Dealing with artificially dichotomized variables in Meta-Analytic Structural Equation Modeling

de Jonge, H.; Jak, S.; Kan, K.-J.

DOI
10.1027/2151-2604/a000395

Publication date
2020

Document Version
Final published version

Published in
Zeitschrift für Psychologie

License
Article 25fa Dutch Copyright Act

Citation for published version (APA):

General rights
It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

Disclaimer/Complaints regulations
If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: https://uba.uva.nl/en/contact, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.

UvA-DARE is a service provided by the library of the University of Amsterdam (https://dare.uva.nl)
Dealing With Artificially Dichotomized Variables in Meta-Analytic Structural Equation Modeling

Hannelies de Jonge, Suzanne Jak, and Kees-Jan Kan

Department of Child Development and Education, University of Amsterdam, The Netherlands

Abstract: Meta-analytic structural equation modeling (MASEM) is a relatively new method in which effect sizes of different independent studies between multiple variables are typically first pooled into a matrix and next analyzed using structural equation modeling. While its popularity is increasing, there are issues still to be resolved, such as how to deal with primary studies in which variables have been artificially dichotomized. To be able to advise researchers who apply MASEM and need to deal with this issue, we performed two simulation studies using random-effects two stage structural equation modeling. We simulated data according to a full and partial mediation model and systematically varied the size of one (standardized) path coefficient ($\beta = .16$, $\beta = .23$, $\beta = .33$), the percentage of dichotomization (25%, 75%, 100%), and the cut-off point of dichotomization (.5, .1). We analyzed the simulated datasets in two different ways, namely, by using (1) the point-biserial and (2) the biserial correlation as effect size between the artificially dichotomized predictor and continuous variables. The results of these simulation studies indicate that the biserial correlation is the most appropriate effect size to use, as it provides unbiased estimates of the path coefficients in the population.

Keywords: meta-analytic structural equation modeling, artificially dichotomized variables, point-biserial correlation, biserial correlation

Meta-analysis (Glass, 1976) is a commonly used statistical technique to aggregate sample effect sizes of different independent primary studies in order to draw inferences concerning population effects. The most common used form of meta-analysis is one in which the relationship between two variables is tested (i.e., “univariate meta-analysis”). To extend the range of research questions that can be answered, new meta-analytic models have been developed, such as meta-analytic structural equation modeling (MASEM) (Becker, 1992, 1995; Cheung, 2014, 2015a; Cheung & Chan, 2005; Jak, 2015; Viswesvaran & Ones, 1995). This increasingly popular method tests several relations between various variables simultaneously. For example, Jansen, Elffers, and Jak (2019) applied MASEM to evaluate the mediating effect of socioeconomic status (SES) on achievement by private tutoring in a path model.

In primary studies, an effect size may represent the strength and direction of the association between any two variables of interest. Such an effect size can be expressed in different ways, for example as Pearson product-moment correlation, Cohen’s $d$, biserial correlation, and point-biserial correlation. How an effect size is expressed depends on the nature of the variables (e.g., continuous or dichotomous), but also on the way the variables are measured or analyzed. If one of the two continuous variables is artificially dichotomized, one may express the effect size as a point-biserial correlation. However, this typically provides a negatively biased estimate of the true underlying Pearson product-moment correlation (e.g., Cohen, 1983; MacCallum, Zhang, Preacher, & Rucker, 2002). The biserial correlation on the other hand should generally provide an unbiased estimate (Soper, 1914; Tate, 1955). Bias in the effect size of any primary study may affect meta-analytic results in the same direction (Jacobs & Viechtbauer, 2017). In the current study, we will evaluate how using point-biserial correlations versus biserial correlations from primary studies may affect path coefficients, their standard errors/confidence intervals, and model fit in MASEM using population models that represent full and partial mediation. Based on the results, we expect to be able to inform researchers about which of the two investigated effect sizes is the most appropriate to use in MASEM-applications and under which conditions.
Meta-Analytic Structural Equation Modeling

MASEM is a statistical technique in which meta-analytical procedures are combined with structural equation modeling (SEM) in order to study aggregated results (Becker, 1992, 1995; Viswesvaran & Ones, 1995). There is a growing interest in MASEM in methodological research (e.g., Cheung & Hafdahl, 2016; Jak & Cheung, 2018a, 2018b; Ke, Zhang, & Tong, 2018) as well as in many different substantive research fields (e.g., Hagger & Chatzisarantis, 2016; Montazemi & Qahri-Saremi, 2015; Rich, Brandes, Mullan, & Hagger, 2015). In contrast to univariate meta-analysis, with MASEM it is possible to test complete hypothesized models including more than two variables, mediational effects, and possibly even latent variables. MASEM does not only provide individual parameter estimates, but also the overall model fit. This model fit provides information on whether the observed data agree with the hypothetical model.

In MASEM, a hypothesized model will typically be fitted to a pooled correlation matrix using SEM (see Cheung, 2015a; Jak, 2015). To estimate a pooled correlation matrix, one needs to express the bivariate effect sizes in the primary studies as correlation coefficients. Since primary studies may report different kinds of effect sizes, based on the nature of the variables and the way the variables are measured or analyzed, these effect sizes thus first need to be converted into a correlation coefficient.

Relationship Between a Dichotomous and Continuous Variable

Many meta-analyses in educational research express their effect size as Cohen’s $d$ (i.e., the standardized mean difference) or the related unbiased estimate Hedges’ $g$ (Ahn, Ames, & Myers, 2012; De Jonge & Jak, 2018), since they are interested in investigating a relationship between an independent dichotomous variable and dependent continuous variable. A variable can be dichotomous by nature or artificially dichotomized. An example of a variable that can be seen as dichotomous by nature is experimental condition (e.g., intervention group or control group). When a variable is artificially dichotomized, the actual variable is continuous but dichotomized by researchers, often for practical purposes. For example, the continuous variable SES is often artificial dichotomized into high SES and low SES.

