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The Pension Composition, a Behavioral Approach

Bernard M.S. van Praag and J. Peter Hop

Abstract

Pensions may be provided for in a modern society by a mix of several methods, namely by voluntary individual savings, mandatory fully-funded occupational pension systems, mandatory social security financed by pay-as-you-go, and old-fashioned hoarding in cash. We call a specific mixture of the four systems a *pension composition*. We assume that individual workers decide on their own individual savings, that the fully-funded occupational system is decided upon by the age cohort of the median worker (MW), and that social security is decided upon by the median voter (MV). We assume that individual and collective pension savings are the only sources of capital supply. When capital supply equals demand from industry there is equilibrium in the capital market with a corresponding equilibrium interest rate and pension composition. In this paper we assume a demography with one hundred age brackets and we investigate how changes in the birth rates, survival rates, and the retirement age affect the pension composition and the capital market equilibrium. Our conclusion is that for a given technology the pension composition and the interest rate are determined by the demography and cannot be modified at will as a long-term political instrument.

Keywords: demography, funded pensions, unfunded pensions, social security, interest rate, overlapping generations, individual savings.

JEL codes: H55, H75, J1, J26

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1. Introduction

Nowadays there is much anxiety over the question of whether the present systems by which old age support is given can be maintained in the future, given the fact that populations are aging. In many countries, such as the USA, France, the United Kingdom, Germany, Spain, Italy, and The Netherlands, there are lively political debates, the majority of which are leading to proposals either to increase occupational mandatory pensions premiums and/or private savings – and as a consequence reduce the mandatory public pension system run on a pay-as-you-go basis – or, inversely, to increase the PAYG-part and as a consequence reduce savings. A closer look at pensions reveals that there are many simultaneous arrangements in order to support the retired. We mostly have a combination of funded mandatory occupational pensions, social security, and private pension insurance, with perhaps some additional cash hoarding. This mix we call the *pension composition* for short. The objective of this paper is to build a general model by which the existence of several old-age provision schedules at the same time, that is the pension composition, can be explained. We assume a stable demography, that is, a demography with a constant growth rate and a constant age distribution over time. We assume the pension composition is the equilibrium outcome of a rather complex system with many players. The players are parliament, represented by the Median Voter, as far as social security is concerned, the trade union,[1] represented by the Median Worker, that decides on the existence and the level of mandatory occupational funded pensions, and the individual households as far as it concerns their voluntary private savings. We do not assume the existence of a social planner who determines the pension system with the intention of optimizing a social welfare function. Rather, we assume a Pareto-type equilibrium where the Median Voter, Median Worker, and the individual private savers try to optimize their decisions simultaneously. The resulting equilibrium is a behavioral equilibrium, bearing the character of a compromise between the age cohorts. We notice that workers simultaneously belong to the group of individual savers, the group of workers, and the group of voters. Thus, the interests of the three groups are not identical but partly coincide.

We show for a number of different demographic parameter sets that there is an equilibrium pension composition and that this composition depends on the demography. The model can be easily transformed into a dynamic version by means of which we could calculate transition paths when the demographic parameters, e.g., birth and survival rates change over time. In this paper we do not look at transition paths. Furthermore, we do not have any intention of mimicking a specific country with a specific demography.

[1] In some countries the employers are also formally involved in the decisions on pensions, but since employers are primarily interested in total wage costs, the division of those costs between present net wage and future pension income is irrelevant for the employers and is left to labor representatives.

Rather, we present a general model where the specific parameters of a country can be filled in. We notice that in reality no country has a purely stable demography. Under *ceteris paribus* (c.p.) conditions the system tends to an equilibrium, i.e., a *pension composition*. We assume a closed economy. Hence, national capital supply is assumed to equal the sum of individual savings and the reserves of the pension funds. Equating the capital supply with the capital demand from firms we find an equilibrium interest rate in the capital market and this closes the model. In a few cases we find that the equating interest rate would be negative. In those cases a corner solution is preferred where the interest rate becomes zero and some savings are hoarded in cash.
A specific part of those c.p. conditions is the demography of the country. When the demography changes the pension composition will change as well, and this is what we observe in reality.
Obviously, our model is a simplification of reality in several aspects. The main simplifications and assumptions are:
First, we consider the final solution of a dynamic model. As already said, in reality we are never in such an equilibrium situation. The demographic parameters vary so quickly that the situation of a stable age distribution with a constant population growth rate is nearly never reached in practice. Typically, from an arbitrary position a demographic process needs several hundred years to converge to the demographic equilibrium. However, knowledge of the latent equilibrium situation remains relevant, as the real behavior of the system may be assumed to *tend* to the equilibrium in the long run.
Second, we make the usual assumption that the population is homogeneous. All individuals have the same utility function and the same labor productivity.
Third, we assume that there are no random disturbances, e.g., no random fluctuations of the interest rate.
Fourth, we do not distinguish a specific class of entrepreneurs and their savings behavior.
Fifth, we are unable to give analytical solutions, but we are able to calculate solutions in the spirit of the seminal contribution by Auerbach and Kotlikoff (1987) if we functionally specify the model and assume plausible values for some fundamental parameters.
However, these simplifications are familiar in the literature. This paper is intended as a first presentation of the basic model in order to explain the existence of pension compositions and their relation with the underlying demographic structure. We intend to relax some of these qualifications in future studies.

The structure of this paper is as follows. In Section 2 we look on the existing literature. In Section 3 we specify the general model. We describe our demographic model, the behavior of the different players, and the different pension systems, i.e., public pensions, funded occupational pensions, private pensions, and hoarding.

The model will be functionally specified in Section 4. In this paper we focus on three sets of parameters, viz., the age-specific birth rates, the age-specific survival rates, and the retirement age. In order to get an idea of the effects of changes in those parameters we calculate solutions for a variety of parameter combinations.
In Section 5 we describe our solution method. In Section 6 we consider and evaluate the outcomes of our model for various parameter combinations. Those outcomes include pension and social security premiums and benefits, wage rates, interest rates, and capital per worker. In Section 7 we consider the political relevance of our findings and position our approach within the literature. The main novelty of our study seems to be the behavioristic Pareto approach, according to which we find an equilibrium pension composition, which explains the co-existence of social security, mandatory occupational pensions, individual savings, and hoarding, and the ensuing importance of demography with respect to wages, interest, and capital.
Although we did our best to choose more or less realistic parameter values and functional specifications, it is not difficult to suggest other values and specifications as being more realistic alternatives. For instance, the demographic sub-model we use is found in the literature, but due to its stylization it does not equal any specific national demography. This is, of course, the price to be paid when one wants to analyze *structural* properties. Actually, our model is very flexible, and can be easily implemented in practice to make predictions on dynamic developments for real national economies.

2. A look at the literature

There is a vast literature on the subject of pensions. An early strand of the literature focuses on the conditions of dynamic efficiency, e.g., Samuelson (1954) and Aaron (1966). A second strand of the literature focuses on risk sharing and the effect of aging. Examples include Gordon and Varian (1988), Bohn (2003), Ball and Mankiw (2007), Beetsma and Bovenberg (2007), Matsen and Thøgerson (2004), and Gollier (2008). In this paper we focus on the role of the demography and on the question of what determines the pension composition. In the literature we find various approaches to the relation between demography and economics. It is beyond the scope of this paper to consider the hundreds of articles written. Those studies differ in many ways. We may distinguish between more theoretical and more applied papers. In the theoretical papers one looks for a dynamic equilibrium, where the demography consists mostly of a few age brackets. In the applied papers one looks at a specific more realistic setting, where the model is calibrated to reality and one tries to predict developments for a specific country. In most papers two sources of old-age provision are considered: individual voluntary savings (IVS) and unfunded social security (SS). Sometimes the interest rate is taken as exogenous, while others take it to be endogenous.

In the theoretical analyses like Samuelson (1954), Aaron (1966), Cooley and Soares (1999a,b), Galasso (1999), Casamatta, Cremer and Pestieau (2000), Galasso and Profeta (2004), Galasso (2008), Gonzalez-Eiras and Niepelt (2007), Mateos-Planas (2008), Cremer et al (2009), Thøgersen (2015), and Alonso-García and Devolder (2016), there is (mostly) a two- or three-period overlapping generation population. Individuals are assumed to save within the constraint of a life budget, while the government is assumed to affect savings behavior by means of taxation and social security in order to reach ~~some~~ an optimal outcome according to a social welfare function. Matters become more complex if we assume three sources for old-age provisions, viz., social security, mandatory funded occupational pensions, and voluntary individual savings. This difference is stressed by Lindbeck and Perssons (2003).

In the more applied papers simulations are performed on real populations with many age cohorts in order to predict the development of the pension system for specific economies (e.g., the seminal Auerbach and Kotlikoff (1987), Miles (1999), Poterba (2001), Barr and Diamond (2006), Krueger and Ludwig (2007), Beetsma and Bovenberg (2009), Bovenberg and Nijman (2009), Börsch-Supan and Ludwig (2010), Lee and Mason (2010), Bell and Hill (1984), and Auerbach and Lee (2011)). For such more realistic worlds analytical results are difficult to find and one has to rely on model simulations, which we will use as well. Recently, there have been some authors who have looked more systematically at the relationship between demography and economics *per se*. They are mostly working in continuous time. The problem with this approach is that unless we use some tractable functional specifications, it becomes impossible to find explicit formulas for the solutions. We mention a.o. d'Albis (2007), Bruce and Turnovsky (2013), Heijdra and Mierau (2011), Cipriani (2016).

