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Gravitational Test beyond the First Post-Newtonian Order with the Shadow of the M87 Black Hole

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The 2017 Event Horizon Telescope (EHT) observations of the central source in M87 have led to the first measurement of the size of a black-hole shadow. This observation offers a new and clean gravitational test of the black-hole metric in the strong-field regime. We show analytically that spacetimes that deviate from the Kerr metric but satisfy weak-field tests can lead to large deviations in the predicted black-hole shadows that are inconsistent with even the current EHT measurements. We use numerical calculations of regular, parametric, non-Kerr metrics to identify the common characteristic among these different parametrizations that control the predicted shadow size. We show that the shadow-size measurements place significant constraints on deviation parameters that control the second post-Newtonian and higher orders of each metric and are, therefore, inaccessible to weak-field tests. The new constraints are complementary to those imposed by observations of gravitational waves from stellar-mass sources.

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Tests of general relativity have traditionally involved solar-system bodies [1] and neutron stars in binaries [2], for which precise measurements can be interpreted with minimal astrophysical complications. In recent years, observations at cosmological scales [3] and the detection of gravitational waves [4] have also resulted in an array of new gravitational tests.

The horizon-scale images of the black hole in the center of the M87 galaxy obtained by the Event Horizon Telescope (EHT) [5] in 2017 offer the most recent addition to the set of observations that probe the strong-field regime of gravity. As an interferometer, the EHT measures the Fourier components of the brightness distribution of the source on the sky at a small number of distinct Fourier frequencies. The features of the underlying image are then reconstructed either using agnostic imaging algorithms or by directly fitting model images to the interferometric data. The central brightness depression seen in the M87 image has been interpreted as the shadow cast by this supermassive black hole on the emission from the surrounding plasma. The observability of the shadow of the black hole in M87 and the one in the center of the Milky Way, Sgr A*, had been predicted earlier based on the properties of the radiatively inefficient accretion flows around these objects and their large mass-to-distance ratios [6].

The outline of a black-hole shadow is the locus of the photon trajectories on the screen of a distant observer that, when traced backwards, become tangent to the surfaces of spherical photon orbits hovering just above the black-hole horizons [7]. The Boyer-Lindquist radii of these spherical photon orbits lie in the range $(1-4)M$, depending on the black-hole spin and the orientation of the angular momentum of the orbit [8] (here M is the mass of the black hole and we have set $G = c = 1$, where G is the gravitational constant, and c is the speed of light). It is the fact that the outlines of black-hole shadows encode in them the strong-field properties of the spacetimes that led to the early suggestion that they can be used in performing strong-field gravitational tests [9–11].

Even though the radii of the photon orbits have a strong dependence on spin, a fortuitous cancellation of the effects of frame dragging and of the quadrupole structure in the Kerr metric causes the outline of the shadow, as observed at infinity, to have a size and a shape that depends very weakly on the spin of the black hole or the orientation of the observer [10]. This cancellation occurs because, due to the no-hair theorem, the magnitude of the quadrupole moment of the Kerr metric is not an independent quantity but is instead always equal to the square of the black-hole spin. For all possible values of spin and inclination, the size of the shadow is $\simeq 5M \pm 4\%$ and its shape is nearly circular to within $\sim 7\%$. For a black hole of known mass-to-distance ratio, the constancy of the shadow size allows for a null-hypothesis test of the Kerr metric [12]. At the same time, the nearly circular shape of the shadow offers the possibility of testing the gravitational no-hair theorem [10].

The first inference of the size of the black-hole shadow in M87 used as a proxy the measurement of the size of the bright ring of emission that surrounds the shadow and calibrated the difference in size via large suites of GRMHD simulations [5]. When this ring of emission is narrow, as is the case for the 2017 EHT image of M87, potential biases in the measurement are small. The inferred size of the M87 black-hole shadow was found to be consistent (to within $\sim 17\%$ at the 68-percentile level) with the predicted size based on the Kerr metric and the mass-to-distance ratio of the black hole derived using stellar dynamics [5,13] (see, however, [14–16]). The agreement between the measured and predicted shadow size does constitute a null-hypothesis test of the general relativity predictions: the data give us no reason to question the validity of the assumptions that went into this measurement, the Kerr metric being one of them. However, using this measurement to place quantitative constraints on any potential deviations from the Kerr metric is less straightforward for two reasons.