The dichotomization of continuous variables leads to a loss of information, typically leading to an underestimation of the true underlying correlation between the artificially dichotomized and continuous variable (e.g., Cohen, 1983; MacCallum et al., 2002). However, in models with multiple predictors, one can also obtain an overestimation of effects if the continuous predictor(s) are artificially dichotomized (see Maxwell & Delaney, 1993; Vargha, Rudas, Delaney, & Maxwell, 1996). Therefore, the dichotomization of continuous variables is not recommended. Nevertheless, researchers still often artificial dichotomize variables. As a result, meta-analysts frequently have to deal with primary studies in which variables have been artificially dichotomized. Since raw data are difficult to obtain from researchers (see Wicherts, Borsboom, Kats, & Molenaar, 2006), for MASEM, a correlation coefficient has to be calculated from the information provided in the primary study. Researchers often provide the sample size, means, and standard deviations of the continuous variable per group. Based on these summary statistics, one could calculate the point-biserial correlation (Lev, 1949; Tate, 1954) and the bivariate correlation (Pearson, 1909). However, meta-analysts may not be aware of the difference between these correlation coefficients and will presumably transform the provided effect size to the point-biserial correlation.

The (Point-)Biserial Correlation

The point-biserial correlation is a special case of the Pearson product-moment correlation and intended to express the association between a natural dichotomous and continuous variable (Lev, 1949; Tate, 1954). It can be obtained by applying the equation for the Pearson product-moment correlation coefficient to the dichotomous variable and the continuous variable. However, for the relationship between an artificially dichotomized and continuous variable, the point-biserial correlation does not provide an accurate estimate of the true underlying population correlation (e.g., Cohen, 1983; MacCallum et al., 2002). The point-biserial correlation does not take into account that one of the variables is artificially dichotomized and interest lies in the underlying continuous variables, leading to biased (pooled) effect sizes. Jacobs and Viechtbauer (2017) showed that the bias increases with larger population correlation coefficients, and with larger imbalance of the groups. Using larger samples does not reduce this bias. Only when the population correlation is zero, the point-biserial correlation provides an unbiased estimate of the population correlation.

In contrast to the point-biserial correlation, the biserial correlation assumes a continuous, normally distributed variable underlying the dichotomous variable (Tate, 1950). At a fixed threshold, the observations are assigned a 1 if they fall on the right side of this threshold and 0 if they fall on the left. For example, if one has to respond to a dichotomous item on a questionnaire used to indicate a specific neurodevelopmental disorder, it is assumed that there exists a continuous normal distribution of attitudes towards answering this item, but a specific threshold.
determines which of the two answer options one chooses. It has been shown mathematically that for the relationship between an artificially dichotomized and continuous variable, the estimated biserial correlation provides an unbiased estimate of the relationship between the two underlying continuous variables (Soper, 1914; Tate, 1955). As may be expected, Jacobs and Viechtbauer (2017) found that pooling biserial correlations in a meta-analysis typically provides unbiased estimates of the correlation between the two underlying continuous variables. If the sample sizes were small and the population correlation was large, the population correlation was marginally underestimated depending on the exact dichotomization method (i.e., adaptive or hard cut-off). However, all bias could be considered negligible when the sample sizes were larger or equal to 60. Hence, the biserial correlation accounts for the artificial dichotomization of a variable and is, therefore, preferred over the point-biserial correlation if the correlation between an artificially dichotomized and continuous variable is included in a meta-analysis (see also Hunter & Schmidt, 1990).

Calculating the (Point-)Biserial Correlation for a Meta-Analysis

The point-biserial and biserial correlation can be computed in different ways depending on which summary statistics are reported in a primary study (Jacobs & Viechtbauer, 2017). If the sample means of the two groups (defined as \( \bar{y}_1 \) and \( \bar{y}_0 \)), the groups sizes (denoted as \( n_1 \) and \( n_0 \)), and the sample standard deviations (defined as \( s_1 \) and \( s_0 \)) are provided in the primary study, Cohen’s \( d \) can be first computed with

\[
d = \frac{\bar{y}_1 - \bar{y}_0}{\sqrt{\frac{(n_1 - 1)s_1^2 + (n_0 - 1)s_0^2}{n_1 + n_0 - 2}}},
\]

Next, one can convert this Cohen’s \( d \) into the point-biserial correlation coefficient using

\[
r_{pb} = \frac{d}{\sqrt{d^2 + h}},
\]

where \( h \) is denoted as \( h = m/n_1 + m/n_0 \) and \( m = n_1 + n_0 - 2 \). If the standard deviations are not provided separately for both groups, but only of the \( y \) scores (denoted as \( s_y \)), the point-biserial correlation coefficient can be computed by

\[
r_{pb} = \left(\frac{\bar{y}_1 - \bar{y}_0}{s_y}\right)\sqrt{\frac{npq}{n-1}},
\]

where \( p = n_1/n \) and \( q = 1 - p = n_0/n \). Alternatively, if only the test statistic of the independent sample \( t \)-test (with test statistic \( t \)) and the two group sizes are reported, one can transform this into the point-biserial correlation coefficient with

\[
r_{pb} = \frac{t}{\sqrt{t^2 + m}}.
\]

Once a point-biserial correlation coefficient is obtained, one can convert this into the biserial correlation coefficient using the following equation

\[
r_b = \sqrt{pq \cdot f(z_p)} r_{pb}.
\]

In this equation, as \( f(z_p) \) indicates the density of the standard normal distribution at value \( z_p \). The value \( z_p \) is the point for which \( P(Z > z_p) = p \), where \( Z \) is a standard normally distributed random variable. The density of the standard normal distribution can be computed using

\[
f(z_p) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z_p^2}{2}}.
\]

Aim and Expectations

Our aim is to investigate the effects of using (1) the point-biserial correlation and (2) the biserial correlation for the relationship between an artificially dichotomized variable and a continuous variable on MASEM-parameters and model fit. Specifically, our interest lies in path coefficients, standard errors/confidence intervals of these coefficients, and model fit. We expect that the use of the point-biserial correlation for the relation between an artificially dichotomized and continuous variable biases the path coefficients in MASEM. In contrast, we expect unbiased path coefficients if one uses the biserial correlation instead.

