Supplementary Material

In this appendix we provide details on the complete set of longitudinal dispersion relations for non-conformal charged isotropic crystals, as well as the holographic renormalisation procedure employed in the letter.

Longitudinal modes.—In the main text, we presented longitudinal modes in case of a conformal charged viscoelastic fluid for simplicity. Here we provide the generic non-conformal expressions. Let us first change the thermodynamic variables from (T,μ) to (ϵ_f, q_f) via

$$\frac{\partial T}{\partial \epsilon_f} = \frac{1}{T\Xi}\frac{\partial q_f}{\partial \mu}, \qquad \frac{\partial T}{\partial q_f} = -\frac{1}{T\Xi}\frac{\partial \epsilon_f}{\partial \mu},$$
$$\frac{\partial \mu}{\partial \epsilon_f} = -\frac{1}{T\Xi}\frac{\partial q_f}{\partial T}, \qquad \frac{\partial \mu}{\partial q_f} = \frac{1}{T\Xi}\frac{\partial \epsilon_f}{\partial T}, \tag{A.1}$$

where $\Xi = \frac{\partial s_f}{\partial T}\frac{\partial q_f}{\partial \mu} - \frac{\partial s_f}{\partial \mu}\frac{\partial q_f}{\partial T}$. The longitudinal sound velocity is given as

$$v_\parallel^2 = \frac{(w_f+w_\ell)\frac{\partial P_m}{\partial \epsilon_f} + (q_f+q_\ell)\frac{\partial P_m}{\partial q_f}}{\chi_{\pi\pi}} + \frac{B + 2\frac{d-1}{d}G - P_\ell}{\chi_{\pi\pi}}, \tag{A.2}$$

whereas the attenuation is

$$\Gamma_\parallel = \frac{\left(\frac{\partial P_m}{\partial q_f}\right)^2 \sigma_q + \frac{1}{\sigma_\phi}\left(\mathcal{F}_1 + \frac{\partial P_m}{\partial q_f}\gamma\right)\left(\mathcal{F}_1 - \frac{\partial P_m}{\partial q_f}\gamma'\right)}{v_\parallel^2 \chi_{\pi\pi}} + \frac{\zeta + 2\frac{d-1}{d}\eta}{\chi_{\pi\pi}}. \tag{A.3}$$

Here $P_m = P_f + P_\ell$ is the mechanical pressure. For clarity of presentation, we have defined

$$\mathcal{F}_1 = w_f\left(v_\parallel^2 - \frac{\partial P_m}{\partial \epsilon_f}\right) - q_f\frac{\partial P_m}{\partial q_f}, \quad \mathcal{F}_2 = T^2\frac{\partial(\mu/T)}{\partial q_f}. \tag{A.4}$$

The quadratic governing the diffusion modes, on the other hand, is given by

$$\frac{D_\parallel}{T\mathcal{F}_2}\left(D_\parallel - \frac{w_f^2}{\sigma_\phi}\frac{2\frac{d-1}{d}G + B - P_\ell}{\chi_{\pi\pi}v_\parallel^2\Xi T\mathcal{F}_2}\right) - \frac{\sigma_q}{T^2}\left[\left(\frac{(w_f+w_\ell)^2}{\chi_{\pi\pi}\Xi T\mathcal{F}_2} + \frac{2\frac{d-1}{d}G + B - P_\ell}{\chi_{\pi\pi}}\right)\frac{D_\parallel}{v_\parallel^2} - \frac{w_f^2}{\sigma_\phi}\frac{2\frac{d-1}{d}G + B - P_\ell}{\chi_{\pi\pi}v_\parallel^2\Xi T\mathcal{F}_2}\right]$$
$$= \frac{D_\parallel}{\sigma_\phi v_\parallel^2}\left[\frac{(w_f+w_\ell)^2}{\chi_{\pi\pi}T\Xi\mathcal{F}_2}\left(\frac{s_f q_\ell - q_f s_\ell}{w_f+w_\ell} + \frac{\gamma}{T}\right)\left(\frac{s_f q_\ell - q_f s_\ell}{w_f+w_\ell} - \frac{\gamma'}{T}\right) + \frac{B + 2\frac{d-1}{d}G - P_\ell}{\chi_{\pi\pi}}\left(\frac{\partial P_f/\partial q_f}{\mathcal{F}_2} - \frac{\gamma}{T}\right)\left(\frac{\partial P_f/\partial q_f}{\mathcal{F}_2} + \frac{\gamma'}{T}\right)\right]. \tag{A.5}$$