A third, political-economic stream emphasizes the importance of aging on election outcomes. Examples include Conesa and Krueger (1999), Cooley and Soares (1999a,b), Rangel and Zeckhauser (1999), Boldrin and Rustichini (2000), Breyer and Stolte (2001), Demange (2005), Gonzalez-Eiras and Niepelt (2007), and D'Amato and Galasso (2010). See Galasso and Profeta (2002) for a still relevant overview of the political economy of social security. More general and influential opinion papers include Feldstein (1997) and Sinn (2000). For an up-to-date survey on macro-economics and aging see Lee (2016). That survey does not deal with pensions in particular.

In this study we differentiate between four age-providing systems, viz. social security on a PAYG-basis, occupational pensions, individual savings, and hoarding in cash. There are only a few studies where the three systems are examined simultaneously. We mention Knell (2010).

In reality, the four systems will mostly exist side by side. When the interest rate is not zero, hoarding is obviously suboptimal. We will distinguish between 45 working age cohorts, where each cohort determines its future individual savings, and where two cohorts have a specific additional role. The median worker (MW) cohort has the choice to save either individually or via a mandatory occupational pension. If the cohort of median workers prefers the mandatory system, all other working cohorts are forced to participate in the mandatory pension system as well. Similarly, the median voter (MV) cohort may choose between individual savings or providing for old age via a mandatory social security system. In the latter case it forces the other cohorts to participate as well. The aggregate capital supply from individual and mandatory savings may sometimes exceed the demand for capital, even at zero interest rate. In that case, we leave open the possibility that individual savings are partly held in cash, as hoarding is less costly than keeping deposits at the bank at a negative interest rate.

The main difference between our approach and those used in the existing literature is, in our opinion, the recognition that there may exist various old-age provision systems side by side, reflecting the fact that some individuals may be better off saving individually, while others may prefer to save through a mandatory funded pension system or by means of mandatory social security on a PAYG-basis. If the interest threatens to become negative, hoarding in cash may be an alternative as well. The resulting pension composition is a compromise in the form of a Pareto equilibrium between the preferences of the different age groups, the workers, and the electorate (including the retired) as a whole. An essential feature is that two of the pension saving systems are mandatory, where the cohorts of the median worker and the median voter, determine the uniform premiums, which have to be paid by all other age cohorts. The resulting system mix is an endogenously determined behavioral equilibrium; macro-economic variables like the wage level, capital per worker, and the interest rate are simultaneously determined as well. The resulting equilibrium pension composition depends on the birth pattern, mortality, and the retirement age.

3. Structure of the equilibrium model

In this section we describe the model firstly in general terms. In the next section we focus on the functional specifications. We consider a homogeneous population with N age cohorts $n = 0,1, \ldots ,N$. The population is assumed to be stable, i.e., the age distribution $p = (p_0, \ldots , p_N)$ is constant and the population grows at a constant growth rate ν. The size of the total population at time t is N_t. We assume that there is no inflow or outflow by migration.

The demographic process depends on a birth pattern $\beta = (\beta_0, \ldots , \beta_N)$ and a survival pattern $\mu = (\mu_0, \ldots , \mu_N)$. It follows that $p = p(\beta, \mu)$ and $\nu = \nu(\beta, \mu)$. In many studies by economists the demography is succinctly described by its growth rate ν only without looking at the underlying birth and survival process. Distinguishing between the two creates a possibility of separately investigating the effects of a declining birth rate or increasing survival rates.

Four age cohorts are pivotal for the analysis. First, the age SW (taken in our numerical analysis at 20) at which one starts working and saving; second, the end of the working period EW (taken in our numerical analysis at 64) after which retirement starts. Pension payment ends at age EP, which we assume in this study to be at 100. For our analysis we mostly consider the population of adults only, belonging to the cohorts $n \geq SW$. We denote the corresponding conditional adult population shares by $p_{n|SW} = p_n / \sum_{j=SW}^{100} p_j$. Similarly, for the conditional distribution of workers only, i.e., the conditional population shares in the interval $[SW, EW]$, are denoted by $p_{n|SW,EW} = p_n / \sum_{j=SW}^{EW} p_j$. The (adult) population share of the workers is denoted by $P_{work|SW} = \sum_{j=SW}^{EW} p_j \Big/ \sum_{j=SW}^{100} p_j$. The population share of the retired is analogously defined and denoted by $P_{ret|SW}$. The third pivotal cohort is that of the median workers (MW), and the fourth pivotal cohort is that of the median voters (MV).

We assume there are four methods of providing for old-age:

Individualistic, voluntary

a. *Voluntary participation in a pension insurance contract* (IVS)

b. *Hoarding in cash* (HO)

Collectivistic, mandatory

c. *Fully-funded occupational pension* (FF)

d. *Pay-as-you-go social security* (SS)

Participation in an individual voluntary pension contract is understood to mean that one voluntarily promises at age n to pay an annual premium $S_n^{(IVS)}$ to the insurance company until retirement in return for which the insurance company promises to pay an annual benefit $B_n^{(IVS)}$ as a pension income to the individual when retired until death.[2] The voluntary pension insurance contracts may start at any working age n. Crucial is the pension/premium ratio $B_n^{(IVS)} / S_n^{(IVS)} = G_n^{(IVS)}$ of the arrangement, which depends on the age at which the insurance contract is started. We denote those revenue rates by $G_{SW}^{(IVS)}, \ldots, G_{EW}^{(IVS)}$, where we assume a pension insurance policy may start in any working year; consequently, an individual may successively enter into a cascade of insurance contracts. Similarly, for hoarding in cash we assume that if the individual at age n decides to hoard, from then on, each coming year, he will put aside an amount $S_n^{(HO)}$, while the

[2] There are many forms of such pension plans. For example, an alternative would be to assume that each year the individual buys a new life insurance contract without obligation to pay a premium for each working year to come. Using other pension plans would not change the qualitative results of our analysis.

collected cash will be consumed in equal parts $B_n^{(HO)}$ during retirement. The hoarding contract is similar to the pension insurance contract except for the fact that it yields a zero interest rate. Also, many alternative hoarding plans are conceivable here.

Moreover, citizens have to participate in a mandatory fully-funded pension fund (if it exists) with a premium $S^{(FF)}$ and an old–age benefit $B^{(FF)}$; similarly for social security they pay a mandatory contribution $S^{(SS)}$ and receive a retirement benefit $B^{(SS)}$. The revenue rates for the collectivistic arrangements are taken to be uniform with respect to age. They are denoted by $G^{(FF)}$, $G^{(SS)}$ respectively.

A second distinction between the four arrangements is whether the arrangement is interest-bearing or not. This yields a classification as in Figure 1.

	Individual	**Collective**
Interest bearing	Pension insurance (IVS)	Mandatory occ. pens. (FF)
Not interest bearing	Hoarding	Social security (SS)

Fig. 1: Classification of pension arrangements.

The decisions about participation in those arrangements are made by different parties/actors. For the individualistic arrangement it is obvious that the various individuals make their own decisions. For the fully-funded occupational pensions the decision is assumed to be in the hands of the trade union and we assume that within the trade union the cohort MW of median workers is decisive. They decide both on whether there will be a mandatory pension arrangement or not and, if that decision is positive, how large the premium and the resulting benefit will be. Similarly, the existence and the size of the social security system is decided by parliament and there the cohort MV of median voters is assumed to be decisive.

The revenue rates G differ per arrangement. How they are calculated is a technical question, but here we can already see that they are not all affected by the same set of variables.

The dependencies are laid out in Figure 2.

	n	β	μ	i
$G_n^{(IVS)}$	x		x	x
$G^{(FF)}$		x	x	x
$G^{(SS)}$		x	x	
$G_n^{(HO)}$	x		x	$(i=0)$

Fig. 2: Determinants of G

The revenue rate of individual policies depends on the age n of the individual when the policy is started, on the expected survival rates μ, and the resulting longevity, and on the interest rate i at which the premiums are invested. For the fully-funded mandatory pension individual ages do not count, but the demography, characterized by birth and survival rates β and μ, is a determinant together with the interest rate i. For social security the demography, that is (β,μ), is relevant but the interest rate is irrelevant. For hoarding, the age n at which the hoarding arrangement starts and the survival pattern μ count. Details are described in the next section.

Decision-making

Decisions on the four arrangements are made by the different actors by optimizing their remaining lifetime utility functions, where they take into account their wage income w, their already-standing other obligations OO_n like pension premiums agreed on in earlier contracts, mandatory pension premiums, and social security contributions, and their already secured other retirement benefits OB_n.