Method

We performed two simulation studies to evaluate the effects of using the point-biserial and biserial correlation in a full and partial mediation model.

Simulation Study 1 – Full Mediation

In the first simulation study, we simulated meta-analytic data according to a full mediation (hence overidentified) population model (see Figure 1), with a continuous predictor variable \( X \), continuous mediator \( M \), and a continuous variable \( Y \) as outcome. Depending on the condition, the predictor variable \( X \) is artificially dichotomized in all or a given percentage of the primary studies. For comparison, we also analyzed the original datasets in which we did not dichotomize the predictor variable \( X \) at all. We chose this population model because in educational research the
median number of variables in a “typical” meta-analysis is three (De Jonge & Jak, 2018) and because mediation is a popular research topic.

Under this population model, random meta-analytic datasets were generated under different conditions. We systematically varied the following: (1) the size of the (standardized) path coefficient between $X$ and $M$ ($\beta_{MX} = .16$, $\beta_{MX} = .23$, $\beta_{MX} = .33$), (2) the percentage of primary studies in which $X$ was artificially dichotomized (25%, 75%, 100%), and (3) the cut-off point at which $X$ was artificially dichotomized (at the median value, so a proportion of .5, or when groups become unbalanced, at a proportion of .1). These choices were mainly based on typical situations in educational research. The size of the path coefficient reflects the minimum, mean/median, and maximum pooled Pearson product-moment correlations in a “typical” meta-analysis in educational research (De Jonge & Jak, 2018). The 75% primary studies that artificially dichotomize the variable $X$, is based on a comparable example of a meta-analysis in educational research (Jansen et al., 2019). We chose a cut-off proportion of .5 because a median split, leading to balanced groups, is a common way to dichotomize variables. The cut-off proportion of .1 leads to unbalanced groups, and is representative of clinical cut-offs for attributes like dyslexia or depression.

To ensure that the model-implied covariance matrix was a correlation matrix, we fixed the residual variances of all three variables at values that lead to total variances of one. We used between-study variances of .01. The number of primary studies in a meta-analysis was fixed at the median number of a “typical” meta-analysis, which is 44 (De Jonge & Jak, 2018). Because we use a random-effects MASEM-method, the assumption is thus that the population comprises 44 subpopulations from which the 44 samples are drawn, and that the weighted mean of the subpopulation parameters equals the population parameter. Given a specific condition and the fixed number of 44 primary studies, we randomly sampled the within primary study sample sizes from a positively skewed distribution as used in Hafdahl (2007) with a mean of 421.75, yielding “typical” sample sizes (De Jonge & Jak, 2018) for every iteration. We imposed 39% missing correlations (Sheng, Kong, Cortina, & Hou, 2016) by (pseudo)randomly deleting either variable $M$ or $Y$ from 26 of the 44 studies.

Simulation Study 2 – Partial Mediation

In the second simulation study, we included an extra fixed direct effect of .16 between the predictor variable $X$ and outcome variable $Y$ ($\beta_{XY}$) in the population model. This value represents a small effect in a “typical” meta-analysis in educational research (De Jonge & Jak, 2018). We used the same conditions as in our first simulation.

Analyses

In each condition, we generated 2,000 meta-analytic datasets drawn from the 44 subpopulations, which we analyzed using (1) the point-biserial and (2) the biserial correlation as effect size between the artificially dichotomized predictor $X$ and the continuous variables ($M$ or $Y$). To simulate the data and to carry out the analyses we used R (version 3.5.3; R Core Team, 2019). The R scripts to reproduce the results can be found in De Jonge, Jak, and Kan (2019a).

The full or partial mediation models were fitted using random-effects two stage structural equation modeling (TSSEM) (Cheung, 2014). Unless stated otherwise, we used default settings of the tssem1() and tssem2() functions within the R package metaSEM (Cheung, 2015b). In the first stage of the random-effects TSSEM, a pooled correlation matrix is estimated using maximum likelihood estimation. In the second stage, the hypothesized structural equation model is fitted to this pooled correlation matrix using weighted least squares estimation. As recommended (Becker, 2009; Hafdahl, 2007), we used the weighted mean correlation across the included primary studies to estimate the sampling variances and covariances of the correlation coefficients in the primary studies.

To make sure that the possible differences in outcomes when analyzing point-biserial versus biserial correlations are due to the use of a different kind of correlation coefficient, these analyses were performed using the same datasets.

