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# Case-Based Reasoning with Precedent Models: Preliminary Report

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**Abstract.** Formalizing case-based reasoning is an important topic in AI and Law, which has been discussed using various approaches, such as formal dialogue games, abstract dialectical frameworks. In this paper we model case-based reasoning by using the formal argument semantics of case models. With the precedent models we present, the validity of legal arguments in the case-based reasoning process can be shown formally. We also present a case study of precedent models in a real legal domain and evaluate the validity of arguments in case-based reasoning.

**Keywords.** legal argumentation, case-based reasoning, precedent

## 1. Introduction

This paper focuses on the formalization of case-based reasoning within the case model formalism developed in [1]. Case-based reasoning is an important topic in AI and Law. Research in this topic is associated with argumentation as arguments are main outcomes in case-based reasoning, which are given by the parties in courts to defend their positions. Notions in case-based reasoning such as case comparisons and legal argument evaluation are good examples of computational argumentation theory.

As summarized by Bench-Capon [2], HYPO and its successors have exercised great influence in the study of case-based reasoning. Ashley and other researchers model legal reasoning by representing cases with factors [3, 4, 5, 6, 7, 8]. With a current situation, users can retrieve precedents according to the shared factors between precedents and the situation, and try to analogize and distinguish them.

Case-based reasoning can be formalized and connected with other reasoning approaches. Hafner and Berman discuss the role of context in case-based reasoning [9]. Wyner and his colleagues discuss the relation between cases and arguments [10]. Prakken and Sartor [11] model case-based reasoning in a formal dialogue game and combine case-based reasoning with rule-based reasoning. Prakken and his colleagues [12] also

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model case-based reasoning with ASPIC+ framework [13]. Horty and Bench-Capon [14] present a formalism on case-based reasoning with reason models that combine facts, rules and outcomes in precedents. Horty formalizes dimensions in case-based reasoning instead of standard factors [15]. Al-Abdulkarim and her colleagues [16] formalize CATO with abstract dialectical frameworks [17]. Other approaches include dialectical arguments [18], ontologies in OWL [19] and abstract argumentation [20].

In [1], Verheij presents a form of argumentation theory which formally defines the validity of arguments through case models [21]. A case model is a set of cases combined with a preference relation. The theory has been implemented in a Prolog program [22]. With case models, the validity of generated arguments and attacks can be distinguished in coherent, presumptive and conclusive validity. It provides formal semantics for combining rules, arguments and cases [1]. With this approach, we can analyze case-based reasoning with logical tools and evaluate arguments formally.

Outline: we show the theory part of the precedent model formalism based on Verheij's approach in Section 2. We give a case study of precedent models in a real legal domain in Section 3. We discuss how arguments' validity can be used in case-based reasoning in Section 4. We also compare our approach with others in the discussion section.

## 2. Precedent models

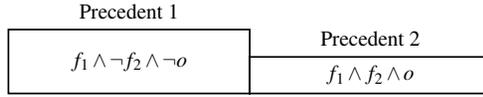
The precedent models defined in this section are based on the case models formalism addressed by Verheij [1]. The formalism introduced in this paper uses a propositional logic language  $L$  generated from a set of propositional constants. We write  $\neg$  for negation,  $\wedge$  for conjunction,  $\vee$  for disjunction,  $\leftrightarrow$  for equivalence. The associated classical, deductive, monotonic consequence relation is denoted  $\models$ .

Precedents consist of factors and outcomes. Both *factors* and *outcomes* are literals. A literal is either a propositional constant or its negation. We use  $F \subseteq L$  to represent a set of factors,  $O \subseteq L$  to represent a set of outcomes. The sets  $F$  and  $O$  are disjoint and consist only of literals. If a propositional constant  $p$  is in  $F$  (or  $O$ ), then  $\neg p$  is also in  $F$  (respectively in  $O$ ). A factor represents an element of a case, namely a factual circumstance. Its negation describes the opposite fact. For instance, if a factor  $\varphi$  is "A kills B", then its negation  $\neg\varphi$  is "A does not kill B". An outcome always favors a side in the precedent, its negation favors the opposite side. For instance, an outcome  $\omega$  is "A is guilty", its negation  $\neg\omega$  is "A is not guilty".

Following existing work in case-based reasoning, a *precedent* is a logical consistent conjunction of factors and outcomes. If a precedent contains an outcome, then we say it is a *proper precedent*. If a precedent doesn't have any outcome, then it is a *situation* that describes a current case. The outcomes of these situations need to be decided upon.

