Off-Axis Dipole Forces in Optical Tweezers by an Optical Analog of the Magnus Effect

Spreeuw, R.J.C.

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It is shown that a circular dipole can deflect the focused laser beam that induces it and will experience a corresponding transverse force. Quantitative expressions are derived for Gaussian and angular top hat beams, while the effects vanish in the plane wave limit. The phenomena are analogous to the Magnus effect, pushing a spinning ball onto a curved trajectory. The optical case originates in the coupling of spin and orbital angular momentum of the dipole and the light. In optical tweezers the force causes off-axis displacement of the trapping position of an atom by a spin-dependent amount up to $\lambda/2\pi$, set by the direction of a magnetic field. This suggests direct methods to demonstrate and explore these effects, for instance, to induce spin-dependent motion.

The optical analog of the Magnus effect occurs when an atom with an angular momentum $m_j = 1$ is circularly polarized by a focused laser beam. The direction of the polarization must be set by the quantization axis $y$, and a magnetic field $B\parallel y$ generates the field at the quantization axis. The dipole then emits a spiral wave that interferes with the incoming wave, resulting in a deflection of the beam and a corresponding reaction force acting on the atom. The direction of this force changes sign with the detuning from the atomic resonance.

In optical tweezers, the off-axis displacement of the trapping potential created by a circular dipole can be observed. For example, when the detuning $\Delta < 0$, the transverse force shifts the trapping position away from the optical axis by an amount $\lambda = \lambda/2\pi$. This effect can be used to trap an atom where the focus is not located, as opposed to the typical scenario where the atom is detected at the focus of the laser beam. The geometric argument based on light scattering shows that the true displacement of the trapping potential is a direct consequence of the apparent deflection of the focused laser beam.

A simple geometric argument based on light scattering shows that the true displacement of the trapping potential is a direct consequence of the apparent deflection of the focused laser beam. This prediction provides a direct method to demonstrate and explore these effects in optical tweezers.
displacement of an emitting circular dipole [2,3]. Tweezer trap displacements have previously been calculated numerically, for specific beam shapes, in terms of vector and tensor light shifts [14,31]. The geometric argument given here directly shows that, for a circular dipole, the displacement is simply \( \pm \lambda / 2 \pi \) and is remarkably independent of many parameters, including the laser detuning, trap frequency, and even the detailed shape of the trap. A comparison of a Gaussian beam with an angular top hat beam illustrates this. This profound insight provides the basis for state-dependent manipulation of atomic motion within the tweezer.

We describe these effects in terms of interference between the focused incident beam with the wave scattered by the circular dipole, see Fig. 1. In the optical theorem, such interference is used to describe the attenuation of light in terms of the forward scattering amplitude [32]. In contrast, here we concentrate on beam deflection, as a consequence of the tilt of the spiral wave front with respect to the incident wave. Two simple atomic-level schemes serve as examples; (i) a \( j = 0 \rightarrow j' = 1 \) and (ii) a \( j = 1 \rightarrow j' = 0 \) transition. Case (i) is conceptually simpler and most suited to observe the Magnus-like deflection of a weak, near-resonant probe beam. Case (ii) offers interesting extra opportunities in the usual far-off-resonance regime of optical tweezer experiments.

Starting with case (i), the \( j = 0 \rightarrow j' = 1 \) transition, we focus a linearly polarized \( (E|y) \), monochromatic laser onto a single atom placed in the origin, see Fig. 1. A magnetic field \( B|\hat{y} \) defines the quantization axis and splits the excited state into three \( |j, m_j\rangle_y \) sublevels, separated by the Zeeman shift \( \sim \mu_B B / \hbar \), with \( \mu_B \) the Bohr magneton [33]. We tune the laser close to the \( \Delta m_j = +1 \) transition, with a detuning \( \Delta = \omega_k - \omega_0 \) small compared to the Zeeman shift, so that the \( \Delta m_j = 0, -1 \) transitions can be neglected (for example, \( \Delta / 2 \pi \approx 10 \) and \( \mu_B B / \hbar \approx 100 \text{ MHz} \)). The emission by the induced circular dipole has a spiral wave front in the \( \hat{xz} \) plane, tilted with respect to the forward \( \hat{z} \) direction of the incident beam.