First, the Kerr metric is the unique black-hole solution to a large number of modified gravity theories that are Lorentz symmetric and have field equations with constant coupling coefficients between the various gravitating fields [17,18]. Only a limited number of black-hole solutions are known for theories with dynamical couplings [19] (e.g., dynamical Chern-Simons gravity and Einstein-dilaton-Gauss-Bonnet gravity [20]) or for Lorentz-violating theories [21]. Despite substantial progress in recent years, this line of work leads to limited theoretical guidance on the form and magnitude of potential deviations from the Kerr metric.

Second, if we instead use an empirical parametric framework to extend the Kerr metric, we would find that most naive parametric extensions lead to pathologies, such as non-Lorentzian signatures, singularities, and closed timelike loops, which render it impossible to calculate photon trajectories in the strong-field regime (see, e.g., [22]). In recent years, this problem has been addressed with the development of a number of parametric extensions of the Kerr metric that are free of pathologies [23–29]. Resolving the pathologies, however, comes at the cost of very large complexity. In principle, we can use the EHT measurement with any of these parametric extensions to place constraints on the specific parameters of the metric we used [30]. However, understanding the physical meaning of such constraints and comparing them with the constraints imposed when other parametric extensions are used are not readily feasible. In addition, the complexity of the various parametric extensions to the Kerr metric hinders the comparison of these gravitational tests with the results of other, e.g., weak-field and cosmological ones and, therefore, the effort to place complementary tests on the underlying gravity theory.

In this Letter, we use analytic arguments as well as numerical calculations of shadows to set new constraints on gravity using the 2017 EHT measurements, elucidate their physical meaning, and compare them with earlier weak-field tests. We find that the EHT measurements place constraints primarily on the tt component of the black-hole spacetime (when the latter is expressed in areal coordinates and in covariant form). This is analogous to the fact that solar-system tests that involve gravitational lensing or Shapiro delay measurements constrain primarily one of the metric components of the parametric post-Newtonian (PPN) framework [1]. However, we show that the constraints imposed by the EHT measurements are of (at least) the second post-Newtonian (PN) order and are, therefore, beyond the reach of weak-field experiments.

The size of the black-hole shadow both in the Kerr metric and in other parametric extensions depends very weakly on the black-hole spin [10,31,32]. For this reason, we start by exploring analytically the shadow size for a general static, spherically symmetric metric of the form

$$ds^2 = g_{tt}dt^2 + g_{rr}dr^2 + r^2 d\Omega. \quad (1)$$

Note that the choice of coordinates we use here is different from the isotropic coordinates of the PPN framework. We made this choice because, as we will show below, the radius of the photon orbit and the size of the shadow depend on only one component of the metric in these coordinates (unlike, e.g., Eq. [101] of Ref. [33], which is written in isotropic coordinates).

Without loss of generality, we consider photon trajectories in the equatorial plane, i.e., set $\theta = \pi/2$. Following Ref. [34], we use two of the Killing vectors of the spacetime to write the components of the momentum of a photon traveling in this spacetime as

$$(k^t, k^r, k^\theta, k^\phi) = \left(\frac{E}{g_{tt}}, \left[-\frac{E^2}{g_{tt}g_{rr}} - \frac{l^2}{g_{rr}r^2}\right]^{1/2}, 0, \frac{l}{r^2}\right), \quad (2)$$

where E and l are the conserved energy and angular momentum of the photon and we have used the null condition $\underset{\sim}{k} \cdot \underset{\sim}{k} = 0$ to calculate the radial component of the momentum.