**Definition 1** (Precedents) A precedent is a logically consistent conjunction of distinct factors and outcomes  $\pi = \varphi_0 \wedge \varphi_1 \wedge \dots \wedge \varphi_m \wedge \omega_0 \wedge \omega_1 \wedge \dots \wedge \omega_{n-1}$ , where  $m$  and  $n$  are non-negative integers. We say that  $\varphi_0, \varphi_1, \dots, \varphi_m$  are the factors of  $\pi$ ,  $\omega_0, \omega_1, \dots, \omega_{n-1}$  are the outcomes of  $\pi$ . If  $n = 0$ , then we say that  $\pi$  is a situation with no outcomes, otherwise  $\pi$  is a proper precedent.

Notice that both  $m$  and  $n$  can be equal to 0. When  $m = 0$ , there is one single factor. When  $n = 0$ , the precedent has no outcome and the empty conjunction  $\omega_0 \wedge \dots \wedge \omega_{n-1}$  is



**Figure 1.** An example of precedent model

equivalent to  $\top$ . Figure 1 shows two precedents.  $f_1, f_2$  and  $\neg f_2$  are factors,  $o$  and  $\neg o$  are outcomes.

We do not assume precedents are complete descriptions. That is, factors may exist which do not occur in the precedent. Furthermore, we do not assume that the negation of a factor holds when the factor does not occur in the precedent.

A *precedent model* is a set of logically incompatible precedents forming a total pre-order that can represent a preference among the precedents. This preference relation is determined by the purpose of the models, for instance, in the precedent model shown in Section 3, precedents are equally preferred because of the setting of HYPO.

**Definition 2** (Precedent models) *A precedent model is a pair  $(P, \geq)$ , such that:  $P$  is a set of precedents; for all  $\pi, \pi' \in P$  with  $\pi \neq \pi'$ ,  $\pi \wedge \pi' \models \perp$ ; and  $\geq$  is a total preorder on  $P$ .*

The strict weak order  $>$  between two precedents  $\pi$  and  $\pi'$  is standardly associated with a total preorder  $\geq$  which is defined as  $\pi > \pi'$  if and only if it is not the case that  $\pi' \geq \pi$  (for  $\pi$  and  $\pi' \in P$ ). When  $\pi > \pi'$ , we say that  $\pi$  is (strictly) preferred to  $\pi'$ . The associated equivalence relation  $\sim$  is defined as  $\pi \sim \pi'$  if and only if  $\pi \geq \pi'$  and  $\pi' \geq \pi$ .

Precedent models are case models as defined in [1, Definition 2.1]:

**Proposition 1** *Let  $(P, \geq)$  be a precedent model. The following properties hold, for all  $\pi, \pi'$  and  $\pi'' \in P$ :*

1.  $\not\models \neg \pi$ ;
2. *If  $\not\models \pi \leftrightarrow \pi'$ , then  $\models \neg(\pi \wedge \pi')$ ;*
3. *If  $\models \pi \leftrightarrow \pi'$ , then  $\pi = \pi'$ ;*
4.  $\pi \geq \pi'$  or  $\pi' \geq \pi$ ;
5. *If  $\pi \geq \pi'$  and  $\pi' \geq \pi''$ , then  $\pi \geq \pi''$ .*

The proof is straightforward and is omitted. Figure 1 shows an example of a precedent model. The preference relation of this model is Precedent 1  $>$  Precedent 2, and is denoted by the size of the boxes directly.

The definitions of arguments, attacks and their validities in case models [1] can be applied to precedent models. Precedents are considered as the cases made by arguments. In Figure 1, Precedent 1 is the case made by the argument from  $f_1 \wedge \neg f_2$  to  $\neg o$ .

**Definition 3** (Arguments [1]) *An argument from  $\chi$  to  $\rho$  is a pair  $(\chi, \rho)$  with  $\chi$  and  $\rho \in L$ . For  $\lambda \in L$ , if  $\chi \models \lambda$ ,  $\lambda$  is a premise of the argument; if  $\rho \models \lambda$ ,  $\lambda$  is a conclusion; if  $\chi \wedge \rho \models \lambda$ ,  $\lambda$  is a position in the case made by the argument. We say that  $\chi$  expresses the full premise of the argument,  $\rho$  the full conclusion, and  $\chi \wedge \rho$  its full position, also referred to as the case made by the argument.*

Arguments have three kinds of validities. If an argument is logically implied by one of the precedents in a precedent model, then the argument is *coherently valid* in the precedent model. If all precedents in a precedent model logically implying an argument's full

premise also logically imply its full conclusion, then the argument is *conclusively valid* in the precedent model. If an argument's conclusion is logically implied by a precedent which is the most preferred among the precedents that logically imply the argument's full premise, then the argument is *presumptively valid* in the precedent model.

