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Elementary excitations of atomic hydrogen gas on liquid helium

M.W. Reynolds^a, G.V. Shlyapnikov^{a,b}^a *Van der Waals–Zeeman Institute, University of Amsterdam, Valckenierstraat 65–67, 1018 XE Amsterdam, The Netherlands*^b *Russian Research Center Kurchatov Institute, Kurchatov Square, 123182 Moscow, Russian Federation*

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Abstract

We discuss the nature of the elementary excitations for a gas of spin-polarized atomic hydrogen adsorbed on the free surface of liquid helium. Below the Kosterlitz–Thouless transition temperature the coupling between above-condensate hydrogen atoms and ripplons through the hydrogen quasicondensate results in the elementary excitations being superpositions of hydrogen quasiparticles (single atom at large momenta and phonon at small momenta) and ripplons. For realistic values of the condensate density this hybridization is appreciable.

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Achieving the Kosterlitz–Thouless (KT) transition and studying the phenomenon of two-dimensional superfluidity in the Bose gas of spin-polarized atomic hydrogen ($H\downarrow$) adsorbed on the surface of liquid helium is an important goal in atomic hydrogen research (see Ref. [1] and references therein). So far, however, the temperatures and densities achieved in experiment are still outside the region of superfluidity. The main problems are connected with the instability and heating of the system due to three-body dipole recombination of $H\downarrow$ atoms [2,3]. These problems are hoped to be circumvented by using sophisticated techniques, such as creating a “cold spot” [4] or “magnetic spot” [5,6] with a much higher $H\downarrow$ density than its surroundings, due to a much lower temperature or a significantly larger magnetic field in this spot. But however the KT transition is realized, the nature of the elementary excitations in adsorbed $H\downarrow$ should be of major importance.

In this paper we show that, below the KT transition

temperature, the existence of a quasicondensate (condensate with fluctuating phase) in adsorbed $H\downarrow$ makes the leading interaction between the excitations of $H\downarrow$ (quasiparticles) and the excitations of the liquid helium surface (ripplons) bilinear in quasiparticle and ripplon operators. This means that the elementary excitations are not pure quasiparticles and ripplons, but superpositions of these excitations. We derive the new elementary excitations and their dispersion relations and address several potentially serious questions.

Locally the quasicondensate has the properties of a true condensate (see Ref. [7]). One can divide the system into blocks of a size much larger than the ordinary correlation length, and there will be a true condensate within each block. Only the phases in far-separated blocks are not correlated, which prevents us from introducing a unified condensate wavefunction. However, considering low temperatures, where the phase correlation length is very much longer than the microscopic length scales of interest, we may ne-

glect the phase fluctuations and proceed as if a true condensate is present. The two-dimensional degenerate gas of interacting hydrogen atoms is represented by the usual Bogoliubov Hamiltonian $H_Q = \sum_q \varepsilon_{aq} a_q^\dagger a_q$ of a gas of noninteracting quasiparticles. Here a_q is the annihilation operator for a quasiparticle with momentum $\hbar q$ in the plane of the surface. The energy $\varepsilon_{aq} = (E_q^2 + 2n_0 \tilde{U} E_q)^{1/2}$, where $E_q = \hbar^2 q^2 / 2m$, m is the hydrogen atom mass, $\tilde{U}$ is the effective vertex of elastic pair interaction between adsorbed hydrogen atoms and n_0 is the two-dimensional condensate number density (H atoms per unit area). Considering n_0 in the range $(1-5) \times 10^{13} \text{ cm}^{-2}$, we take $\tilde{U} = 100 \text{ K } \text{\AA}^2$ [1].

The above-condensate atom operators for nonzero momentum are expressed in terms of hydrogen quasiparticle operators as

$$c_q = \cosh x_q a_q - \sinh x_q a_{-q}^\dagger, \quad (1)$$

where $\tanh x_q = (\varepsilon_{aq} - E_q)^2 / 2n_0 \tilde{U} E_q$ decreases from unity for $E_q \ll 2n_0 \tilde{U}$ (phonon-like excitations) to zero as $n_0 \tilde{U} / 2E_q$ for $E_q \gg 2n_0 \tilde{U}$ (particle-like excitations).