The location of the circular photon orbit is the solution of the two conditions $k^r = 0$ and $dk^r/dr = 0$. Combining them, we write the radius $r_{\rm ph}$ of the photon orbit as the solution to the implicit equation

$$r_{\rm ph} = \sqrt{-g_{tt}}\left(\left.\frac{d\sqrt{-g_{tt}}}{dr}\right|_{r_{\rm ph}}\right)^{-1}. \quad (3)$$

The radius $r_{\rm sh}$ of the black-hole shadow as observed at infinity is the gravitationally lensed image of the circular photon orbit. This effect was calculated in Ref. [34] (Eq. [20]) and, when applied to the size of the photon orbit, leads to

$$r_{\rm sh} = \frac{r_{\rm ph}}{\sqrt{-g_{tt}(r_{\rm ph})}}. \quad (4)$$

As advertised earlier, both the radius of the photon orbit and the size of the black-hole shadow depend only on the tt component of the metric (1) written in areal coordinates and in covariant form.

In order to connect the strong-field constraints from black-hole shadows to the weak-field tests, we expand the tt component in powers of r^{-1} as

$$-g_{tt} = 1 - \frac{2}{r} + 2\left(\frac{\bar{\beta} - \bar{\gamma}}{r^2}\right) - 2\left(\frac{\zeta}{r^3}\right) + \mathcal{O}(r^{-4}). \quad (5)$$

Hereafter, we set $G = c = M = 1$, where G is the gravitational constant, c is the speed of light, and M is the black-hole mass. In this equation, we have employed the usual PPN parameters $\bar{\beta}$ and $\bar{\gamma}$ and added a 2 PN term parametrized by the quantity ζ. Weak-field tests have placed strong constraints on the 1 PN parameters to be equal to unity to within a few parts in 10^5 [1]. Even though modified gravity theories may not satisfy Birkhoff's theorem and, therefore, the values of the 1 PN parameters may be different outside the Sun and outside a black hole, we make here the very conservative assumption that the Solar System limits are applicable to the external spacetimes of astrophysical black holes and set $\bar{\beta} - \bar{\gamma} \simeq 0$. If the tt component of the black-hole metric has indeed a vanishing 1 PN term, as required by the weak-field tests, and terminates at the 2 PN term, the radius of the circular photon orbit would be

$$r_{\rm ph} = 3 + \frac{5}{9}\zeta \quad (6)$$

and the size of the black-hole shadow would be

$$r_{\rm sh} = 3\sqrt{3}\left(1 + \frac{1}{9}\zeta\right). \quad (7)$$

This is a quantitative demonstration of the fact that the size of the black-hole shadow probes the behavior of the spacetime at least at the 2 PN order. Moreover, the size of the black-hole shadow depends linearly on the magnitude of the 2 PN term.

To explore in detail the constraints imposed by the EHT results, we will consider, as concrete examples of regular, parametric extensions to the Kerr metric, the metrics developed in Refs. [22,23] (hereafter the Johannsen-Psaltis (JP) metric) and in Refs. [24,35] [hereafter the modified gravity bumpy Kerr (MGBK) metric]. Table I shows the 1 PN and 2 PN parameters [see Eq. (1)] for these metrics, when the spin parameter is set to zero and only leading orders of the parameters are considered. From the analytic argument above, we expect the shadow sizes to be determined primarily by the parameters that control the 2 PN and higher-order terms for these metrics. Hereafter, we define the spin of a given metric as the dimensionless ratio J/M^2 of the lowest-order current moment, i.e., the angular momentum, to the square of the lowest-order mass moment, i.e., the Keplerian mass, of the spacetime.

The JP metric has four lowest-order parameters to describe possible deviations from Kerr [22]. The outlines of black-hole shadows for this metric have been calculated in Refs. [31,32] and were shown to depend very weakly on

TABLE I. PPN expansions of various parametric extensions to the Kerr metric.

