I. QUANTIFICATION OF DISORDER

In order to quantify and compare how well annealed are the glasses prepared by SWAP (POLY system) or conventional quench protocols (LJ system), we have borrowed a measure of structural disorder that is based on the phonon broadening of the density of states [1]. In this method, the disorder follows

$$\chi = \frac{\Delta \omega}{\omega} \sqrt{\frac{N}{n_q}},$$  \hspace{1cm} (1)

where $\Delta \omega$ is the spectral width of a phonon peak of mean frequency $\omega$ and degeneracy $n_q$. For the first and second shear waves, we have $n_1 = 4$ and $n_2 = 4$, respectively. In Fig. 1(a-b), we show the low vibrational region of the density of state $D(\omega)$ of the LJ and POLY systems. For both systems, we find a shift to high frequencies of the first and second shear waves for the better annealed glasses, which is explained from an increase of the bulk shear modulus consistent with a previous work [2]. We also observe a gradual narrowing of the phonon width for less ductile glasses, indicating a decrease of structural disorder, i.e. a narrower distribution of the local shear modulus. In Fig. 1(c), we show $\chi$ for the two different systems and various quench protocols. Fig. 1(c) tells us that the LJ system prepared by a gradual quench (GQ) from $T_g^{\text{sim}}$ is roughly as stable as the POLY system prepared at $T_{\text{ini}} = 0.085$, and less surprising, that the LJ liquid quenched from roughly 9 times $T_g^{\text{sim}}$ compares in disorder with the POLY system equilibrated by SWAP at the highest temperature $T_{\text{ini}} = 0.3$. Finally, we compare the metric $\chi$ with the distance to the first instability $\langle \gamma_{\text{min}} \rangle$. We find a very good correlation between this two indicators. We expect the same collapse for larger systems with a shift of $\langle \gamma_{\text{min}} \rangle$ towards smaller values.

FIG. 1. Quantifying structural disorder from phonon broadening. Low vibrational spectrum of the LJ (a) and POLY (b) systems for various glass stabilities. (c) Measure of the structural disorder $\chi$ extracted from the phonon broadening of the first and second shear waves (Gaussian fits). (d) Comparison between $\chi$ and the distance to the first instability $\langle \gamma_{\text{min}} \rangle$. 
II. EFFECT OF A COARSE-GRAINING LENGTH

For some indicators, the sharp local fluctuation of a structural field could induce a decrease in the correlation as we select only one particle to define the core of a plastic instability. Several studies have addressed this issue by smoothing the structural field with a coarse-graining length ranging from 2 to 5 particle diameters [3, 4]. We have carefully checked the influence of such a procedure on the prediction score of various indicators. Our protocol works as follows: a particle receives a new value equals to the average of the field within a coarse-graining radius \( l \). In fig. 2(a) to (c), we show how does \( l \) affect the vibrality field \( \Psi \). Plotting \( C_{\text{min}} \) as a function of \( l \) for both the LJ (fig. 2(d)) and POLY (fig. 2(e)) systems, we find that the correlation remains approximatively constant for radius close to the first local minimum of the radial distribution function (\( \sim 1.5\sigma \)). Beyond the correlation decreases, as one exception the local yield stress, which is explained due to the coarse-grained nature of this measure for which particles within a cavity of radius \( 5\sigma \) share the same value and thus are barely affected by a coarse-graining of \( l = 1.5 \) to \( 4\sigma \). As some indicators (\( \Theta, S, naf\mu \)) can be very sensitive to \( l \), we choose to work with bare structural fields, \textit{i.e.} \( l = 0 \).

III. ANELASTIC EVENTS IN VERY STABLE GLASSES

In Fig. 3(a-e), we show the stress-strain curve of the loading and unloading after the first plastic event for 5 samples. The fact that these events are reversible indicates that the saddle annihilates along the elastic branch and thus is not present in the potential energy landscape at \( \gamma = 0 \). This result explains why Saddle Point Sampling performs badly at predicting plasticity in very stable glasses.
In the main text, we compare different family of indicators for the most and least ductile glasses. Here, we provide additional data by comparing for each indicator the change in the decay of correlation $C_{\gamma}$ for various material protocol, here the SWAP equilibrium parent temperature $T_{ini}$, see Fig. 4. As the glass becomes more stable, the correlation decay is postponed at larger strains. This trend can be explained by plotting the energy dissipation density $\Gamma$ as a function of the strain. We find a lot of dissipation in ductile glasses which corresponds to a large plastic activity. As a consequence, the microscopic structure is quickly reshuffled.

**FIG. 4.** Decay correlation and energy dissipation. (a-e) Decay correlation $C_{\gamma} = C(0, \gamma)$ for various indicators and glass stabilities. (f) Energy dissipation density $\Gamma$ as a function of the strain from ductile to brittle samples.

