don't we send our children to Germany to be educated? (Haegeman [2003])
b. There are biscuits on the sideboard if you want them. (Austin [1956])

The goal of this paper is to summarize the state of art with respect to the meaning of indicative conditionals, as defined above. This is not easily done. First of all, there is an immense literature on the semantics of indicative conditionals. Additionally, recently some other (excellent) handbook articles on the topic have been published ([Gillies, 2012, Fintel, 2011b]). In view of this, we needed to make some choices. We decided not to write a general overview article, but rather focus on some central problems that need to be solved by any approach to the meaning of indicative conditionals. In the discussion of these problems we will try to put emphasis on developments that either are relatively young or have, in our eyes, not gotten the attention they deserve by the linguistic community.

In our discussion of the meaning of indicative conditionals we will focus on two challenges that we consider central in the discussion. The first is the challenge to account for the non-monotonic inference pattern that conditionals in general display. The second is to account for the interaction with probability claims (and adverbs of quantification in general) and the related triviality results.

2 Challenge 1: The non-monotonicity of conditionals

2.1 From material implication to dynamic strict conditionals

A first idea for how to formalize indicative conditionals could be to just use material implication. There is a lot that speaks for this. Material implication validates many of the inferences that seem intuitively valid for indicative conditionals. In fact, in some formally very precise sense material implication appears to be the only viable option to formalize indicative conditionals. It can be shown that if we assume that indicatives are at least as strong as the material implication and add some very basic validities for indicatives, then material implication is the only binary connective that meets these constraints.[1]

Compelling as this might be, there are also serious and well-known **empirical problems** of the material implication account of indicative conditional sentences. We won't go into the extensive literature on this topic, but only focus on one problem: indicative conditionals appear to be intensional expressions. We want to say that the conditional *If you push the red button, the alarm will go off* is true or acceptable not simply because nobody pressed or will press the button, but because in the possibility that somebody did press the button, the alarm would be triggered. Indicative conditionals talk about alternative possitilities. One can also make this point with the following examples involving embedded indicative conditionals (see Fintel and Iatridou [2002], Higginbotham

[1] The result can be found in difference forms in Stalnaker [1975], Gibbard [1981], Veltman [1985] and McGee [1989].

[2003], Huitink [2010]).

(4) a. Every student will succeed if she works hard.
b. Every student who works hard will succeed.

Suppose, there is a student, Steve, who is just not smart enough to pass no matter how hard he works. On top of that Steve is also extremely lazy and not working at all. In such a situation sentence (4-b) can still be true, but Steve seems to be a clear counterexample to (4-a). Even though he doesn't work hard in the actual world, the fact that he would still fail in a counterfactual world where he was working hard, makes the sentence intuitively false. This intensionality of indicative conditionals makes an analysis as material implication, or any other truth-functional analysis, a non-starter.

The intensionality of indicative conditionals is one of the reasons that motivated an analysis as *strict implication* (cf. Lewis [1918]), $\Box(A \rightarrow B)$. The material implication $A \rightarrow B$ should be true in all worlds. Unfortunately, the resulting analysis seems to be too strong. According to the strict conditional analysis the following four inferences are all valid, if we represent an indicative conditional by $A \Rightarrow B$.

1. *hypothetical syllogism* ($B \Rightarrow C; A \Rightarrow B \models A \Rightarrow C$),
2. *contraposition* ($A \Rightarrow B \models \neg B \Rightarrow \neg A$),
3. *strengthening of the antecedent* ($A \Rightarrow C \models (A \wedge B) \Rightarrow C$),
4. the inference *from-Or-to-If* ($A \vee B \models \neg A \Rightarrow B$).

It is widely assumed that these inferences are not valid for *subjunctive* conditionals, and Stalnaker [1968] and Lewis [1973] developed their similarity-based analysis of counterfactuals in order *not* to validate these inferences. It is noteworthy to observe, however, that also for *indicative* conditionals these predictions seem wrong, because (5-a)-(5-c), (6-a)-(6-b), (7-a)-(7-b) and (8-a)-(8-b) seem clear counterexamples to them (cf. Adams [1975], Cooper [1985]):

(5) a. If Jones wins the election, Smith will retire to private life.
b. If Smith dies before the election, Jones will win it. therefore
c. If Smith dies before the election, he will retire to private life.

(6) a. If it is after 3 o'clock, it is not much after 3 o'clock. therefore
b. If it is much after 3 o'clock, it is not after 3 o'clock.

(7) a. If there is sugar in the coffee, then it will taste good. therefore
b. If there is sugar in the coffee and diesel-oil as well, then it will taste good.

(8) a. Hitler started the second world war or some Martians did. therefore
b. If Hitler didn't start the second world war, some Martians did.

Notice that the invalidity of *strengthening of the antecedent* shows that conditionals behave non-monotonic: by adding information to the antecedent a conditional can become intuitively false of unacceptable.

Explaining the invalidity of these inferences is what we consider to be the first central challenge in accounting for the meaning of indicative conditionals. Quite a number of different strategies have been employed to deal with this challenge.

Pragmatics to explain away 'counterexamples' If one assumes that semantics is all there is to meaning, then the invalidity of these inferences falsify the proposed analysis of indicative conditionals and it should thus be rejected. But maybe we can explain away these 'counterexamples' pragmatically, appealing to the assumption that the speaker is rational and cooperative? This kind of Gricean reasoning [Grice, 1989] has indeed been used to deal with the problems discussed above, notably by Warmbrod [1981] and Veltman [1986]. However, as stated above, hypothetical syllogism, contraposition, strengthening of the antecedent and *from-If-to-Or* are problematic for other classes of conditionals as well. In the literature on subjunctive conditionals or counterfactuals, these inferences are often given as the main motivation to give up the strict conditional approach and try something else. One might wonder how plausible it is to account for the same observation with a pragmatic explanation in case of indicatives and a semantic explanation in case of subjunctives. Such a strategy implicitly embraces the assumption that the pattern we observe here is accidental. But this is not very plausible, given that we talk about conditional sentences in both cases.

Another pragmatic alternative: dynamic strict conditionals In the pragmatic solution sketched in the previous paragraph, we appealed to principles of cooperative conversation to account for the invalidity of the inferences. But there is another pragmatic explanation that allows us to keep the strict conditional approach to indicatives. Frank [1997] and Fintel [1999, 2011a] proposes that we should make the strict conditional analysis context dependent by letting the context set the domain the necessity modal quantifies over. We can combine this with a notion of entailment that requires the domain of quantification to be fixed for all premises and the conclusion (Von Fintel dubs this 'Strawson Downward Entailment'). Such a notion of inference can account for the counterexamples discussed above. Take our counterexample for the inference schema of hypothetical syllogism, for instance. Both premises appear to talk about different domains of quantification. While in case of (5-a) the domain doesn't seem to include worlds in which Smith dies before the election, the domain of (5-b) certainly does. So, we have a context shift between both premises and, consequently, the conclusion doesn't follow. A strong point of this approach is that it can also explain why in normal cases all four inference schema's appear to be valid. The reason is that normally, you can assume the domain of quantification to be constant for subsequent sentences.

