A Unique Multi-Messenger Signal of QCD Axion Dark Matter

Supplementary Material

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This Supplementary Material is organized as follows: In App. I we discuss both the gravitational wave and radio signals dependence on the Dark Matter (DM) spike parameters. Appendix II discusses the velocity distribution of the DM, highlighting its limitations and how it can be addressed in future work. Finally, App. III discusses the dependence of the radio signal on the Neutron Star (NS) parameters. Here, we also speculate about the amplification of the radio signal if the neutron star had magnetic field strengths up to 10^{15} G or spin periods down to 0.1 s.

I. GRAVITATIONAL WAVES AND SPIKE DEPENDENCE

The phase difference between a vacuum inspiral and the one considered here is given by,

$$\Delta\psi = \tilde{\phi} - \phi\,, \tag{S1}$$

where $\phi = -\frac{3}{4}\left(8\pi G_N \mathcal{M}_c f/c^3\right)^{-5/3}$ is the Newtonian vacuum phase evolution and $\tilde{\phi}$ is the phase evolution including dynamical friction, as given by Eq. (28) of Ref. [57]. The phase evolution of the gravitational wave signal provides a fundamental insight into the dynamics of the binary system. As shown in Fig. S1, the presence of a DM spike (with $\alpha > 2.0$) produces a considerable phase shift when compared to the evolution of a vacuum inspiral. Again for $\alpha > 2.0$, the specific phase evolution of any particular system can therefore be used to constrain α to high precision. For our baseline scenario of $\alpha = 7/3$, constraints on α correspond to an error on the DM density of $\mathcal{O}(0.01\%)$ at $r \approx 1.3 \times 10^{-8}$ pc. For $\alpha < 2.0$, the phase difference becomes increasingly difficult to probe. Assuming the masses of the two objects can be independently measured to high precision, the constraint on α provides a direct constraint on the DM density local to the position of the NS [97]. We do not account for errors on the overall normalisation of the DM density profile directly. The normalisation can also be measured, though it is degenerate with $M_{\rm NS}$ and $M_{\rm BH}$. To resolve the individual masses, higher order effects close to merger need to be accounted for. The errors associated with the individual mass determinations may dominate the error on the DM density normalisation at larger radii, but this is beyond the scope of the paper. We will address this in future work.

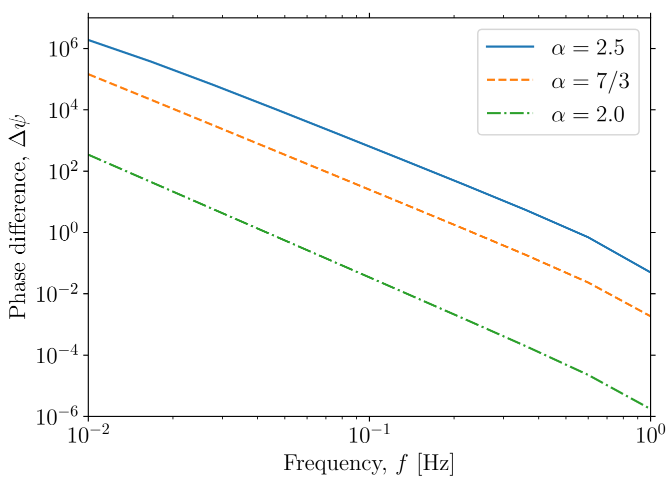

FIG. S1. **Phase difference between vacuum and DM spike inspiral.** We show the difference in the phase evolution of the IMBH-DM-NS system compared to a vacuum IMBH-NS inspiral for $\alpha = \{2.0, 7/3, 2.5\}$. As α is increased, the phase difference becomes larger. Similarly, the phase difference continues to be significant for higher frequencies when $\alpha > 7/3$. This persistent phase shift for large α is reflected as tighter constraints on the DM density, as seen in Fig. S2.

Figure S2 shows the constraint on the DM density from the GW signal, as described in the main text. The left and right panels show the constraint for $\alpha = 2.0$ and $\alpha = 2.5$, respectively. The error bars become larger for lower values of α; this can easily be understood from Fig. S1. As the inspiral progresses, the GW frequency becomes larger (equivalently, the radius decreases). Similarly, the phase difference becomes ever smaller, gradually approaching the vacuum inspiral phase evolution and therefore providing no probe of the DM density. As α is increased, the large phase differences persist further into the inspiral, allowing one to probe the DM density closer to the IMBH.