Decision-making by the individual worker

We start by looking for the individual savings pattern $\left(S_{20}^{(IVS)},S_{21}^{(IVS)},\ldots,S_{EW}^{(IVS)}\right)$, where we assume that individuals start working at the age of 20 and where we set *EW*=64, and where we also assume that negative saving is impossible. If one buys a pension insurance contract at the age of 20 and a second contract at the age of 21, he will pay at the age of 21 a total premium amount of $S_{20}^{(IVS)}+S_{21}^{(IVS)}$. For simplicity we assume that the individual assumes that his wage *w* and savings will not change over the coming years. Of course, this can be replaced by assuming a variable wage profile, but this will not substantially change the results of our paper. Moreover, for most individuals the constant-wage-assumption seems to be a plausible behavioral assumption. On the other hand, each succeeding year the remaining lifetime utility function will change, at least due to the fact that the period till retirement is reduced by one year. The remaining lifetime utility function[3] $\bar{U}_n$ of the decision-maker at age n looks like

$$\bar{U}_n = W_n.U(w-S_n^{(IVS)}-OO_n)+(1-W_n).U(S_n^{(IVS)}.G_n^{(IVS)}(i)+OB_n) \tag{3.1}$$

where $U(.)$ stands for the instantaneous utility function, with $W_n(>0)$ the weight attached to the remaining working life and $1-W_n(>0)$ the weight attached to the retirement

[3] The constant wage assumption simplifies the utility function. If we drop this assumption the remaining lifetime utility function would be

$$\bar{U}_n = \sum_{t=n}^{EW}\varpi_n.U(w_n-S_n^{(IVS)}-OO_n)+\sum_{t=EW+1}^{EP}\varpi_n.U(S_n^{(IVS)}.G_n^{(IVS)}(i)+OB_n) \text{ with } \sum_{t=n}^{EW}\varpi_n = W_n$$

period. When the individual grows older the weight $1-W_n$ on the retirement period will increase and the complementary weight W_n on the remaining working period will decrease[3].

The utility when working is $U(w-S_n^{(IVS)}-OO_n)$, where the argument stands for the net consumption of workers and where $S_n^{(IVS)}$ is the decision variable. The amount $S_n^{(IVS)}.G_n^{(IVS)}$ is the annual pension benefit derived from the new pension insurance. Clearly, the premium/benefit ratio $G_n^{(IVS)}(i)$ is an increasing function of the prevailing interest rate i. The individual maximizes (3.1) with respect to $S_n^{(IVS)}$. Since $G_n^{(IVS)}$ increases with the interest rate, savings are a decreasing function in interest. The working individual has to take into account that there may be a mandatory occupational pension premium $S^{(FF)}$ and a mandatory social security contribution $S^{(SS)}$ to be paid as well, and perhaps premiums on voluntary pension insurance contracts closed in previous years, say $S_{20}^{(IVS)},\ldots,S_{n-1}^{(IVS)}$. We call these amounts *other obligations* OO_n for short. Similarly, we define *other benefits* OB_n, consisting of occupational pension $B^{(FF)}=S^{(FF)}.G^{(FF)}$, social security benefit $B^{(SS)}=S^{(SS)}.G^{(SS)}$ and voluntary pensions $S_{20}^{(IVS)}.G_{20}^{(IVS)},\ldots,S_{n-1}^{(IVS)}.G_{n-1}^{(IVS)}$ stemming from earlier pension contracts.

Here, we notice that we do not assume that an individual optimizes over a 45-period budget set, that is, he would apply dynamic programming at the age of 20 to plan his future consumption and savings at ages 10 or 30 years ahead. We stick to the relatively more realistic assumption that the individual will continue to save at the same rate for the years ahead. On the other hand the individual has the possibility to adapt his/her savings pattern each working year, triggered by the fact that his utility function changes with age n as the retirement period draws closer.

If the instantaneous utility function is concave, i.e., the second-order derivative $U''<0$, then it follows that the remaining lifetime utility function (3.1) is concave in $S_n^{(IVS)}$ and consequently has a unique maximum. Since we exclude negative savings, the optimum may be a corner solution with $S_n^{(IVS)}=0$, in words, zero savings. There is still another instance where engaging in a voluntary pension insurance is not the first choice. If i <0, a case which nowadays is not merely hypothetical in some countries, the individual would be better off hoarding the savings in cash rather than investing the savings in a voluntary pension contract at a negative interest rate. In that case, hoarding will yield $G_n^{(HO)}=G_n^{(IVS)}(0)$ per dollar saved. In that case, the individual will maximize

$$\bar{U}_n=W_n.U(w-S_n^{(HO)}-OO_n)+(1-W_n).U(S_n^{(HO)}.G_n^{(HO)}+OB_n). \qquad (3.1a)$$

We see from (3.1) and (3.1a) that both an increase in OO_n and in OB_n will have a negative effect on the new savings.

The individual differentiates between four situations:

a. If $i > 0$ and $\left[\frac{d\bar{U}_n}{dS_n^{(IVS)}}\right]_{S_n^{(IVS)}=0} \geq 0$, then $S_n^{(IVS)} \geq 0$ and zero hoardings

b. If $i > 0$ and $\left[\frac{d\bar{U}_n}{dS_n^{(IVS)}}\right]_{S_n^{(IVS)}=0} < 0$, then $S_n^{(IVS)} = 0$ and zero hoardings

c. If $i = 0$ and $\left[\frac{d\bar{U}_n}{dS_n^{(HO)}}\right]_{S_n^{(HO)}=0} \geq 0$, then $S_n^{(HO)} \geq 0$

d. If $i = 0$ and $\left[\frac{d\bar{U}_n}{dS_n^{(HO)}}\right]_{S_n^{(HO)}=0} < 0$, then $S_n^{(HO)} = 0$

Cases b. and d. may occur if the sum of pensions(FF and/or SS), built up thus far, is deemed already sufficient.

Finally, there is the border case $i = 0$, where the individual is indifferent between saving in pension contracts or hoarding in cash. These conditions have to be satisfied for each working cohort. For instance, if individuals start working at 20 and stop at 65, it implies 45 decisions to be made.

The median worker (*MW*)

Consider now the special position of the median worker (MW). As an individual the median worker may either save individually or hoard according to the behavioral rules just specified. However, as a median worker he is also the deciding cohort in the population of workers, that is, the age cohorts from 20 to 65. The decision here is whether there should be a mandatory funded occupational pension or not, and if so, what should be the size of that pension premium $S^{(FF)}$? The pension-premium ratio for an individual pension starting at working age n is denoted by $G_n^{(IVS)}$. If the median worker opts for a fully-funded mandatory occupational pension the corresponding pension-premium ratio is denoted by $G^{(FF)}$. If the mandatory pension covers all workers, it is identical to a pension contract starting at the beginning of the working period, i.e., at 20. Therefore, the corresponding pension/premium-ratio is then $G^{(FF)} = G_{20}^{(IVS)}$. It is now obvious that $G_n^{(IVS)}$ is decreasing in n. More specifically, there holds $G_{MW}^{(IVS)} < G_{20}^{(IVS)}$. It follows that the median worker's first choice, if he is inclined to make additional pension savings, will be in favor of the mandatory occupational pension framework, since this presents better value for money than the individual contract would give. Hence, the median worker will not go for an individual pension contract. However, it may be that the median worker feels he already has enough pension contracts collected anyhow, and he will then abstain from the new mandatory contract as well. As a consequence the negative decision would imply that there would be no mandatory occupational pension fund, because the median worker is the one who decides on its existence. If he goes for the mandatory pension fund by choosing a (for him) optimum premium $S^{(FF)}$, then every worker, old and young, will have to participate in it, because it is a *mandatory* pension arrangement. If $i>0$, the optimal premium is found by optimizing the remaining lifetime utility (3.1) with $n=MW$ and $G_n^{(IVS)}$ replaced by $G_{20}^{(IVS)}$. If $i<0$, the median worker will hoard as well and optimize (3.1a).

The median voter

For the behavior of the median voter the situation is a bit different. If individual savings yield a better pension, that is, when $G_{MV}^{(IVS)} > G^{(SS)}$, then the median voter will opt for the individual arrangement, that is, either IVS or hoarding if $i<0$, if he wants to create an additional pension. Then, there will not exist social security. If, on the contrary, $G_{MV}^{(IVS)} < G^{(SS)}$, he will choose an additional social security pension if he wants to create additional pension. The optimal premium is found by optimizing the remaining lifetime utility (3.1) with $n=MV$ and $G_n^{(IVS)}$ replaced by $G^{(SS)}$. We notice that social security functions according to a PAYG-system. We have

$$S^{(SS)}.P_{work|20} = B^{(SS)}.P_{ret|20}$$

Consequently $G^{(SS)} = \dfrac{B^{(SS)}}{S^{(SS)}} = \dfrac{P_{work|20}}{P_{ret|20}}$ is the support ratio, i.e., the inverse of the old-age dependency ratio.