Evaluation Criteria

We estimated the relative percentage bias in all path coefficients, calculated as $100 \times (\hat{\beta} - \beta)/\beta$. If the estimation bias was less than 5%, we considered it as negligible
(Hoogland & Boomsma, 1998). Additionally, we estimated the bias in the standard error of the direct effects. We calculated the relative percentage bias of the standard errors as $\frac{100 \times (\overline{SE}(\hat{\beta}) - SD(\hat{\beta}))}{SD(\hat{\beta})}$. In this equation, $\overline{SE}(\hat{\beta})$ is the average standard error of the parameter estimate across replications and $SD(\hat{\beta})$ is the standard deviation of the parameter estimate across replications. If the relative percentage bias of the standard errors was less than 10% we considered it as acceptable (Hoogland & Boomsma, 1998). We tested the significance of the indirect effects using 95% likelihood-based confidence intervals, which are recommended over the Wald confidence intervals in this case (Cheung, 2009). Over the 95% likelihood-based confidence intervals we calculated the coverage percentages, which is the percentage confidence intervals that includes the population parameter. For comparison, we also calculated the coverage percentages of the 95% Wald confidence intervals and likelihood-based confidence intervals of the direct effects. To evaluate model fit, we calculated the rejection rates (i.e., proportion of significant test results) of the chi-square statistic of the full mediation model of Stage 2 ($df = 1, \alpha = .05$). We tested whether the rejection rate significantly differed from the nominal $\alpha$-level with the proportion test (using $\alpha = .05$). By means of QQplots and the Kolmogorov-Smirnov test (using $\alpha = .05$), we compared the theoretical chi-square distribution with one degree of freedom with the empirical chi-square distributions.

**Results**

The simulation results are based on the datasets that converged in Stage 1 and Stage 2 of the random-effects TSSEM. In most conditions, there were no convergence problems, as noted in Tables 1–4.

**Simulation Study 1 – Full Mediation**

**Direct Effects**

Table 1 shows that when the point-biserial correlation was used, the relative percentage bias in the estimated path coefficient between the predictor and mediator ($\beta_{MX}$) was between $-41.70\%$ and $-5.05\%$. The relative percentage bias in this path coefficient exceeded the set boundary of $5\%$ in all conditions, representing substantial bias. This bias was negative in all conditions. When the percentage of primary studies in which the predictor variable $X$ was artificially dichotomized increased, the bias in $\beta_{MX}$ also increased. In the conditions with a cut-off proportion of .1, the relative percentage bias in $\beta_{MX}$ was always larger compared to the conditions in which the cut-off proportion was .5. There was no systematic difference in the relative percentage bias in this path coefficient between conditions with different population values of $\beta_{YM}$. The relative percentage bias in the path coefficient between the continuous mediator and continuous outcome variable ($\beta_{YM}$) was below $5\%$ in all conditions, which can be considered negligible. The relative percentage bias in the standard errors of both path coefficients was below the set boundary of $10\%$ in all conditions. However, note that the bias in the standard error of $\beta_{YM}$ was negative in all conditions. The coverage percentages of the 95% likelihood-based confidence interval of $\beta_{MX}$ were between $0.00\%$ and $92.10\%$ and of $\beta_{YM}$ between $91.15\%$ and $93.38\%$ (see Table 3/SIM1 in De Jonge, Jak, & Kan, 2019b). The coverage percentages of the 95% Wald confidence intervals were roughly the same for both effects (see Table 4/SIM1 in De Jonge et al., 2019b). The low coverage percentages of the confidence intervals in some conditions are not surprising given the bias in the point estimates caused by using the point-biserial correlation coefficient.

When the biserial correlation was used instead of the point-biserial correlation, the relative percentage bias in both path coefficients in the model ($\beta_{MX}$ and $\beta_{YM}$) was below $5\%$ in all conditions (see Table 1). The relative percentage bias in the standard errors of both path coefficients was less than the set boundary of $10\%$ in all conditions. Therefore, there was no substantial bias in the parameter estimates or the standard errors according to the criteria that were applied. However, note that the relative percentage bias in the standard errors of both path coefficients was negative in all conditions. In accordance, the coverage percentages of the 95% likelihood-based as well as the Wald confidence intervals were slightly below $95\%$ for both path coefficients (see Tables 3–4/SIM1 in De Jonge et al., 2019b).

**Indirect Effect**

Since indirect effects are the product of two direct effects, any bias in the direct effects will induce bias in the indirect effect. Indeed, Table 2 shows that if the point-biserial correlation was used, the bias in the indirect effect was always negative and above the set boundary of $5\%$. The bias in the indirect effect increased according to the same patterns as when the bias in $\beta_{MX}$ increased. The coverage percentages of the 95% likelihood-based confidence intervals of the indirect effect were between $0.05\%$ and $92.50\%$. If the biserial correlation was used instead, the bias in the indirect effect was always below $5\%$. The coverage percentages were between $92.55\%$ and $95.05\%$.

**Model Fit**

The rejection rates of the chi-square test of model fit at Stage 2 of the random-effects TSSEM ($df = 1, \alpha = .05$) are

© 2020 Hogrefe Publishing
### Table 1. Simulation Study 1 (full mediation): Percentages estimation bias in the direct effects and their standard errors