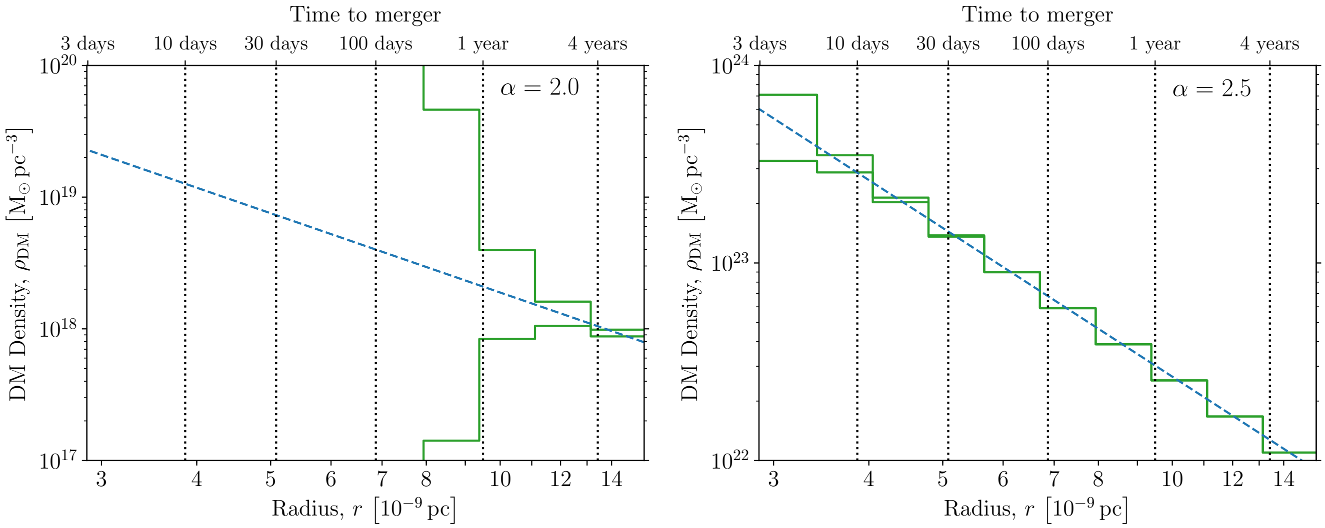

FIG. S2. **Error on the DM density from GW measurements of $\boldsymbol{\alpha}$.** Green bands show the 1σ uncertainties on the reconstructed DM density from analysing the GW waveform (for a system at $d = 0.01\,{\rm Gpc}$) over 10 bins in radius (measured from the position of the $10^4\,M_\odot$ IMBH). The fiducial density profiles are shown as a blue dashed line with $\alpha = \{2.0,\ 7/3,\ 2.5\}$ on the left and right respectively. Along the top axis we also label the approximate time-to-merger as a function of radius in the vacuum case.

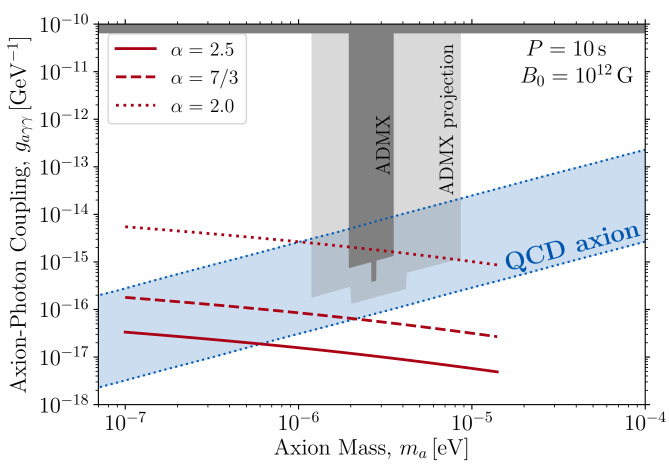

FIG. S3. **Projected reach in axion-photon coupling from radio observations.** Sensitivity curves of the SKA2 telescope (100 hours of observation) to the axion-photon coupling as a function of the axion mass for $\alpha = \{2.0,\ 7/3,\ 2.5\}$. Here, we assume a radial separation of $r = 3 \times 10^{-9}\,{\rm pc}$, $d = 0.01\,{\rm Gpc}$ for the distance to the system, $B_0 = 10^{12}\,{\rm G}$ for the NS magnetic field, $P = 10\,{\rm s}$ as the NS spin period, and $\theta = 90^\circ$. The predicted range of parameters for the QCD axion are represented by the blue band, while the vertical and horizontal gray bands show the ADMX [30, 31] and CAST [32] limits, respectively.