As observed by Frank [1997], Fintel [1999, 2011a], the dynamic strict conditional approach can account naturally for the observation that so called 'Sobel sequences' (the conjunction '$A \Rightarrow B$ but $(A \wedge C) \Rightarrow \neg B$' can be true) cannot be reversed (the conjunction '$(A \wedge C) \Rightarrow \neg B$ but $A \Rightarrow B$' cannot be true). This can be accounted for by postulating a restriction claiming that the domain of quantification can only grow in discourse, but not shrink. Another argument, which von Fintel brings forward for this analysis, is that it can explain why Negative Polarity Items (NPIs) are licensed in the antecedent of indicative conditionals. Using the notion of Strawson entailmen, the antecedent of a dynamic strict conditional is a downward entailing context. According to the standard few this is exactly the kind of context in which NPIs can be used.

2.2 Non-monotonic semantics: The similarity approach

Instead of explaining away the non-monotonic behavior of conditionals by pragmatics, we can also account for it semantically. In fact, this is exactly what Stalnaker [1968, 1975] originally proposed. According to him, we should use the famous similarity approach, which Lewis only applied to subjunctive conditionals, to capture the meaning of indicative conditionals. So, how does the similarity approach work? The central idea here is to describe the meaning of indicative conditionals in terms of an order relation over possible worlds. We will distinguish here two variants of the approach.

Acceptability in terms of a plausibility order. We start with a model $M = \langle W, \preceq, V \rangle$ where W is a set of possible worlds (considered possible by the speaker), V a valuation function, and $\preceq$ is a preorder (a binary relation that is reflexive and transitive). In terms of this preorder we can define a selection function f as a function that maps a proposition p to the stronger proposition $f(p)$ only containing worlds minimal with respect to the order.[2] In terms of these models, we can say that $A \Rightarrow B$ is *accepted* in model M iff $f([\![A]\!]) \subseteq [\![B]\!]$ (where $[\![A]\!] =_{def} \{w \in W : V_w(A) = 1\}$).

Although the above analysis bears a close resemblance to Stalnaker [1968]'s and Lewis [1973]'s analyses of (subjunctive) conditionals, it is important to realize that there is a crucial difference: In contrast to what Stalnaker and Lewis assume, this theory doesn't assign truth conditions to conditional given a world, the conditional is only *accepted* or not given a model, M. Thus, the analysis under discussion assumes that indicative conditionals do *not express propositions*. We are also not dealing with a similarity order that compares worlds with respect to how similar they are to the evaluation world (because there is no evaluation world). Instead $\preceq$ can best be understood as a plausibility order selecting the most acceptable worlds among those making a sentence true. Intuitively, it now holds that $A \Rightarrow B$ is assertible iff $A \wedge \neg B$ is implausible given A.

[2] We assume here, as Stalnaker does, that such minimal exists.

Be that as it may, if we define logical consequence in the standard way, but now as maintenance of acceptability, this approach can make predictions about inferences involving conditionals. The resulting logic is known as system **P** and can be characterized by the following rules (Burgess [1981]):

If $\models A \leftrightarrow B$, then $A \Rightarrow C \models B \Rightarrow C$	Left Logical Equivalence
If $\models A \rightarrow B$, then $C \Rightarrow A \models C \Rightarrow B$	Right Weakening
$\models A \Rightarrow A$	Reflexivity
$A \Rightarrow C, B \Rightarrow C \models (A \vee B) \Rightarrow C$	Or
$(A \wedge B) \Rightarrow C, A \Rightarrow B \models A \Rightarrow C$	Cut
$A \Rightarrow B, A \Rightarrow C \models (A \wedge B) \Rightarrow C$	Cautious Monotonicity
$A \Rightarrow B, A \Rightarrow C \models A \Rightarrow (B \wedge C)$	And

Importantly, some inferences that are wrongly predicted to be valid on a material or strict implication analysis of the conditional, are now no longer predicted to be valid. Among others, the four inferences we observed to be invalid above: *hypothetical syllogism*, *contraposition*, *strengthening of the antecedent*, and the inference *from Or-to-If*, just as desired. Because the monotonic inference *strengthening of the antecedent* is *not* valid for the conditional $\Rightarrow$, system **P** is standardly taken to be appropriate for 'non-monotonic' logics.

Truth in terms of a similarity order. Especially for linguists, a theory that denies that indicative conditionals have truth conditions doesn't look very attractive. Why should this particular class of sentences behave so differently? Furthermore, such a stance makes it impossible to account for the interaction of indicative conditionals with other construction. Just to pick one example: embeddings of indicative conditionals don't make sense. Following the lead of Stalnaker (1968) and Lewis (1973), we can overcome this hurdle. One can stick to the analysis given above, but now make the ordering relation world-dependent. For the rest we keep everything as before: the (world dependent) selection function $f_w([\![A]\!])$ picks out the worlds minimal with respect to the world dependent order $\preceq_w$ that make A true, and we say that $A \Rightarrow B$ is true in world w iff $f_w([\![A]\!]) \subseteq [\![B]\!]$. The ordering relation is still going to be reflexive and transitive. But that is not quite enough now: Modus Ponens is not guaranteed to hold. To validate Modus Ponens, we have to impose another constraint on the ordering relation (or the selection-function), i.e., the *centering* assumption, assuring that if $w \in [\![A]\!]$, then $w \in f_w([\![A]\!])$.[3] This is the similarity approach as we know it from subjunctive conditionals.

Default inferences and Context Dependence. Where pragmatics was used by proponents of the material and strict implication accounts to get rid of undesired predicted validities, proponents of the similarity account seek to 'validate' some inferences that are predicted not to be valid semantically. For

[3]We can validate the inference from $A \wedge B$ to $A \Rightarrow B$ by assuring that $f_w([\![A]\!]) = \{w\}$, if $w \in [\![A]\!]$.

instance, we now need to explain why in most cases *hypothetical syllogism*, *contraposition*, *strengthening of antecedent* and the inference *from Or-to-If* are valid inference patterns for indicative conditionals. Different strategies are possible here.[4] We want to point out one strategy in particular, which in involves context dependence, because it relates very nicely to the dynamic strict approach discussed before.