<table>
<thead>
<tr>
<th>Condition</th>
<th>Converged</th>
<th>Bias in $\beta_{MX}$</th>
<th>Bias in SE of $\beta_{MX}$</th>
<th>Bias in $\beta_{MY}$</th>
<th>Bias in SE of $\beta_{MY}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$r_{pb}$</td>
<td>$r_b$</td>
<td>$r_{pb}$</td>
<td>$r_b$</td>
</tr>
<tr>
<td>25</td>
<td>.1</td>
<td>.16</td>
<td>2,000</td>
<td>2,000</td>
<td>-10.656</td>
</tr>
<tr>
<td></td>
<td>.23</td>
<td>.2000</td>
<td>-10.513</td>
<td>-0.118</td>
<td>0.426</td>
</tr>
<tr>
<td></td>
<td>.33</td>
<td>.2000</td>
<td>-10.619</td>
<td>-0.217</td>
<td>4.126</td>
</tr>
<tr>
<td></td>
<td>.5</td>
<td>.16</td>
<td>2,000</td>
<td>1,999</td>
<td>-5.046</td>
</tr>
<tr>
<td></td>
<td>.23</td>
<td>.2000</td>
<td>-5.197</td>
<td>-0.159</td>
<td>-0.762</td>
</tr>
<tr>
<td></td>
<td>.33</td>
<td>.2000</td>
<td>-5.375</td>
<td>-0.311</td>
<td>-0.382</td>
</tr>
<tr>
<td>75</td>
<td>.1</td>
<td>.16</td>
<td>1,999</td>
<td>2,000</td>
<td>-31.412</td>
</tr>
<tr>
<td></td>
<td>.23</td>
<td>.1999</td>
<td>-31.382</td>
<td>-0.164</td>
<td>1.123</td>
</tr>
<tr>
<td></td>
<td>.33</td>
<td>.2000</td>
<td>-31.224</td>
<td>-0.010</td>
<td>8.956</td>
</tr>
<tr>
<td></td>
<td>.5</td>
<td>.16</td>
<td>2,000</td>
<td>1,999</td>
<td>-14.976</td>
</tr>
<tr>
<td></td>
<td>.33</td>
<td>.2000</td>
<td>-15.175</td>
<td>0.026</td>
<td>-0.769</td>
</tr>
<tr>
<td>100</td>
<td>.1</td>
<td>.16</td>
<td>1,994</td>
<td>2,000</td>
<td>-41.290</td>
</tr>
<tr>
<td></td>
<td>.23</td>
<td>.1995</td>
<td>-41.509</td>
<td>-0.121</td>
<td>-4.528</td>
</tr>
<tr>
<td></td>
<td>.33</td>
<td>.1990</td>
<td>-41.702</td>
<td>-0.355</td>
<td>-1.463</td>
</tr>
<tr>
<td></td>
<td>.5</td>
<td>.16</td>
<td>2,000</td>
<td>2,000</td>
<td>-20.091</td>
</tr>
<tr>
<td></td>
<td>.23</td>
<td>.1999</td>
<td>-20.251</td>
<td>-0.058</td>
<td>-6.689</td>
</tr>
<tr>
<td></td>
<td>.33</td>
<td>.2000</td>
<td>-20.473</td>
<td>-0.340</td>
<td>-3.935</td>
</tr>
</tbody>
</table>

Note. DICH = percentage of primary studies in which X was artificially dichotomized; CO = cut-off point at which X was artificially dichotomized; ES = size of the systematically varied (standardized) path coefficient between X and M; Converged = number of datasets that converged in Stage 1 and Stage 2 of the random-effects TSSEM; Bias in $\beta_{MX}$ = relative percentage bias in the path coefficient between X and M; Bias in SE of $\beta_{MX}$ = relative percentage bias in the standard error of the path coefficient between X and M; Bias in $\beta_{MY}$ = relative percentage bias in the path coefficient between M and Y; Bias in SE of $\beta_{MY}$ = relative percentage bias in the standard error of the path coefficient between M and Y; $r_{pb}$ = point-biserial correlation; $r_b$ = biserial correlation.

### Table 2. Simulation Study 1 (full mediation): Percentages estimation bias in the indirect effect and the coverage percentages of the 95% likelihood-based confidence interval

<table>
<thead>
<tr>
<th>Condition</th>
<th>Converged</th>
<th>Bias in indirect</th>
<th>Coverage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$r_{pb}$</td>
<td>$r_b$</td>
<td>$r_{pb}$</td>
</tr>
<tr>
<td>25</td>
<td>.1</td>
<td>.16</td>
<td>2,000</td>
</tr>
<tr>
<td></td>
<td>.23</td>
<td>.2000</td>
<td>-10.972</td>
</tr>
<tr>
<td></td>
<td>.33</td>
<td>.2000</td>
<td>-10.402</td>
</tr>
<tr>
<td></td>
<td>.5</td>
<td>.16</td>
<td>2,000</td>
</tr>
<tr>
<td></td>
<td>.23</td>
<td>.2000</td>
<td>-5.199</td>
</tr>
<tr>
<td></td>
<td>.33</td>
<td>.2000</td>
<td>-5.529</td>
</tr>
<tr>
<td>75</td>
<td>.1</td>
<td>.16</td>
<td>1,999</td>
</tr>
<tr>
<td></td>
<td>.23</td>
<td>.2000</td>
<td>-31.184</td>
</tr>
<tr>
<td></td>
<td>.33</td>
<td>.2000</td>
<td>-31.178</td>
</tr>
<tr>
<td></td>
<td>.5</td>
<td>.16</td>
<td>2,000</td>
</tr>
<tr>
<td></td>
<td>.23</td>
<td>.2000</td>
<td>-15.116</td>
</tr>
<tr>
<td></td>
<td>.33</td>
<td>.2000</td>
<td>-15.558</td>
</tr>
<tr>
<td>100</td>
<td>.1</td>
<td>.16</td>
<td>1,994</td>
</tr>
<tr>
<td></td>
<td>.23</td>
<td>.1995</td>
<td>-41.725</td>
</tr>
<tr>
<td></td>
<td>.33</td>
<td>.1990</td>
<td>-41.750</td>
</tr>
<tr>
<td></td>
<td>.5</td>
<td>.16</td>
<td>2,000</td>
</tr>
<tr>
<td></td>
<td>.23</td>
<td>.1999</td>
<td>-20.530</td>
</tr>
<tr>
<td></td>
<td>.33</td>
<td>.2000</td>
<td>-20.428</td>
</tr>
</tbody>
</table>