We represent the light fields by their angular spectrum [34,35], using spherical \( k \)-space coordinates \( \langle k, \theta, \phi \rangle \). For monochromatic light, with \( k = \omega / c \) fixed, the incident field can be written as \( \frac{1}{2} E_{in}(\Omega) e^{i\omega t} + \text{c.c.} \), with \( \Omega = (\theta, \phi) \). The total field is the sum of the incident and scattered waves. Writing only the positive frequency \( (\sim e^{-i\omega t}) \) components, the total field reads

\[
E(\Omega) = E_{in}(\Omega) + E_{sc}(\Omega), \tag{1}
\]

with \( E_{sc}(\Omega) \) the scattered wave.

We define the radiant intensity

\[
J(\Omega) = |E(\Omega)|^2 / 2Z_0, \tag{2}
\]

with \( Z_0 = 1 / \varepsilon_0 c \), so that \( J(\Omega) d\Omega \) is the power flowing out of an infinitesimal solid angle \( d\Omega = \sin \theta d\theta d\phi \) around

\[
\mathbf{u}_\Omega = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta). \tag{3}
\]

Combining Eqs. (1) and (2), the total radiant intensity is the sum of three terms,

\[
J(\Omega) = J_{in}(\Omega) + J_{sc}(\Omega) + J_{if}(\Omega). \tag{4}
\]

The interference term

\[
J_{if}(\Omega) = \frac{1}{2Z_0} [E_{in}^{(coh)}(\Omega) \cdot E_{in}^{(coh)}(\Omega) + \text{c.c.}] \tag{5}
\]

contains only the coherent component of the scattered field. An incoherent component would contribute to \( J_{sc}(\Omega) \) but not to \( J_{if}(\Omega) \). For simplicity we assume that the scattered field is entirely coherent, essentially restricting ourselves to the low-saturation limit (see Supplemental Material [35]).

The deflection of the light beam can be expressed as the change in average wave vector \( \langle \mathbf{k} \rangle - \langle \mathbf{k} \rangle_{in} \) between the total (incident plus scattered) and the incident wave, using

\[
\langle \mathbf{k} \rangle_{in} = k \int u_{\Omega} J_{in}(\Omega) d\Omega, \tag{6}
\]

and similar for \( \langle \mathbf{k} \rangle \), omitting the subscript. Assuming (again for simplicity) that nonradiative decay is absent, we shall write \( P_{in} = P \) throughout.

The deflection is entirely determined by the interference term \( J_{if}(\Omega) \). The scattered light itself does not contribute, due to the symmetry of the dipole radiation pattern, \( J_{sc}(\mathbf{r}, \mathbf{p}, \theta, \phi) = J_{sc}(\mathbf{r} - \mathbf{p}, \pi - \theta, \pi + \phi) \), so that \( \int u_{\Omega} J_{sc}(\Omega) d\Omega = 0 \). For the deflection, we therefore have

\[
\delta(\mathbf{k}) = \langle \mathbf{k} \rangle - \langle \mathbf{k} \rangle_{in} = \frac{k}{\mathbf{p}} \int u_{\Omega} J_{if}(\Omega) d\Omega, \tag{7}
\]

and for the force on the atom, by momentum conservation,

\[
\mathbf{F} = -\frac{P}{\omega} \delta(\mathbf{k}) = -\frac{1}{c} \int u_{\Omega} J_{if}(\Omega) d\Omega. \tag{8}
\]

While this expression includes the forward radiation pressure force, in the cases of interest here the main force will be transverse to the optical axis, \( \mathbf{F} \approx F_x \hat{x} \). Then (approximately) \( \delta(\mathbf{k}) \perp \langle \mathbf{k} \rangle_{in} \) and with \( \langle \mathbf{k} \rangle_{in} \approx k \mathbf{u}_z \) the deflection angle is

\[
|\delta \theta| \approx \frac{|\delta(\mathbf{k})|}{k}. \tag{9}
\]

We will choose \( \delta \theta > 0 \) if \( F_x < 0 \).