The idea here is to account for the default inferences by making the selection function, and thus the meaning of the connective '$\Rightarrow$' **context dependent**. This is basically what Stalnaker [1975] proposed. In particular, he assumed that every sentence is interpreted relative to a context set, thought of as a set of possible worlds, which represents the believed or presupposed background of the speaker. Then he famously came up with the following constraint for the appropriate use of indicative conditionals:

> *If an indicative conditional is being evaluated at a world in the context set, then the world selected*[5] *must, if possible, be within the context set as well.*

Thus, the proposition expressed by $A \Rightarrow B$ might depend on the believed or presupposed background of the speaker. If the latter is represented by K, the most straightforward way to satisfy the constraint is to make the selection function f dependent on K, by defining f^K constrained as follows: $f^K_w(A) = f_w(A \cap K)$, if $A \cap K \neq \emptyset$. Stalnaker shows that by making use of this constraint, he can account for the *from-Or-to-If* inference. Stalnaker [1975] claims that it can be shown that on the same assumptions, contraposition and the hypothetical syllogism can be explained in similar ways as well.[6,7]

Stalnaker [1975] suggested that the context-dependence of the proposition expressed by an indicative conditional is rather objective: it depends only on what is presupposed by the members of the conversational situation. That would imply that if it is clear to each member what is presupposed, it is uncontroversial between the members of the conversation what is expressed by the conditional. This seems important for communication using conditional sentences. Unfortunately, that are reasons to believe that things cannot be that simple. Notice that on the present analysis the *principle of conditional non-*

[4] See, for instance, Burgess [2009] who uses the **conventional implicature** that the antecedents of the conditionals in the whole argument have to be *seriously possible*.

[5] According to Stalnaker the selection function always selects one singular world as the most similar to the evaluation world.

[6] See Morreau [1992] for a similar approach. Proponents of premise-semantics like Harper [1976], Veltman [1976, 1985] and Kratzer [2012] make the meaning of conditionals context dependent as well. Distinctive about premise-semantics is that they define the ordering —and thus the meaning of $\Rightarrow$ — from a set of independently given propositions (what Kratzer called an 'ordering source'). The orderings can be derived in slightly different ways resulting in weak orders Harper [1976] or partial ones Veltman [1976, 1985], Kratzer [2012]. But the details won't matter much for us here.

[7] Harper [1976], for instance, shows that on such an analysis conditionals of the form $A \Rightarrow (B \Rightarrow C)$ turn out to be equivalent to ones of the form $(A \wedge B) \Rightarrow C$.

contradiction ($A \Rightarrow B$ is inconsistent with $A \Rightarrow \neg B$) is predicted to be valid.[8] Gibbard [1981]'s famous Riverboat example strongly suggests that we would accept both 'If A, then B' and 'If A, then $\neg B$, asserted by two different speakers, which violates the principle of conditional non-contradiction. Gibbard [1981] notes that the problem can be solved if we can make the propositions expressed depend on the individual beliefs of the speakers, but he rejects that view as stretching the notion of truth-conditions too much.[9] Whether we should agree with Gibbard or not, we must leave to the reader.

Notice that if one assumes that the selection function depends on what is presupposed, it immediately follows that this selection function changes during discourse because more information becomes presupposed. A discourse-dependent selection function can also be used to explain the asymmetry of Sobel sequences (cf. van Rooij [1997]) and the fact that negative polarity items are licenced in antecedents of conditionals. The idea would be that the conditional 'If A, then B' changes the selection function f to f^A, and that (something like Strawson-) entailment (e.g. from $A \Rightarrow B$ to $(A \wedge C) \Rightarrow B$) is defined with respect to a selection function that doesn't change anymore after the interpretation of any of the relevant sentences. The latter constraint means that the selected A-worlds of the first conditional will all be $A \wedge C$-worlds such that the downward monotonicity inference is predicted to be valid, just like in the dynamic strict conditional account.

3 Challenge 2: Conditional Probabilities and Quantifiers

3.1 Stalnaker's constraint and Triviality

The second challenge we want to discuss concerns the relation of indicative conditionals to statements about probabilities. Consider, for instance, the following example due to Grice [1989]. Yog and Zog play chess, Yog has black 9 out of 10 times, and draws are not allowed. We don't know who won what game, but we do know that of the hundred games they played up to now, Yog won 80 times when he had black and lost all 10 times that he had white. Under these circumstances, the following sentences are, intuitively, both true:

(9) a. If Yog had black, there is a probability of 8/9 that he won.
b. If Yog didn't win, there is a probability of 1/2 that he didn't have black.

Grice's examples (9-a) and (9-b) suggest that the conditional should not validate

[8] In this, it crucially differs from an analysis in terms of material implication: from $A \rightarrow B$ and $A \rightarrow \neg B$, you cannot derive a contradiction but instead conclude $\neg A$.

[9] This is certainly in line with what Lewis [1981], p. 138 believed: 'presumably our indicative conditional has a fixed interpretation, the same for speakers with different beliefs, and for one speaker before and after a change in his beliefs. Else how are disagreements about a conditional possible, or changes of mind possible?'

contraposition. The strict conditional approach, which we discussed earlier does. So, the example seems to provide evidence against this line of approach. However, also for the similarity approach it is not obvious how it can account for this example. Maybe we first need to approach the problem from a more general point of view: what is the relation between the meaning of an indicative conditional and it's probability? To what degree would you, for instance, believe the following sentences, given that a card has been picked at random from a standard 52 card deck:

(10) a. The selected card is a king, *if* it's red.
b. It's diamonds, *if* it's black.
c. It's spades, *if* it is a nine.

The obvious answers are $\frac{1}{13}$, $\frac{1}{2}$, and $\frac{1}{4}$, respectively. These degrees are predicted if they each are calculated as the **conditional probability** of the consequent given the antecedent of the above conditional sentences. This strongly suggests that the belief in a conditional sentence 'If A, then B' should equal one's conditional probability of the consequent, given the antecedent of the conditional, $P(B|A) = \frac{P(A \wedge B)}{P(A)}$. This idea has been corroborated by empirical research mostly due to psychologists (see for instance, Over and Evans [2003], Over et al. [2007], Oaksford et al. [2000], Oaksford and Chater [2007]). Moreover, it seems that what (9-a) and (9-b) express can best be interpreted in terms of conditional probability, for the probability of B given A, $P(B|A)$ can be very different from the probabilty of $\neg A$ given $\neg B$, $P(\neg A|\neg B)$. Indeed, if 'Y' and 'B' stand for 'Yog Won' and 'Yog has black', respectively, $P(Y|B) = \frac{8}{9}$, while $P(\neg B|\neg Y) = \frac{1}{2}$ in Grice's example.

The idea that we should interpret conditionals in terms of conditional probabilities had for related reasons already been proposed in the nineteen-sixties by Jeffrey [1964] and Adams [1965]. The hypothesis that there should be a binary connective '$\Rightarrow$' such that $A \Rightarrow B$ expresses a context-independent proposition for which the equation $P(A \Rightarrow B) = P(B|A)$ holds was explicitly made by Stalnaker [1970]. We will refer to it as **Stalnaker's constraint**. To account for this basic intuition about the probability of indicative conditionals we consider the second major challenge of approaching this class of conditionals.