The inner equilibrium

Since the joint optimization model just sketched consists of 2*(65-20)+2(2)=94 interdependent first-order conditions in the 94 unknown $S^{(IVS)}$, $S^{(FF)}$, $S^{(SS)}$.and $S^{(HO)}$ and corner solutions are possible, an analytical solution of this system is out of the question. Combining the conditions for $S^{(IVS)}$, $S^{(FF)}$, $S^{(SS)}$ and $S^{(HO)}$ the question is whether there is a numerical solution to the system

$$\begin{aligned}
S^{(IVS)} &= f_{IVS}(S^{(FF)}, S^{(SS)}, S^{(HO)} \,|\, w, i) \\
S^{(FF)} &= f_{FF}(S^{(IVS)}, S^{(SS)}, S^{(HO)} \,|\, w, i) \\
S^{(SS)} &= f_{SS}(S^{(IVS)}, S^{(FF)}, S^{(HO)} \,|\, w, i) \\
S^{(HO)} &= f_{HO}(S^{(IVS)}, S^{(FF)}, S^{(SS)} \,|\, w, i)
\end{aligned} \qquad \forall S \geq 0 \qquad (3.2)$$

for a given value interest rate i, where the $f(.)$s are short-hand notations for the optimization outcomes above. It appears that it is possible to find an equilibrium through iteration. In practice, as we will see later on, we always found a unique equilibrium. This equilibrium is found under the assumption that the interest rate i is exogenously fixed.

We call this equilibrium solution the 'inner' equilibrium. This assumption of an exogenously fixed interest rate is perhaps valid for a small open economy, but this assumption is not so realistic anymore, given the global mobility of nowadays. Dutch pension funds invest about 95% of their investments abroad. Rather, we have to see the whole world as a closed economy. Hence, we have to take into account that the equilibrium interest rate is endogenous as well. This 'outer' equilibrium is found by varying the interest rate over the corresponding inner equilibria until we find the rate for which capital demand equals capital supply. If this equilibrium entails a negative interest rate, we assume that the actual rate will fall to 0%, and that the remaining savings will be hoarded, since no one is interested in investing capital at a negative interest rate.[4]

Capital, interest, and wages; the outer equilibrium

We ~~notice~~ have to endogenize the interest rate i and the corresponding wage rate w which had been taken to be exogenous up to this point. Thus, we introduce a capital market. We assume for simplicity that capital supply is only provided by individual savings and by the reserves of the occupational pension fund. Hence, we may write for the individual and collective accumulated savings per head of the population

$$\begin{aligned} K^{(IVS)} &= K^{(IVS)}(S^{(IVS)}; i, D) \\ K^{(FF)} &= K^{(FF)}(S^{(FF)}; i, D) \end{aligned} \tag{3.3}$$

where we summarize the demographic variables by D for the moment. This total capital supply per head of the population yields the capital per worker of

$$k_{\sup} = (K^{(IVS)} + K^{(FF)}) / P_{work|SW} \tag{3.4}$$

We notice that this capital supply is a function of the interest rate i, since $G^{(IVS)}$ and $G^{(FF)}$ depend on i. Looking at (3.1) it can be seen that an interest increase leads to a decline in voluntary and mandatory savings. It follows that capital supply is decreasing in interest. We close the model by introducing a capital demand function per worker, denoted by $k_{dem}(i)$, standing for the demand of an optimizing firm owner. The demand function $k_{dem}(i)$ is monotonically decreasing in i as well. For a stable full-employment equilibrium $k_{dem}(i)$ is derived by maximizing the profit per worker. The marginal condition is

$$f'(k) - (i + \delta + \nu) = 0 \tag{3.5}$$

where we assume a production $f(k)$ per worker. Capital costs consist of three components, viz., interest, depreciation, and new investment to cope with population growth (or decline) to ensure a constant capital per head of the population.

[4] We ignore the fact that bank deposits even at a negative interest may be better protected than keeping the money at home in cash at zero interest. Then, the negative interest rate may be interpreted as protection costs.

An outer equilibrium is there where $k_{dem}(i) = k_{\sup}(i)$. In this general model we cannot exclude that there will be more than one equilibrium. In our model to be specified hereafter we find that both capital supply and demand fall with increasing interest, but that demand for zero interest is much higher than supply, while the demand curve is much more steeply falling than the supply curve with increasing interest.

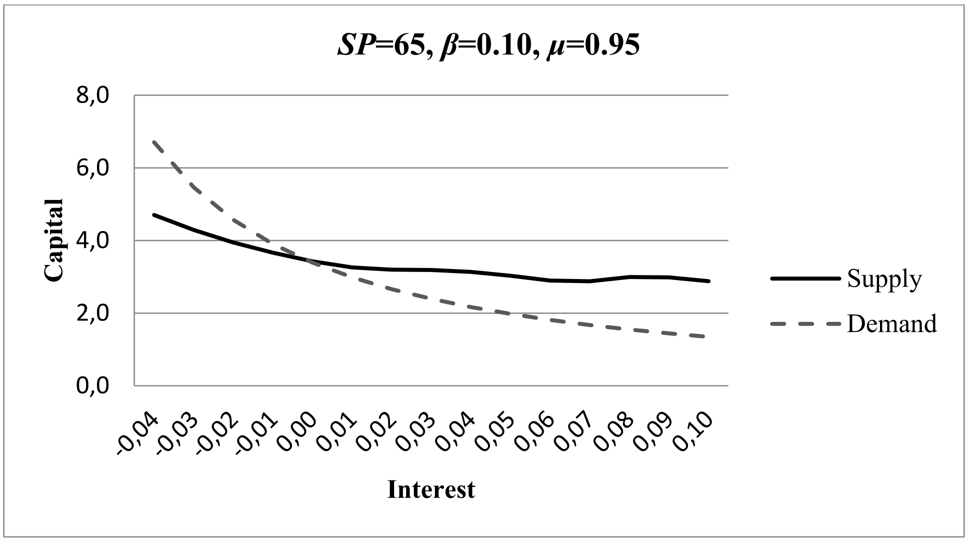

Fig. 3: Capital demand and supply curves.

Consequently, in the model specified hereafter, we found only one point of intersection where $k_{dem}(i) = k_{\sup}(i)$; the equalizing value of i is the equilibrium interest rate. An example is sketched in Fig. 3 for a retirement age of 65, a birth rate of 0.10 (two children per couple) and an annual survival rate of 95% during retirement. This equilibrium is called the 'outer' equilibrium.

It stands to reason that solving this for the equilibrium interest in the capital market requires an iterative solution. It follows that we have two sequential iteration processes: the inner loop converging to the inner equilibrium, which gives the total capital supply function per worker as a function of the interest rate i, and the outer loop where the interest i is varied until the equilibrium interest rate has been found at which supply and demand curves intersect each other. That final result is called the outer equilibrium.

In the next section we will specify the model by choosing specific functions and parameter values. If the supply and demand curves intersect each other for a negative interest rate i, hoarding of part of the supply becomes relevant. The interest rate reaches its lower bound at $i = 0$ and the difference between supply and demand at $i = 0$ is hoarded. Whether the hoarding is done wholly by individuals or by the pension fund, or through a mixture of both, is not determined.

4. Specifications

Now we have to assume a specific demographic model with numerically specified parameters, a specific instantaneous utility function, a production function, and we have to calculate the multiplication factors G. But as soon as we specify these ingredients we may meet the objection that the demography studied is not realistic, that utility functions and production functions should be replaced by others, etc. Let us explicitly repeat here that the model we use is not intended to be realistic in the sense that it predicts the development of a specific country. This is also impossible because no country has a stable population, that is, in which birth rates and survival rates are constant over time. However, the stable population reflects a population toward which the present population would tend if present birth rates and survival rates were to remain constant from now on into the future. The same holds for the choice of production functions and utility functions. There are many different estimates of those functions. We shall make a choice such that the resulting model is plausible. If one wants to use different parameter values or functional specifications the theoretical model and the computer program are easily adaptable.

Demography

The population at time t is described by a vector $N_t^{'} = (N_{0,t},\dots,N_{100,t})$ where $N_{n,t}$ stands for the number of people of age n at time t. The population develops according to the well-known Leslie (1945)-model (see also Lotka (1907)) described by the matrix equation system

$$\begin{aligned} N_{0,t+1} &= \beta' N_{-,t} \\ N_{-,t+1} &= M N_{t,-} \end{aligned} \tag{4.1}$$

where $N_{0,t}$ stands for the number of newborns at time t, $N'_{-,t} = (N_{1,t},\ \dots\ ,N_{100,t})$ stands for the vector of age cohorts from 1 to 100, $N'_{t,-} = (N_{0,t},\ \dots\ ,N_{99,t})$ for the vector of age cohorts from 0 to 99,where β stands for a vector of (age-specific) birth rates, and where M stands for a diagonal (100×100)-matrix of (age-specific) survival rates. The diagonal elements of M are also denoted as the vector μ. We assume that there is a fertility period of 10 years during which individuals may have children. This fertility period starts at the age of 25 and ends at 34. During that period the annual birth rate is taken to be constant at β per individual. Since no difference is made between males and females, at β =10% a couple is just reproducing (the expected number of children is 2(=2*10*β), if we exclude child mortality, as we do. Consequently, the population growth rate for β =10% is ν =0%. In order to investigate the effect of changes in the birth rate we simulate the model for β =7.5%, 10%,...,30% where β =7.5% stands for 1.5 children per couple and β =30% stands for six children per couple.