**Definition 4** (Validity of arguments [1]) *Let  $(P, \geq)$  be a precedent model. We define, for all  $\chi, \rho \in L$ :*

1. *argument  $(\chi, \rho)$  is coherently valid with respect to the precedent model if and only if  $\exists \pi \in P : \pi \models \chi \wedge \rho$ . We then write  $(P, \geq) \models (\chi, \rho)$ ;*
2. *argument  $(\chi, \rho)$  is conclusively valid with respect to the precedent model if and only if  $\exists \pi \in P : \pi \models \chi \wedge \rho$  and for all  $\pi \in P$ : if  $\pi \models \chi$ , then  $\pi \models \chi \wedge \rho$ . We then write  $(P, \geq) \models \chi \Rightarrow \rho$ ;*
3. *argument  $(\chi, \rho)$  is presumptively valid with respect to the precedent model if and only if  $\exists \pi \in P$ :*
  - (a)  *$\pi \models \chi \wedge \rho$ ; and*
  - (b) *for all  $\pi' \in P$ : if  $\pi' \models \chi$ , then  $\pi \geq \pi'$ .**We then write  $(P, \geq) \models \chi \rightsquigarrow \rho$ .*

In the precedent model of Figure 1, the following are true:

1.  $(P, \geq) \models (f_1, o)$ , as Precedent 2 logically implies this argument;
2.  $(P, \geq) \models f_2 \Rightarrow o$ , as all precedents in the model which logically imply  $f_2$  also logically imply  $o$ .
3.  $(P, \geq) \models f_1 \rightsquigarrow \neg o$ , as Precedent 1 logically implies  $\neg o$  and that is the most preferred precedent which logically implies  $f_1$  in the model.

**Definition 5** (Successful attacks [1]) *Let  $(P, \geq)$  be a precedent model, and  $(\chi, \rho)$  be a presumptively valid argument:*

1.  *$\tau \in L$  is a successful attack of argument  $(\chi, \rho)$  if and only if  $(\chi \wedge \tau, \rho)$  is not a presumptively valid argument, then we say  $(P, \geq) \models \chi \rightsquigarrow \rho \times \tau$ ;*
2. *If argument  $(\chi \wedge \tau, \rho)$  is not coherently valid, then we say successful attack  $\tau$  is excluding;*
3. *If  $\exists \pi \in P$ , and  $\pi \models \chi \wedge \tau$ , then we say  $\pi$  provides grounding for the successful attack  $\tau$ ;*

In the precedent model shown above, we have  $(P, \geq) \models f_1 \rightsquigarrow \neg o \times f_2$ , it is an excluding successful attack, Precedent 2 provides grounding for this successful attack.

The comparisons are based on Definition 3.1 and 3.4 in [1]. *Analogies* between two precedents are the properties that follow logically from both two precedents. *Distinctions* are the unshared properties between two precedents, namely the properties that only follow logically from one of the precedents but not the other one.

**Definition 6** (Precedent comparisons) *Let  $\pi, \pi' \in L$  be two precedents, we define:*

1. *a sentence  $\alpha \in L$  is an analogy between  $\pi$  and  $\pi'$  if and only if  $\pi \vee \pi' \models \alpha$ .*
2. *a sentence  $\delta \in L$  is a  $\pi$ - $\pi'$  distinction if and only if  $\pi \models \delta$  and  $\pi' \not\models \delta$ .*

*A  $\pi$ - $\pi'$  distinction is a distinction in  $\pi$  with respect to  $\pi'$ . Both  $\pi$ - $\pi'$  distinctions and  $\pi'$ - $\pi$  distinctions are called the distinctions between  $\pi$  and  $\pi'$ .*

Comparing the two precedents in Figure 1,  $f_1$  is an analogy between them,  $f_2$  is a distinction in Precedent 2 with respect to Precedent 1. Note that the outcomes are also distinctions between them as these precedents are decided differently.