Stalnaker's constraint cannot be accounted for without further ado. Take, for instance, a material implication approach to indicative conditionals. We cannot say that the degree of belief in a conditional is simply the probability of the material implication being true, $P(A \rightarrow B)$, because many times $P(A \rightarrow B) \neq P(B|A)$. If $P(A) = \frac{3}{4}$ and $P(B) = 0$, for instance, $P(A \rightarrow B) = \frac{1}{4} \neq 0 = P(B|A)$.[10] An analysis of indicatives as strict implication runs into similar problems.

Even worse, Lewis's (1975a) famous **Triviality Result** showed that Stalnaker's constraint in combination with some other natural assumptions about

[10] In general, $P(B|A) \leq P(A \rightarrow B)$.

the meaning of indicative conditionals results in trivial interpretations.[11] Lewis shows that if one also assumes that pairs of sentences of the form '$A \Rightarrow (B \Rightarrow C)$' and '$(A \wedge B) \Rightarrow C$' are equivalent (the import-export assumption) and that the meaning of the conditional connective '$\Rightarrow$' is context-independent, it follows that for all A and B, $P(B|A) = P(A \Rightarrow B) = P(B)$, meaning that all propositions are *independent* of each other, which trivializes the probability function.

Bradley [2000] shows that the triviality result is not dependent on the import-export assumption. He notices that if we assume that an indicative conditional $A \Rightarrow B$ expresses a context-independent proposition, standard probability theory demands that $P(A \Rightarrow B) = P(A \Rightarrow B|B) \times P(B) + P(A \Rightarrow B|\neg B) \times P(\neg B)$. He then observes that if we make the following, seemingly quite innocent, assumptions, $P(A \Rightarrow B|B) = 1$ and $P(A \Rightarrow B|\neg B) = 0$,[12] it immediately follows that $P(A \Rightarrow B) = 1 \times P(B) + 0 \times P(\neg B) = P(B)$, something we definitely don't want. Russell and Hawthorne [2016] simplify this triviality result. They show that the result depends on the assumption that incompatibility is a symmetric relation.

3.2 Reactions to Triviality

From the very beginning, several reactions to the triviality results have been proposed. We will give here an overview over the main solutions proposed in the literature before turning in the next subsection to one of the solutions in particular, which basically consists of accepting the result and giving up on the assumptions that conditionals express propositions.

Lewis: back to material implication. Because Stalnaker's constraint does not hold for material implication Lewis [1975b] proposed that indicative conditionals express the material implication after all. He, and Jackson [1991], proposed that together with some Gricean pragmatics, this could still explain the intuition behind Stalnaker's constraint. Although such a defense of the material implication-account might seem appealing, we believe that they ultimately fail. The reason is that in Grice's example (9-a) and (9-b) we are explicitly dealing with conditional probabilities in the *truth conditions*. It seems very unlikely—certainly because contraposition is valid for material implication—that we should account for these *semantic* facts by relying almost exclusively on *pragmatic* reasoning.

Belnap: multiple truth values. As Lewis [1975b] noted himself already, the triviality result could be escaped if we make use of more than two truth-values. If we treat conditionals as conditional assertions as proposed by Belnap [1984], using a three-valued logic, one could still say that the conditional probability of

[11]Since Lewis made his triviality proof public in 1972, a host of new triviality results followed, and still follow. We will ignore most of them.

[12]Rumfitt [2016] has argued against this assumption: $P(A \Rightarrow \bot|\neg\bot)$ need not be 0, but should be equated with $P(\neg A)$. He claims that conditionals of the form '$A \Rightarrow \bot$' are counterexamples to Stalnaker's constraint.

consequent given antecedent is basically the probability of (something like) the context-independent propositions expressed by the conditional. A somewhat similar proposal has been made later by Jeffrey and Stalnaker [1994], making use of (many) expected values instead of (just two) truth values. Also they are able to preserve Stalnaker's constraint that $P(A \Rightarrow B) = P(B|A)$. We will come back to these analyses later.

Imaging. Another reaction was that the probability of a conditional should not be equated with the corresponding conditional probability, but rather with what Lewis (1976) calls 'imaging': $P(A \Rightarrow B) = P_A(B)$, where $P_A(B) = \sum_{w \in B} P_A(w)$ and $P_A(w) = P(v)$ iff $f_v(A) = w$, the closest world to v where A is true. In modern terms due to Pearl [2000], $P(A \Rightarrow B)$ is not $P(B|A)$, but rather (something like) $P(B|do(A))$. Whereas $P(B|A)$ has a purely evidential reading, $P(B|do(A))$ has a causal one. On this reading, Bradley's preservation conditions can be violated: it can be that $P(A \Rightarrow B|B) \neq 1$ and $P(A \Rightarrow B|\neg B) \neq 0$. Of course, it is standard to assume that the similarity account works only for subjunctive conditionals, but Stalnaker always claimed that it works for indicative conditionals as well. Moreover, Gibbard and Harper [1981] and Kaufmann [2004] have argued that the future oriented decision-relevant conditionals should be accounted for in terms of something like the *do*-calculus. An appealing way to illustrate the difference between $P(B|A)$ and $P(B|do(A))$ is by making use of partitions (Skyrms [1980], Pearl [2000], Kaufmann [2005], Yablo [2016]). According to standard probability theory, $P(B|A) = \sum_i [P(B|X_i \wedge A) \times P(X_i|A)]$, with $\{X_i\}$ a partition of the state space. Instead of proposing that $P(A \Rightarrow B) = P(B|A)$, it is natural to propose that $P(A \Rightarrow B) = P(B|do(A)) = \sum_i [P(B|X_i \wedge A) \times P(X_i)]$, where the X_i are the causally relevant alternative hypotheses.[13] Notice that although in general $P(B|A) \neq P(B|do(A))$, they come to the same if the antecedent A is probabilistically independent of the issue of which hypothesis in fact obtains, i.e., if for all X_i, $P(X_i|A)$ is the same as $P(X_i)$. Although we find such an analysis appealing, we wonder how natural it is to assume that indicative conditionals in general violate Bradley's preservation conditions.[14]

Appealing to context-dependence. A very popular reaction was to give up on the assumption that indicative conditionals express *context-independent* propositions. We have seen in Section 2 that there is independent motivation for this move. Harper [1976] and Fraassen [1976] even show that once we make the selection function, and thus the meaning of $\Rightarrow$, depend on the probability function P (perhaps of the speaker), Stalnaker's constraint (or a special case of this, for Harper) can be preserved. These analyses have problems, however, to account for the fact that we appear to use conditionals to make claims about

[13]This is the way Pearl [2000] estimates $P(B|do(A))$ when no explicit intervention, or experiment, is possible. X is thought of as the confounding variable that should be controlled.