Metric	$\bar{\beta}-\bar{\gamma}$ (1 PN)	ζ (2 PN)
Kerr	0	0
JP	0	α_{13}
MGBK	$-\gamma_{1,2}/2-\gamma_{4,2}\to 0$	$-\gamma_{1,2}-4\gamma_{4,2}\to\gamma_{1,2}$

the black-hole spin. Setting the spin to zero, we use Eq. (4) for the full metric to derive the shadow size as a function of the deviation parameters. We find that, in this limit, the shadow size depends entirely on one of the deviation parameters, α_{13}, which is also the one that controls the 2 PN terms of the metric. The complete expression is very complicated to display here but a power-law expansion is

$$r_{\mathrm{sh,JP}}=3\sqrt{3}\left[1+\frac{1}{27}\alpha_{13}-\frac{1}{486}\alpha_{13}^2+\mathcal{O}(\alpha_{13}^3)\right]. \quad (8)$$

Note that the coefficient of the deviation parameter α_{13} is different from what we would have expected from Eq. (7) because the JP metric does not terminate at the 2 PN order. Requiring that the shadow size is consistent to within 17% with the 2017 EHT measurement for M87 places a bound on the deviation parameter $-3.6<\alpha_{13}<5.9$. The left panel of Fig. 1 shows the corresponding limits on α_{13} obtained numerically from the full JP metric, when the black-hole spin is taken into account and the second metric parameter that affects the shadow size for a spinning black hole, i.e., α_{22}, is varied. As evident here, the constraints on α_{13} change only mildly when effects that introduce deviations from spherical symmetry are included. Therefore, for the JP metric, the EHT measurement constrains predominantly the deviation parameter α_{13}, which controls the 2 PN terms.

The MGBK metric has four lowest-order parameters to describe possible deviations from Kerr [35] without requiring the 1 PN deviation to vanish (see Table I). The outlines of black-hole shadows have been calculated in Ref. [32] and their overall sizes were shown to depend primarily on the parameters $\gamma_{3,3}$, $\gamma_{1,2}$, and $\gamma_{4,2}$ (see Fig. 8 of [32]). In its original formulation, the parameter $\gamma_{3,3}$ describes frame dragging in a manner that remains finite even for nonspinning black holes (see Eq. [17] of [35]). Here, we scale this parameter with spin, i.e., write $\gamma'_{3,3}=\gamma_{3,3}a$ to remove the divergent behavior of the shadow size with $a\to 0$ found in Ref. [32]. We also set $\gamma_{4,2}=-\gamma_{1,2}/2$ for this metric to be consistent with Solar System tests at the 1 PN order. In this case, the magnitude of potential 2 PN deviations becomes equal to $\zeta_{\mathrm{MGBK}}=\gamma_{1,2}$.

With these redefinitions, the size of the shadow for the MGBK metric depends primarily on parameter $\gamma_{1,2}$ and only weakly on spin. As before, we calculate analytically the shadow size for this metric using Eq. (4) having set the spin equal to zero. We again display only an expansion of the size in the deviation parameter $\gamma_{1,2}$:

$$r_{\mathrm{sh,MGBK}}=3\sqrt{3}\left[1+\frac{1}{27}\gamma_{1,2}+\mathcal{O}(\gamma_{1,2}^3)\right]. \quad (9)$$

Requiring that the shadow size is consistent to within 17% with the 2017 EHT measurement for M87 places a bound on the deviation parameter $-5.0<\gamma_{1,2}<4.9$. The right panel of Fig. 1 shows the corresponding constraints obtained numerically from the full solution, when the black-hole spin is taken into account and the other deviation parameters are varied. Again, the constraints