Note. DICH = percentage of primary studies in which X was artificially dichotomized; CO = cut-off point at which X was artificially dichotomized; ES = size of the systematically varied (standardized) path coefficient between X and M; Converged = number of datasets that converged in Stage 1 and Stage 2 of the random-effects TSSEM; Bias in indirect = relative percentage bias in the indirect effect of X on Y ($\beta_{MX} \times \beta_{XY}$); Coverage = percentage of confidence intervals that includes the population parameter of the indirect effect of X on Y; $r_{pb}$ = point-biserial correlation; $r_b$ = biserial correlation.
Table 3. Simulation Study 2 (partial mediation): Percentages estimation bias in the direct effects and their standard errors

<table>
<thead>
<tr>
<th>Condition</th>
<th>DICH CO ES</th>
<th>Converged</th>
<th>Bias in $\beta_{XX}$</th>
<th>Bias in $\beta_{XY}$</th>
<th>Bias in $\beta_{YY}$</th>
<th>Bias in $\beta_{XM}$</th>
<th>Bias in $\beta_{YM}$</th>
<th>Bias in $\beta_{XY}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 .1 .16</td>
<td>2,000 2,000</td>
<td>-10.552 -0.134</td>
<td>0.967 -0.193</td>
<td>2.039 -0.188</td>
<td>-5.573 -5.611</td>
<td>-11.338 -0.544</td>
<td>-0.539 -3.152</td>
<td></td>
</tr>
<tr>
<td>.23</td>
<td>2,000 2,000</td>
<td>-10.502 -0.172</td>
<td>0.715 -4.349</td>
<td>2.915 -0.360</td>
<td>-4.026 -4.063</td>
<td>-11.621 -0.255</td>
<td>-0.047 -4.018</td>
<td></td>
</tr>
<tr>
<td>.33</td>
<td>2,000 2,000</td>
<td>-10.660 -0.299</td>
<td>5.625 -2.604</td>
<td>5.035 -0.016</td>
<td>-4.300 -4.313</td>
<td>-12.083 0.798</td>
<td>1.260 -2.738</td>
<td></td>
</tr>
<tr>
<td>.5</td>
<td>2,000 2,000</td>
<td>-5.093 -0.015</td>
<td>-0.537 -1.059</td>
<td>1.169 0.053</td>
<td>6.179 -6.083</td>
<td>-5.299 -0.057</td>
<td>-2.234 -3.302</td>
<td></td>
</tr>
<tr>
<td>.23</td>
<td>2,000 2,000</td>
<td>-5.249 -0.209</td>
<td>-0.237 -1.169</td>
<td>1.713 0.072</td>
<td>-6.874 -6.832</td>
<td>-5.679 -0.147</td>
<td>-2.956 -3.630</td>
<td></td>
</tr>
<tr>
<td>.33</td>
<td>2,000 2,000</td>
<td>-5.417 -0.361</td>
<td>1.479 -0.373</td>
<td>2.437 -0.070</td>
<td>-5.324 -5.320</td>
<td>-6.320 -0.131</td>
<td>0.788 -0.513</td>
<td></td>
</tr>
</tbody>
</table>

Note. DICH = percentage of primary studies in which X was artificially dichotomized; CO = cut-off point at which X was artificially dichotomized; ES = size of the systematically varied (standardized) path coefficient between X and M; Converged = number of datasets that converged in Stage 1 and Stage 2 of the random-effects TSSEM; Bias in $\beta_{XX}$ = relative percentage bias in the path coefficient between X and M; Bias in SE of $\beta_{XX}$ = relative percentage bias in the standard error of the path coefficient between X and M; Bias in $\beta_{XY}$ = relative percentage bias in the path coefficient between M and Y; Bias in SE of $\beta_{XY}$ = relative percentage bias in the standard error of the path coefficient between M and Y; Bias in SE of $\beta_{XY}$ = relative percentage bias in the standard error of the path coefficient between X and Y; $r_{pb}$ = point-biserial correlation; $r_{bs}$ = biserial correlation.

Table 4. Simulation Study 2 (partial mediation): Percentages estimation bias in the indirect effect and the coverage percentages of the 95% likelihood-based confidence interval

<table>
<thead>
<tr>
<th>Condition</th>
<th>DICH CO ES</th>
<th>Converged</th>
<th>Bias in indirect</th>
<th>Coverage</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 .1 .16</td>
<td>2,000 2,000</td>
<td>-8.824 -0.461</td>
<td>90.400 94.600</td>
<td></td>
</tr>
<tr>
<td>.23</td>
<td>2,000 2,000</td>
<td>-7.895 -0.558</td>
<td>88.400 92.750</td>
<td></td>
</tr>
<tr>
<td>.33</td>
<td>2,000 2,000</td>
<td>-6.173 -0.344</td>
<td>90.300 92.450</td>
<td></td>
</tr>
<tr>
<td>.5</td>
<td>2,000 2,000</td>
<td>-4.093 -0.088</td>
<td>93.250 93.850</td>
<td></td>
</tr>
<tr>
<td>.23</td>
<td>2,000 2,000</td>
<td>-3.674 -0.195</td>
<td>92.250 93.100</td>
<td></td>
</tr>
<tr>
<td>.33</td>
<td>2,000 2,000</td>
<td>-3.163 -0.493</td>
<td>93.000 93.450</td>
<td></td>
</tr>
</tbody>
</table>