Let us now introduce specific field patterns to calculate \( J_{if}(\Omega) \). We take the dipole to be circular, \( \mathbf{p} = pe^{i\alpha} \mathbf{u}_+ \), with \( \mathbf{u}_\pm = (\hat{x} \mp i \hat{z}) / \sqrt{2} \) denoting spherical unit vectors, and \( \alpha \) the phase of the \( p_z \) component of the dipole, relative to the local driving field. The field radiated by a coherent dipole [32], in angular coordinates, takes the form [35]
\[ \mathbf{E}_{\text{sc}}(\Omega) = \mathcal{E}_{\text{sc}} e^{i\Delta}(\mathbf{u}_0 \times \mathbf{u}_+) \times \mathbf{u}_\Omega, \]  
(10)

with corresponding \( J_{\text{sc}}(\Omega) \) given by Eq. (2). Here \( \mathcal{E}_{\text{sc}} = p k^2 / 4\pi\epsilon_0 > 0 \) is a real-valued amplitude. Assuming the steady state of the optical Bloch equations for a two-level system, \( \cot \alpha = -\Delta / \gamma \), where \( \Delta = \omega - \omega_0 \) the detuning from the \( \Delta n_j = +1 \) transition, and \( \gamma = \hbar \alpha D^2 / 8\pi\epsilon_0 \hbar \) the half width of the transition, with \( D \) the transition dipole moment.

For comparison, we consider two different types of incident beams, Gaussian \((G)\) and “angular top hat” \((\Pi)\), where the latter approximates the output of a uniformly illuminated focusing lens. The field for these two beams can be written as

\[ \mathbf{E}^{(G)}_m(\Omega) \approx \mathcal{E}^{(G)}_0 \exp(-\theta^2 / w_\theta^2) \mathbf{u}_x(\Omega), \]  
(11)

\[ \mathbf{E}^{(\Pi)}_m(\Omega) = \mathcal{E}^{(\Pi)}_0 \Pi(\theta / 2r_\theta) \mathbf{u}_x(\Omega), \]  
(12)

with amplitudes \( \mathcal{E}^{(G)}_0 \) and \( \mathcal{E}^{(\Pi)}_0 > 0 \). The Gaussian beam has an angular width \( w_\theta \), which is related to the minimum waist \( w_0(1 / \epsilon^2 \text{ spatial radius of intensity}) \) as \( w_0 \omega_0 = \lambda / \pi \). For the angular top hat, \( \Pi(\theta / 2r_\theta) \) is the rectangular function with angular half-width \( r_\theta \) and unit amplitude. Its spatial profile near the focus is the familiar Airy disk pattern. Note that neither propagation phases nor the Gouy phase are recognized for the plane wave limit, as the above expressions are in angular coordinates.

The polarization vector \( \mathbf{u}_x(\Omega) \) is transverse to \( \mathbf{u}_\Omega \); it is obtained by corotating \( \hat{\mathbf{x}} \) when rotating \( \hat{\mathbf{z}} \rightarrow \mathbf{u}_\Omega \), i.e., rotating by \( \theta \) around an axis \( \hat{\mathbf{z}} \times \mathbf{u}_\Omega \) \([11,36]\),

\[ \mathbf{u}_x(\Omega) = \begin{pmatrix} \cos \theta \cos^2 \phi + \sin^2 \phi \\ \cos \theta - \sin \phi \cos \phi \\ - \sin \theta \cos \phi \end{pmatrix}. \]  
(13)

When combining Eq. (10) with Eq. (11) or (12) in Eq. (5), the interference term contains the amplitude product \( \mathcal{E}^{(G)}_0 \mathcal{E}_{\text{sc}} \) or \( \mathcal{E}^{(\Pi)}_0 \mathcal{E}_{\text{sc}} \). In the low-saturation limit, the amplitude \( \mathcal{E}_{\text{sc}} \) is proportional to \( \mathcal{E}^{(G)}_0 \) or \( \mathcal{E}^{(\Pi)}_0 \). Their ratio can be obtained by requiring energy conservation \([35]\).

Upon insertion of the resulting ratios \( \mathcal{E}_{\text{sc}} / \mathcal{E}^{(G)}_0 \) and \( \mathcal{E}_{\text{sc}} / \mathcal{E}^{(\Pi)}_0 \) into Eq. (5), the interference term \( J_m(\Omega) \) becomes proportional to the total power; the deflection angle is then independent of power.

In Fig. 2 we show \( J_m(\Omega) \) in the plane of the dipole \((\phi = 0)\), together with the total radiant intensity \( J(\Omega) \). For the Gaussian beam, the effect of \( J_m(\Omega) \) is to shift the peak and the average of the direction of propagation away from \( \theta = 0 \). For the angular top hat, the interference leads to an intensity gradient across the angular width of the beam, whereas the edges stay at the same angle. In this case, the intensity gradient leads to a change in average beam direction.