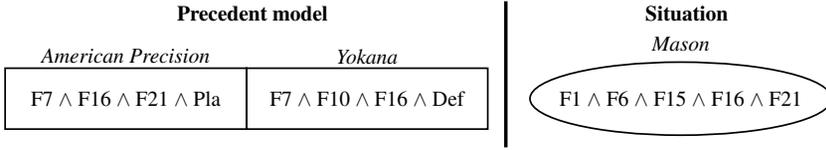


Figure 2. Precedent model for the *Mason* case

### 3. Case study: HYPO in precedent models

In this section, we give a case study of precedent models using a legal domain studied in HYPO, namely the trade secret law in the United States [3, 23].

The precedent model  $(P, P \times P)$  in this case study contains two precedents (i.e. the *Yokana* case<sup>2</sup> and the *American Precision* case<sup>3</sup>), which have been discussed in [23, Chapter 3.3.2]. Notice that  $P \times P$  denotes the trivial preference relation where all precedents are equivalent. The current situation is adapted from the *Mason* case<sup>4</sup>. The precedents in this model have equal preference. We assume the set of outcomes  $O = \{\text{Pla}, \text{Def}\}$  and  $\models \text{Def} \leftrightarrow \neg \text{Pla}$ . Pla stands for plaintiff wins the claim, Def stands for defendant wins the claim. The *Yokana* case favors defendant and the *American Precision* case favors plaintiff. As shown in Figure 2, both of them share some factors with the *Mason* case.

In the case-based reasoning process with these two precedents, arguments [23, Figure 3.2] can be generated for discussing the current situation:

1. (F16, Def) Defendant cites the *Yokana* case in order to give a statement that defendant should win the case. F16 is an analogy between the *Yokana* case and situation.
2. (F10, Pla) and (F6  $\wedge$  F15  $\wedge$  F21, Pla) Plaintiff distinguishes the *Yokana* case by pointing out distinctions (F10 and F6  $\wedge$  F15  $\wedge$  F21) in order to suggest that the situation should be decided differently.
3. (F16  $\wedge$  F21, Pla) Plaintiff also cites a more on point counterexample (the *American Precision* case) which shares more factors (F16  $\wedge$  F21) with the situation.
4. (F1, Def) and (F7, Def) Defendant distinguishes the counterexample, namely the *American Precision* case, by using factor F1 and F7.

The evaluation of these arguments is shown as below:

$$\begin{array}{ll}
 (P, P \times P) \models \text{F16} \rightsquigarrow \text{Def} & (P, P \times P) \not\models (\text{F10}, \text{Pla}) \\
 (P, P \times P) \not\models (\text{F6} \wedge \text{F15} \wedge \text{F21}, \text{Pla}) & (P, P \times P) \models \text{F16} \wedge \text{F21} \Rightarrow \text{Pla} \\
 (P, P \times P) \not\models (\text{F1}, \text{Def}) & (P, P \times P) \models \text{F7} \rightsquigarrow \text{Def}
 \end{array}$$

The evaluation shows that arguments for analogizing the *Yokana* case and the *American Precision* case with the *Mason* case are at least presumptively valid in the model, while most of the arguments for distinguishing them with the situation are incoherent.

Some factors can be considered as successful attacks of arguments in case-based reasoning. For instance:

$$\begin{array}{ll}
 (P, P \times P) \models \text{F16} \wedge \text{F21} \rightsquigarrow \text{Pla} \times \text{F1} & (P, P \times P) \models \text{F16} \rightsquigarrow \text{Def} \times \text{F6} \\
 (P, P \times P) \models \text{F16} \rightsquigarrow \text{Def} \times \text{F15} & (P, P \times P) \models \text{F16} \rightsquigarrow \text{Def} \times \text{F21}
 \end{array}$$

<sup>2</sup>Midland-Ross Corp. v. Yokana, 293 F.2d 411 (3rd Cir.1961)

<sup>3</sup>American Precision Vibrator Co. v. National Air Vibrator Co., 764 S.W.2d 274 (Tex.App.-Houston [1st Dist.] 1988)

<sup>4</sup>Mason v. Jack Daniel Distillery, 518 So.2d 130 (Ala.Civ.App.1987)

According to Definition 5, all these attacks are excluding. The *American Precision* case provides grounding for the attack F21 on argument (F16, Def), and there is no precedent that can provide grounding for other successful attacks above.

#### 4. Preliminary evaluation of arguments' validity in case-based reasoning

In this section, we discuss the validity of arguments in case-based reasoning. The approach we use to construct precedent models allows us to evaluate the arguments' validity in case-based reasoning, which can give reasons for why such argument moves in the reasoning process can be taken. For instance, the argument citing the *Yokana* case is presumptively valid, but the one based on *American precision* is even conclusive, and hence stronger in a formal sense.