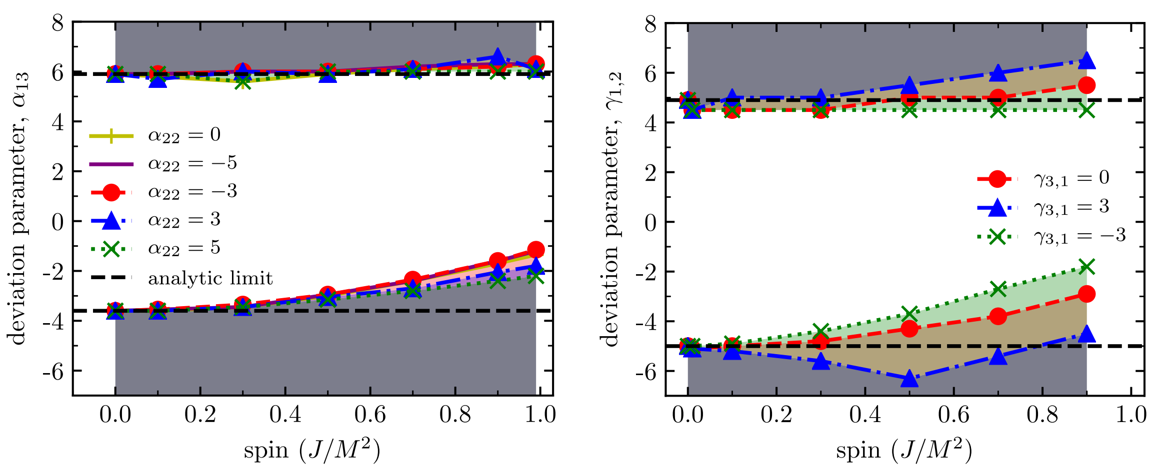

FIG. 1. Bound on the deviation parameters (left) α_{13} of the JP metric and (right) $\gamma_{1,2}$ for the MGBK metric, as a function of spin (J/M^2) and for different values of the other metric parameters, placed by the 2017 EHT observations of M87. The shaded areas show the excluded regions of the parameter space. The dashed line shows the analytic result obtained for zero spin. The EHT measurements place constraints predominantly on α_{13} (for JP) and $\gamma_{1,2}$ (for MGBK), which control the 2 PN expansion of the corresponding metrics (see Table I).

on $\gamma_{1,2}$ change only mildly when effects that introduce deviations from spherical symmetry are included.

Even though the complex functional forms of the various elements in the two metrics we considered here are very different from each other, in both cases the predicted size of the black-hole shadow depends almost exclusively (and in a very similar manner) on the deviation parameter that controls the 2 PN and higher-order terms for each metric. This conclusion remains the same when we use, e.g., the Rezzolla-Zhidenko (RZ) metric [29], for which the deviations from Kerr are introduced by a sequence of parameters, with a_i controlling primarily the $i+1$ PN order. For this metric, $\zeta = -4\alpha_1$ and requiring that the predicted shadow size is consistent with the EHT measurements leads to the constraint $-1.2 < \alpha_1 < 1.3$. This supports our conclusion that an EHT measurement of the size of a black hole leads to metric tests that are inaccessible to weak-field tests.

In this Letter we have allowed for only one of the high-order PN parameters of the g_{tt} component of each metric to deviate from its Kerr value in order to show that significant constraints can be obtained even with the current EHT results. However, if more than one PN parameters of the same metric component are included, then the size measurement of the black-hole shadow will instead lead to a constraint on a linear combination of these parameters. Similar constraints will be possible in the very near future with EHT observations of the black hole in the center of the Milky Way, for which there is no ambiguity in the inferred mass. In that case, monitoring of individual stellar orbits has provided very precise measurements of its mass-to-distance ratio [36] leading to a prediction of 47–53 μas for its shadow diameter, depending on the black-hole spin.

Observations of double neutron stars [2] and of coalescing black holes with LIGO/VIRGO [4] also probe the strong-field properties of their gravitational fields and lead to post-Newtonian constraints of similar magnitude as the ones we obtain here. The mass and curvature scale of the stellar-mass sources are eight orders of magnitude different from those of the M87 black hole, thereby probing a very different regime of gravitational parameters [5,11]. It is this combination of gravitational tests across different scales that will provide complementary and comprehensive constraints on possible modifications of the fundamental gravitational theory.

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