Finally, in Fig. S3 we present the radio sensitivity for $\alpha = \{2.0,\ 7/3,\ 2.5\}$. As expected, the varying density, as shown in Fig. S2, amplifies or decreases the density close the IMBH. For $\alpha = 2.0$, it is still possible to probe a small range of the QCD axion parameter space, although the constraint on the DM density becomes significantly worse (see left panel of Fig. S2). When $\alpha = 2.5$, the density can be constrained extremely well down to small radii. The density is also increased by an order of magnitude compared to the $\alpha = 7/3$ scenario, subsequently increasing the sensitivity by a similar amount.

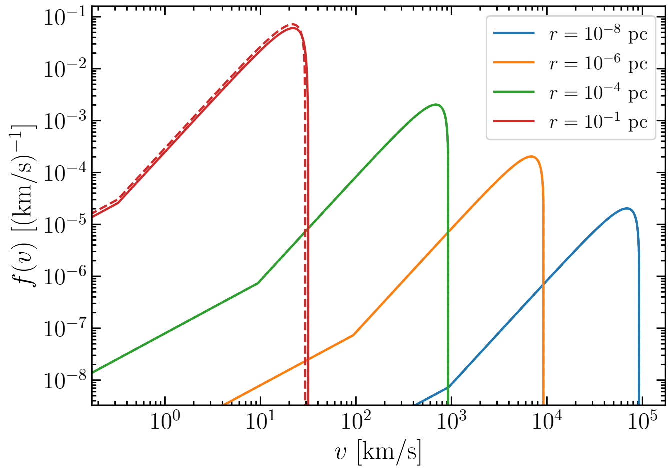

FIG. S4. **Dark Matter Speed distributions.** DM speed distribution derived from the Eddington Inversion formula, Eq. (S2), at different distances r from the central IMBH, $M_{\mathrm{IMBH}} = 10^4\, M_\odot$. Solid lines show the full calculation accounting for the potential due to the DM halo itself, while dashed lines show the approximate result, Eq. (S5), including only the potential due to the IMBH. Here, we assume $\alpha = 7/3$.

II. DARK MATTER VELOCITY DISTRIBUTION

We assume that the distribution of DM around the central black hole is spherically symmetric and that the velocity distribution of DM particles is isotropic. In this case, we can calculate the DM distribution function using Eddington's Inversion Formula:

$$f(\mathcal{E}) = \frac{1}{\sqrt{8}\pi^2} \int_0^{\mathcal{E}} \frac{\mathrm{d}\Psi}{\sqrt{\mathcal{E} - \Psi}} \frac{\mathrm{d}^2\rho}{\mathrm{d}\Psi^2}\,. \tag{S2}$$

Here, $\rho(r)$ is the density profile of the DM particles, while $\Psi(r)$ is the total gravitational potential, which in general includes a contribution from both the central mass and the mass enclosed in the DM halo. However, at small radii, the enclosed DM mass is small and we can typically neglect the contribution of the DM halo itself to the gravitational potential. We thus write $\Psi(r) = G_N\, M_{\mathrm{BH}}/r$ and re-express the density in terms of the potential:

$$\rho(\Psi) = \rho_{\mathrm{sp}} \left(\frac{r_{\mathrm{sp}}}{G_N\, M_{\mathrm{BH}}}\right)^{\alpha} \Psi^{\alpha}\,. \tag{S3}$$

We therefore find:

$$\begin{aligned} f(\mathcal{E}) &= \frac{\alpha(\alpha-1)}{\sqrt{8}\pi^2}\rho_{\mathrm{sp}} \left(\frac{r_{\mathrm{sp}}}{G_N\, M_{\mathrm{BH}}}\right)^{\alpha} \int_0^{\mathcal{E}} \Psi^{\alpha-2} \frac{\mathrm{d}\Psi}{\sqrt{\mathcal{E}-\Psi}} \\ &= \frac{\alpha(\alpha-1)}{(2\pi)^{3/2}}\rho_{\mathrm{sp}} \left(\frac{r_{\mathrm{sp}}}{G_N\, M_{\mathrm{BH}}}\right)^{\alpha} \frac{\Gamma(\alpha-1)}{\Gamma(\alpha-\frac{1}{2})}\mathcal{E}^{\alpha-3/2}\,. \end{aligned} \tag{S4}$$