Finally, the deflection angle is obtained by integration as in Eq.(7),

\[ \delta \theta \approx \frac{3}{4} \frac{\gamma \Delta}{(\gamma^2 + \Delta^2)} \times \begin{cases} \frac{w_\theta^2}{4} & \text{(Gaussian)} \\ \frac{r_\theta^4}{4} & \text{(angular top hat)} \end{cases} \]  
(14)

and the reaction force as

\[ F_x \approx -\frac{P}{c} \delta \theta. \]  
(15)

The results are given as the leading order in \( w_\theta \) and \( r_\theta \). The deflection angle reaches maximal values of \( \delta \theta = \pm 3w_\theta^2 / 8 \) and \( \pm 3r_\theta^4 / 32 \), respectively, for \( \Delta = \pm \gamma \); it vanishes in the plane wave limit, \( w_\theta, r_\theta \rightarrow 0 \). In this central result, we recognize in the detuning dependence that the force is essentially a dipole force \([37]\), arising from polarization gradients near the focus of a linearly polarized light beam \([7–9,12,14,24,31]\).

We now address the question of how we can observe the deflection of a laser beam, either directly or via the reaction force on the atom. As shown by Eq. (14), the angle of deflection by a single atom is small compared to the divergence angle, \( \delta \theta \ll r_\theta, w_\theta \). A direct observation will thus require sufficiently high signal-to-noise ratio, similar to what was achieved in the recent observation of apparent \( \lambda \) displacement of an emitter \([3]\). With maximal signal occurring near resonance \((\Delta = \pm \gamma)\), where the photon scattering rate is high, the best approach would be to hold the atom in an independent trap, such as an ion trap or a tight optical tweezer. One can then look for the deflection of a weak, near-resonant probe beam. A larger deflection angle may be obtained if multiple atoms cooperate. For example, one may consider dense clouds of subwavelength size, containing tens to hundreds of atoms, that have been
observed to show collective scattering properties [38,39]. Another possibility may be to use elongated, (quasi)one-dimensional samples with tight ($\lesssim 3\lambda$) radial confinement, achievable, e.g., in optical lattices [40–42] and on atom chips [43].

The second mode of observation, via the force on the atom, provides extra opportunities to manipulate spin-dependent atomic motion in an optical tweezer. To see this we consider case (ii): an optical tweezer trapping an atom with a $j = 1 \to j' = 0$ transition. The $|m_j = \pm 1\rangle_y$ states now couple to the $(\sigma^\pm)_y$ components of the light field and therefore experience opposite forces $F_x$. In this configuration, there is no need for a separate probe beam [24], the far off-resonance light ($\Delta/2\pi \sim 1–10$ THz) of the tweezer itself is sufficient. The photon scattering and associated heating rates can thus be kept as low as in typical tweezer experiments. In this case, we assume that the Zeeman shift is large compared to the trap depth $U_0$ (for example, $\mu_B B/h \sim 10$ and $U_0/h \sim 1$ MHz.) Looking at the spiral wave of a $u_x$ dipole shown in Fig. 1, we can readily see that the relative tilt of the forward wave fronts will vanish if we displace the atom by $\lambda$ in the $x$ direction. By thus aligning the wave fronts, the transverse force should vanish. An atom in the $|m_j = -1\rangle_y$ sublevel will therefore find an equilibrium position in the tweezer at a displaced off-axis location $x_{eq} = \lambda$. By the same reasoning, the $|m_j = +1\rangle_y$ sublevel will have the opposite displacement, so that for the $j = 1 \to j' = 0$ transition

$$x_{eq} = -(m_j) \lambda.$$  

(16)

The tweezer thus traps the atom off axis, where the focus is not, in a spin-dependent location. For the situation considered here, the $|m_j = 0\rangle_y$ state would be untrapped, for a lack of $\pi$ component in the laser polarization. This could be changed by rotating B. In particular, setting the angle between $E_{in}$ and B to $\arctan(\sqrt{2})$, the polarization components $\sigma^+, \pi$, and $\sigma^-$ would become equal. At this “magic angle” all three spin components would be trapped with a Stern-Gerlach type separation [24,44].

These simple geometric arguments are backed up by a calculation [35], that shows that Eq. (14) for the beam deflection is multiplied by $1 \equiv kd$, for a $u_x$ dipole displaced by $d$ in the $x$ direction, to lowest order in $d$. Thus, the transverse force indeed vanishes for a transverse displacement of $d = k^{-1} = \lambda$ in the $x$ direction. Remarkably, the size of the displacement is independent of the detuning, the beam divergence angle, the trap frequency, or even the precise shape of the beam (Gauss vs angular top hat). This profound insight follows from the geometric properties of the scattering problem.