Table 1. Effect of demography on the demographic key variables

birth rate (β)	survival rate (μ)	retirement age (SP)	population growth %	life expectancy	age median worker	age median voter	support ratio
0.075	0.95	65	-0.9	81	44	55	1.92
0.10	0.95	65	0.0	81	42	50	2.67
0.15	0.95	65	1.3	81	39	44	4.29
0.20	0.95	65	2.3	81	36	40	6.05
0.25	0.95	65	3.0	81	35	38	7.94
0.30	0.95	65	3.6	81	34	36	9.95
0.10	**0.93**	65	0.0	77	42	49	3.40
0.10	**0.94**	65	0.0	79	42	49	3.03
0.10	**0.95**	65	0.0	81	42	50	2.67
0.10	**0.96**	65	0.0	83	42	52	2.34
0.10	**0.97**	65	0.0	87	42	53	2.03
0.10	**0.98**	65	0.0	90	42	55	1.74
0.10	0.95	**63**	0.0	81	41	50	2.28
0.10	0.95	**64**	0.0	81	41	50	2.47
0.10	0.95	**65**	0.0	81	42	50	2.67
0.10	0.95	**66**	0.0	81	42	50	2.90
0.10	0.95	**67**	0.0	81	43	50	3.15
0.10	0.95	**68**	0.0	81	43	50	3.42
0.10	0.95	**69**	0.0	81	44	50	3.71
0.10	0.95	**70**	0.0	81	44	50	4.02
0.10	0.95	**71**	0.0	81	45	50	4.36
0.10	0.95	**72**	0.0	81	45	50	4.72
0.075	**0.98**	**70**	-0.9	90	47	61	1.65

It is well known from demographic theory that along the equilibrium path the population is growing at a constant rate ν and has an age distribution $p = (p_0, \ldots, p_{20}, \ldots, p_{65}, \ldots, p_{100})$. For ease of exposition we shall assume no mortality, that is, μ =1, before 65 and a constant annual survival rate μ<1 from the age of 65 onwards. We will vary the annual survival rate from 0.93 to 0.98. We assume that individuals start working SW (StartWork) at the age of 20 and retire when they have reached the retirement age SP (StartPension). Their last working year is EW (EndWork= $SP-1$).

Assuming that individuals younger than 20 do not work, from here on we denote the adult population share of the workers in the age interval $[SW, EW]$ by P_{work} and the share of the retired in the age interval $[SP,100]$ by P_{ret}, where we normalize such that $P_{work} + P_{ret}$ =1. We will vary SP from 63 to 72.

In order to gain insight into the effects of key demographic parameters, we present Table 1. We include the retirement age as a demographic variable, although strictly speaking it is not a demographic variable but is mostly determined by lawmakers. It is obvious that we can opt for a more sophisticated demographic model where the birth rate and survival patterns vary continuously with age; computationally, this is no problem. However, it implies that birth and survival patterns cannot be easily characterized by only one parameter each, which would obscure our analysis.

Pension systems

In modern societies we mostly find a mixture of those systems simultaneously present, although the sizes of those systems differ between economies. The pension composition (PC) may be described by a vector

$$(S_{SW}^{(IVS)},...,S_{EW}^{(IVS)},B_{SW}^{(IVS)},...,B_{EW}^{(IVS)},S^{(FF)},B^{(FF)},S^{(SS)},B^{(SS)},S_{SW}^{(HO)},...,S_{EW}^{(HO)},B_{SW}^{(HO)},...,B_{EW}^{(HO)})$$

or more briefly by $(S^{(IVS)},S^{(FF)},S^{(SS)},S^{(HO)})$. We assume that under voluntary individual saving the individual at age n $(n = SW,...,EW)$ may buy a pension insurance contract according to which he agrees to pay a premium $S_n^{(IVS)}$ for the rest of his working life in exchange for an annual pension of $B_n^{(IVS)}$, starting at the retirement age. The link between premiums and benefits is given by the actuarial balance equation

$$S_n^{(IVS)}.\left[1+\frac{1}{1+i}+...+\left(\frac{1}{1+i}\right)^{EW-n}\right] = B_n^{(IVS)}.\left[\left(\frac{1}{1+i}\right)^{SP-n}.\mu+...+\left(\frac{1}{1+i}\right)^{EP-n}.\mu^{EP-EW}\right] \quad (4.2)$$

We define the benefit-premium ratio $G_n^{(IVS)}$ for the contract by $B_n^{(IVS)} = G_n^{(IVS)}.S_n^{(IVS)}$. Benefits are proportional to the premium paid. The sum of those benefits for all voluntary pension contracts at the start of retirement, that is, the total individual pension, will be denoted by $\hat{B}_{EW}^{(IVS)} = \sum_{n=SW}^{EW} B_n^{(IVS)}$. For the hoarding benefits we get, similarly, $\hat{B}_{EW}^{(HO)} = \sum_{n=SW}^{EW} B_n^{(HO)}$. In a similar way we denote the mandatory funded pension by its premium $S^{(FF)}$ and the corresponding benefit by $B^{(FF)}$. Since all age groups from $SW = 20$ onwards are obliged to participate in the mandatory system, this mandatory insurance is identical to the voluntary insurance in which we may participate at the age of 20. It follows that $G^{(FF)} = G_{20}^{(IVS)}$.

In Fig. 4 we sketch the behavior of the G's as functions of the interest rate i. The social security contribution is $S^{(SS)}$ and the corresponding benefit $B^{(SS)}$. There holds $G^{(SS)} = \frac{B^{(SS)}}{S^{(SS)}} = \frac{P_{work|20}}{P_{ret|20}}$.

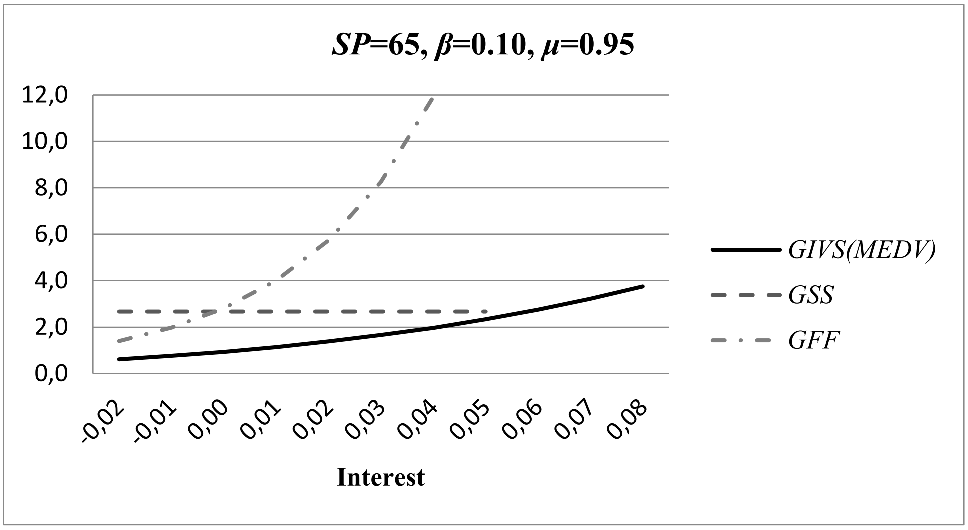

Fig. 4: The behavior of $G_{MV}^{(IVS)}, G^{(SS)}, G^{(FF)}$ as a function of the interest rate.

The median voter may make a choice between IVS and social security. If $G^{(SS)} < G_{MV}^{(IVS)}$, he will prefer to save individually instead of contributing to a social security system. This may be the case for high interest rates. In Fig. 4 this occurs if the interest rate exceeds about 5.5%. If the median voter prefers the individual pension, there will not be (a majority for) a social security system in the society.

In some societies the institutional structure may be such that not all three systems are at work. For instance, in Chile there is no social security arrangement for old-age pensions on a PAYG-basis. In other countries occupational pensions are mostly run on a pay-as-you-go basis. Hoarding in cash is a primitive last method of saving for old age. We refer to OECD (2017) for an international survey. In countries lacking a banking system, individual saving may be nearly impossible.

In this paper we assume that all four pension schedules are accessible, even if some of those schedules are not actually used in the equilibrium. We assume that all voluntary and mandatory savings by individuals are eventually aimed at safeguarding an old-age pension. The retirement age SP is fixed here at 65. Later on we shall also vary the retirement age.

Capital supply.

The resulting aggregate of individual saving reserves per working adult of age $n \leq EW$, where EW is set at 64, is

$$RES_n^{(IVS)} = \sum_{j=SW}^{n} S_j^{(IVS)} . \sum_{m=j}^{n} (1+i)^{m-j} \tag{4.3a}$$

The individual reserves for a retiree at age $n \geq 65$ are the present values of the future benefit flow

$$RES_n^{(IVS)} = \hat{B}^{(IVS)}.\sum_{t=n}^{EP-1}\left(\frac{\mu}{1+i}\right)^{t-n} \qquad n \ge SP \tag{4.3b}$$

It follows that the average IVS reserve per head in the adult population is

$$RES^{(IVS)} = \sum_{n=SW}^{EW} p_{n|SW}.RES_n^{(IVS)} + \sum_{n=SP}^{EP} p_{n|SW}.RES_n^{(IVS)} \tag{4.3c}$$

The per capita reserve in the mandatory FF-system is calculated likewise. It equals the individual pension contract for n =20, where the premium $S^{(FF)}$ is determined by the median worker. Hence, we get

$$RES^{(FF)} = S^{(FF)}\sum_{j=SW}^{EW} p_{j|SW}\cdot\sum_{m=j}^{EW}(1+i)^{m-j} + S^{(FF)}.G^{(FF)}.\sum_{j=SP}^{EP} p_{j|SW}\cdot\sum_{t=j}^{EP-1}\left(\frac{\mu}{1+i}\right)^{EP-j-1} \tag{4.3d}$$

For other values of the retirement age SP the formulas have to be changed slightly because mortality may start before retirement when individuals are still at work.