The DM speed distribution at a radius r is then given by

$$f(v|r) = 4\pi v^2 \frac{f(\Psi(r) - \frac{1}{2}v^2)}{\rho(r)} = \frac{4}{\sqrt{\pi}} \frac{\Gamma\left(\alpha+1\right)}{\Gamma\left(\alpha-\frac{1}{2}\right)} \frac{v^2}{v_{\mathrm{max}}^{2\alpha}} \left(v_{\mathrm{max}}^2 - v^2\right)^{\alpha-3/2}\,. \tag{S5}$$

Here, we have defined $v_{\mathrm{max}} = v_{\mathrm{max}}(r) = \sqrt{2\Psi(r)}$, and we set the speed distribution to zero for $v > v_{\mathrm{max}}(r)$. With this definition, the speed distribution is normalised to one at any given radius:

$$\int_0^{v_{\mathrm{max}}(r)} f(v|r)\,\mathrm{d}v = 1\,. \tag{S6}$$

The peak velocity (far from the NS) is obtained by setting $\partial f(v|r)/\partial v = 0$, giving:

$$v_{\mathrm{peak}}^2 = v_{\mathrm{max}}^2 \left[\alpha - \frac{1}{2}\right]^{-1} = \frac{2G_N M_{\mathrm{BH}}}{r} \left[\alpha - \frac{1}{2}\right]^{-1}\,. \tag{S7}$$

In Fig. S4, we show the DM speed distribution at several radii r, assuming $\alpha = 7/3$ and $M_{\rm IMBH} = 10^4\, M_\odot$. Solid lines show the speed distribution derived from a full numerical calculation of $f(\mathcal{E})$ (using Eq. (S4) and including self-consistently the potential due to the DM halo). Dashed lines show the approximate speed distribution given in Eq. (S5) (neglecting the potential of the DM halo itself). We see that in all cases of interest to us, $r \lesssim 10^{-8}\,$pc, Eq. (S5) provides an excellent approximation to the full expression.

We note that as $r \to r_{\rm ISCO}$, the maximum DM speed tends towards the speed of light. As we discuss in the main text, the dominant effect from dynamical friction typically occurs at larger radii, where the DM speeds are lower and the non-relativistic formalism should apply. However, at a radius $r = 3 \times 10^{-9}\,$pc, the maximum DM speed is $v_{\rm max} \approx 1.7 \times 10^5\,$km/s $\approx 0.56\,c$. Adding also the infall velocity toward the conversion radius, the DM particles are accelerated up to $\sim 0.8\,c$. This suggests that towards the end of the inspiral, our non-relativistic formalism would over-estimate the speeds which can be reached by the DM particles. However, even at $r = 3 \times 10^{-9}\,$pc, the DM speeds are still only mildly relativistic ($\gamma \sim 1.7$), suggesting that this should be a small effect. We leave a more detailed analysis – including relativistic effects, boosting in the NS rest-frame and anisotropy of the infalling DM flux – to future work.

Finally, we have checked that the DM halo should survive the merger itself; the work done by dynamical friction during the five year inspiral is only a few percent of the total gravitational binding energy of the halo. A more detailed study of feedback on the DM halo in different systems is in preparation [97].

III. NEUTRON STAR PARAMETERS

In the Goldreich-Julian model for the NS magnetosphere [110], the amplitude of the radio signal (produced by the resonant axion-photon conversion) is completely determined by the magnetic field strength at the NS poles B_0 and the spin period P. In particular, they determine the number density of charged particles around the NS (Eq. (7)), and consequently the plasma frequency (6) and the conversion radius (8). By plugging Eq. (10) into Eq. (9), we find the scaling relatation of the radiated power with respect to B_0 and P to be

$$\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}\Omega} \propto B_0\, P \left(\frac{3\cos^2\theta + 1}{|3\cos^2\theta - 1|}\right) \left[g_{a\gamma\gamma}^2\, m_a\, \rho_{\rm DM}(r_c)\, v_c\right] . \tag{S8}$$

where the quantities in the squared parentheses are almost independent of the NS parameters. Hence, for a given axion mass, the larger the NS magnetic field and spin period, the larger the radiated power. Furthermore, the radiated power can be significantly larger for $\cos\theta = 1/\sqrt{3}$. We note that by neglecting the second term in the expression (12) for the velocity v_c, one can show that the minimum detectable axion-photon coupling from Eq. (14) scales with the axion mass as $g_{a\gamma\gamma}^{\rm min} \sim m_a^{-1/2}$.