[14]Khoo [2016] recently suggested that $\{X_i\}$ could be *any* partition, or question under discussion. But that seems wrong (cf. Korzukhin [2016] and Douven [2008]).

the world, and not just about our beliefs. McGee [1989]) and Bradley [2012] partly solve this problem by making *meanings more fine-grained*. We will come back to both strategies below.

3.3 Zooming in on non-propositional accounts

In this subsection we will discuss two approaches that give up the assumption that conditionals express propositions. According to the first, the *probabilistic* analysis, this is something special about conditionals. According to the second, the *dynamic* analysis, this holds for sentences in general.

3.3.1 The probabilistic analysis

We have discussed already one approach back in Section 2.2 on the similarity approach that gives up the assumption that indicative conditionals express propositions: the analysis using plausibility orders. A much more popular analysis along these lines, however, is one that claims that conditionals are assertable because the speaker's conditional probability, his or her conditional belief, of B given A, $P(B|A)$, is high (see Adams [1966], Edgington [1995], Gibbard [1981]) and others). On this proposal a natural language indicative conditional of the form $A \Rightarrow B$ has no truth-value, and the thesis that their degree of belief is $P(B|A)$ holds only for conditionals whose antecedent A and consequent B do not contain other conditionals.

Although the analysis seems very natural, and also easily accounts for Grice's examples (9-a)-(9-b), the analysis immediately raises a serious problem. If some sentences don't express propositions, how can we *reason* with them? So, the challenge will be to **generalize standard logic** with probabilistic reasoning.

Note, first, that one can easily prove the following general theorem connecting logical entailment with probability preservation:

- $A_1, \cdots, A_n \models B$ iff for all P and all A_i, if $P(A_i) = 1$, then also $P(B) = 1$.

In these cases one only looks at arguments with premises that are absolutely certain. But what happens to probability preservation if we allow for uncertainty? Suppes [1966] shows that if each of the premises $A_1, \cdots, A_n$ has probability of at least $1 - \epsilon$ and these premises logically imply B, then $P(B) \geq 1 - n\epsilon$.[15]

Given the above, we could *define* a new notion of probabilistic entailment, ***p*-entailment**:[16]

[15]In general the lower bound of $1 - n\epsilon$ cannot be improved on, i.e., equality holds in some cases whenever $1 - n\epsilon \geq 0$. Notice also that the upper bound on the uncertainty of the conclusion depends on the number of premises. If a valid argument has a small number of premises, each of which has a high certainty, then its conclusion will also have a reasonably high certainty. Conversely, if a valid argument has premises with high certainties, then its conclusion can only be highly uncertain if the argument has a large number of premises. A famous illustration of this converse principle is Kyburg [1961]'s lottery paradox.

[16]Almost equivalently, one can state the same definition in terms of **uncertainty**, mostly

- $A_1, \cdots, A_n \models^p B$ iff for all P and ϵ, if for all A_i, $P(A_i) \geq 1 - \epsilon$, then $P(B) \geq 1 - n\epsilon$.

By taking $\epsilon = 0$, one can see that the above mentioned **general theorem** follows from this notion of entailment. But this means that classical entailment is a special case of p-entailment. In fact, p-entailment is an *extension* of classical entailment in the sense that (i) any classical entailment is a p-entailment as well (including all entailments involving material implication), but (ii) some arguments are now also p-valid that were not valid classically. These extra validities are entailments involving conditionals, interpreted as conditional probabilities. As it turns out (Adams [1975]), the p-inferences involving conditionals are characterized exactly by system **P** as discussed in Section 2.2, meaning that it gives rise to a standard non-monotonic logic.

Embeddings of conditionals. The probabilistic analysis is not without problems. The most obvious problem of the probabilistic account is due to the fact that it predicts that conditionals do not express propositions. As discussed in Section 2.2, this has as immediate consequence that embeddings of conditionals within (truth-conditional) connectives cannot be made sense of. This prediction is, on the one hand, welcome, because many embeddings of conditionals are troublesome to make sense of: who knows, for instance, Gibbard [1981] wonders what is meant with (11-a).

(11) a. If Kripke was there if Strawson was, then Anscombe was there.
b. If the cup broke it it was dropped, it was fragile. (Fraassen [1976])

On the other hand, many embeddings seems to be fine, and this requires an explanation. Some cases seem to allow for a straight forward solution.[17] But embedded conditionals remain problematic and some instances seem to be interpretable, such as (11-b). The natural proposal to account for such examples is to – well, explain them away. Gibbard [1981], for instance, acknowledges that (11-b) is appropriate, but claims that the embedded conditional expresses a dispositional property, and that it, thereby, doesn't contain an embedded (indicative) conditional after all.[18] But other embeddings are much more natural. This is in particular the case for sentences of the form 'If A, then if B, then C'. For such examples, proponents of the non-propositional account typically propose to translate them into other sentences that do make sense under the probabilistic treatment, for instance, 'If A and B, then C'.

Default validities. When we discussed the similarity approach in Section 2.2 we pointed out that another challenge of this line of approach is to explain why normally the inference patterns *hypothetical syllogism*, *contraposition*, *strengthening of antecedent* and the inference *from Or-to-If* 'seem' to be valid. The

used by Adams [1965]. We say that the uncertainty of A w.r.t. P, $U_P(A)$, as $1 - P(A)$. Now we can define $A_1, \cdots, A_n \models^p B$ iff for all $P : U_P(B) \leq \sum_{i \leq n} U_P(A_i)$.

[17] See Edgington [1995] for negation.

[18] Edgington [1995] claims that something similar holds for disjunctions of conditionals.

probabilistic approach faces the same challenge. As in the case of the similarity approach the answer here is an appeal to pragmatics (cf. Adams [1983], Pearl [1988, 1990]). One might think of Adams [1983]'s proposal, for instance, as making use of a new *pragmatic* probabilistic consequence relation, call it $\models^{ppr}$, defined roughly as follows:

- $A_1, \cdots, A_n \models^{ppr} B$ iff for all ϵ and all **relevant** P:
  if for all A_i, $P(A_i) \geq 1-\epsilon$, then $P(B) \geq 1-n\epsilon$.

The *relevant* probability functions are those where the premises (and perhaps the conclusion) could be asserted appropriately. Thus, when checking the pragmatic validity of an argument, not *all* the probability functions should be considered, but only those according to which what was asserted could be done so appropriately.