The evaluation of arguments' validity provides a strategy to manipulate arguments in case-based reasoning, which is aiming for improving arguments' validity. By this action, the arguer's standpoint can be more acceptable to the judge. For instance, in the case study, if defendants find a favorable precedent which contains factor F7 and F1, then the level of validity of the argument they used for distinguishing the *American Precision* case with the *Mason* case will become conclusive, which makes their distinction more acceptable.

Using an incoherent argument can make sense and break new ground. A decision based on such an argument can be considered as going beyond the current legal status modeled in the precedent model. After adding a precedent incorporating such a groundbreaking decision, the previously incoherent argument can become coherent in the adapted model. For instance, if *Mason's* decision favors plaintiff, then in the current precedent model ( $F1 \wedge F6 \wedge F15 \wedge F16 \wedge F21, Pla$ ) is incoherent, but in the precedent model with the decided version of *Mason* included as third precedent it is coherent.

Precedents can be compared not only by the shared factors, but also by the preference relation in precedent models. The precedent model in Section 3 has equal preference, but this relation can change if the precedents are from different court level. Even with the same facts, a higher level court can make a decision which is opposite to the decision of a lower level court. Assume for instance that the precedent model in the case study has another precedent from a higher court level with the same factors as the *American Precision* case but opposite outcome, and it is more preferred than other precedents, then the argument ( $F16 \wedge F21, Def$ ) is presumptively valid, while ( $F16 \wedge F21, Pla$ ) is only coherent. Although they share the same factors with the situation, the precedent from higher court level can still be considered as a better one, since the argument for citing it has stronger validity.

#### 5. Discussion

In this section, we compare our precedent models with other relevant research.

Starting with HYPO, we observe that HYPO represents factors with dimensions, which can represent graduality or strength (very low - low - neutral - high - very high). In precedent models, factors are more similar to the notion in CATO [7], namely all factors are binary, and either can be found in a case or not. A pair of opposite factors in our models can be considered as the two extremes (very low and very high) of a dimension.

Precedent models are an extension of the case model formalism in [1], which briefly discussed case analogies and distinctions. Cases in case models are abstract in the sense that factors and outcomes are not distinguished as in our notion of precedents. With the precedent models, we therefore are able to describe elements of case-based reasoning in formal logic while staying closer to notions studied in HYPO.

In other case-based reasoning models [11, 12, 14, 16], case-based reasoning has been modeled in terms of a formal dialogue game [11], in terms of ASPIC+ framework [12], in terms of a reason model [14], and in terms of abstract dialectical frameworks [16]. The theory we use is in terms of a propositional logical language.

Precedents in [11] are represented with sets of rules, expressing which factors favor an outcome and which detract from it. They also describe factors as a kind of rules in order to represent the conflict resolution between the pro and con factors. Precedents in [12] are sets of factors, they use predicates in a first-order language to describe factors in precedents. Horty and Bench-Capon [14] represent precedents as a combination of rules, facts and outcome. The representation method used by Al-Abdulkarim and her colleagues is related to the factor hierarchy used in CATO [7]. In our precedent models, precedents are represented with conjunctions of factors and outcomes instead of sets or hierarchies. Rules can be translated to our arguments, and therefore the validity of rules can vary in our models.

The meaning of the preference relation is also different. In [11, 12, 14], the preference relation is used inside a precedent, it is either between rules [11] which is determined by which rule has the priority, or between factors [12, 14] which is determined by the outcome of the precedent. A similar notion of preference relation is also used in [16], which comes from prioritized abstract dialectical frameworks. It is used for comparing factors in the hierarchy. In our approach, the preference relation is a relation between precedents. For instance, in our precedent model for HYPO-style reasoning (Section 3), precedents are with equal preference. In the case model for Dutch tort law [1], the preference of cases are different in order to represent which cases are exceptional.

## 6. Conclusion

In this paper, we study case-based reasoning with precedent models and how to formalize arguments' validity with the formal logical semantics of precedent models. This approach shows that Verheij's formalism [1], which uses a formal, logical language, can be used to formally model elements of case-based reasoning. The use of precedent models allows for the logical evaluation of arguments grounded in past cases. Since past cases can be considered as a kind of data, and valid arguments show patterns that hold in the data, the approach provides a step in the development of hybrid AI systems that combine structured knowledge grounded in data [24].

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