The total capital supply per workplace is the sum of individual and collective savings. We have

$$k_S(i) = \left(RES^{(IVS)}(i) + RES^{(FF)}(i)\right)/P_{work|SW} \tag{4.4}$$

Since social security is run on a pay-as-you-go basis it does not generate a reserve. The same holds for hoarding.

Parameter values

The choice of specific parameter values is a delicate one. There are many different estimates and they also vary between countries, between moments of estimation, and between the empirical estimation methods used. Since we are developing a general theory and our numerical simulations are only intended to get qualitative insights, we abstain from calibrating our parameter values in order to fit one specific country at a specific moment in time.

For the instantaneous utility function we take the well-known Constant Relative Risk Aversion (CRRA) specification $U(y) = y^{1-\gamma}/(1-\gamma)$, where we take $\gamma = 3$. In the literature there are many estimates for γ, but they vary over a great range. See e.g., Gandelman and Hernandez-Murillo (2015) and the recent survey by Outreville (2015). See also Booij and Van Praag (2009). The value of 3 is somewhere in the middle of recent empirical estimates, but there is much uncertainty about it. The time weights are assumed to be

$$W_n = \sum_{m=n}^{EW}\rho^{m-n} \Big/ \sum_{m=n}^{EP}\rho^{m-n}$$

where the subjective time discount rate ρ is set equal to 0.89. There is a host of different estimates for ρ as well, but for macro-economic long-term decision settings this value seems to be in the middle of the range. (see Shane, Loewenstein, and O'Donoghue (2002)). It appears that the outcomes of the model are very sensitive with respect to the value of ρ. We therefore tried several values.

For the production function we take the traditional Cobb-Douglas function $Y = C.K^{\alpha}.L^{1-\alpha}$ where we use the traditional value α =0.25. This value is debatable, too, since the capital elasticity varies a lot between industries and seems to increase over the years (see Piketty 2014, Karabarbounis and Neiman 2014, and OECD 2015). Finally, we assume the depreciation rate to be δ =10%. Also, here the value of the macro-depreciation rate is rather uncertain. We refer to Nadiri and Prucha (1996) and a recent very down-to-earth but detailed catalogue of depreciation rates as prescribed by the New Zealand tax authorities (Taake 2017).

5. Description of the numerical solution

We start to solve the system (3.2) by iteration according to the schedule $S_{m+1} = f(S_m | w_m, i_m)$ with

$$\begin{aligned} S_{m+1}^{(IVS)} &= f_{IVS}(S_m^{(FF)}, S_m^{(SS)}, S_m^{(HO)} | w_m, i_m) \\ S_{m+1}^{(FF)} &= f_{FF}(S_m^{(IVS)}, S_m^{(SS)}, S_m^{(HO)} | w_m, i_m) \\ S_{m+1}^{(SS)} &= f_{SS}(S_m^{(IVS)}, S_m^{(FF)}, S_m^{(HO)} | w_m, i_m) \\ S_{m+1}^{(HO)} &= f_{HO}(S_m^{(IVS)}, S_m^{(FF)}, S_m^{(SS)} | w_m, i_m) \end{aligned} \qquad \forall S \geq 0 \tag{5.1}$$

where m is the iteration step. We start with $S_0^{(FF)}$, $S_0^{(SS)} = 0$, $i_0 = -(\nu + \delta)$ and k_D, w defined below by (5.2), (5.3) for $i_0 = -(\nu + \delta)$. In practice, the system (5.1) always converges to a unique equilibrium for every value of i, although we were unable to prove this analytically. Mostly, the iteration process takes about six rounds. For a given i we hence find $S_m^{(IVS)}(i)$, $S_m^{(FF)}(i)$, $S_m^{(SS)}(i)$, $S_m^{(HO)}(i)$. Finally, we calculate the capital supply $k_S(i)$ per worker according to (4.2), (4.3) and (4.4). We call this iteration process (5.1) the 'inner loop,' and the resulting equilibrium the 'inner' equilibrium. It depends on the interest rate i.

Assuming a Cobb-Douglas production function and capital costs consisting of interest, depreciation, and net investment so that the capital per worker keeps pace with population growth ν, the demand per worker for capital by a profit-maximizing firm for a given i is found by solving the first-order-condition $f'(k_D(i)) = (i + \nu + \delta).k_D(i)$. We notice that there has to hold $(i + \nu + \delta) \geq 0$. It follows that a negative interest would be possible, where $i \geq -(\nu + \delta)$. We get a capital demand function

$$k_D(i) = \left(\frac{\alpha}{i + \nu + \delta} \right)^{1/(1-\alpha)} \tag{5.2}$$

while the corresponding wage rate is

$$w(i) = f(k_D(i)) - (i + \nu + \delta).k_D(i) \tag{5.3}$$

Now, we have to compare capital demand and supply in the capital market. There is equilibrium in the capital market if $k_S(i) = k_D(i)$. This equilibrium rate of interest is also found by iteration, which we call the 'outer loop.' It normally takes only a few rounds. The corresponding value of i, say $\bar{i}$, is the equilibrium interest, and from it (5.1) provides us with the equilibrium values $\bar{S}^{(IVS)}(\bar{i})$, $\bar{S}^{(FF)}(\bar{i})$, $\bar{S}^{(SS)}(\bar{i})$, $\bar{S}^{(HO)}(\bar{i})$. This is called the 'outer' equilibrium. In Fig. 3 above we sketched the demand and supply curve for β =0.10, μ =0.95, SP =65 and ρ =0.89 and δ =0.10, α =0.25.

We will see from our numerical examples in the next section that the equilibrium interest rate thus found might turn out to be negative. There are examples of 'old' populations, that is, with a low birth rate and/or high life expectancy, where the equilibrium rate would be negative. We give one example in Table 2. This is, of course, not attractive for savers and pension funds. In such a situation hoarding at an effective rate of interest of 0% is favored above bringing the money to the bank or the capital market where the revenue would be negative. Hence, there is an effective lower bound on the interest rate at 0%. It implies that there may be an oversupply of capital, where part of the savings is hoarded, since not all capital supply can be invested at a non-negative interest rate. Whether this hoarding is done by individuals or by pension funds, or both, is irrelevant. We will find one instance in the numerical results below.

6. Outcomes for a closed economy

In this section we present the equilibria for different parameter constellations. We take as a starting point a birth rate of β =0.10, equivalent to, on average, two children per couple and zero population growth, a survival rate of μ =0.95 and a retirement age EW =64, i.e., pension payments start at 65. The subjective time preference rate ρ will be taken at 0.89, the depreciation rate at δ =10% and capital productivity at α =0.25. Our policy will be to vary one parameter, while leaving the other values unchanged. Similarly, we will look at the effects for values β =0.075, 0.10, 0.15, … , 030 and the effects when the retirement age is increased from 63 up to 72.

The subjective time preference rate

We start by varying ρ from 0.88 up to 0.92, while setting β =0.10, μ =0.95 and SP =65. An increase in ρ implies that all parties put more weight on the retirement period. This will result in more capital and/or more social security. More savings will be reflected in more capital per worker. We see that if ρ increases from 0.88 to 0.92, capital per worker increases by 27% from 2.656 to 3.393. This implies a capital elasticity with respect to ρ in the order of 5. It seems to imply, intuitively not implausibly, that the outcomes are rather sensitive with respect to ρ. We see that with an increase in ρ the equilibrium interest rate falls from 2% to 0%.

Table 2. The effect of subjective time preference on the pension composition

Subjective time preference rate (ρ)	Indiv. savings start/ finish %	Private pension %	Premium fully funded %	Fully funded pension %	Premium social security %	Social security pension %	Net wage %	Total pension %	Interest rate %	Gross wage	Capital-income ratio	Capital	Hoarding capital %
	1	2	3	4	5	6	7	8	9	10	11	12	13
ρ=0.88	0.7 11.1	23.6	2.7	15.5	2.2	5.8	84.1	44.8	2.0	0.957	2.774	2.656	-
ρ=0.89	1.0 8.5	23.0	3.0	13.8	3.0	8.1	85.5	45.0	1.4	0.974	2.916	2.839	-
ρ=0.90	1.7 8.1	22.7	3.3	12.2	3.9	10.4	84.7	45.4	0.8	0.993	3.092	3.069	-
ρ=0.91	2.6 9.5	22.6	3.5	10.6	4.8	12.8	82.2	46.0	0.2	1.012	3.277	3.317	-
ρ=0.92	3.8 11.1	22.5	3.8	9.2	5.6	15.0	79.4	46.6	-0.4	1.033	3.486	3.602	-
	4.0 10.8	25.3	3.7	10.4	4.9	13.1	80.5	48.9	0.0	1.018	3.333	3.393	10.5

Birth rate (β)=0.10, survival rate (μ)=0.95 and age of retirement (SP)=65. For legend see Appendix.