In Fig. 5 below, we provide additional data showing the correlation decay plotted as a function of the number of plastic events $n_{pl}$ occurring in the system.

**FIG. 5.** Decay correlation and plastic activity. (a-e) Decay correlation $C_{\gamma} = C(0, \gamma_{pl}(n_{pl}))$ for various indicators and glass stabilities. (f) Average plastic strain $\langle \gamma_{pl} \rangle$ as a function of the number of plastic events $n_{pl}$ from ductile to brittle samples.
V. DECAY AND GROWTH CORRELATION: LJ DATA

In Fig. 6, we provide the decay $C_\gamma$ and growth $C_{\Delta \gamma}$ correlation for the LJ system. We find similar results as the one found in glasses prepared by SWAP. We find a faster decay of $C_\gamma$ with respect to $\gamma$ for the most ductile glasses. Close to a plastic event, we find that indicators constructed from harmonic vibrational modes perform extremely well for both the most ductile (HTL) and well annealed (GQ) glasses. In contrast, we find the purely structural indicator $\Theta$ performing much better for well annealed glasses.

**FIG. 6.** Decay and growth correlation of the LJ data. Correlation decay $C_\gamma = C(0, \gamma)$ between a structural field computed at $\gamma_0$ and plastic events occurring at strain $\gamma$. Correlation growth $C_{\Delta \gamma} = C(\gamma_{pl} - \Delta \gamma, \gamma_{pl})$ between a structural field $\Delta \gamma$ away from a plastic event located at $\gamma_{pl}$. Results are for (HTL) ductile glasses (a-b) and (GQ) well annealed glasses (c-d). The different colors and symbols corresponds to the structural indicators labeled in (d).

In Fig. 7 below, we provide a copy of Fig. 6 showing the correlation decay plotted as a function of the number of plastic events $n_{pl}$ occurring in the system.

**FIG. 7.** Decay and growth correlation of the LJ data. Correlation decay $C_\gamma = C(0, \gamma_{pl}(n_{pl}))$ between a structural field computed at $\gamma_0$ and plastic events occurring at strain $\gamma$. Correlation growth $C_{\Delta \gamma} = C(\gamma_{pl} - \Delta \gamma_{pl}, \gamma_{pl})$ between a structural field $\Delta \gamma$ away from a plastic event located at $\gamma_{pl}$. Results are for (HTL) ductile glasses (a-b) and (GQ) well annealed glasses (c-d). The different colors and symbols corresponds to the structural indicators labeled in (d). The gray gradient in (a) and (c) indicates the change in strain from $\gamma_0 = 0$ (transparent) to $\gamma_{Yielding} \approx 7\%$ (opaque).

VI. DETECTION OF SOFT EXCITATIONS

In the harmonic approximation, the low-frequency regime of the density of state is dominated by phonons for large enough systems (a couple of thousand of particles in 2D). As a consequence, quasilocalized vibrational modes are fully hybridized with plane waves and one cannot extract their exact position as well as their respective frequency. In the present paper, we have gone beyond the harmonic approximation and extract non-linear vibrational modes (see appendix). These excitations do not suffer from hybridization and are the true representative of soft modes [5, 6]. This framework allows extracting precisely the location of soft vibrational modes and their frequency without being polluted by phonons.

In Fig. 8(a), we show the normalized histogram $D(\omega)$ of non-linear excitations of frequency $\omega$. We observe a tail at low frequencies that is consistent with the universal scaling $D(\omega) \sim \omega^4$ found in structural glasses [7, 8]. We fix the threshold used in Fig 8 of the main text as the onset frequency for which $D(\omega) \sim \omega^4$ breaks down (indicated by dashed lines in Fig. 8(a)). Under shear, we find a big increase of low-frequency excitations as it can be seen in Fig. 8(b) by
comparing $D(\omega)$ for quiescent configurations ($\gamma_0 = 0$) and configurations close but below yielding ($4\% < \gamma < 6\%$). This increase corresponds to a softening of excitations already present at $\gamma_0$. Quantifying in detail the actual change in the population of soft modes under shear will be left for upcoming work.

![Graph](image)

FIG. 8. **Soft excitations.** (a) Density of state of non-linear excitations in glasses prepared by SWAP. The black dashed line indicates the scaling $D(\omega) \sim \omega^4$. The vertical colored lines indicate the frequency threshold used to define low-energy excitations, $\omega^* = 0.7$ and $\omega^* = 1.2$ for the most ductile $T_{ini} = 0.3$ and least ductile $T_{ini} = 0.05$ glasses, respectively. (b) $D(\omega)$ for a finite $\gamma$ ($4\% < \gamma < 6\%$), comparing with $\gamma = 0\%$, for $T_{ini} = 0.05$.