The elementary excitations of the free surface of liquid helium are ripples, or quantized capillary waves. It is a good approximation here to consider the liquid as incompressible, with density ρ_0 and surface tension σ . The ripplon Hamiltonian is $H_R = \sum_q \varepsilon_{bq} b_q^\dagger b_q$, where b_q is the annihilation operator for a ripplon with momentum $\hbar q$. The ripplon energy $\varepsilon_{bq} = \hbar \omega_q$, with $\omega_q = (\sigma / \rho_0)^{1/2} q^{3/2}$ (we neglect gravity and the influence of the substrate). Ripples cause a vertical displacement of the liquid surface given by the operator

$$\zeta(\mathbf{r}) = S^{-1/2} \sum_q \xi_q e^{iq \cdot \mathbf{r}} (b_{-q}^\dagger + b_q), \quad (2)$$

where S is the area of the surface and $\xi_q = (\hbar q / 2\rho_0 \omega_q)^{1/2}$.

The most important interaction between adsorbed H atoms and ripples involves the processes of ripplon emission and absorption by the H atoms ($\text{H} \rightleftharpoons \text{H} + \text{R}$) [8,9]. In second quantization the interaction Hamiltonian

$$H_{\text{int}} = S^{-1/2} \sum_{p,q} F_q \xi_q c_{p+q}^\dagger c_p (b_{-q}^\dagger + b_q), \quad (3)$$

where, again, c_q and b_q are annihilation operators of atoms and ripples, respectively. We calculate F_q using the analytical model of Refs. [8,9]. For $q < 0.2 \text{ \AA}^{-1}$, we find that $F_q \simeq -Cq^2 - Dq^4$ with $C = -7.8 \text{ K } \text{\AA}$ and $D = 39 \text{ K } \text{\AA}^3$. The expansion of F_q in a power-series expansion in q^2 admits a simple physical interpretation. In particular, the coefficient C describes the dependence of the atom energy on the curvature of the surface: the energy of the atom is shifted by an amount $C \nabla^2 \zeta(\mathbf{r})$ from its energy on a flat surface. Our value for C is in essential agreement with Ref. [9] except for the sign.

Assuming that the number of H atoms in the quasicondensate, N_0 , is much larger than the number of above-condensate particles (which is the case well below the KT transition temperature), we discard all terms in H_{int} excepting those involving $c_0^\dagger$ or c_0 , and replace these operators by $\sqrt{N_0}$. Then, representing the above-condensate atom operators c_q in H_{int} in terms of operators a_q for hydrogen quasiparticles using (1), we arrive at the total Hamiltonian $H = H_Q + H_R + H_{\text{int}}$, bilinear in the operators of hydrogen quasiparticles and ripples,

$$H = \sum_q \{ \varepsilon_{aq} a_q^\dagger a_q + \varepsilon_{bq} b_q^\dagger b_q + A_q (a_q^\dagger b_{-q}^\dagger + a_q^\dagger b_q + a_{-q} b_{-q}^\dagger + a_{-q} b_q) \}, \quad (4)$$

with $A_q = F_q \xi_q n_0^{1/2} \exp(-x_q)$. The quantity

$$\exp(-x_q) \simeq (E_q / 2n_0 \tilde{U})^{1/4}$$

for $E_q \ll 2n_0 \tilde{U}$ and tends to unity for large E_q .

The third term in (4) couples the hydrogen quasiparticle and ripplon fields. The physical origin of this coupling is the character of the quasiparticles as fluctuations in the $\text{H}_\downarrow$ density. The existence of such a coupling leads to a modification of the nature and dispersion relations of the elementary excitations. To determine the new elementary excitations the Hamiltonian (4) is reduced to the diagonal form

$$H = \sum_q E_{\alpha q} \alpha_q^\dagger \alpha_q + E_{\beta q} \beta_q^\dagger \beta_q \quad (5)$$

by a straightforward transformation. The new excitations are defined by their annihilation operators α_q and β_q , which are superpositions of hydrogen quasiparticle and ripplon operators,

Fig. 1. Energy of the combined elementary excitations as a function of q for $n_0 = 2 \times 10^{13} \text{ cm}^{-2}$. Dashed curves correspond to pure hydrogen quasiparticles (linear at small q) and pure ripples. Also shown (dotted) is the free atom dispersion relation E_q .

$$\alpha_q = d_{\alpha a} a_q + d_{\alpha b} b_q + \tilde{d}_{\alpha a} a_{-q}^\dagger + \tilde{d}_{\alpha b} b_{-q}^\dagger, \quad (6a)$$

$$\beta_q = d_{\beta a} a_q + d_{\beta b} b_q + \tilde{d}_{\beta a} a_{-q}^\dagger + \tilde{d}_{\beta b} b_{-q}^\dagger. \quad (6b)$$

The linear combination coefficients d are real functions of q .