3.3.2 A Dynamic account

As shown by Russell and Hawthorne [2016], the triviality proof is partly based on the assumption that the notion of (in)compatibility is symmetric. This is natural if we model meanings in terms of static truth conditions. According to dynamic semantics, however, the meaning of a sentence is the way it *updates* contexts. To account for anaphora, presupposition satisfaction, and the analysis of epistemic modals, proponents of dynamic semantics claim that it is crucial that updates do *not* behave *symmetrically.* For sentences involving epistemic modals like 'might', $\Diamond A$, this is due to the fact that in contrast to 'factual' sentences, $\Diamond A$ is interpreted as a *global test* (cf. Veltman [1996]). This accounts for the fact that ' $\Diamond A \cdots \neg A$' is an appropriate discourse, but '$\neg A \cdots \Diamond A$' is not. If indicative conditionals were treated as (something) like global tests as well (cf. Gillies [2004]), we might be able to account for the intuition that although 'If A, then B' would be rejected after we learned that $\neg B$ is the case, this doesn't mean that we would accept $\neg B$ after we learned 'If A, then B'. We think this is an appealing idea. Unfortunately, we don't see how it helps to explain the intuition behind Stalnaker's constraint.

In Section 2.2 we discussed another dynamic proposal: the selection function that determines the meaning of '$\Rightarrow$' changes during a discourse. The selection function is constrained by, and thus depends on, the common ground, which changes during a conversation. If K models the common ground, the selection function should be something like f^K as defined there. If the common ground is updated with $\neg B$ that is compatible with, but not entailed by, K, the new common ground is $K \cap \neg B$ and the new selection function is $f^{K \cap \neg B}$. But this means that after $\neg B$ is accepted, '$A \Rightarrow B$' is false in any (relevant) world, including world w that, let us assume, makes both A and B false. Still, it could be that $A \Rightarrow B$ was true in w with respect to the original selection function f^K. Thus, whereas not all sentences of the discourse '$\neg B$ ... $A \Rightarrow B$' are true in w, this might be the case for '$A \Rightarrow B$ ... $\neg B$', showing that in contrast to the first, the second discourse is consistent. This proposal helps to explain the intuition behind Stalnaker's constraint, because it is, to some extent, the idea behind 'citeFraassen76's and Harper [1976]'s tenability results.

3.4 If-clauses as restrictors of binary quantifiers

From a linguistic point of view it might look as if this entire discussion concerning the triviality result is not really relevant for linguistic concerns and can, therefore, be ignored. But the problem easily translates to a general problem concerning the compositional semantics of conditional sentences.

We started the present section by discussing Grice's famous Yog and Zog example. The problem with this example was that it is unclear how to account for the way the conditional construction combines with the construction *there is a probability of X that*, at least if you assume that indicative conditionals should be analyzed as based on a binary propositional connective. Considering this example led us to the formulation of Stalnaker's constraint, which, then, came with the triviality results. However, the problem we are confronted with given Grice's example generalizes to sentences involving *adverbs of quantification* like (12-a), (12-b) and (12-c).

(12) a. Always, *if* John comes, Mary comes (too).
b. Sometimes, *if* John comes, Mary comes (too).
c. Often, *if* John comes, Mary comes (too).

Again, the question is how to account for the meaning of these constructions by combining the quantifier expressed by the adverb with our account of indicative conditionals. Or more generally, assume any analysis of an indicative conditional as a proposition that is constructed by combining two propositions (antecedent and consequent) with a binary connective '$\Rightarrow$'. Can we come up with a meaning of this connective '$\Rightarrow$', such that (12-a)-(12-c) could be represented by something like '$Q(A \Rightarrow B)$'? As Lewis [1975b] observed, the answer to this question is negative once we make some standard assumptions.[19]

In light of Lewis [1975b] triviality result, this was, perhaps, only to be expected. Just as there is no binary connective $\Rightarrow$ with a context-independent meaning such $P(A \Rightarrow B) = P(B|A)$, there is also no such connective that gives $Q(A \Rightarrow B)$ its intuitive meaning. Lewis [1975b] concludes that "the *if* of our restrictive if-clauses should not be regarded as a sentential connective. It has no meaning apart from the adverb it restricts. ... It serves merely to mark an argument-place in a polyadic construction." [Lewis [1975b], p. 184-185].[20] This view is reminiscent to Adams [1966]'s proposal of how to deal with conditional

[19]Higginbotham [1986] pointed out a similar problem for quantified conditional sentences like (i) 'Every student will succeed if they work hard' and (ii) 'No student will succeed if they goof of'. Higginbotham notes that in contrast to (i), the 'if' in (ii) cannot be treated as material implication to account for the intuitively correct reading. He concludes that if the connective is truth-functional, the meaning of 'if' has to depend on the quantificational context in which it is embedded, giving rise to a counterexample to compositionality. But, of course, the problem is not restricted to truth-functional meanings of 'if'.

[20]Given Lewis [1975b]'s reaction to the triviality result as discussed in Section 3.2, this proposal is rather surprising.

sentences, and a similar conclusion emerges a few years later as well in generalized quantifier theory (cf. Barwise and Cooper [1981]): they'd better be treated as *polyatic* rather than *unary* constructions.

Kratzer [1979, 2012] proposes to take Lewis's analysis of conditionals embedded under adverbs of quantification as the principled analysis of (indicative) conditionals. In other words, she gives up on the idea that there is something like a conditional proposition. "The persistent belief that there could be such 'conditional propositions' is based on a simple syntactic mistake. *If*-clauses need to be parsed as adverbial modifiers that restrict operators that might be silent and a distance away. This is what we might call the '**restrictor view** of *if*-clauses' [Kratzer [2012], p. 107]. The examples (12-a)-(12-c) are represented as $Always(\text{if } A, B), Sometimes(\text{if } A, B)$, and $Often(\text{if } A, B)$, if the antecedent and consequent of the embedded clauses of (12-a)-(12-c) are represented as A and B, respectively.

Kratzer claims that also conditionals without explicit adverb, or (probabilistic) modal, should be treated as sketched above, by assuming that they contain a covert adverb of quantification, or (probabilistic) modal, after all. Thus, *all* indicative conditionals should be represented by something of the form '$Q(\text{if } A, B)$'. Kratzer's analysis of the conditional has become highly influential, even dominant, within linguistic semantics. There can be little doubt that such an approach captures the truth conditions of such sentences correctly.

Kratzer's approach is obviously closely related to Adams [1965]'s proposal. What sets them apart is that Kratzer assumes that, in the end, indicative conditionals are still true or false in a world, and thus express propositions. As noted by Charlow [2016], this fact makes the approach in principle vulnerable to triviality results. If we assume that $P(Q(\text{if } A, B)|B) = 1$ and $P(Q(\text{if } A, B)|\neg B) = 0$, from Bradley's argument it immediately follows that Kratzer's analysis of indicative conditionals results in triviality just as much as any other propositional analysis of conditionals does. This is the case at least if we read Q in an epistemic way and assume that $Q(\text{if } A, B)$, given B, or in a context where B is assumed, is true, and $Q(\text{if } A, B)$, given $\neg B$ is false. However, the restrictor approach additionally assumes the meaning of conditionals to be context-dependent.[21] Thereby, Bradley's argument cannot be applied anymore. Because of this inherent context dependence of the approach, the restrictor approach does not only provide an elegant way to deal with adverbs of quantification, but also offers a way out of the triviality results, without that we need to give up the assumptions that indicative conditionals express propositions. But like earlier context-dependent analyses, this comes with a new challenge: the need of an explanation for how disagreement about a conditional is possible.