Holographic renormalisation.—In order for the action (18) to be finite on the class of black brane solutions in (19), we need an additional Gibbons-Hawking-York counter-term at the boundary along with a boundary potential for the scalars. To wit,

$$S_{\text{counter}} = \int_{r=r_c} \mathrm{d}^3x\sqrt{-\gamma}\left(K - 2 + \bar{V}(\bar{X})\right), \tag{A.6}$$

where we have defined the induced metric $\gamma_{\mu\nu}$ at the location of the cutoff surface $r = r_c$. We have assumed the boundary to be flat and avoided any curvature dependent terms. Here $\bar{X} = \delta_{IJ}\frac{1}{2}\gamma^{\mu\nu}\partial_\mu\Phi^I\partial_\nu\Phi^J$ and $K = G^{ab}\mathrm{D}_a n_b$ is the mean extrinsic curvature, where n_a is an outward pointing normal vector to the surface and D_a the covariant derivative compatible with the bulk metric G_{ab}. For the onshell action not to have any divergences, the potential must take the form

$$\bar{V}(\bar{X}) = 2\left(1 - \sqrt{1 - U(\bar{X})}\right) - \sum_n \frac{\mathcal{M}_n}{r_c^3}(r_c^2\bar{X})^n. \tag{A.7}$$

In (A.7) we have introduced $\mathcal{M}_n$, which are arbitrary cutoff dependent renormalisation scale parameters taken to be regular in the limit $r_c \to \infty$. Their presence spoils the conformal symmetry of the holographic model by imposing non-conformal boundary conditions. Different values of $\mathcal{M}_n$ describe different physical theories, or different points in the RG flow of the same physical theory. In the letter, we have only turned on $\mathcal{M} = \mathcal{M}_1$ for simplicity.

Assuming the bulk potential falling as $V(X) \sim X \sim r^{-2}$ near the boundary, we can read out the boundary value of the fields

$$g_{\mu\nu} = \lim_{r_c\to\infty}\frac{1}{r_c^2}\gamma_{\mu\nu}, \quad A_\mu = \lim_{r_c\to\infty}\mathcal{A}_\mu, \quad \phi^I = \lim_{r_c\to\infty}\Phi^I. \tag{A.8}$$

For potentials falling off more quickly, the qualitative behaviour of the scalars is different; see [41] for more details. Fields in eq. (A.8) serve as sources for the respective operators obtained by varying the total onshell action

$$\delta S^{\text{onshell}}_{\text{bulk+counter}} = \int_{r=r_c} \mathrm{d}^3x\sqrt{-g}\left(\frac{1}{2}T^{\mu\nu}\delta g_{\mu\nu} + J^{\mu}\delta A_{\mu} - \Pi_I\delta\phi^I\right), \quad \text{(A.9)}$$

where

$$T^{\mu\nu} = \lim_{r_c\to\infty} r_c^5\bigg(K\gamma^{\mu\nu} - K^{\mu\nu} - 2\gamma^{\mu\nu} + \bar{V}(\bar{X})\gamma^{\mu\nu} - \delta_{IJ}\bar{V}'(\bar{X})\gamma^{\mu\rho}\gamma^{\nu\sigma}\partial_\rho\Phi^I\partial_\sigma\Phi^J\bigg),$$

$$J^{\mu} = \lim_{r_c\to\infty}\frac{r_c^3}{2}\mathcal{F}^{ua}n_a,$$

$$\Pi_I = \lim_{r_c\to\infty} r_c^3\delta_{IJ}\bigg(V'(X)n^a\partial_a\Phi^J + \frac{1}{\sqrt{-\gamma}}\partial_\mu\left(\sqrt{-\gamma}\,\gamma^{\mu\nu}\bar{V}'(\bar{X})\partial_\nu\Phi^J\right)\bigg). \quad \text{(A.10)}$$

We have arrived at the holographic formula for $T^{\mu\nu}$ and J^μ. In this picture, however, the fields ϕ^I serve as sources leading to explicit breaking of translations. In order to describe spontaneous symmetry breaking, we deform the theory with a surface action $S_{\text{alternative}} = \int \mathrm{d}^3x\sqrt{-g}\,\Pi_I\phi^I$ implementing alternative quantisation and switching the roles of Π_I and ϕ^I. In this picture, $K_I^{\text{ext}} \equiv \Pi_I$ are the background sources coupled to the dynamical fields ϕ^I. Finally, the boundary conditions imposed on our holographic model for spontaneously broken translations are $g_{\mu\nu} = \eta_{\mu\nu}$, $A_\mu = \mu\delta^t_\mu$, and $\Pi_I = 0$.