In Fig. S5 we report the projected sensitivity curves of SKA2 for three different values for the NS magnetic field strength (left panel) and three different values for the NS spin period (right panel), while fixing $r = 3 \times 10^{-9}$ pc, $d = 0.01$ Gpc, $\theta = 90^\circ$ and $\alpha = 7/3$. As expected, the larger the magnetic field and the spin period, the smaller the axion-photon coupling that can be probed by SKA2. Moreover, larger axion masses can be explored for larger magnetic fields or smaller spin periods. Increasing B_0, or decreasing P, causes the axion-photon conversion to occur at a larger radius (see Eq. (8)). The requirement that the the conversion radius is larger than the size of the NS ($r_c \geq r_{\rm NS} = 10$ km) is then satisfied for a wider range of axion masses.

We note that the values for B_0 and P considered here cover almost all the possible properties of active NSs, according to the ATNF pulsar catalog [117]. On the other hand, old dead NSs are expected to have low magnetic fields and large spin periods, providing weaker sensitivities. However, their properties are quite uncertain and model-dependent [50, 118, 119]. In all cases, we have also verified that the plasma remains bound to the NS. Even down to the innermost stable circular orbit, the forces from the NS magnetosphere dominate over the gravitational force from the BH by many orders of magnitude.

Finally, we conclude this appendix by discussing how the radio signal depends on the model for the NS magnetosphere. In deriving the expression for the conversion radius (8), we have assumed that the NS plasma is dominated by electron and positrons. The presence of ions would reduce the conversion radius once all the other quantities are fixed, since the plasma frequency (6) would be suppressed by the larger mass m_c. This would increase the radio signal according to Eq. (9) at the expense of probing smaller axion masses. Moreover, different analytic models and numerical simulations of the NS magnetosphere lead to different profiles for the plasma frequency [120]. This might increase or reduce the amplitude of the radio signal. For example, Ref. [50] pointed out that the electrosphere model instead of the Goldreich-Julian model for the NS plasma [121] provides larger or smaller radio signals depending on

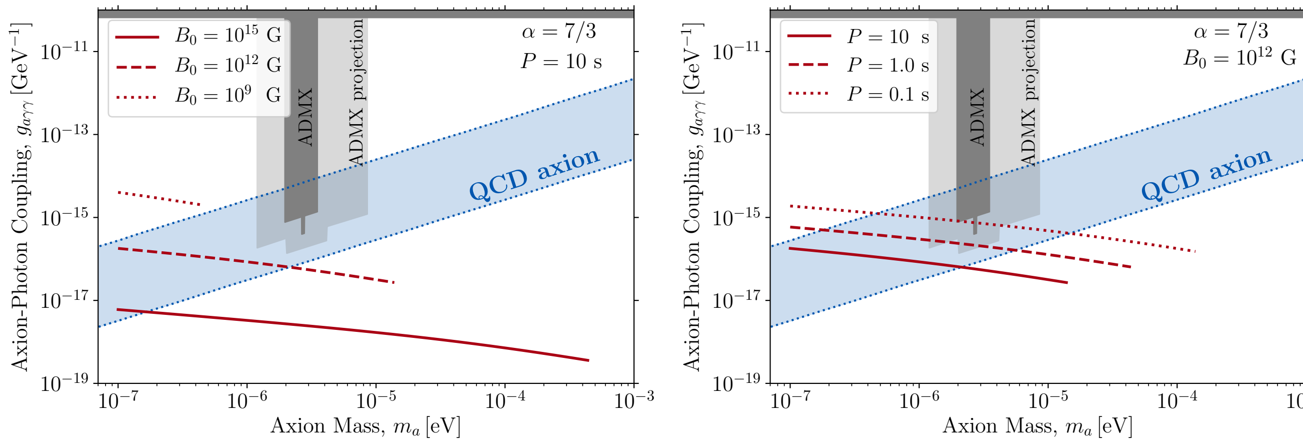

FIG. S5. **Projected reach in axion-photon coupling from radio observations.** Sensitivity curves of SKA2 telescope (100 hours of observation) to the axion-photon coupling as a function of the axion mass for three values of the NS magnetic field strength (left panel) and three values of the NS spin period (right panel). Here, we assume $\alpha = 7/3$ for the slope of the DM spike, $r = 3 \times 10^{-9}$ pc, $d = 0.01$ Gpc, and $\theta = 90°$.

the polar angle and the misalignment between the magnetic dipole axis and the rotation axis.

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