Modeling evolution of dark matter substructure and annihilation boost

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Abstract. The structure of dark matter halo is hierarchical. Among them, small-scale structures in dark matter halo (so-called subhalos) can enhance dark matter annihilation signals. It is necessary to quantify boost factors by those subhalos to derive the property of dark matter with current/future gamma-ray observations. In order to derive the subhalo boost factors, calculations of halo structure covering more than 20 orders-of-magnitude in the halo mass up to a redshift of $z \sim 10$ are required. This is beyond the capability of the current state-of-art cosmological N-body simulation which is a widely-adopted method to study the halo structure. In this talk, I introduce our analytical approach for the formalism of subhalo evolutions and the resultant boost factors. I show that the constraints on the annihilation cross-section obtained by isotropic $\gamma$-ray observations can be updated by several factors by taking the contribution from subhalos into account.

1. Introduction
Weakly Interacting Massive Particle (WIMP) is a strong candidate for dark matter (DM). It naturally achieves the relic abundance with the so-called freeze-out mechanism [1]. The mass $m_{DM}$ and the annihilation cross-section $\langle \sigma v \rangle$ of WIMP into the standard model particles should be around $m_{DM} \sim \mathcal{O}(1)$ GeV - $\mathcal{O}(1)$ TeV and $\mathcal{O}(10^{-26})$ cm$^3$ s$^{-1}$, respectively. Currently, the strongest bounds on the annihilation cross-section are obtained from the $\gamma$-ray observations of dwarf spheroidal galaxies [2]. The obtained upper limits on the annihilation cross-section is already smaller than the canonical value of $\sim 3 \times 10^{-26}$ cm$^3$ s$^{-1}$ for $m_{DM} \sim 100$ GeV.

WIMP is categorized as cold dark matter (CDM) when we consider the structure formation. In cosmological simulations of CDM structure formation, small-mass CDM halos are abundantly produced. Those small halos reside in larger halos, i.e., exist as subhalos. Subhalos lying on our lines-of-sight should boost the $\gamma$-ray flux from DM annihilations. However, quantitative discussion about the subhalo boost of the annihilation signal is difficult due to the limited numerical resolution. We have to count the minimum mass halos of $m \sim \mathcal{O}(10^{-6}) M_\odot$ in the largest halos corresponding to the galaxy cluster scale, $M \sim \mathcal{O}(10^{16}) M_\odot$. Numerical calculations
about the subhalo populations and the boost factor are available for certain ranges of the mass-scale and the redshift but not in the full range of the interest. Then, previous works (e.g. [3]) have estimated the boost factor by extrapolating the numerical results at certain ranges of the host mass wider. Depending on the model of the subhalo mass function, \( dN_{sh}/dm_{sh} \propto m_{sh}^{-\alpha} \) where \( m_{sh} \) denotes the subhalo mass, the boost factor changes from \( O(1) \) to \( O(10^3) \).

In this work [4, 5], we develop a new analytical model of the subhalo abundance. In this way, we can estimate the boost factor without adopting the extrapolation of the mass function, which causes variations of the boost factor. Evolutions of the subhalo under the tidal force of the host are described with a simple physically-motivated model [7]. By combining the halo accretion history in the extended Press-Schechter formalism [6], we derive the mass function and the boost factor.

2. Method

Subhalos lose their mass after the accretion due to the gravitational force of their hosts. The tidal mass-loss of the subhalo is most significant at their pericenter. Assuming a Navarro-Frenk-White (NFW) profile [8] for both of the host and subhalos, the pericenter radius, the orbital period, and the tidal truncation radius are determined for given orbital parameters of the subhalo. We include the distribution of the subhalo orbit which is parameterized with the energy and the angular momentum of the orbital motion by Monte-Carlo simulation. Fig. 1 shows the mass-loss rate of the subhalo averaged over the first orbital period after the accretion. This simplified model agrees well with the N-body calculations [9, 10], which are shown in the squares with error bars in this figure. By performing the calculations with different host mass and the redshift, we derive the host mass and the redshift dependence of the subhalo tidal mass-loss rate

\[
\dot{m}_{sh}(z) = -A(M_{\text{host}}, z) \frac{m_{sh}(z)}{\tau_{\text{dyn}}(z)} \left[ \frac{m_{sh}(z)}{M_{\text{host}}(z)} \right]^\xi
\]

(1)

where \( \tau_{\text{dyn}} \) is a dynamical timescale.