Note. DICH = percentage of primary studies in which X was artificially dichotomized; CO = cut-off point at which X was artificially dichotomized; ES = size of the systematically varied (standardized) path coefficient between X and M; Converged = number of datasets that converged in Stage 1 and Stage 2 of the random-effects TSSEM; Bias in indirect = relative percentage bias in the indirect effect of X on Y ($\beta_{XX} \times \beta_{XY}$); Coverage = percentage of confidence intervals that includes the population parameter of the indirect effect of X on Y; $r_{pb}$ = point-biserial correlation; $r_{bs}$ = biserial correlation.
provided in Table 5/SIM1 in De Jonge et al. (2019b). The rejection rates of the chi-square test were slightly above the nominal α-level (.05) in almost all conditions, no matter if the biserial or point-biserial correlation was used. For both types of correlations, the difference between the rejection rate and the nominal α-level was statistically significant in 5 of the 18 conditions. Based on chance (with α = .05), we would expect a significant difference in one condition. The results of the Kolmogorov-Smirnov test provided in Table 6/SIM1 in De Jonge et al. (2019b) and the QQplots in De Jonge et al. (2019b) show that when the biserial correlation was used, there was a statistically significant difference between the empirical chi-square distribution and the theoretical chi-square distribution with one degree of freedom in 5 of the 18 conditions. When the point-biserial correlation was used, there was a significant difference in the same 5 conditions plus in 3 other conditions. There seems to be no systematic pattern in which conditions the distributions differed significantly or not.

Simulation Study 2 – Partial Mediation

Direct Effects

Table 3 shows that when the point-biserial correlation was used, the relative percentage bias in βMX was between −41.68% and −5.09%. The bias was always negative and exceeded the set boundary of 5% in all conditions. The pattern of the bias across conditions was very similar to the findings in simulation Study 1. The relative percentage bias in βYM was between 1.17% and 15.56%. This bias was positive in all conditions and exceeded 5% in 10 of 18 conditions. When the percentage of primary studies in which the predictor variable X was artificially dichotomized increased, the bias in βYM also increased. The relative percentage bias in βYM was always larger in conditions in which the cut-off proportion was .1 compared to conditions with a cut-off proportion of .5. If the population value of βMX increased the overestimation of βYM also increased. The relative percentage bias in the path coefficient between the predictor X and outcome variable Y (βXY) was between −45.85% and −5.30%. The bias was always negative and exceeded 5% in all conditions. The bias in βXY increases according the same patterns as when the relative percentage bias in βYM increases.

Except from one condition, the relative percentage bias in the standard errors of all path coefficients was less than 10%, representing insubstantial bias. However, note that the bias in the standard error of βYM was always negative. The coverage percentages of the 95% likelihood-based confidence intervals of βMX and βYM were slightly below 95% and of βXY between 92.85% and 95.10% (see Table 3/SIM2 in De Jonge et al., 2019b). The coverage percentages of the 95% Wald confidence intervals were very similar to the results from the likelihood-based confidence intervals (see Table 4/SIM2 in De Jonge et al., 2019b).

When the biserial correlation was used instead, the relative percentage bias in all path coefficients (βMX, βYM, and βXY) was below 5% (see Table 3). This bias could be considered as negligible according to the criteria that were applied. The relative percentage bias in the standard errors of all path coefficients was less than 10%, representing no substantial bias. However, note that the bias in the standard errors of βMX and βYM was always negative. The coverage percentages of the 95% likelihood-based confidence intervals of βMX and βYM were slightly below 95% and of βXY between 92.85% and 95.10% (see Table 3/SIM2 in De Jonge et al., 2019b). The coverage percentages of the 95% Wald confidence intervals were roughly the same as the likelihood-based confidence intervals (see Table 4/SIM2 in De Jonge et al., 2019b).

Indirect Effect

Table 4 shows that the relative percentage bias in the indirect effect of X on Y (βMX × βYM) was between −37.40% and −3.16%, if the point-biserial correlation was used. In 15 of the 18 conditions this bias was above the set boundary of 5%, representing substantial bias. Only in conditions in which the percentage of primary studies which artificially dichotomized variable X was 25% and had a cut-off proportion of .5, the bias was below 5%. The bias in the indirect effect increased when the percentage of primary studies that artificially dichotomize variable X increased. The bias in the indirect effect was always larger in conditions with a cut-off proportion of .1, compared with conditions in which the cut-off proportion was .5. When the population value of βMX increased, the bias in the indirect effect decreased. The coverage percentages of the 95% likelihood-based confidence intervals of this indirect effect were between 5.42% and 93.25%. If the biserial correlation was used instead, the relative percentage bias in the indirect effect of X on Y was below 5% in all conditions (see Table 4). The coverage percentages of the 95% likelihood-based confidence intervals of the indirect effect were slightly below 95%.

Discussion

We performed two Monte Carlo simulation studies in order to advise researchers how to deal with primary studies with artificially dichotomized predictor variables in MASEM. When the point-biserial correlation for the relation between the artificially dichotomized predictor X and the continuous variables was used, the path coefficient between the predictor and mediator (βMX) was systematically underestimated.
Underestimation of the Standard Errors

The relative percentage bias in the standard errors of all path coefficients could typically be considered as insubstantial. For the MASEM-analyses, the sampling variance and covariance of the correlation coefficients in the primary studies are estimated to meta-analyze the correlation matrices. To estimate the sampling variance of the biserial correlation, Jacobs and Viechtbauer (2017) showed that the so-called Soper’s (1914) exact and approximate methods have the best overall performance for practical use in meta-analysis. However, to our knowledge, there are no existing formulas to estimate the sampling covariance between two biserial correlations, between a biserial and point-biserial correlation, and between a biserial and Pearson product-moment correlation. In our study, we therefore used the existing formulas for the sampling (co)variance of Pearson product-moment correlations, as implemented in the metaSEM package, and plugged in the biserial correlation coefficients instead. Our results suggest that using the formulas for Pearson product-moment correlations for the biserial correlation has, under the investigated conditions, no serious consequences in meta-analytic practice, since the relative percentages bias in the point estimate and standard errors of the direct effects could be considered negligible according to the criteria that were applied.

