Road traffic network design and control using a multi-class diffusion model

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1. Motivation

Over the past decades a wide variety of traffic flow models has been proposed. Their primary relevance lies in the evaluation of the impact of control measures on key performance metrics, enabling the design and control of the system in such a way that the realized performance is optimized.

The majority of the existing traffic flow models utilizes physical laws to describe the dynamics of vehicles, and is therefore of a deterministic nature. The pinnacle example is the class of kinematic wave models that combines the principle of conservation laws with the fundamental diagram, i.e. the functional relation between vehicle density and flow, to describe traffic flows through a partial differential equation; examples are Richards (1956), Lighthill and Whitham (1955), Drake, Schofer, and May (1967), Smulders (1990), Daganzo (1995). However, as argued in in e.g. Qu, Zhang, and Wang (2017), besides physical laws, traffic flows are also strongly affected by a wide range of microscopic variables, such as the different perceptions, moods, responses, and driving habits of individual car drivers. As such, there is a consensus in the literature (Qu, Zhang, and Wang 2017, Section 1) that these microscopic variables should be modeled as random variables. It means that, due to the inherent stochastic nature of road traffic, the efficacy and impact of control measures can only be correctly assessed using *stochastic* traffic flow models. They enable the evaluation of the probability distribution of performance metrics (such as travel times), rather than just providing the mean. The resulting framework facilitates the development of guidelines for network design as well as performance optimizing control mechanisms.

Setting up a suitable probabilistic model has not proved to be an easy feat though. It should be consistent with the physical laws incorporated in deterministic models, but at the same time it should remain sufficiently tractable. On a macroscopic scale, attempts to strike this balance have been made, but, as pointed out in Jabari and Liu (2012), most approaches lead to undesirable features, such as inconsistencies with deterministic models and the possibility of negative sample paths. On a microscopic scale, one can explicitly model stochastic features of individual vehicles (cf. Maerivoet and De Moor (2005), Nagel and Schreckenberg (1992), van Wageningen-Kessels et al. (2015)), but in these models the computation of the vehicle density distribution (and other metrics) is typically challenging for instances of a realistic size.

The discrete-space stochastic model proposed in Jabari and Liu (2012, 2013), is referred to by Qu, Zhang, and Wang (2017) as one of the rare examples of a probabilistic model that allows explicit analysis (under a specific parameter scaling) while being consistent with kinematic wave models. It however lacks flexibility in that it only considers single-class macroscopic fundamental diagrams, while in the recent literature (Wong and Wong (2002), Chanut and Buisson (2003), Logghe and Immers (2008)) the importance of a multi-class setup has been explicitly stressed. Another limitation is that Jabari and Liu (2012, 2013) consider isolated road segments rather than more general networks.

We develop a stochastic traffic flow model that remedies the issues mentioned above. It has the following properties: (i) it is flexible enough to incorporate physical phenomena, inherent to vehicle dynamics, (ii) provides an explicit analysis under an appropriate scaling, and (iii) covers multiple vehicle classes and general network structures. The developed model allows us to assess the efficacy of various traffic control measures, thus enabling us to optimize the performance. In addition, it can play a crucial role in assessing possible network designs.

2. Approach

Our approach aligns, to some extent, with the one followed in Jabari and Liu (2012, 2013): after a natural scaling of time and space, the vehicle densities in individual cells, at a given point in time, are shown to be accurately described by means of a multivariate normal random vector. There are, however, a few crucial differences. In the first place, unlike in the single-class model of Jabari and Liu (2012, 2013), we focus on a setup capable of handling *multi-class* macroscopic fundamental diagrams. Secondly, we extend the model from a road segment setting to a network of roads of arbitrary size. As such, we have constructed a stochastic traffic flow model that satisfies the desirable properties discussed above: flexibility, consistency with physical laws, and being analytically tractable. In our framework the vehicle density process is approximated by a Gaussian process of the form of Eqn. (6.1) in Section 5.6 of Karatzas and Shreve (2012). In particular, this leads to a Gaussian approximation of the vehicle density distribution, both spatially and temporally, with explicit expressions for the means and (co)-variances that can be computed in a numerically efficient manner.

Besides remedying modelling omissions, our approach also allows for an approximation of the vehicles' travel-time distribution, between any origin-destination pair in the network. This quantity, not covered in Jabari and Liu (2012, 2013), is an important metric when evaluating the performance achieved by control measures.

3. Results

We formulate a stochastic traffic flow model that covers both state-of-the-art multi-class macroscopic fundamental diagrams, satisfying mild regularity conditions, and general networks structures. The model is embedded in the rigorous framework of *spatial population processes* developed in Kurtz (1981), which facilitates the establishment, under a natural scaling, of *fluid* and *diffusion limits.* The fluid limit describes the model's mean behavior, which is consistent with kinematic wave models, and thus aligns with the physical laws deterministic models built upon. The diffusion limit can be used to approximate the joint vehicle density distribution, both in the spatial as in the temporal sense, by a Gaussian distribution. Moreover, a similar analysis leads to approximations of the distribution of the travel time experienced by vehicles moving through the network, which allows for numerically efficient computation.

Our results are then used to numerically assess various design issues, as well as the impact of traffic control mechanisms. As an example, one wonders how to (potentially dynamically) adapt the maximum speed to prevent or mitigate delays due to traffic jams. Changing the velocity at which vehicles are allowed to drive corresponds to an adaptation of the fundamental diagram. As such, we can analyze the effect of changing the maximum speed on the vehicle density and travel-time distributions. It facilitates the computation of an optimal control policy.

In the second place, we can compare the travel-time distributions along multiple routes. While conventionally such decisions are solely based on the *mean* travel time pertaining to the routes, we can now generate route selection advice based on *distributional* predictions. Concretely, a driver may want to select the route that minimizes the mean travel time increased by some positive constant c times the standard deviation of the travel time, where c is an individually chosen parameter reflecting the driver's risk aversion. We present examples which show how the selected route is affected by the value of c.

A third application concerns the impact of changes in the network layout. For instance, when adding lanes or an extra connection between two nodes in the network, one wishes to quantify its effect on e.g. the vehicle density distribution and the travel-time distribution. We demonstrate how the underlying computations can be performed. Our examples include an instance exhibiting the well-known *Braess paradox.*

Fourthly, we compare how changing the rate of arrival for one class of vehicles, has impact on the performance of the other vehicle classes. For example, we analyze how the vehicle classes' travel-time distributions change when reducing or raising the number of trucks that use a certain road segment during a given period of time. This type of analysis can be used in optimal decision making for controlled-access highways with *ramp metering.*

4. Conclusions

Based on our flexible and generally applicable stochastic traffic flow model, we have developed an accurate and efficient device to evaluate the distributions of multiple-type vehicle densities and travel times. We have showcased the usefulness of our framework in the context of traffic control and network design.

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