[21] The quantifier that the antecedent of a conditional restricts is assumed to take two contextually determined arguments: the contextual background, which defines the domain of quantification (and is restricted by the antecedent) and an ordering source, which introduces a standard of normalcy with respect to which the modal claim made is interpreted.

4 The search for a meaning of '⇒'

There can be little doubt that Lewis's and Kratzer's restrictor view of *if*-clauses is linguistically speaking very successful. Still, because it gives up the traditional (and according to many, intuitively correct) way to think about indicative conditionals, many people have looked for a (in some sense) more traditional alternative using the binary connective '⇒'. In this section we will see that this is actually possible, although it demands sometimes a rather complicated analysis of the conditional connnective.

4.1 Conditionals as conditional assertions

Belnap [1984] proposed that 'if' does express a binary truth-conditional connective, '⇒', but instead of *two* truth-values, he uses *three-valued logic* to state its meaning.[22]

- $V_w(A \Rightarrow B) \quad = \quad V_w(B)$, if $V_w(A) = 1$ (i.e., if $A \Rightarrow B$ is defined)
  $\quad = \quad$ undefined otherwise

According to this proposal, a conditional always expresses a conditional assertion: it asserts the consequent on the condition that the antecedent is true, and nothing is asserted otherwise. Lewis [1975a] noted already that this three-valued analysis of the conditional could be used to account for the correct truth-conditions of sentences like (12-a)-(12-c) involving adverbs of quantification. We can represent a sentence like (12-c) as $Often(A \Rightarrow B)$ and say that it is true iff for most worlds, or cases, v for which $V_v(A \Rightarrow B)$ is defined, $V_v(A \Rightarrow B) = 1$. Because $V_v(A \Rightarrow B)$ is defined, i.e. has a classical truth-value, only for worlds or cases in which A is true, this comes down to an analysis according to which B is true in most worlds or cases in which A is true, just as desired. Sentences like (12-a) and (12-b) are handled similarly.

McDermott [1996] and Milne [1997b] note that on this analysis Stalnaker's constraint, $P(A \Rightarrow B) = P(B|A)$, almost immediately follows, without leading to triviality, if A and B are non-conditional (with $[\![A]\!] = \{v \in W : V_v(A) = 1\}$ and $(\!(A)\!) = \{v \in W : V_v(A) \in \{1, 0\}\}$):

$$P(A \Rightarrow B) \quad = \quad \frac{P([\![A \Rightarrow B]\!])}{P((\!(A \Rightarrow B)\!))} \quad = \quad \frac{P(\{w \in W : V_w(A \wedge B) = 1\})}{P(\{w \in W : V_w(A) = 1\})} \quad = \quad P(B|A).$$

It follows that sentences with adverbs of quantification can be analyzed as follows:

1. $V_w(Allways(A \Rightarrow B)) = 1 \quad$ iff$_{df} \quad P(A \Rightarrow B) = 1.$
2. $V_w(Some(A \Rightarrow B)) = 1 \quad$ iff$_{df} \quad P(A \Rightarrow B) > 0$
3. $V_w(Often(A \Rightarrow B)) = 1 \quad$ iff$_{df} \quad P(A \Rightarrow B) > 0.5$

[22]As observed by Milne [1997a], a similar idea had been put forward by de Finetti much earlier.

Notice that in contrast to the proposals of Lewis [1975a] and Kratzer [1979, 2012], on this analysis the (meaning of the) adverb takes as input not two, but only one (open) proposition. Of course, the adverb of quantification doesn't have to 'quantify' over worlds, it can 'quantify' over individuals, sequences of individuals (Lewisean 'cases'), times, or (minimal) situations as well, as in the famous 'Often, if a farmer owns a donkey, he beats it'. Although the resulting analysis looks very appealing, Lewis [1975a] dismissed it almost as soon as he brought it up. It was only a few years ago that Geurts and Huitink [2008] re-vitalized it and developed it further. Huitink argues that, in contrast to other analyses, it can explain why in Gibbard [1981]'s riverboat example, we conclude from two assertions of the forms '$A \Rightarrow B$' and '$A \Rightarrow \neg B$' that A is false. Similarly, this analysis can explain why from a conditional '$B \Rightarrow \bot$' we conclude that B is false.

Unfortunately, the analysis is not without its problems. Among other things, this analysis is – like material implication – non-intensional, and can thus not account for the difference between (4-a) and (4-b).

4.2 Conditionals and expected values

Many of the problems of the Belnap-conditional are due to the fact that $V_w(A \Rightarrow B)$ is undefined in case $V_w(A) \neq 1$. What if we give $V_w(A \Rightarrow B)$ another value in that case? Jeffrey [1991] and Jeffrey and Stalnaker [1994] (followed by Kaufmann [2005, 2009] proposed that this value should be $P(B|A)$:[23]

- $V_w(A \Rightarrow B) \quad = \quad V_w(B)$, if $V_w(A) = 1$
  $\quad = \quad P(B|A)$ otherwise.

Jeffrey and Stalnaker propose to look at the *expected value* of the sentence. The expected value of $A, E(A)$, is defined as: $\sum_w P(w) \times V_w(A)$, where $P(w)$ denotes the probability of world w. While for any non-conditional sentence A this comes down to the probability that A is true: $E(A) = P(A)$, for conditional sentences it has the appealing consequence that $E(A \Rightarrow B) = P(B|A)$ (if A and B are sentences that are true or false in a world).[24]

Of course, one can use this analysis also to account for the interaction of conditionals with adverbs of quantification, i.e., sentences represented by 'ADV(if A, then B)'. The idea is that with an adverb like 'often' the sentence is true iff $E(A \Rightarrow B) > 0.5$.[25] Observe that like the proposal discussed in the previous

[23]Below we will be sloppy and assume that probabilities are world-independent. This is wrong. Instead, we should assume that our agents are probabilistically introspective, which means that $\forall w$: if $P(w) > 0$, then $P_w(\cdot) = P(\cdot)$. As a result, $P_w(A)$ will be the same for each (relevant) w. We will ignore this complication here.

[24]To show this notice that world w can be of three types: (i) it makes $A \wedge B$ true, and thus $V_w(A \Rightarrow B) = 1$, (ii) it is an $A \wedge \neg B$-world, and $V_w(A \Rightarrow B) = 0$, or (iii) A is false in w, and $V_w(A \Rightarrow B) = P(B/A)$. It follows that $E(A \Rightarrow B)$, comes down to $P(A \wedge B) \times 1 + P(A \wedge \neg B) \times 0 + P(\neg A) \times P(B|A) = P(B|A) \times P(A) + 0 + P(B|A) \times P(\neg A) = P(B|A)$.