However, we noticed that the relative percentage bias in the standard error of the path coefficient between the predictor and mediator ($\beta_{YM}$) seems systematically negatively biased when the biserial correlation was used. In accordance, the coverage percentages of the 95% likelihood-based and Wald confidence intervals were always slightly underestimated. This is in accordance with the study of Jacobs and Viechtbauer (2017), who showed that if the biserial correlation is plugged in into the formula for the sampling variance of the Pearson product-moment correlation, this generally leads to an underestimation of the true sampling variance. In contrast, when the point-biserial correlation was used in our simulation studies, the relative percentage bias in the standard error of $\beta_{YM}$ seems not systematically negative. Additional simulations, in which we did not dichotomize the predictor variable $X$ at all so we could use Pearson product-moment correlations, also showed that the bias in the standard error of $\beta_{MX}$ seems also not systematically negative (results see Table 1/SIM1 and Table 1/SIM2 in De Jonge et al., 2019b). Future research is needed to further investigate this issue and to develop formulas to calculate the sampling covariance between two biserial correlations, between a biserial and point-biserial correlation, and between a biserial and Pearson product-moment correlation.

Possibly, using an “incorrect” formula for the sampling (co)variances is not the only possible explanation of the negative bias in standard errors with the biserial correlation. By inspecting our results, we found that the relative percentage bias in the standard error of the path coefficient between the continuous variables $M$ and $Y$ ($\beta_{YM}$) seems also systematically negative regardless whether the point-biserial or biserial correlation was used. In accordance, the coverage percentages of the 95% likelihood-based as well as the Wald confidence intervals were always underestimated.

When the data were not dichotomized at all and the Pearson product-moment correlation was used, the relative percentage bias in the standard error of $\beta_{YM}$ seems also systematically negative (results see Table 1/SIM1 and Table 1/SIM2 in De Jonge et al., 2019b) and accordingly the coverage percentages were also always slightly below 95% (results see Tables 3–4/SIM1 and Tables 3–4/SIM2 in De Jonge et al., 2019b). We found the same patterns in the relative percentage bias in the standard errors of the pooled correlation coefficients between the continuous variables $M$ and $Y$ in Stage 1 (results see Table 8/SIM1 and Table 6/SIM2 in De Jonge et al., 2019b).

One tentative cause of the underestimated standard errors and confidence intervals could be that the sampling (co)variances from the primary studies are treated as known in MASEM (similar to V-known models in univariate meta-analysis), while they are actually estimated. A similar underestimation in standard errors is found in univariate random-effects meta-analysis as a consequence of not taking into account the uncertainty due to estimating the between-study and sampling variance (Sánchez-Meca & Marín-Martínez, 2008; Viechtbauer, 2005). Note, however, that the bias that we found was within the limit of 10% and often even below 5% in all conditions. Future research...
would be needed to verify the robustness of our results and to further investigate this issue.

Model Fit

In most conditions, the rejection rate of the chi-square test of model fit at Stage 2 of the random-effects TSSEM was slightly above the nominal α-level, no matter if the point-biserial or biserial correlation was used. When the predictor was not dichotomized at all and the Pearson product-moment correlation was used, the rejection rate was also slightly above the nominal α-level. When the predictor was artificially dichotomized, the rejection rate was slightly higher than .05 (results see Table 5/SIM1 in De Jonge et al., 2019b). This finding is in accordance with simulation studies about fixed-effects TSSEM (Jak & Cheung, 2018a; Oort & Jak, 2016). Further research is needed to investigate why the chi-square test in TSSEM may provide rejection rates slightly above the nominal α-level.

Strengths, Limitations, and Recommendations

This is the first simulation study in which the effect of using the point-biserial correlation versus the biserial correlation for the relation between an artificially dichotomized variable and continuous variable on MASEM-parameters and model fit is investigated. We chose two realistic population models and 18 different realistic conditions which can occur in educational research. This provides a clear first impression of the effect of using the point-biserial correlation versus the biserial correlation in mediation models using MASEM. However, further research is needed to investigate the effect in other research settings in which MASEM will be applied, for example, investigating the effect in more complex models, which include more than three variables or moderation effects.

Future research is also needed with regard to further assumptions of the models. Maximum likelihood, for instance, assumes that the population distributions of the endogenous variables are multivariate normally distributed (Kline, 2015). In these simulation studies, we artificially dichotomized the exogenous variable (i.e., predictor variable X), which does not lead to a violation of this assumption. Therefore, further research is needed to investigate the effect of using the point-biserial versus the biserial correlation if an endogenous variable is artificially dichotomized, so that the normality assumption is violated.

Conclusion

We advise researchers who want to apply MASEM and want to investigate mediation to convert the effect size between any artificially dichotomized predictor and continuous variable to a biserial correlation, not to a point-biserial correlation.

References


History

Received May 14, 2019
Revision received October 13, 2019
Accepted November 16, 2019
Published online March 31, 2020

Open Data

R scripts for simulation study 1 (full mediation) and simulation study 2 (partial mediation) are available in De Jonge, H., Jak, S., & Kan, K. J. (2019a). Tables with additional results of both simulation studies as well as QQplots of simulation study 1 are available in De Jonge, H., Jak, S., & Kan, K. J. (2019b).

Hannelies de Jonge

Department of Child Development and Education

University of Amsterdam

PO Box 15776

1001NG Amsterdam

The Netherlands

H.deJonge@uva.nl

H. de Jonge et al., Artificially Dichotomized Variables in MASEM

Zeitschrift für Psychologie (2020), 228(1), 25–35

© 2020 Hogrefe Publishing