In the fifth line of the table the interest rate would become negative if we excluded the possibility of hoarding. In the last line the outcomes are presented when hoarding is possible, i.e., when the interest rate is fixed at a lower limit of 0%. The amount hoarded in the last situation is about 10% according to the last column.[5] The net benefit-ratio is approximately 48.9/80.5, that is, around 60%. The total savings ratio is about 19.4%, of which 4.9% is spent on social security. Finally, we look at voluntary savings behavior over life. The individual with ρ=0.88 starts at 20 with a tiny individual savings ratio of 0.7% which increases over life to 11.1% just before retirement. For an individual with a higher time preference of 0.92 the corresponding ratios are 4% and 10.8%, respectively.

Increasing longevity

We will now consider in Table 3 how the equilibrium changes if the survival rate μ is varied from 0.93 up to 0.98, keeping β=0.10, SP=65 and ρ=0.89. Here, the interesting changes are seen in savings behavior. Individual savings dwindle when life expectation increases while the mandatory schedules gain in weight. The occupational pension premium increases from 2.5% to 3.5%, but the major change is in the role of social

[5] Notice that in this model direct hoarding by individuals or deposits at 0% in the bank yields the same result. If we assume that banks will charge for hoarding costs, that is tantamount to a negative interest, e.g., -1%, individuals will prefer to hoard at home.

security. The social security premium rises from 0.6% to 9.1%, while the ratio of a fully-funded pension to the social security benefit 19.1/2.1=9.1 for μ =0.93 (life expectancy 77) that ratio changes into 11.1/15.9=0.70 for μ =0.98 (life expectancy 90).

Table 3. The effect of aging on the pension composition

Survival rate (μ)	Indiv. savings start/ finish %	Private pension %	Premium fully funded %	Fully funded pension %	Premium social security %	Social security pension %	Net wage %	Total pension %	Interest rate %	Gross wage	Capital-income ratio	Capital
	1	2	3	4	5	6	7	8	9	10	11	12
μ=0.93	1.5 11.8	33.9	2.5	19.1	0.6	2.1	85.0	55.1	2.1	0.955	2.751	2.627
μ=0.94	1.5 11.7	28.6	2.8	15.5	1.7	5.2	83.8	49.3	1.6	0.969	2.871	2.781
μ=0.95	1.0 8.5	23.0	3.0	13.8	3.0	8.1	85.5	45.0	1.4	0.974	2.916	2.839
μ=0.96	0.3 6.2	17.9	3.1	13.3	4.9	11.4	85.8	42.7	1.6	0.970	2.881	2.793
μ=0.97	0.0 6.3	14.0	3.3	11.5	6.7	13.5	83.6	39.0	1.3	0.976	2.943	2.873
μ=0.98	0.0 5.3	9.9	3.5	11.1	9.1	15.9	82.0	36.9	1.5	0.972	2.904	2.824

Birth rate (β)=0.10, age of retirement (*SP*)=65, and time discount rate(ρ)=0.89. For legend see Appendix.

Or in other words, individual pensions amount to 61% of total pension, the mandatory occupational pension to 35% and social security to a meager 4% of total pension for μ =0.93. For a rather old population these fractions are 27%, 30%, and 43% respectively. Total pension as a fraction of gross wage falls from 55.1% to 36.9% and the net-benefit ratio falls rather dramatically from about 65% to 45%. The situation of workers does not change dramatically, but the situation for pensioners does deteriorate dramatically. If we may believe these figures, at least qualitatively, the future for an aging society appears bleak.

Increasing birth rate

The effect of varying the birth rate is not so straightforward. A consequence of a rising birth rate is a strongly growing labor force. The effect when capital is unchanged is that the capital per worker becomes scarcer. It follows that the interest rate will increase while the gross wage will fall. Indeed, we see that the interest rate increases from a moderate 1.4% to 15.2% when the number of children increases from 1.5 to a, for developed economies, unusual six children per couple. Actually, the interest rate rises much faster than the population growth rate. Gross wages fall from 1.002 to 0.715 and capital per job falls from 3.186 to 0.826. This capital thinning is due to the fact that the ratio of workers to retired increases (see Table 1) from 1.92 to 9.95.

Table 4. The effect of changes in the birth rate on the pension composition

Birth rate (β)	Indiv. savings start/ finish %	Private pension %	Premium fully funded %	Fully funded pension %	Premium social security %	Social security pension %	Net wage %	Total pension %	Interest rate %	Gross wage	Capital-income ratio	Capital
	1	2	3	4	5	6	7	8	9	10	11	12
β=0.075	0.0 6.6	19.2	3.4	15.8	5.4	10.4	84.6	45.5	1.4	1.002	3.179	3.186
β=0.10	1.0 8.5	23.0	3.0	13.8	3.0	8.1	85.5	45.0	1.4	0.974	2.916	2.839
β=0.15	1.8 14.6	30.4	2.3	13.3	1.2	5.3	81.9	49.0	2.1	0.923	2.486	2.295
β=0.20	2.1 15.8	40.3	1.6	13.9	0.3	1.7	82.3	55.9	3.2	0.880	2.156	1.898
β=0.25	0.0 0.0	0.0	0.2	139.7	0.0	0.0	99.8	139.7	14.8	0.724	1.197	0.866
β=0.30	0.1 0.1	81.6	0.1	81.5	0.0	0.0	99.8	163.1	15.2	0.715	1.155	0.826

Survival rate (μ)=0.95, age of retirement (SP)=65, and time discount (ρ)=0.89. For legend see Appendix.

The tremendous increase in the interest rate to about 15% makes voluntary and mandatory saving very profitable with, as a result, very tiny savings, while social security vanishes. When the birth rate rises the situation of the retired relative to that of the workers improves a great deal and to such an extent that retirees' pensions are much larger than net wages, which is indeed surprising. For Western countries where the birth rate hovers around 0.10 or below we get rather low interest rates. Countries where the birth rate is still 0.20 or above are nowadays the less developed economies. In those countries the whole pension system is clearly different from the one in our model as frequently there is not a well-developed IVS-, FF-, and/or SS-system. Moreover, the demography is rather different from ours with respect to the survival rate and the same probably holds for the subjective time discount rate ρ.

Increasing retirement age

Finally, let us consider the effect of the retirement age. We assume here that individuals of 72 are as efficient workers as those of 63, which is improbable in reality. We see a similar phenomenon as when the birth rate increases. Capital has to be spread over more workers with the effect that gross wage decreases and the interest rate increases. When the retirement age increases, there is a decline in the inequality between workers and the retired. If the retirement age increases to 71 we find that the retired become even better off than the workers due to the increase of the interest rate to 11.7%.

Table 5. The effect of changes in the retirement age on the pension composition

Retire-ment age (SP)	Indiv. savings start/ finish %	Private pension %	Pre-mium fully funded %	Fully funded pension %	Pre-mium social security %	Social security pension %	Net wage %	Total pension %	Interest rate %	Gross wage	Capital-income ratio	Capital
	1	2	3	4	5	6	7	8	9	10	11	12
SP=63	0.7 7.3	18.7	3.3	11.9	4.9	11.3	84.4	42.0	1.2	0.980	2.977	2.919
SP=64	1.0 6.7	20.7	3.0	13.0	4.1	10.1	86.2	43.7	1.4	0.973	2.914	2.837
SP=65	1.0 8.5	23.0	3.0	13.8	3.0	8.1	85.5	45.0	1.4	0.974	2.916	2.839
SP=66	1.1 9.9	25.9	2.7	15.2	2.1	6.0	85.4	47.1	1.7	0.965	2.841	2.741
SP=67	1.2 10.9	28.9	2.6	16.1	1.3	4.1	85.1	49.1	1.7	0.965	2.844	2.746
SP=68	1.3 11.5	33.2	2.3	17.6	0.4	1.5	85.7	52.3	2.1	0.956	2.763	2.642
SP=69	0.2 2.5	28.9	1.6	35.5	0.0	0.0	95.8	64.4	4.5	0.899	2.300	2.069
SP=70	0.0 0.0	0.0	0.5	96.5	0.0	0.0	99.5	96.5	9.5	0.815	1.709	1.393
SP=71	0.0 0.0	0.0	0.5	108.0	0.0	0.0	99.5	108.0	9.4	0.816	1.716	1.400
SP=72	0.0 0.0	0.0	0.2	137.1	0.0	0.0	99.8	137.1	11.7	0.786	1.535	1.206

Birth rate (β)=0.10, survival rate (μ)=0.95, and time discount (ρ)=0.89. For legend see Appendix.

7. Discussion and evaluation

In this section we aim to answer the following questions:

a. How realistic is the model?
b. How does it fit with the economics literature? What is new?
c. What is the political relevance?