The off-axis trapping locations offer interesting opportunities to manipulate the motion of atoms in the tweezer, see Fig. 3. Let us imagine an atom trapped in the $|m_j = 1\rangle_y$ state. As we slowly rotate the magnetic field in the $yz$ plane, the orientation of the atom will adiabatically follow the rotating quantization axis. After rotating the field $y \to z \to -y \to -z$, the spin will have maintained its orientation relative to B, i.e., $|m_j = 1\rangle_y \to |m_j = -1\rangle_y$, since B has changed direction. The space-referenced spin flip implies that the atom must have moved to the other side of the optical axis. Thus, by rotating the magnetic field in the $yz$ plane at a frequency $\omega_B$, we effectively shake the trap back and forth: $x_{eq} = -(m_j) \lambda \cos \omega_B t$. The $m_j = \pm 1$ levels are shaken with opposite phase.

Shaking the trap at an amplitude $\lambda$ is equivalent to a harmonic driving force $F_x = m\omega^2 \lambda \cos \omega_B t$, with $\omega$ the trap frequency. Resonant shaking, $\omega \approx \omega_B$, will induce an oscillatory motion in the trap. For example, for a tweezer with a laser wavelength of $\lambda \approx 0.8$ $\mu$m, a Gaussian waist of 2 $\mu$m, holding an atom of mass $m = 88 u$ in a 20 $\mu K$ deep trap, the trap frequency will be $\omega \approx 2\pi \times 7$ kHz. In a simple driven harmonic oscillator model, only 3.5 drive cycles would impart enough energy to kick the atom out of the trap, corresponding to a velocity of $\sim 6$ $cm/s$. In reality one would of course need to take anharmonicity into account. The point here is that magnetic field modulation can easily induce oscillatory motion in the trap, which can then be detected either as trap loss, or by using time-of-flight imaging methods. For the required magnetic field, a few gauss should be sufficient to ensure that the Larmor

FIG. 3. Optical tweezer operating on a $j = 1 \to j' = 0$ transition, leading to $\pm \lambda$ off-axis displacements for the $(m_j)_y = \mp 1$ sublevels (upper left). The four panels show, in clockwise order, the effect of a rotation of the quantization axis (B), through a cycle $y \to z \to -y \to -z$. While the B-referenced $(m_j)_y$ of an atom is conserved, the space-referenced $(m_j)_y$ is not. The locations of the $(m_j)_y = \pm 1$ traps move up and down along the $x$, axis, in antiphase. If B is rotated at the trap frequency, spin-dependent oscillatory motion in the tweezer can be induced.
frequency is large compared to the trap frequency. Rotating the field at frequencies of \( \sim 10 \, \text{kHz} \) is well possible, being comparable to what is used in time-averaged, orbiting potential traps [45].

Many available atomic-level systems should be suitable to display off-axis tweezer trapping. For example, in \( ^{88}\text{Sr} \) the transition \( ^{3}\text{P}_{2} \rightarrow ^{3}\text{S}_{1} \) would provide a \( j = 2 \rightarrow j' = 1 \) transition. The outer \( \langle m_{F} \rangle_{y} = 2(-2) \) state couples only to the \( \sigma^{-}(\sigma^{+}) \) polarization component, so its spatial shift will be \( \pm \hbar(\pm \lambda) \). Using \( ^{87}\text{Rb} \) one could operate a tweezer red detuned to the \( D_{1} \) line (795 nm), driving the two hyperfine lines \( F = 2 \rightarrow F' = 1, 2 \). Also in this case the outer state \( \langle m_{F} \rangle_{y} = 2(-2) \) is displaced by \( \pm \hbar(\pm \lambda) \), as long as the detuning stays small compared to the fine structure splitting of the \( D \) lines.

In summary, it is predicted that a circular dipole can deflect a focused laser beam, similar to a spinning ball deflecting a stream of air in the Magnus effect. The reaction force on the atom leads to spin-dependent, off-axis displacement of atoms trapped in an optical tweezer. For a pure circular dipole, the displacement is \( \pm \hbar \lambda \), independent of many trap parameters. An external magnetic field can be used to induce spin-dependent motion or to perform Stern-Gerlach type analysis of the spin states of the atom in the tweezer.

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* r.j.c.spreeuw@uva.nl


[27] See also [28] for a very different kind of analogy of the Magnus effect.