[25]In order to get the correct results, it should be assumed that $V_w(A) = 0$, for instance by stipulating that A is about the future.

section, but unlike the one of Lewis [1975b] and Kratzer [1979, 2012], this analysis is in accordance with the traditional way of thinking about conditionals. As a result, we can think of the adverb as taking just one proposition. Just like in the previous section, the adverb of quantification doesn't have to 'quantify' over worlds. If we allow for quantification over sequences of individuals as well, the analysis is also able to account for donkey-sentences.

This proposal faces a couple by now well-known challenges. Edgington [1995] complains that this type of analysis makes the meaning of the conditional too context-dependent, because dependent on (subjective) probability function P. Another challenge is to account for entailments involving conditionals. If entailment were defined as preservation of truth it would give rise to all the problems the Belnap-connective gave rise to.[26] An familiar *empirical* challenge is how to extend it to embedded conditionals.(see e.g. Edgington [1991] and Kaufmann [2005]).

4.3 Context-dependence or fine-grained propositions

Can't we have it all? An analysis according to which indicative conditionals express two-valued propositions such that $P(A \Rightarrow B) = P(B|A)$? Fraassen [1976] shows this is actually possible, if we make the selection function, and thus the meaning of $\Rightarrow$, context-dependent, i.e., dependent on the probability function P.[27] In that way we could account for adverbs of quantification just like in the Belnap analysis. On Fraassen [1976] analysis, the problems of the Belnap-connective discussed in Section 4.1 do not arise: Modus Ponens is predicted to be valid, $\neg(A \Rightarrow B) \not\models A \wedge \neg B$, and $(A \Rightarrow A) \wedge (\neg A \Rightarrow B)$ might well be true.

Although appealing, the analysis has some severe limitations. First, it requires a rather weak logic of $\Rightarrow$. Stalnaker [1981], for instance, has shown that van Fraassen's result is not possible for the logic proposed in Stalnaker [1968]. Second, it has been shown (cf. Hajek and Hall [1976]) that van Fraassen's construction can work only in case the domain of P (i.e., the set of possible worlds) is uncountable. Finally, Korzukhin [2016] shows that when we assume that (i) for each world w, the selected A-worlds depends on nothing else than the A-worlds in $\{w \in A : P(w) > 0\}$, (ii) it is possible to learn that $A \Rightarrow B$ without learning $A \wedge B$, and (iii) strong centering, triviality results, after all.

Bradley [2012] — following an earlier suggestion of McGee [1989] — proposes that Stalnaker's constraint can be saved if we take meanings to be *more fine-grained* than sets of possible worlds. Instead, meanings are sets of *possibilities*, pairs consisting of a world and a selection function, $\langle w, f\rangle$, because it are such possibilities that make (conditional) sentences true or false.[28] We say that $V_{\langle w,f\rangle}(A \Rightarrow B) = 1$ if $f_w(A) \in B$, 0 otherwise. Then $P(A \Rightarrow B) =$

[26] A natural alternative is to use a close analogue to Adams' probabilistic entailment.

[27] For a recent simplication, see Bacon [2015].

[28] More precisely, Bradley [2012] proposes that the meaning of a conditional is a set of *sequences* of worlds. This idea is actually closely related to that of Fraassen [1976], also made use of by Jeffrey and Stalnaker [1994] and Kaufmann [2009]. At present, we don't know exactly how closely related.

$\sum_{\langle w,f\rangle} P(\langle w,f\rangle) \times V_{\langle w,f\rangle}(A \Rightarrow B)$. By some constraints on what are proper selection functions, it follows that $P(A \Rightarrow B) = P(B|A)$, without giving rise to the triviality results of Lewis and Bradley.[29] This proposal avoids Bradley's own triviality result, for instance, because $P(A \Rightarrow B|B) = 1$ only if A entails B.[30]

5 Conclusion

In this paper we have concentrated on two challenges for any analysis of indicative conditionals: (i) indicative conditionals, like subjunctive conditionals, seem to behave in a *non-monotonic* way; and (ii) the probability of an indicative conditional should, at least under many circumstances, be closely related to the corresponding conditional probability. We have seen that both problems can be met easily when we give up the assumption that indicative conditionals express propositions. The challenge for such an analysis is to explain how we seem to be able to disagree over conditional sentences. There are several alternative possibilities when we assume that conditionals *do* express propositions. The problem that then should be faced is how to escape on Lewis's triviality problem. Almost by necessity this means that we have to make the meaning of the conditional *context dependent*. The main open question, we believe, is how best to account for this context dependence: either (i) following Lewis and Kratzer and assume that if-clauses function as restrictors of context dependent (perhaps probabilistic) quantifiers, or (ii) assume that a conditional expresses a proposition even without the quantifier, but make the selection function that helps to determine which proposition is expressed depend on context. Finally, the problem has to be addressed how different the propositional analysis will be, in the end, from a non-propositional one, for instance, to account for disagreement over indicative conditionals.

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[29] To illustrate, suppose that $W = \{w_1, w_2, w_3, w_4\}$, $F = \{f, g\}$ and $[\![A]\!] = \{w_1, w_2\} \times F$ and $[\![B]\!] = \{w_1, w_3\} \times F$. The selection functions are such that $f_{w_1}(A) = w_1 = g_{w_1}(A)$, $f_{w_2}(A) = w_2 = g_{w_2}(A)$ (by strong centering), and $f_{w_3}(A) = f_{w_4}(A) = w_1$ and $g_{w_3}(A) = g_{w_4}(A) = w_2$. (To be precise, we should say that $g_{w_4}(A) = \langle w_2, g\rangle$.) This model has $|W| \times |F| = 8$ possibilities, 4 of which make $A \Rightarrow B$ true: $\{\langle w_1, f\rangle, \langle w_1, g\rangle, \langle w_3, f\rangle, \langle w_4, f\rangle\}$. Thus $P(A \Rightarrow B) = \frac{1}{2} = P(B|A)$. This model also shows that Lewis's triviality result can be avoided, for $P(A \Rightarrow B|B) = \frac{P((A \Rightarrow B) \wedge B)}{P(B)} = \frac{|\{\langle w_1,f\rangle,\langle w_1,g\rangle,\langle w_3,f\rangle\}|}{|\ [\![B]\!]\ |} = \frac{3}{4} \neq 1$, and $P(A \Rightarrow B|\neg B) = \frac{|\{\langle w_4,f\rangle\}|}{|\ [\![B]\!]\ |} = \frac{1}{4} \neq 0$.

[30] Whether Bradley's proposal can account for donkey sentences involving adverbs of quantification remains an interesting challenge.

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