Realism

The dilemma behind economic modeling is that we have to choose between realism and transparency. If we opt for realism, we may end up with a jungle of details, actors, relationships, and variables. At the other extreme we may have a simple elegant and transparent model, but it is so far simplified and stylized that it cannot be seen as a relevant description of reality. Moreover, by leaving out essential variables we may find strongly biased effects of the remaining variables. We have looked for a compromise. Hence, some of our readers will object that our model is not realistic enough while others will complain that given the multiple simultaneous non-linear relationships in the model we do not always get monotonic clear-cut effects which can be economically interpreted.

However, the model in this paper can be easily extended to a more realistic demography, a heterogeneous labor force, a heterogeneous industrial sector, etc. It has to be seen as a first step. The main objective is to present a fresh way of thinking on the genesis of the pension composition as a mix of private savings (and incidentally hoarding) and the two main mandatory systems, which may be a stepping-stone to investigating the effects of changing demographics and retirement ages.
In this study we assume a stable demography, that is, a fixed population growth rate (which is negative for β<0.10) and a fixed age distribution. Clearly, this is unrealistic since the demographic parameters, i.e., birth and mortality rates, are never constant over time. However, since all demographic parameters change from one year to another, it is also not helpful to start from a specific population, say the American or the British, in a specific year, say 2016, and follow that population over a time period, when one wants to gain some insight into the general effects of demographic changes. The model which we have developed can be reformulated into a dynamic version, not starting from a stable equilibrium. But, even if we are not in an equilibrium situation and we assume birth and survival rates stay constant from now on, reaching the equilibrium path from the present disequilibrium would take many decades or even centuries. The attractiveness and the usefulness of studying an equilibrium model is that one can abstract from the specific peculiarities of different real situations, random shocks, intertemporal changes in values of model parameters, or in specifications of behavioral equations. We consider the stable equilibrium as the basic structure behind the reality. For country-specific studies that start from an actual demographic disequilibrium we refer a.o. to Krueger and Ludwig (2007), Börsch-Supan and Ludwig (2010), Miles (1999).

What is novel?

One central, and to our knowledge novel point in our analysis is that we admit for the possibility of four simultaneously existing old-age support arrangements, viz., individual voluntary savings, funded occupational pensions, social security on a pay-as-you-go-basis, and, as a residual component, the hoarding option. We call that mix the pension composition. That composition is not exogenously determined, but it is the joint result of the independent decisions of several parties, viz., all individual workers deciding on their individual savings, the median worker (or trade union) as representative of the body of workers deciding on the mandatory occupational pension system, and the median voter as representative of the electorate deciding on the existence and the size of social security (see also e.g., Galasso 2008, Galasso and Profeta 2004, Bruce and Turnovsky 2013). All decision-makers act against the background of a specific demography and this demography determines their decisions in the last resort. And therefore, taking utility functions and the production function as given, the main macro-economic variables like wages, interest, and investments are in this model in the end determined by the

demography as well. This stress on the different behavior of age cohorts and of two partly overlapping social classes, viz. active workers represented by the median worker and the electorate as a whole, including the retired, represented by the median voter, and the interpretation of the resulting pension composition as a Pareto equilibrium, seems to be novel as well.
We assume that in the economy the sources for capital investment are voluntary savings and mandatory savings for old age. In our time the weight of institutional pension funds, pension insurance companies, and institutional savings funds is becoming overwhelming. We refer to Boeri et al. (2006), Bijlsma, Van Ewijk, and Haaijen (2014), Conference Board (2010), and Mitchell (2008). It would have been possible within this model to make an extension such that individuals could also save for private investment without the explicit goal of old-age provision, but this would not have changed the essential message of this paper. Moreover, our addition of a mandatory funded occupational pension, where the median worker decides on the existence and the size of the pension, is also novel. With respect to individual savings most authors assume that utility is maximized subject to an intertemporal budget constraint. It is assumed that savings from one year may be used for consumption in the following year in order to smooth consumption, and that the citizen plans his/her savings and dis-savings for each future period over the remaining lifetime, given knowledge of his future annual incomes as well. This depicts a perfectly rational individual who has perfect knowledge of his future using the Euler conditions. But is such an assumption realistic when we face a future of about 45 years with 45 possible decision moments? Apart from the heroic assumption of the decision-making capacities of the individual, is it possible to know what the situation will be in the future many decades ahead? Instead, we make the rather naïve saving assumption that individuals expect their wages and their annual savings to remain constant over the years ahead. Each year to come, the individual will revise his savings decision based on the most recent situation. Although both assumptions do not seem perfectly realistic, we think that our assumption might be nearer to the truth in describing the savings behavior of ordinary humans than assuming an individual with perfect foresight on his lifetime 45-period budget equation. Our model can be generalized to encompass more general savings assumptions. For instance, we may assume that individuals decide each year on their savings for the current year only.

The main politically relevant results of our study are:

a. The finding that the room for governmental pension policy is rather restricted, because demography is the main determinant for the long-term equilibrium. It seems there are only a few possible political measures which all deal with the structure of old-age provisions: we may exclude one or more of the channels IVS, FF, or SS.[6] In this paper we looked only at the triple combination IVS+FF+SS. Moreover, we may change the legal retirement age.

b. The demography appears to be a fundamental determinant of macro-economics, having effects on the wage rate, the interest rate, and capital per worker. This suggests that (the now frequently tabooized) population policy could (or even should) be a powerful instrument for reaching macro-economic targets (cf. Lee and Mason 2010).

c. Aging of the population will result in a severe worsening of the net income of the retired.

d. Aging will also strongly increase the inequality between net wages and pensions to the disadvantage of the retired.

e. Increasing the retirement age would not have much effect on the financial situation of workers but it would improve the position of the retired.

f. Fertility increases will have a strong increasing effect on the interest rate.

g. Fertility increases may weaken social security and above a certain fertility rate social security may even vanish.

h. Fertility increases might strongly improve the situation of the retired.

Notwithstanding that this paper is based on a model which is oversimplified with respect to a number of issues, we believe that the way of thinking about the demographic problem in this paper sheds new light on one of the most threatening questions of our time: How do we provide for our old age, and what are the possibilities if we stick to the present institutional setup, where the pension composition is the joint result of the decisions of a number of parties?

Obviously, the model may be extended in many ways but already within the present setup its potential political relevance may be demonstrated by looking at a prediction for developed countries when we assume that they will stay at a low under-reproduction fertility level of 1.5 children per couple and a high survival rate of 0.98, that is a life expectancy of about 90 in our model, and that in the future 70 will be the new retirement age. Still, one step further would be to increase the maximum age in our model from 100 to 120 in order to reflect the increasing longevity in the present century. In Table 6 in the first line we present the situation with a maximum age of 100 and in the second line the outcomes of the model when the maximum age is increased to 120.

When the maximum age is kept at 100 the rough prediction of our model would be a *real* interest rate of about 1.6%, an aggregate premium of about 16.4% of gross wage and a pension/net wage ratio of 39.8/83.6=48% (see Table 6). The capital per worker would be

[6] It is possible in our model to assume that only the channels IVS and FF exist or only IVS and SS. Such restrictions exist in reality. For instance, in Chile there is no SS-system. The only case which seems impossible is when both IVS and FF are blocked, for then there would be no source of capital in the economy. In this paper we ignored these possibilities to focus on the main message.

high at about 3.12. When we assume a maximum age of 120, which implies a lengthening of the potential retirement period from 30 to 50 years, the interest rate would increase to 2.7% and the aggregate premium to 21.8%. The pension/net wage ratio would again be about ½ while the gross wage would decrease by about 3.5%. Capital per job would decrease by about 10%. Individual voluntary savings would be nearly non-existent, while the social security premium would lean toward 18%.

Table 6. A look at a bleak future.

SP=70 β=0.075 μ=0.98 ρ=0.89	Indiv. savings start/ finish %	Private pension %	Pre-mium fully funded %	Fully funded pension %	Pre-mium social security %	Social security pension %	Net wage %	Total pension %	Interest rate %	Gross wage	Capital-income ratio	Capital
	1	2	3	4	5	6	7	8	9	10	11	12
Max. age=100	0.0 4.9	11.2	3.3	15.1	8.2	13.5	83.6	39.8	1.6	0.997	3.133	3.123
Max. age=120	0.0 0.2	0.3	3.0	18.3	18.7	20.6	78.2	39.2	2.7	0.963	2.822	2.717

For legend see Appendix.

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Appendix

Legend:

1. Individual voluntary savings (IVS), initial and final, as percentage of gross wage
2. Private pension (IVS) as percentage of gross wage
3. Premium mandatory fully funded (FF) as percentage of gross wage
4. Mandatory funded pension (FF) as percentage of gross wage
5. Premium social security (SS) as percentage of gross wage
6. Social security pension (SS) as percentage of gross wage
7. Net wage as percentage of gross wage
8. Total pension as percentage of gross wage (benefit-income ratio)
9. Interest rate at equilibrium
10. Gross wage
11. Capital demand as percentage of gross wage (capital-income ratio)
12. Capital demand
13. Hoarding capital as percentage of capital demand (only if applicable)