**VII. YIELDING PATHWAY IN A BRITTLE GLASS FROM THE SOFTNESS FIELD $S$**

We provide a copy of the Fig.8 of the main text with the Softness field $S$ instead of the indicator $\Theta$. In Fig.9(c), we show the distribution $P(S)$ of the most and least ductile glasses. We observe an increase in the population of particles being characterized as "soft" in the most ductile glass in agreement with other indicators. This increase can be seen in the two most-left bottom snapshots (state point (1)) where we compare a ductile sample (red) and a brittle sample (blue). Here white particles correspond to particles having $S > 0$. At the onset of shear banding (state point (3)), we observe a more anisotropic spatial distribution with an increase of softness where the band is going to form, quantitatively confirmed in the profiles shown in the main text. We also find that Softness is able to characterize the shear band after yielding, which would enable to track how its size changes as a function of the strain.

![Graph](image)

FIG. 9. **Yielding pathway.** (a) Stress-strain curve of a brittle glass prepared via SWAP Monte Carlo at $T_{ini} = 0.05$. (b) Probability distribution function of the (strain) distance to threshold $\Delta \tau_y/\mu$ at zero strain for ductile (red) and brittle (blue) glasses. (c) Probability distribution of softness $S$. (d) Snapshots highlight the spatial distribution of soft regions (white color) with particles having $\Delta \tau_y/\mu < 3\%$ and $S > 0$, respectively. Black crosses show the location of low-energy excitations.
VIII. CLASSIFICATION OF STRUCTURAL INDICATORS

<table>
<thead>
<tr>
<th>Family</th>
<th>Reference</th>
<th>Features</th>
<th>Remarks</th>
<th>Proposed for</th>
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<tbody>
<tr>
<td><strong>Structural</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$\rho, \phi$: local density or local free volume</td>
<td>[9]</td>
<td></td>
<td>- can be applied for large variety of systems that go way beyond simple pairwise interactions</td>
<td>COLLOIDAL GLASSES, FOAMS, COMPUTER GLASSES</td>
</tr>
<tr>
<td>$s_2$: local excess entropy</td>
<td>[9]</td>
<td></td>
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<tr>
<td>$P, Q$: divergence of Voronoi cell anisotropy vector</td>
<td>[10]</td>
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<td></td>
<td></td>
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<tr>
<td>$\Theta$: generalized bond orientational order</td>
<td>[3]</td>
<td></td>
<td>- only valid for steric packings</td>
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<tr>
<td><strong>Machine Learning</strong></td>
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<tr>
<td>$S$: softness field</td>
<td>[11]</td>
<td></td>
<td>- require to set the number of modes used which can vary with the system size as well as the glass stability</td>
<td>COLLOIDAL GLASSES, FOAMS, COMPUTER GLASSES</td>
</tr>
<tr>
<td><strong>Linear response</strong></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$\Lambda$: soft modes</td>
<td>[12, 13]</td>
<td></td>
<td>- can be extend to provide the coupling of a soft mode with respect to an arbitrary deformation</td>
<td>COMPUTER GLASSES</td>
</tr>
<tr>
<td>$c_{\alpha}$: local heat capacity</td>
<td>[4, 14]</td>
<td></td>
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<tr>
<td>$\Psi$: vibrality</td>
<td>[15]</td>
<td></td>
<td>- do not require to set a number of modes, but the computational time scales as $\sim N^3$</td>
<td>COMPUTER GLASSES</td>
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<tr>
<td><strong>Linear response + coupling</strong></td>
<td></td>
<td></td>
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<tr>
<td>$\mu$: local shear modulus</td>
<td>[16]</td>
<td></td>
<td>- require a coarse-graining procedure</td>
<td>COMPUTER GLASSES</td>
</tr>
<tr>
<td>naf $\mu$: Atomic shear non-affinity</td>
<td>[17]</td>
<td></td>
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<td>$\dot{x}$: non-affine velocity</td>
<td></td>
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<tr>
<td><strong>Non-linear response</strong></td>
<td></td>
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<td></td>
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<tr>
<td>$\pi$: non-linear modes</td>
<td>[13]</td>
<td></td>
<td>- can be used to traceback the change in stiffness and angle of a STZ approaching a plastic event</td>
<td>COMPUTER GLASSES</td>
</tr>
<tr>
<td>SPS: saddle point sampling</td>
<td>[18]</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$\Delta \tau_y$: residual plastic strength</td>
<td>[19, 20]</td>
<td></td>
<td>- the local yield stress can be decompose as a function of the angle of the shear deformation, require to set a cavity radius and a strain step</td>
<td></td>
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**Symbols** – Efficiency/Parameters: $\bigstar$ fast / $\bigcirc$ expensive / $\bigcirc$ parameters free / $\bigtriangledown$ need to deform the sample.

Landscape: $\blacksquare$ provide estimates for activation barriers: local yield stress, stiffness, threshold strain.

Deformation coupling: $\bigtriangleup$ provide STZ angle.

Finite temperature: $\bigstar$ valid beyond the athermal limit.

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