The NFW profile parameters, i.e., the characteristic density \( \rho_s \) and the scale radius \( r_s \) of the NFW profile, are simultaneously determined assuming the relation tuned against the N-body simulation [11]. By integrating the mass evolution of a given subhalo, we obtain the mass and the profile parameters of that halo at an arbitrary redshift between \( z \sim 7 - 0 \). The number distribution of the accreting subhalo \( d^2N_{sh}/d\ln m_{sh,\text{acc}}dz_{\text{acc}} \) is obtained from the extended Press-Schechter formalism. We use the suffix acc to denote the mass at the accretion, which is an
Figure 2. Subhalo mass function obtained in analytical calculations. We compare two cases of the host mass of $M_{\text{host}} = 10^{12} M_\odot$ and $10^{14} M_\odot$ at $z = 0$. Thinner lines are results from N-body calculations. The dependence on the subhalo mass is quantified.

unevolved one. We adopt the model III of Ref. [6] and calculate the mass and the redshift distribution of the accreting subhalo. The luminosity of the halo $L$ depends on the NFW parameter as $L \propto \rho_s^2 r_s^3$. The weights $w_i$ of the accreting subhalo is proportional to the accreting subhalo distribution, $d^2 N_{\text{sh}}/d \ln m_{\text{acc}} dz_{\text{acc}}$ and the normalization is $\sum_i w_i = N_{\text{tot}}$. Hence the boost factor $B_{\text{sh}}$ becomes

$$B_{\text{sh}} = \frac{L_{\text{sh, tot}}}{L_{\text{host}}}$$  \hspace{1cm} (2)$$

where $L_{\text{sh, tot}}$ is a luminosity including subhalo contributions and the $L_{\text{host}}$ is that of the smooth host.

3. Results

The subhalo mass function after the evolution under the tidal force of the host is derived. Fig. 2 shows the case assuming hosts of $M_{\text{host}} = 10^{12.26} M_\odot$ and $M_{\text{host}} = 10^{14.77} M_\odot$ at $z = 0$. Thinner lines in the figure are results obtained in N-body simulations [9, 10, 12, 13, 14]. These results indicate that the slope of the mass function, which is a source of the significant uncertainties in the boost factor, is a function of the subhalo mass as well as the host mass. Obtained mass functions are consistent with those in N-body simulations within the mass range of subhalos resolved in N-body simulations. Mass fraction of the subhalo is also obtained by integrating the mass function in subhalo mass and also shows a good agreement with the N-body result. Subhalo occupies $\sim 10\%$ in mass fraction for hosts of $M_{\text{host}} \sim 10^{12} M_\odot$.

Based on the calculation of the subhalo mass function and the NFW parameters after the tidal stripping, we derive the boost factor up to $z \sim 5$ for $M_{\text{host}} = 10^{-3} - 10^{15} M_\odot$. At $z = 0$, we expect a factor of 10 enhancement of the DM annihilation signal considering observations of galaxy clusters (see Fig. 3). The boost factor gets higher in the larger redshift. The boost factor, especially at higher redshifts, directly updates the current bound on the DM annihilation cross-section obtained from isotropic $\gamma$-ray background observations. For example, we obtained a comparably strong limits on the DM annihilation cross-section into $bb$ pairs for $m_{\text{DM}} \sim 100\text{GeV}$ from isotropic $\gamma$-ray background observations.

4. Conclusion

We have developed a new analytical model for the evolution of the DM substructure. This model is based on physical considerations capturing the important phenomenological features. The tidal mass-loss rate, the subhalo mass function, and the subhalo mass fraction obtained in
Figure 3. The boost factor from $z = 0$ to $z = 5$. We expect up to a factor of $\sim 10$ enhancement of the DM annihilation signal for DM halo of the galaxy cluster scale. By applying these boost factors to the constraints from isotropic $\gamma$-ray background observations the upper limits are improved by several factors.

This formalism are consistent with those in N-body calculations when we compare in the resolved regime. The boost factor is derived without violent extrapolations using this model. We expect a factor of 10 enhancement of DM annihilation signals in the halos of the galaxy-cluster scale. Also, our model has predictability at a higher redshift. We have shown that by including the boost factor up to $z \sim 5$, current constraints on the DM annihilation cross-section from isotropic $\gamma$-ray background observations are improved by several factors, and could be comparably strong in a certain range of DM mass.

References