The commutation relation $[\gamma_q, H] = E_{\gamma q} \gamma_q$, where γ denotes annihilation operator α or β , gives a quadratic equation for $E_{\gamma q}^2$. The energies of the new elementary excitations are given by (α is the upper branch, β the lower)

$$E_{\gamma q}^2 = \frac{1}{2} (\varepsilon_{aq}^2 + \varepsilon_{bq}^2) \pm \frac{1}{2} [(\varepsilon_{aq}^2 - \varepsilon_{bq}^2)^2 + 16 A_q^2 \varepsilon_{aq} \varepsilon_{bq}]^{1/2}. \quad (7)$$

The density $n_0 = 2 \times 10^{13} \text{ cm}^{-2}$ leads to an essential degeneracy of the H quasiparticle and ripplon dispersion relations and maximizes the hybridization. The dispersion relations (7) for this density are shown in Fig. 1. Clearly, the hybridization is strong over an appreciable range of momenta.

The surface would be unstable if for some q we would have $E_{\beta q}^2 < 0$, which requires $2A_q > (\varepsilon_{aq} \varepsilon_{bq})^{1/2}$. This does not occur for reasonable values of parameters describing atomic hydrogen on liquid helium. The H-ripplon interaction is too weak compared to the surface tension of liquid helium and the (repulsive) self-interaction of the hydrogen gas. To realize the instability at $n_0 = 10^{13} \text{ cm}^{-2}$, for example, F_q would have to be increased by a factor of ten.

Fig. 2. Coefficients d expressing the new excitations (α and β) in terms of hydrogen quasiparticles (a , solid) and ripples (b , dashed), plotted as a function of q . The density of the quasicondensate $n_0 = 2 \times 10^{13} \text{ cm}^{-2}$.

The coefficients d are normalized to $d_{\gamma a}^2 + d_{\gamma b}^2 - \tilde{d}_{\gamma a}^2 - \tilde{d}_{\gamma b}^2 = 1$ in order to preserve the commutation relation $[\gamma_q, \gamma_q^\dagger] = 1$ between the creation and annihilation operators. Fig. 2 shows the coefficients for $n_0 = 2 \times 10^{13} \text{ cm}^{-2}$. For the chosen parameters, the reverse-momentum coefficients $\tilde{d}$ are always small.

At small q the diagonalization of the quadratic Hamiltonian (4) solves the classical problem of a two-dimensional compressible gas film on the surface of an incompressible liquid. The interaction between the liquid and the film is such that the gas would prefer to lie in the troughs of waves (since $C < 0$), with an interaction energy proportional to the product of the gas density and the curvature of the liquid surface. As a result of the interaction, instead of capillary waves of the liquid and sound waves in the gas film, the two normal modes of the system will correspond to the coupled oscillation of the liquid surface and the gas density. These are the new “elementary excitations” which we found in our second quantization treatment.

The terms discarded in the derivation of Hamiltonian (4) correspond to ripplon absorption or emission in which the hydrogen atom remains an above-condensate particle. Retention of these terms would lead to the decay of the combined elementary excitations, but the width of the dispersion curves will be much smaller than their separation provided most atoms are in the quasicondensate.

Despite the fact that the hybridization is apprecia-

ble, we find by explicit calculation that it does not change the superfluid density in adsorbed $H\downarrow$ by more than several percents. The influence of hybridization on both the sticking coefficient for H incident on the surface and on the energy transfer between the surface excitations and the bulk phonons of liquid helium is only slightly larger. This means that the kinetic and thermodynamic properties of $H\downarrow$ below the KT transition temperature can be described in terms of the usual picture of bare ripplons and $H\downarrow$ quasiparticles.

The hybridization will manifest itself most clearly in momentum or energy resolved experiments. For example, consider probing the system by coupling to the hydrogen. The system will respond to a probe of given momentum at the two distinct energies given by the α and β dispersion relations. The strength of the response will depend on the projection of the probed branch of excitations onto the bare hydrogen state. Observation of the two branches of excitations may provide a tool to study the superfluid phase of adsorbed $H\downarrow$.

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